Recent theoretical progress in the study of Generalized Parton Distributions

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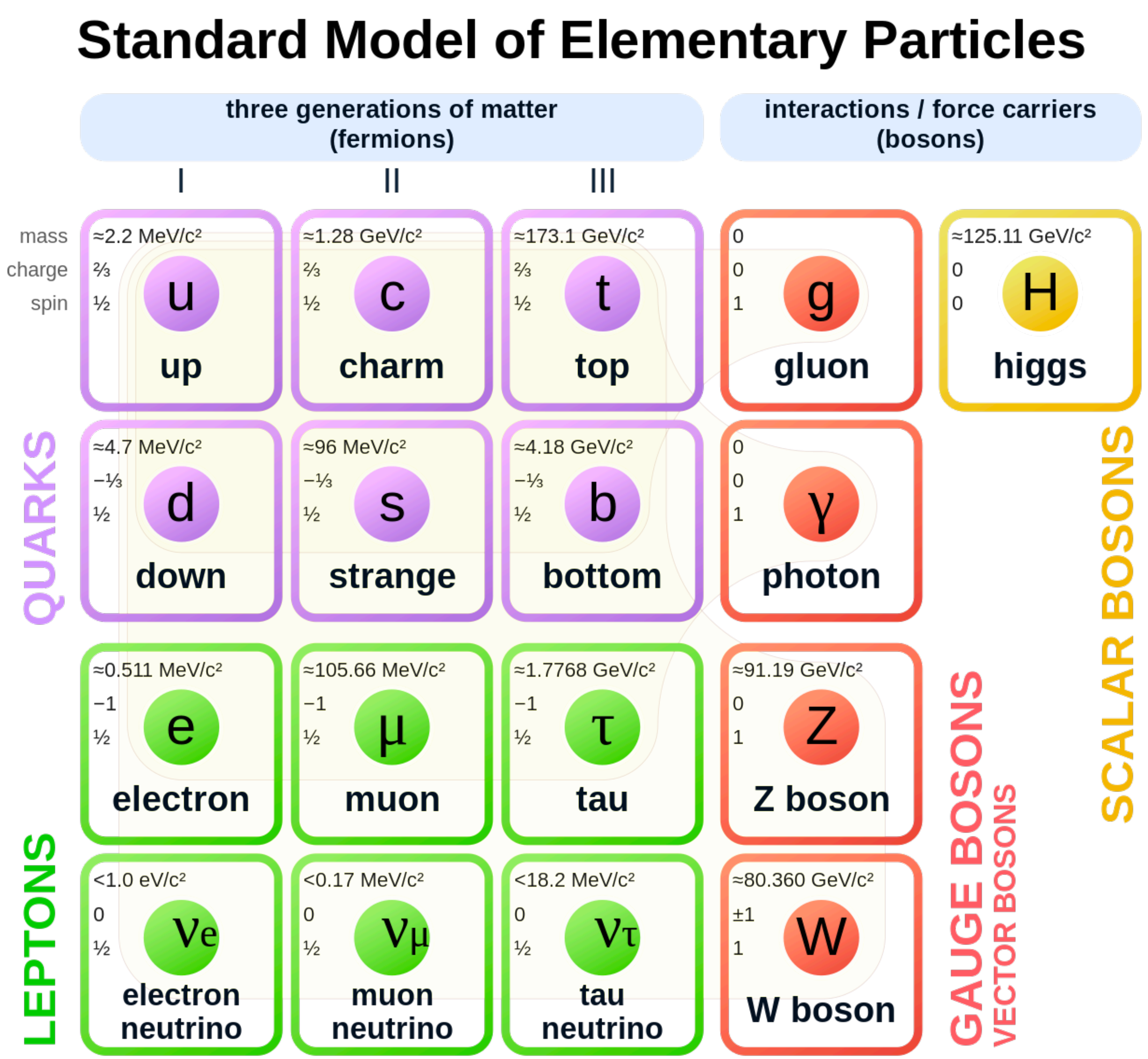
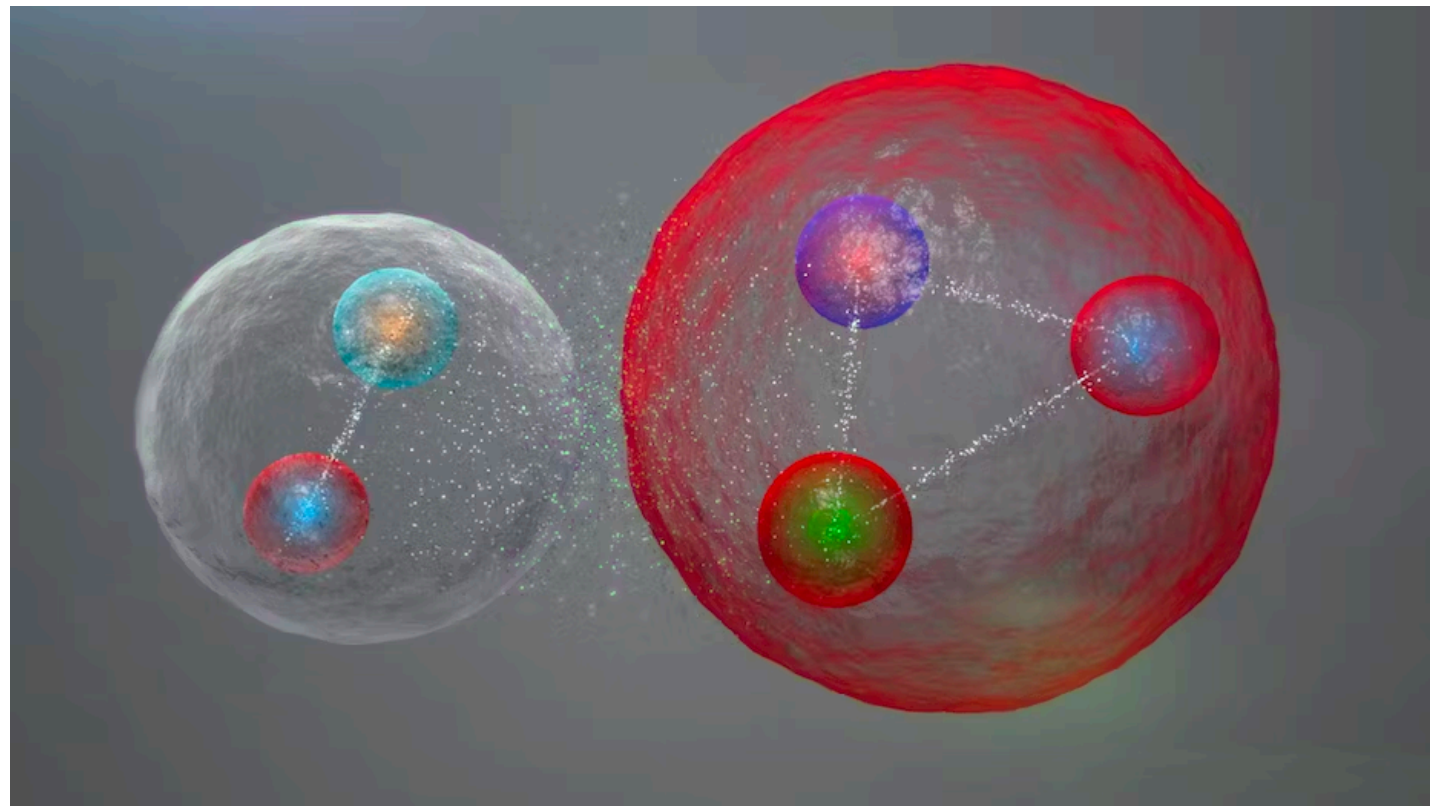
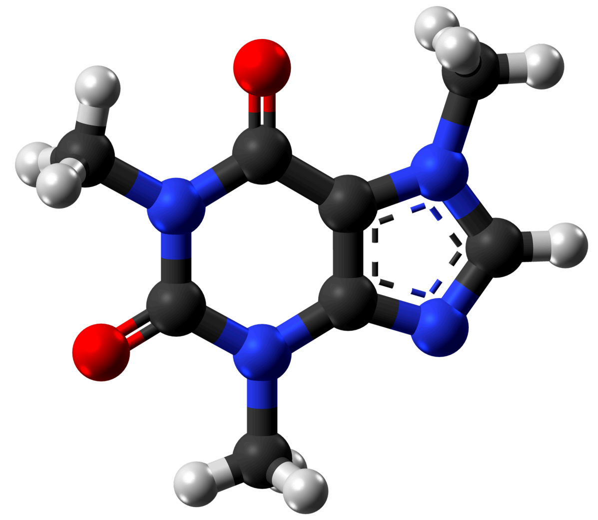
Building blocks of our visible universe

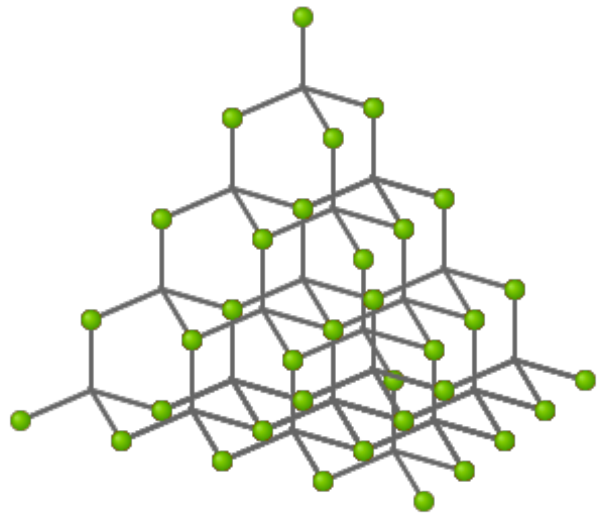


theory for strong interaction:
Quantum Chromodynamics (QCD)



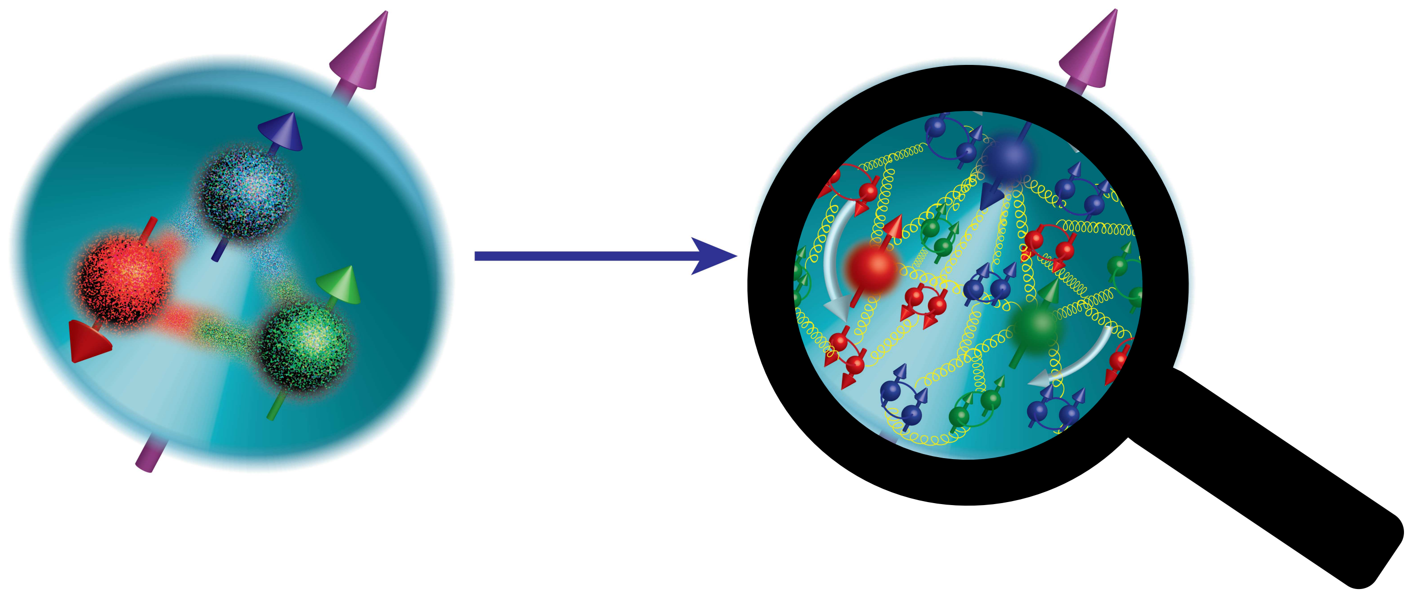
$C_8H_{10}N_4O_2$, C





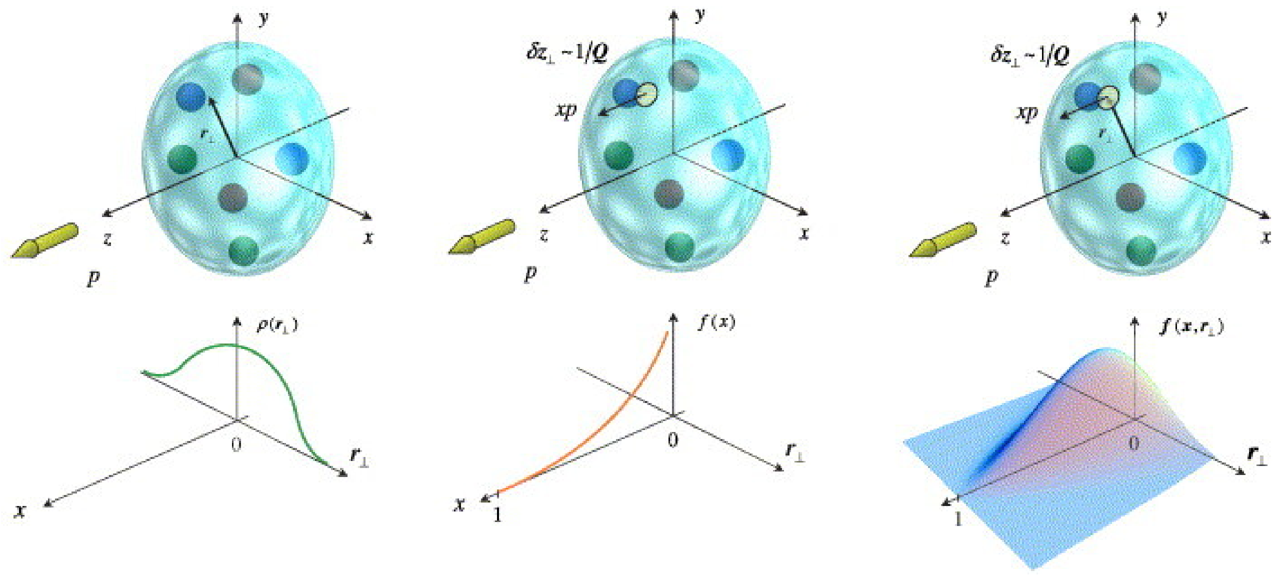
Internal structures of hadrons

- Hadrons are composite particles with complicated internal structures

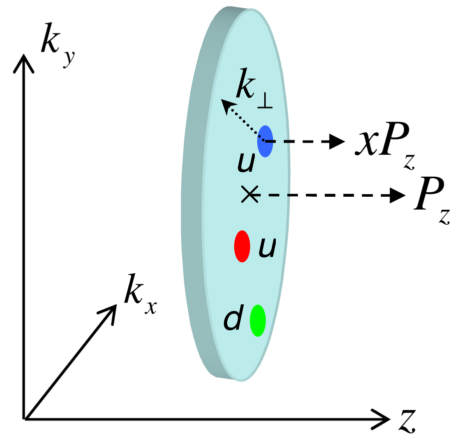


- PDFs only encode longitudinal information; GPDs and TMDs include transverse information

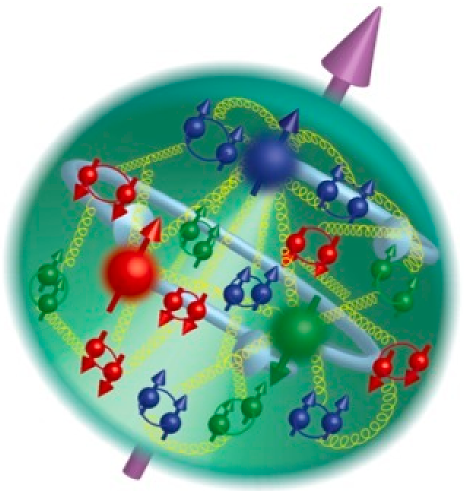
GPD:
tomography of hadrons



TMD:



- GPDs are crucial for understanding the spin structure of hadrons

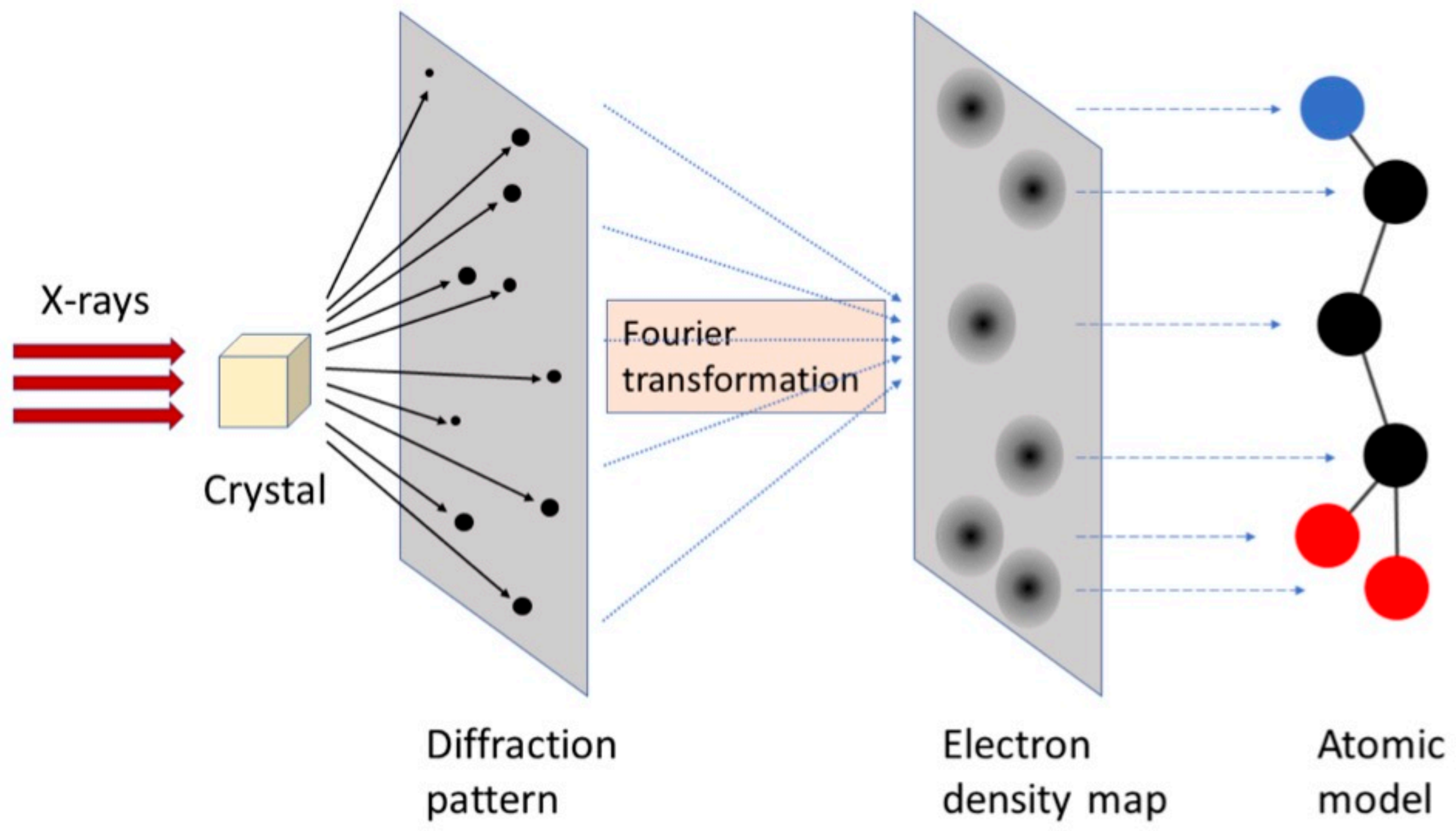
Σ  $\stackrel{?}{=} \frac{1}{2}$

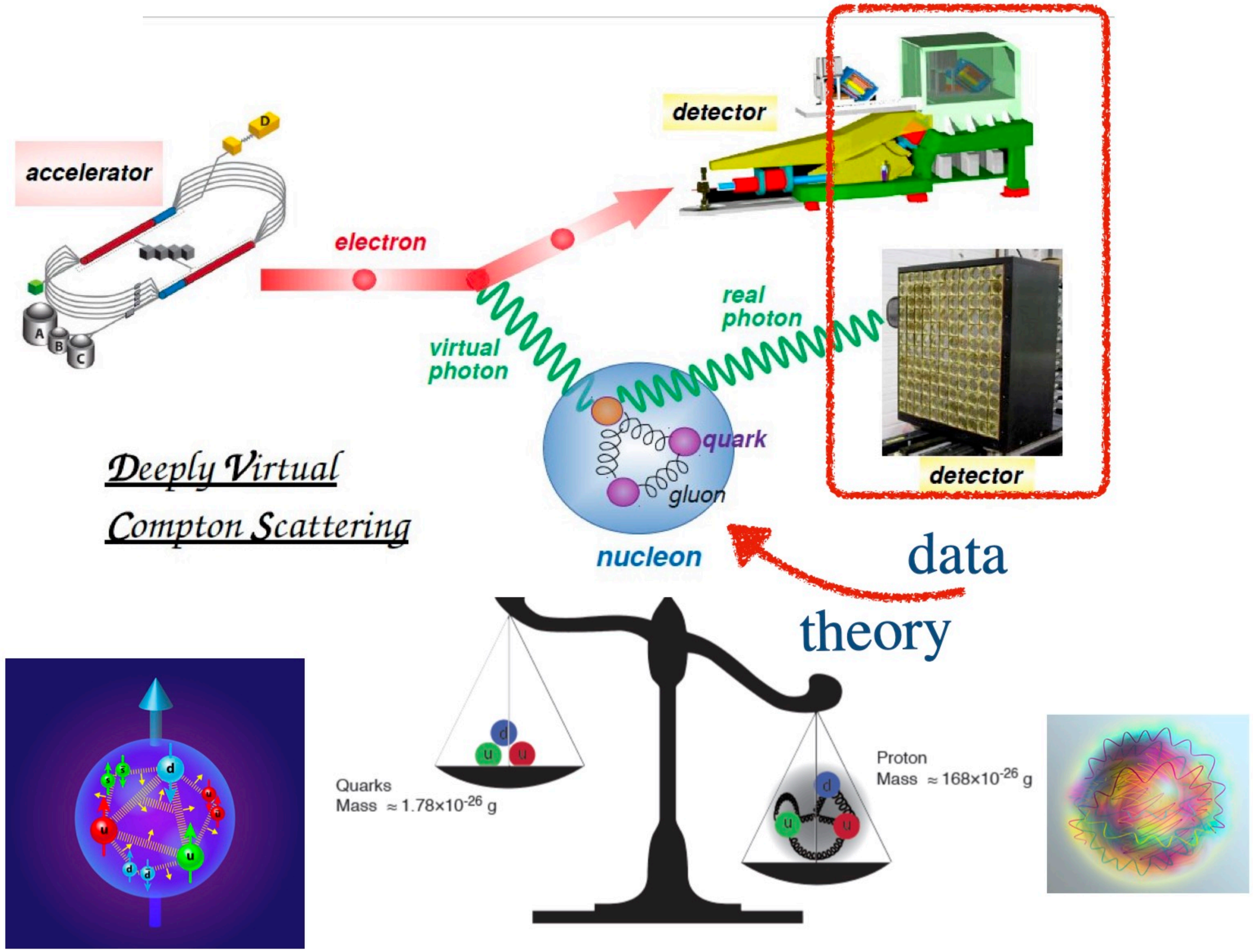
spin sum-rule:
A. Jaffe, A. Manohar (1990)
X. Ji (1996)

Unsolved problem in physics:

? *How do the quarks and gluons carry the spin of protons?*

Structure information from experiment



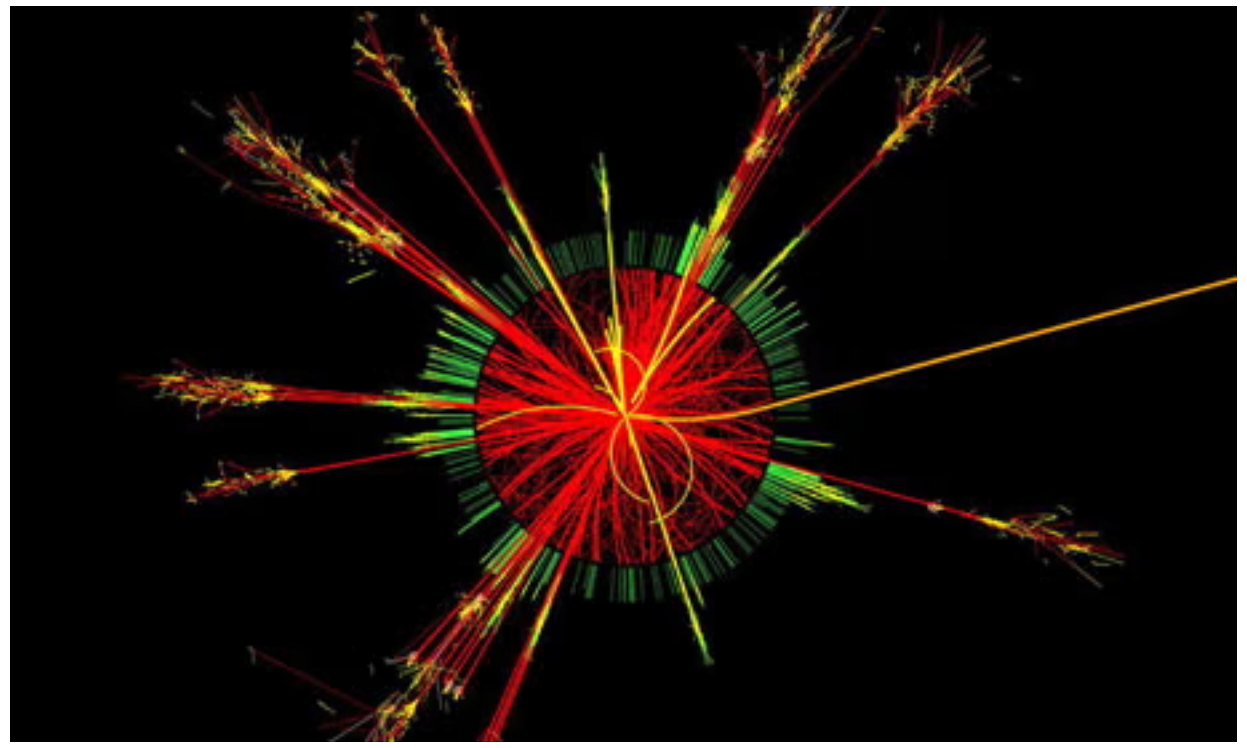


current and future facilities: JLab-12 GeV, EIC, EIcC

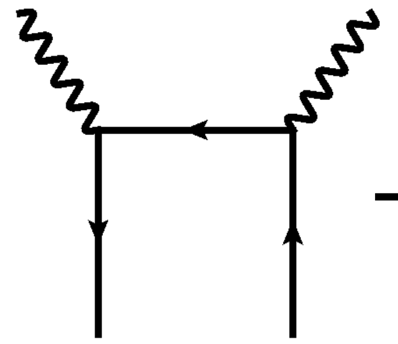
Structure information from experiment

- Hadrons structure is accessible via factorization of exclusive processes

$\mu_{\text{UV}} \longleftarrow$ $\longrightarrow \Lambda_{\text{QCD}}$

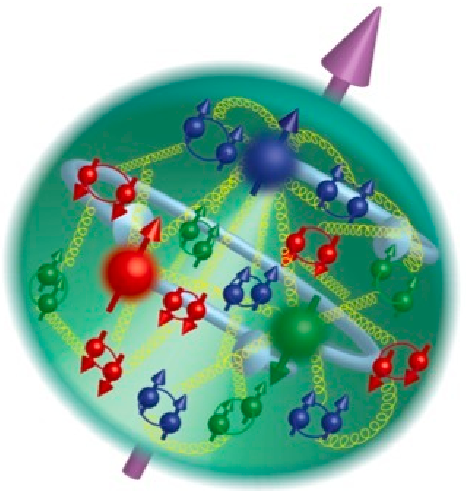


$\sim \int_R C$



$+\cdots$

$\times$ GPD



experimental data

DVCS

DDVCS

DVMP…

perturbative matching

analytically calculable

process-dependent

structure information

no analytic method

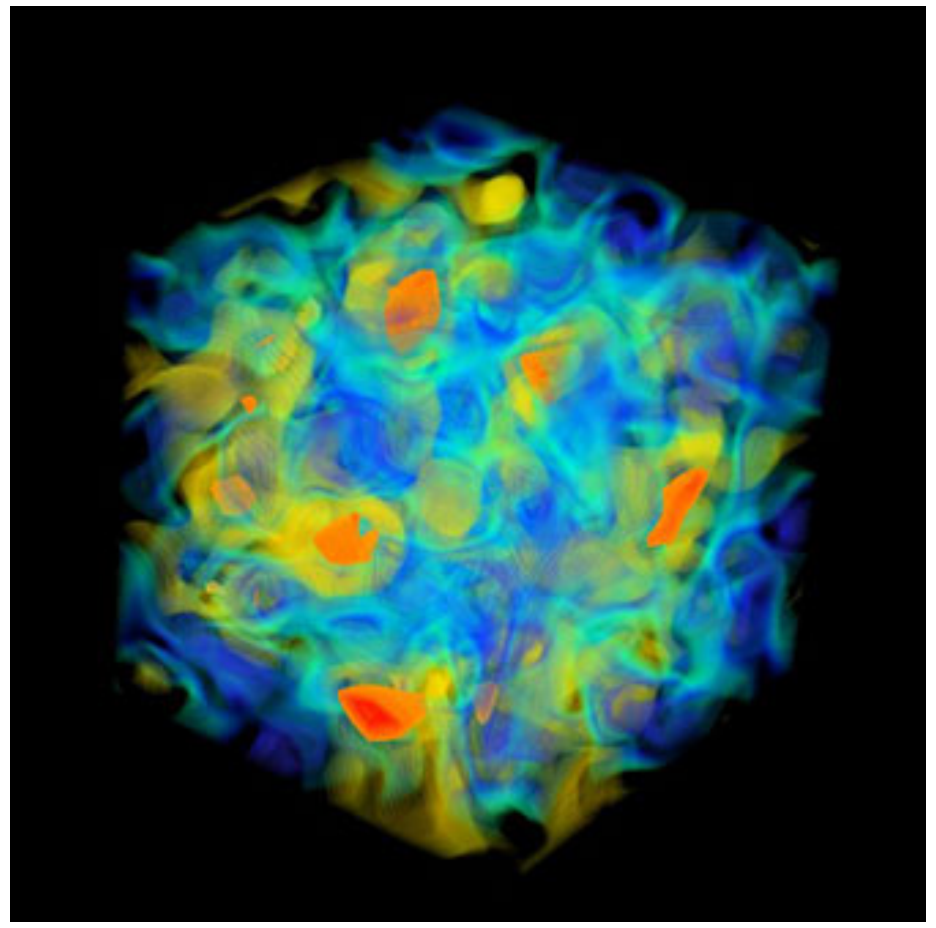
universal

Hadronic Structural information from first principles

- Hadrons structure is also accessible through numerical (lattice) simulations

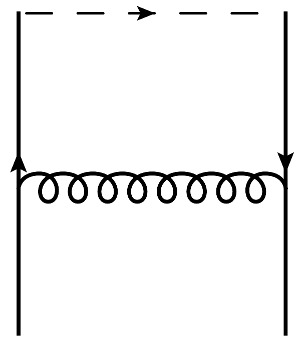
X. Ji (2013)

$\mu_{\rm UV}$ ← → $\Lambda_{\rm QCD}$



quasi-GPD

$\sim \int_R \boxed{C}$

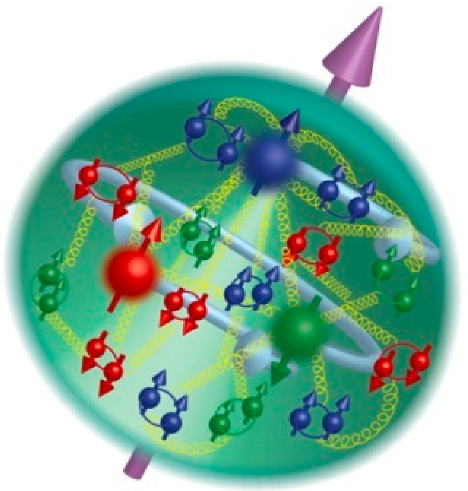


$+\cdots$

perturbative matching

$\times$

GPD

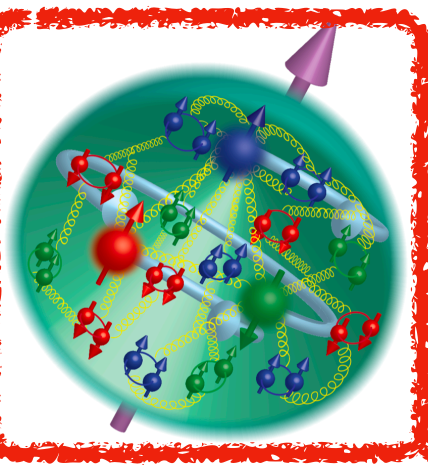


structure information

DVCS: the golden channel for accessing GPDs

DVCS

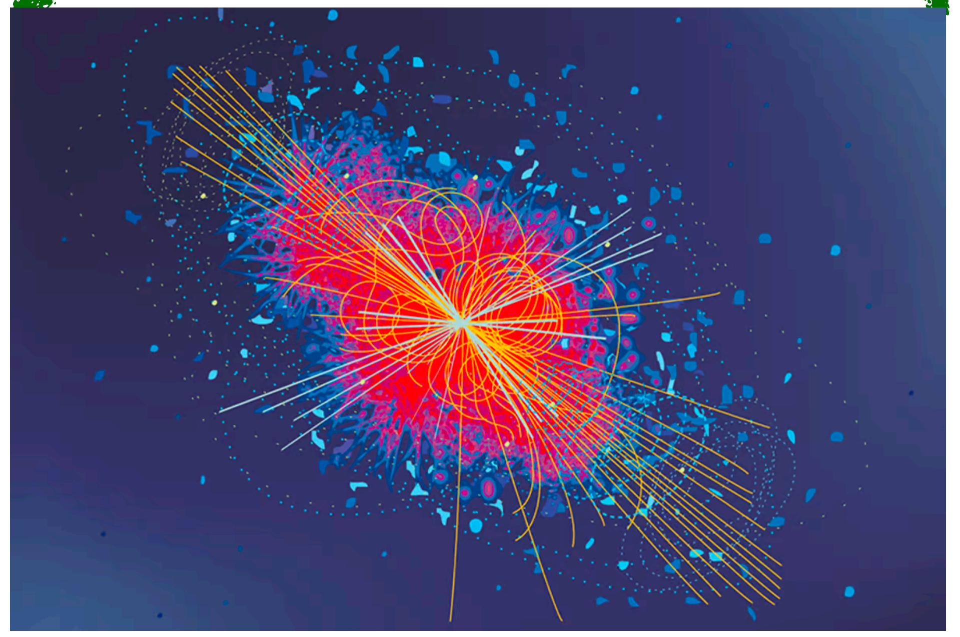
$e + N \rightarrow e + N + \gamma$

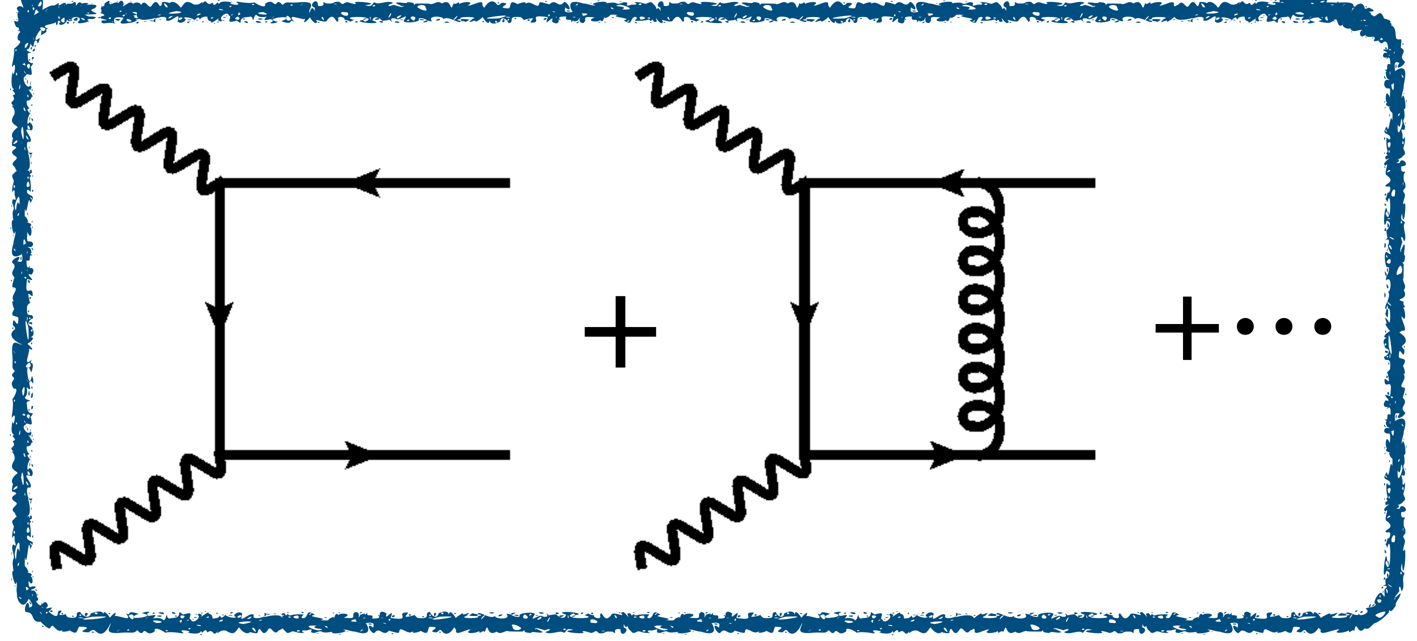


V. Braun, YJ, A. Manashov (2020), (2022), (2025) (via CFT)

kinematic power corrections: known to $1/Q^4$ at LO

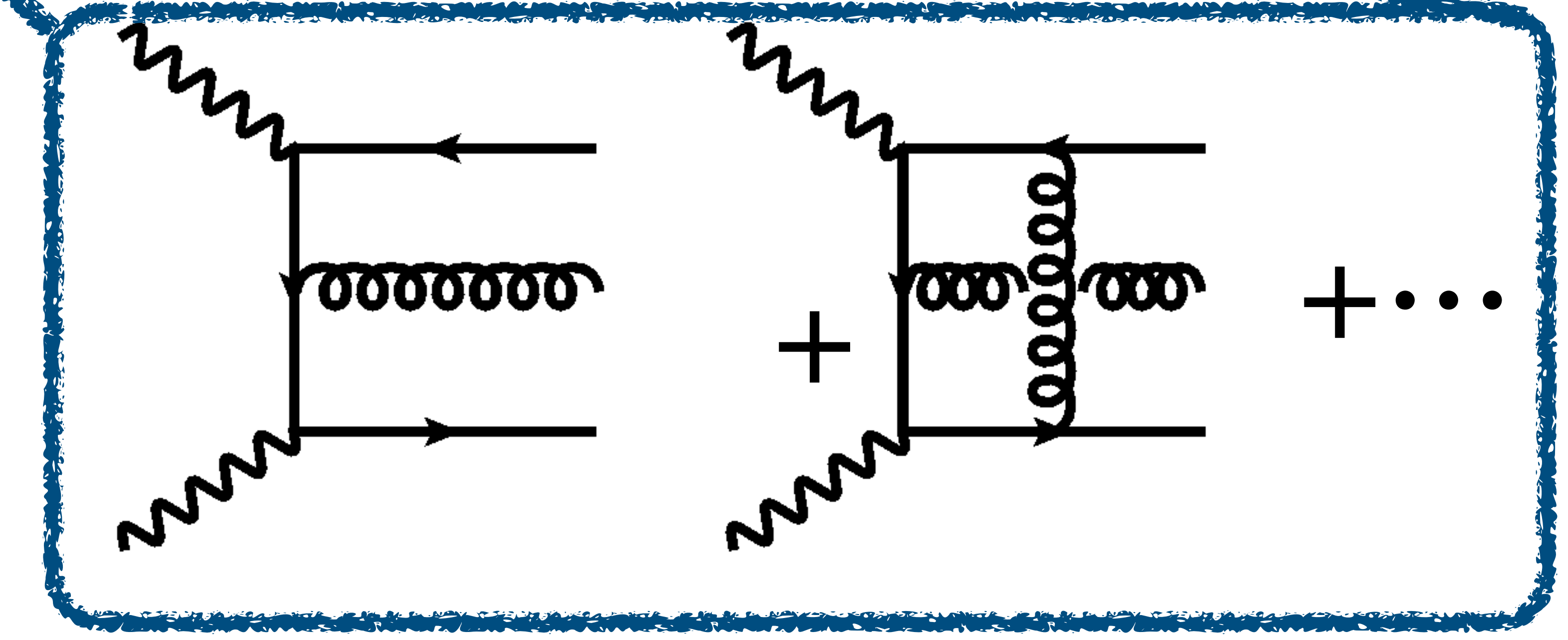
$$d\sigma \sim \left| C_1 \otimes f_1 + \frac{1}{Q} C_2 \otimes f_2 + \frac{1}{Q} \widetilde{C}_1 \otimes f_1 + \cdots \right|^2$$





completely known at two-loop order

V. Braun, A. Manashov, J. Schoenleber (2020), (2021)
J. Gao, T. Huber, YJ, Y-M. Wang (2021)
V. Braun, YJ, J. Schoenleber (2022)
YJ, J. Schoenleber (2023)



$1/Q$ factorization proven

J. Schoenleber, R. Szafron (2024)

one-loop computation underway

YJ, Y-F. Wang

Other exclusive processes for extracting GPDs

- GPD is also accessible through various other exclusive processes

DVMP: $\gamma^* + p \to n + \pi^+ \,,$ $\qquad \gamma^* + p \to p + \pi^0 \,,$ $\qquad \cdots$

known at two-loop:
W. Chen, F. Feng, Y. Jia, Q-T. Song, G. Tang, Z-Y. Wang (2026)
L-B. Chen, Y-X. Guo, YJ soon

known at two-loop:
W. Chen, F. Feng, Y. Jia, Q-T. Song, G. Tang, Z-Y Wang (2026)

DDVCS: $\gamma^* + p \to \gamma^* + p$

vector channel at 2-loop:
V. Braun, H-Y. Jiang, A. Manshov, A. Manteuffel (2024)
other channels to be completed

$2 \to 3$ processes : e.g. $M_1 + h \to h + M_2 + M_3 \,,$ $\qquad M_i = \pi, \gamma, \cdots$

Factorization proven:
J-W. Qiu, Z. Yu (2022)

Evolution of GPDs

- Scale dependence (evolution) of GPDs (e.g., $\mathscr{H}$) are governed by RGEs

$$\mu\frac{\partial}{\partial\mu}\mathscr{H}(x,\xi,\mu) + \beta\frac{\partial}{\partial a_s}\mathscr{H}(x,\xi,\mu) + \mathscr{K}\otimes\mathscr{H}(x,\xi,\mu) = 0$$

resum large logs of $\sim \alpha_s \ln(\mu_0/\mu)$

- $\mathscr{K}$ is the evolution kernel computable perturbatively, diagrammatically or CFT

two-loop: completely known

three-loop: partial known for nonsinglet channel, singlet completely unknown V. Braun, A. Manashov, S. Moch, M. Strohmaier (2017)

- $\mathscr{K}$ is easily reducible to DGLAP and ERBL

- Analytic and numerical solution is only achieved at one-loop

Perturbative calculations for qGPDs

- **quasi-GPD is computable from first principles, needs perturbative coefficient to extract (lightcone) GPD**

$$\mathrm{qGPD} = C \otimes \mathrm{GPD} \qquad \Longrightarrow \qquad \mathrm{GPD} = C^{-1} \otimes \mathrm{qGPD}$$

C at one-loop: completely known F. Yao, YJ, J-H. Zhang (2022); J. P. Ma, Z. Y. Pang, G. P. Zhang (2022)
X. Ji, A. Schäfer, X. Xiong, and J.-H. Zhang (2015), ….

C at two-loop: known (in position space) for nonsinglet unpolarized channel, unknown for singlet channel YJ, F. Yao, J-H. Zhang (2025)

- **quasi-GPDs simulated on lattice harbor large power corrections** $\sim \Lambda_{\mathrm{QCD}}/(xP_z),\ m/P_z$

Such corrections are systematically analyzed through renormalons V. Braun (2023); V. Braun, M. Koller, J. Schoenleber (2024); V. Braun, H-Y. Jiang (2026)

reduces large power corrections for extracting GPDs from qGPDs

- **resummation of large logs at the endpoints**

Global extraction of GPDs across different approaches

- **Rapid accumulation of experimental and lattice data require global analysis for GPD determination**

GPDs through universal moment parameterization (GUMP)

Y. Guo , X. Ji, K. Shiells (2022), Y. Guo, X. Ji, M. Santiago, K. Shiells, J. Yang (2023), Y. Guo, F. Aslan, X. Ji, M. Santiago (2025)

The parameterization is based on conformal moment expansion (Gegenbauer polynomials)

- **Evolution in the conformal basis becomes a more prominent topic**

The conformal sum generally becomes divergent after scale evolution, universal analytic techniques needed + numerics

M. Kirch, A. Manashov, A. Schäfer (2005)

Conclusion and outlook

- DVCS has been systematically obtained to NNLL
- Perturbative matching coefficients for qGPD start to advance to two-loop order

- Complete three-loop evolution kernel for GPDs and higher-order solutions
- DDVCS at NNLL for axialvector and singlet channels
- DVCS at NNNLO
- $2 \to 3$ processes at NLO
- kinematic power corrections at NLO
- Complete two-loop coefficient functions for qGPDs
- …