Baryon-baryon and meson-baryon interactions in Lorentz invariant ChEFT

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In collaboration with

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RUHR UNIVERSITÄT BOCHUM

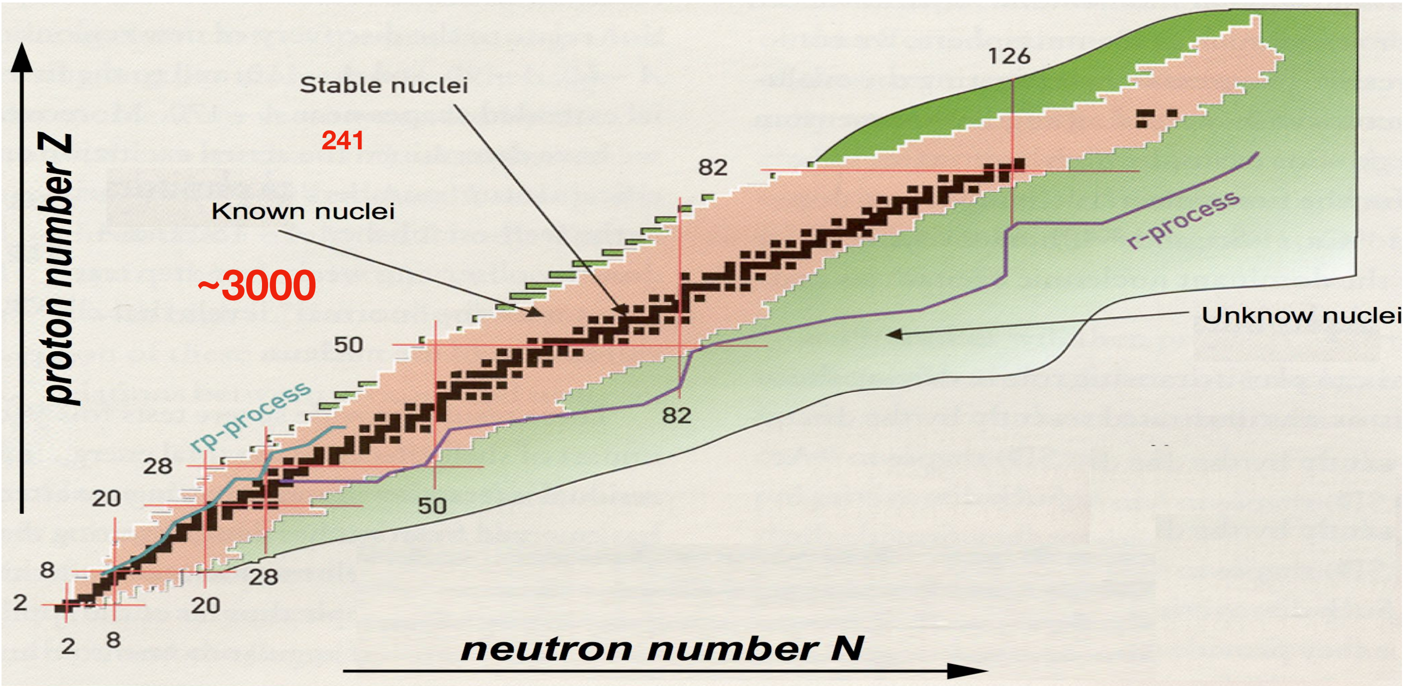
UNIVERSITÄT BONN

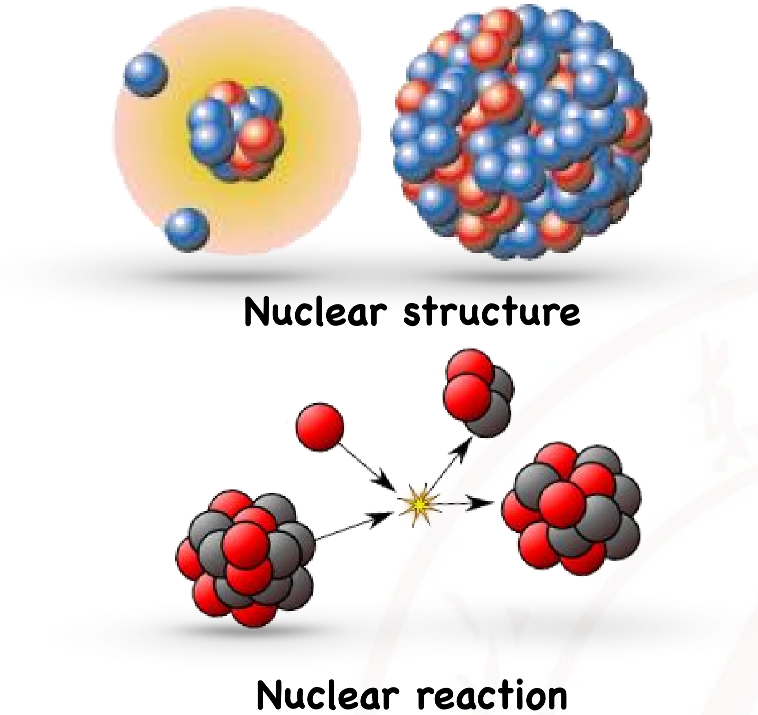
OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary and outlook

(Hyper)Nuclear Chart

- Nuclear Chart (2D –- N, Z)





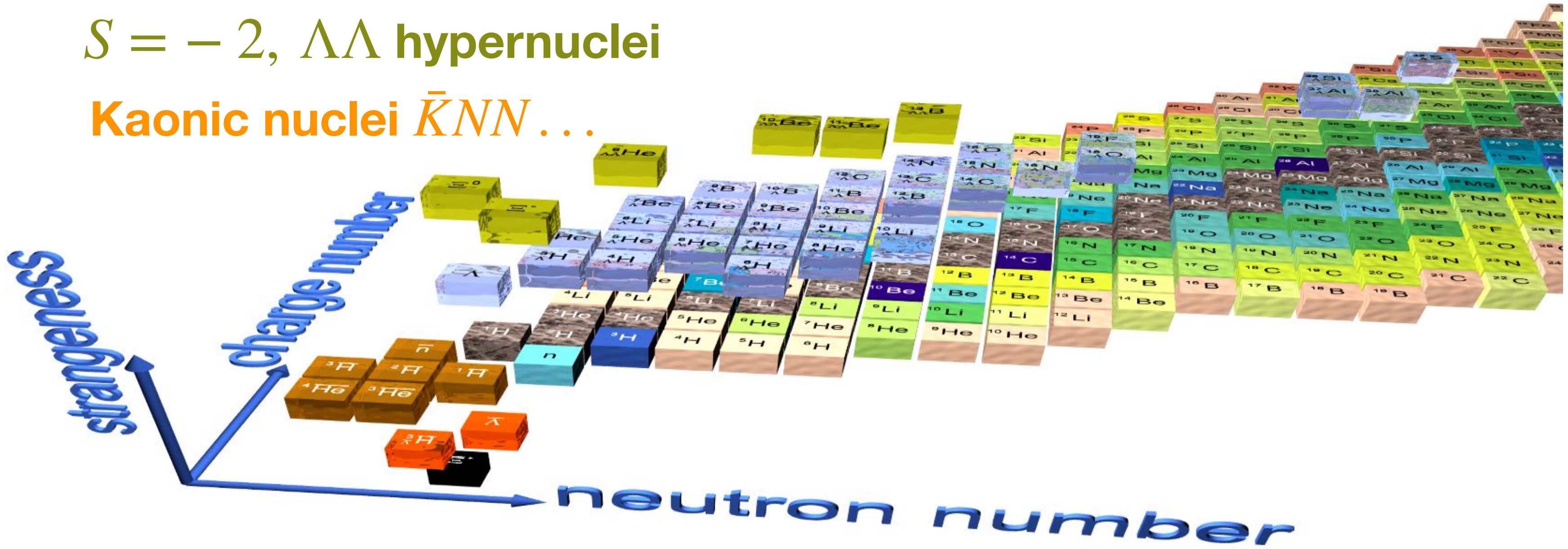
- Detailed understanding of the strong nuclear force (**NN**) is essential!

- Hypernuclear Chart (3D –- N, Z, Strangeness)

$S = -1$, Λ, Ξ **hypernuclei**

$S = -2$, $\Lambda\Lambda$ **hypernuclei**

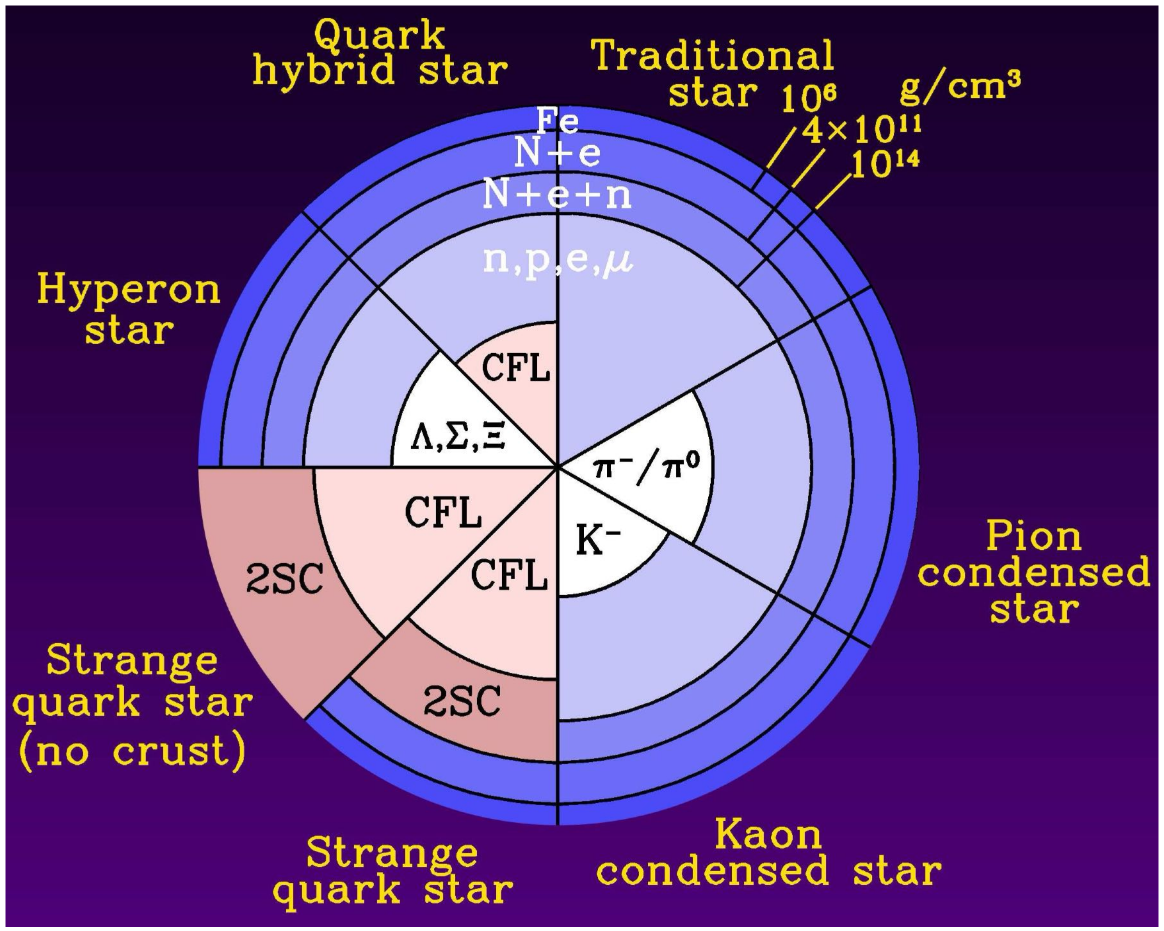
Kaonic nuclei $\bar{K}NN\ldots$



- Hyperon-nucleon (**YN**)
- Hyperon-hyperon (**YY**)
- Antikaon-nucleon (**$\bar{\text{K}}$N**) interactions

Neutron star

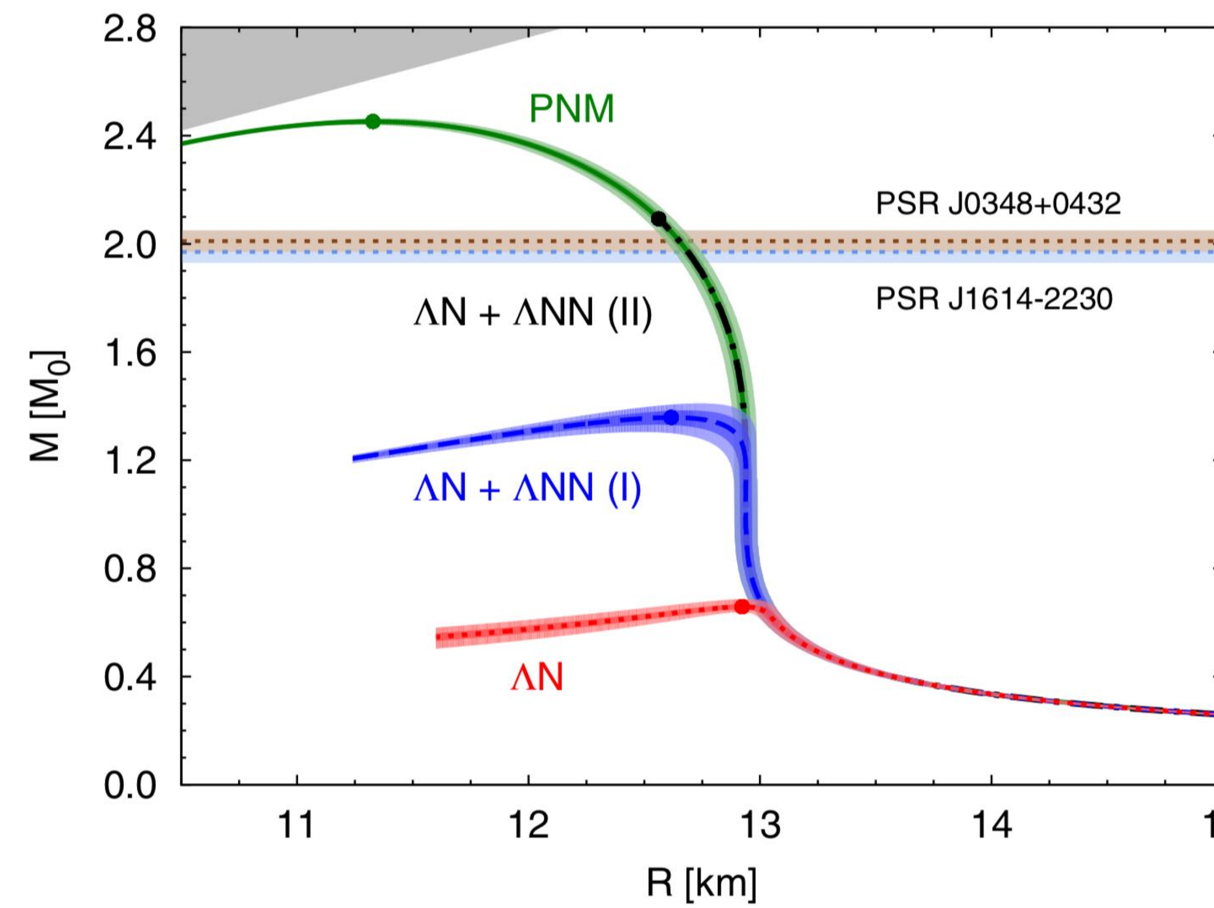
Unknown interior



- Inner Core:
  - ✓ Hyperon star ?
  - ✓ Kaon-condensed star ?
  - ✓ Strange quark star ?

Equation of state

- Hyperon puzzle



D.Lonardoni et al., PRL 114 (2015) 092301

Precise knowledge of YN and $\bar{K}N$ interactions needed

BB and MB scatterings in ChEFT

- BB interactions extensively studied in **non-relativistic** framework

	2NF	3NF	4NF
LO (Q^0)	S. Weinberg, PLB 1990, NPB1990	—	—
NLO (Q^2)	U. van Kolck et al, PLB1992, PRL1994 N. Kaiser et al., NPA1997	—	—
N^{2}LO (Q^3)	U. van Kolck et al., PRC1994 E. Epelbaum et al., NPA1998, 2000	U. van Kolck et al., PRC1994	—
N^{3}LO (Q^4)	R. Machleidt et al., PRC2003 E. Epelbaum et al., NPA2005	S. Ishikwas et al., PRC2007 V. Bernard et al., PRC2007	E. Epelbaum, PLB2006, EPJA2007
N^{4}LO (Q^5)	R. Machleidt et al., PRC2015 E. Epelbaum et al., PRL2015	H. Krebs et al, PRC2012,2013	

	Two-baryon force	Three-baryon force	Four-baryon force
LO (Q^0)	S= -1,-2: H. Polinder et al., NPA2006,PLB2007 S= -3,-4: J. Haidenbauer & U.-G. Meißner PLB2010	—	—
NLO (Q^2)	S= -1: J. Haidenbauer et al., NPA2013, EPJA2020 S= -2: J. Haidenbauer et al., NPA2016	—	—
N^{2}LO (Q^3)	S= -1: J. Haidenbauer et al., arXiv:2301.00722	ΛNN: S. Petschauer et al., PRC2016, NPA2017	—

- Solve Lippmann-Schwinger equation

$$T(\boldsymbol{p}', \boldsymbol{p}) = V(\boldsymbol{p}', \boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}', \boldsymbol{k}) \frac{m_N}{\boldsymbol{p}^2 - \boldsymbol{k}^2 + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

- Renormalization issue is still under debate!

Nuclear Forces for Precision Nuclear Physics: A Collection of Perspectives Few-Body Syst (2022) 63:67

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N^{3}LO (Q^4)	••• *R. Machleidt et al., PRC2003* *E. Epelbaum et al., NPA2005*	••• *S. Ishikwas et al., PRC2007* *V. Bernard et al., PRC2007*	••• *E. Epelbaum, PLB2006, EPJA2007*
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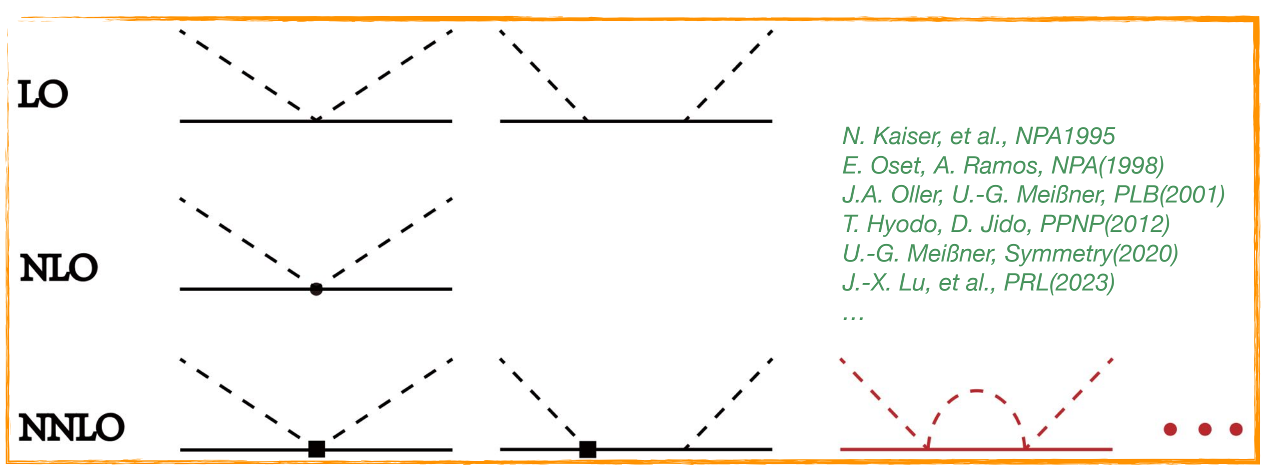
- MB interactions and **resonances** (usually) studied in **covariant** framework



- Solve Bethe-Salpeter equation

$$T(p', p) = V(p', p) + i \int \frac{d^4 k}{(2\pi)^4} V(p', k)\, G(k)\, T(k, p)$$

- On-shell factorization
- Model dependence: Finite cutoff or subtraction constants

In this work

- We tentatively proposed **a unified framework** for baryon-baryon and meson-baryon interactions within time-ordered perturbation theory (TOPT) using the Lorentz invariant chiral Lagrangians

  - Formulate NN and YN interactions at leading order

    V. Baru, E. Epelbaum, J. Gegelia, XLR*, Phys. Lett. B 798,134987 (2019)

    XLR, E.Epelbaum, J.Gegelia, Phys. Rev. C 101,034001 (2020)

  - Extend NN interaction to **next-to-next-to-leading order**

    XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)

    XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 113, 034002(2026)

  - Investigate $\bar{K}N, KN$ interactions and $\Lambda(1405)$ at LO/NLO

    XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406

    XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582

    XLR, Phys. Lett. B 855 (2024) 138802

    XLR, Phys. Rev. D 113 (2026) 114027

    XLR, et al., in preparation (2026)

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Time-ordered perturbation theory

- Re-express the Feynman integral in a form that makes the connection with on-mass-shell (off-energy shell) state explicit.
- This form is called **TOPT (or old-fashioned perturbation theory)**

XLR, PoS(CD2021)007

- External lines

Spin 0 boson (in, out)

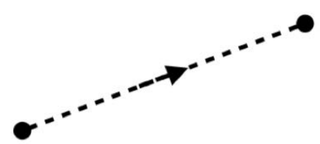
$$\mathbf{1}$$

Spin 1/2 fermion (in, out)

$$u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$$

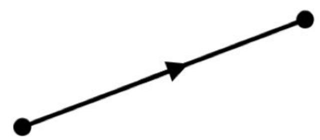
- Internal lines

Spin 0 (anti-)boson



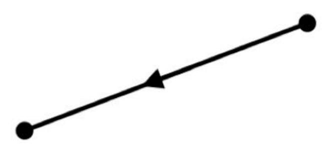
$$\frac{1}{2\,\epsilon_q} \qquad \epsilon_q \equiv \sqrt{\mathbf{q}^2 + M^2}$$

Spin 1/2 fermion



$$\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) \qquad \omega_p \equiv \sqrt{\mathbf{p}^2 + m^2}$$

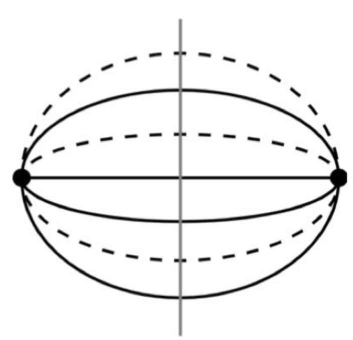
anti-fermion



$$\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$$

- Intermediate state

A set of lines between two vertices



$$\frac{1}{E - \sum_i \omega_{p_i} - \sum_j \epsilon_{q_j} + i\epsilon}$$

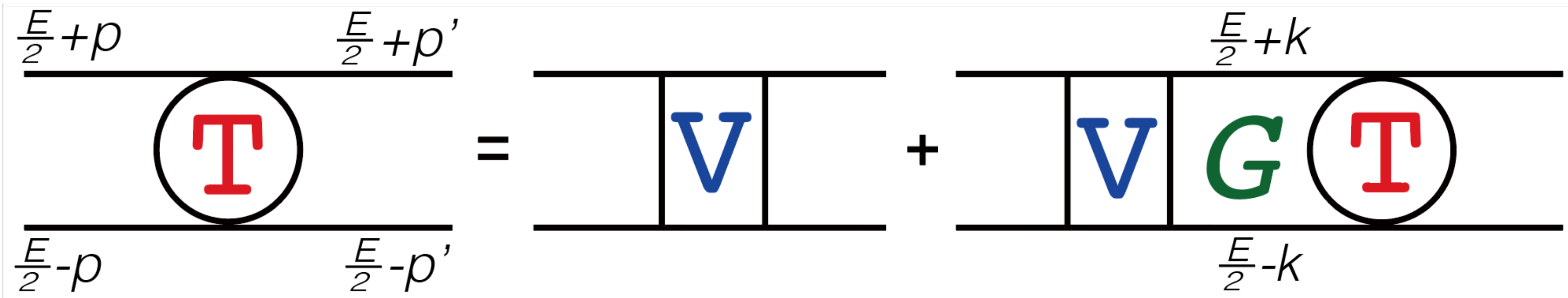
- Interaction vertices: **the standard Feynman rules**
  - Take care of zeroth components of integration momenta

✓ particle $p^0 \to \omega(p, m)$

✓ antiparticle $p^0 \to -\omega(p, m)$

Baryon-baryon scattering in TOPT

- **Baryon-baryon** scattering amplitude



- Potential V: sum up the two-baryon **irreducible time-ordered diagrams**
  - ✓ Employ the Weinberg power counting to perturbatively calculate potential

$$V = \begin{pmatrix} V_{NN,NN} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & V_{\Lambda N,\Lambda N} & V_{\Lambda N,\Sigma N} & 0 & 0 & 0 & 0 \\ 0 & V_{\Sigma N,\Lambda N} & V_{\Sigma N,\Sigma N} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & V_{\Lambda\Lambda,\Lambda\Lambda} & V_{\Lambda\Lambda,\Xi N} & V_{\Lambda\Lambda,\Sigma\Sigma} & V_{\Lambda\Lambda,\Sigma\Lambda} \\ 0 & 0 & 0 & V_{\Xi N,\Lambda\Lambda} & V_{\Xi N,\Xi N} & V_{\Xi N,\Sigma\Sigma} & V_{\Xi N,\Sigma\Lambda} \\ 0 & 0 & 0 & V_{\Sigma\Sigma,\Lambda\Lambda} & V_{\Sigma\Sigma,\Xi N} & V_{\Sigma\Sigma,\Sigma\Sigma} & V_{\Sigma\Sigma,\Sigma\Lambda} \\ 0 & 0 & 0 & V_{\Sigma\Lambda,\Lambda\Lambda} & V_{\Sigma\Lambda,\Xi N} & V_{\Sigma\Lambda,\Sigma\Sigma} & V_{\Sigma\Lambda,\Sigma\Lambda} \end{pmatrix}$$

S= 0

S=-1

S=-2

- Two-baryon Green function (TOPT rules)

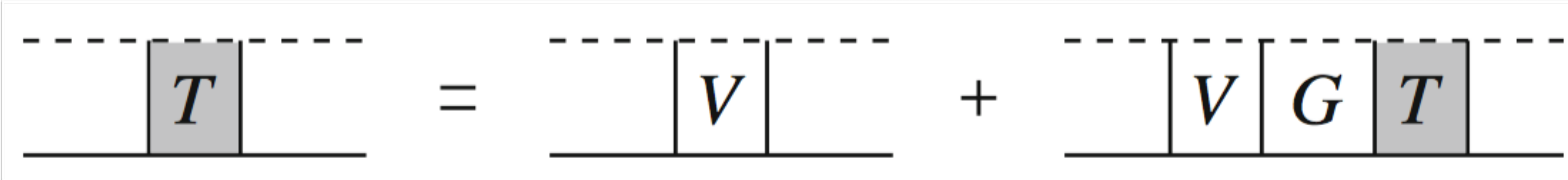
$$G_{ij}^{BB}(E) = \frac{m_i\, m_j}{\omega(k,m_i)\,\omega(k,m_j)} \frac{1}{E - \omega(k,m_i) - \omega(k,m_j) + i\epsilon}$$

✓ This is the generalized Kadyshevsky propagator of NN scattering *V. Kadyshevsky, NPB (1968)*

✓ **SELF-CONSISTENTLY obtained** in TOPT

Meson-baryon scattering in TOPT

☐ **Meson-baryon** scattering amplitude



- Interaction kernel/ potential V
  - ✓ **Define:** sum up the one-meson & one-baryon **irreducible time-ordered diagrams**
  - ✓ **Power counting:** Q/Λ_χ **systematic ordering of all graphs**

- Meson-baryon Green functions in TOPT:

$$G_{MB}(E) = \frac{m}{2\omega(k,M)\,\omega(k,m)} \frac{1}{E - \omega(k,M) - \omega(k,m) + i\epsilon}$$

- Coupled-channel integral equation for T-matrix

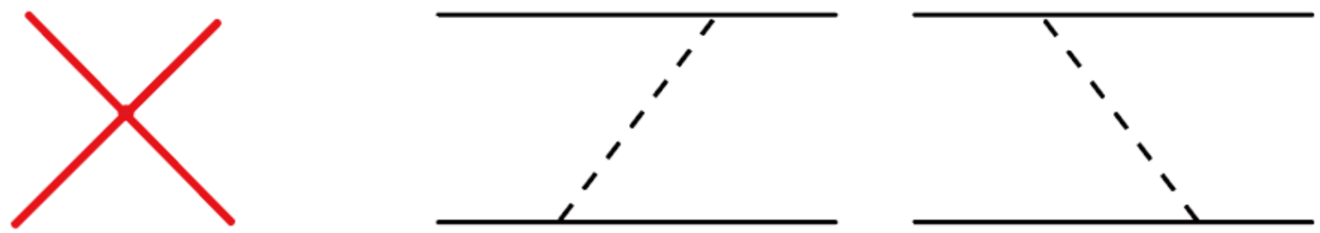
$$\begin{aligned} T_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) &= V_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) \\ &+ \sum_{MB}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} V_{M_jB_j,MB}(\boldsymbol{p}',\boldsymbol{k};E)\,G_{MB}(E)\,T_{MB,M_iB_i}(\boldsymbol{k},\boldsymbol{p};E) \end{aligned}$$

OUTLINE

NN and YN interactions at LO

LO potential

Weinberg power counting

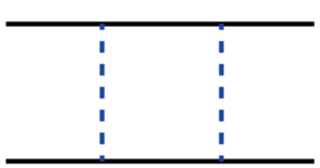


- NN interaction **XLR, et al., 1905.02116**

$$V_{LO,C} = (C_S + C_V) - (C_{AV} - 2C_T)\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$$V_{\rm OPE} = -\frac{g_A^2}{4f_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{4m_N^2}{\omega(q, M_\pi)\,(m_N + \omega(p, m_N))\,(m_N + \omega(p', m_N))} \times \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q}}{\omega(p, m_N) + \omega(p', m_N) + \omega(q, M_\pi) - E - i\epsilon}$$

- **Milder UV behaviour than that of the non-relativisitc OPEP**



$$\begin{cases} I_{VGV}^{\rm Our} \to \int dk^3 \frac{1}{k} \frac{1}{k^3} \frac{1}{k} = \int dk^3 \frac{1}{k^5} \\ I_{VGV}^{\rm NR} \to \int dk^3\, 1\, \frac{1}{k^2}\, 1 = \int dk^3 \frac{1}{k^2} \end{cases}$$

NN and YN interactions at LO

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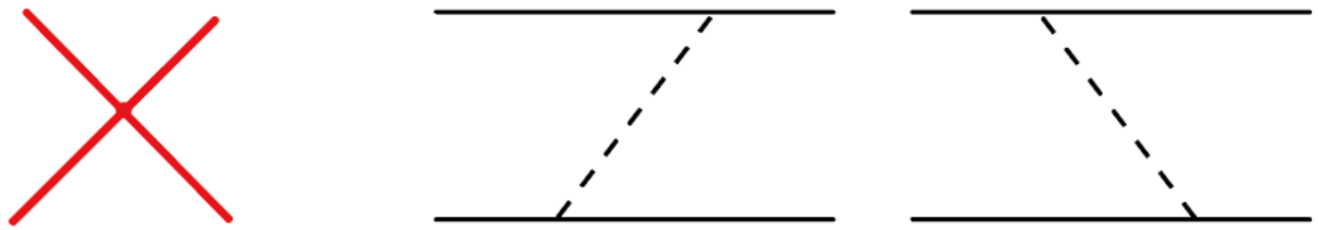
Weinberg power counting



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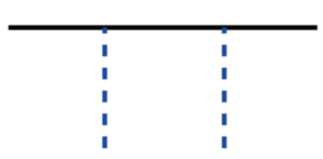

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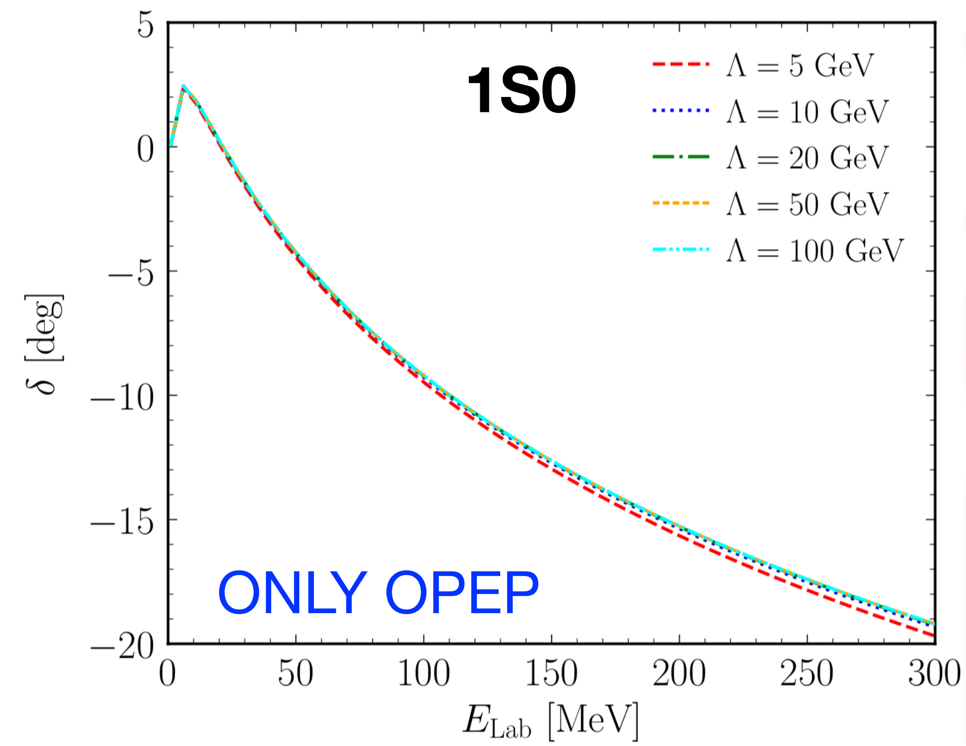

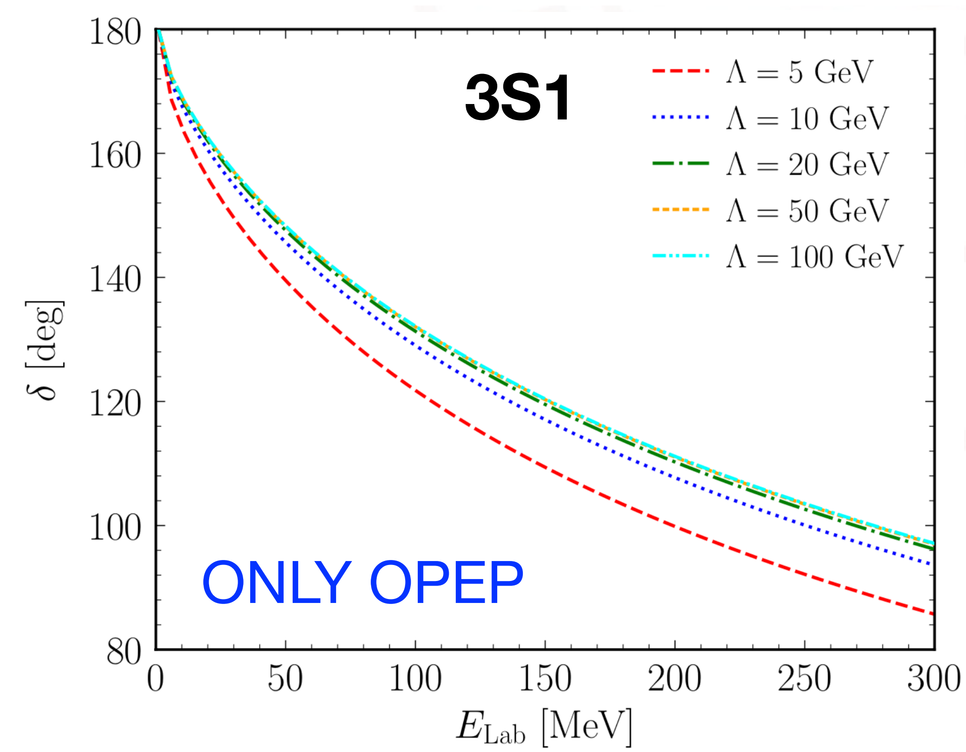

NN and YN interactions at LO

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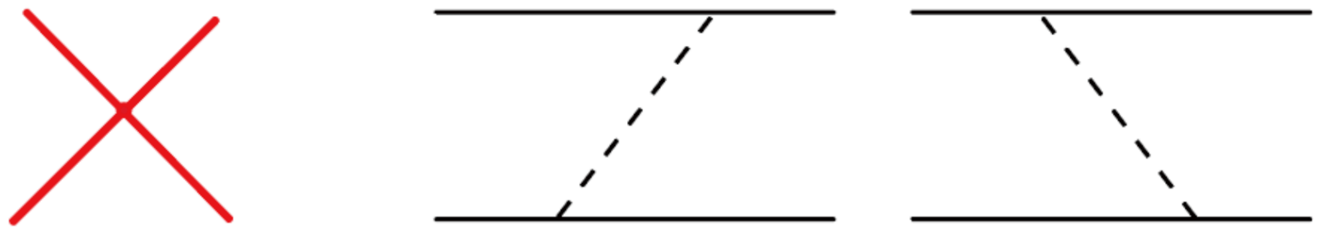

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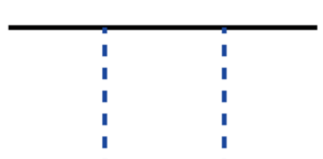

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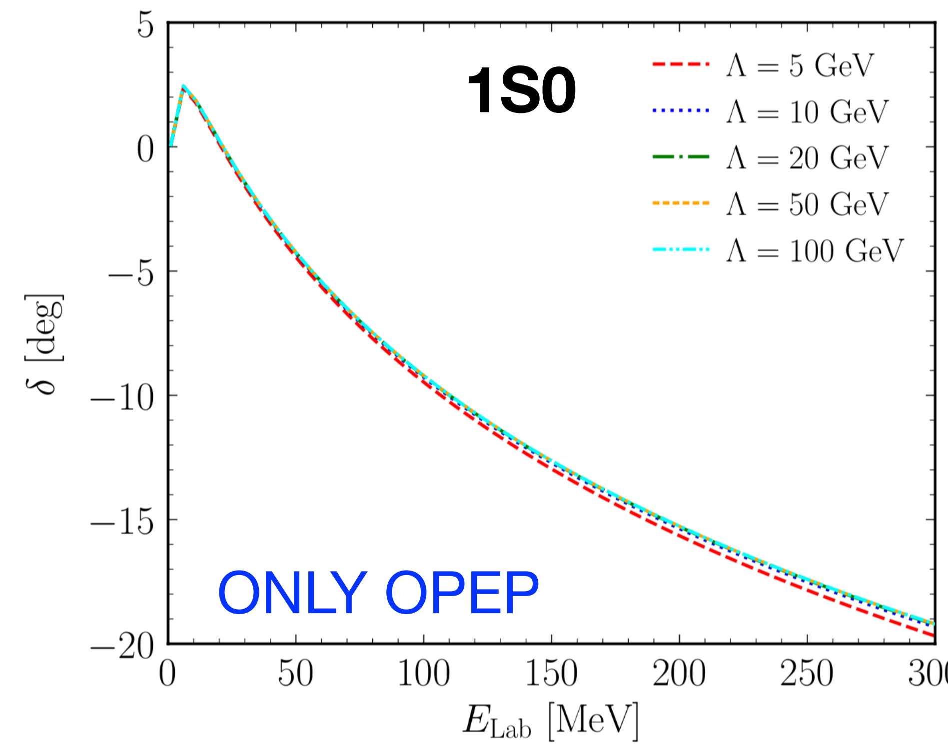

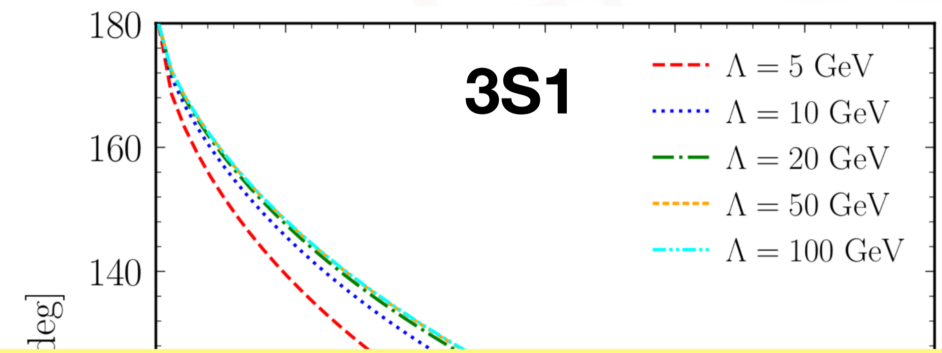

→Our LO potential is renormalizable!

- Unique solutions for all partial waves, no limit-cycle behavior
- Avoid finite-cutoff artefacts inherent to the conventional NR framework

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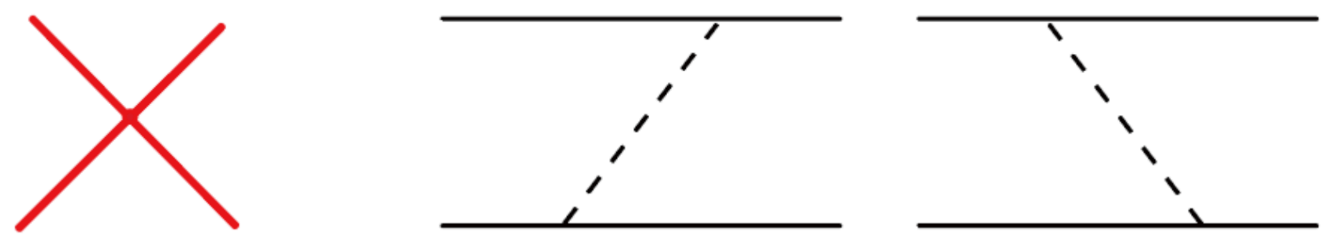
Weinberg power counting



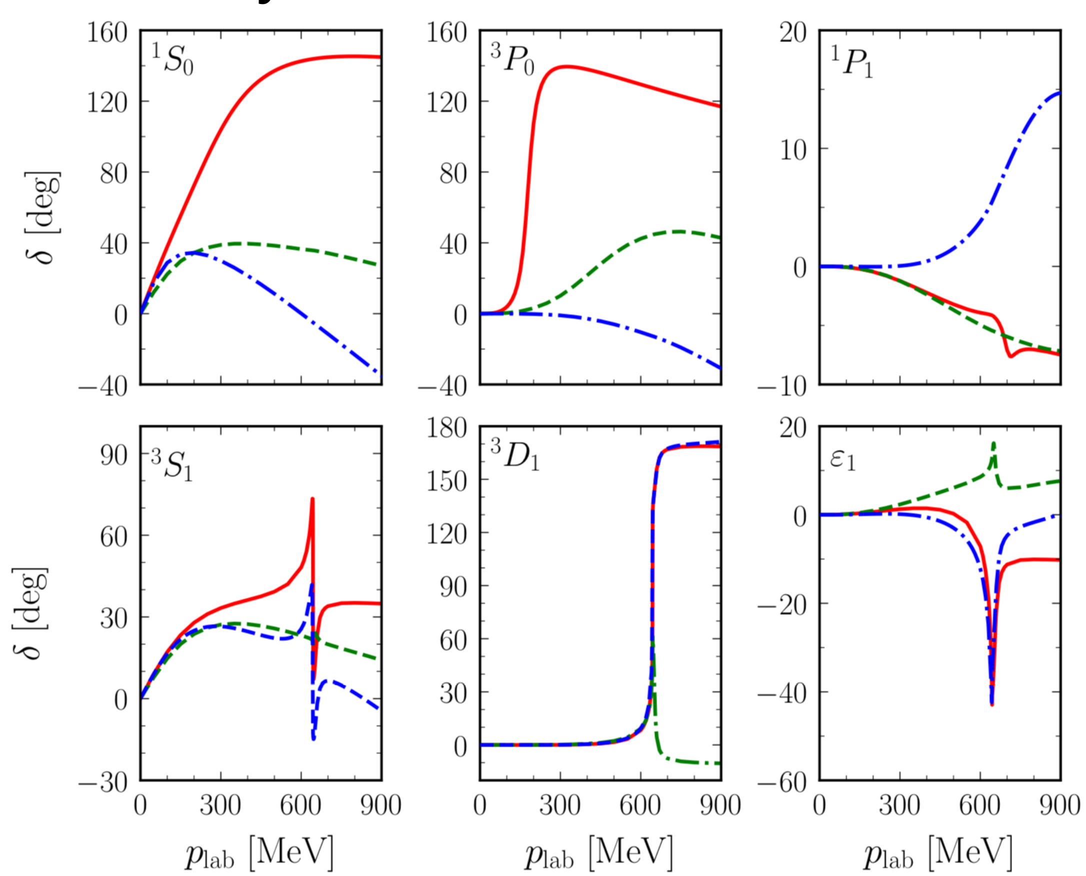
- Extend the renormalizalbe framework to the ΛN-ΣN interaction

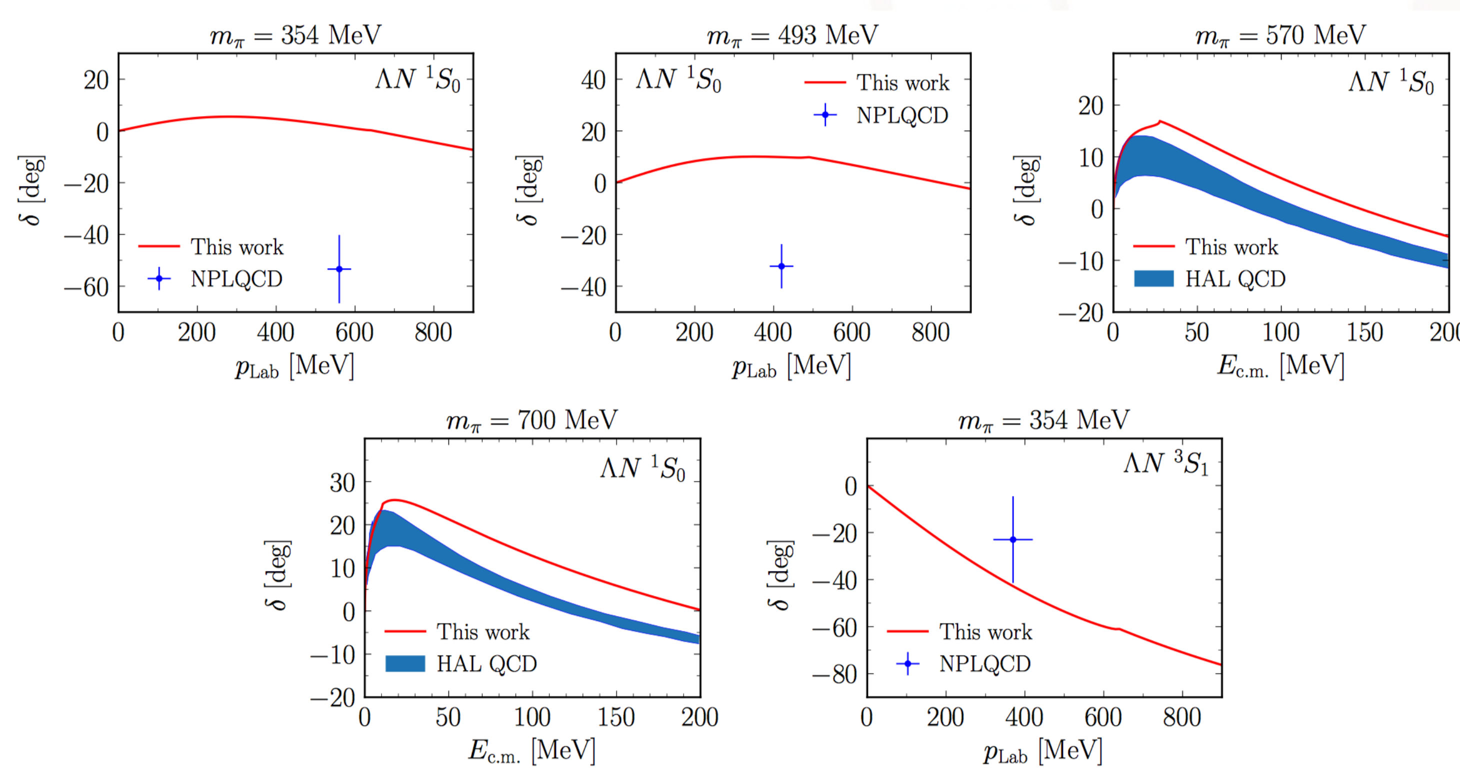
$$V_{\rm LO} = V_{\rm C} + V_{\rm OME} = \begin{pmatrix} V_{\Lambda N,\Lambda N}, & V_{\Lambda N,\Sigma N} \\ V_{\Sigma N,\Lambda N}, & V_{\Sigma N,\Sigma N} \end{pmatrix}$$

XLR, et al., 1911.05616

✓ 3 unknown LECs for each partial waves (1S0 and 3S1)

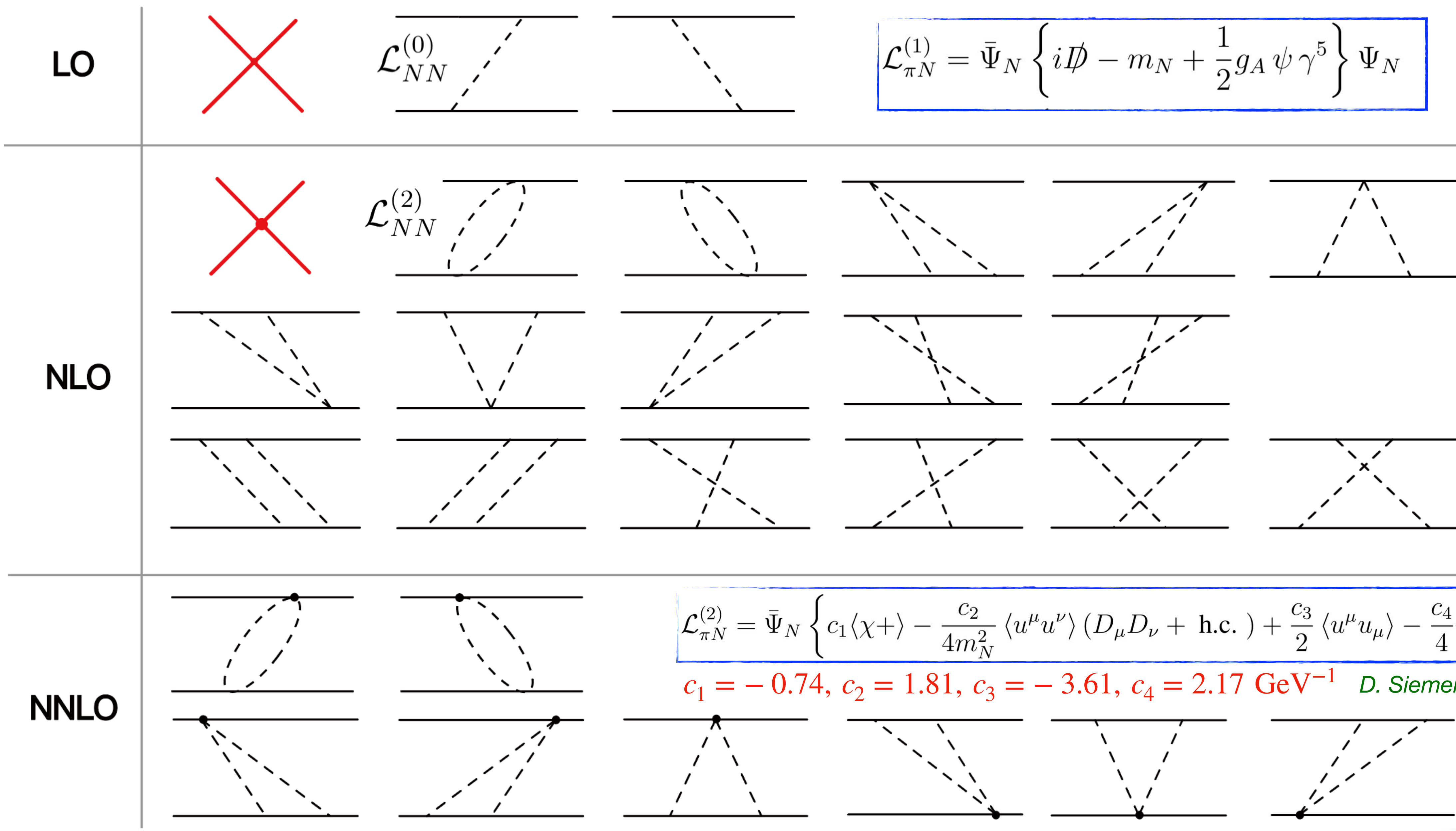
✓ Carry out the renormalization





NN interaction up to NNLO

- Two-nucleon **irreducible time-ordered diagrams**



$$\mathcal{L}^{(1)}_{\pi N} = \bar{\Psi}_N \left\{ i\slashed{D} - m_N + \frac{1}{2} g_A \slashed{u}\gamma^5 \right\} \Psi_N$$

$$\mathcal{L}^{(2)}_{\pi N} = \bar{\Psi}_N \left\{ c_1 \langle \chi_+ \rangle - \frac{c_2}{4m_N^2} \langle u^\mu u^\nu \rangle \left(D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[u_\mu, u_\nu \right] \right\} \Psi_N$$

$c_1 = -0.74,\ c_2 = 1.81,\ c_3 = -3.61,\ c_4 = 2.17\ \text{GeV}^{-1}$ *D. Siemens, et al.,PLB(2017)*

XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)

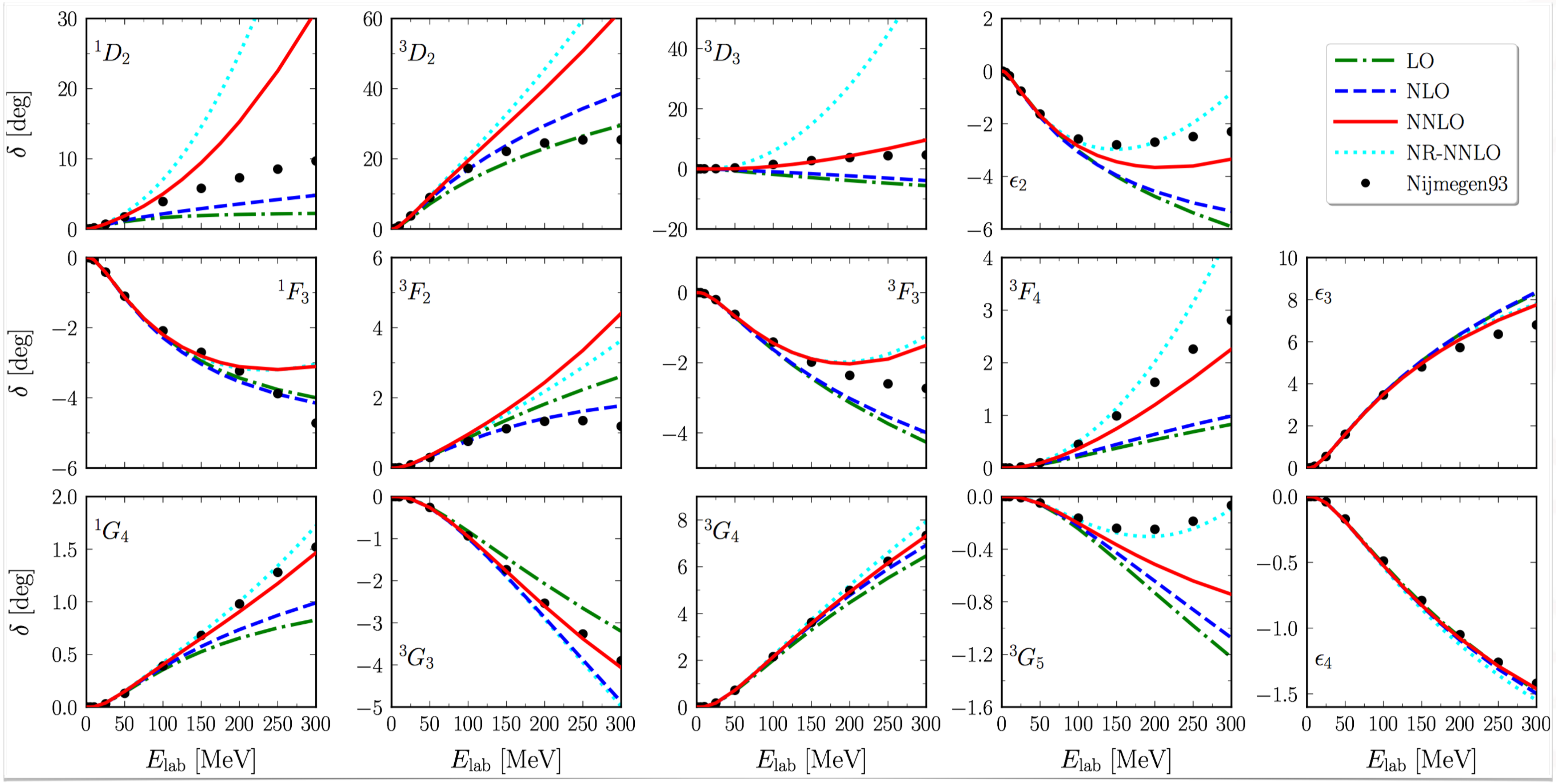
XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 113, 034002 (2026)

NN phase shifts at NLO and NNLO

- On-shell T-matrix under the Born approximation

$$T(p',p) = V_{\mathrm{OPE}}(p',p) + V^{(2)}_{2\pi,irr}(p',p) + V^{(3)}_{2\pi,irr}(p',p) + V_{\mathrm{OPE}}\,G\,V_{\mathrm{OPE}}$$

- **Prediction: phase shifts of D, F, G waves**



✓ Improve the description of D waves; globally similar results for F, G waves

- $^3G_5$: non-rel. result is accidental, c_i/m_N effect (N^{4}LO) is large *D. Entem, et al., PRC 91, 014002 (2015)*

XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)

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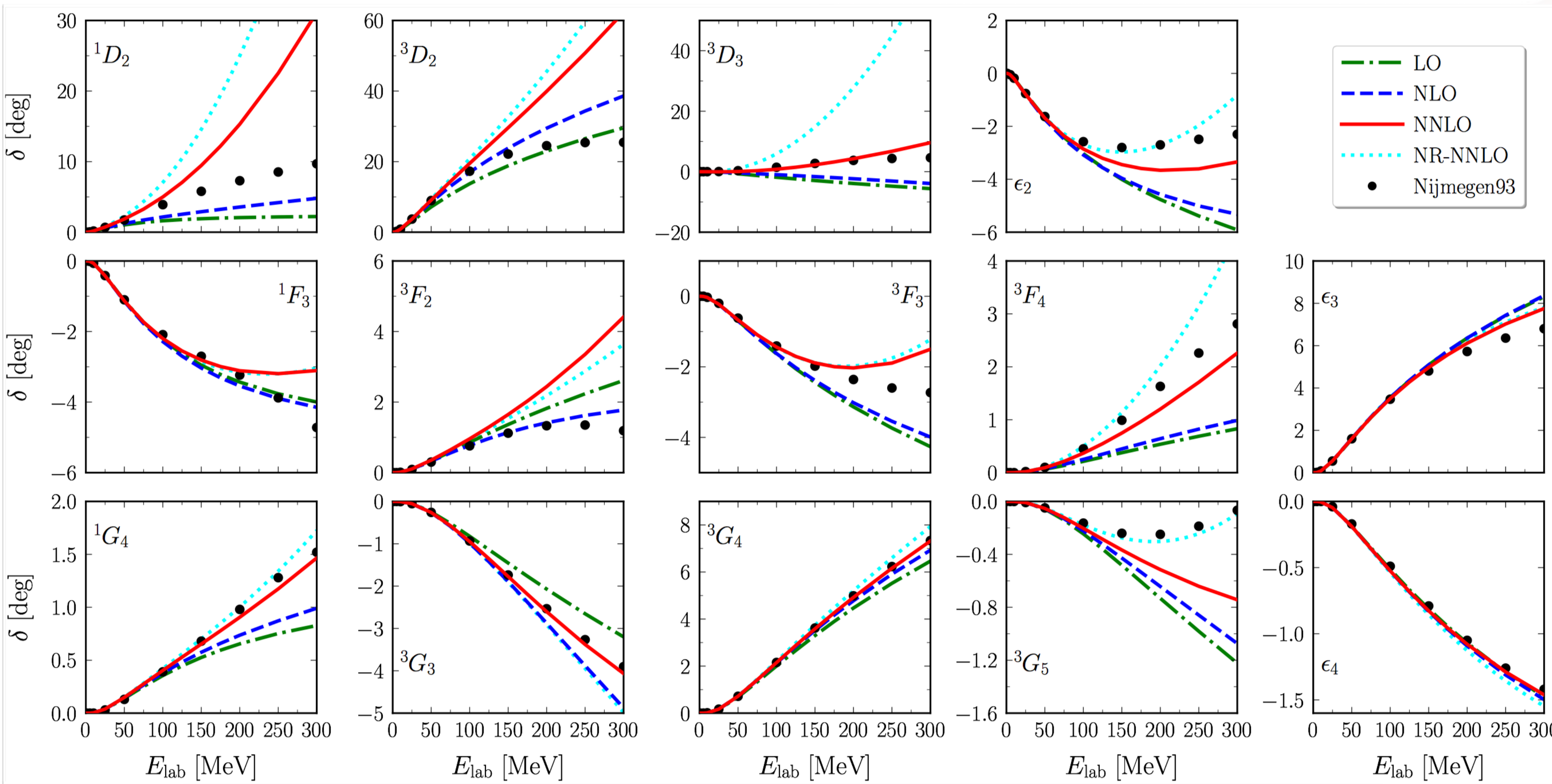
$$T(p', p) = V_{\rm OPE}(p', p) + V^{(2)}_{2\pi,irr}(p', p) + V^{(3)}_{2\pi,irr}(p', p) + V_{\rm OPE}\, G\, V_{\rm OPE}$$

- **Prediction: phase shifts of D, F, G waves**



✓ Improve the description of D waves; globally similar results for F, G waves

- $^3G_5$: non-rel. result is accidental, c_i/m_N effect (N^{4}LO) is large *D. Entem, et al., PRC 91, 014002 (2015)*

XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)

NN phase shifts at NLO and NNLO

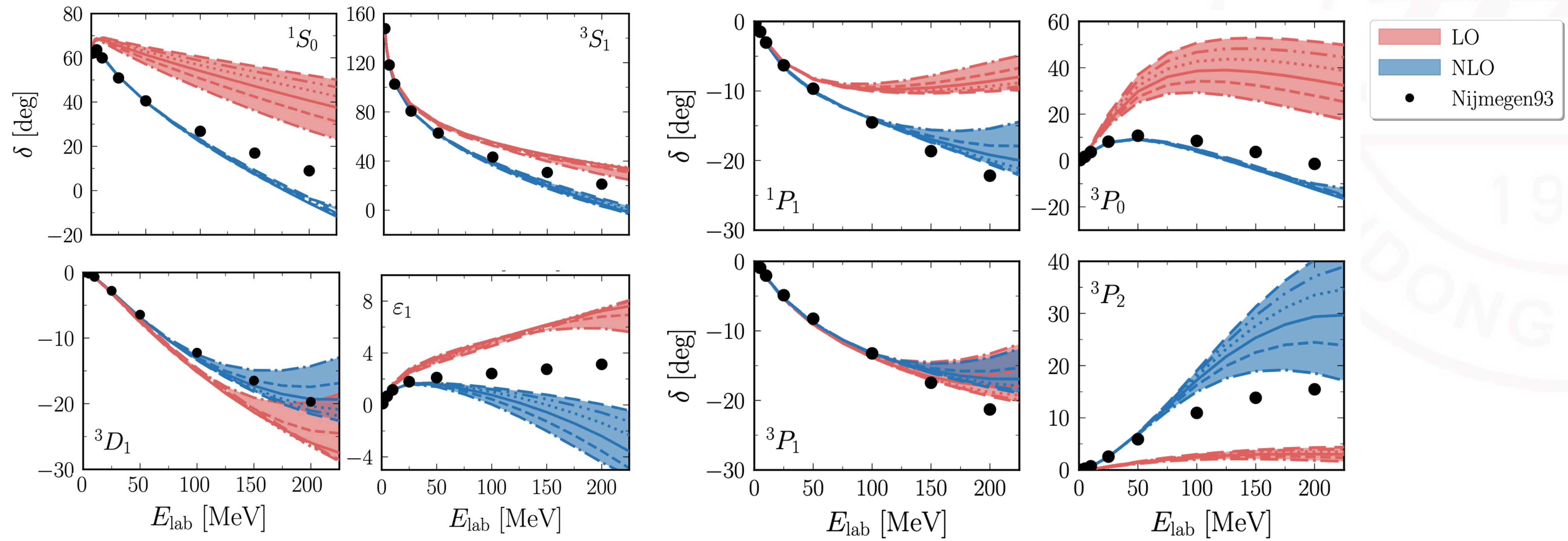
- Partial wave T-matrix to describe S, P waves
  - $V_{\text{NLO, NNLO}}$ non-perturbatively iterated in the Kadyshevsky equation

$$T_{ll'}^{sj}(p',p) = V_{ll'}^{sj}(p',p) + \sum_{l''}\int \frac{d^3k}{(2\pi)^3} V_{ll''}^{sj}(p',k)\,\frac{m_N^2}{2(k^2+m_N^2)}\,\frac{1}{\sqrt{p^2+m_N^2}-\sqrt{k^2+m_N^2}+i\epsilon}\,T_{l''l'}^{sj}(k,p)$$

  - Exponential regulator

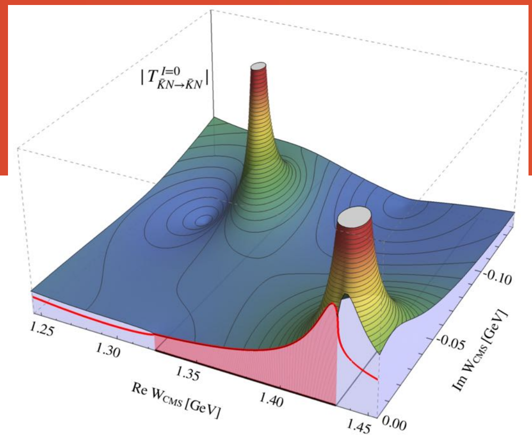
$$F(p) = \exp(-p^{2n}/\Lambda^{2n}),\ \text{with } n=2 \quad \Lambda = 400,\ldots,650 \text{ MeV}$$

- Phase shifts
  - Fit **NPWA ($E_{\text{lab}} \leq 50$ MeV)**: *np* scattering Nijmegen 93

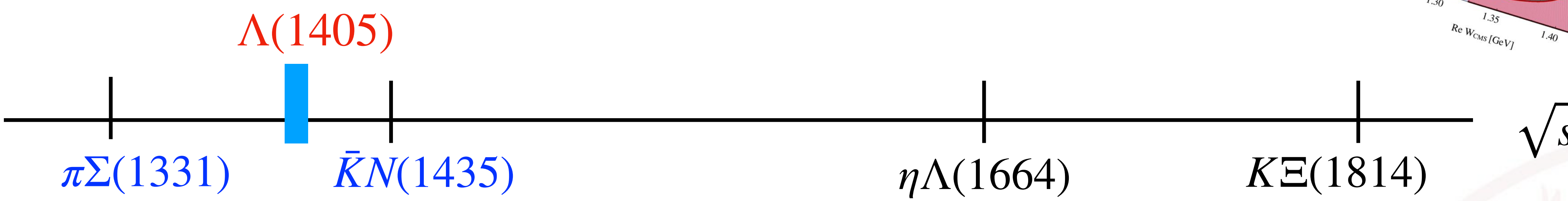


XLR, E. Epelbaum, J. Gegelia, PRC 113, 034002 (2026)

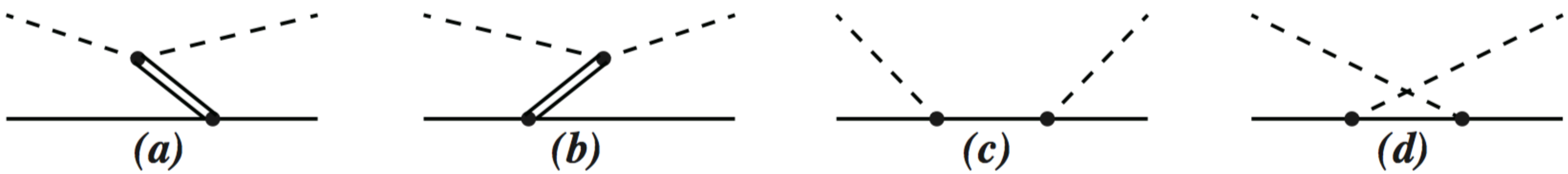
$\bar{K}N$ interaction at LO



- Four coupled channels $\bar{K}N, \pi\Sigma, \eta\Lambda, K\Xi$



- Focus on the S-wave potential



  - **Vector mesons included as explicit degrees of freedom**
    - ✓ VME potential instead of the Weinberg-Tomozawa term   Weinberg:1968de, Borasoy:1995ds
    - ✓ Improve the ultraviolet behaviour without changing the low-energy physics

- Solve coupled-channel scattering equation   **XLR, et al. EPJC 80 (2020) 406**
  - ✓ Taking into account the off-shell effects of potential
  - ✓ Using subtractive renormalization to obtain the renormalized T-matrix

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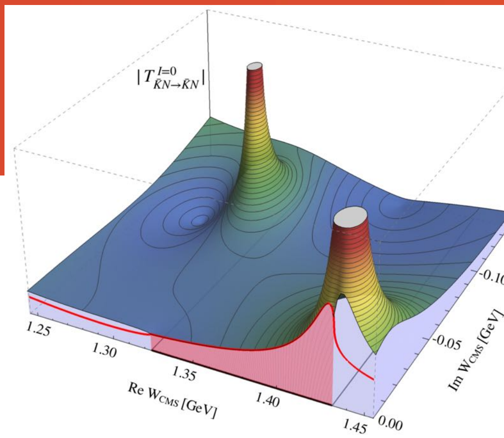

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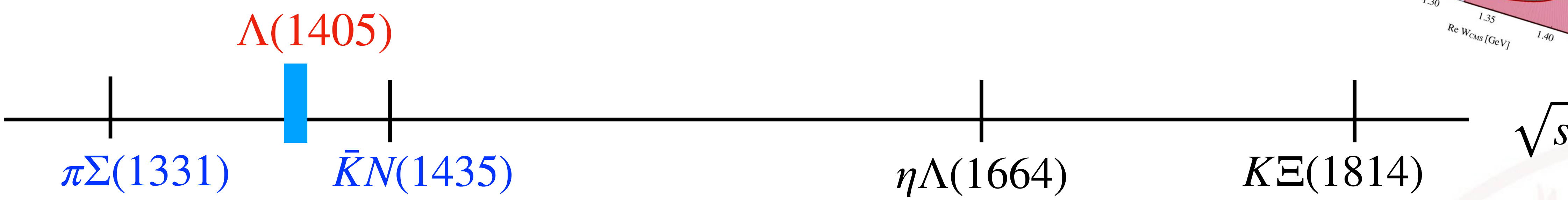

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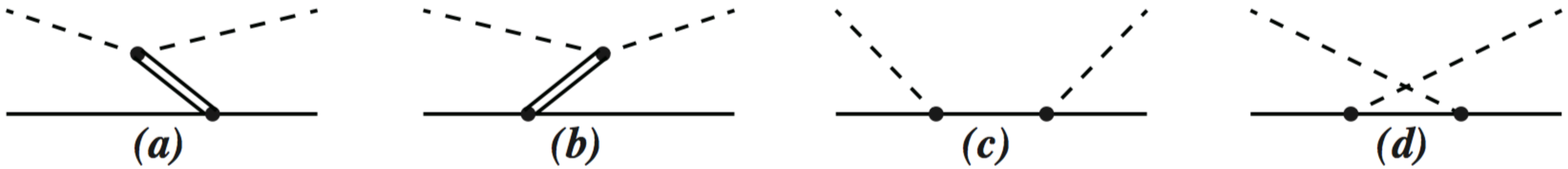

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No free parameters needed to be fitted!

$\bar{K}N$ interaction in TOPT

- Two pole positions of $\Lambda(1405)$ **XLR, et al. EPJC81 (2021) 582**

		lower pole	higher pole
This work (LO)	$F_0 = F_\pi$	$1337.7 - i\,79.1$	$1430.9 - i\,8.0$
	$F_0 = 103.4$	$1348.2 - i\,120.2$	$1436.3 - i\,0.7$
NLO	*Y. Ikeda,NPA(2012)*	$1381^{+18}_{-6} - i81^{+19}_{-8}$	$1424^{+7}_{-23} - i26^{+3}_{-14}$
	Z.-H.Guo,PRC(2013)-Fit II	$1388^{+9}_{-9} - i114^{+24}_{-25}$	$1421^{+3}_{-2} - i19^{+8}_{-5}$
	M.Mai,EPJA(2015)-sol-2	$1330^{+4}_{-5} - i56^{+17}_{-11}$	$1434^{+2}_{-2} - i10^{+2}_{-1}$
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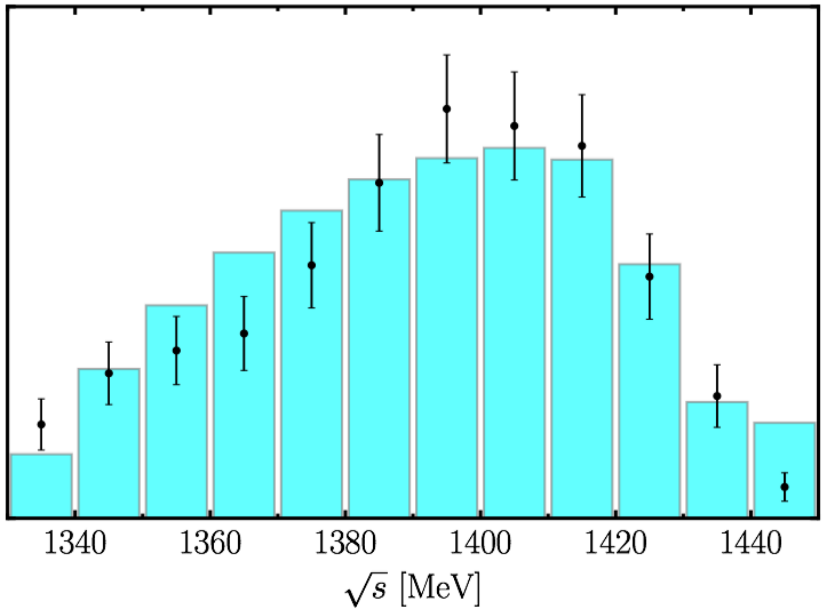
πΣ invariant mass spectrum



- **<u>Consistent with the NLO study by M. Mai at physical point</u>**

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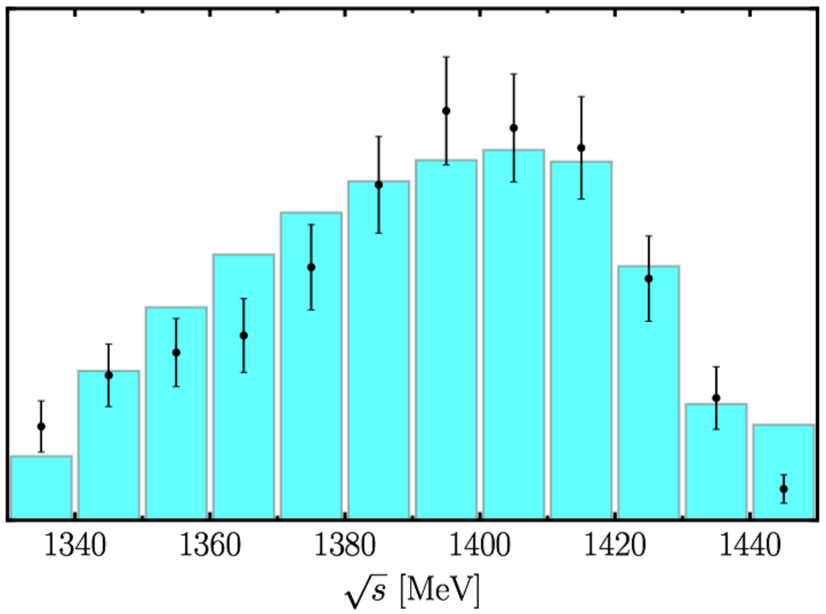


- **Consistent with the NLO study by M. Mai at physical point**

- Extend to the unphysical quark mass region $m_\pi \approx 200$ MeV, $m_K \approx 487$ MeV
  - **Consistent with the BaSc results** PRL 132, 051901 (2024)

	BaSc [PRL2024]	This work			
$\Lambda(1405)$	z_R [MeV]	z_R [MeV]	$g_{\pi\Sigma}$	$g_{\bar{K}N}$	$\lvert g_{\pi\Sigma}\rvert/\lvert g_{\bar{K}N}\rvert$
Lower pole	**1392(18)**	1387.14	$0.021 + i1.87$	$0.017 + i1.55$	1.21
Higher pole	**$1455(21) - i11.5(6.0)$**	$1469.86 - i4.71$	$0.038 + i0.98$	$1.51 - i1.22$	0.50

XLR, Phys. Lett. B 855 (2024) 138802

Meson-baryon intearction @ NLO

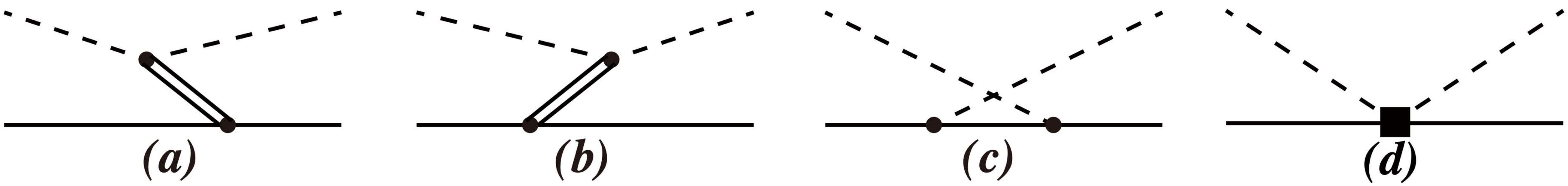
- **Maintain the scattering T-matrix renormalizable**
  - Take LO potential non-perturbatively
  - Include higher order corrections perturbatively
- Up to next-to-leading order
  - Potential: $V = V_{\rm LO} + V_{\rm NLO}$
  - T-matrix: $T = T_{\rm LO} + T_{\rm NLO}$

$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO} G T_{\rm LO}$$ (non-perturbative)

$$T_{\rm NLO} = V_{\rm NLO} + T_{\rm LO} G V_{\rm NLO} + V_{\rm NLO} G T_{\rm LO} + T_{\rm LO} G V_{\rm NLO} G T_{\rm LO}$$

  - Use **the subtractive renormalization** to remove divergent terms and power-counting breaking terms

KN interaction up to NLO



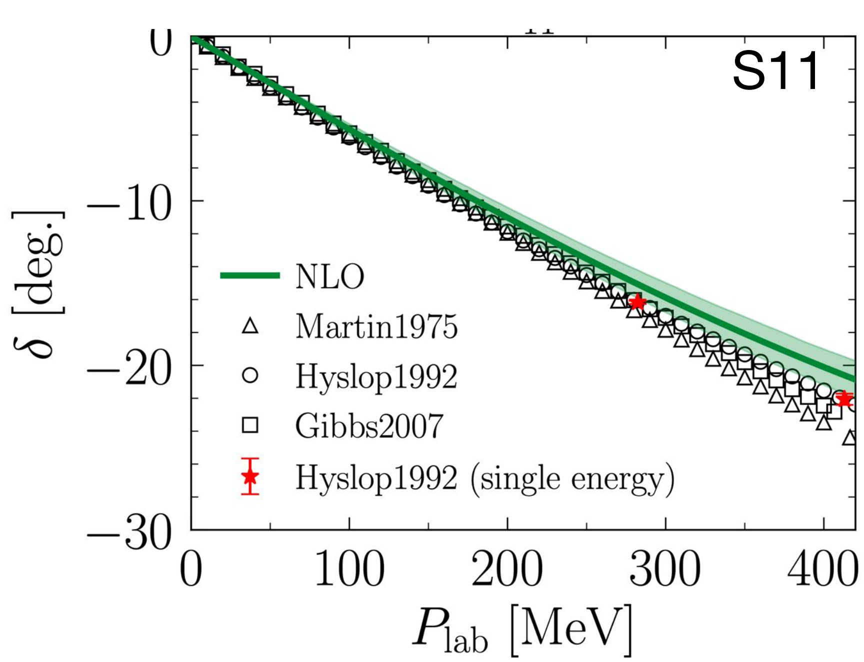
- Chiral effective Lagrangians at NLO
  - For S-wave scattering

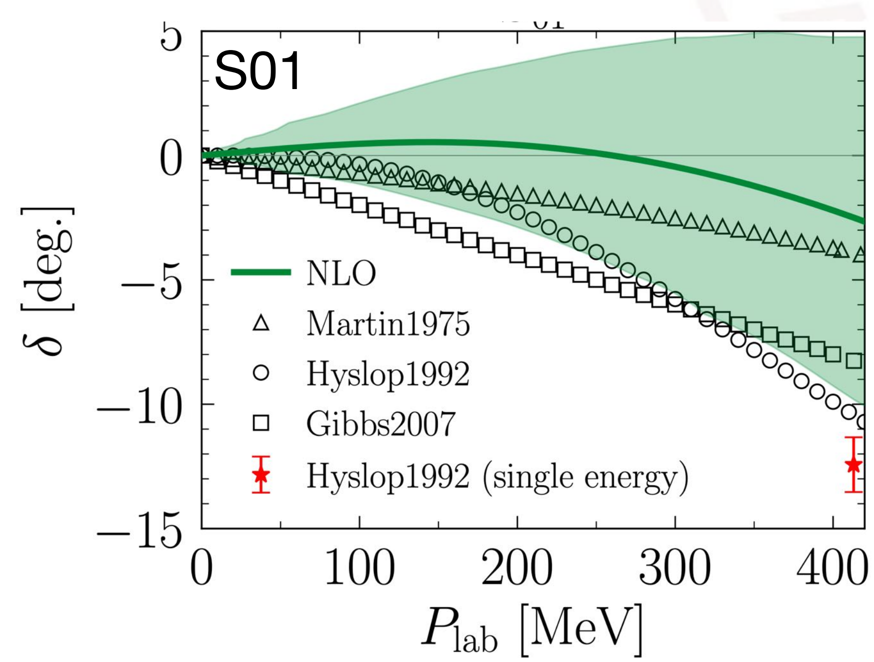
$$\mathscr{L}_{\rm NLO} = b_0\langle\chi_+\rangle\langle\bar{B}B\rangle + b_D\langle\bar{B}\{\chi_+,B\}\rangle + b_F\langle\bar{B}[\chi_+,B]\rangle$$
$$+ d_1\left\langle\bar{B}\{u_\mu,[u^\mu,B]\}\right\rangle + d_2\left\langle\bar{B}\left[u_\mu,[u^\mu,B]\right]\right\rangle$$
$$+ d_3\langle\bar{B}u_\mu\rangle\langle u^\mu B\rangle + d_4\langle\bar{B}B\rangle\langle u^\mu u_\mu\rangle,$$

➡ Four parameters at NLO

- Fixed by RQCD's $b_{0,D,F}$
- Determined by S_{11} and S_{01} scattering lengths

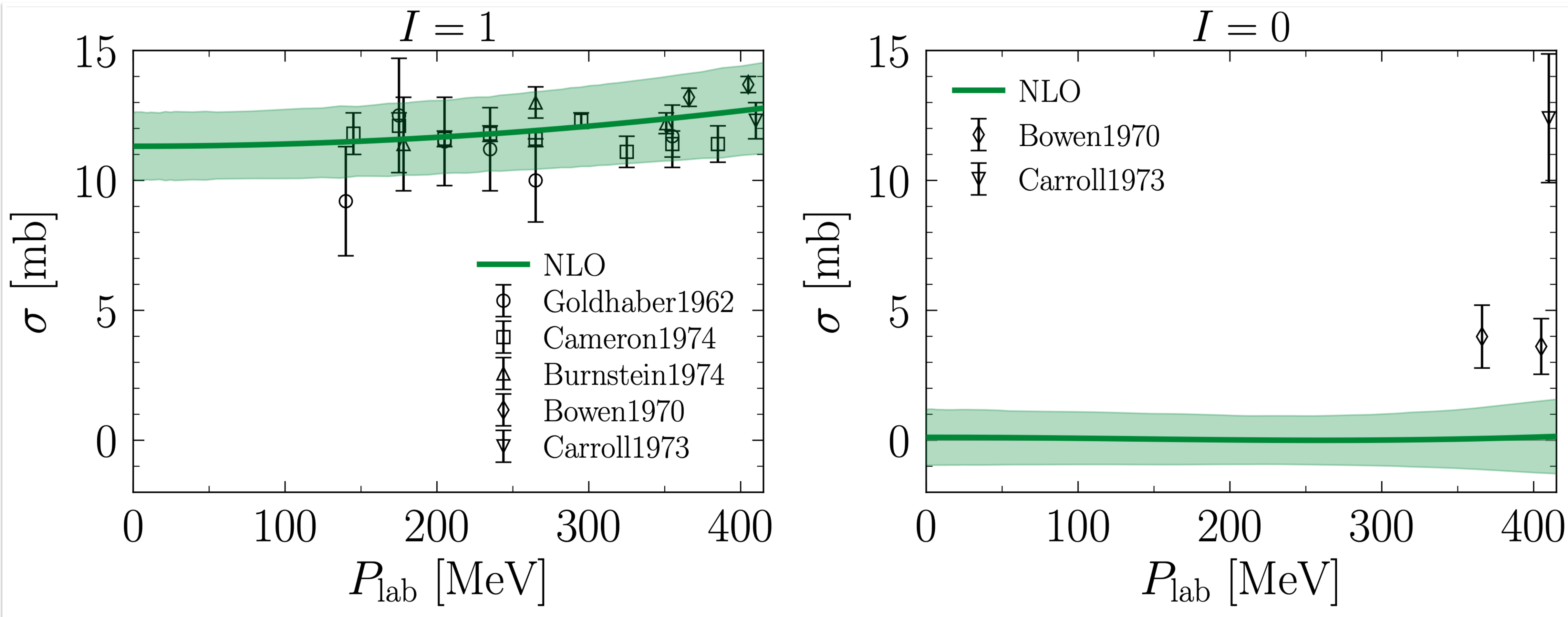
- Prediction of s-wave phase shifts **XLR, Phys. Rev. D 113 (2026) 114027**



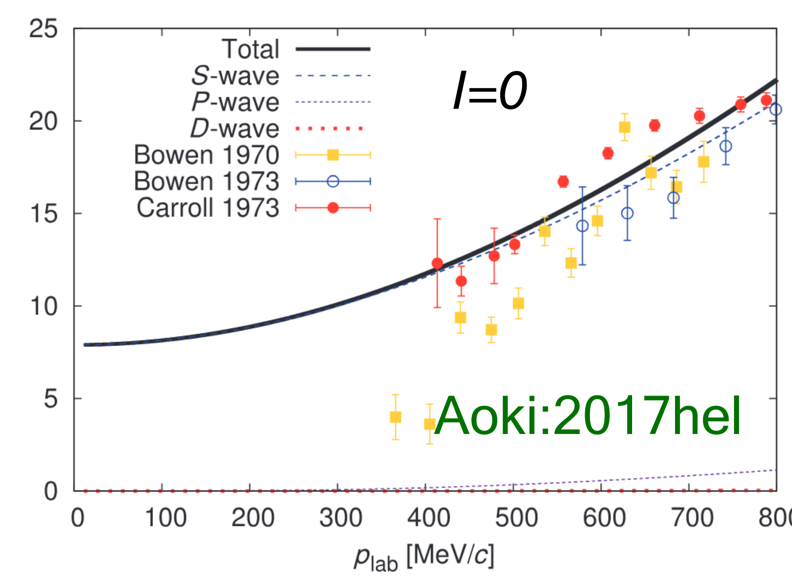


KN interaction up to NLO

- S-wave contributions to the total cross sections



- **<u>I=1 channel</u>**: **agrees well** with experimental data and slowly increases with Plab
- **<u>I=0 channel</u>**: NLO s-wave contribution small
  - ✓ P-wave contribution dominant !



XLR, Phys. Rev. D 113 (2026) 114027

Summary

- Tentatively proposed **a unified framework** to formulate the baryon-baryon and meson-baryon interactions
  - based on time-ordered perturbation theory
  - using covariant chiral effective Lagrangians
- Baryon-baryon interactions
  - Obtained **non-singular** LO BB potential
    - ✓ Avoid finite-cutoff artefacts and take cutoff $\Lambda \to \infty$
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Thank you for your attention!