Fragmentation Functions for the Production of P-wave Bc Mesons

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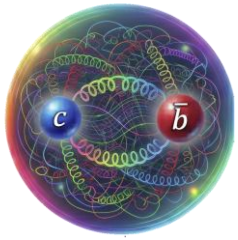
中国物理学会高能物理分会学术年会@ South China Normal University, 2026.07.16

1. Background



➢ Unique Properties of Bc family

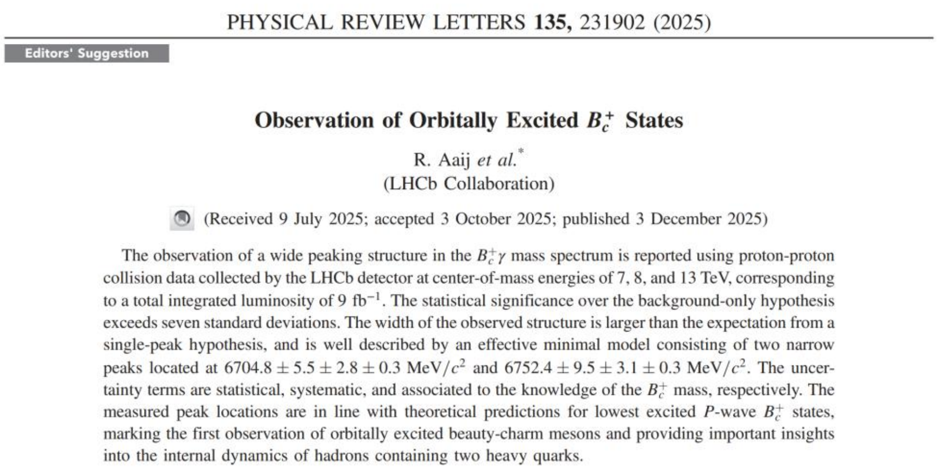
- The only meson system containing two different heavy flavors
- Possesses a rich spectrum of excited states cascading down via radiative or hadronic transitions

➢ Production Dynamics

- Can be systematically described within NRQCD factorization
- Provides a cleaner test for NRQCD, as its production strictly requires the simultaneous creation of two heavy-quark pairs ($c\bar{c}$ and $b\bar{b}$)

- ➢ Two structures consistent with orbitally excited Bc states have been observed.

1998: Bc ground state ⟶ **2019: 2S states** ⟶ **2025: orbital excitations**


PHYSICAL REVIEW LETTERS **135**, 231902 (2025)

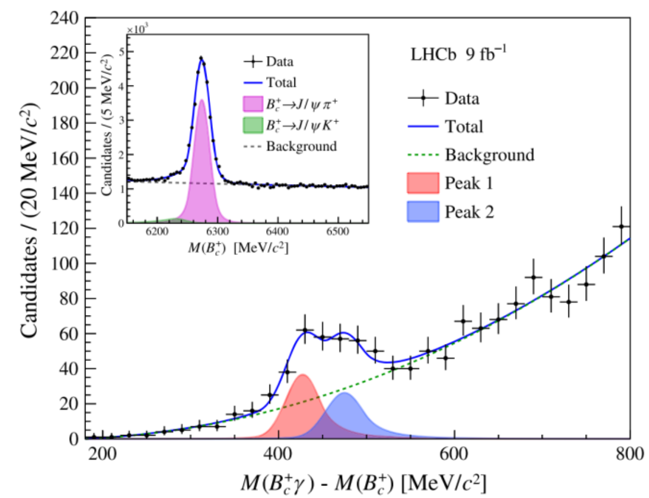
Editors' Suggestion

Observation of Orbitally Excited B_c^+ States

R. Aaij *et al.*[*]
(LHCb Collaboration)

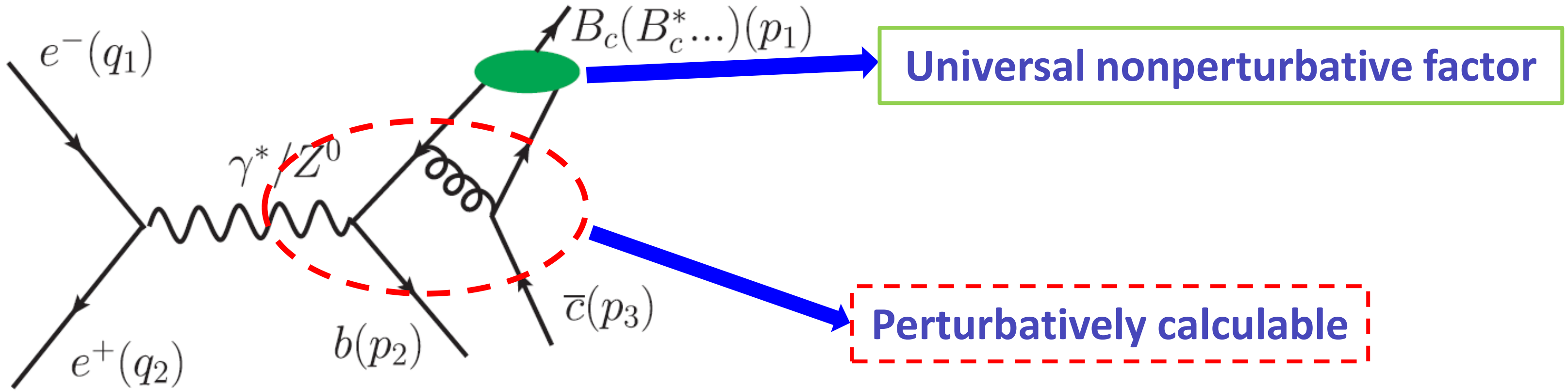
(Received 9 July 2025; accepted 3 October 2025; published 3 December 2025)

The observation of a wide peaking structure in the $B_c^+\gamma$ mass spectrum is reported using proton-proton collision data collected by the LHCb detector at center-of-mass energies of 7, 8, and 13 TeV, corresponding to a total integrated luminosity of 9 fb^{-1}. The statistical significance over the background-only hypothesis exceeds seven standard deviations. The width of the observed structure is larger than the expectation from a single-peak hypothesis, and is well described by an effective minimal model consisting of two narrow peaks located at $6704.8 \pm 5.5 \pm 2.8 \pm 0.3$ MeV$/c^2$ and $6752.4 \pm 9.5 \pm 3.1 \pm 0.3$ MeV$/c^2$. The uncertainty terms are statistical, systematic, and associated to the knowledge of the B_c^+ mass, respectively. The measured peak locations are in line with theoretical predictions for lowest excited P-wave B_c^+ states, marking the first observation of orbitally excited beauty-charm mesons and providing important insights into the internal dynamics of hadrons containing two heavy quarks.



Bc spectroscopy has entered the era of orbital excitations

- NRQCD factorization



$$d\sigma(e^+ + e^- \to Bc + b + \bar{c})$$
$$= \sum_n d\hat{\sigma}(e^+ + e^- \to c\bar{b}[n] + b + \bar{c}) \langle O^{Bc}(n) \rangle$$

NRQCD factorization

Short-distance coefficients

Long-distance matrix elements

- NRQCD factorization

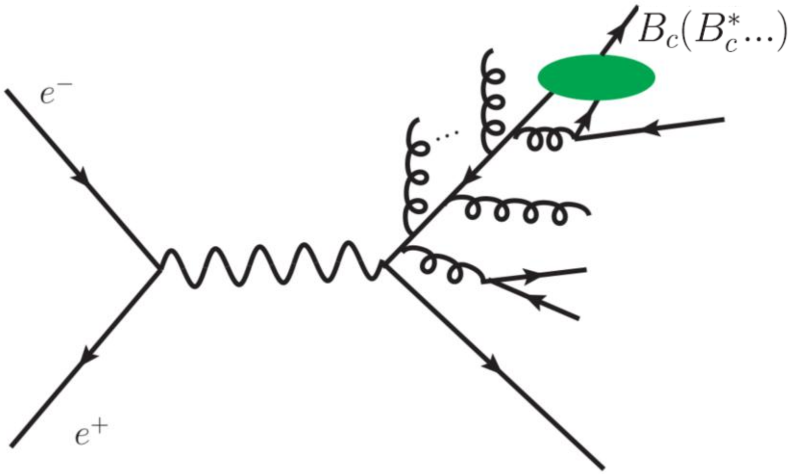
Energy scales: $\sqrt{s}, m_Q$

$$d\sigma(e^+ + e^- \to Bc + b + \bar{c})$$
$$= \sum_n d\sigma(e^+ + e^- \to (c\bar{b})[n] + b + \bar{c}) \langle O^{Bc}(n) \rangle$$

Log-terms appear in short-distance coefficients:

$$\boxed{\alpha_s^m \sum_{n=0}^{\infty} \alpha_s^n \ln^n(s/m_Q^2)}$$

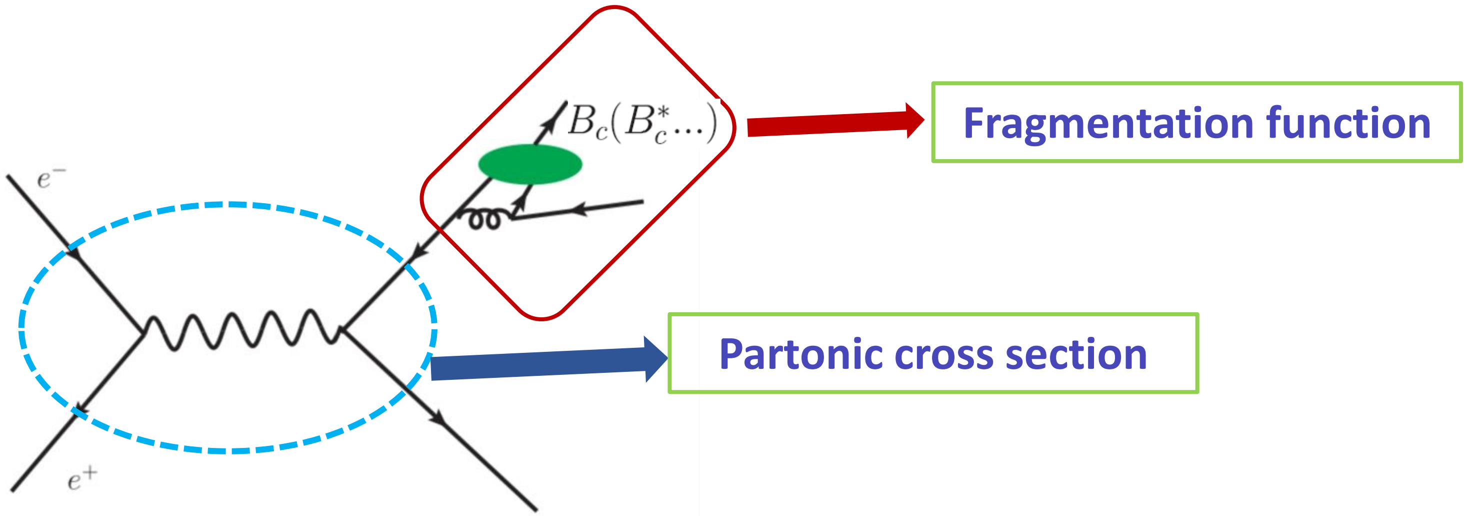
Collinear emission



Spoil or weaken the convergence of the series

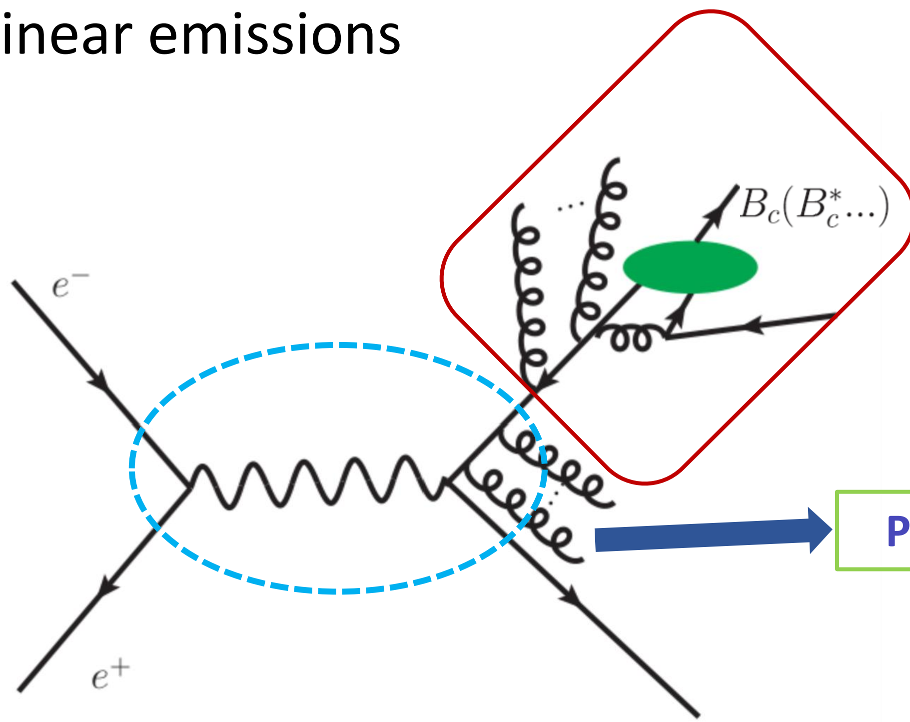
$\ln(p_t^2/m_Q^2)$ appearing in the production at a hadron collider

- Fragmentation-function approach



$$d\sigma(e^+ + e^- \to Bc(p) + b + \bar{c})$$
$$= \sum_i d\hat{\sigma}(e^+ + e^- \to i + X)(p/z, \mu_F) \otimes D_{i\to Bc}(z, \mu_F) + \mathrm{O}(m_Q^2/s)$$

Collinear emissions



Fragmentation function

Partonic cross section

Large Logarithms of s/m_Q^2?

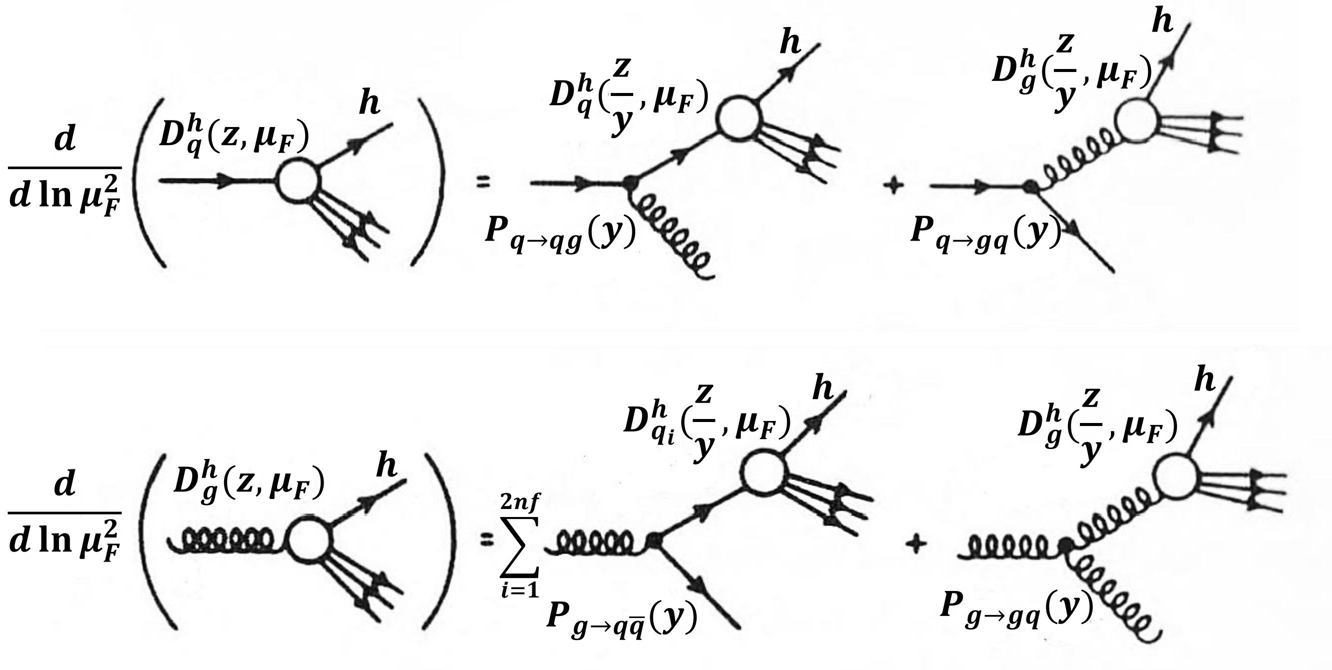
Involving $\ln(s/\mu_F^2)$

$\mu_F = O(\sqrt{s}\,)$

$$d\sigma(e^+ + e^- \to Bc(p) + b + \bar{c})$$
$$= \sum_i \boxed{d\hat{\sigma}(e^+ + e^- \to i + X)(p/z, \mu_F)} \otimes \boxed{D_{i\to Bc}(z, \mu_F)} + \mathrm{O}(m_Q^2 / s)$$

Involving $\ln(\mu_F^2/m_Q^2)$

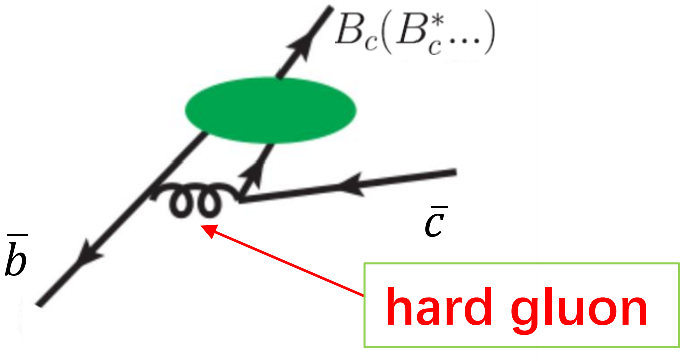
DGLAP evolution



Collinear log-terms can be resummed through the DGLAP evolution.

Boundary condition: $D_i^h(z,\mu_{F0})$ with $\mu_{F0} = O(m_Q)$

NRQCD factorization for FFs:



The fragmentation process contains perturbatively calculable information

$$D_{i\to Bc}(z,\mu_{F0})=\sum_n d_{i\to c\bar{b}[n]}(z,\mu_{F0})\langle O^{Bc}(n)\rangle$$

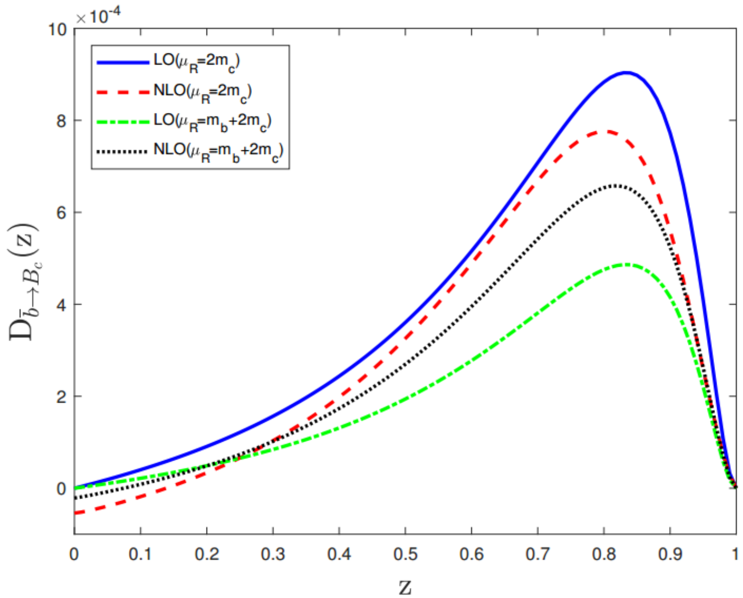
Involving $\ln(\mu_{F0}^2/m_Q^2)$ $\mu_{F0}=O(m_Q)$

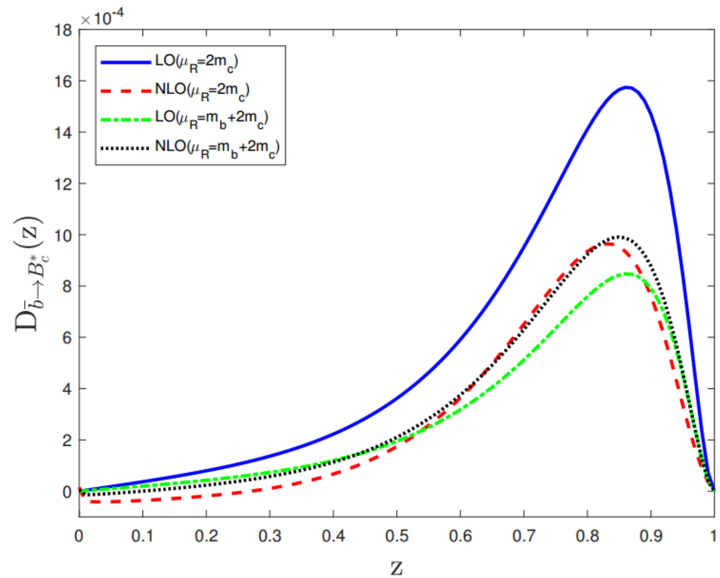
The FFs at a higher scale can be obtained by solving the DGLAP equations

2. FFs for S-wave Bc mesons

PRD 100, 034004, (2019),
X.-C. Zheng, C.-H. Chang, T.-F Feng, X.-G. Wu.

NLO fragmentation functions for $\bar{b} \to B_c$ and $\bar{b} \to B_c^*$



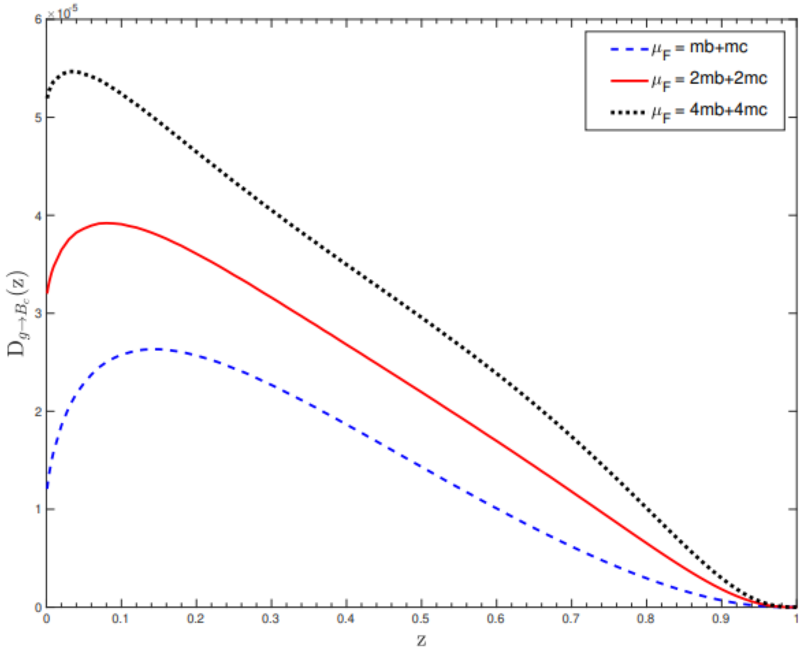


$$D_{\bar{b}\to Bc}(z, \mu_F = m_b + 2m_c)$$

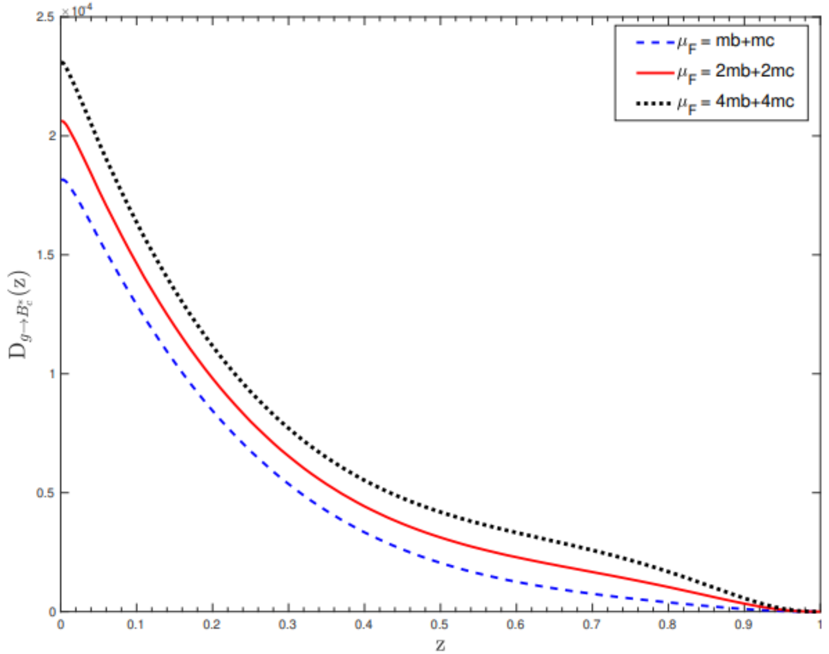
$$D_{\bar{b}\to B_c^*}(z, \mu_F = m_b + 2m_c)$$

g->Bc(Bc*) FFs

JHEP 05, 036, (2022),
X.-C. Zheng, C.-H. Chang, X.-G. Wu.



g->Bc FFs



g->Bc* FFs

PHYSICAL REVIEW D **106,** 054030 (2022)

Gluon fragmentation into $B_c^{(*)}$ in NRQCD factorization

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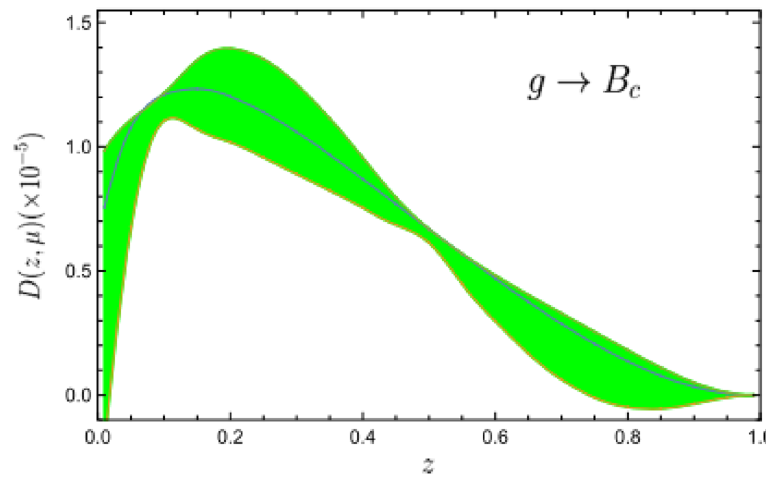
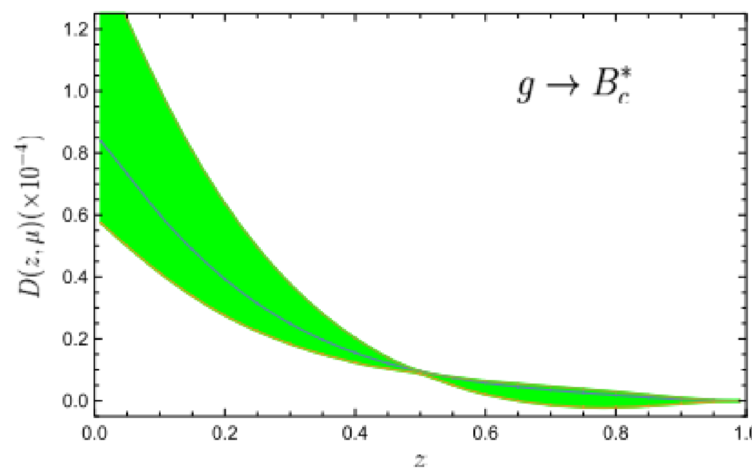
The universal fragmentation functions of gluon into the flavored quarkonia B_c and (polarized) B_c^* are computed within NRQCD factorization framework at the lowest order in velocity expansion and strong coupling constant. It is mandatory to invoke the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi renormalization program to render the NRQCD short-distance coefficients UV finite in a pointwise manner. The calculation is facilitated with the sector decomposition method, with the final results presented with high numerical accuracy. This knowledge is useful to enrich our understanding toward the large-p_T behavior of $B_c^{(*)}$ production at LHC experiment.

DOI: 10.1103/PhysRevD.106.054030

$c_1^{Bc}(z)$	z=0.1	Z=0.2	Z=0.5	Z=0.8	Z=0.9
Zheng et al	0.2323	0.2311	0.1282	0.02612	0.006144
Feng et al	0.2324	0.2311	0.1282	0.02612	0.006143

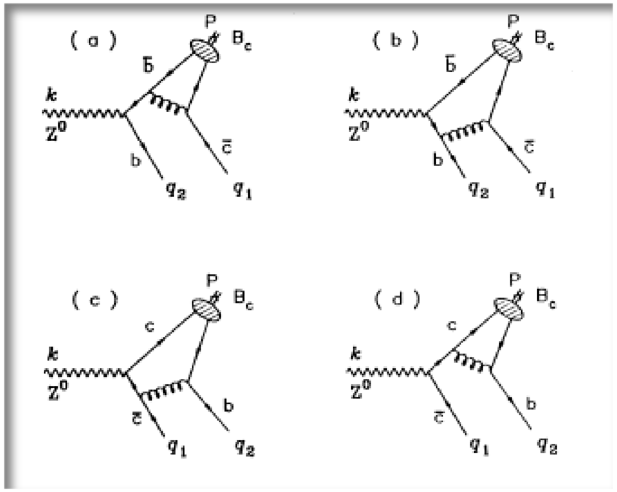
$c_1^{Bc^*}(z)$	z=0.1	Z=0.2	Z=0.5	Z=0.8	Z=0.9
Zheng et al	1.155	0.7554	0.1822	0.03412	0.009589
Feng et al	1.155	0.7550	0.1822	0.03411	0.009586

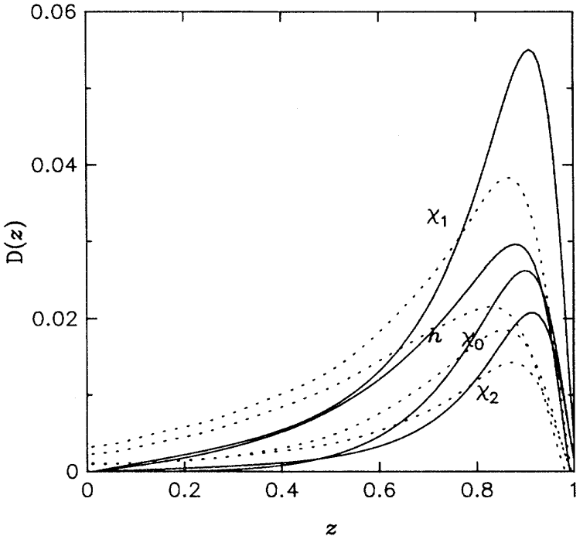




3. FFs for P-wave Bc mesons

- FFs for a heavy quark into P-wave Bc mesons
  - Y.-Q. Chen, PRD 48, 5181, (1993);
  - T. C. Yuan, PRD 50, 5664, (1994).

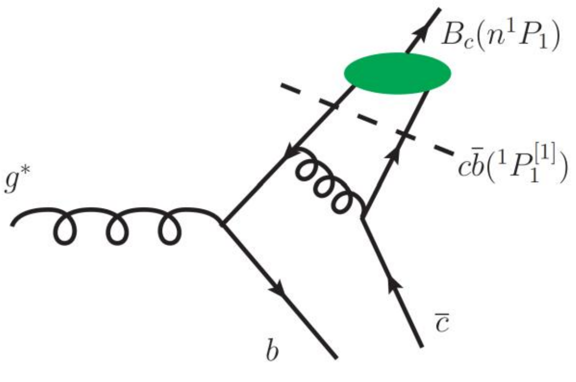




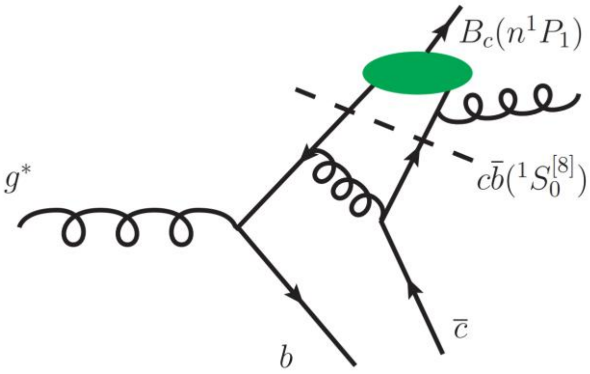
g->Bc(nP) FFs

At LO in v, the FFs can be factorized as

$$
\begin{aligned}
D_{g\to B_c(n^1P_1)}(z,\mu_F) = &\; d_{g\to(c\bar{b})[^1P_1^{[1]}]}(z,\mu_F)\langle\mathcal{O}^{B_c(n^1P_1)}(^1P_1^{[1]})\rangle \\
&+d_{g\to(c\bar{b})[^1S_0^{[8]}]}(z,\mu_F)\langle\mathcal{O}^{B_c(n^1P_1)}(^1S_0^{[8]})\rangle, \\
D_{g\to B_c(n^3P_J)}(z,\mu_F) = &\; d_{g\to(c\bar{b})[^3P_J^{[1]}]}(z,\mu_F)\langle\mathcal{O}^{B_c(n^3P_J)}(^3P_J^{[1]})\rangle \\
&+d_{g\to(c\bar{b})[^3S_1^{[8]}]}(z,\mu_F)\langle\mathcal{O}^{B_c(n^3P_J)}(^3S_1^{[8]})\rangle,
\end{aligned}
$$



Color-singlet mechanism



Color-octet mechanism

Calculation of SDCs

Applying NRQCD factorization to an on-shell quark pair:

$$D_{g\to(c\bar{b})[n]}(z,\mu_F)=\sum_{n'} d_{g\to(c\bar{b})[n']}(z,\mu_F)\langle\mathcal{O}^{(c\bar{b})[n]}(n')\rangle.$$

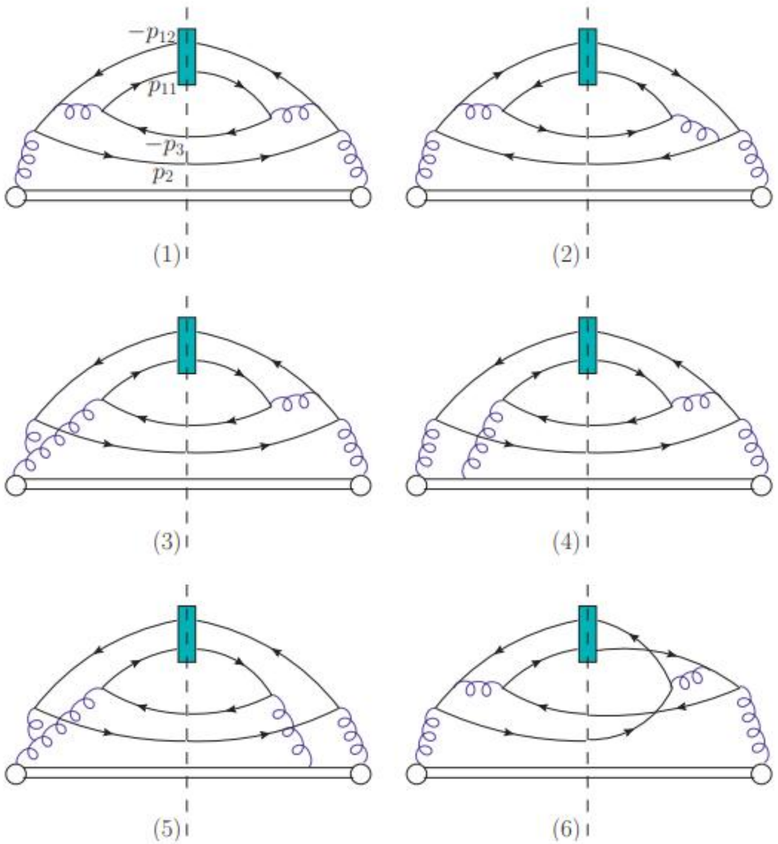
Calculated through Feynman diagrams

Calculated from the operator definition

$$\begin{aligned}
\langle\mathcal{O}^{(c\bar{b})[^1S_0^{[8]}]}(^1S_0^{[8]})\rangle &= N_c^2-1,\\
\langle\mathcal{O}^{(c\bar{b})[^3S_1^{[8]}]}(^3S_1^{[8]})\rangle &= (d-1)(N_c^2-1),\\
\langle\mathcal{O}^{(c\bar{b})[^1P_1^{[1]}]}(^1P_1^{[1]})\rangle &= 2(d-1)N_c\,\boldsymbol{q}^2,\\
\langle\mathcal{O}^{(c\bar{b})[^3P_J^{[1]}]}(^3P_J^{[1]})\rangle &= 2\,(2J+1)N_c\boldsymbol{q}^2,
\end{aligned}$$

The SDCs can be determined through **matching**.

Cut diagrams



末态有三个粒子，且相空间积分存在紫外发散。

Six of the 49 cut diagrams

Estimate of LDMEs

- The color-singlet LDMEs can be estimated by wave function:

$$\langle \mathcal{O}^{B_c(n^1P_1)}({}^1P_1^{[1]})\rangle \approx \frac{9N_c}{2\pi}|R'_{nP}(0)|^2.$$

- The RGE of the color-octet LDMEs:

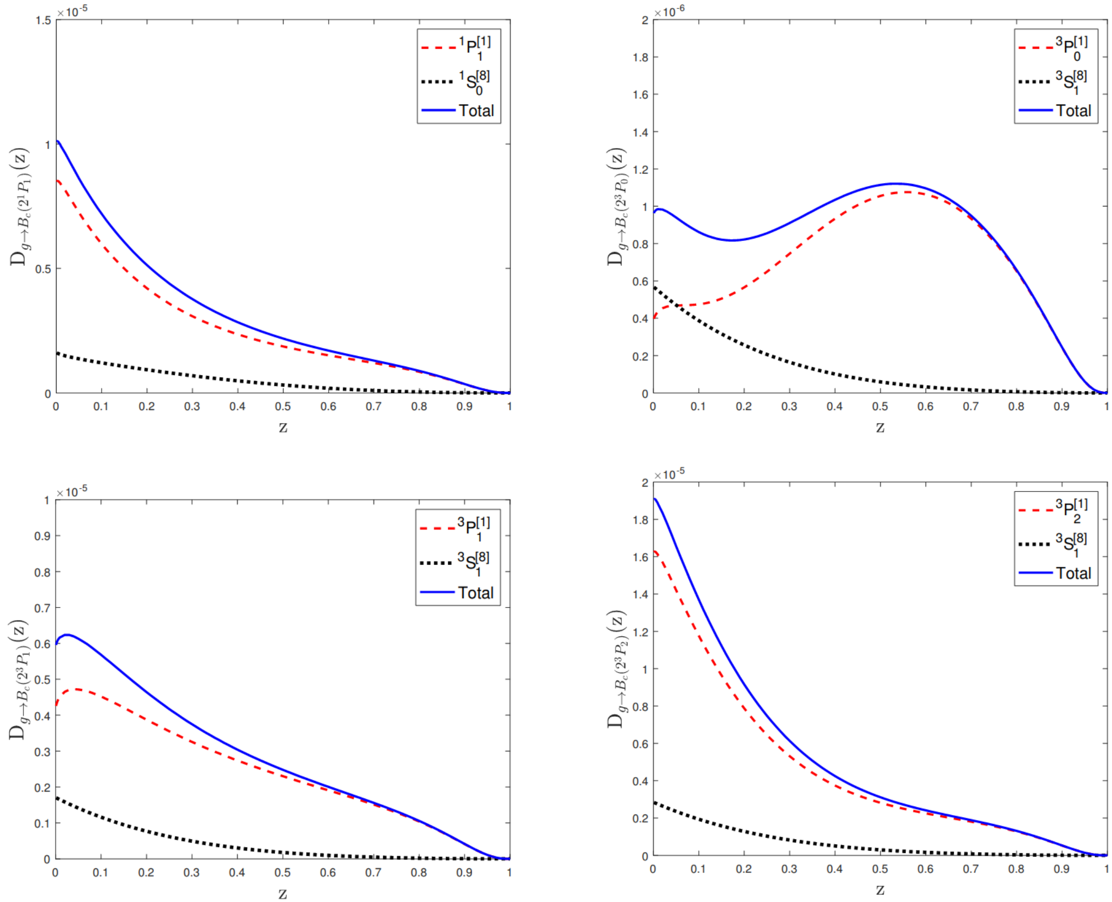
$$\mu_\Lambda \frac{d}{d\mu_\Lambda}\langle \mathcal{O}^{B_c(n^1P_1)}({}^1S_0^{[8]})\rangle = \frac{4\alpha_s(\mu_\Lambda)}{27\pi m_{\rm red}^2}\langle \mathcal{O}^{B_c(n^1P_1)}({}^1P_1^{[1]})\rangle,$$

$$\langle \mathcal{O}^{B_c(n^1P_1)}({}^1S_0^{[8]})\rangle_{\mu_\Lambda} = \langle \mathcal{O}^{B_c(n^1P_1)}({}^1S_0^{[8]})\rangle_{\mu_{\Lambda 0}} + \frac{8}{27\beta_0 m_{\rm red}^2}\ln\left(\frac{\alpha_s(\mu_{\Lambda 0})}{\alpha_s(\mu_\Lambda)}\right)\langle \mathcal{O}^{B_c(n^1P_1)}({}^1P_1^{[1]})\rangle,$$

The color-octet LDMEs are estimated using their RGE together with a boundary assumption.

FFs for P-wave Bc mesons

- FFs for 2P Bc excited states **arXiv: 2602.01211, X.-C. Zheng, X.-G. Wu.**



- Fragmentation probabilities arXiv: 2602.01211, X.-C. Zheng, X.-G. Wu.

State	$P(\mu_F = M)$	$P(\mu_F = 2M)$	$P(\mu_F = 4M)$
$B_c(2\,^1P_1)$	2.23×10^{-6}	3.02×10^{-6}	3.80×10^{-6}
$B_c(2\,^3P_0)$	3.68×10^{-7}	8.18×10^{-7}	1.27×10^{-6}
$B_c(2\,^3P_1)$	1.82×10^{-6}	2.77×10^{-6}	3.72×10^{-6}
$B_c(2\,^3P_2)$	4.12×10^{-6}	5.17×10^{-6}	6.21×10^{-6}

$M = m_b + m_c$

Fragmentation probabilities are sensitive to the factorization scale.

- Average momentum fraction

State	$\langle z\rangle(\mu_F = M)$	$\langle z\rangle(\mu_F = 2M)$	$\langle z\rangle(\mu_F = 4M)$
$B_c(2\,^1P_1)$	0.23	0.27	0.29
$B_c(2\,^3P_0)$	0.49	0.44	0.42
$B_c(2\,^3P_1)$	0.29	0.31	0.33
$B_c(2\,^3P_2)$	0.21	0.25	0.27

Comparison with $\bar{b}$-quark fragmentation

- Fragmentation probabilities for gluon into P-wave Bc mesons

State	$P(\mu_F = M)$	$P(\mu_F = 2M)$	$P(\mu_F = 4M)$
$B_c(2\,^1P_1)$	2.23×10^{-6}	3.02×10^{-6}	3.80×10^{-6}
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arXiv: 2602.01211,
X.-C. Zheng, X.-G. Wu.

- Fragmentation probabilities for $\bar{b}$ quark into P-wave Bc mesons

$$P_{\bar{b}\to\bar{b}c(2^3P_0)} \approx 2.3 \times 10^{-5} \,,$$

$$P_{\bar{b}\to\bar{b}c(2\,1^{+\prime})} \approx 4.4 \times 10^{-5} \,,$$

$$P_{\bar{b}\to\bar{b}c(2\,1^{+})} \approx 4.8 \times 10^{-5} \,,$$

$$P_{\bar{b}\to\bar{b}c(2^3P_2)} \approx 5.6 \times 10^{-5} \,.$$

- Fitting functions for FFs: **arXiv: 2602.01211, X.-C. Zheng, X.-G. Wu.**

$$d_{g\to(c\bar{b})[^{2S+1}L_J^{[1(8)]}]}(z,\mu_F)=\frac{\alpha_s}{2\pi}\ln\frac{\mu_F^2}{4M^2}\int_z^1\left[\sum_{Q=\bar{b},c}P_{Qg}(y)d^{\rm LO}_{Q\to(c\bar{b})[^{2S+1}L_J^{[1(8)]}]}(z/y)\right]\frac{dy}{y}+\frac{\alpha_s^3}{M^{(2L+3)}}f_{[^{2S+1}L_J^{[1(8)]}]}(z),$$

$$f_{[^{2S+1}L_J^{[1(8)]}]}(z)=N\,z^{\alpha}(1-z)^{\beta}\left(1+\sum_{i=1}^{6}a_i\,z^i\right).$$

State	N	α	β	a_1	a_2	a_3	a_4	a_5	a_6
$^1P_1^{[1]}$	6.705	0.01418	2.172	-2.293	7.423	-18.93	47.56	-65.72	41.25
$^3P_0^{[1]}$	1.297	0.06646	2.269	0.7862	4.369	34.07	140.1	-381.9	336.1
$^3P_1^{[1]}$	3.753	0.03236	2.138	2.702	-15.01	51.22	-66.48	26.77	18.66
$^3P_2^{[1]}$	7.580	0.01234	2.114	-1.318	-2.371	14.21	-17.72	4.866	8.175
$^1S_0^{[8]}$	1.420	-0.01096	3.101	0.3302	7.717	-33.55	86.58	-108.5	56.07
$^3S_1^{[8]}$	1.619	0.001004	1.675	-2.113	2.244	-2.408	2.907	-2.043	0.4914

Summary

- Fragmentation provides a systematic description of high-energy P-wave Bc production and **resums collinear logarithms**;
- We calculated the **gluon fragmentation functions** into the 2P Bc states at $O(\alpha s^3)$ within NRQCD;
- The FFs show distinctive **state-dependent z distributions**, with integrated values of $O(10^{-7}–10^{-6})$ at the chosen input scale;
- These **fragmentation functions** may be studied at the high energy colliders, such as CEPC, FCC-ee, etc.

Thank you！

Backup

- **Phase-space integrals**

UV divergences, dimensional regularization.

$$D_{g\to Bc}(z) = \int N_{CS}\, d\phi_3 [A_{(1-49)} - A_S] + \int N_{CS}\, d\phi_3 A_S$$

Calculated in 4 dimensions

Calculated in d dimensions

Various types of subtraction terms need to be integrated!

- **Renormalization of the operator**

$$D_{g\to(c\bar{b})[n]}(z,\mu_F) = D^{(1-49)}_{g\to(c\bar{b})[n]}(z) - \frac{\alpha_s}{2\pi}\left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\frac{4\pi\mu^2}{\mu_F^2}\right] \times \int_z^1 \frac{dy}{y}\left[\sum_{Q=\bar{b},c} P_{Qg}(y) D^{\mathrm{LO}}_{Q\to(c\bar{b})[n]}(z/y)\right],$$