Huazhong University of Science and Technology
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Progress on Inclusive Charm Decay

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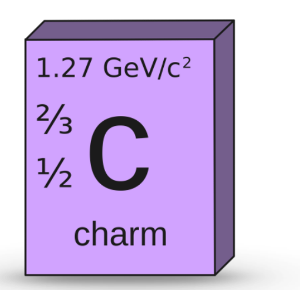

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Content

Motivation: Inclusive Charm Decay as a Probe

- **Why Inclusive ?**

Compare with exclusive : Better theoretical control

More important with more experiments

BESIII、STCF、Belle II

- **Why Charm ?**

Theoretical aspect

QCD & High Power correction don’t be ignored

Test the Convergence of QCD & HQE under some quark mass scheme

As a probe to seek for new physics

Motivation: Inclusive Charm Decay as a Probe

- **Why Inclusive ?**

  Compare with exclusive : Better theoretical control

  More important with more experiments

  BESIII、STCF、Belle II、LHCb

- **Why Charm ?**

  Phenomenological aspect

  - **Determine the SM fundamental Parameters ——CKM matrix**
  - **Determine the HQE parameters**

    ——As necessary inputs for Phenomenon Predictions
  - **Test the Lepton Flavor Universe**
  - **…**

Theoretical Progress in Charm Inclusive Decay

Optical theorem

$$\sum \langle D|H|X\rangle\langle X|H|D\rangle \propto \mathrm{Im}\int \mathrm{d}^4x\langle \mathrm{D}|\mathrm{T}\{\mathrm{H(x)H(0)}\}|\mathrm{D}\rangle$$

Operator product expansion (OPE)

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0) \to 1 + \frac{\Lambda_{\mathrm{QCD}}}{m_c} + \frac{\Lambda^2_{\mathrm{QCD}}}{m_c^2} + \ldots$$

Perturbative corrections: Wilson coefficients expanded in α_s

Power corrections: local operators organized in 1/m_c

$$W_i = W_i^{(0,0)} + W_i^{(\pi,0)}\frac{\mu_\pi^2}{2m_c^2} + W_i^{(G,0)}\frac{\mu_G^2}{2m_c^2} + W_i^{(D,0)}\frac{\rho_D^3}{2m_c^3} + W_i^{(LS,0)}\frac{\rho_{LS}^3}{2m_c^3}$$
$$+\frac{\alpha_s\left(\mu_s\right)C_F}{\pi}\left[W_i^{(0,1)} + W_i^{(\pi,1)}\frac{\mu_\pi^2}{2m_c^2} + W_i^{(G,1)}\frac{\mu_G^2}{2m_c^2} + W_i^{(D,1)}\frac{\rho_D^3}{2m_c^3} + W_i^{(LS,1)}\frac{\rho_{LS}^3}{2m_c^3}\right]$$
$$+\cdots$$

Under Pole mass scheme

P. Gambino et al. 2023

Theoretical Progress in Charm Inclusive Decay

Our Results : Eur.Phys.J.C 85 (2025) 9, 1011

$$D \to X_q e^+ \nu_e; q = d, s$$

Theoretical results for decay width and energy moments (NNLO&dim-6)

NLO analytical integration

NNLO numerical results provided by authors of [Chen,Chen,Guan,Ma,'23]

$$\Gamma_{D_i \to X_q} = \hat{\Gamma}_0 \left|V_{cq}\right|^2 m_c^5 \Big\{ 1 + \frac{\alpha_s(\mu)}{\pi} \frac{2}{3}\left(\frac{25}{4} - \pi^2\right) + \frac{\alpha_s^2(\mu)}{\pi^2}\left[\frac{\beta_0}{4}\frac{2}{3}\left(\frac{25}{4} - \pi^2\right)\log\left(\frac{\mu^2}{m_c^2}\right) + 2.14690 n_l - 29.88311\right]$$

$$- 8\rho\delta_{sq} - \frac{1}{2}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{3}{2}\frac{\mu_G^2(D_i)}{m_c^2} + \left(6 + 8\log\left(\frac{\mu^2}{m_c^2}\right)\right)\frac{\rho_D^3(D_i)}{m_c^3} + \frac{\tau_0(D_i \to X_q)}{m_c^3} + \ldots \Big\}$$

Dim-5, $\Lambda_{\rm QCD}^2/m_c^2$

Dim-6, $\Lambda_{\rm QCD}^3/m_c^3$

Under Pole mass scheme

For Dim-6 four quark operators

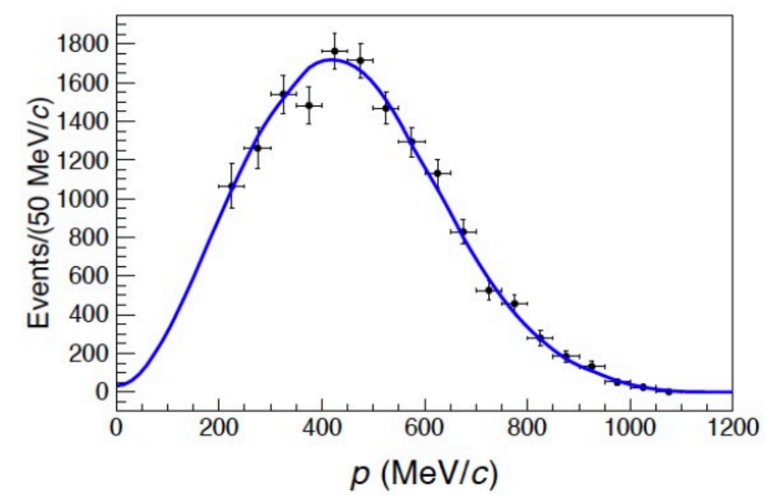
In practical phenomenological analysis

1. **Vacuum-Insertion_Approximation (VIA)**
2. **HQET Sum Rules**

Theoretical Progress in Charm Inclusive Decay

- **Electron Energy spectrum**

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = 12(1-y)y^2\theta(1-y)$$
$$+\frac{2\mu_\pi^2}{m_c^2}\left[-10y^3\theta(1-y) + 2\delta(1-y)\right.$$
$$-\frac{2\mu_G^2}{3m_c^2}\left[6y^2(6-5y)\theta(1-y)\right] + O\left(\alpha_s, \frac{\Lambda^3}{m_c^3}\right)$$



[BESIII, '21]

- **Up to finite power, the obtained differential decay rate is NOT the experimental spectrum**

[Neubert, 1995] [Mannel et al,'94]

- **Observables require integration over final states.**

Smearing

$$\Gamma = \int \frac{d\Gamma}{dy}dy, \left\langle E_\ell^n \right\rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy}E_\ell^n dy, n = 1,2,3,4$$

[Gambino, Kamenik, '10]

- **The first four order energy moments also are provided by us**

Theoretical Progress in Charm Inclusive Decay

The width of semi-leptonic inclusive decays in D mesons is highly sensitive to the charm quark mass.

- **1S mass scheme**: **well perturbative behaviors** $1/2\ \mathrm{J}/\psi$ mass

$$\Gamma/\Gamma_{\mathrm{LO}} \approx 1 - 13.1\% - 4.8\% + 1.8\,\%$$

[Hoang,Ligeti,Manohar, '98; Hoang,Teubner, '99]

- $\overline{\mathrm{MS}}$ **mass scheme**: **slow** convergence

$$\Gamma/\Gamma_{\mathrm{LO}} = 1 + 1.35\alpha_s + 3.02\alpha_s^2 + 7.69\alpha_s^3 \approx 1 + 52\% + 46\% + 44\,\%$$

[Melnikov,van Ritbergen, '99]

- **Pole mass scheme: no convergence**

Naive parameters of HQET

$$\Gamma/\Gamma_{\mathrm{LO}} = 1 - 0.77\alpha_s - 2.38\alpha_s^2 - 10.73\alpha_s^3 \approx 1 - 30\% - 36\% - 62\,\%$$

- **Kinetic mass scheme**: $\left(\alpha_s/\pi\right)\mu^n/m_c^n$ cut-off scale **somewhat subtle**

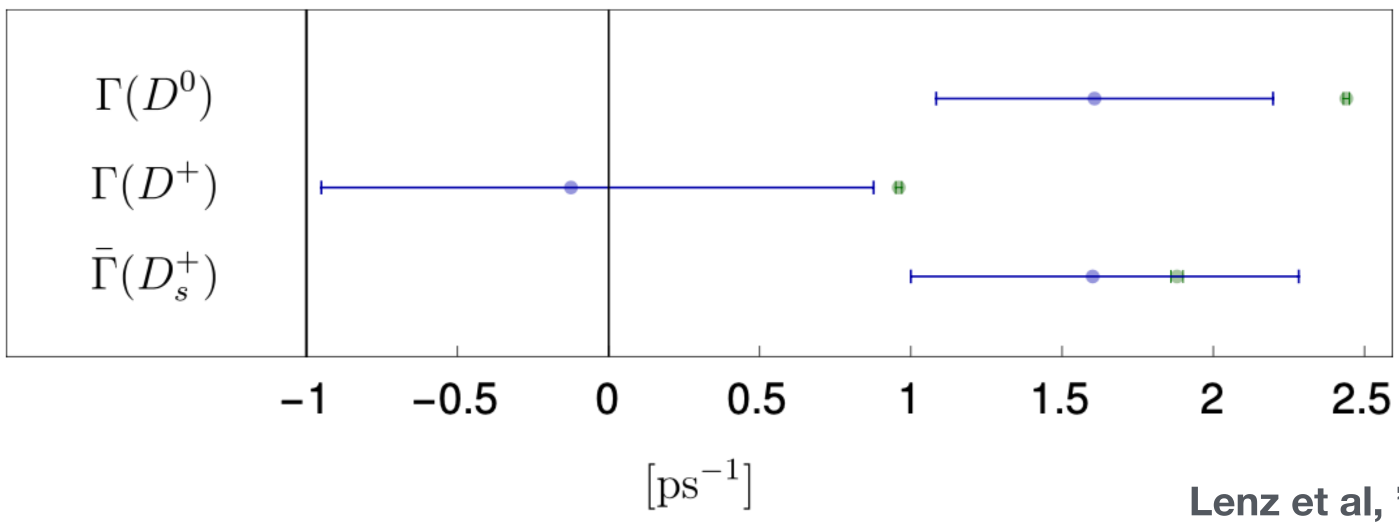
[Fael,Schönwald, Steinhauser, '20]

From HQE parameters to CKM physics

Eur.Phys.J.C 85 (2025) 9, 1011

- **Determine the HQE parameters**

Motivation:



Lenz et al, '22]

Meson

Experiment： High precision

Theory： significant uncertainty

Theoretical predictions are limited by non-perturbative inputs.

The extraction of model-independent non-perturbative matrix elements will play an important role

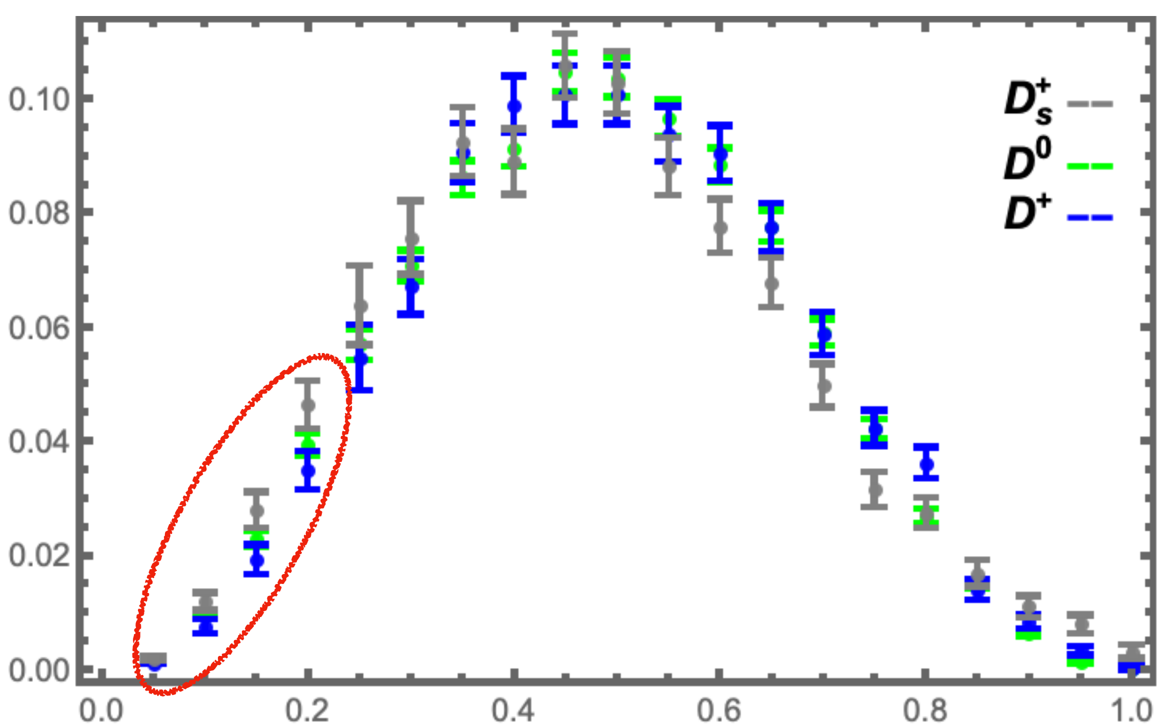
From HQE parameters to CKM physics

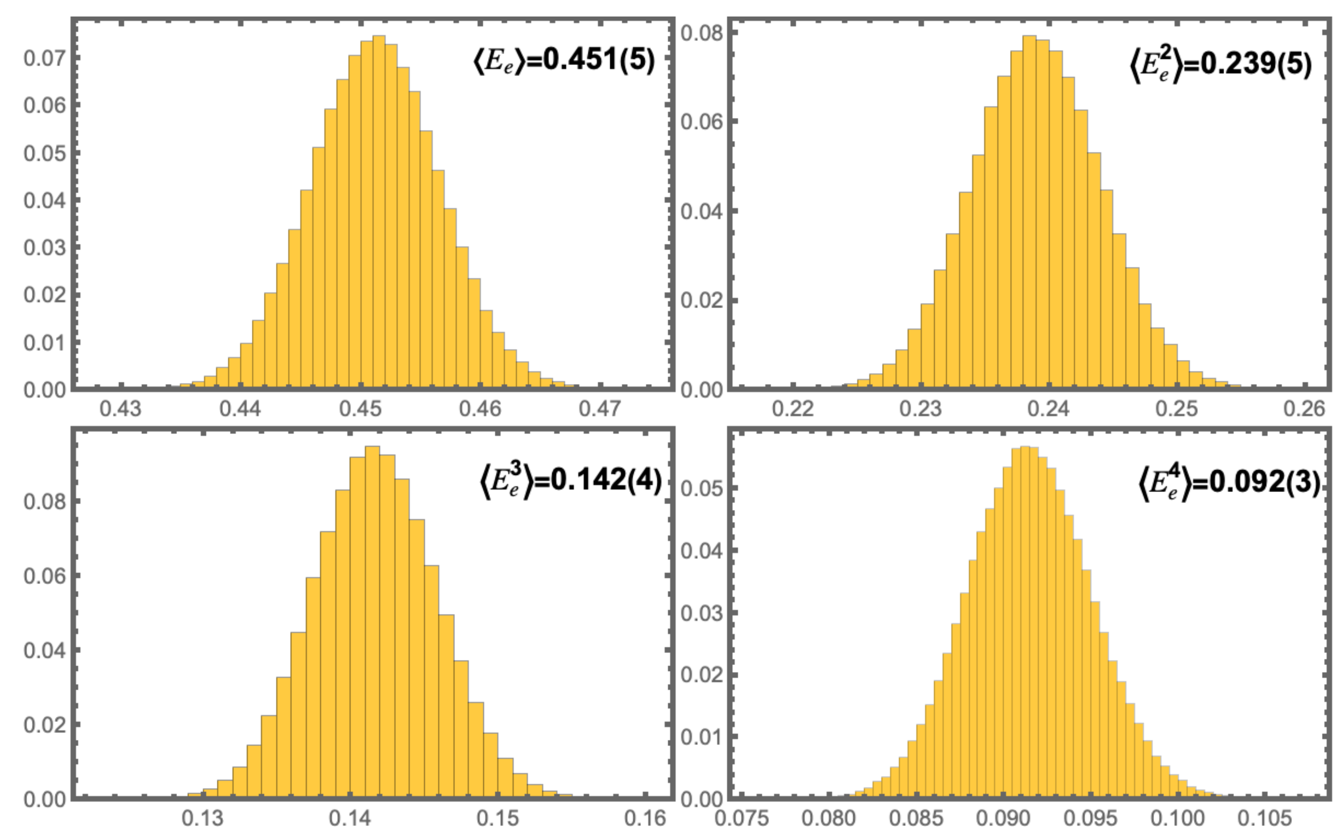
Eur.Phys.J.C 85 (2025) 9, 1011

- **Determine the HQE parameters**

Our approach:

- **The electron energy spectrum and branch ratio has measured by CLEO** [CLEO, '09]
- **The data of Ds meson has been updated by BESIII** [BESIII, '21]





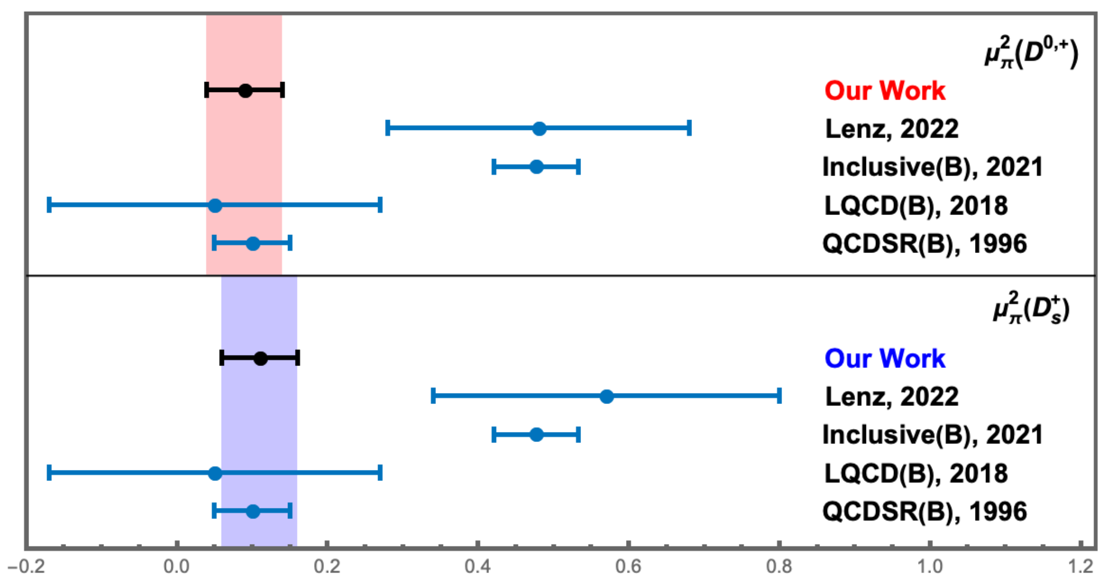
From HQE parameters to CKM physics

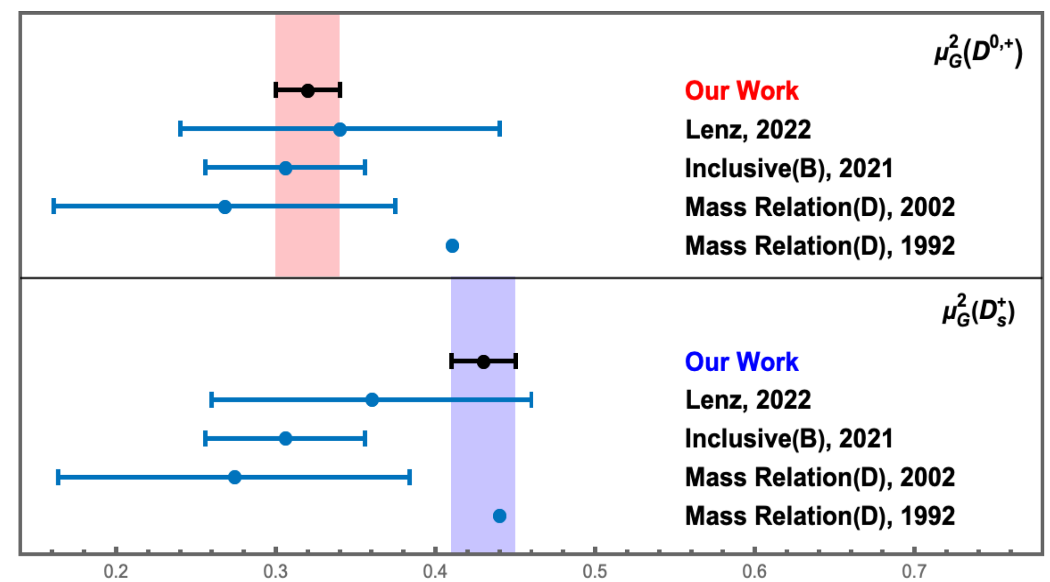
Eur.Phys.J.C 85 (2025) 9, 1011

Preliminary Results

- **Determine the HQE parameters**

Our Results (1S mass scheme) :

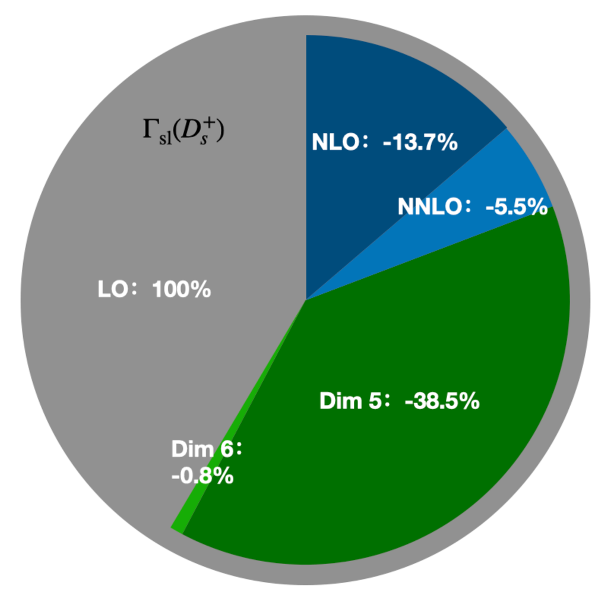


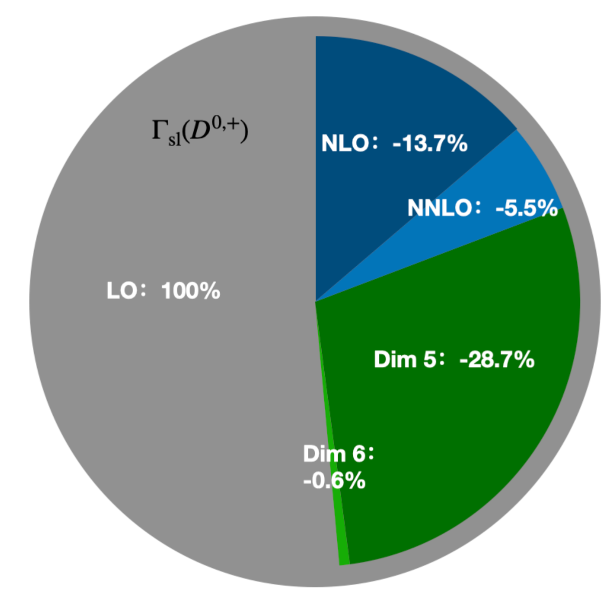


$$\Gamma_{\rm sl}^{D^{0,+}} \sim 1 - 0.137 - 0.055 - 0.287 - 0.006 \cdots$$

$$\Gamma_{\rm sl}^{D_{\rm sl}^+} \sim 1 - 0.137 - 0.055 - 0.385 - 0.008 \cdots$$

LO NLO NNLO Dim-5 Dim-6





- **This approach has been extended to Λ_c^+ by us** *Chin.Phys.C* 50 (2026) 6, 063001

From HQE parameters to CKM physics

ArXiv: 2509.11404

Preliminary Results

Determine the CKM $|V_{cs}|$ & $|V_{cd}|$

1. Treat V_{cs} and V_{cd} as free parameters rather than fixed inputs.
2. Difficult for incl. measurements to X_s and X_d

	D^+	D^0	D_s
$X_s e^+ \nu_e$	$14.60 \pm 0.16\%$	$5.81 \pm 0.06\%$	$5.58 \pm 0.14\%$
$X_d e^+ \nu_e$	$0.96 \pm 0.03\%$	$0.45 \pm 0.01\%$	$0.49 \pm 0.03\%$

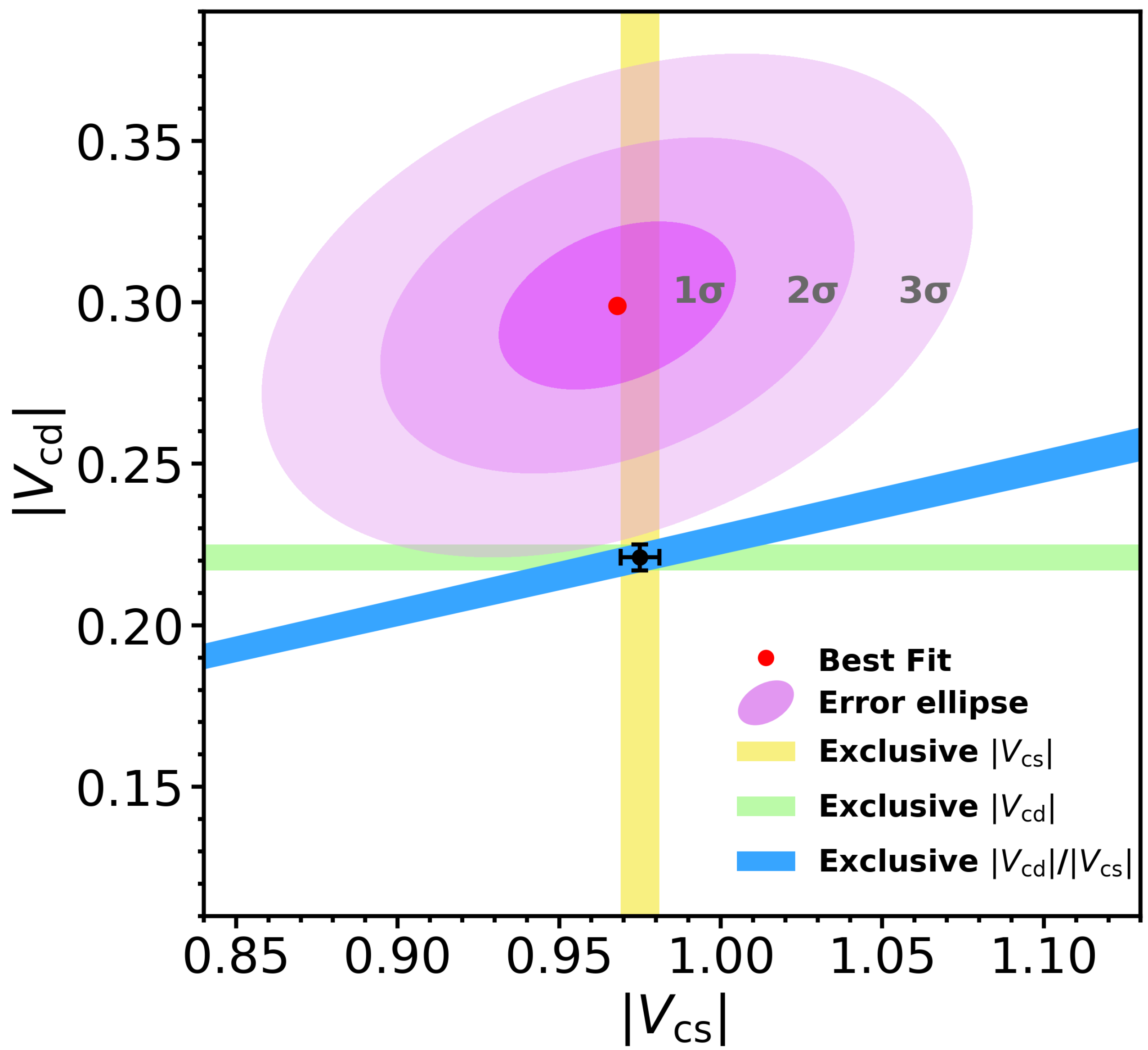
TABLE I. The branching ratios of semi-leptonic inclusive D decays, derived from the sum of the experimentally well-measured exclusive decay modes.

$$B(D^+ \to X_{d+s} e^+ \nu_e) = 0.1602(32),$$
$$B(D^0 \to X_{d+s} e^+ \nu_e) = 0.0636(15),$$
$$B(D_s \to X_{d+s} e^+ \nu_e) = 0.0631(14),$$

In agreement with 1～2σ

$$|V_{cs}| = 0.968 \pm 0.022 \pm 0.026 \pm 0.014$$
$$|V_{cd}| = 0.299 \pm 0.025 \pm 0.007 \pm 0.002$$

Sum of excl. channels



From HQE parameters to CKM physics

ArXiv: 26xx.xxxxx

Preliminary Results

Test the LFU in D meson

1. The finial leptonic mass couldn't be ignored
2. More Complex phase space will be included

$$R_{\mu/e}(D) = \frac{\Gamma(D \to X\mu^+\nu_\mu)}{\Gamma(D \to Xe^+\nu_e)}.$$

$$\frac{\mathrm{d}^3\Gamma}{\mathrm{d}q^2\mathrm{d}E_\mu\mathrm{d}E_{\nu_\mu}} = \frac{G_F^2|V_{cq}|^2}{2\pi^3}\left\{W_1(q^2-m_\mu^2) + W_2\left(2E_\mu E_{\nu_\mu} - \frac{q^2-m_\mu^2}{2}\right)\right.$$
$$\left. - W_3\left[\left(q^2(E_\mu - E_{\nu_\mu}) - m_\mu^2(E_\mu + E_{\nu_\mu}\right)\right] + W_4\frac{m_\mu^2(q^2-m_\mu^2)}{2} + W_5(2m_\mu^2 E_{\nu_\mu})\right\}$$

$$D \to X_q \mu\bar{\nu}, q = d, s$$

$$R_{\mu/e}(D) = \frac{\Gamma(D \to X\mu^+\nu_\mu)}{\Gamma(D \to Xe^+\nu_e)}.$$

$D^{0,+} \to X_s\mu^+\nu_\mu$	0.964 ± 0.003
$D^{0,+} \to X_d\mu^+\nu_\mu$	0.965 ± 0.003
$D_s^+ \to X_s\mu^+\nu_\mu$	0.962 ± 0.004
$D_s^+ \to X_d\mu^+\nu_\mu$	0.963 ± 0.004

$$\Gamma = \Gamma_0\left(\Gamma^{(0)} + \Gamma^{(\hat{\mu}_\pi^2)}\frac{\hat{\mu}_\pi^2}{m_c^2} + \Gamma^{(\hat{\mu}_G^2)}\frac{\hat{\mu}_G^2}{m_c^2} + \Gamma^{(\hat{\rho}_D^3)}\frac{\hat{\rho}_D^3}{m_c^3} + \Gamma^{(\hat{\rho}_{LS}^3)}\frac{\hat{\rho}_{LS}^3}{m_c^3}\right)$$

$$\langle E_\mu^n\rangle \quad \langle (q^2)^n\rangle \quad n = 1,2,3,4$$

Up to Dim-6 two operators contributions at LO

Outlook: Toward Precision Inclusive Charm Decay

$$D \to X_q \eta \bar{\nu}_\eta, q = d, s; \eta = e, \mu$$

- **More high power corrections** Dim-6 four quark operators
- **NLO&Power corrections** Dim-5 &NLO
- **More Observables**

Electron energy spectrum
q^2spectrum、M_x spectrum

smearing $\rightarrow$

$< E^n >$ $< E^n >_{cut}$

$< (q^2)^n >$ $< (q^2)^n >_{cut}$

$< M_x^n >$ $< M_x^n >_{cut}$

...

Thanks!

Back up

Theoretical Framework and Development

- **Optical theorem**

$$\sum \langle D|H|X\rangle\langle X|H|D\rangle \propto \mathrm{Im}\int \mathrm{d}^4x\langle \mathrm{D}|\mathrm{T}\{\mathrm{H(x)H(0)}\}|\mathrm{D}\rangle$$

- **Operator product expansion (OPE)**

  - Short distance : $x \sim 1/m_c$
  - Fluctuation in D meson $\sim \Lambda_{\mathrm{QCD}}$

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0) \to 1 + \frac{\Lambda_{\mathrm{QCD}}}{m_c} + \frac{\Lambda_{\mathrm{QCD}}^2}{m_c^2} + \dots$$

Systematic OPE in HQET.

- **Heavy Quark Effective Theory**

$$h_v(x) \equiv e^{-im_c v\cdot x}\frac{1+\gamma\cdot v}{2}c(x) \qquad v = (1,0,0,0)$$

$$L \ni \bar{h}_v iv\cdot Dh_v - \bar{h}_v\frac{D_\perp^2}{2m_c}h_v - a(\mu)g\bar{h}_v\frac{\sigma\cdot G}{4m_c}h_v + \dots$$

Subtract the big intrinsic momentum,

Leave only ~Λ_{QCD} degrees of freedom.

Theoretical Framework and Development

- **OPE**

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0)$$

$C_n(x)$

- ☆ LO: $\alpha_s^0(m_c)$
- ☆ NLO: $\alpha_s(m_c)$
- ☆ ...

$O_n(0)$

- ☆ Dim-3: $\bar{h}_v h_v$ $(\bar{c}\gamma^\mu c) \rightarrow$ **partonic decay rate.**
- ☆ Dim-5: $\bar{h}_v D_\perp^2 h_v$, $g\bar{h}_v \sigma \cdot G h_v$.
- ☆ Dim-6: $\bar{h}_v D_\mu (v \cdot D) D^\mu h_v$, $(\bar{h}_v \Gamma_1 q)(\bar{q}\Gamma_2 h_v), \ldots$
- ☆ ...

- **Contribute to inclusive decay rate and lifetime**

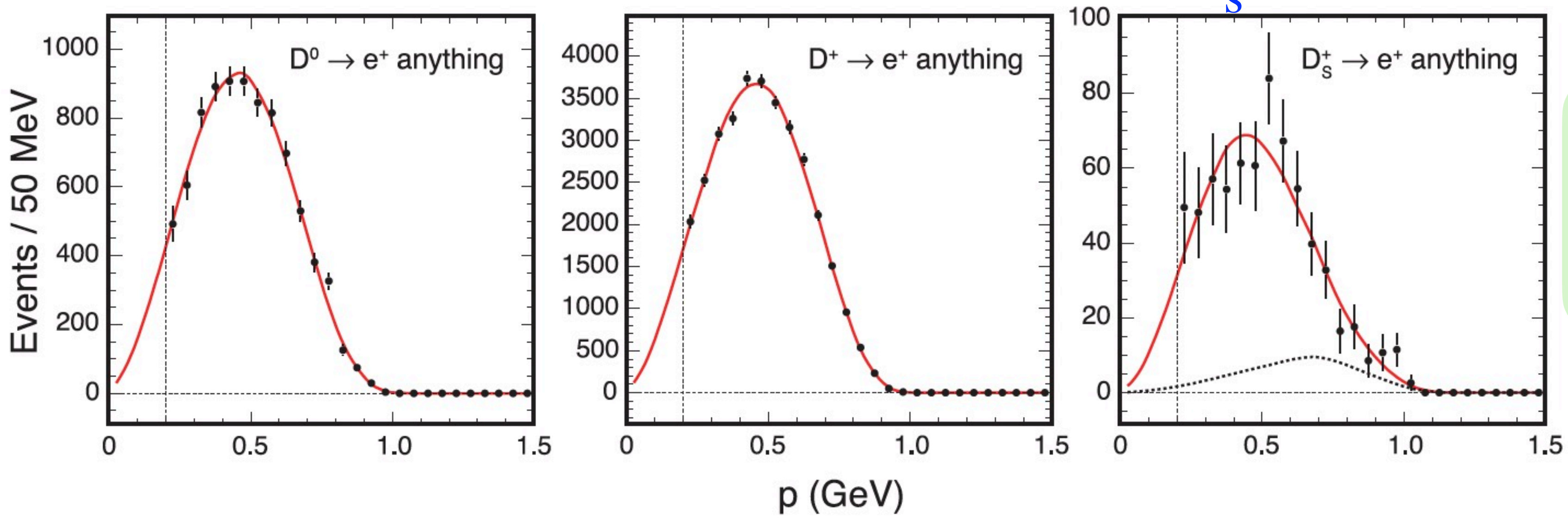
1. Matrix elements of the **same operators** (SL& NL)
2. Only different short-distance coefficients

$$\boxed{\begin{aligned} \mu_\pi^2 &= -\frac{1}{2M_D}\langle D(v) \,|\, \bar{c}_v (iD)^2 c_v \,|\, D(v)\rangle \\ \mu_G^2 &= \frac{1}{6M_D}\langle D(v) \,|\, \bar{c}_v \frac{g_s}{2} G_{\mu\nu}\sigma^{\mu\nu} c_v \,|\, D(v)\rangle \end{aligned}}$$

Experimental status

CLEO measurements

$D^0 \to e^+X$ $\quad D^+ \to e^+X$ $\quad D_s^+ \to e^+X$

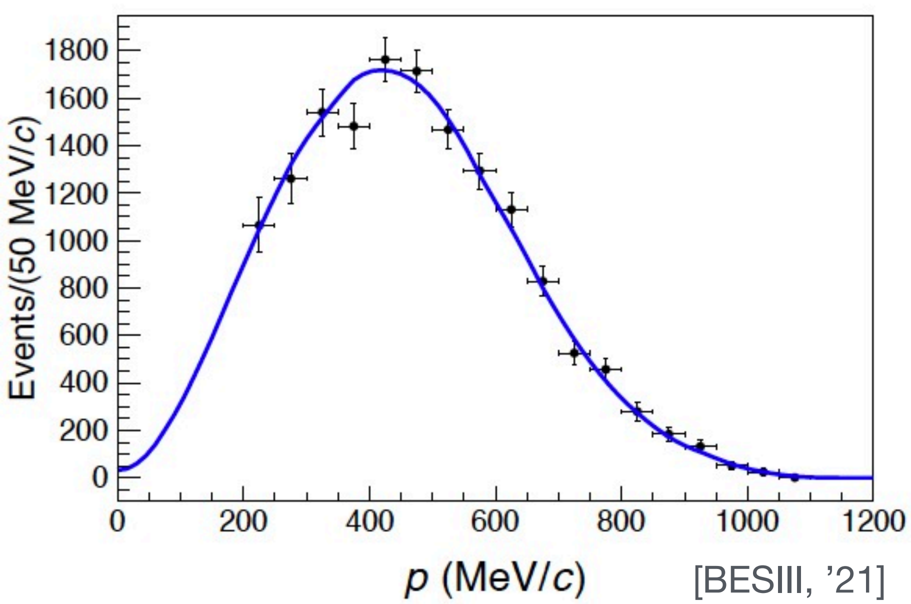


$$B\left(D^0 \to Xe^+\nu_e\right) = (6.46 \pm 0.09 \pm 0.11)\,\%$$
$$B\left(D^+ \to Xe^+\nu_e\right) = (16.13 \pm 0.10 \pm 0.29)\,\%$$
$$B\left(D_s^+ \to Xe^+\nu_e\right) = (6.52 \pm 0.39 \pm 0.15)\,\%$$

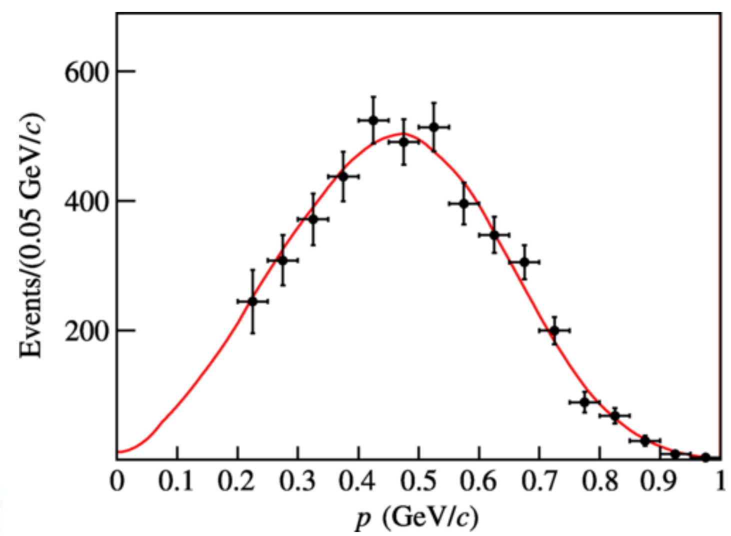
[CLEO, '09]

BESIII measurements

$D_s^+ \to e^+X$ $\quad \Lambda_c^+ \to e^+X$



[BESIII, '21]



[BESIII, '23]

$$B\left(D_s^+ \to Xe^+\nu_e\right) = (6.30 \pm 0.13 \pm 0.10)\,\%$$

[BESIII, '21]

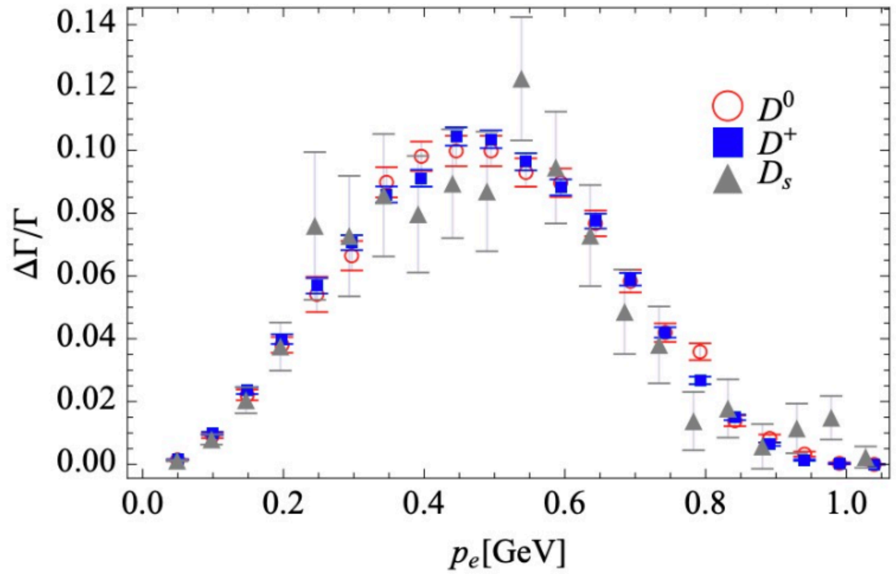
$$B\left(\Lambda_c^+ \to Xe^+\nu_e\right) = (4.06 \pm 0.10 \pm 0.09)\,\%$$

[BESIII, '23]

2% precision!

Phenomenological Progress

- Four quark operators were estimated



$$B_{\rm WA}^{(0),s}(D^0) = -0.001(3)\text{GeV}^3\,, \qquad B_{\rm WA}^{(0),s}(D^+) = -0.001(3)\text{GeV}^3\,,$$
$$\bar{B}_{\rm WA}^{(1),s}(D^0) = -0.0001(6)\text{GeV}^3\,, \qquad \bar{B}_{\rm WA}^{(1),s}(D^+) = -0.0001(6)\text{GeV}^3\,,$$
$$\bar{B}_{\rm WA}^{(2),s}(D^0) = -0.0001(10)\text{GeV}^3\,, \qquad \bar{B}_{\rm WA}^{(2),s}(D^+) = -0.0002(10)\text{GeV}^3\,,$$
$$\bar{B}_{\rm WA}^{(\sigma),s}(D^0) = -0.0000(7)\text{GeV}^3\,, \qquad \bar{B}_{\rm WA}^{(\sigma),s}(D^+) = -0.0000(7)\text{GeV}^3\,,$$

[Gambino, et al '10]

- Strong interaction running coupling constant have been estimated

$$\alpha_s = 0.377 \pm 0.008 \pm 0.114$$

[Wu, et al '24]

- Assuming non-perturbative parameters are **identical to those of B meson**
- Global Fitting under **kinetic mass scheme**

?

Starting from dimension-5!

Weak Annihilation Uncertainty

❖ Redo the fits, adopting the HQET SR calculation for the weak annihilation contributions

$$\tau_0(D_d \to X_d) = \tau_0(D_s \to X_s) = \tau_{\text{val}} = (-0.18 \pm 0.65)\ \text{GeV}^3$$ [King, Lenz, Piscopo, Rauh,'19]

$$\tau_0(D_u \to X_{d,s}) = \tau_0(D_d \to X_s) = \tau_0(D_s \to X_d) = \tau_{\text{nonval}} = (0.45 \pm 2.10)\ \text{GeV}^3$$

$$\chi^2(\boldsymbol{\theta}) = \sum_{l=u,d,s} \sum_{i=1}^{5} \sum_{j=1}^{5} (y_{i,l} - \eta_{i,l})\, V_{ij,l}^{-1}\, (y_{j,l} - \eta_{j,l}) + \left(\frac{\tau_{\text{nonval}} - (-0.18)}{0.65}\right)^2 + \left(\frac{\tau_{\text{val}} - 0.45}{2.10}\right)^2$$

$$\mu_\pi^2(D^{0,+}) = 0.08\text{GeV}^2\ ,\ \mu_G^2(D^{0,+}) = 0.33\text{GeV}^2\ ,\ \rho_D^3(D^{0,+}) = -0.003\text{GeV}^3\ ,\ \rho_{LS}^3(D^{0,+}) = 0.004\text{GeV}^3$$
$$\mu_\pi^2(D_s) = 0.15\text{GeV}^2\ ,\ \mu_G^2(D_s) = 0.38\text{GeV}^2\ ,\ \rho_D^3(D_s) = -0.005\text{GeV}^3\ ,\ \rho_{LS}^3(D_s) = 0.006\text{GeV}^3\ ,$$
$$\tau_{\text{val}} = -0.11\text{GeV}^3,\ \tau_{\text{nonval}} = 0.002\text{GeV}^3\ .$$

The best-fit values only slightly change.

Phenomenological Analysis

1. Treat V_{cs} and V_{cd} as free parameters rather than fixed inputs.
2. Difficult for incl. measurements to X_s and X_d → **Sum of excl. channels**

D^+ decays		D^0 decays		D_s decays	
Mode	BR(%)	Mode	BR(%)	Mode	BR(%)
$D^+ \to \bar{K}^0 e^+ \nu_e$	8.72 ± 0.09	$D^0 \to K^- e^+ \nu_e$	3.549 ± 0.026	$D_s \to \phi e^+ \nu_e$	2.34 ± 0.12
$(K^-\pi^+)_{\bar{K}^*(892)^0} e^+ \nu_e$	3.54 ± 0.09 [23]	$(\bar{K}^0\pi^-)_{\text{S-wave}} e^+ \nu_e$	0.079 ± 0.017	$D_s \to \eta e^+ \nu_e$	2.27 ± 0.06
$(K^-\pi^+)_{\text{S-wave}} e^+ \nu_e$	0.228 ± 0.011	$D^0 \to K^*(892)^- e^+ \nu_e$	2.04 ± 0.047 [24]	$D_s \to \eta' e^+ \nu_e$	0.81 ± 0.04
$D^+ \to \bar{K}_1(1270)^0 e^+ \nu_e$	0.230 ± 0.026	$D^0 \to \bar{K}_1(1270)^- e^+ \nu_e$	0.101 ± 0.018	$D_s \to f_0(980) e' \nu_e$	0.164 ± 0.013
$D^+ \to \eta e^+ \nu_e$	0.111 ± 0.007	$D^0 \to \pi^- e^+ \nu_e$	0.291 ± 0.004	$D_s \to K^0 e^+ \nu_e$	0.288 ± 0.026
$D^+ \to \pi^0 e^+ \nu_e$	0.372 ± 0.017	$D^0 \to \rho(770)^- e^+ \nu_e$	0.145 ± 0.007	$D_s \to K^*(892)^0 e^+ \nu_e$	0.205 ± 0.020
$D^+ \to \pi^+\pi^- e^+ \nu_e$	0.245 ± 0.008	$D^0 \to a(980)^- e^+ \nu_e$	$0.0133^{+0.0034}_{-0.0030}$		
$D^+ \to \pi^0\pi^0 e^+ \nu_e$	0.0315 ± 0.0027 [25]				
$D^+ \to \omega e^+ \nu_e$	0.169 ± 0.011				
$D^+ \to \eta' e^+ \nu_e$	0.020 ± 0.004				
$D^+ \to a(980)^0 e^+ \nu_e$	0.017 ± 0.008				

TABLE III. Branching ratios of selected exclusive decays of the D^+, D^0, and D_s mesons in units of percentage. All the values without references are taken from the PDG [21].

	D^+	D^0	D_s
$X_s e^+ \nu_e$	$14.60 \pm 0.16\%$	$5.81 \pm 0.06\%$	$5.58 \pm 0.14\%$
$X_d e^+ \nu_e$	$0.96 \pm 0.03\%$	$0.45 \pm 0.01\%$	$0.49 \pm 0.03\%$

TABLE I. The branching ratios of semi-leptonic inclusive D decays, derived from the sum of the experimentally well-measured exclusive decay modes.

$$B(D^+ \to X_{d+s} e^+ \nu_e) = 0.1602(32),$$
$$B(D^0 \to X_{d+s} e^+ \nu_e) = 0.0636(15),$$
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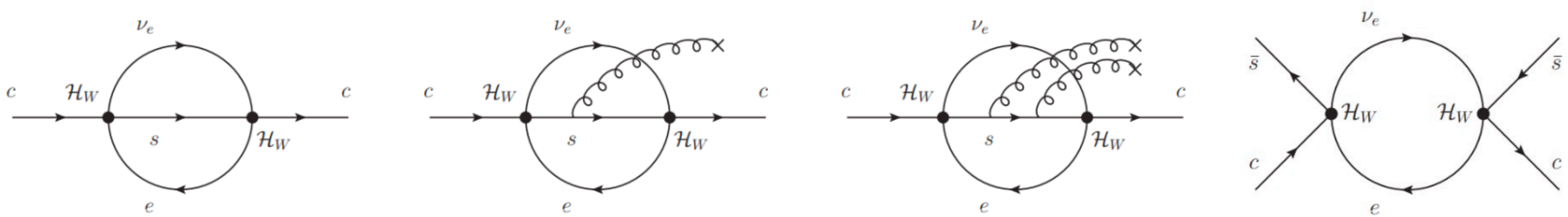
In agreement with 1～2σ

Theoretical Progress in Charm Inclusive Decay

Heavy quark expansion up to dimension-7 operators **Tree level**

1. QCD and HQET operator matching by calculating quark-gluon diagrams



2. The transition c to d likes b to u, specially, c to s unlikes b to c

m_s will appears both the wilson coefficient and local operators

3. The operators mixing will appears at tree level

Fael et al. 2019

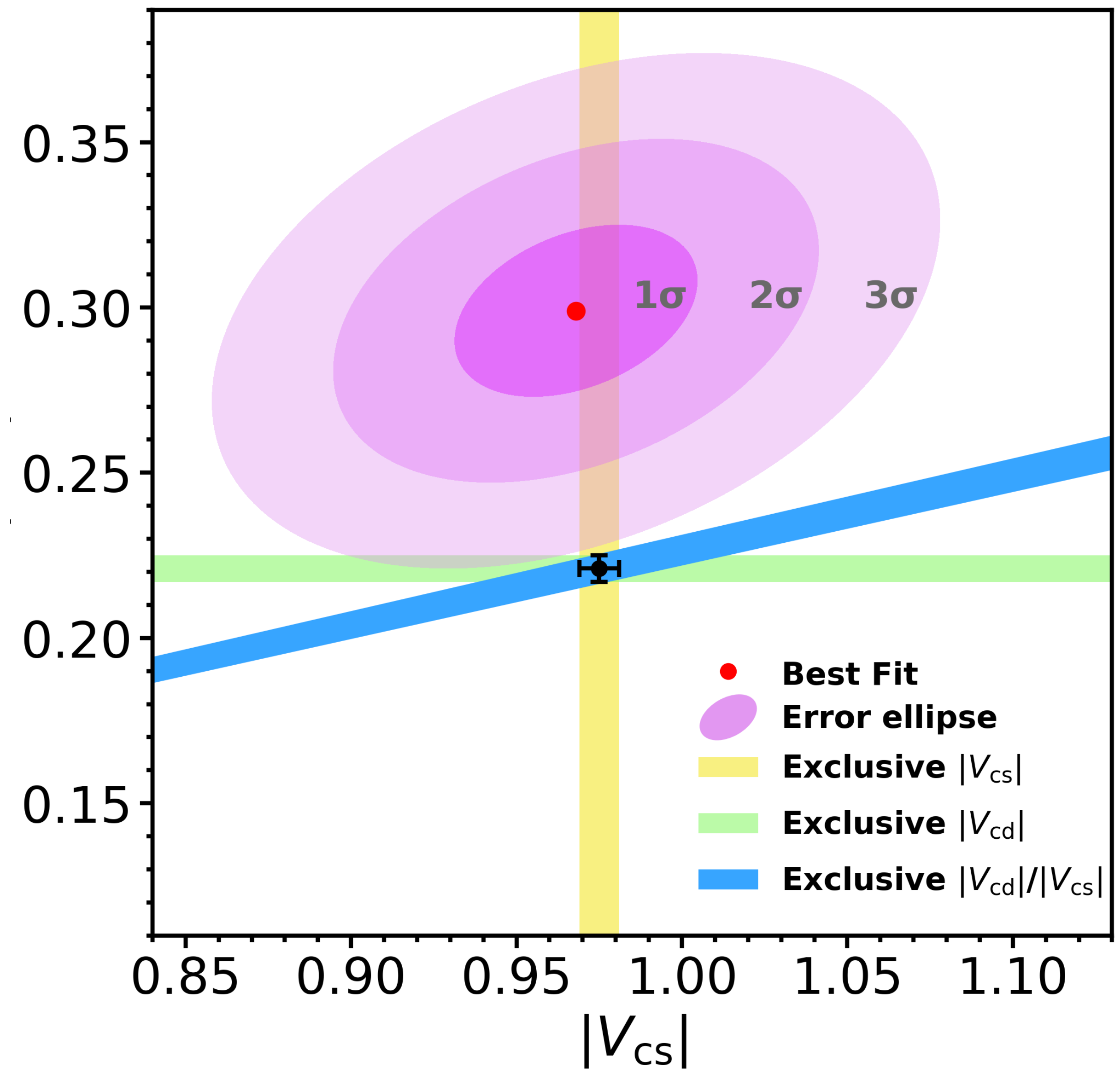
Phenomenological Analysis

- Data choice strategy: all inclusive X_{s+d} data are used
  - **S1**: Sum-of-exclusive X_s data, plus sum-of-exclusive X_d data,
  - **S2**: Sum-of-exclusive X_s data
  - **S3**: Sum-of-exclusive X_d data
- For each strategy, two scenarios for weak-annihilation contributions
  - **Scenario 1**: VIA, $\tau_0 = 0$
  - **Scenario 2**: HQET SR, input the previous best-fit values $\tau_{\rm val} = -0.11 \mathrm{GeV}^3, \ \tau_{\rm nonval} = 0.002 \mathrm{GeV}^3$.

Phenomenological Analysis

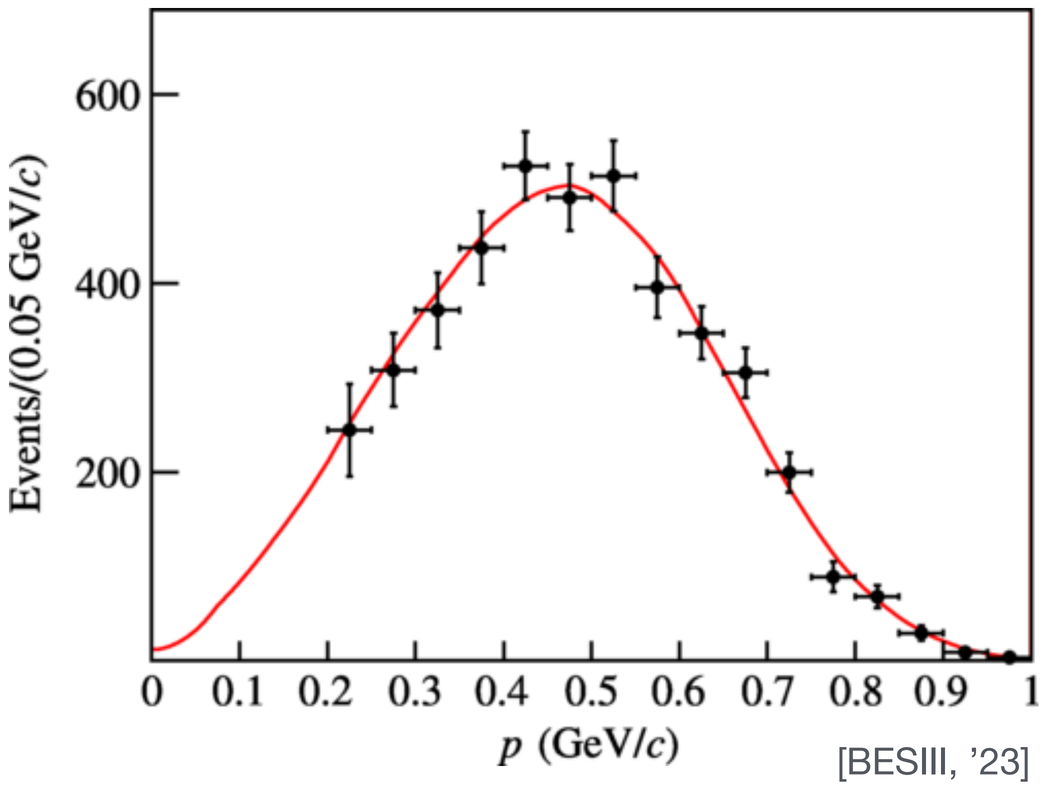
S1 $(X_{s,d})$	χ^2/d.o.f.	$\lvert V_{cs}\rvert$	$\lvert V_{cd}\rvert$
Scenario 1	3.24	$0.977 \pm 0.022 \pm 0.026$	$0.252 \pm 0.006 \pm 0.006$
Scenario 2	1.12	$0.969 \pm 0.022 \pm 0.026$	$0.253 \pm 0.006 \pm 0.006$
S2 (X_s)	χ^2/d.o.f.	$\lvert V_{cs}\rvert$	$\lvert V_{cd}\rvert$
Scenario 1	0.43	$0.974 \pm 0.022 \pm 0.026$	$0.279 \pm 0.031 \pm 0.007$
Scenario 2	0.29	$0.960 \pm 0.021 \pm 0.026$	$0.278 \pm 0.031 \pm 0.006$
S3 (X_d)	χ^2/d.o.f.	$\lvert V_{cs}\rvert$	$\lvert V_{cd}\rvert$
Scenario 1	4.08	$0.982 \pm 0.023 \pm 0.027$	$0.253 \pm 0.006 \pm 0.005$
Scenario 2	1.24	$0.974 \pm 0.022 \pm 0.026$	$0.254 \pm 0.006 \pm 0.006$

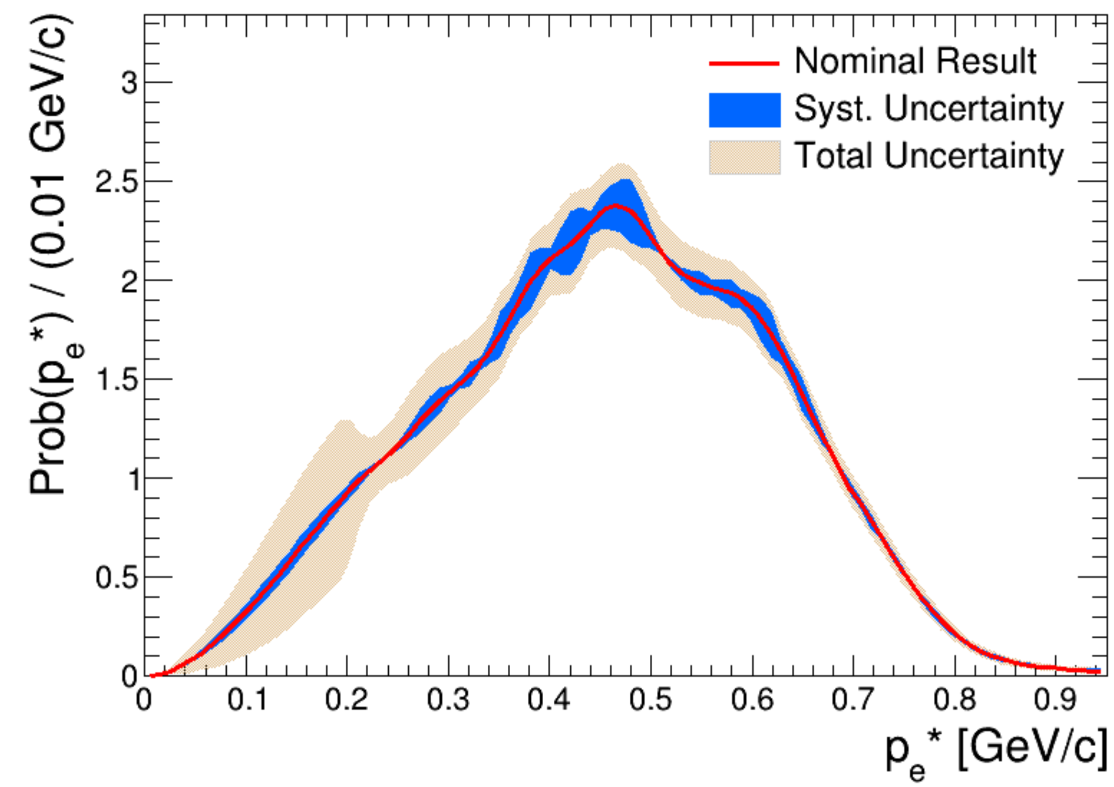
$$|V_{cs}| = 0.968 \pm 0.022 \pm 0.026 \pm 0.014$$
$$|V_{cd}| = 0.299 \pm 0.025 \pm 0.007 \pm 0.002$$



https://arxiv.org/abs/2512.08559

Phenomenological Analysis





$\mu_\pi^2\,[10^{-1}\,\mathrm{GeV}^2]$	$\rho_D^3\,[10^{-4}\,\mathrm{GeV}^3]$	$\tau_{\mathrm{WA}}\,[10^{-1}\,\mathrm{GeV}^3]$	$\chi^2/\mathrm{d.o.f.}$	N_{data} v.s. N_{param}
$1.33 \pm 0.18 \pm 0.02 \pm 0.40$	$-2.95 \pm 2.59 \pm 0.36 \pm 1.18$	$-2.69 \pm 0.33 \pm 0.12 \pm 0.81$	0.53	5 v.s. 3

- **Agree with ones in D mesons**
- **Differ from bag-model and wave-functions approachs**

Energy Moment	Value
$\langle E_e \rangle[\mathrm{GeV}]$	$(4.55 \pm 0.16 \pm 0.03) \times 10^{-1}$
$\langle E_e^2 \rangle_{\mathrm{center}}[\mathrm{GeV}^2]$	$(2.75 \pm 0.37 \pm 0.07) \times 10^{-2}$
$\langle E_e^3 \rangle_{\mathrm{center}}[\mathrm{GeV}^3]$	$(-3.67 \pm 4.39 \pm 0.91) \times 10^{-4}$
$\langle E_e^4 \rangle_{\mathrm{center}}[\mathrm{GeV}^4]$	$(1.89 \pm 0.45 \pm 0.08) \times 10^{-3}$

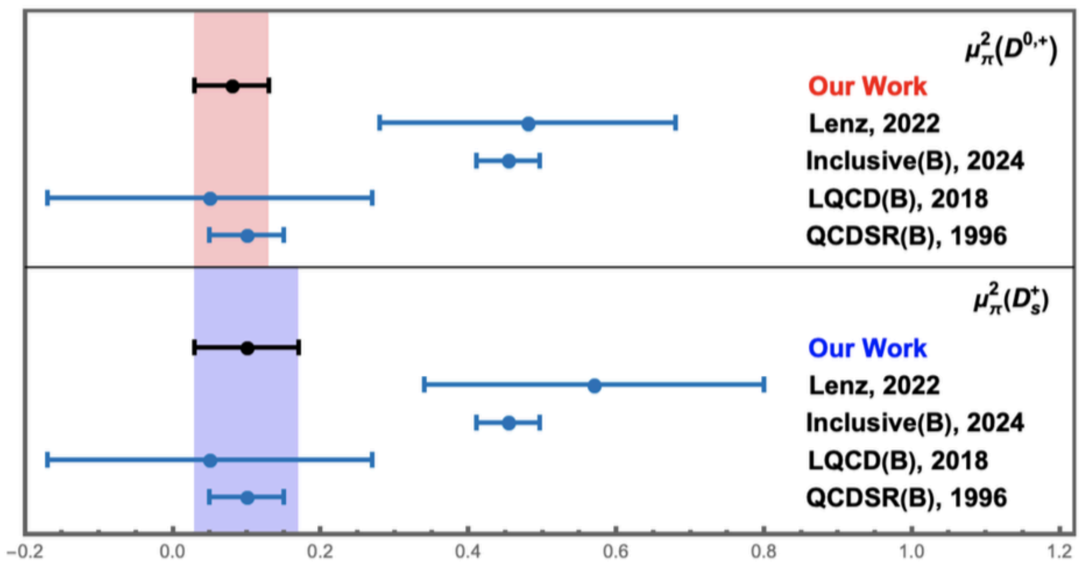
Summary

Phenomenology of inclusive D decays

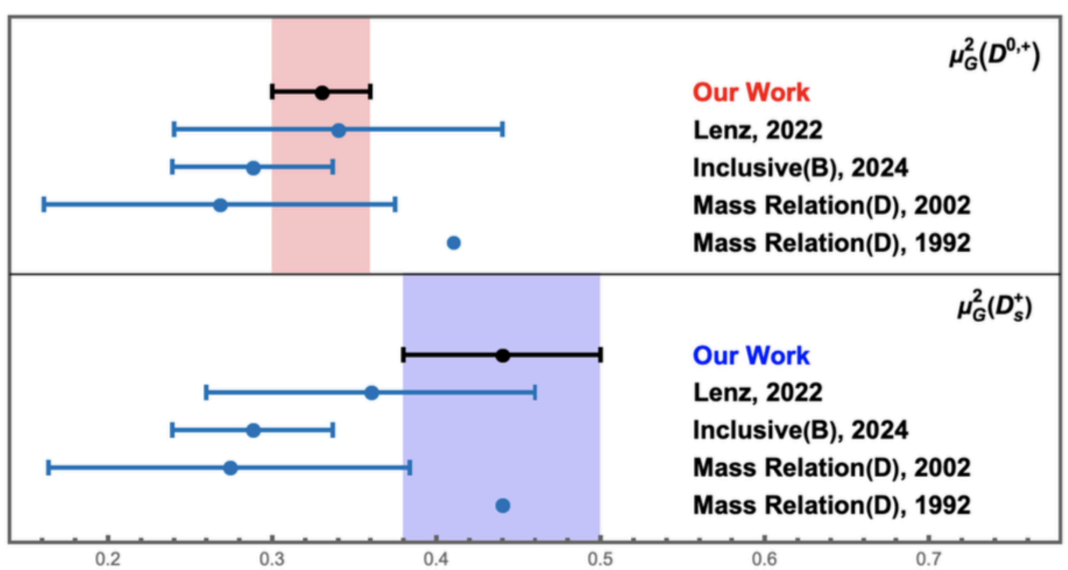
❖ By fitting the HQET formulas for inclusive D decays to data, we determine for the first time

➡ The **HQE parameters** for D mesons

$\boxed{\mu_\pi^2}$



$\boxed{\mu_G^2}$



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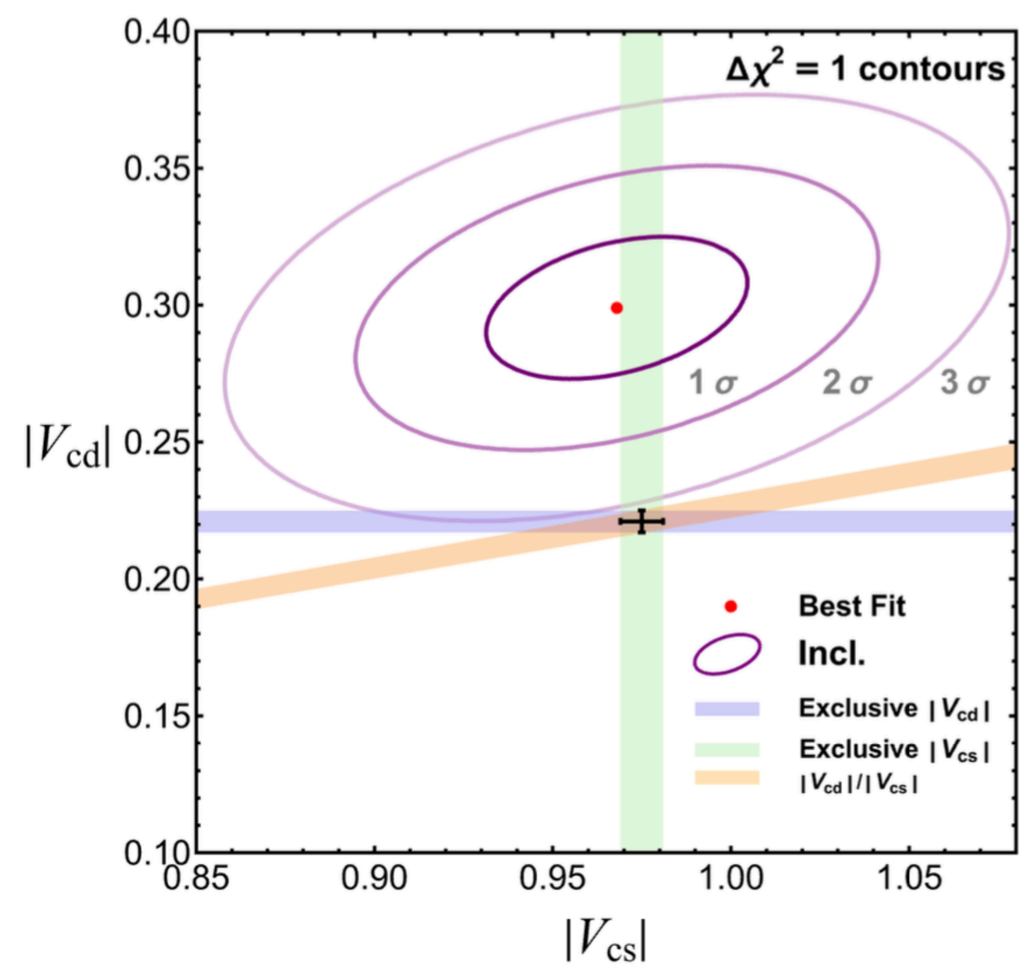
➡ The **CKM matrix elements** $V_{cs,cd}$ (inclusive)



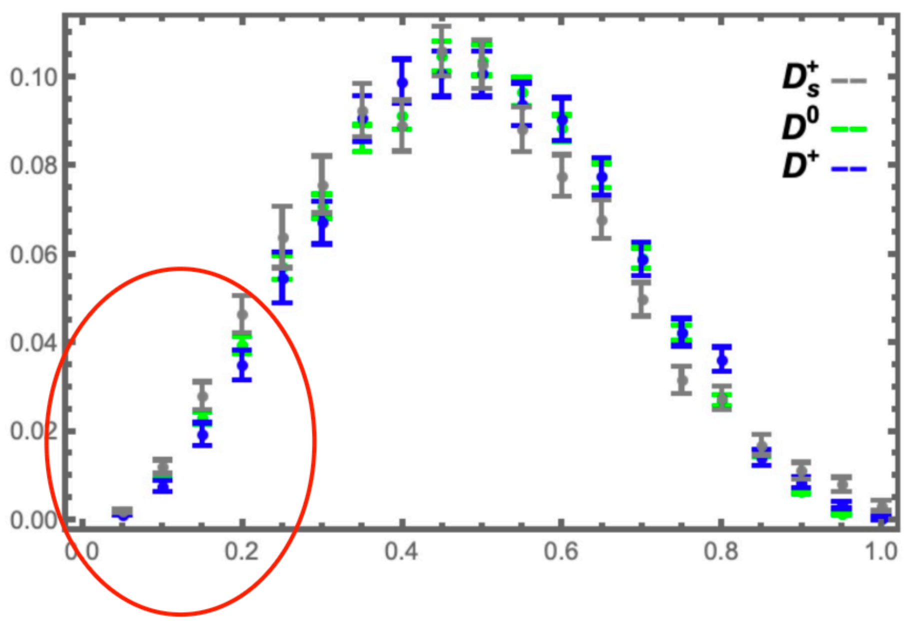
[Shao,Huang,**QQ**,2502.05901]

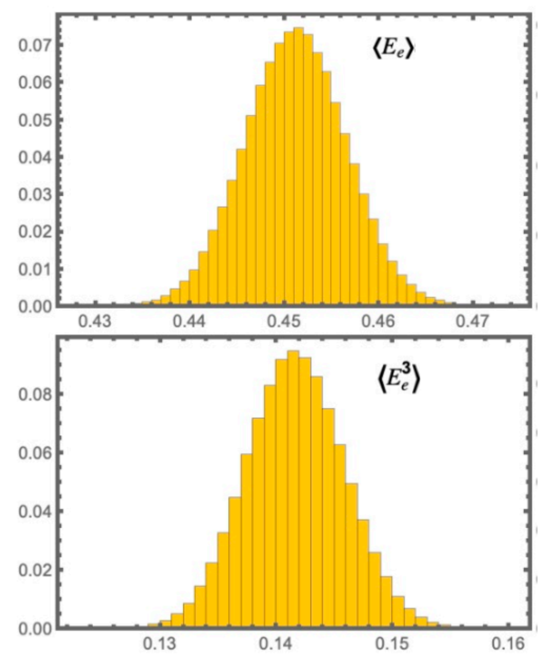
[Shao,Feng,Liu,**QQ**,Sun,Yu,2509.11404]

Data Aspect

$$\frac{d\Gamma}{dy} = ay^2(1+by)(1-y)$$

Original data → **Extrapolation** [Gambino,Kamenik, '10] → **All-bin distribution** → **Lorentz boost** → **Rest frame** → **MC Simulation** → **Electron energy moments**





$$\langle E_e \rangle_{exp}^{D_s} = 0.437(6)\text{GeV}, \quad \langle E_e^2 \rangle_{exp}^{D_s} = 0.220(5)\text{GeV}^2$$
$$\langle E_e \rangle_{exp}^{D^0} = 0.462(5)\text{GeV}, \quad \langle E_e^2 \rangle_{exp}^{D^0} = 0.242(5)\text{GeV}^2$$
$$\langle E_e \rangle_{exp}^{D^+} = 0.455(4)\text{GeV}, \quad \langle E_e^2 \rangle_{exp}^{D^+} = 0.236(4)\text{GeV}^2$$

$$\langle E_e^3 \rangle_{exp}^{D_s} = 0.121(4)\text{GeV}^3, \quad \langle E_e^4 \rangle_{exp}^{D_s} = 0.072(3)\text{GeV}^4$$
$$\langle E_e^3 \rangle_{exp}^{D^0} = 0.138(4)\text{GeV}^3, \quad \langle E_e^4 \rangle_{exp}^{D^0} = 0.084(3)\text{GeV}^4$$
$$\langle E_e^3 \rangle_{exp}^{D^+} = 0.134(3)\text{GeV}^3, \quad \langle E_e^4 \rangle_{exp}^{D^+} = 0.081(3)\text{GeV}^4$$

<u>Statistical uncertainties uncorrelated,</u>
<u>Systematic uncertainties fully correlated.</u>

Fitting Results

❖ Global fit in two mass schemes, each with **Scenario 1** ($\Lambda^2_{\mathrm{QCD}}/m_c^2$) and **Scenario 2** ($\Lambda^3_{\mathrm{QCD}}/m_c^3$)

$\overline{\mathrm{MS}}$ scheme	χ^2/d.o.f.	D_i	μ_π^2/GeV^2	μ_G^2/GeV^2	$\rho_{\mathrm{D}}^3/\mathrm{GeV}^3$	$\rho_{LS}^3/\mathrm{GeV}^3$
Scenario 1	4.5	$D^{0,+}$	0.09 ± 0.01	0.27 ± 0.14	-	-
		D_s	0.09 ± 0.02	0.39 ± 0.12	-	-
Scenario 2	2.1	$D^{0,+}$	0.11 ± 0.02	0.26 ± 0.14	-0.002 ± 0.002	0.003 ± 0.002
		D_s	0.12 ± 0.02	0.38 ± 0.13	-0.003 ± 0.002	0.005 ± 0.002

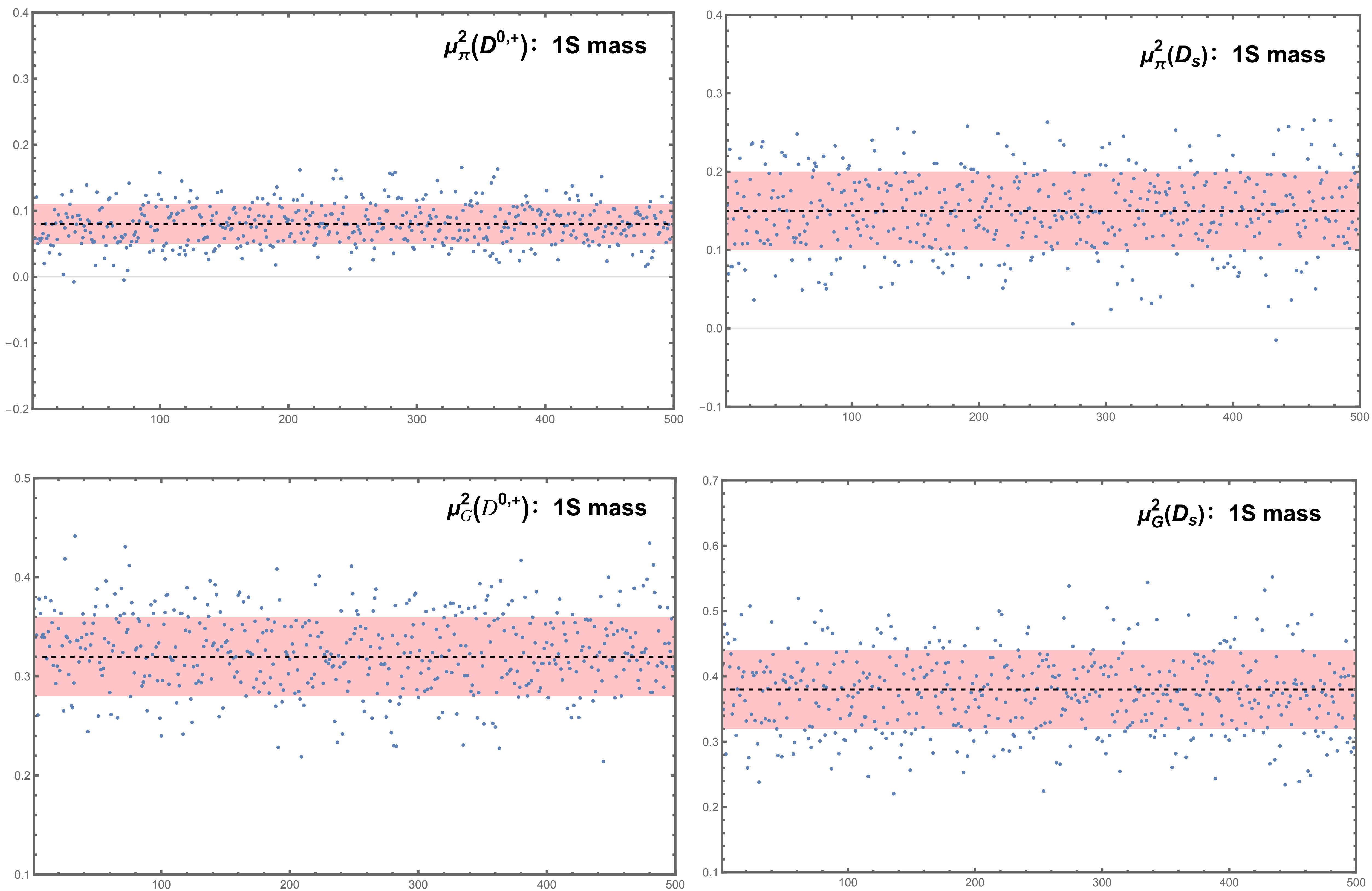
1S scheme	χ^2/d.o.f.	D_i	μ_π^2/GeV^2	μ_G^2/GeV^2	$\rho_{\mathrm{D}}^3/\mathrm{GeV}^3$	$\rho_{LS}^3/\mathrm{GeV}^3$
Scenario 1	4.9	$D^{0,+}$	0.04 ± 0.01	0.33 ± 0.02	-	-
		D_s	0.06 ± 0.02	0.44 ± 0.02	-	-
Scenario 2	0.33	$D^{0,+}$	0.09 ± 0.02	0.32 ± 0.02	-0.003 ± 0.002	0.004 ± 0.002
		D_s	0.11 ± 0.02	0.43 ± 0.02	-0.004 ± 0.002	0.005 ± 0.002

4-q operator contributions vanish under VIA, $\tau_0 = 0$.

Reliable perturbative calculation ensures a good fit!

Differences between Scenarios 1 and 2 as systematic uncertainties.

1S mass scheme robust Testing： scenario 3



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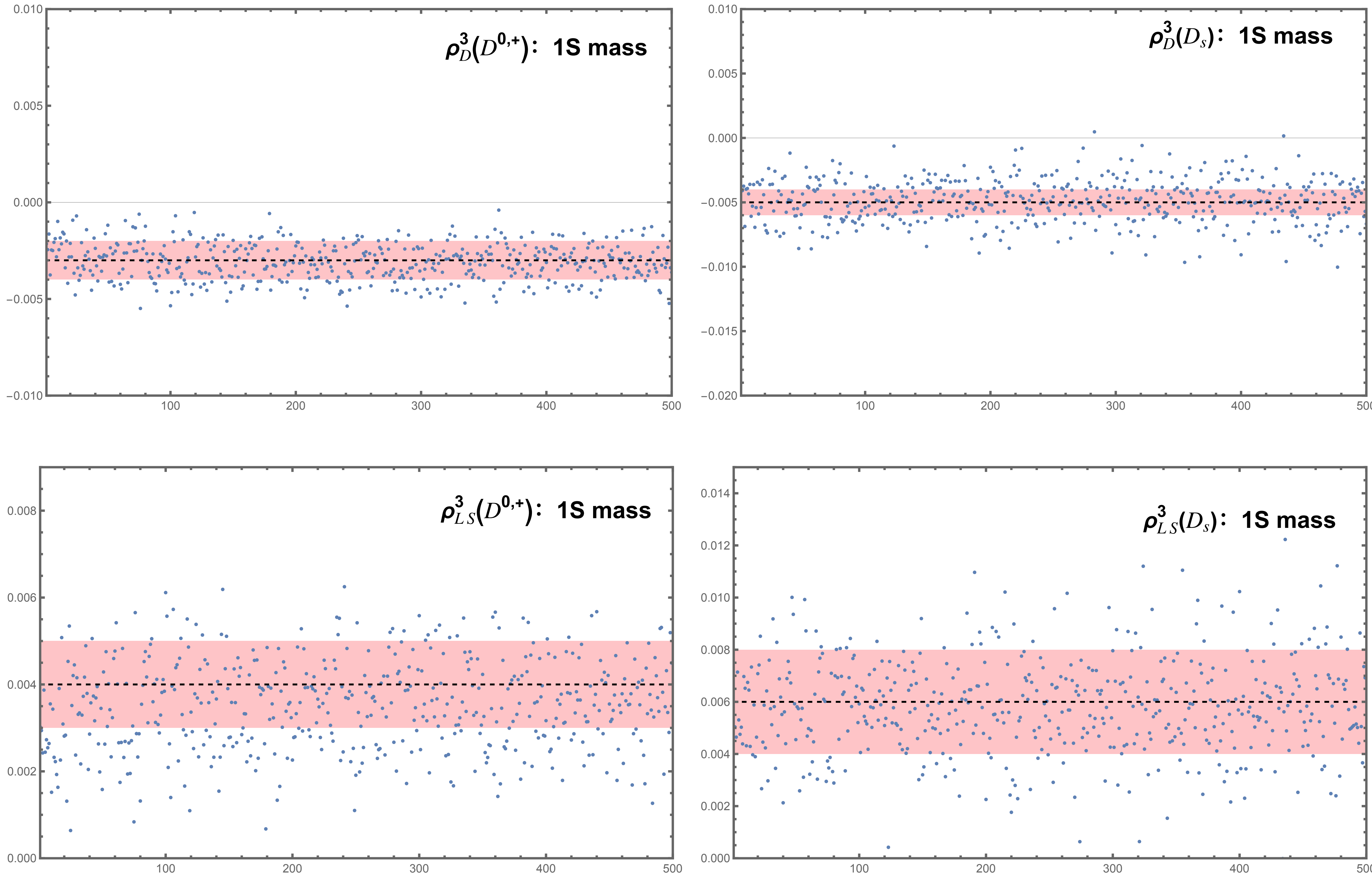

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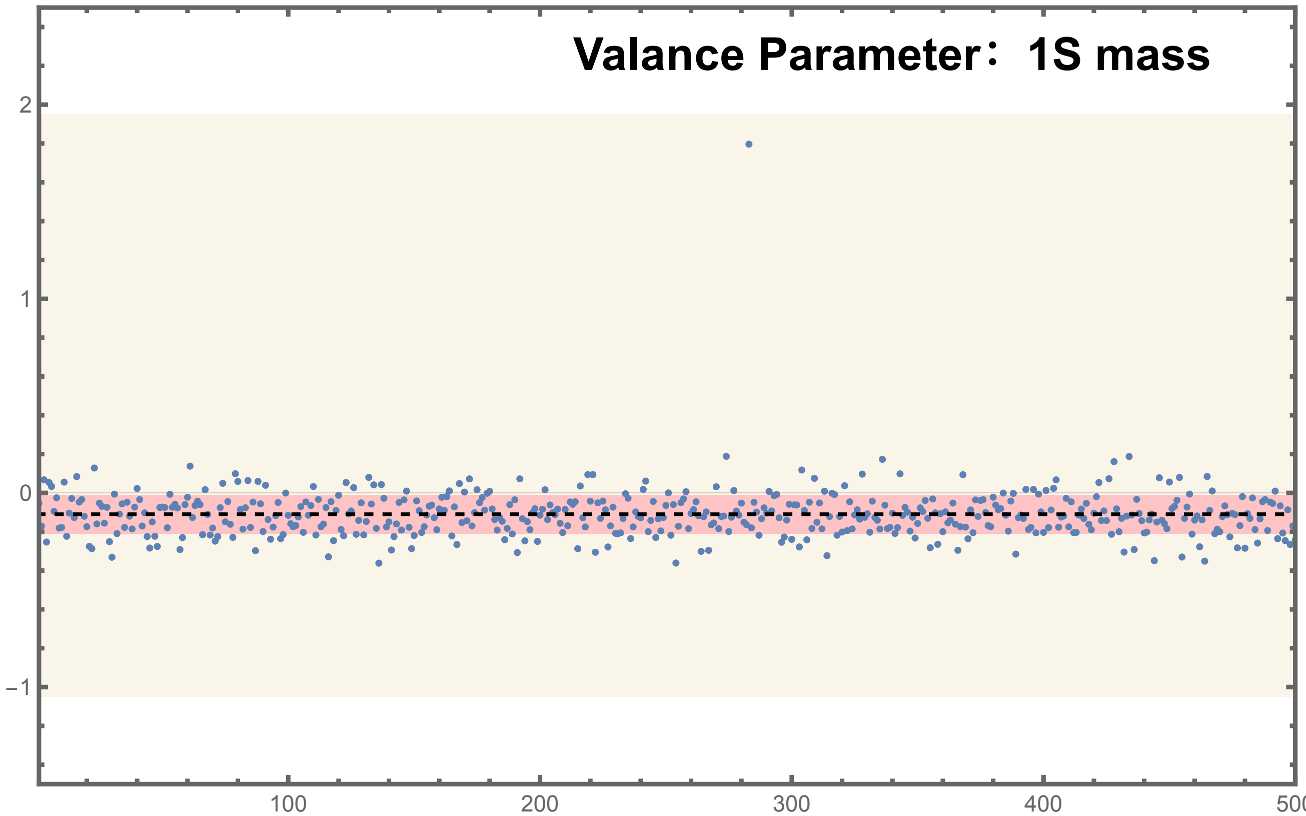
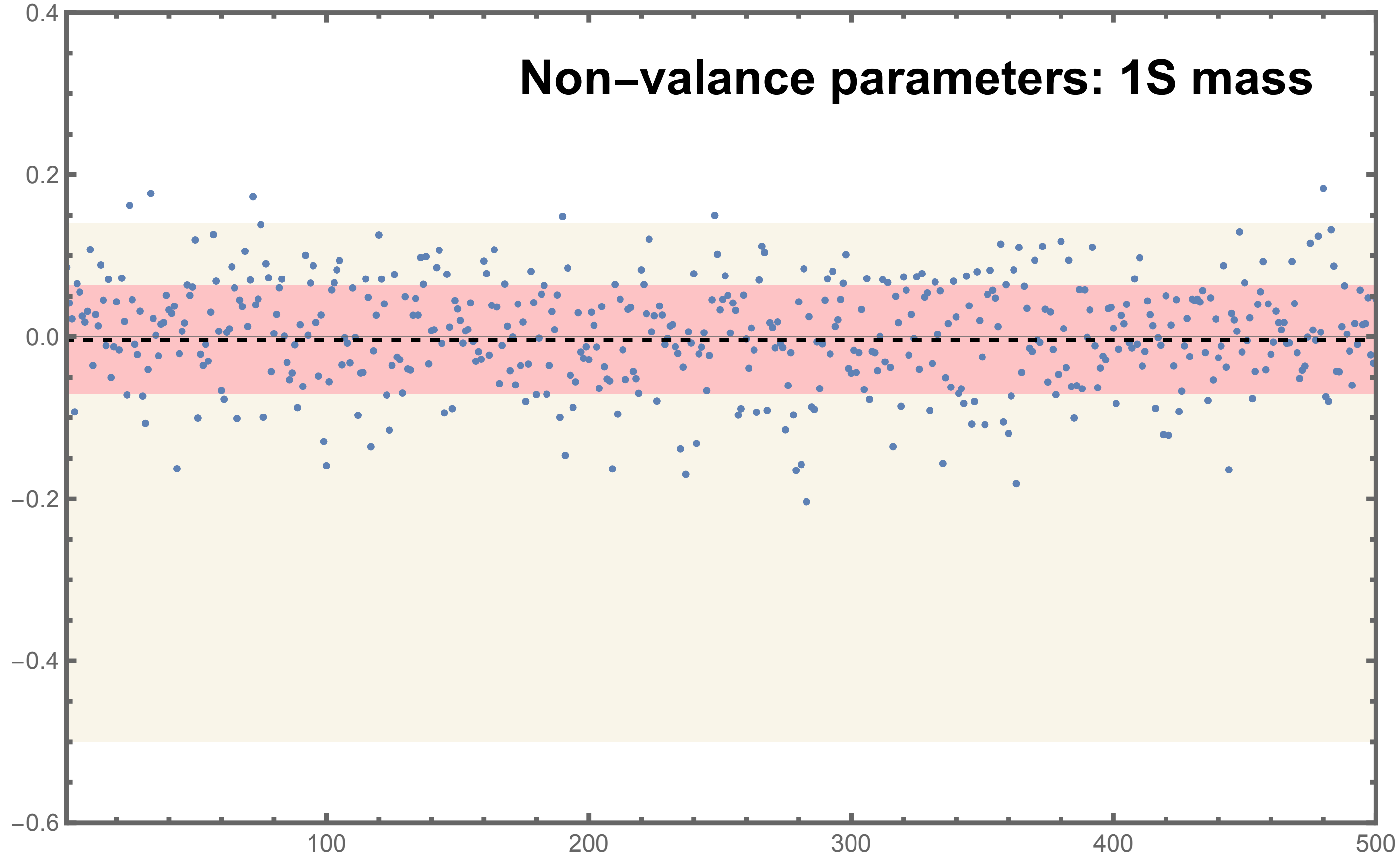




形状函数

b to u 跃迁电子能谱

$$\frac{1}{2\Gamma_b}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = yF(y)\Theta(1-y) - \frac{\lambda_1 + 33\lambda_2}{6m_b^2}\delta(1-y) - \frac{\lambda_1}{6m_b^2}\delta'(1-y)$$

$$\frac{1}{2\Gamma_b}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = yF(y)\frac{1}{N}\sum_{i=1}^{N}\Theta\left(1-y+\varepsilon_i\right)$$

$$\delta y = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i = -\frac{\lambda_1 + 33\lambda_2}{6m_b^2}, \quad \sigma_y^2 = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i^2 = -\frac{\lambda_1}{3m_b^2}$$

$$\frac{1}{N}\sum_{i=1}^{N}\Theta\left(1-y+\varepsilon_i\right) \xrightarrow{N\to\infty} \vartheta(y) = \Theta(1-y) + S(y)F(1)/F(y)$$

$$S(y) = \left\langle \Theta\left[1 - y + \frac{2}{m_b}(v-\hat{p})\cdot iD\right] - \Theta(1-y)\right\rangle + \text{ less singular terms}$$

Fig.3(b)

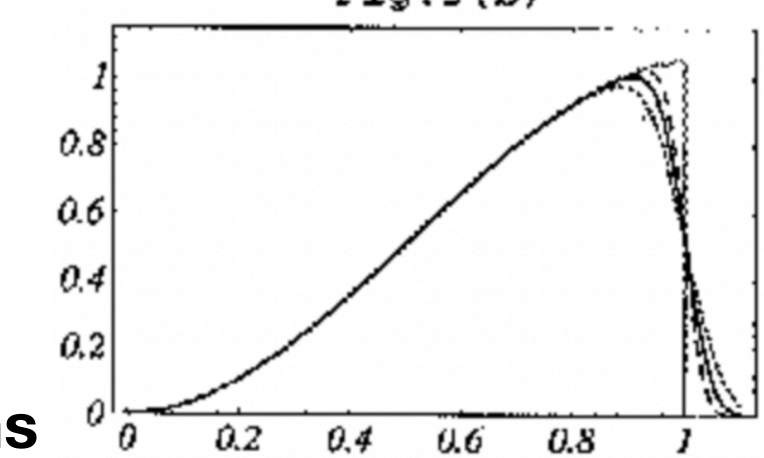
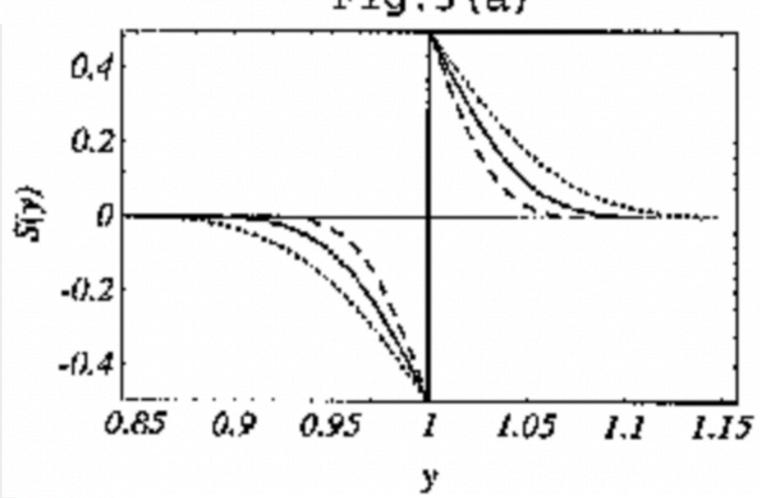


Fig.3(a)



P: 轻子动量

D：b夸克动量

部分子模型

束缚态效应 $\phi\left(\left|\vec{p}_b\right|\right)$

忽略胶子场 A.C.M. 模型

$$\phi\left(\left|\vec{p}_b\right|\right) = \frac{4}{\sqrt{\pi}p_F^3}\exp\left(-\frac{\left|\vec{p}_b\right|^2}{p_F^2}\right)$$

$$S_{\mathrm{ACM}}(y) = \int \cdots\phi\left(\left|\vec{p}_b\right|\right)\cdots\mathrm{d}\vec{p}_b = \left[\frac{1}{2} - \Theta(1-y)\right]\Phi\left(\frac{m_b}{p_F}|y-1|\right)$$

粲介子寿命与半轻单举衰变宽度

- 同位旋对称性: D^0, D^+

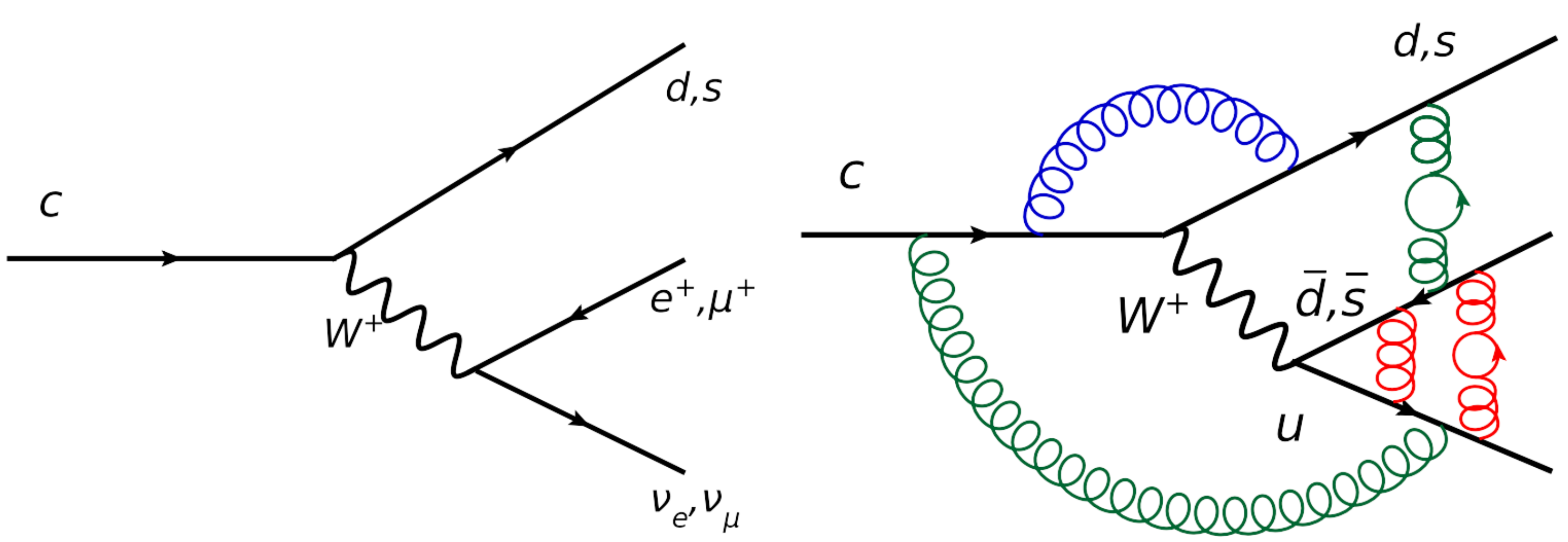
强子化过程中

Quantity	D^0	D^+	D_s^+
$\tau\ [ps]$	0.4101 ± 0.0015	1.040 ± 0.007	0.504 ± 0.004
$\Gamma\ [ps^{-1}]$	2.438 ± 0.009	0.962 ± 0.006	1.984 ± 0.0016
$BR(D_i \to Xe\nu)\ [\%]$	6.49 ± 0.16	16.07 ± 0.30	6.30 ± 0.16
$\Gamma(D_i \to Xe\nu)\ [ps^{-1}]$	0.158 ± 0.004	0.155 ± 0.003	0.125 ± 0.003

旁观者效应贡献很小

HQETSR, VIA

- 粲介子寿命差异: 旁观者夸克效应：弱湮灭、弱交换和泡利干涉项



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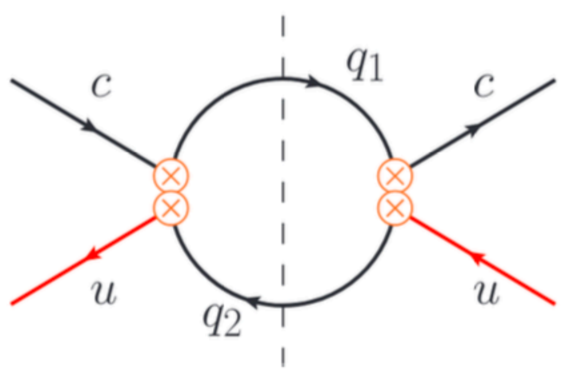
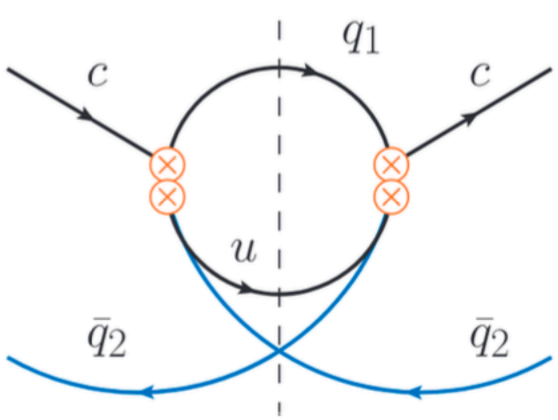
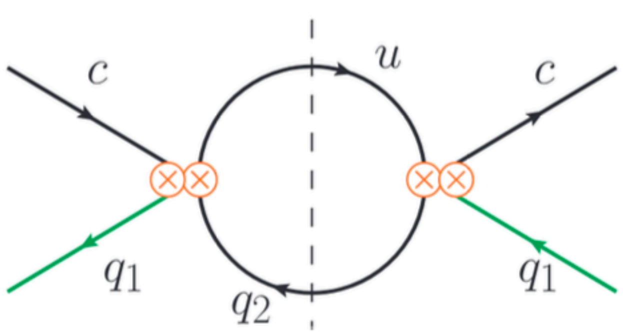


弱交换



泡利干涉



弱湮灭

半轻衰变过程仅包含弱湮灭过程

粲介子寿命与半轻单举衰变宽度

- 动力学质量方案下粲介子衰变宽度对非微扰矩阵元依赖形式为（$\mu = 0.5\mathrm{GeV}$）：

$$\Gamma\left(\mathrm{D}^0\right) = \Gamma_0[\underbrace{6.15}_{c_3^{\mathrm{Lo}}} + \underbrace{2.95}_{\Delta c_3^{\mathrm{NLO}}} - 1.66\frac{\mu_\pi^2(D)}{\mathrm{GeV}^2} + 0.13\frac{\mu_G^2(D)}{\mathrm{GeV}^2} + 23.6\frac{\rho_D^3(D)}{\mathrm{GeV}^3}$$

$$-1.60\tilde{B}_1^q + 1.53\tilde{B}_2^q - 21.0\tilde{\epsilon}_1^q + 19.2\tilde{\epsilon}_2^q + \underbrace{0.00}_{\mathrm{dim-7,VIA}}$$

$$-10.7\tilde{\delta}_1^{qq} + 1.53\tilde{\delta}_2^{qq} + 54.6\tilde{\delta}_3^{qq} + 0.13\tilde{\delta}_4^{qq} - 29.2\tilde{\delta}_1^{sq} + 28.8\tilde{\delta}_2^{sq} + 0.56\tilde{\delta}_3^{sq} + 2.36\tilde{\delta}_4^{sq}]$$

$$= 6.15\Gamma_0\Big[1 + 0.48 - 0.13\frac{\mu_\pi^2(D)}{0.48\mathrm{GeV}^2} + 0.01\frac{\mu_G^2(D)}{0.34\mathrm{GeV}^2} + 0.31\frac{\rho_D^3(D)}{0.082\mathrm{GeV}^3}$$

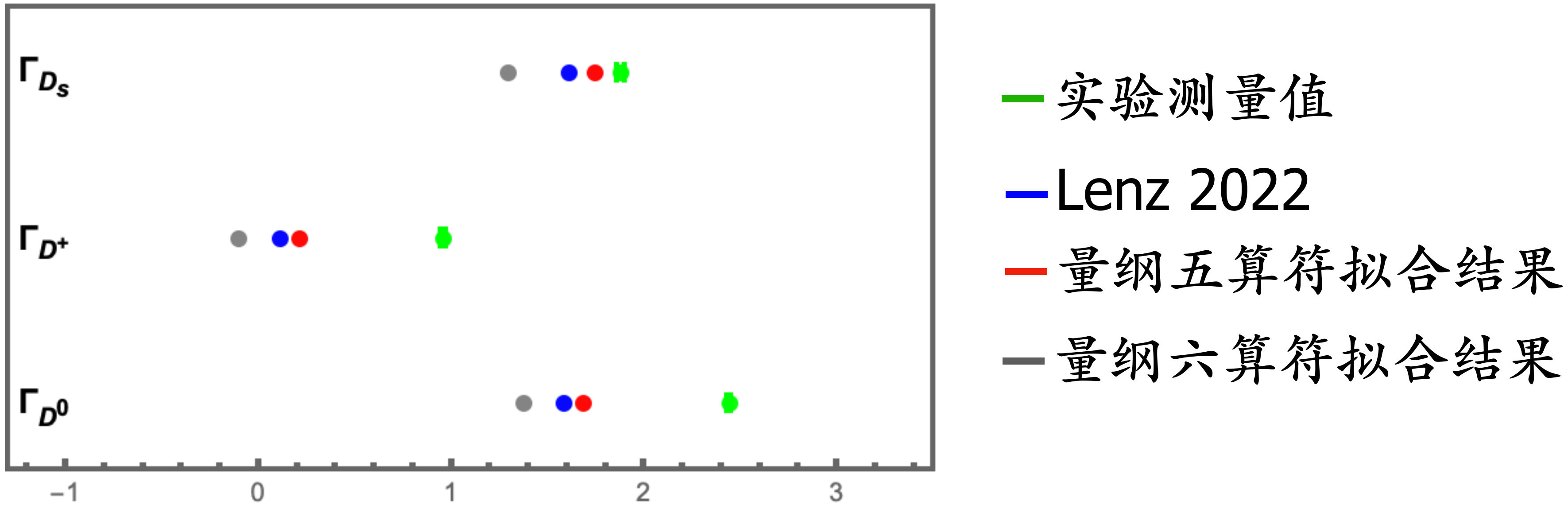
$$-\underbrace{0.01}_{\mathrm{dim\text{-}6,VIA}} - 0.005\frac{\delta\tilde{B}_1^q}{0.02} + 0.005\frac{\delta\tilde{B}_2^q}{0.02} + 0.137\frac{\tilde{\epsilon}_1^q}{-0.04} - 0.125\frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{0.00}_{\mathrm{dim-7,VIA}}$$

$$-0.0045r_1^{qq} - 0.0004r_2^{qq} - 0.0035r_3^{qq} + 0.0000r_4^{qq}$$

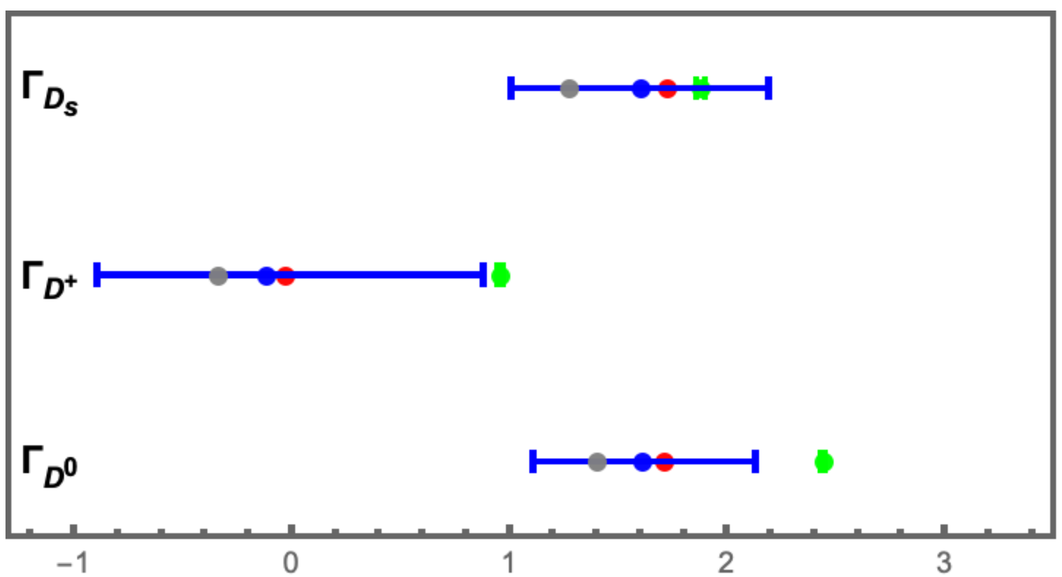
$$-0.0109r_1^{sq} - 0.0079r_2^{sq} - 0.0000r_3^{sq} + 0.0001r_4^{sq}\Big].$$

粲介子寿命的初步定性讨论

动力学质量方案下基于VIA对粲介子衰变宽度的理论预言



动力学质量方案下基于HQETSR对粲介子衰变宽度的理论预言



- 实现了量纲五算符非微扰矩阵元模型无关的高精度全局拟合估计
- 该估计值使得衰变宽度更靠近实验测量值
- 该现象没有模型依赖