

Precision measurement of branching fraction $\psi(2S) \rightarrow \tau^+ \tau^-$ at BESIII

B.X. Zhang, X.H. Mo, J.Y. Zhang, P. Wang

Institute of High Energy Physics , CAS
(for the BESIII Collaboration)

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Outline



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Introduction

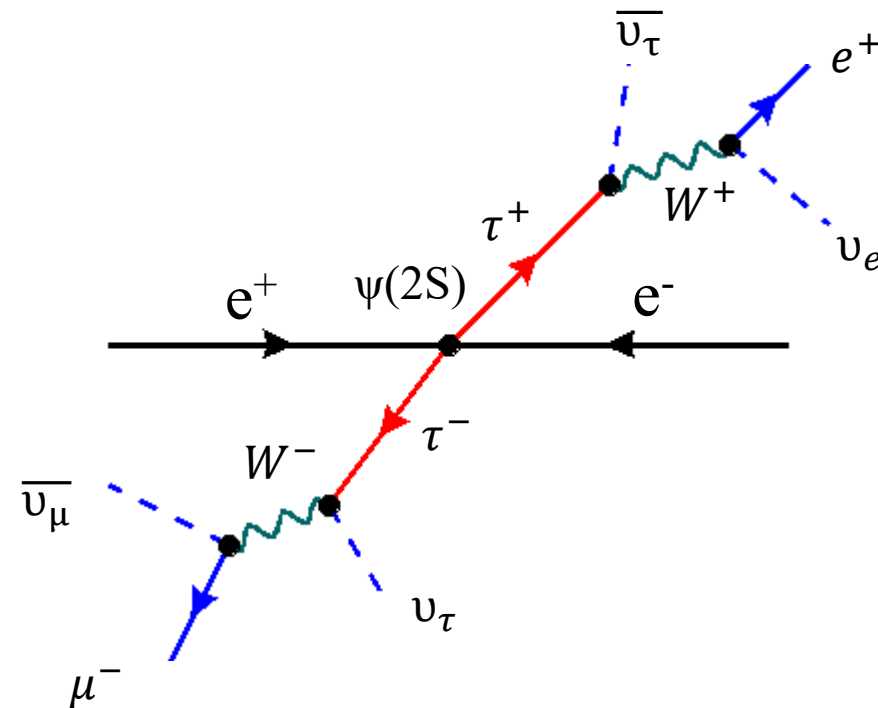
- $\psi(2S)$ decay to lepton pairs, the relationship of their branching fractions of these decays with Lepton Flavor Universality (LFU) hypotheses: $B_{ee} \cong B_{\mu\mu} \cong \frac{B_{\tau\tau}}{0.3890} \equiv B_{ll}$
- The combined results of B factory and LHCb show a deviation from the SM prediction by 3.1σ . the ratio read $R(D^*) = \frac{\Gamma(B \rightarrow D^{(*)} \tau^+ \nu_\tau)}{\Gamma(B \rightarrow D^{(*)} l^+ \nu_l)} \quad (l = e, \mu)$
- BESIII Collaboration have collected about 3B $\psi(2S)$ events, more precise measurements are expected



Data sets and MC simulation

- Data sets : $\psi(2S)$ 2259.3M (2021)
- Signal MC: $\psi(2S) \rightarrow \tau^\pm \tau^\mp (e^\pm \nu_e \nu_\tau \mu^\mp \nu_\mu \nu_\tau)$ (ConExc)

- Particle vpho 3.686 0.00093
- Decay vpho
- 1.0 ConExc -2 15 -15;
- Enddecay
- Decay tau-
- 1.0000 e- anti-nu_e nu_tau PHOTOS TAULNUNU;
- Enddecay
- Decay tau+
- 1.0000 mu+nu_mu anti-nu_tau PHOTOS TAULNUNU;
- Enddecay
- End





Data sets and MC simulation

- Background simulations:

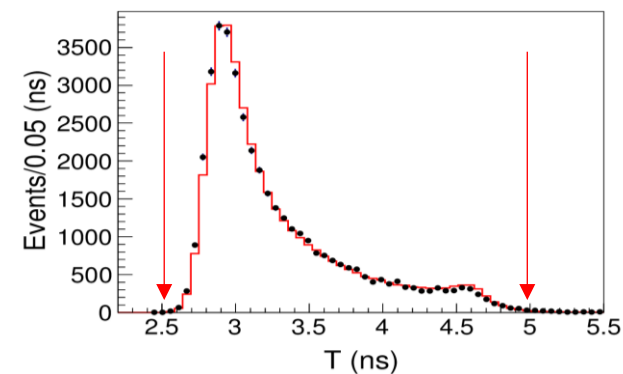
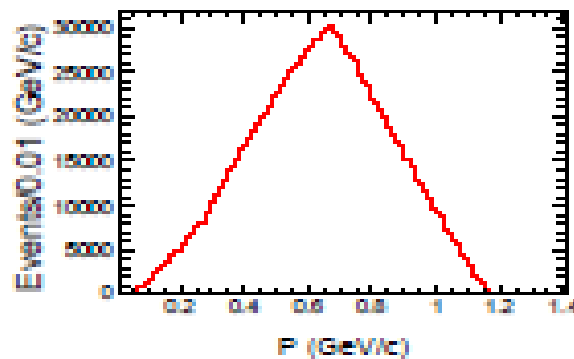
processes	MC generator	σ [nb]	No. of events	processes	MC generator	σ [nb]	No. of events
$\psi(2S) \rightarrow e^+ e^-$	PHOTOS VLL	PDG	500K	$e^+ e^- \rightarrow e^+ e^-$	Babayaga	544.13	500K
$\psi(2S) \rightarrow \mu^+ \mu^-$	PHOTOS VLL	PDG	500K	$e^+ e^- \rightarrow \mu^+ \mu^-$	Babayaga	6.70	500K
$\psi(2S) \rightarrow \pi^+ \pi^-$	PHOTOS VLL	PDG	500K	$e^+ e^- \rightarrow \gamma \gamma$	Babayaga	25.89	500K
$\psi(2S) \rightarrow anything$	Lundcharm	PDG+Model	1 Lum	$e^+ e^- \rightarrow e^+ e^- e^+ e^-$	DIAG36	414.0	1M
$\psi(2S) \rightarrow e \pi 3 \nu$	PHOTOS VLL	PDG	1M	$e^+ e^- \rightarrow e^+ e^- \mu^+ \mu^-$	DIAG36	30.49	1M
$\psi(2S) \rightarrow e \pi \pi^0 3 \nu$	PHOTOS VLL	PDG	1M	$e^+ e^- \rightarrow e^+ e^- \eta$	EKHARA	0.29	1M
$\psi(2S) \rightarrow \mu \pi 3 \nu$	PHOTOS VLL	PDG	1M	$e^+ e^- \rightarrow e^+ e^- \eta'$	EKHARA	0.26	1M
$\psi(2S) \rightarrow \pi \pi 2 \nu$	PHOTOS VLL	PDG	1M	$e^+ e^- \rightarrow e^+ e^- K^+ K^-$	GALUGA2.0	0.18	1M
				$e^+ e^- \rightarrow e^+ e^- \pi^+ \pi^-$	GALUGA2.0	2.33	1M



Event Selection

➤ Track level

- Charged track ($V_r < 1\text{cm}$, $|V_z| < 10\text{cm}$, $|\cos\theta| < 0.93$, $p_{\text{trk}} < 1.2\text{GeV}$, vertex fit)
- Neutral track ($E_{\text{mc}} > 25\text{MeV}$ (barrel), $E_{\text{mc}} > 50\text{MeV}$ (Endcap), $0 < T_{\text{EMC}} < 14$ (50ns), $\theta_{\gamma, \text{trk}} > 10^\circ$)
- PID for electron ($0.8 < E/p < 1.2$, $\chi^2_{dE/dx} < 4$, $\Delta\text{tof} < 0.3\text{ns}$)
- PID for muon ($E/p < 0.7$, $\chi^2_{dE/dx} < 4$, $\Delta\text{tof} < 0.3\text{ns}$, $\mu_{\text{dep}} > 81 \cdot (p_{\text{trk}} - 0.65)\text{cm}$)
- $2.5 < \text{Tof} < 5\text{ ns}$

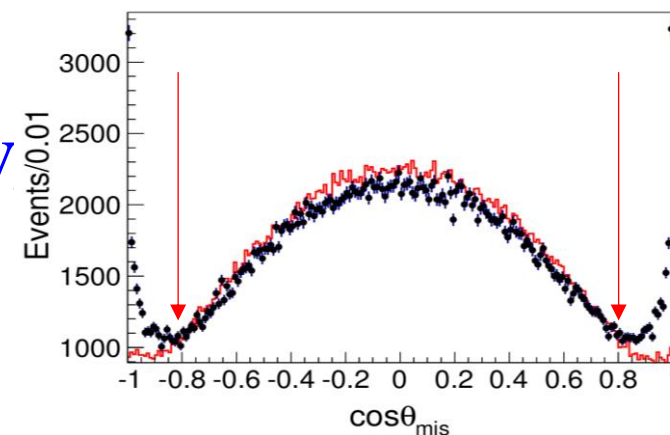
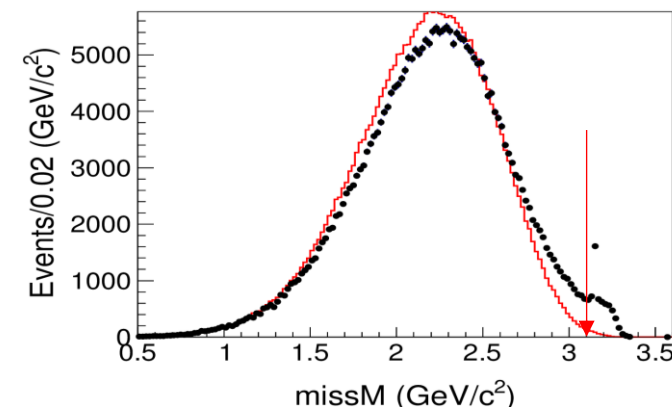
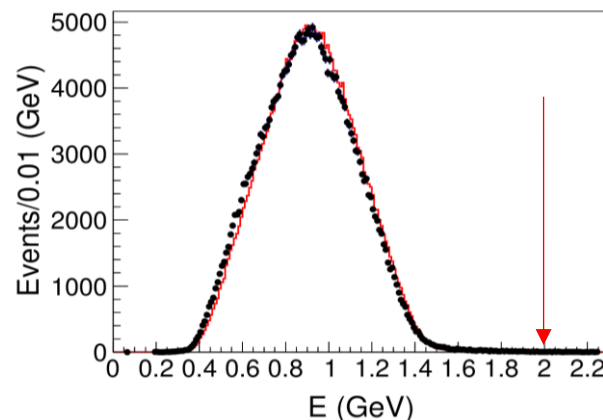




Event Selection(II)

➤ Event level

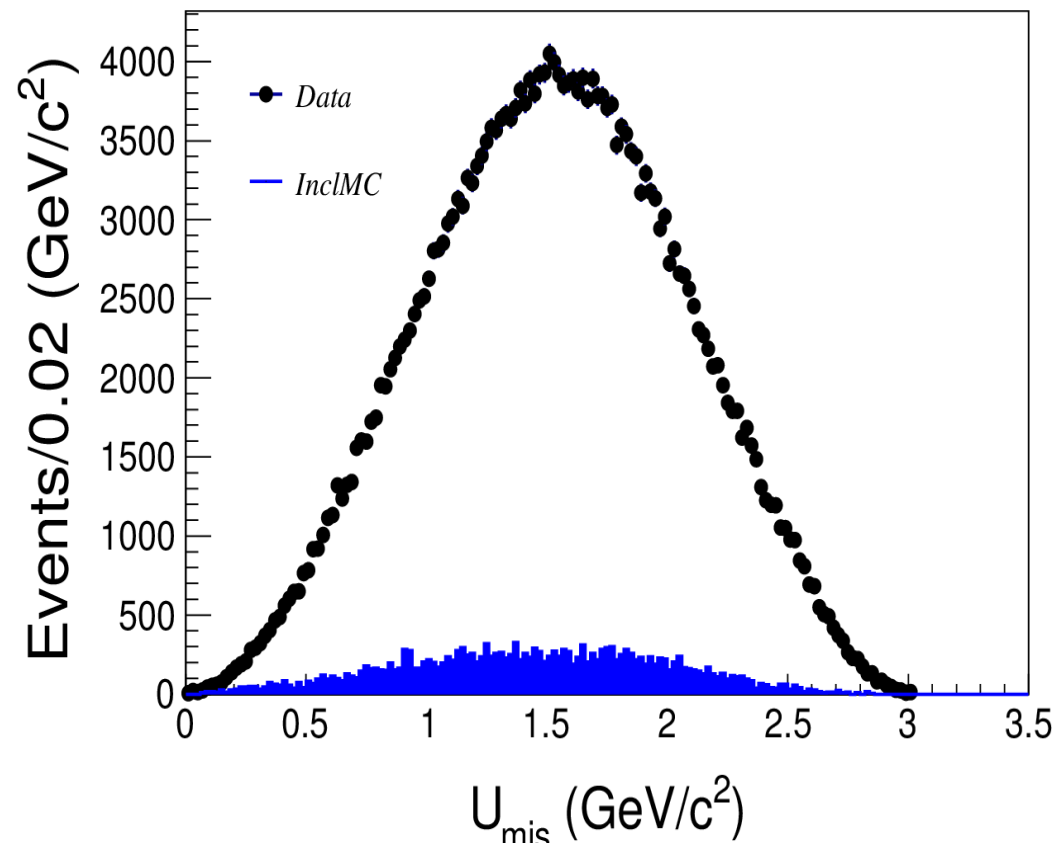
- $N_{\text{charged}}=2$
- $N_{\gamma}=0$
- Sum of deposited energy of the two charged tracks less than 2.0 GeV
- Mass of the missing particles less than 3.05 GeV ($M_{\text{mis}} < 3.05 \text{ GeV}$)
- Polar angle of the missing particles $|\cos\theta| < 0.8$





Background analysis

No.	decay chain	final states	iTopology	nEvt	nTot
0	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau, \tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau$	$e^+\bar{\nu}_\mu\bar{\nu}_\tau\nu_e\mu^-\nu_\tau$	0	64245	64245
1	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \mu^+\nu_\mu\bar{\nu}_\tau, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$	$\bar{\nu}_e\mu^+\bar{\nu}_\tau e^-\nu_\mu\nu_\tau$	1	63262	127507
2	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau, \tau^- \rightarrow \pi^-\nu_\tau$	$e^+\bar{\nu}_\tau\pi^-\nu_e\nu_\tau$	2	5045	132552
3	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \pi^+\bar{\nu}_\tau, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$	$\bar{\nu}_e\bar{\nu}_\tau e^-\nu_\tau\pi^+$	4	4855	137407
4	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \mu^+\nu_\mu\bar{\nu}_\tau, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau\gamma_{FSR}$	$\bar{\nu}_e\mu^+\bar{\nu}_\tau e^-\nu_\mu\nu_\tau$	5	4166	141573
5	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau, \tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau\gamma_{FSR}$	$e^+\bar{\nu}_\mu\bar{\nu}_\tau\nu_e\mu^-\nu_\tau$	3	4141	145714
6	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau, \tau^- \rightarrow \pi^-\nu_\tau\gamma_{FSR}$	$e^+\bar{\nu}_\tau\pi^-\nu_e\nu_\tau$	10	382	146096
7	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \pi^+\pi^0\bar{\nu}_\tau, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau\gamma_{FSR}$	$\bar{\nu}_e\bar{\nu}_\tau e^-\pi^0\nu_\tau\pi^+$	18	325	146421
8	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau, \tau^- \rightarrow \nu_\tau\pi^-K^0, K^0 \rightarrow K_L$	$e^+\bar{\nu}_\tau\pi^-\nu_eK_L\nu_\tau$	14	191	146612
9	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \mu^+\nu_\mu\bar{\nu}_\tau\gamma_{FSR}, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau\gamma_{FSR}$	$\bar{\nu}_e\mu^+\bar{\nu}_\tau e^-\nu_\mu\nu_\tau$	12	163	146775
10	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau\gamma_{FSR}\gamma_{FSR}, \tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau$	$e^+\bar{\nu}_\mu\bar{\nu}_\tau\nu_e\mu^-\nu_\tau$	11	153	146928
11	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau, \tau^- \rightarrow \pi^-\pi^-\pi^+\nu_\tau$	$e^+\bar{\nu}_\tau\pi^-\pi^-\nu_e\nu_\tau\pi^+$	13	141	147069
12	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \bar{\nu}_\tau\pi^+K^0, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau, K^0 \rightarrow K_L$	$\bar{\nu}_e\bar{\nu}_\tau e^-\pi^+K_L\nu_\tau\pi^+$	19	132	147201
13	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \pi^+\pi^+\pi^-\bar{\nu}_\tau, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$	$\bar{\nu}_e\bar{\nu}_\tau\pi^-\pi^-\nu_\tau\pi^+\pi^+$	22	124	147325
14	$\psi' \rightarrow \tau^+\tau^-\gamma_{FSR}, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau, \tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau$	$e^+\bar{\nu}_\mu\bar{\nu}_\tau\nu_e\mu^-\nu_\tau$	17	52	147377
15	$\psi' \rightarrow \tau^+\tau^-\gamma_{FSR}, \tau^+ \rightarrow \mu^+\nu_\mu\bar{\nu}_\tau, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$	$\bar{\nu}_e\mu^+\bar{\nu}_\tau e^-\nu_\mu\nu_\tau$	24	52	147429
16	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \mu^+\nu_\mu\bar{\nu}_\tau, \tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau$	$\mu^+\bar{\nu}_\mu\bar{\nu}_\tau\mu^-\nu_\mu\nu_\tau$	20	51	147480
17	$\psi' \rightarrow J/\psi\pi^+\pi^-, J/\psi \rightarrow e^+e^-\gamma_{FSR}$	$e^+\pi^-\pi^+\pi^+$	8	43	147523
18	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \pi^+\pi^0\bar{\nu}_\tau, \tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau$	$\bar{\nu}_\mu\bar{\nu}_\tau\pi^0\mu^-\nu_\tau\pi^+$	40	34	147557
19	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow K^+\bar{\nu}_\tau, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$	$\bar{\nu}_e\bar{\nu}_\tau e^-\nu_\tau K^+$	9	31	147588
20	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau, \tau^- \rightarrow K^-\nu_\tau$	$e^+\bar{\nu}_\tau K^-\nu_e\nu_\tau$	63	24	147612
21	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \mu^+\nu_\mu\bar{\nu}_\tau, \tau^- \rightarrow \pi^-\nu_\tau$	$\mu^+\bar{\nu}_\tau\pi^-\nu_\mu\nu_\tau$	16	22	147634
22	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau\gamma_{FSR}, \tau^- \rightarrow \pi^-\nu_\tau$	$e^+\bar{\nu}_\tau\pi^-\nu_e\nu_\tau$	21	16	147650
23	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \bar{\nu}_\tau\pi^+K^0\pi^0, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau\gamma_{FSR}, K^0 \rightarrow K_L$	$\bar{\nu}_e\bar{\nu}_\tau e^-\pi^0K_L\nu_\tau\pi^+$	56	13	147663
24	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$	$e^+\bar{\nu}_\tau e^-\nu_\tau\nu_\tau$	41	11	147674
25	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \bar{\nu}_\tau\pi^+\pi^-\pi^+\pi^0, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau\gamma_{FSR}$	$\bar{\nu}_e\bar{\nu}_\tau\pi^-\pi^-\pi^+\pi^+\pi^0\nu_\tau\pi^+\pi^+$	54	11	147685
26	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau\gamma_{FSR}, \tau^- \rightarrow \pi^-\pi^-\pi^+\nu_\tau$	$e^+\bar{\nu}_\tau\pi^-\pi^-\nu_e\nu_\tau\pi^+$	15	9	147694
27	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow \pi^+\pi^0\bar{\nu}_\tau, \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau\gamma_{FSR}\gamma_{FSR}$	$\bar{\nu}_e\bar{\nu}_\tau e^-\pi^0\nu_\tau\pi^+$	64	9	147703
28	$\psi' \rightarrow \tau^+\tau^-, \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau\gamma_{FSR}, \tau^- \rightarrow \nu_\tau\pi^-K^0, K^0 \rightarrow K_L$	$e^+\bar{\nu}_\tau\pi^-\nu_eK_L\nu_\tau$	55	7	147710
29	$\psi' \rightarrow J/\psi\pi^+\pi^-, J/\psi \rightarrow e^+e^-\gamma_{FSR}\gamma_{FSR}$	$e^+\pi^-\pi^+\pi^+$	42	7	147717



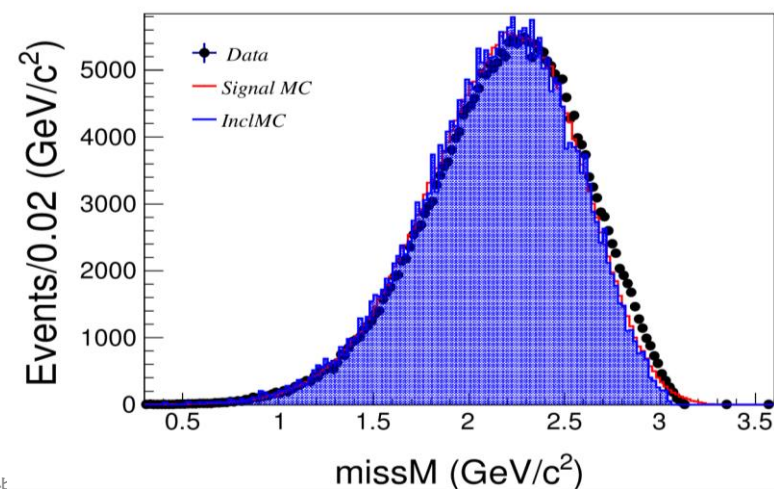
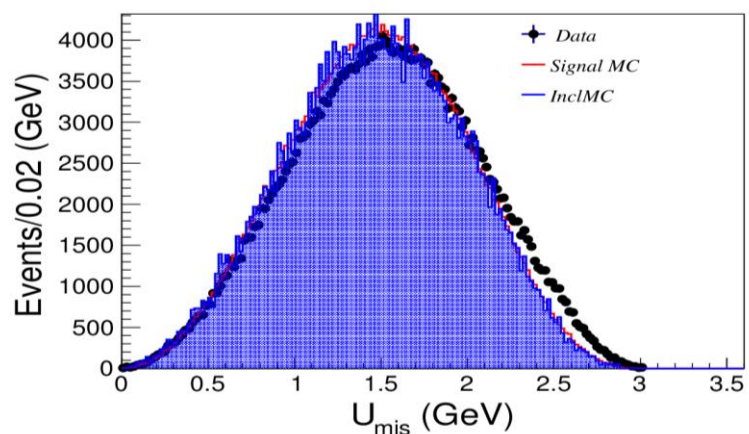
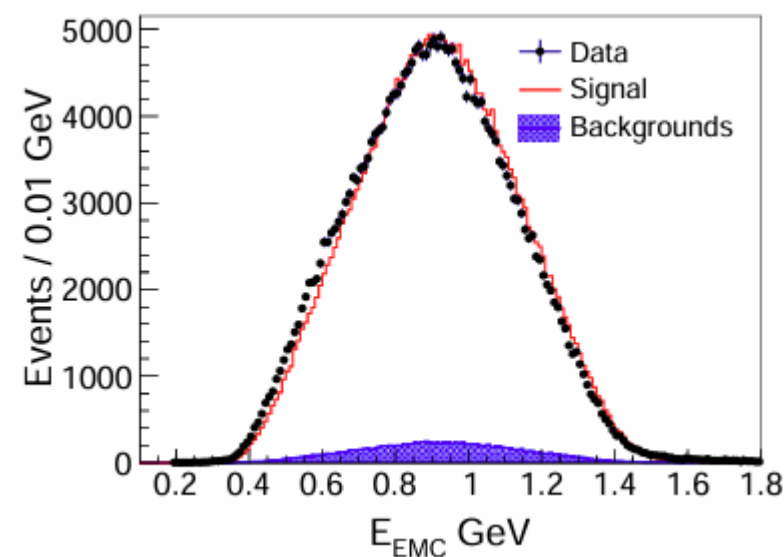
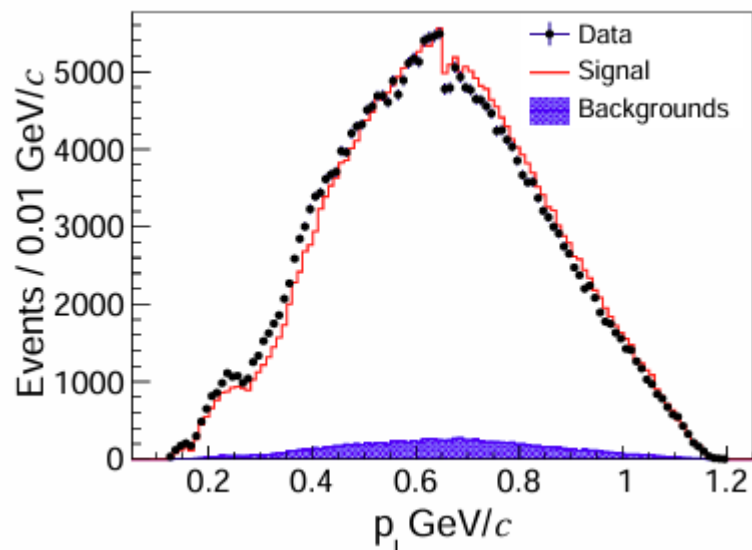
$$N_{\text{bkg}} = 11531$$

Cut flow

Criteria	efficiency	requirements
Basic selection	0.8206	Ntrk=2, $ \cos\theta <0.93$, $\Sigma Q=0$, vertexFit Emc>0.025 GeV(Barrel) Emc>0.050GeV
emuDecay=1	0.4245	Electron muon events requirement
Tof requirement 1	0.4233	$2.5<TOF1<5.0$
Tof requirement 2	0.4221	$2.5<TOF2<5.0$
Ngam = 0	0.3503	Number of photon is zero
$M_{\text{mis}}<3.05 \text{ GeV}$	0.3491	Missing mass requirement
$ \cos\theta_{\text{mis}} <0.8$	0.3086	Absolute value of missing $\cos\theta$
EnergyDep<2.0	0.3065	Deposited energy of two tracks in EMC



Comparison of MC and Data





Cross section of ($e^+e^- \rightarrow \tau\tau$)



$$\sigma_{\text{exp}}(s, m_\tau, \Delta) = \int_0^\infty d\sqrt{s'} G(\sqrt{s'}, \sqrt{s}) \times \int_0^{1 - \frac{4m_\tau^2}{s'}} dx F(x, s') \frac{\bar{\sigma}_{\text{tot}}(s'(1-x), m_\tau)}{|1 - \Pi(s'(1-x))|^2} \quad [1]$$

Δ Energy spread
 $\sqrt{s}$ nominal C.M. energy
 $\sqrt{s'}$ actual C.M. energy

$$F(x, s) = \beta x^{\beta-1} \left[1 + \frac{3}{4}\beta + \frac{\alpha}{\pi} \frac{\pi^2}{2} - \frac{1}{2} + \beta^2 \frac{9}{32} - \frac{1}{12}\pi^2 \right] - \beta \left(1 - \frac{1}{2}x \right) + \frac{1}{8}\beta^2 [4(2-x) \ln \frac{1}{x} - \frac{1+3(1-x^2)}{x} \ln(1-x) - 6+x]$$

$$\beta = \frac{2\alpha}{\pi} \left(\ln \frac{s}{m_e^2} - 1 \right)$$

[1] E. A. Kuraev, V. S. Fadin and S. Victor Sov. J. Nucl. Phys. 41, 466 (1985).



Cross section of ($ee \rightarrow \tau\tau$)(II) BESIII

$$G(\sqrt{s'}, \sqrt{s}) = \frac{1}{2\pi\Delta} e^{-\frac{(\sqrt{s}-\sqrt{s'})^2}{2\Delta}} \quad [2]$$

$$\bar{\sigma}_{\text{tot}}(s) = F_s(s) \cdot (\sigma_{\text{con}}(s) + \sigma_{\text{int}}(s) + \sigma_{\text{res}}(s)) / \sigma_{\text{con}}(s)$$

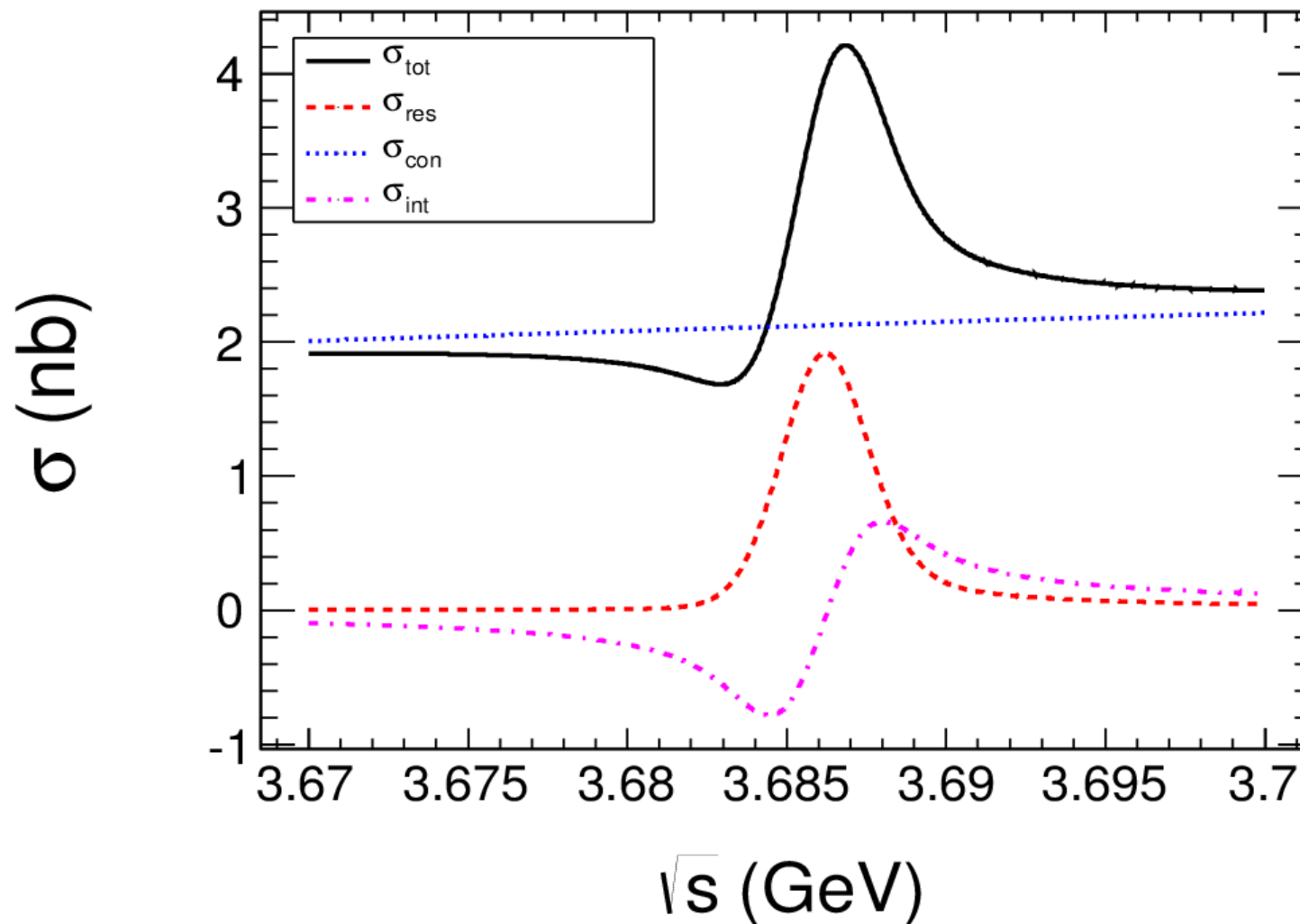
$$\begin{aligned} \sigma_{\text{con}}(s) &= \frac{4\pi\alpha^2}{3s}, & F_s(s) &= \sigma_{\text{con}}(s) \cdot \frac{\pi(3-v^2)}{1 - \exp(-\pi\alpha/v)} \\ \sigma_{\text{res}}(s) &= \frac{12\pi\Gamma_e^2}{(s - M_R^2)^2 + M_R^2\Gamma_\tau^2}, & v^2 &\equiv 1 - \frac{4m_\tau^2}{s}. \\ \sigma_{\text{int}}(s) &= \frac{8\alpha\Gamma_e}{\pi\sqrt{s}} \frac{s - M_R^2}{(s - M_R^2)^2 + M_R^2\Gamma_\tau^2} \end{aligned}$$

[2] P. Wang, C. Z.Yuan, X. H. Mo and D. H. Zhang Phys. Lett. B 593, 89 (2004).



Cross section ($ee \rightarrow \tau\tau$)

BESIII



$$\sigma_{Q+I} = 2.125 \text{ nb}$$



Branching ratio calculation

$$B(\tau\tau) = \frac{N_{e\mu} - N_{bg}}{B\epsilon} - \frac{\sigma_{Q+I}\mathcal{L}}{N_{\psi(2S)}}$$

This term is estimated by QED calculation

- B fraction of $\tau^+\tau^-$ events yielding the $e\mu$ topology. 0.6190 (PDG)
- $N_{e\mu}, N_{bg}, N_{\psi(2S)}$ Events number of $e\mu$, background and $\psi(2S)$
- ϵ detection efficiency
- σ_{Q+I} QED production cross section 2.125nb
- $\mathcal{L}$ the accumulated luminosity $\psi(2S)$



Branching ratio calculation(II)



Nobs	Nbkg	$\mathcal{L}$ (pb^{-1})	ϵ	$\sigma(\text{nb})$	$N_{\psi'}(10^6)$	$\text{Br}(10^{-3})$
280412	11839	3208.5	0.3065	2.125	2259.3	$3.240 \pm 0.023(\text{sta.})$ $\pm 0.081(\text{sys.})$
This work $(3.240 \pm 0.084) \times 10^{-3}$						

$(3.1 \pm 0.4) \times 10^{-3}$ (PDG)



Systematic uncertainty



• tracking efficiency	1%	[3][4]
• $\psi(2S)$ total number	0.5%	[5]
• MC statistic $\frac{1}{\sqrt{N}} \frac{\sqrt{(1-\varepsilon)}}{\sqrt{\varepsilon}}$	0.1%	
• Tau branching fraction	0.3% (PDG)	[6]
• Luminosity	1.1%	[7]

[3] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 92, 112008 (2015).

[4] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 122, 071802 (2019).

[5] M. Ablikim et al. (BESIII Collaboration), Chin. Phys. C 48, 093001 (2024).

[6] R. L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022).

[7] C.M. Carloni Calame, G. Montagna, O. Nicrosini and F. Piccinini Nucl. Phys. B - (Proc. Suppl.) 131, 48 (2004).



Systematic uncertainty (II)

- PID for e & μ 1.2%
- μ and π separation 0.5%
- N_γ requirement 1.0%
- M_{mis} requirement 0.8%
- $|\cos\theta_{mis}|$ requirement 0.1%
- Background rejection 0.3%
- cross section calculation 0.4%
- Total systematic uncertainty 2.5%

[8] 2502.19850[hep-ex]

Branching ratio of lepton pairs

$B_{ee} (10^{-3})$	$B_{\mu\mu}(10^{-3})$	$B_{\tau\tau} / 0.3890 (10^{-3})$
7.93 ± 0.17	8.00 ± 0.60	8.33 ± 0.24





Summary

- The branching fraction $\mathcal{B}_{\tau\tau}$ of $\psi(2S) \rightarrow \tau^+\tau^-$ has been measured with improved precision.
- Assuming lepton universality, the average value of branching fraction of lepton pairs $\mathcal{B}_{ll}$ are determined and total width of resonance $\psi(2S)$ has been calculated.
- This result, along with previous data of $\mathcal{B}_{ee}$ and $\mathcal{B}_{\mu\mu}$ is in agreement with the relation predicted by the sequential lepton hypothesis.

Thanks for your attention !

Systematic uncertainty study (back up 1)

To estimate the uncertainties due to the particle identification of the electron and muon, the two processes $e^+e^- \rightarrow \gamma e^+e^-$ and $e^+e^- \rightarrow \gamma \mu^+\mu^-$ are selected as control samples with $\psi(2S)$ data. For the former, the corresponding MC sample is generated with the BABAYAGA [23]. And for the latter process, the KKMC generator is used. The difference of the PID efficiency between data and MC for the e and μ tracks is obtained using a weighted method and is found to be 0.9% and 0.8%, respectively. Accordingly, 1.2% is taken as the combined systematic uncertainty from the PID of the electron and muon.

e & μ PID' uncertainty

To investigate the uncertainty associated with the requirement applied to distinguish between pions and muons, $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ and $J/\psi \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$ control samples are used, and corresponding MC samples are generated. The maximum relative difference of the detection efficiencies between data and MC simulation obtained with and without this additional requirement is found to be 0.2%. Taking into account the pions background contamination estimated in Sec. III, the systematic uncertainty on the branching fraction is calculated to be 1.0% in the decay $\psi(2S) \rightarrow \tau^+\tau^-$.

π & μ difference uncertainty

Systematic uncertainty study (back up 2)

The uncertainty associated with the number of photons is estimated from the efficiencies between data and inclusive MC simulation with and without the photon number requirement, and the difference is found to be 0.2%.

$N_\gamma=0$ uncertainty

The uncertainty associated with the background contamination is estimated by changing the branching fractions of the corresponding background processes by one standard deviation. The resulting change in the measurement result, 0.3%, is considered as the systematic uncertainty.

Background uncertainty

To evaluate the systematic uncertainty associated with the missing mass M_{mis} and $\cos\theta_{\text{mis}}$, we vary the missing mass from 3.05 to 3.10 GeV/c^2 or $3.00/c^2$ GeV . The maximum difference of the measured branching fraction, 0.8%, is taken as the systematic uncertainty. Similarly, the relative difference is regarded as the uncertainty by varying $\cos\theta_{\text{mis}}$ from 0.80 to 0.75 or 0.85, which is approximately 0.1%.

M_{mis} & $|\cos\theta|$ uncertainty

Taking into account the uncertainty associated with the c.m. energy measurement, which is 1 MeV from the energy measurement system and 0.4 MeV from the energy spread conservatively, a resulting difference in the QED cross section calculation, 0.4%, is regarded as the uncertainty.

$\sqrt{s}$ and Δ uncertainty