The transverse-spin asymmetry $A_{UUT}^{\sin\phi_{S_\Lambda}}$ in Λ hyperon production in SIDIS within the collinear framework

杨永亮

青岛大学物理科学学院

In collaboration with Zhun Lu

中国物理学会高能物理分会第十二届全国会员代表大会暨学术年会

7.13-19, 2026. Guangzhou, Guangdong Province

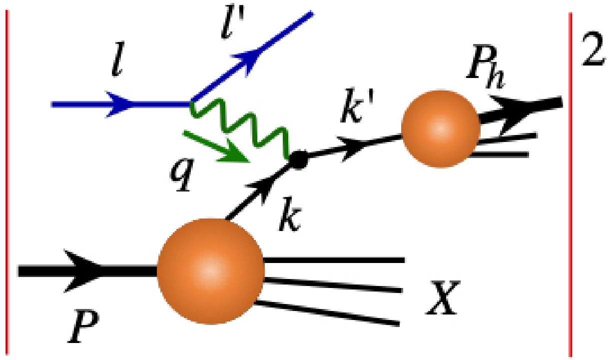
Outline

- Fragmentation and motivation
- Model information for the twist-3,T-odd FF $\widetilde{D}_T$
- Prediction on the $sin\phi_{S_\Lambda}$ asymmetry of Λ production in SIDIS
- Conclusion

Fragmentation functions

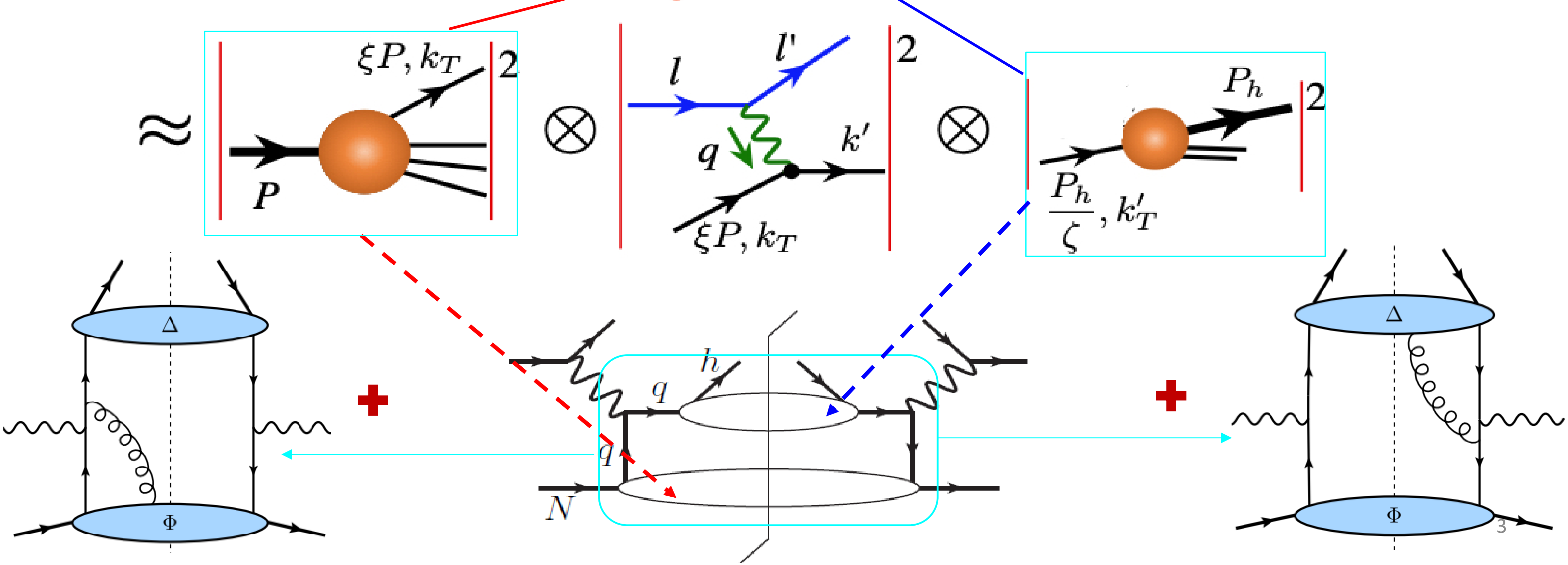
- ➢ 碎裂函数（FFs）描述部分子的强子化过程;
- ➢ SIDIS中的强子产生:

$\sigma_{\mathrm{SIDIS}} \propto$



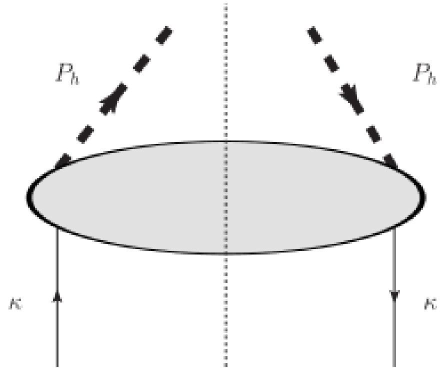
$h = \pi, K, p, \Lambda \ldots$

$$d\sigma \sim d\hat{\sigma} \otimes PDF \otimes FF$$



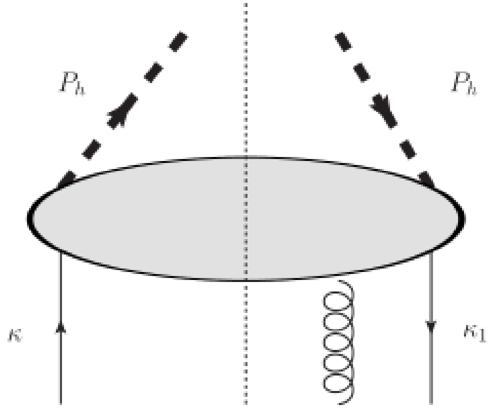
Fragmentation functions

qq关联函数:

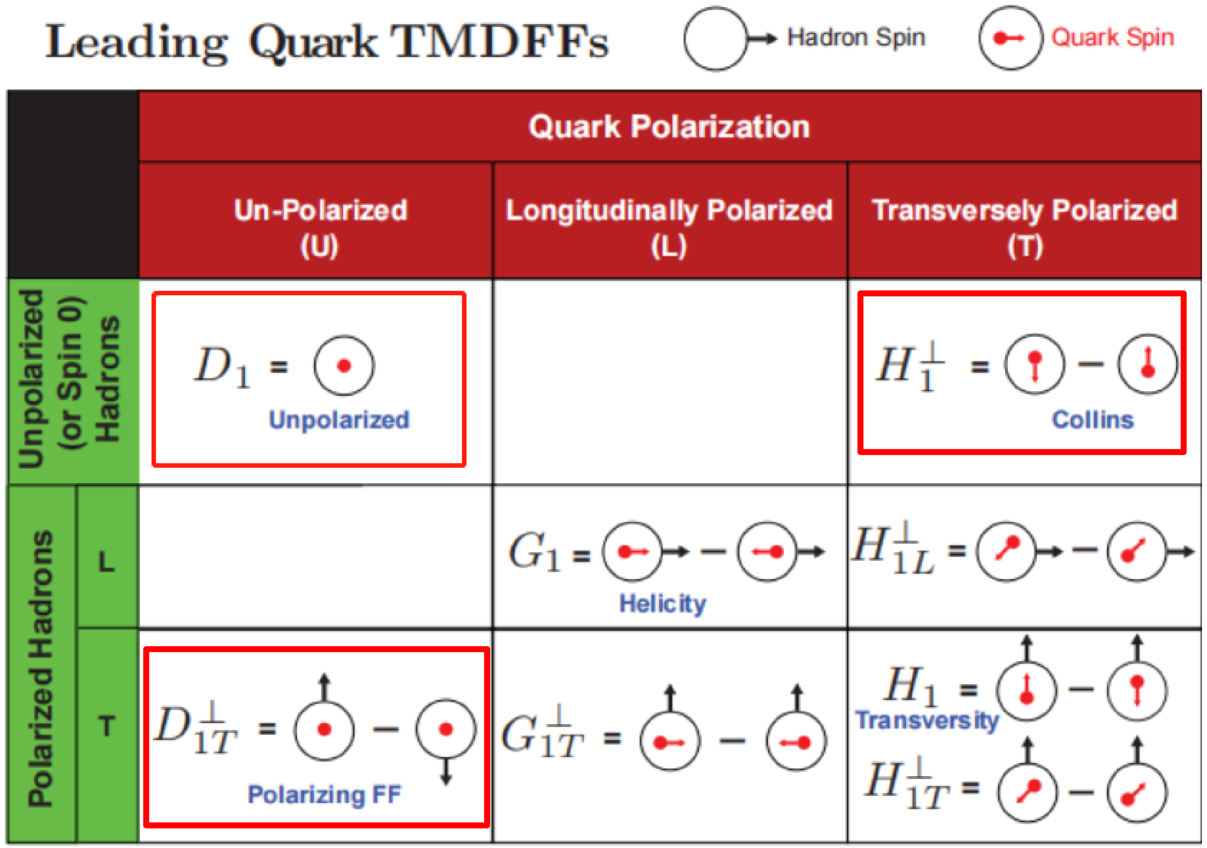


- 相比D_1、$D_{1T}^{\perp}$ 和 Collins function, twist-3的夸克–胶子–夸克相关碎裂函数仍缺乏有效信息。

qgq关联函数:

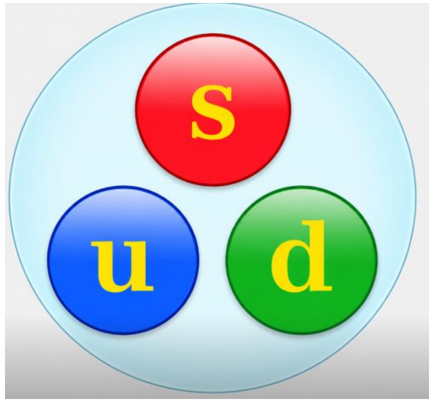


领头阶碎裂函数的几率解释



Motivation

- Λ 超子：$|uds\rangle$.



Y.Yang, Z. Lu; Phys. Rev. D 95, 074026 (2017)

- 在SIDIS中，一个结构函数主要由$f_1^q(x)\tilde{D}_T^{q\to\Lambda}(z)$ 表示

$$F_{UUT}^{\sin\phi_{S_\Lambda}}(x,z) = x\sum_a e_a^2 \frac{2M_\Lambda}{Q} f_1(x)\frac{\tilde{D}_T(z)}{z},$$

- $\tilde{D}_T$来源于 twist-3 自旋相关的夸克–胶子–夸克关联函数$\tilde{\Delta}_{A\alpha}$,可在横向极化Λ产生过程中引起横向自旋相关的不对称度：

$$A_{UUT}^{\sin\phi_{S_\Lambda}}$$

- 该不对称度能够同时探测 twist-3 碎裂机制以及质子中的s夸克结构。

Model information for the twist-3,T-odd FF $\widetilde{D}_T$

- 夸克–胶子–夸克碎裂关联函数$\widetilde{\Delta}_{A\alpha}$

$$\tilde{\Delta}_A^{\alpha}(z,k_T;S_\Lambda)=\sum_X \frac{1}{2z}\int\frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3}\int e^{ik\cdot\xi}\langle 0|\int_{\pm\infty^+}^{\xi^+} d\eta^+ \mathcal{U}^{\boldsymbol{\xi}_T}_{(\infty^+,\eta^+)}$$

$$\times\, gF_\perp^{-\alpha}(\eta)\mathcal{U}^{\boldsymbol{\xi}_T}_{(\eta^+,\xi^+)}\psi(\xi)|P_\Lambda,S_\Lambda;X\rangle\langle P_\Lambda,S_\Lambda;X|\bar{\psi}(0)\mathcal{U}^{\boldsymbol{0}_T}_{(0^+,\infty^+)}\mathcal{U}^{\infty^+}_{(\boldsymbol{0}_T,\boldsymbol{\xi}_T)}|0\rangle\Big|_{\substack{\eta^-=\xi^-=0\\ \eta_T=\xi_T}}$$

- $\widetilde{D}_T$的关系式:

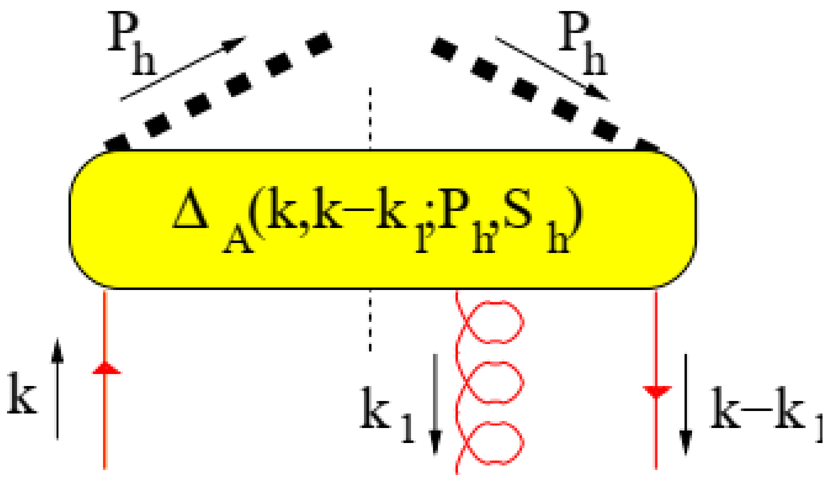
$$\tilde{D}_T=\tilde{D}'_T-\frac{k_T^2}{2M_\Lambda^2}\tilde{D}_T^\perp$$

- $\widetilde{D}'_T(z,k_T^2)$ 和$\widetilde{D}_T^\perp(z,k_T^2)$ 来自于:

$$\frac{z}{2M_\Lambda}\mathrm{Tr}[\tilde{\Delta}_{A\alpha}(g_T^{a\alpha}-i\epsilon_T^{a\alpha}\gamma_5)\gamma^-]=\epsilon^{aS_T}(\tilde{D}'_T+i\tilde{G}'_T)-\frac{k_T\cdot S_T}{M_\Lambda^2}\epsilon^{ak_T}(\tilde{D}_T^\perp+i\tilde{G}_T^\perp)$$

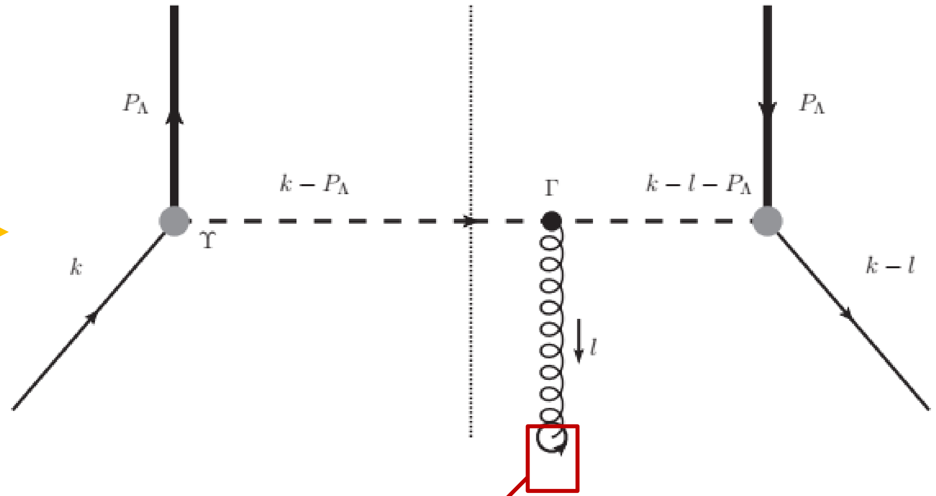
Diquark spectator model

➢ 图像：



(qgq碎裂关联函数)

Feynman diagram



$$F^{\alpha\beta} \to (l^\alpha g_T^{\rho\beta} - l_T^\alpha g^{\beta\rho})$$

➢ 矩阵元：

$$\langle P_\Lambda, S_\Lambda; X|\bar{\psi}(0)|0\rangle = \begin{cases} \bar{U}(P_\Lambda, S_\Lambda)\Upsilon_s \frac{i}{\not{k}-m_q} & \text{scalar diquark,} \\ \bar{U}(P_\Lambda, S_\Lambda)\Upsilon_v^\mu \frac{i}{\not{k}-m_q}\varepsilon_\mu & \text{axial-vector diquark} \end{cases}$$

Diquark spectator model

- 标量和轴矢量的耦合顶角:

$$
\begin{cases}
\Upsilon_s = \mathbf{1} g_s(k^2)\,, \\
\Upsilon_v^\mu = \dfrac{g_v(k^2)}{\sqrt{3}}\gamma_5\left(\gamma^\mu + \dfrac{P_\Lambda^\mu}{M_\Lambda}\right)
\end{cases}
$$

- 极化求和:

$$d^{\mu\nu} = \sum_{\lambda_a}\epsilon_{\lambda_a}^{\mu}\epsilon_{\lambda_a}^{\nu} = -g^{\mu\nu} + \frac{P_\Lambda^\mu P_\Lambda^\nu}{M_\Lambda^2}$$

- 胶子-双夸克耦合顶角Γ:

$$
\begin{aligned}
\Gamma^\rho &= i\,(2k - 2P_\Lambda - l)^\rho; \\
\Gamma^{\rho\mu\nu} &= -\, i\left[(2k - 2P_\Lambda - l)^\rho\, g^{\mu\nu} - (k - P_\Lambda - l)^\mu g^{\nu\rho} - (k - P_\Lambda)^\nu g^{\rho\mu}\right.
\end{aligned}
$$

- 表达式:

$$\tilde{\Delta}_A^\alpha = \frac{C_F\alpha_S}{4(2\pi)^2 2(1-z)P_\Lambda^-}\frac{1}{k^2-m^2}\int\frac{d^4l}{(2\pi)^4}\frac{(l^- g_T^{\alpha\rho} - l^\alpha n_+^\rho)(\not{k} - \not{l} + m)\bar{\mathcal{Y}}^\nu(\not{P}_\Lambda + M_\Lambda)(\gamma_5\not{S}_\Lambda)\mathcal{Y}^\mu(\not{k} + m)}{((k-l)^2 - m^2)(l^2 - i\epsilon)((k - l - P_\Lambda)^2 - m_s^2)(-l^- \pm i\epsilon)}d^{i\mu}d^{j\nu}\bar{\Gamma}_{ij\rho}$$

The model calculation of the FF $\widetilde{D}_T(z, \boldsymbol{k}_T^2)$

T-odd FF $\widetilde{D}_T(z, \boldsymbol{k}_T^2)$ 来自于散射振幅的虚部：

- 函数虚部 ⟵ Cutkosky cut：

$$\frac{1}{l^2 + i\varepsilon} \to -2\pi i\delta(l^2),$$

$$\frac{1}{(k-l)^2 - m^2 + i\varepsilon} \to -2\pi i\delta((k-l)^2 - m^2).$$

- SU(6) spin-flavor symmetry

$$D^{u\to\Lambda} = D^{d\to\Lambda} = \frac{1}{4}D^{(S)} + \frac{3}{4}D^{(V)}, \quad D^{s\to\Lambda} = D^{(S)}$$

- 模型结果：

Scalar part:

$$\tilde{D}_T^S(z, \boldsymbol{k}_T^2) = \frac{C_F\alpha_S}{4\,M_\Lambda(2\pi)^2(1-z)}\frac{1}{k^2 - m^2}\tilde{D}_{TS}(z, \boldsymbol{k}_T^2)$$

$$\tilde{D}_{TS}(z, \boldsymbol{k}_T^2) = 2g_s^2(z\,m + M_\Lambda)\{\mathcal{A}[M_\Lambda^2(z-1) - zm_D^2] + \mathcal{B}M_\Lambda^2(z-1) - \boldsymbol{k}_T^2 z[\mathcal{C}k^-(1-z) + \mathcal{I}_2]\}$$

The model calculation of the FF $\widetilde{D}_T(z, \boldsymbol{k}_T^2)$

$$\tilde{D}_T^V(z,\boldsymbol{k}_T^2) = \frac{z\,C_F\alpha_S}{4\,M_\Lambda(2\pi)^2(1-z)}\frac{1}{k^2-m^2}\{\tilde{D}_{TV}(z,\boldsymbol{k}_T^2)+\tilde{D}_{TV0}(z,\boldsymbol{k}_T^2)+\tilde{D}_{TV1}(z,\boldsymbol{k}_T^2)+\tilde{D}_{TV2}(z,\boldsymbol{k}_T^2)\}$$

axial-vector part:

$$\tilde{D}_{TV} = -\frac{2g_v^2}{3z}(z\,m+M_\Lambda)\{\mathcal{A}[M_\Lambda^2(z-1)-zm_D^2]+\mathcal{B}M_\Lambda^2(z-1)-\boldsymbol{k}_T^2 z[\mathcal{C}k^-(1-z)+\mathcal{I}_2]\}$$

$$\begin{aligned}\tilde{D}_{TV0} =& \frac{2g_v^2}{3M_\Lambda}\{(k\cdot P_\Lambda+mM_\Lambda)[(\mathcal{A}\mathcal{A}+z\mathcal{A}\mathcal{B})k\cdot P_\Lambda+M_\Lambda^2(\mathcal{A}\mathcal{B}+z\mathcal{B}\mathcal{B})+z\mathcal{W}_1]+\boldsymbol{k}_T^2M_\Lambda^2(\mathcal{A}\mathcal{A}+z\mathcal{A}\mathcal{B})\\ &-(\mathcal{A}+z\mathcal{B})[2mM_\Lambda k\cdot P_\Lambda+(k\cdot P_\Lambda)^2+M_\Lambda^2(m^2+2\boldsymbol{k}_T^2)]\}\end{aligned}$$

$$\begin{aligned}\tilde{D}_{TV1} =& \frac{g_v^2}{3M_\Lambda}\Big\{M_\Lambda\{z[-\boldsymbol{k}_T^2(m\mathcal{A}\mathcal{A}+2M_\Lambda\mathcal{A}\mathcal{B}-4m\mathcal{A}-2M_\Lambda\mathcal{B})+(M_\Lambda\mathcal{B}-m\mathcal{I}_2)(k^2+3m^2)+2m\mathcal{W}_1]\\ &+2M_\Lambda[-\mathcal{A}\mathcal{A}\boldsymbol{k}_T^2+3\mathcal{A}\boldsymbol{k}_T^2+M_\Lambda(M_\Lambda\mathcal{B}\mathcal{B}-m\mathcal{B})+3\mathcal{W}_1]\}\\ &+k\cdot P_\Lambda\{z[-\mathcal{A}\mathcal{A}\boldsymbol{k}_T^2+(\mathcal{A}-\mathcal{I}_2)(k^2+3m^2)-2\mathcal{W}_1]+2k\cdot P_\Lambda(\mathcal{A}\mathcal{A}-3\mathcal{A}+2\mathcal{I}_2)\\ &+2M_\Lambda(2\mathcal{A}\mathcal{B}M_\Lambda-\mathcal{A}m-3\mathcal{B}M_\Lambda+2\mathcal{I}_2m)\}\Big\}\end{aligned}$$

$$\begin{aligned}\tilde{D}_{TV2} =& -\frac{g_v^2}{3M_\Lambda^2}\Big\{\frac{z^2}{2}\{-\boldsymbol{k}_T^2[\mathcal{C}\mathcal{C}k^-(k^2-m^2)(m+M_\Lambda)+2mM_\Lambda(\mathcal{C}\mathcal{D}k^-M_\Lambda-2\mathcal{C}k^-m)-m\frac{(k^2-m^2)}{2k^2}\mathcal{I}_1]\\ &+\mathcal{W}_2k^-(k^2-m^2)(m+M_\Lambda)+m\frac{k^2-m^2}{2}\mathcal{I}_1\}\\ &+M_\Lambda^2z[-\boldsymbol{k}_T^2(2\mathcal{C}\mathcal{C}k^-(zm+M_\Lambda)+\mathcal{C}\mathcal{D}k^-M_\Lambda z)+4\mathcal{W}_2k^-(zm+M_\Lambda)]\\ &+z\{-\boldsymbol{k}_T^2\mathcal{C}\mathcal{E}k^-k^-z(M_\Lambda-zm)-k\cdot P_\Lambda[4\mathcal{W}_2k^-(M_\Lambda+zm)+\boldsymbol{k}_T^2(2\mathcal{C}k^-zm-\mathcal{C}\mathcal{C}k^-(3zm+M_\Lambda))]\}\Big\}\end{aligned}$$

Fit model parameters

- 利用 $D_1(z)$ 和 $G_1(z)$ 的模型结果： Y. Yang, Z. Lu and I. Schmidt, Phys. Rev. D 96(3), 034010 (2017)

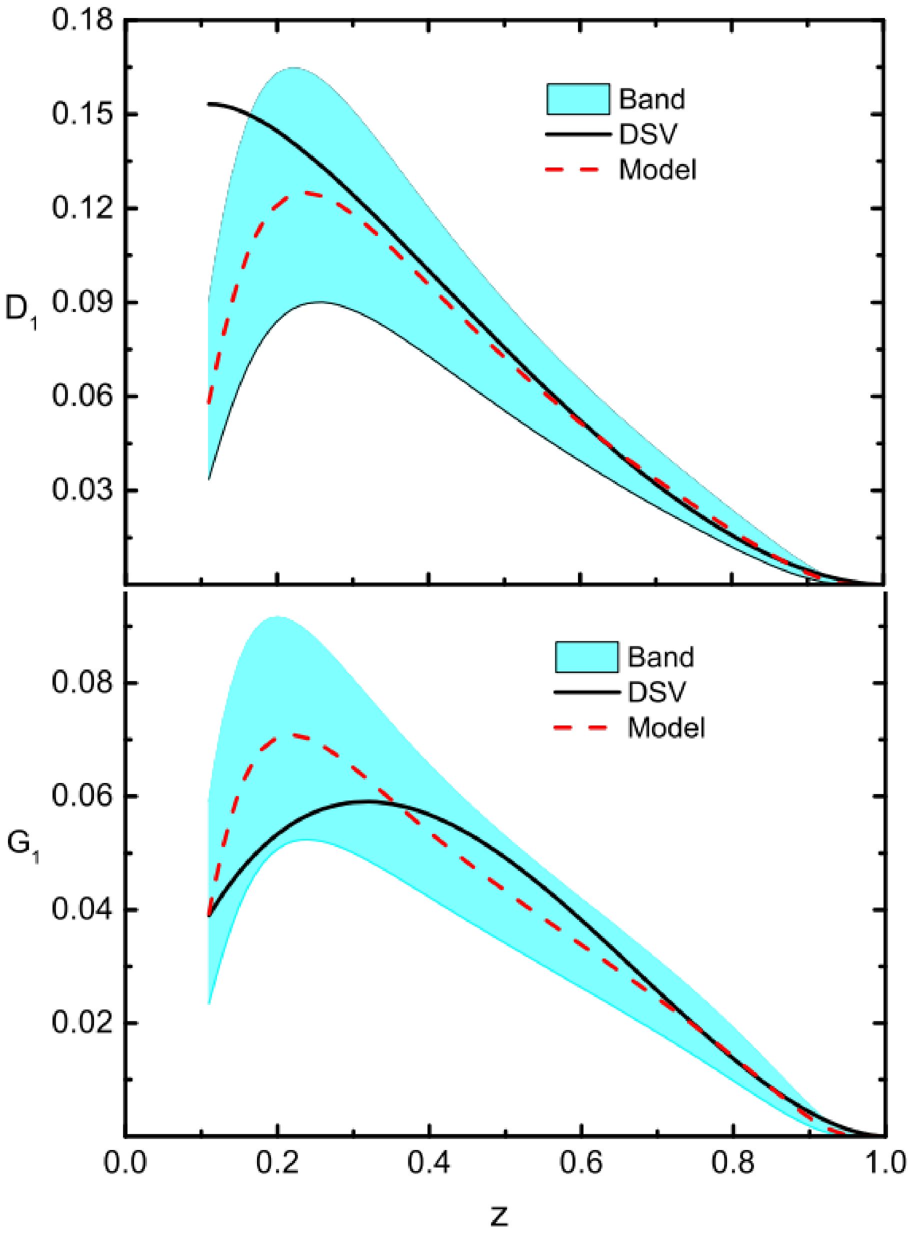
$$
\begin{aligned}
D_1^\Lambda(z) = {} & \frac{g_s^2}{4(2\pi)^2}\frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4L^2}\Bigg\{z(1-z)((m_q+M_\Lambda)^2-m_D^2) \\
& \times \exp\left(\frac{-2zL^2}{(1-z)\Lambda^2}\right) \\
& +((1-z)\Lambda^2-2((m_q+M_\Lambda)^2-m_D^2)) \\
& \times \frac{z^2L^2}{\Lambda^2}\Gamma\left(0,\frac{2zL^2}{(1-z)\Lambda^2}\right)\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
G_1^\Lambda(z) = {} & \frac{g_s^2}{4(2\pi)^2}\frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4L^2}\Bigg\{(1-z)[M_\Lambda^2(2-z)+2zm_qM_\Lambda \\
& +z(m_D^2+m_q^2(2z-1))]\exp\left(\frac{-2zL^2}{(1-z)\Lambda^2}\right) \\
& -z[2M_\Lambda^2(2-z)+4zm_qM_\Lambda \\
& +z((1-z)\Lambda^2+2(m_D^2+m_q^2(2z-1)))] \\
& \times\frac{L^2}{\Lambda^2}\Gamma\left(0,\frac{2zL^2}{(1-z)\Lambda^2}\right)\Bigg\}.
\end{aligned}
$$

$$\text{where } \Lambda^2 = \lambda^2 z^\alpha (1-z)^\beta$$

- 与LO-DSV 参数化拟合， 初始能标：$\mu_{LO}^2 = 0.23\ GeV^2$

Fit model parameters



- 对D_1^{Λ}: $D_1^{u\to\Lambda} = D_1^{d\to\Lambda} = D_1^{s\to\Lambda} \equiv D_1^{\Lambda}.$

- 只有$G_1^{s\to\Lambda}$：（DSV’ scenario-1）

$$G_{1L}^{\mathrm{u}\to\Lambda}(z, k_T^2) = 0, \qquad G_{1L}^{\mathrm{d}\to\Lambda}(z, k_T^2) = 0,$$

$$G_{1L}^{\mathrm{s}\to\Lambda}(z, k_T^2) = G_{1L}^{(s)}(z, k_T^2).$$

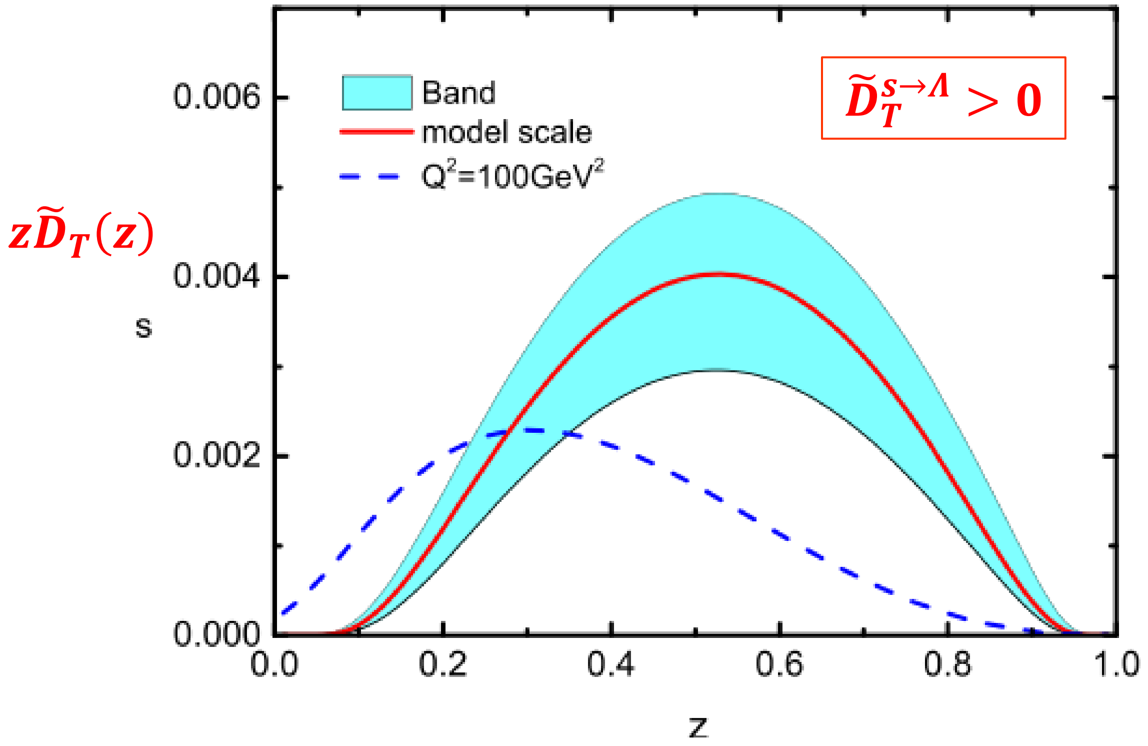
  - masses of u, d, s： $m = 0.36\ GeV$;
  - Λ hyperon mass to be $1.116\ GeV$;

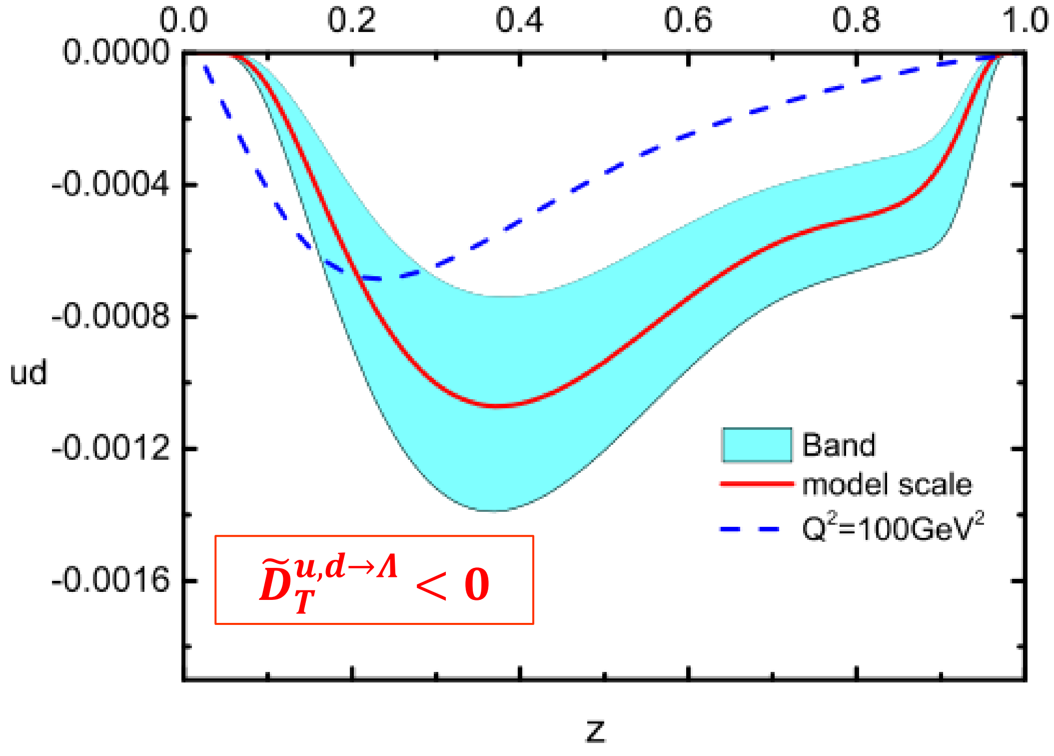
- 拟合模型参数:

g_s	λ (GeV)	m_D (GeV)	m (GeV)	α	β
$2.83^{+0.12}_{-0.1}$	$3.567^{+0.23}_{-0.21}$	$0.397^{+0.03}_{-0.03}$	0.36 (fixed)	$0.2^{+0.04}_{-0.04}$	$0.52^{+0.08}_{-0.08}$

Numerical result for $\widetilde{D}_T$

共线函数 $\widetilde{D}_T(z) = z^2 \int d^2\boldsymbol{k}_T\, \widetilde{D}_{\mathrm{T}}(z, \boldsymbol{k}_T^2)$





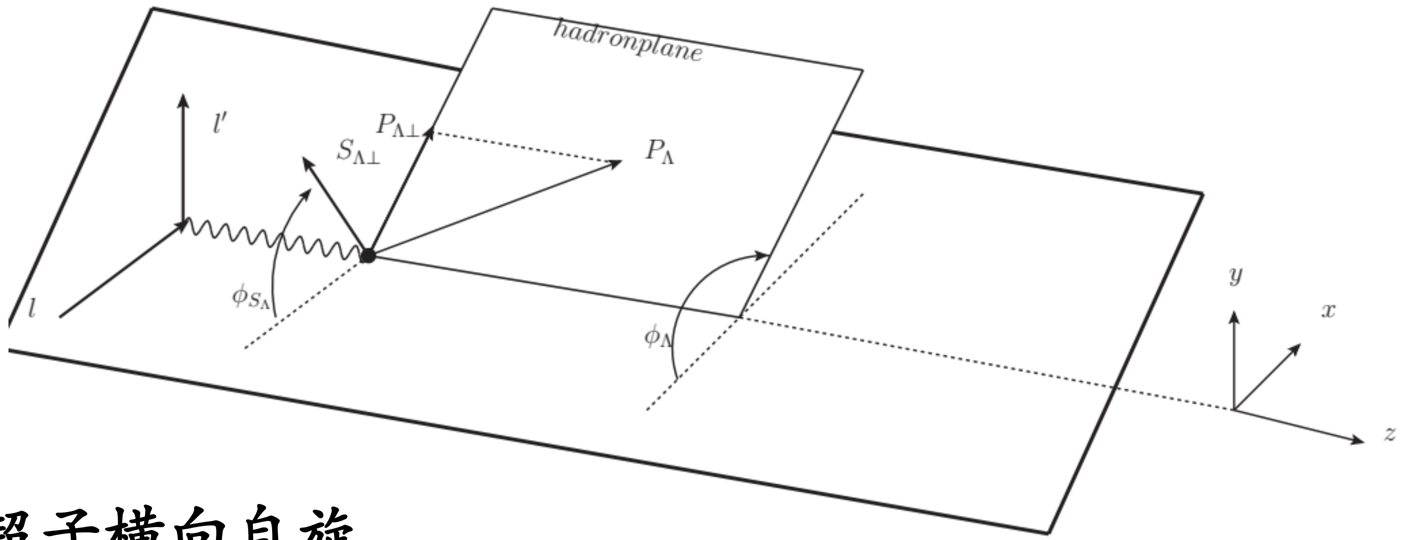
$\alpha_s(Q_0^2) = 0.817, \mu_0^2 = 0.23\ GeV^2$.

- $\boldsymbol{u(d)} \to \boldsymbol{\Lambda}$: 负值，小的，包含标量和轴矢量贡献
- $\boldsymbol{s} \to \boldsymbol{\Lambda}$: 正值，振幅大于$\boldsymbol{u(d)}$，只有标量结果贡献
- QCD演化从$Q^2 = 0.23 \to 100\ GeV^2$，采取与$D_1$相同的演化核近似

Prediction on the $sin\phi_{S_\Lambda}$ asymmetry of Λ production in SIDIS

SIDIS 过程中的横向极化 Λ超子产生

$$\ell + N \rightarrow \ell' + \Lambda^{\uparrow} + X$$



- 方位角 ϕ_{S_Λ}表示轻子散射平面与 Λ超子横向自旋方向之间的夹角。

$$\sin\phi_{S_\Lambda} = -\frac{l_\mu S_{\Lambda\nu}\epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_{\Lambda\perp}^2}} .$$

- 散射截面中的变量

$$x = \frac{Q^2}{2P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot l}, \qquad z = \frac{P\cdot P_h}{P\cdot q},$$

$$\gamma = \frac{2Mx}{Q}, \qquad W^2 = (P+q)^2, \qquad s = (P+l)^2$$

Prediction on the $sin\phi_{S_\Lambda}$ asymmetry of Λ production in SIDIS

- z-和x-相关的$sin\phi_{S_\Lambda}$不对称度:

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2 y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2 y^2}.$$

$$\begin{cases} A_{UUT}^{\sin\phi_{S_\Lambda}}(z) = \dfrac{\int dx \int dy \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})\sqrt{2\varepsilon(1+\varepsilon)}F_{UUT}^{\sin\phi_{S_\Lambda}}(x,z)}{\int dx \int dy \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})F_{UUU}(x,z)} \\ A_{UUT}^{\sin\phi_{S_\Lambda}}(x) = \dfrac{\int dy \int dz \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})\sqrt{2\varepsilon(1+\varepsilon)}F_{UUT}^{\sin\phi_{S_\Lambda}}(x,z)}{\int dy \int dz \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})F_{UUU}(x,z)}, \end{cases}$$

- 其中的结构函数分别为

$$F_{UUU}(x,z) = x\sum_q e_q^2 \boxed{f_1^q(x)D_1^q(z)},$$

$$F_{UUT}^{\sin\phi_{S_\Lambda}}(x,z) = x\sum_q e_q^2 \frac{2M_\Lambda}{Q}\boxed{f_1^q(x)\frac{\tilde{D}_T^q(z)}{z}},$$

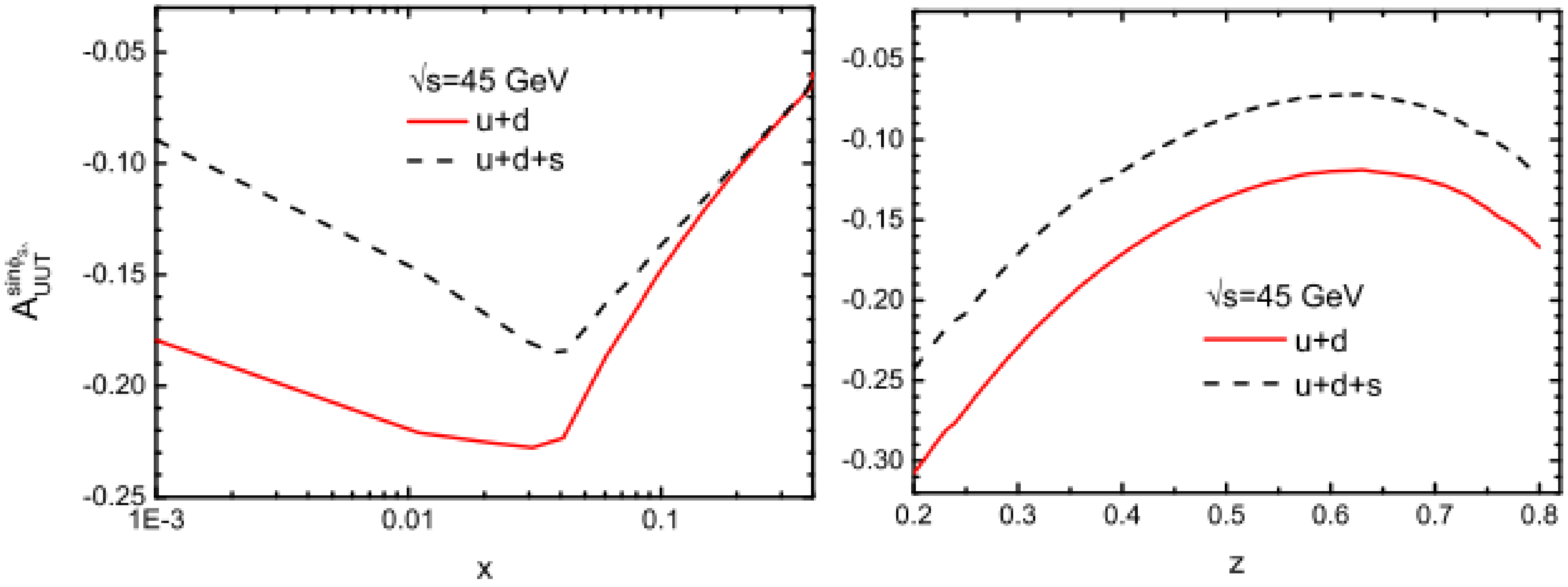
- 分子分母中都包含PDF:$f_1^q(x)$.
- twist-3 结构函数包含$\frac{2M_\Lambda}{Q}$因子.

Predictions at EIC

EIC,运动学区域：

$$0.001 < x < 0.4, \quad 0.01 < y < 0.95, \quad 0.2 < z < 0.8,$$
$$Q^2 > 1\mathrm{GeV}^2, \quad \sqrt{s} = 45\ \mathrm{GeV}, \quad W > 5\ \mathrm{GeV}.$$

x-和z-不对称度的数值结果：



- 在x和z分布中，不对称度均为负且数值较大。
- 由于s夸克道的$\widetilde{D}_T$符号相反，计入s夸克贡献后，不对称度的绝对值减小。

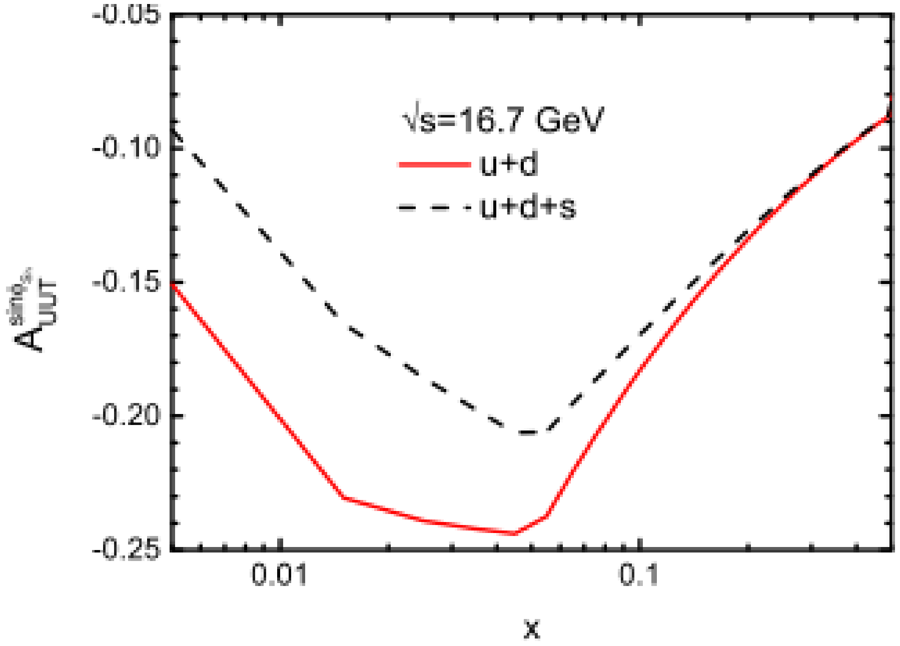
不对称度的绝对值在$x \sim 0.04$附近达到最大，随后随着x增大而减小

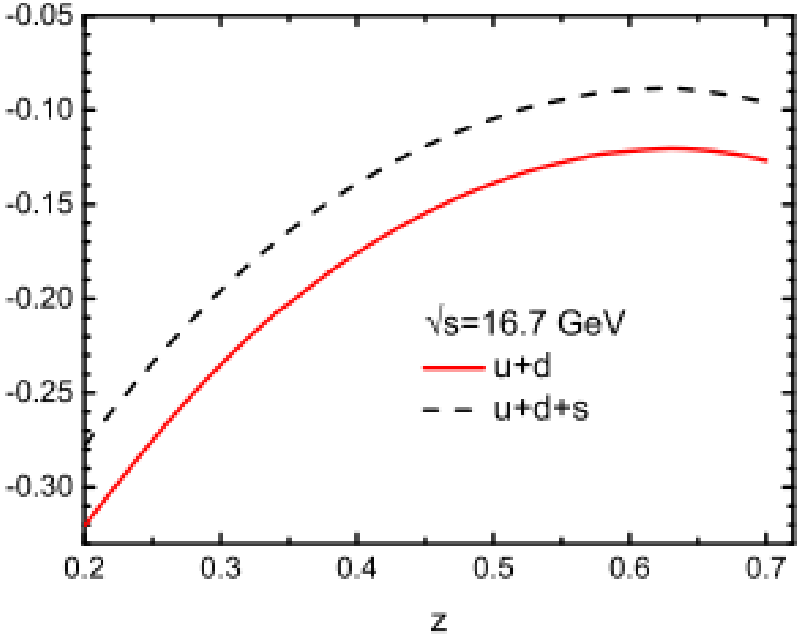
Predictions at EicC

EicC,运动学区域:

$$0.005 < x < 0.5, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.7,$$
$$Q^2 > 1\text{GeV}^2, \quad \sqrt{s} = 16.7\ \text{GeV}, \quad W > 2\text{GeV}.$$

x-和z-不对称度的数值结果:



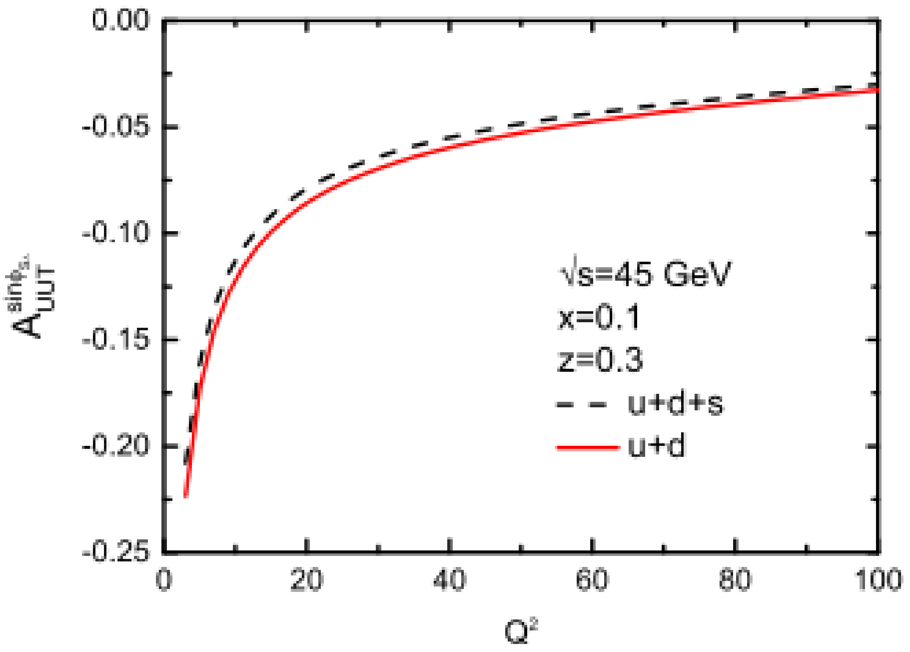


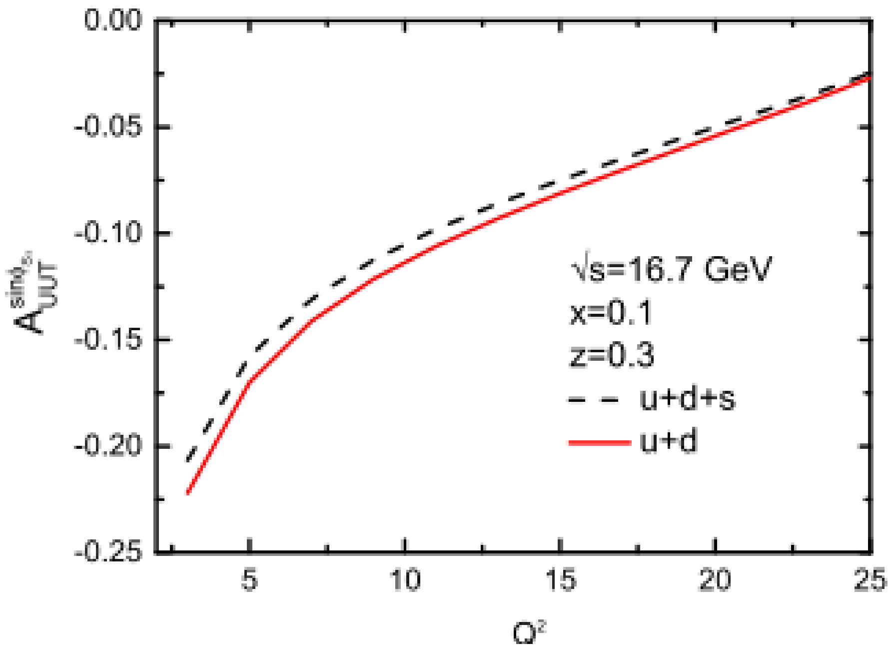
- 其符号和总体变化趋势与 EIC 情形一致。

尽管s夸克的 PDF f_1较小,但s夸克贡献仍会对结果产生明显影响。

Q^2 dependence

固定$x = 0.1$和$z = 0.3$，Q^2依赖的$A_{UUT}^{\sin\phi_{S_\Lambda}}$





- 由 $Q^2 = xys$ 可知，由于 EicC 的质心能量较低，其可探测的 Q^2范围更受限制，最高约为 25 GeV^2。

- 随着Q^2增大，不对称度的绝对值迅速减小。
- 这一变化主要由运动学因子$\frac{2M_\Lambda}{Q}$所引起。

Discussion of the results

➢ 在不对称度大小方面

- 圈图贡献的$\widetilde{D}_T$与强耦合常数α_s成正比。模型能标下：

$$\mu_0^2 = 0.23\ \mathrm{GeV}^2, \alpha_s(\mu_0^2) = 0.817.$$

- 对于Λ超子（$M_\Lambda \simeq 1.116\ \mathrm{GeV}$），twist-3 运动学因子$\frac{2M_\Lambda}{Q}$在低、中$Q$区域是弱抑制的。

➢ s夸克的影响

- 分子中的味道贡献由$f_1^q(x)\,\widetilde{D}_T^{q\to\Lambda}(z)$所决定，包含s夸克贡献。

- $s \to \Lambda$ 道会部分抵消 u, d 夸克道的负贡献，从而减小不对称度$A_{UUT}^{\sin\phi_{S_\Lambda}}$的绝对值。

Summary

- 利用双夸克旁观者模型，计算了 Λ超子的 twist-3 naive-T-odd qgq碎裂函数 $\widetilde{D}_T$。非零，且 $u(d)$夸克与 s夸克贡献的符号相反。
- 模型参数通过D_1和G_1进行拟合。近似采用与 D_1相同的演化核，对$\widetilde{D}_T$的 QCD 演化效应进行了分析。
- 在所考虑的x、z 和Q^2 区域内，EIC 和 EicC 上预测的$A_{UUT}^{\sin\phi_{S_\Lambda}}$ 不对称度均为负，并可达到较大的量级。其精确数值受到模型和演化方案不确定性的影响。
- s夸克道会对不对称度产生明显修正。在共线框架下，该过程还可能为探测质子中的s夸克分布函数 $f_1^s(x)$提供补充方案。

THANKS!

Backup

The functions $A, B, C, D, E,$ AA, AB, BB, CC, CD, CE, and W_1 come from the following double-l integral

$$\int d^4l \frac{l^\mu \, \delta(l^2)\, \delta((k-l)^2 - m^2)}{(k - P_h - l)^2 - m_s^2} = \mathcal{A}\, k^\mu + \mathcal{B}\, P_h^\mu$$

$$\int d^4l \frac{l^\mu \, \delta(l^2)\, \delta((k-l)^2 - m^2)}{((k - P_h - l)^2 - m_s^2)(-l \cdot n_+ + i\epsilon)} = \mathcal{C}\, k^\mu + \mathcal{D}\, P_h^\mu + \mathcal{E} n_+^\mu$$

$$\int d^4l \frac{l^\mu l^\nu \delta(l^2)\delta((k-l)^2 - m^2)}{((k - P_h - l)^2 - m_s^2)} = \mathcal{AA}k^\mu k^\nu + \mathcal{BB}P^\mu P^\nu + \mathcal{AB}(k^\mu P^\nu + P^\mu k^\nu) + g^{\mu\nu}\mathcal{W}_1$$

$$\int d^4l \frac{l^\mu l^\nu \delta(l^2)\delta((k-l)^2 - m^2)}{((k - P_h - l)^2 - m_s^2)(-l \cdot n_+ + i\epsilon)} = \mathcal{CC}_f k^\mu k^\nu + \mathcal{DD}_f P^\mu P^\nu + \mathcal{EE}_f n_+^\mu n_+^\nu$$
$$+ \mathcal{CD}_f (k^\mu P^\nu + P^\mu k^\nu) + \mathcal{CE}_f (k^\mu n_+^\nu + n_+^\mu k^\nu) + \mathcal{DE}_f (n_+^\mu P^\nu + P^\mu n_+^\nu) + g^{\mu\nu}\mathcal{W}_2.$$