



华南师范大学
SOUTH CHINA NORMAL UNIVERSITY



Lattice Parton
Collaboration

Toward a Unified Understanding of Hadron Light-Cone Distribution Amplitudes from Lattice QCD

华俊 华南师范大学

On behalf of Lattice Parton Collaboration

07/16 @ 中国物理学会高能物理分会

OUT LINE

01 Overview about LCDAs

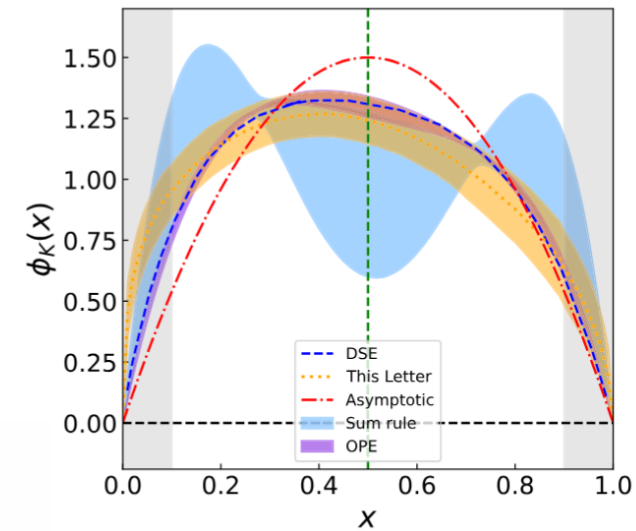
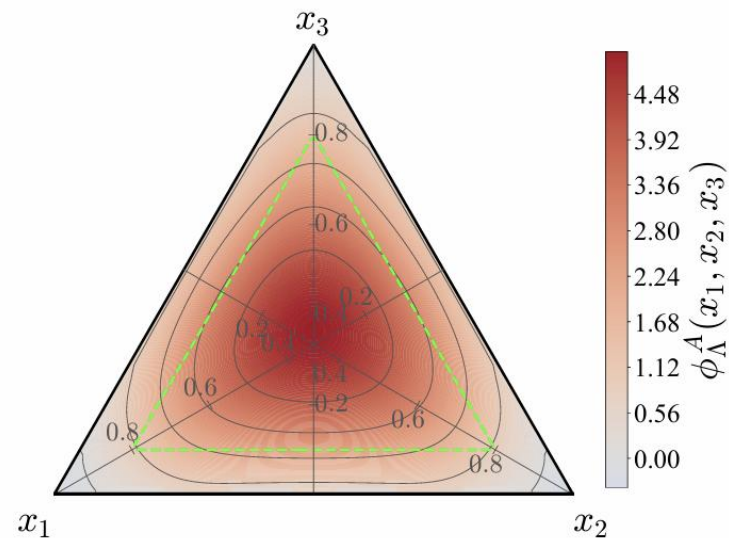
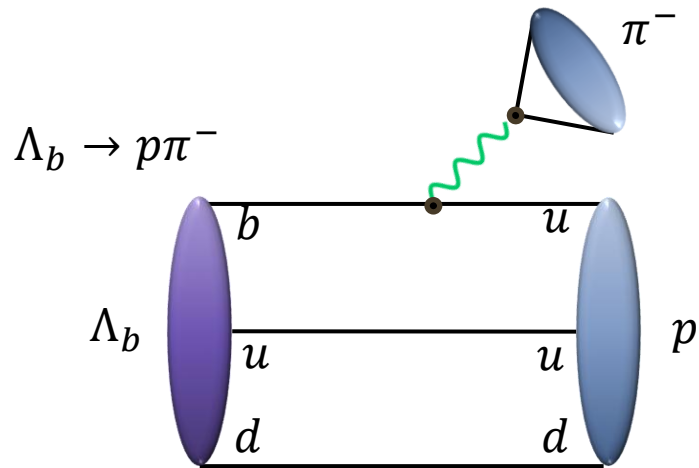
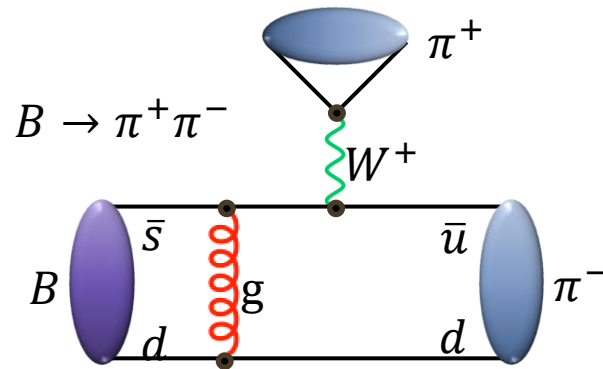
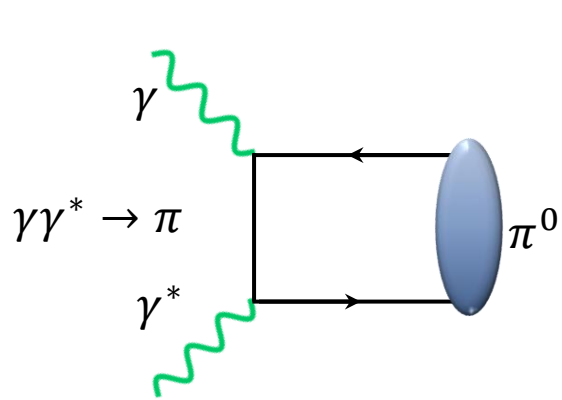
02 Recent Progress on Light Meson LCDA

03 Recent Progress on Light Baryon LCDA

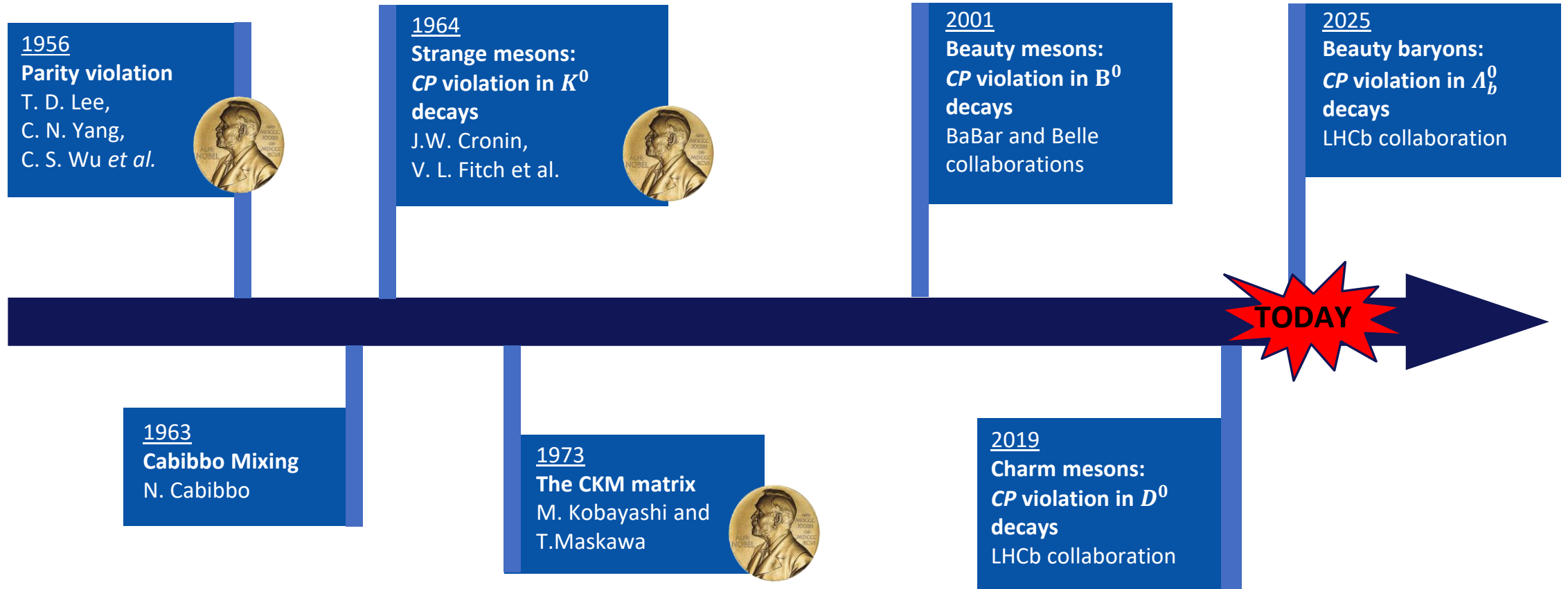
04 Summary and Outlook

About LCDA

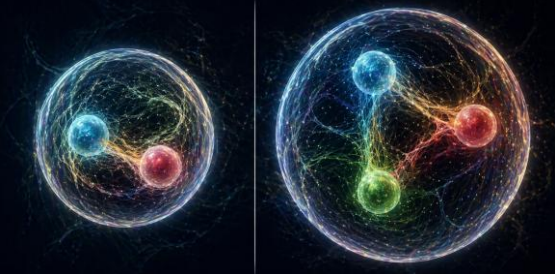
- Richer QCD dynamical information and most important input during describing
Exclusive scattering — LCDAs



About LCDA



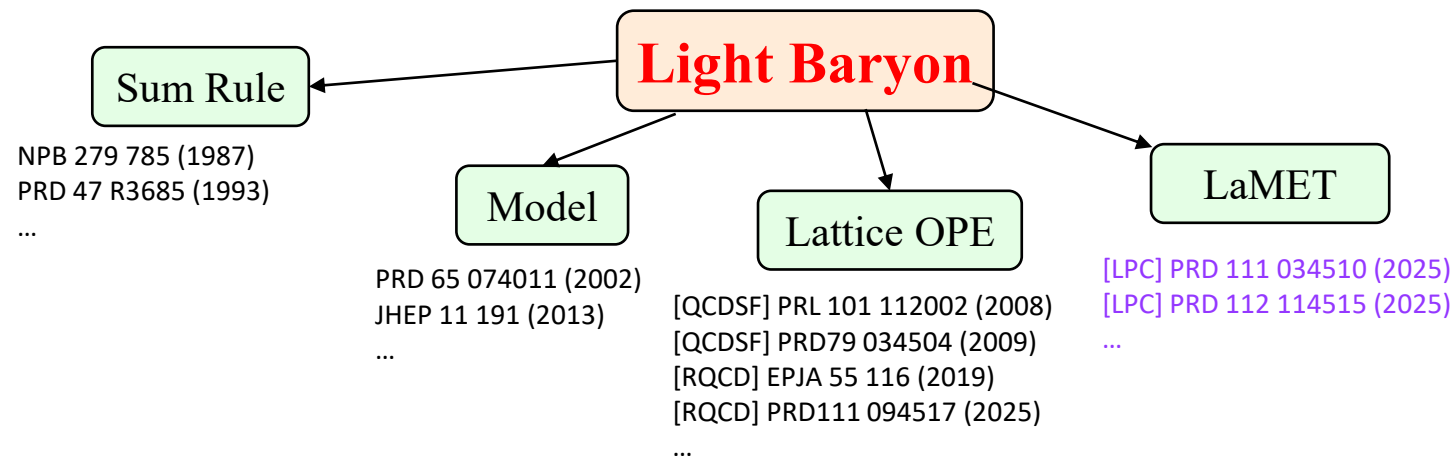
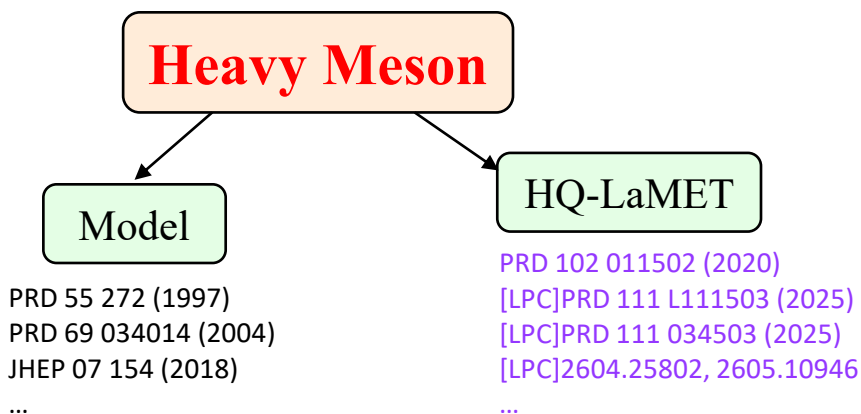
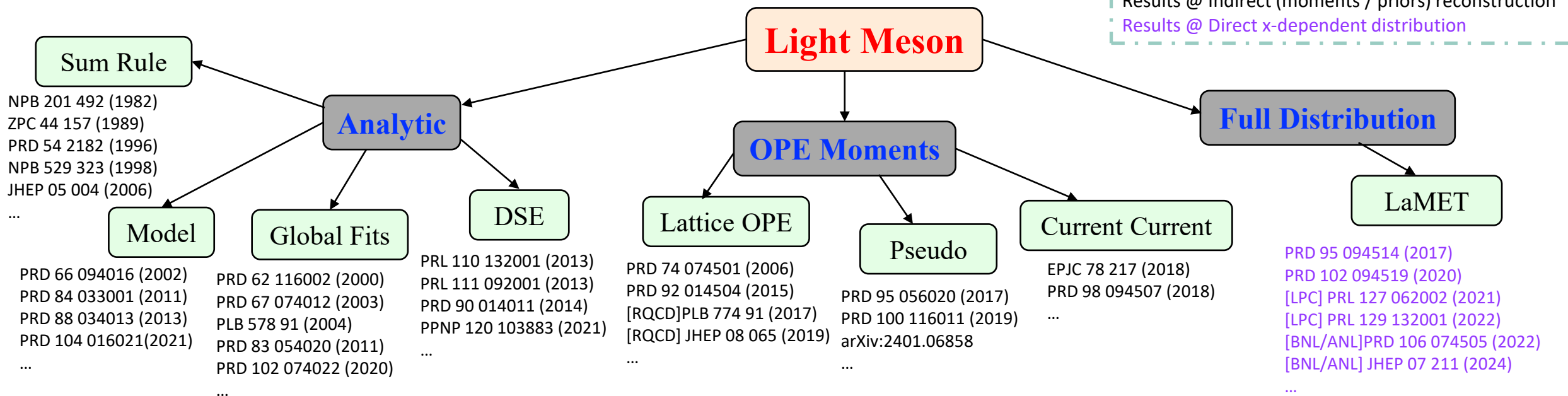
- Among these, the LCDAs serve as a key theoretical cornerstone for investigating CP violation



LCDAs of Meson & Baryon: dynamical progress

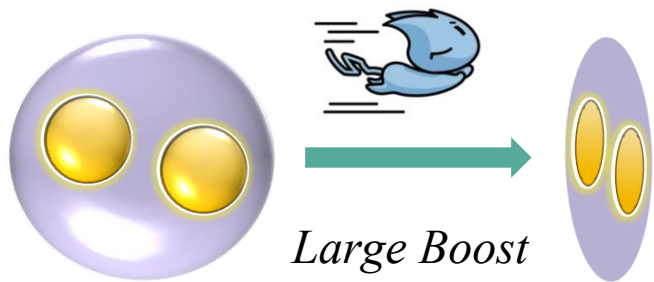
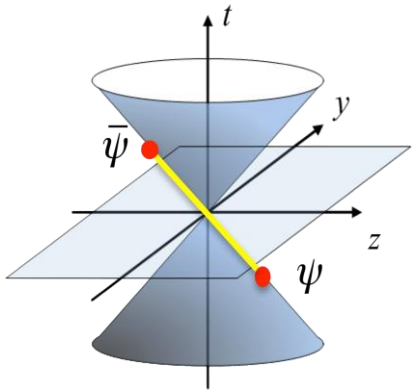


Results @ Indirect (moments / priors) reconstruction
Results @ Direct x-dependent distribution



LQCD for LCDAs OPE VS LaMET

LQCD is formulated as a Feynman path integral on a **4D Euclidean grid**,
 with a Wick rotation from real time to imaginary time.



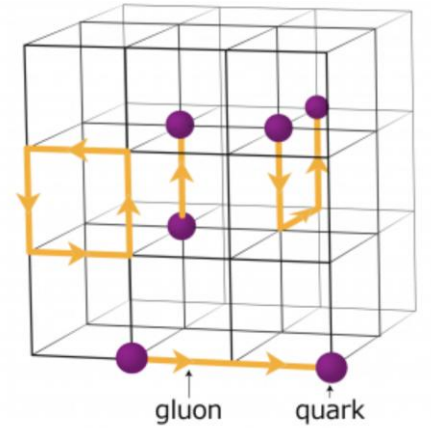
π LCDA: $\langle 0 | \bar{q}_2(\xi^+) \gamma^+ \gamma_5 U(0, \xi^-) q_2(\xi^-) | \pi \rangle$

- OPE** $\sim$ in local limit: $\rightarrow$ moments of DA

$$\langle 0 | \bar{q}_2(\xi^+) \gamma^+ \gamma_5 U(\xi^+, \xi^-) q_2(\xi^-) | \pi \rangle \xrightarrow{\xi \rightarrow 0} C_m(z)^m \bar{q}_2(0) (\gamma^t + \gamma^z) \gamma_5 (i n_z \cdot \overleftrightarrow{D})^m q_1(0)$$

- LaMET** $\sim$ in large P^z limit: $\rightarrow$ shape of DA

$$\langle 0 | \bar{q}_2(\xi^+) \gamma^+ \gamma_5 U(\xi^+, \xi^-) q_2(\xi^-) | \pi \rangle \xrightarrow{P^z \rightarrow \infty} \langle 0 | \bar{q}_2(z_+) (\gamma^t + \gamma^z) \gamma_5 U(z_+, z_-) q_2(z_-) | \pi(P^z) \rangle$$



From Meson LCDA

Meson LCDAs by LaMET: (2021~ ...)

- π, K LCDA $a \rightarrow 0$

R.Zhang, H.W.Lin et.al. PRD 102,094519

- $K^*, \phi; \pi, K$ LCDA $a \rightarrow 0, m_\pi \rightarrow 130\text{MeV}$

(LPC) PRL 127 062002(2021); PRL 129 132001(2022)

- Hybrid and Self renormalization

(LPC) NPB 964 115311(2021); NPB 969 115443(2021)...

- RGR resummation & Renormalon resummation (LRR)

Y.S.Su et.al. NPB 993 116282(2023)

- Threshold resummation

Y.S.Su et.al. JHEP 03 045(2025)

- Two loop matching

F.Yao et.al. arXiv:2504.09367; PRD 113 014505(2026)

- Heavy Meson DA (HQ-LaMET)

(LPC) PRD L111503, 111 034503; 2604.25802,2605.10946

- Not physical π mass, RI/MOM scheme, λ truncation

- Physical π mass with $a \rightarrow 0$

- Hybrid scheme, Self renormalization

- Towards high precision

- Not yet done with lattice results

- A new framework including 2-step matching

- High precision is challenge

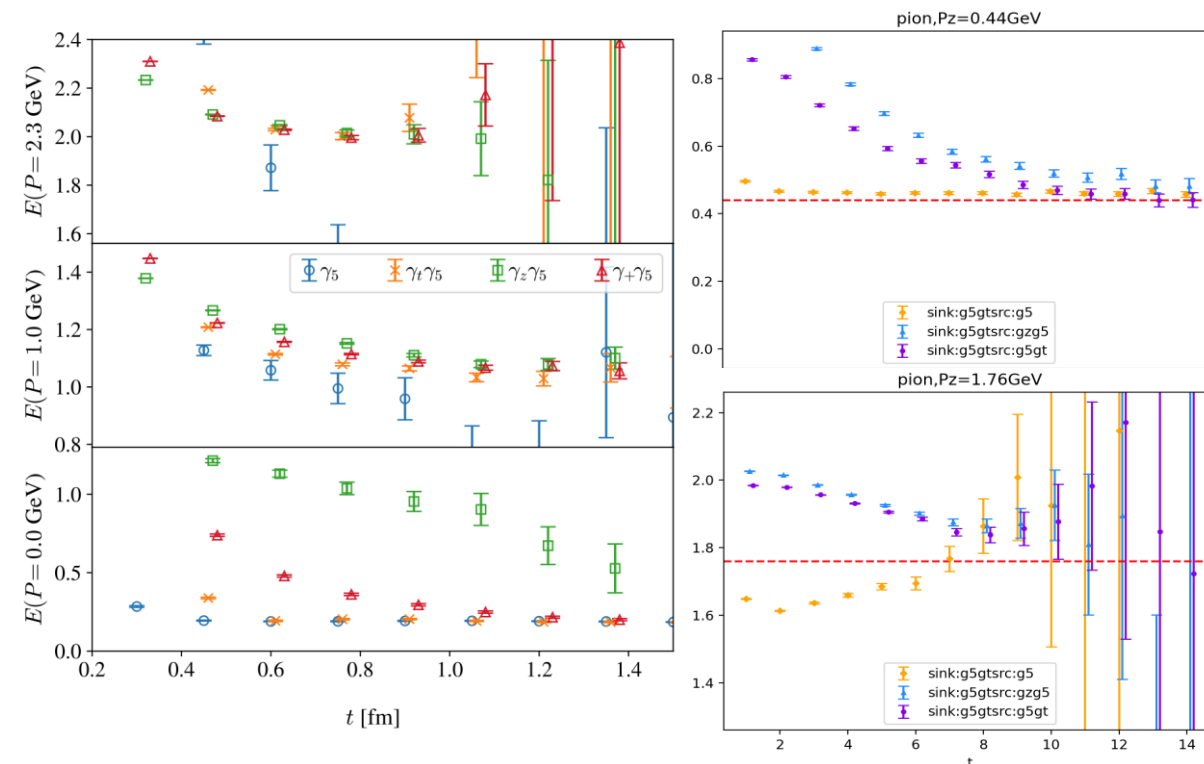
From Meson LCDA

Lattice Advances: (2021 ~ ...)

- Kinematically-enhance
PRD 112 L051502(2025), arXiv: 2606.02447
- Precision check for meson LCDA
... JHW et.al.
- Large λ Extrapolation
PRD 113 014509(2026), arXiv: 2601.12189
- CLQCD ensembles & Pyquda
CLQCD PRD 109(2024) , arXiv: 2411.08461...
- ... Gradient flow
JHEP 06 210 (2024), PRD 112 094504(2025)...

➤ Precision !

Signal Improve



Self renorm & Renormalon resum *JH et.al. NPB 993 116282(2023), XDJ et.al. JHEP 03 045(2025)*

Linear divergence



Multi-a, extract linear-div



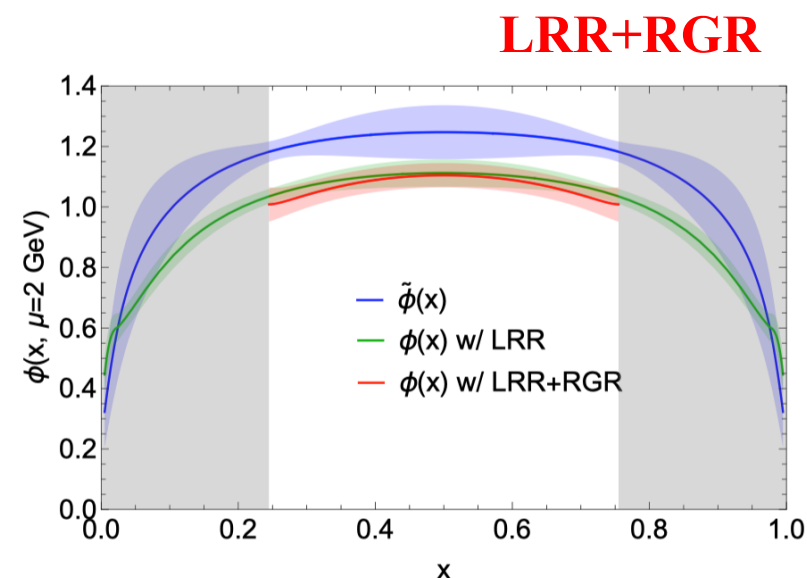
Match with perturbative $\overline{MS}$

$$Z(z, a)_R = \exp \left[\frac{kz}{a \ln [a \Lambda_{\text{OCD}}]} - m_0 z + f(z)a + \frac{3C_F}{b_0} \ln \left[\frac{\ln [1/(a \Lambda_{\text{QCD}})]}{\ln [\mu/\Lambda_{\text{QCD}}]} \right] + \ln \left[1 + \frac{d}{\ln (a \Lambda_{\text{QCD}})} \right] \right]$$

- Extract from multi lattice spacings of zero momentum matrix
- Extract from matching with perturbative $\overline{MS}$ quasi

Renormalon ambiguity occurs

A LRR guarantee a more comprehensive self renormalization



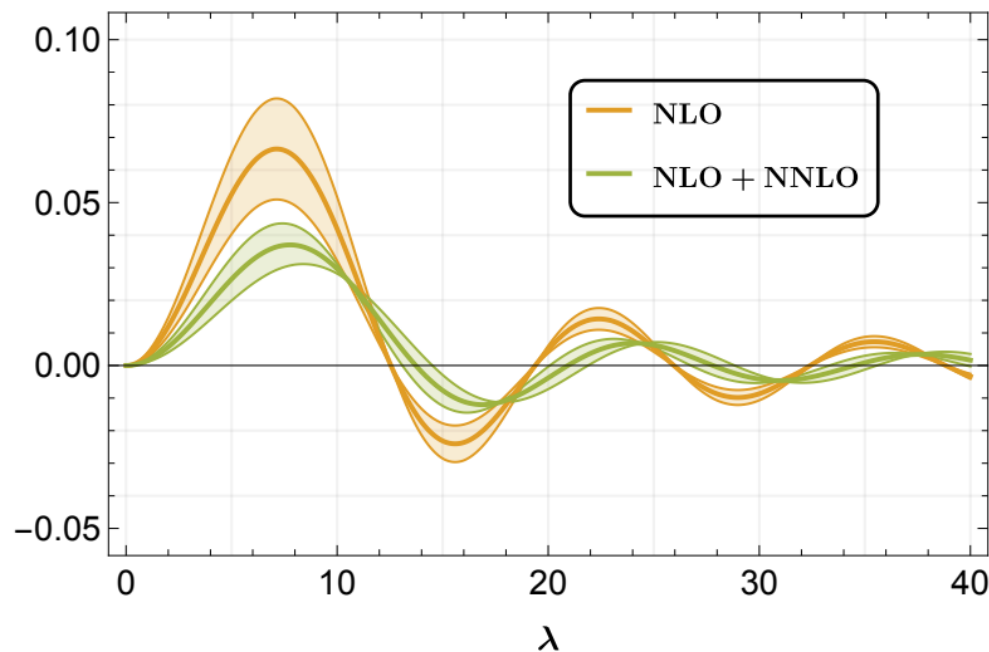
NNLO corrections:

YJ, FY & JHZ arXiv:2504.09367

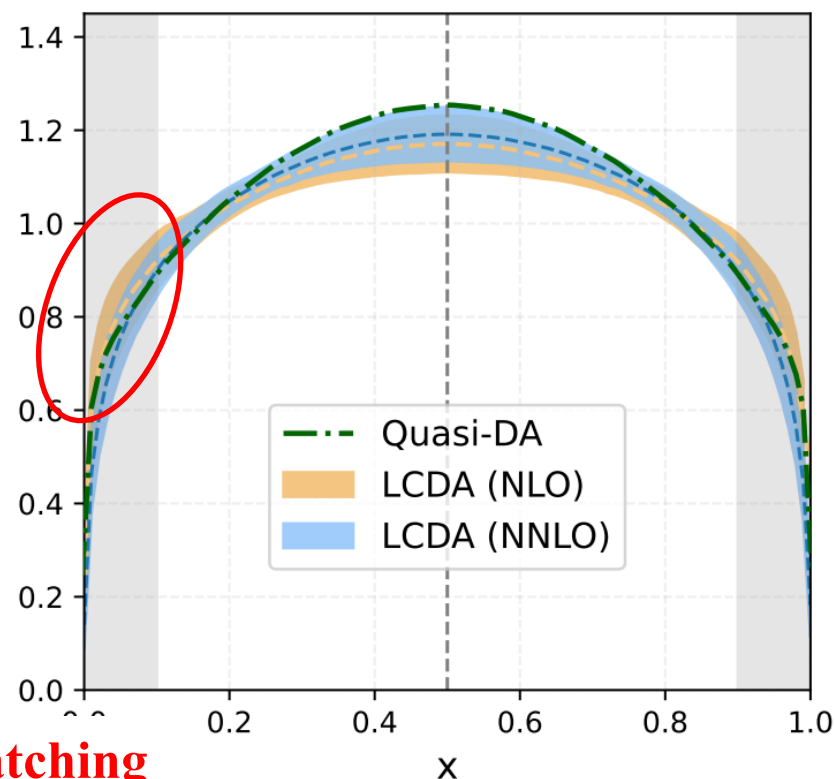
$$\phi_R(x, P^z) = \int dy C\left(x, y, \frac{\mu}{P^z}\right) \tilde{\phi}_R(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{(P^z)^2}\right)$$

Matching kernel

Coordinate space: $\mu = 2 \text{ GeV}, z = 0.2 \text{ fm}$

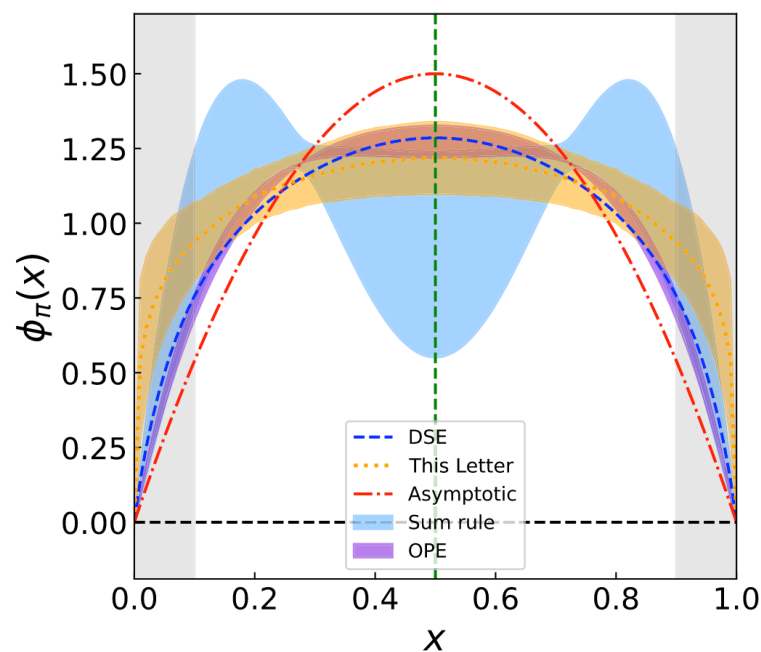


Momentum space:



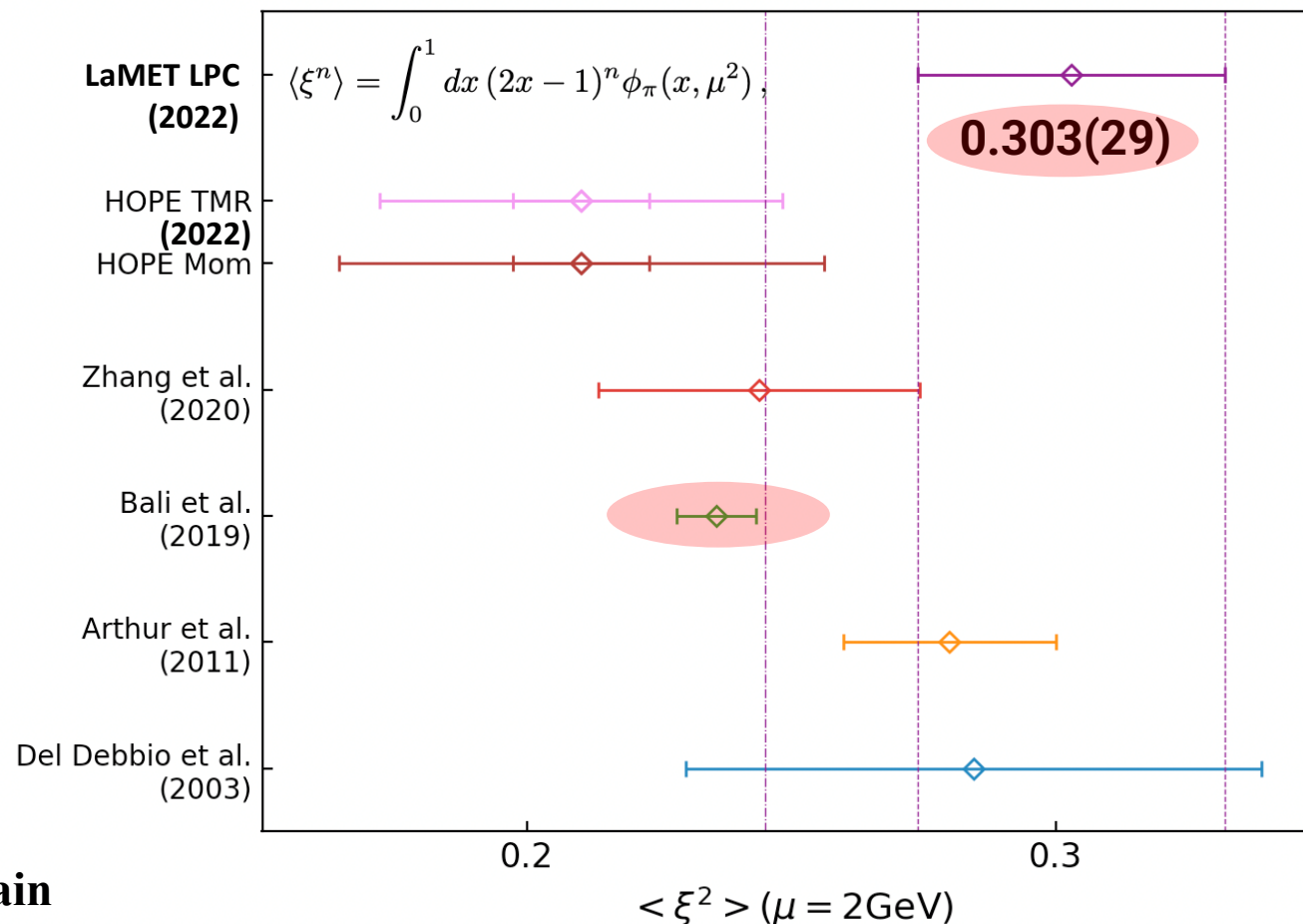
A high precision matching

Moments tension: Why higher precision for light meson LCDA



(LPC) PRL 129,132001 (2022)

- **High precision** under going in LaMET
- Moments **tension** between LaMET & OPE remain



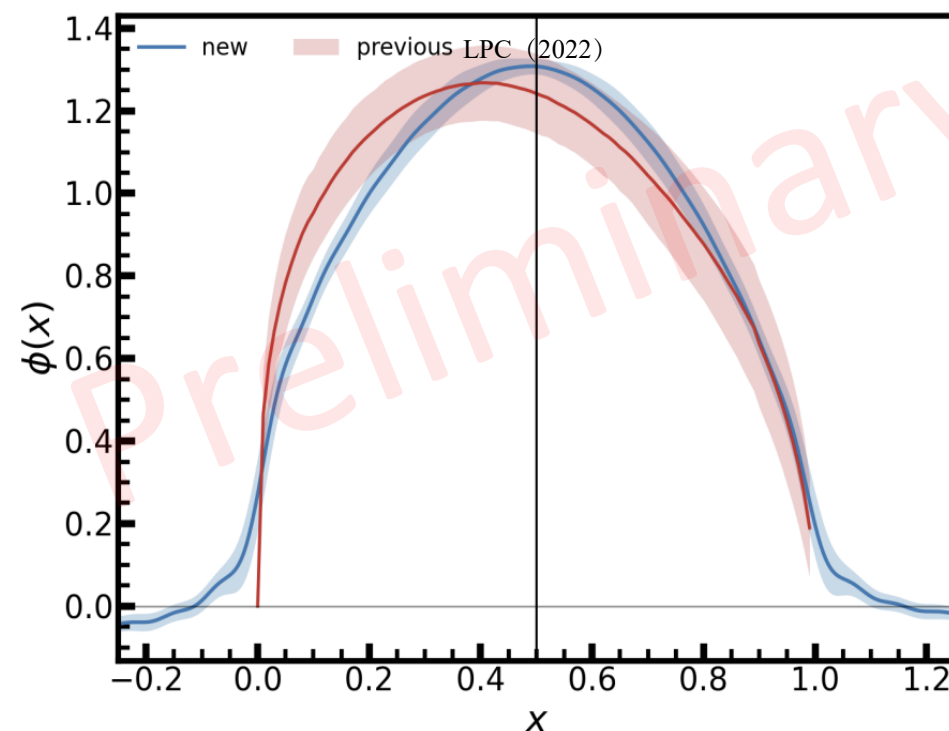
Benchmark moments & LaMET

(LPC) arXiv:26XXX

➤ Benchmark of Local Moments on CLS & MILC ensembles

	$a_\pi^{(2)}$	$a_K^{(1)}$	$a_K^{(2)}$	
RBC/UKQCD [22]	0.23(3)(6)	0.036(1)(2)	0.18(3)(6)	
RQCD [24]	0.116^{+19}_{-20}	0.0525^{+31}_{-33}	0.106^{+15}_{-16}	
LPC (2026)	0.187(15)	0.0481(23)	0.158(14)	
	$a_\rho^{(2)}$	$a_{K^*}^{(1)}$	$a_{K^*}^{(2)}$	$a_\phi^{(2)}$
RBC/UKQCD [22]	0.20(3)(6)	0.037(1)(2)	0.15(6)(6)	0.15(6)(3)
RQCD [25]	0.132(27)			
LPC (2026)	0.102(18)	0.0609(15)	0.085(17)	0.069(17)
	$a_\rho^{(2)T}$	$a_{K^*}^{(1)T}$	$a_{K^*}^{(2)T}$	$a_\phi^{(2)T}$
RQCD [25]	0.101(22)			
LPC (2026)	0.133(20)	0.0301(16)	0.087(19)	0.041(19)

➤ Updated Kaon LCDA



• LaMET vs OPE 2022

$$a_{2,K}^{\text{LaMET,integral}} = 0.170(14)(44) \quad a_{1,K}^{\text{LaMET,integral}} = 0.108(14)(15)$$

$$a_{2,K}^{\text{OPE(RQCD)}} = 0.106^{+15}_{-16} \quad a_{1,K}^{\text{OPE(RQCD)}} = 0.0525^{+31}_{-32}$$

• LaMET vs OPE 2026

$$a_{2,K}^{\text{LaMET,derviative}} = 0.158(38) \quad a_{1,K}^{\text{LaMET,integral}} = 0.037(10)$$

$$a_{2,K}^{\text{LaMET,integral}} = 0.126(11) \quad a_{1,K}^{\text{LaMET,integral}} = 0.034(26)$$

$$a_{2,K}^{\text{OPE}} = 0.158(14) \quad a_{1,K}^{\text{OPE}} = 0.0477(23) \quad 12$$

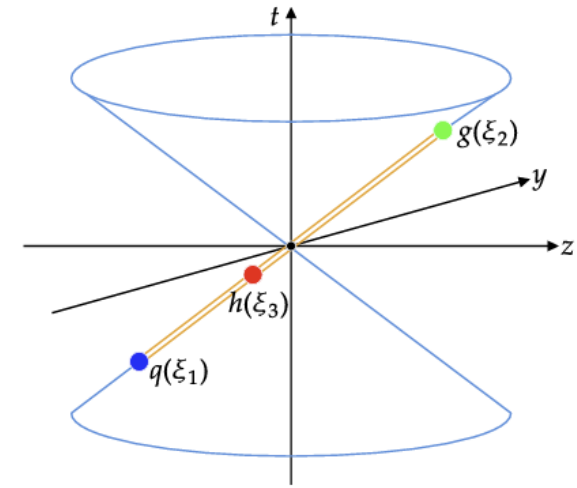
Light Baryon LCDA

Light Baryon LCDAs: (on lattice ~ ...)

- Previous
All results based on models ...
- Lattice QCD with OPE
QCDSF, 2008, 2009; RQCD, 2016, 2019(2025)
- Lattice QCD with LaMET

LPC, 2024,2025;

PRD 111 034510, PRD 112 114515

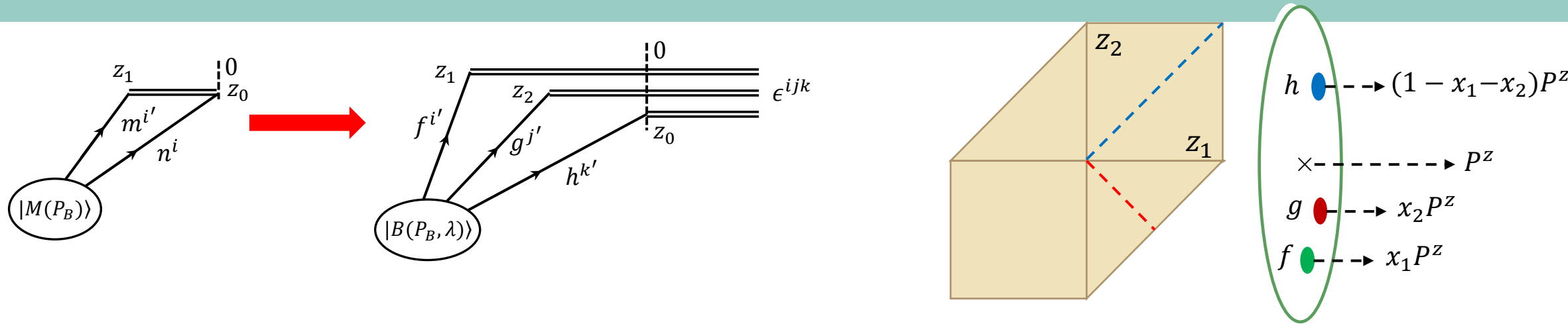


$$\begin{aligned}
 & \langle 0 | f_\alpha(z_1 n) g_\beta(z_2 n) h_\gamma(z_3 n) | B(P_B, \lambda) \rangle \\
 &= \frac{1}{4} f_V \left[(P_B C)_{\alpha\beta} (\gamma_5 u_B)_\gamma V^B(z_i n \cdot P_B) + (P_B \gamma_5 C)_{\alpha\beta} (u_B)_\gamma A^B(z_i n \cdot P_B) \right] \\
 &+ \frac{1}{4} f_T (i \sigma_{\mu\nu} P_B^\nu C)_{\alpha\beta} (\gamma^\mu \gamma_5 u_B)_\gamma T^B(z_i n \cdot P_B),
 \end{aligned}$$

Octet	n	p	Λ
fgh	ddu	uud	uds

Light Baryon LCDA

Challenge from light meson LCDA to light baryon LCDA



- More than a thousand times harder on SNR

➤ Start from hyperon “ Λ ”

- CPV in Baryon decays (*LHCb Nature, 643 1223(2025)*)
- Hyperson EDM *PRL 136 051902, 2026*

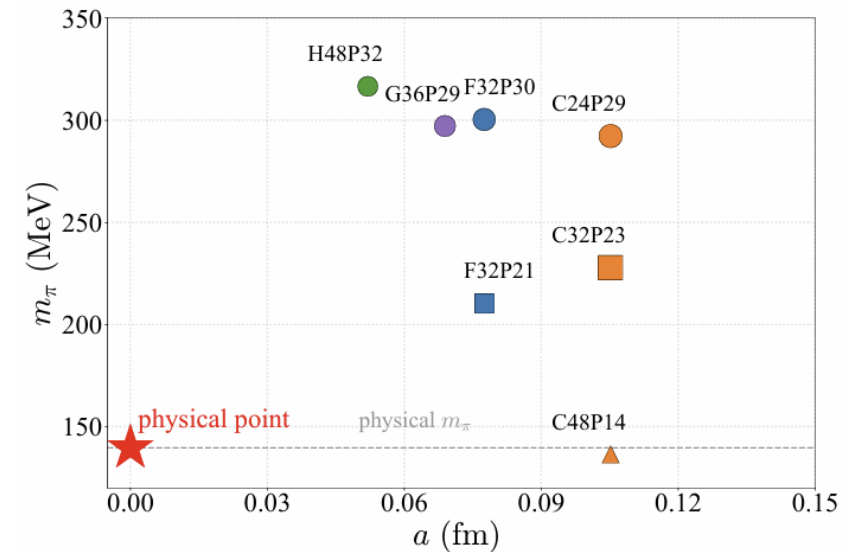
Light Baryon LCDA

CLQCD Ensembles — fully exploiting available



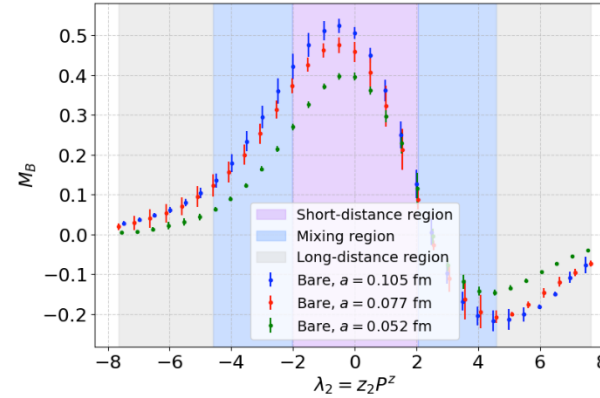
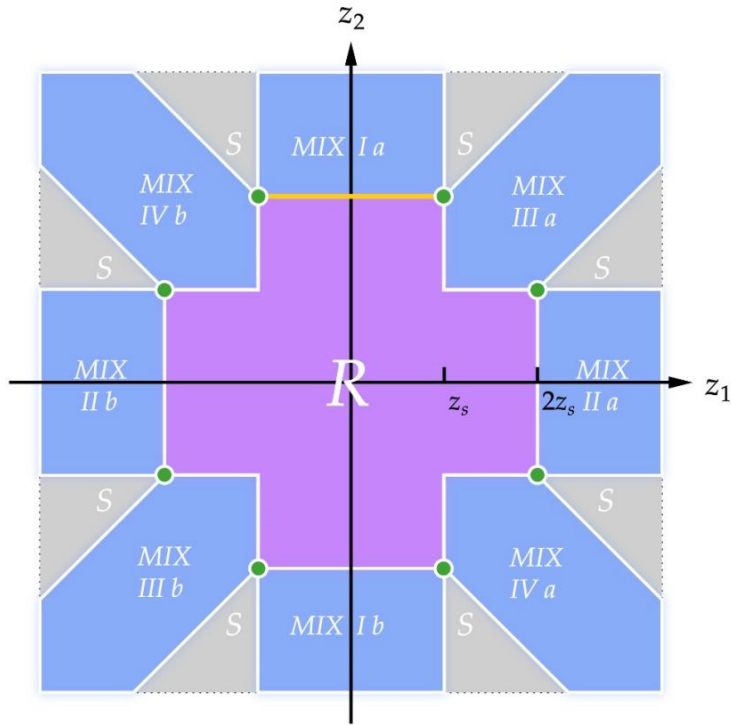
Ensemble	a (fm)	m_π (MeV)	P^z (GeV)	n_{stat}
C24P29	0.1052	292.3	1.96, 2.45, 2.94	864×36
C32P23	0.1052	227.9	1.84, 2.21, 2.57, 2.94	954×32
C48P14	0.1052	136.4	1.96, 2.45, 2.94	302×64
F32P30	0.0775	300.4	2.00, 2.49, 2.99	777×32
F32P21	0.0775	210.3	2.00, 2.49, 2.99	459×64
G36P29	0.0689	297.2	2.00, 2.50, 3.00	656×32
H48P32	0.0520	316.6	1.98, 2.48, 2.98	550×54

(CLQCD) PRD 109 054507 (2024)

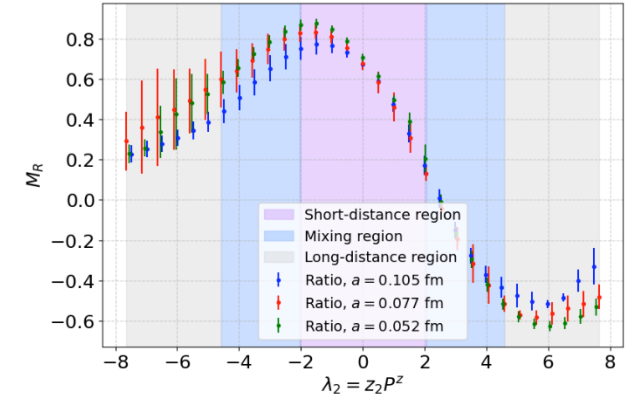


Light Baryon LCDA

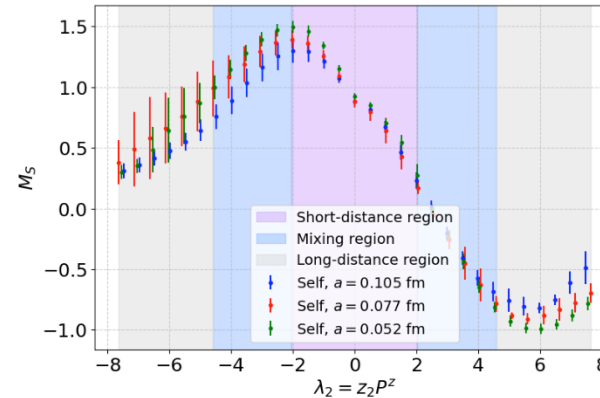
Hybrid renormalization — Done



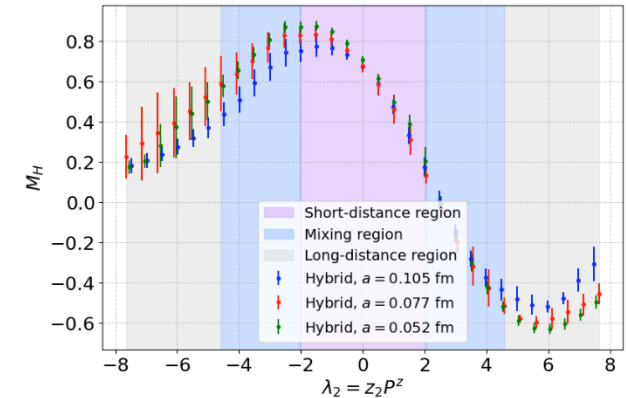
(a) Bare result of Λ at $P = 2.0$ GeV



(b) Ratio scheme result of Λ at $P = 2.0$ GeV



(c) Self renormalized result of Λ at $P = 2.0$ GeV



(d) Hybrid scheme result of Λ at $P = 2.0$ GeV

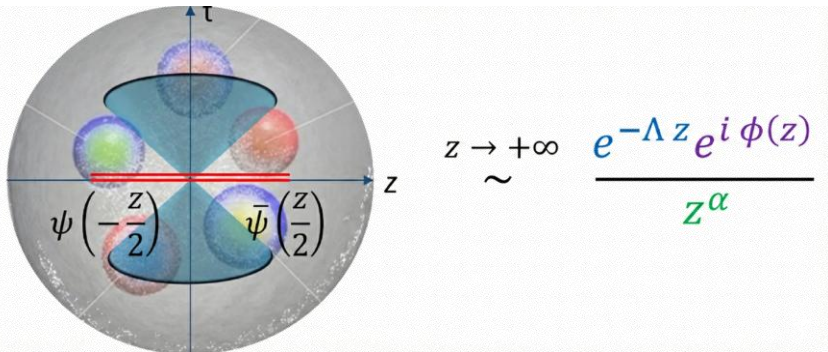
(LPC) PRD 112 114515 (2025)

Light Baryon LCDA

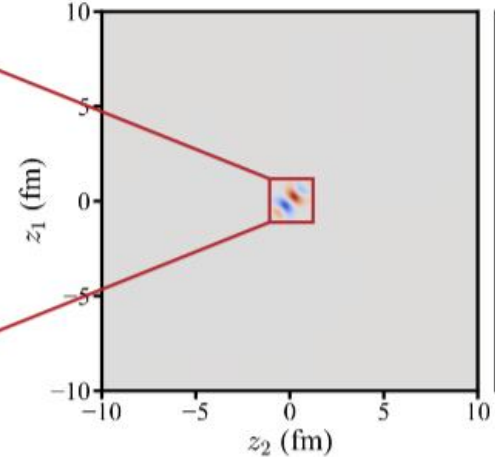
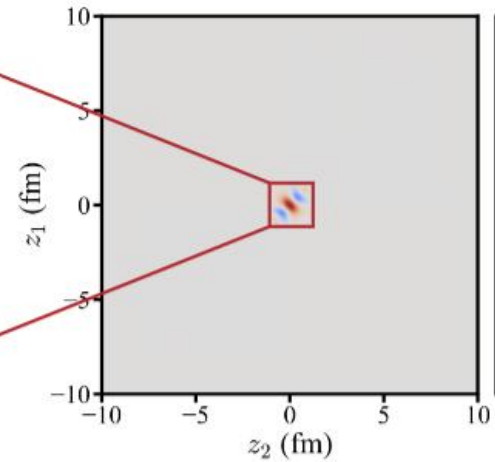
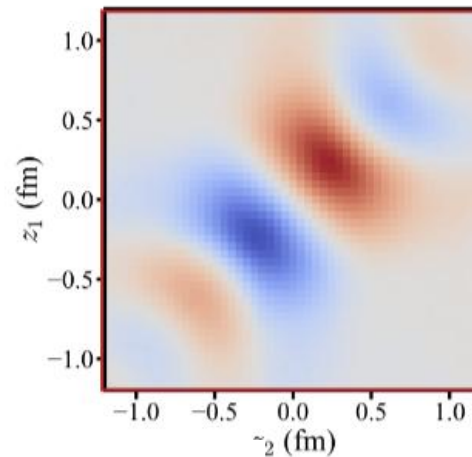
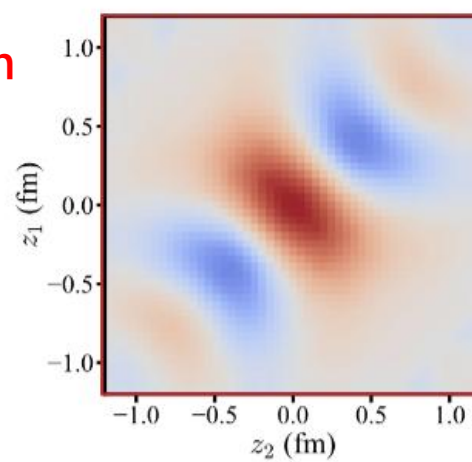
New λ -Extrapolation — Done

Asymptotic Long-Distance **Expansion of Euclidean Correlators** in Lattice Parton Applications

X. Ji, Y. Liu, Y. Su, arXiv:2601.12189



1.2 fm
× 1.2 fm

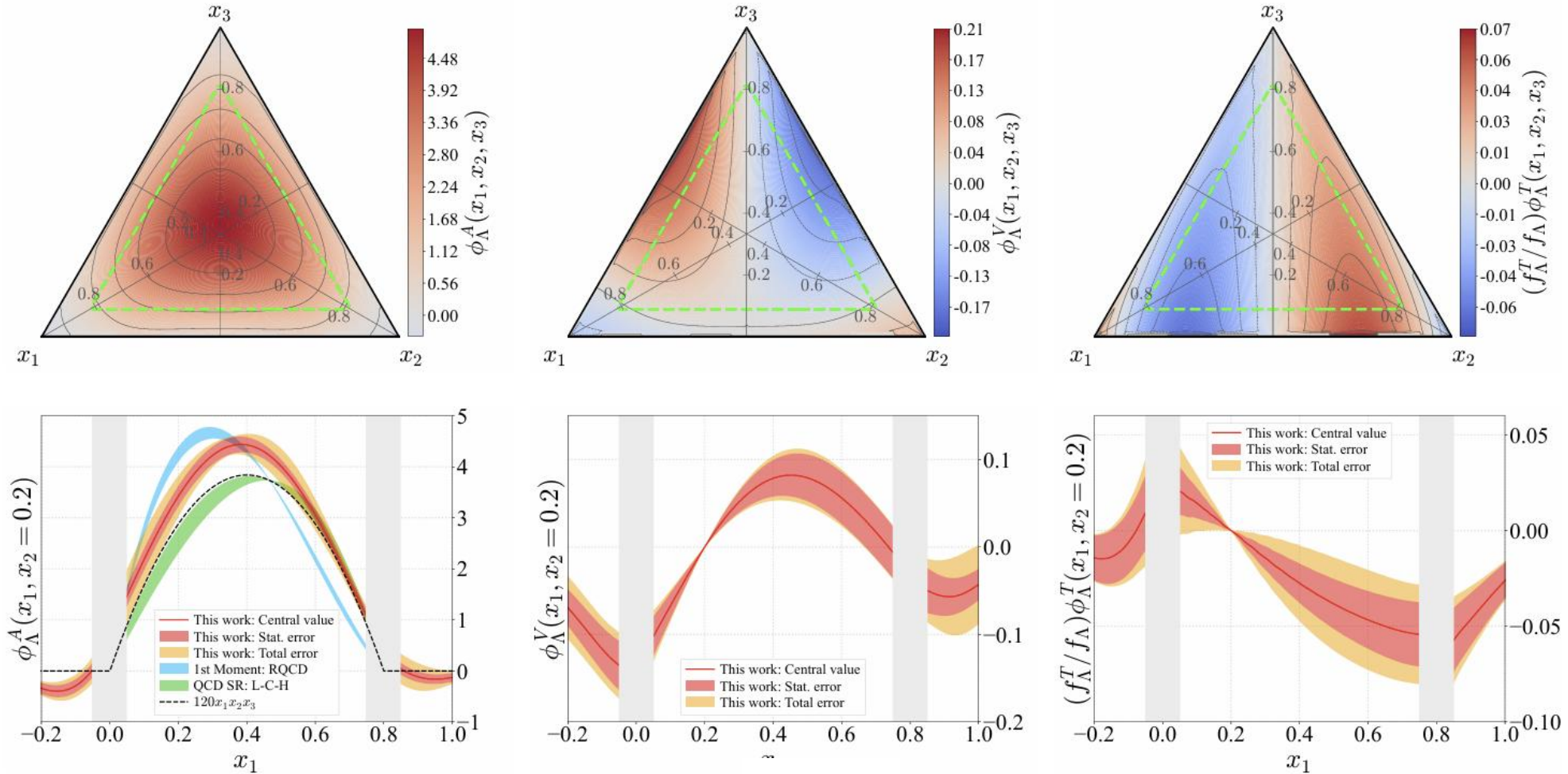


10 fm
× 10 fm

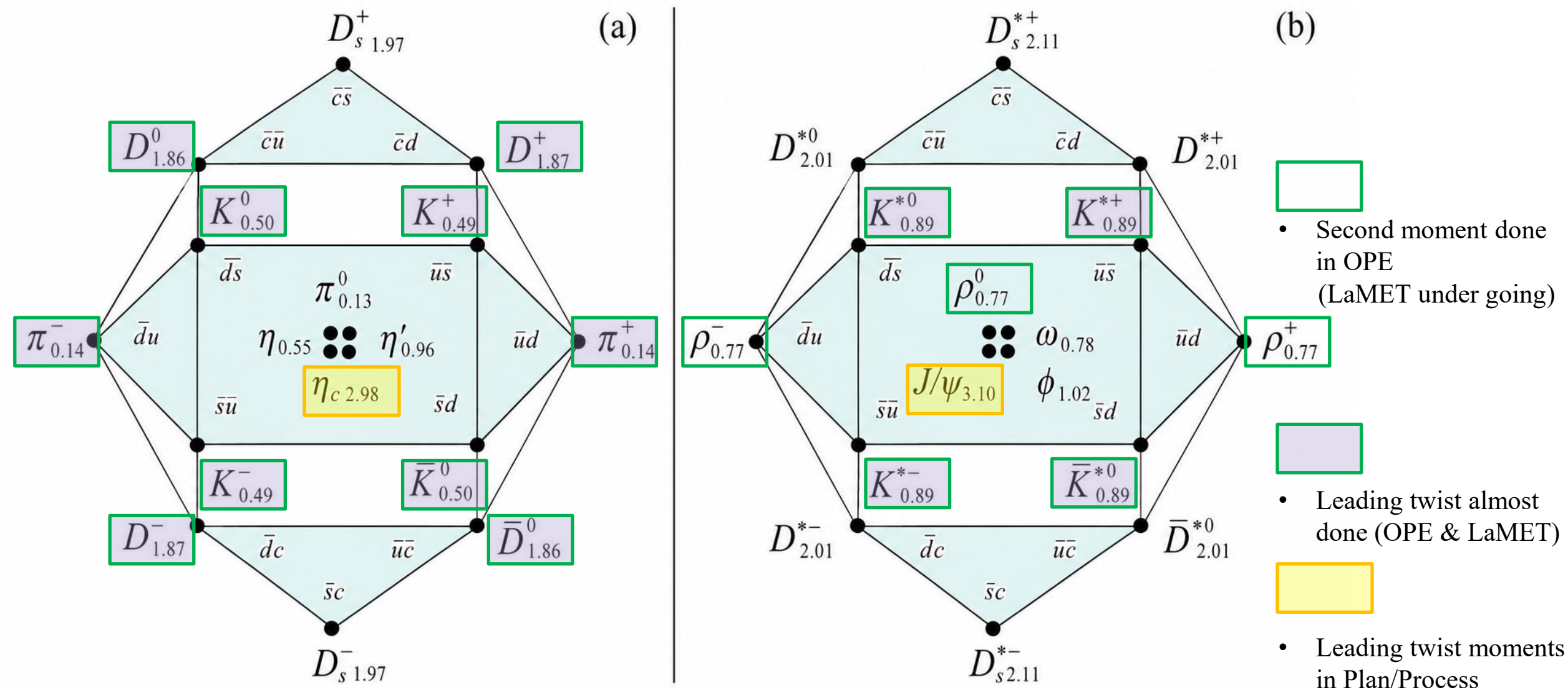
Light Baryon LCDA

Numerical Results of Λ LCDA

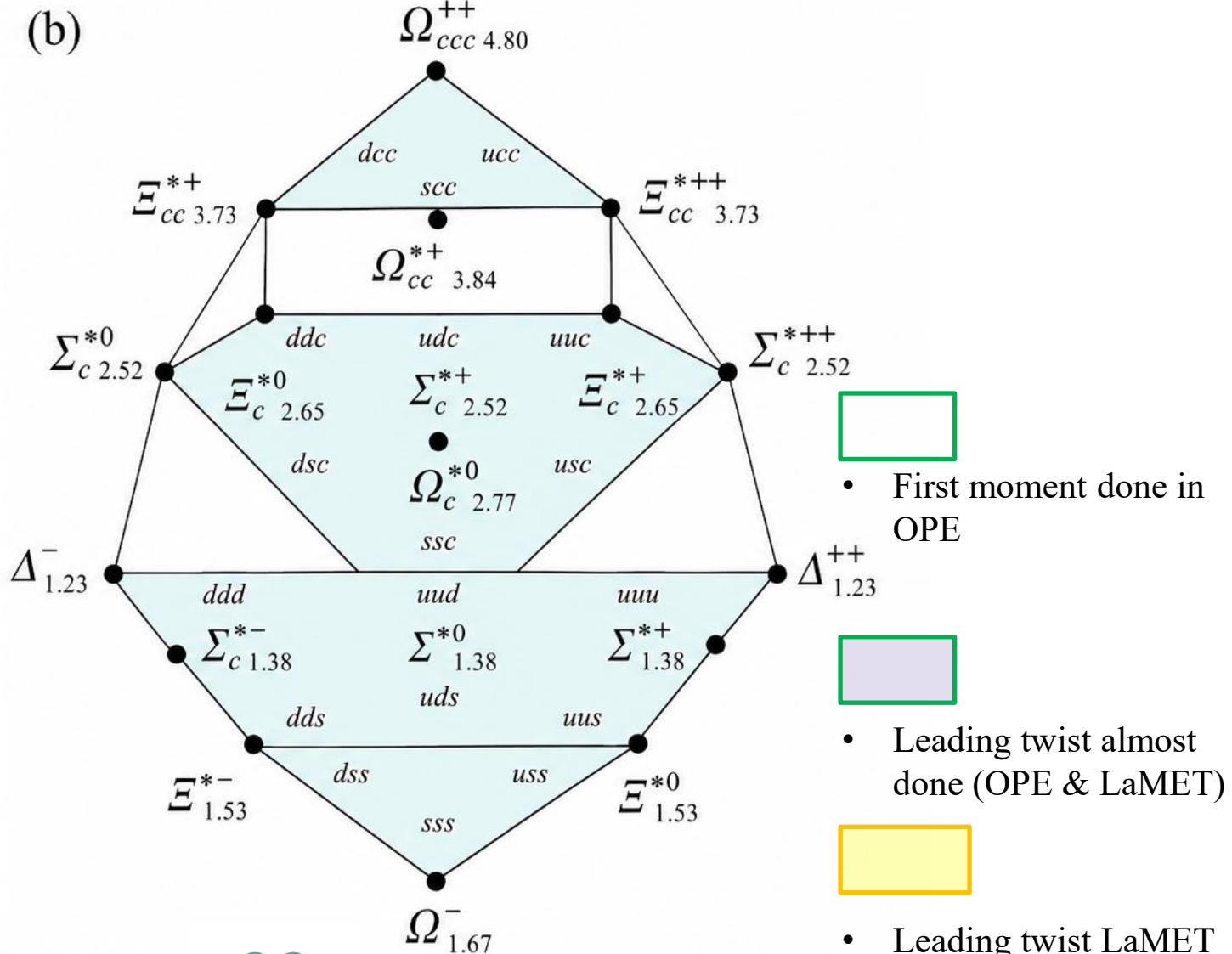
(LPC) arXiv:2606.29597, 2606.30387



Light Meson Map — LaMET



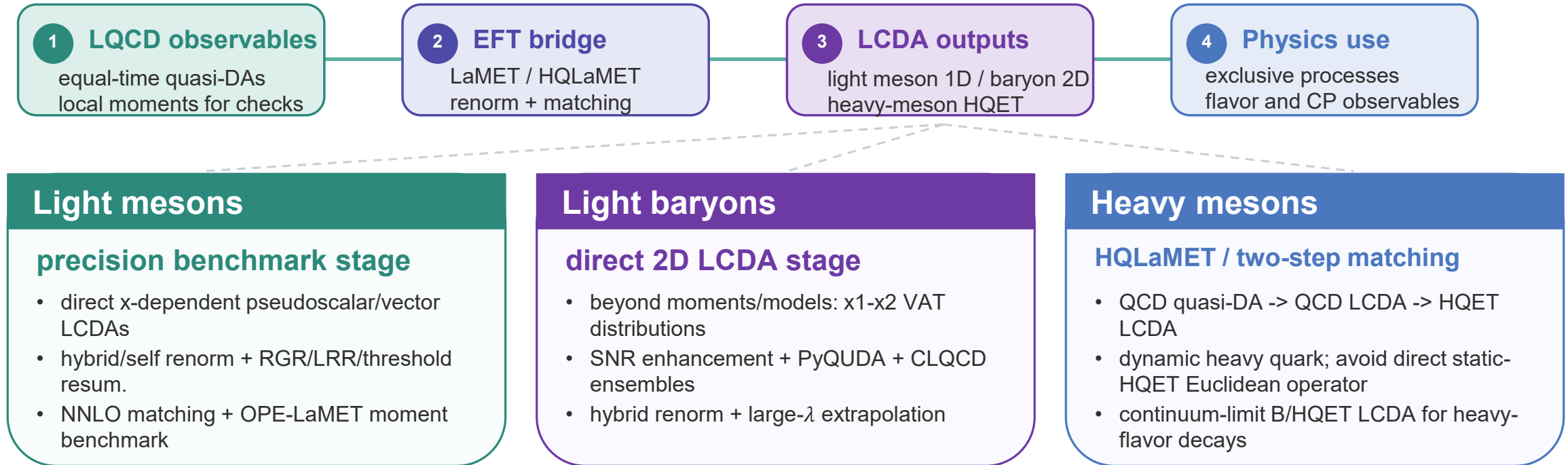
Light Baryon Map — LaMET



20

- Leading twist LaMET in Plan/Process

Summary & Outlook



Long-term goal: *From individual LCDAs to a systematic first-principles LCDA atlas*

Outlook

Precision

multi- $a/m_\pi/P_z$ data; higher-order matching; stable tails/endpoints

Wider LCDA map

light mesons, baryon octet, heavy mesons and shape functions

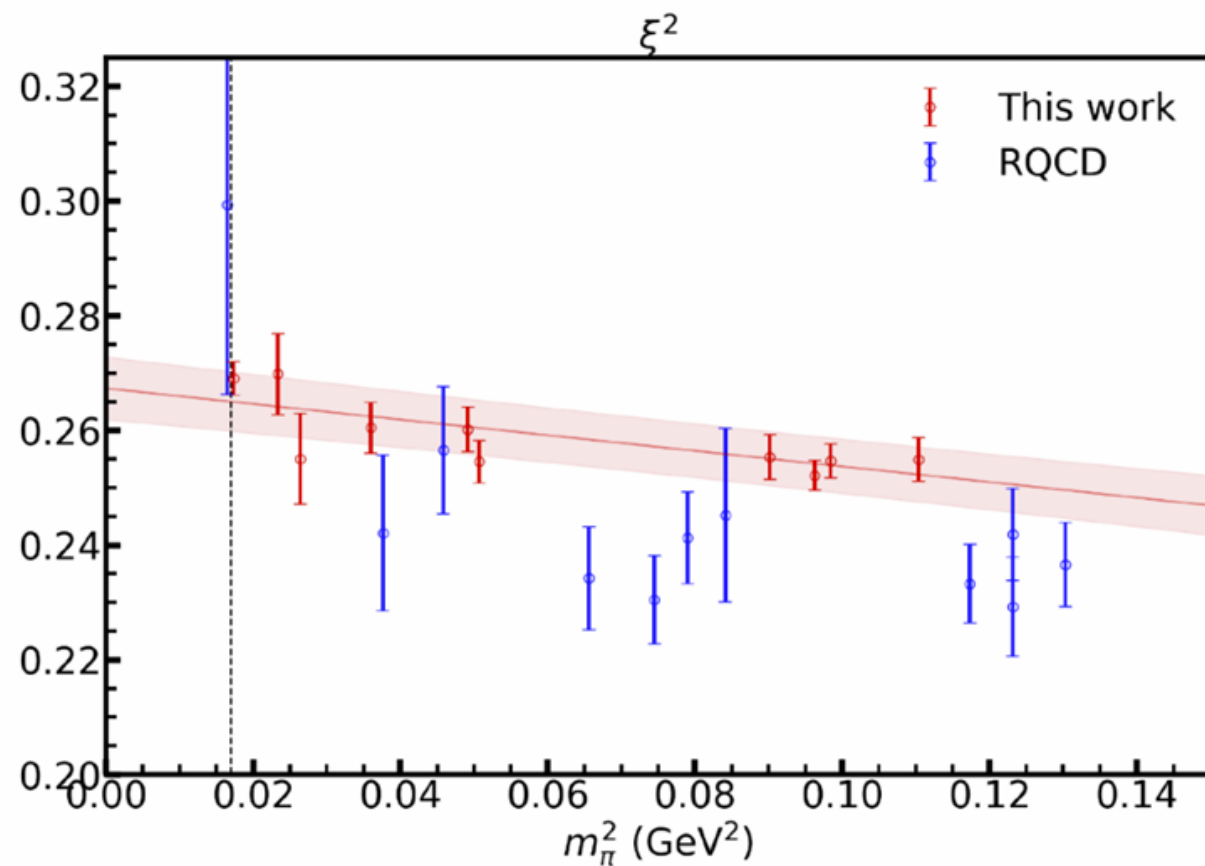
Phenomenology

exclusive form factors; B-decay uncertainties; flavor/CP tests

21 Thanks for Your Attention !

Back Up

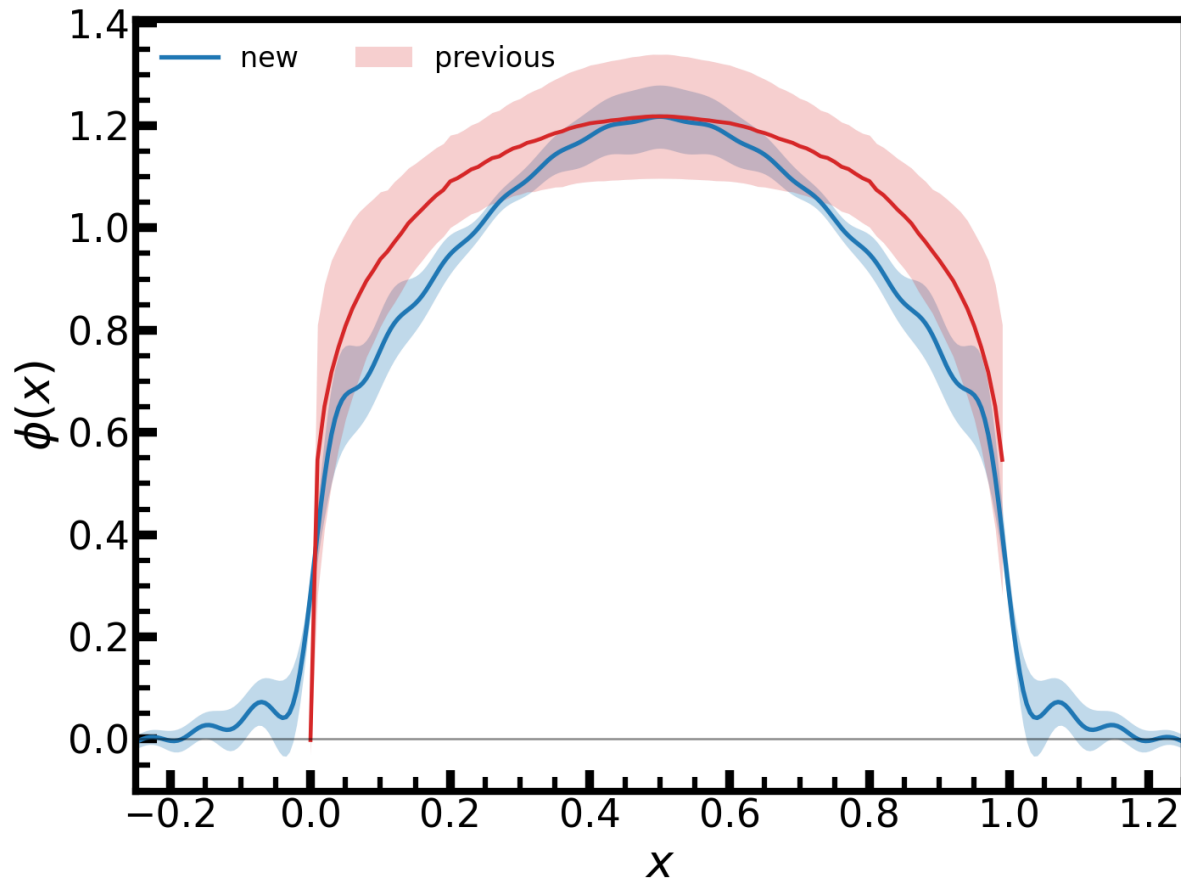
Pion mass extrapolation for $\pi(\xi_1)$:



Physical point problem within RQCD

Back Up

Pion



- **LaMET vs OPE 2022**

$$a_{2,\pi}^{\text{LaMET,intgral}} = 0.258(70)(52)$$

$$a_{2,\pi}^{\text{OPE(RQCD)}} = 0.116_{-20}^{+19}$$

- **LaMET vs OPE 2026**

$$a_{2,\pi}^{\text{LaMET,derviative}} = 0.174(62)$$

$$a_{2,\pi}^{\text{LaMET,intgral}} = 0.186(46)$$

$$a_{2,\pi}^{\text{OPE}} = 0.187(15)$$

Consistent