

Revising the Mass of Light Hybrid Mesons:

NLO QCD Sum Rules Point to $\phi(2170)$ as a Prime Candidate

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Outline

- **Introduction:** history & conflict among different methods
- **NLO Calculation:** renormalization & NLO correction
- **Impact of Vector Meson:** how 1^{--} vector meson affects the prediction
- **Sum Rules Analysis:** Laplace & Gaussian sum rules
- **Conclusion**

History & conflict

$J^{PC} = 1^{--}$ hybrid:

Masses of $\bar{q}qG$ hybrid states (in GeV) and the corresponding values of $\sqrt{s_0}$						
J^{PC}	$\bar{b}bG$	$\sqrt{s_0}$	$\bar{c}cG$	$\sqrt{s_0}$	$\bar{s}sG$	$\sqrt{s_0}$
0^{++}	10.9	11.6	5.0	5.4	3.5	3.8
$*0^{--}$	11.4	12.0	5.4	5.8	3.7	4.0
1^{+-}	10.6	11.2	4.1	4.4	<u>2.8</u>	3.2
$*1^{-+}$	<u>10.6</u>	11.5	<u>4.4</u>	5.0	<u>2.5</u>	3.0
0^{-+}	<u>10.5</u>	11.6	<u>4.7</u>	5.4	<u>3.1</u>	3.8
$*0^{+-}$	<u>11.0</u>	12.0	<u>5.1</u>	5.8	<u>3.4</u>	4.0
1^{--}	<u>10.4</u>	<u>11.2</u>	<u>4.1</u>	<u>4.4</u>	<u>2.9</u>	3.2
1^{++}	10.9	11.5	4.7	5.0	<u>2.8</u>	3.0

LO QCD sum rules prediction: $\approx 2.9\text{GeV}$ ($s\bar{s}g$)

Nuclear Physics B262 (1985) 575-592

Phys. Rev. D 100, 034012 (2019)

different states?
missing something?

Flux tube model:

1.8 – 1.9GeV ($u\bar{u}g$);

2.1 – 2.2GeV ($s\bar{s}g$)

Phys. Rev. D 52, 5242 (1995)

Lattice QCD:

2.2 – 2.3GeV ($I = 1$);

2.4 – 2.5GeV ($I = 0$)

Phys. Rev. D 84, 074023 (2011)

Candidate:

$\phi(2170), I^G(J^{PC}) = 0^-(1^{--})$

Decays into $\phi f_0(980), \phi\eta, \dots$

QCD (SVZ) Sum Rules of Hybrid

$$J^\mu = g \bar{\Psi} \gamma_\rho \gamma^5 \tilde{G}^{\rho\mu} \Psi; \quad \tilde{G}^{\rho\mu} = \frac{1}{2} \varepsilon^{\rho\mu\alpha\beta} G_{\alpha\beta}^n T^n; \quad T^n = \frac{\lambda^n}{2}$$

$$\Pi^{\mu\nu}(q) = i \int d^4x e^{-ipx} \langle \Omega | T J^\mu(x) J^\nu(0) | \Omega \rangle = \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) \Pi_v(q^2) + \frac{q^\mu q^\nu}{q^2} \Pi_s(q^2)$$

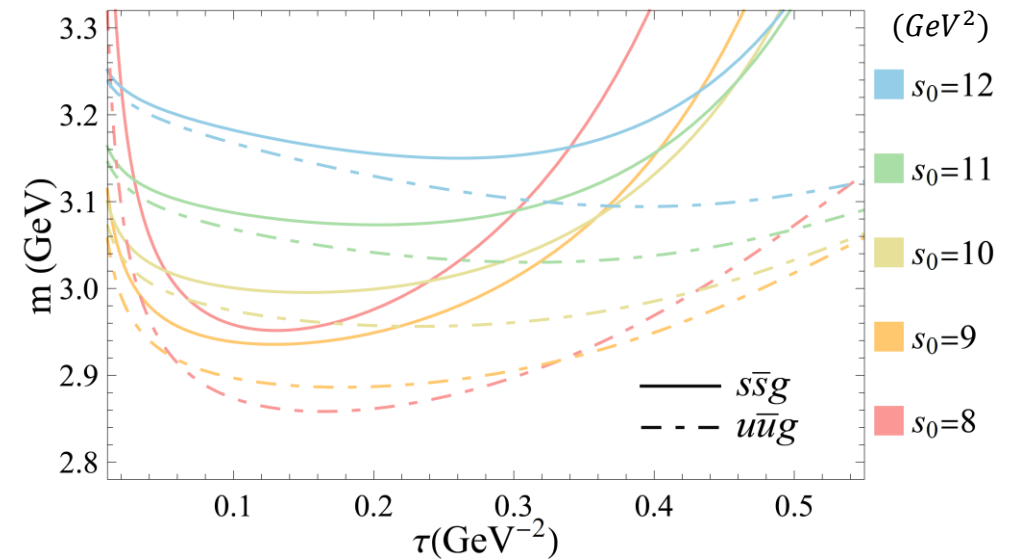
Dispersion relation, “ δ + continuum” ansatz

$$\frac{1}{\pi} \text{Im} \Pi(s) \sim f^2 \delta(s - m^2) + \theta(s - s_0) \rho(s)$$

Laplace sum rules (Borel transformation)

$$\mathcal{M}^n = \frac{1}{\pi} \int_0^{s_0} ds s^n e^{-\tau s} \text{Im} \Pi(s) = f^2 m^{2n} e^{-\tau m^2}$$

$$\mathcal{R}^n = \frac{\mathcal{M}^{n+1}}{\mathcal{M}^n} = m^2$$

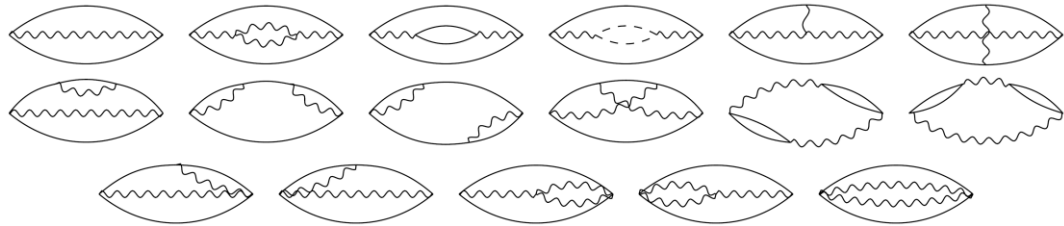


LO Laplace sum rules prediction,
including dimension-8 condensate

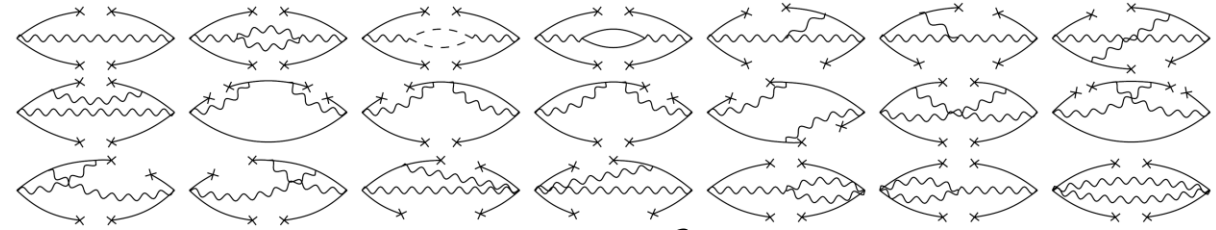
NLO Calculation

$$\begin{aligned}\Pi_v(q^2) &= \sum_n C^n(q^2) \langle O_n \rangle \\ &= C^0(q^2) + C_1^4(q^2) \langle GG \rangle + C_2^4(q^2) m \langle \bar{q}q \rangle + C_1^6(q^2) \langle \bar{q}q \rangle^2 + C_2^6(q^2) m \langle \bar{q}Gq \rangle + C^8(q^2) \langle \bar{q}q \rangle \langle \bar{q}Gq \rangle + \dots \\ &\quad + \alpha_s C^{0'}(q^2) + \alpha_s C_1^{4'}(q^2) \langle GG \rangle + \dots \\ &\quad + \dots\end{aligned}$$

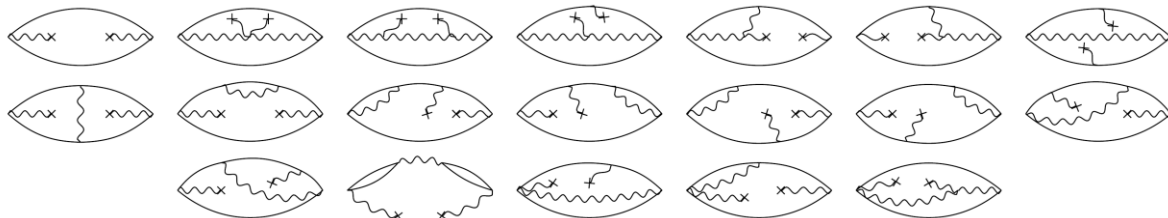
Higher order correction



perturbative

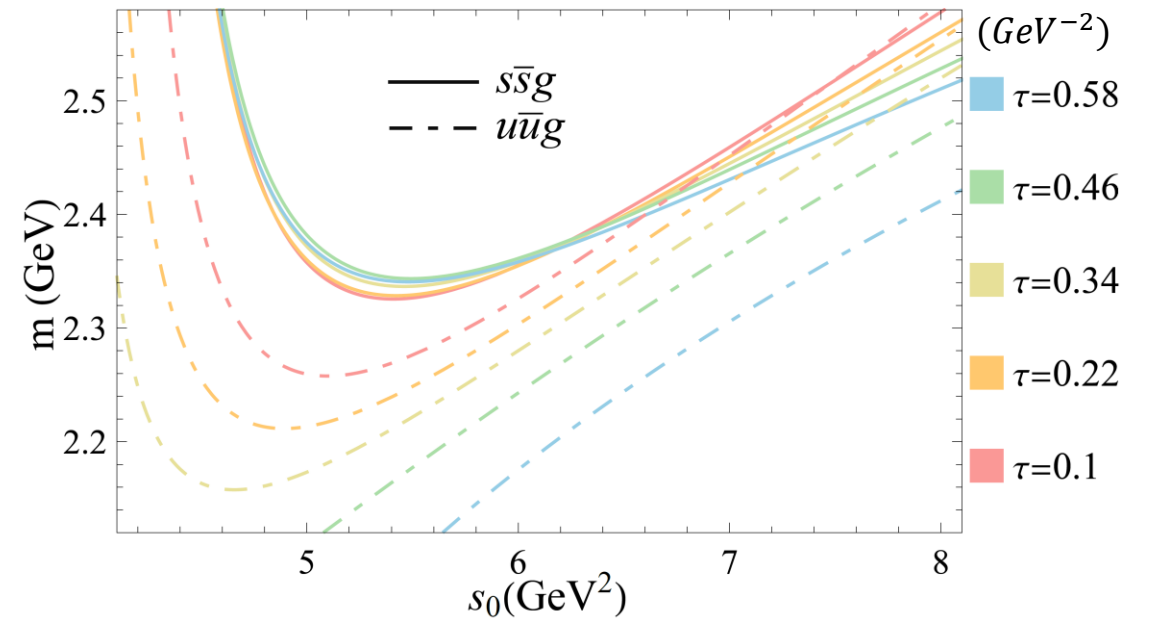
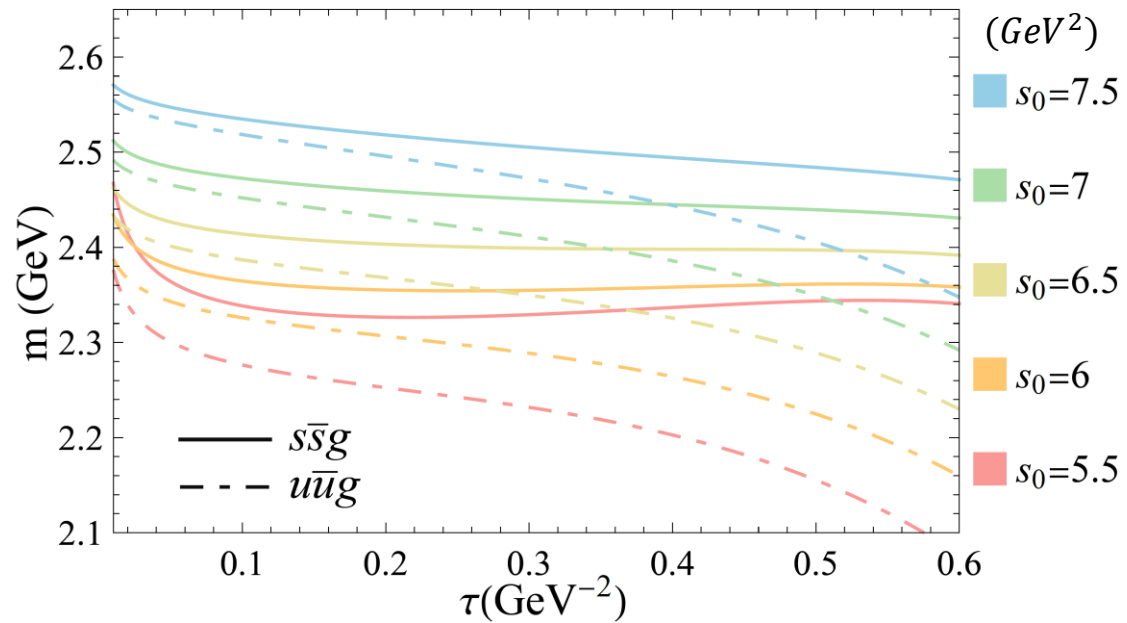


$\langle \bar{q}q \rangle^2$



$\langle GG \rangle$

NLO Result



Lattice QCD:

2.2 – 2.3 GeV ($I = 1$);

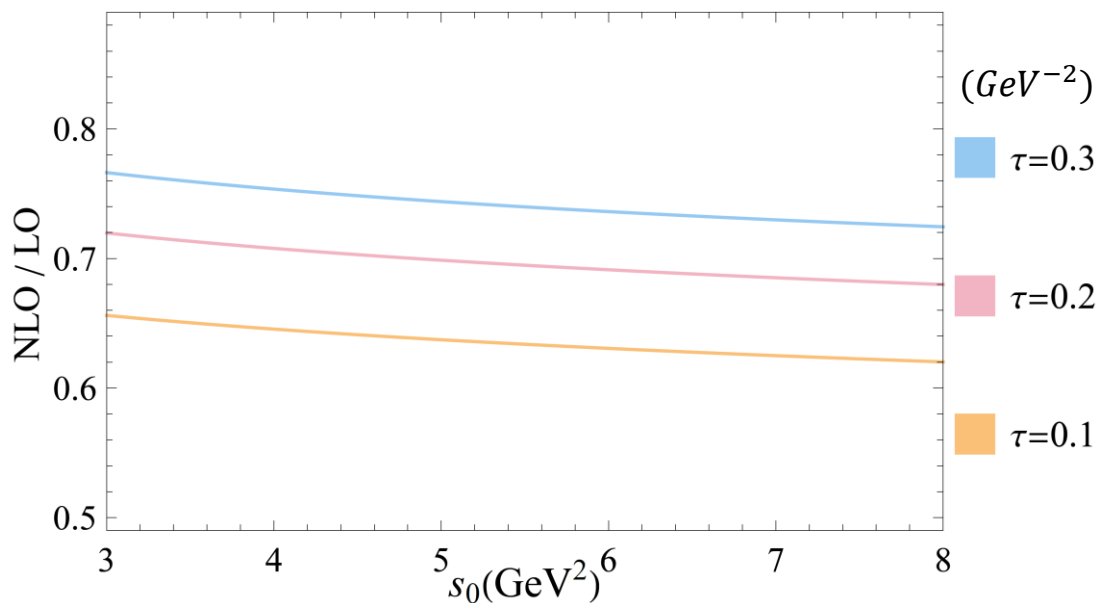
2.4 – 2.5 GeV ($I = 0$)

Phys. Rev. D 84, 074023 (2011)

NLO Impact

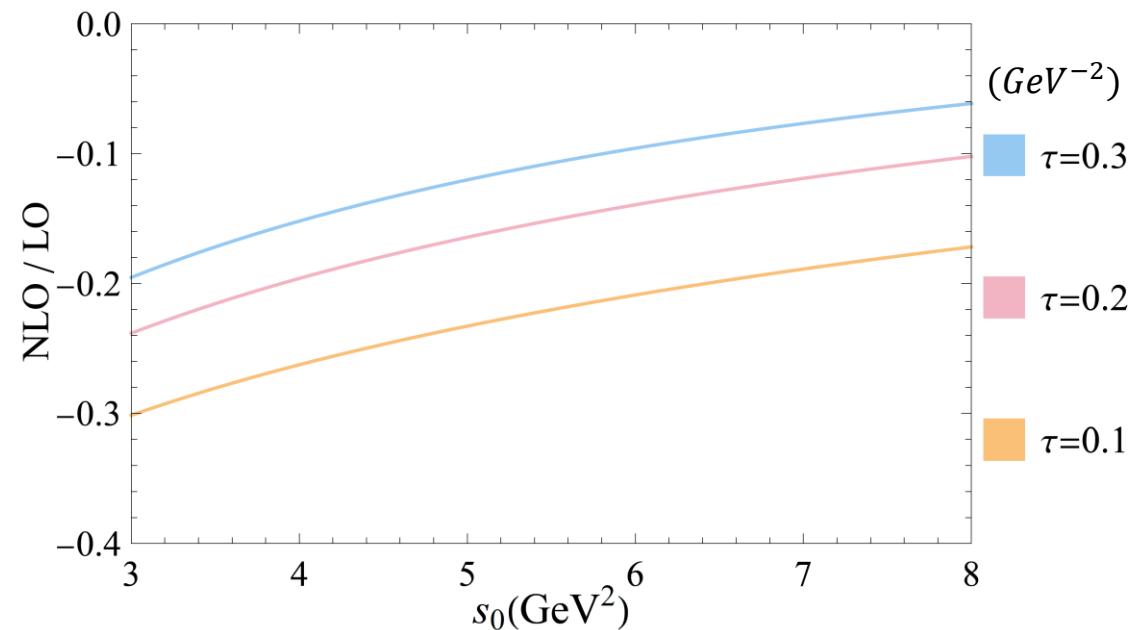
NLO contribution vs LO contribution (after Borel transformation)

NLO perturbative / LO perturbative



$$\frac{\widehat{B} \alpha_s C^{0'}(q^2)}{\widehat{B} C^0(q^2)}$$

NLO $\langle GG \rangle$ / LO $\langle GG \rangle$



$$\frac{\widehat{B} \alpha_s C_1^{4'}(q^2) \langle GG \rangle}{\widehat{B} C_1^4(q^2) \langle GG \rangle}$$

Impact of Vector Meson

$J_H^\mu = g \bar{\Psi} \gamma_\rho \gamma^5 \tilde{G}^{\rho\mu} \Psi$ couples to 1^{--} meson as well

$$\frac{1}{\pi} \text{Im} \Pi(s) \sim f_1^2 \delta(s - m_1^2) + f_2^2 \delta(s - m_2^2) + \theta(s - s'_0) \rho(s)$$

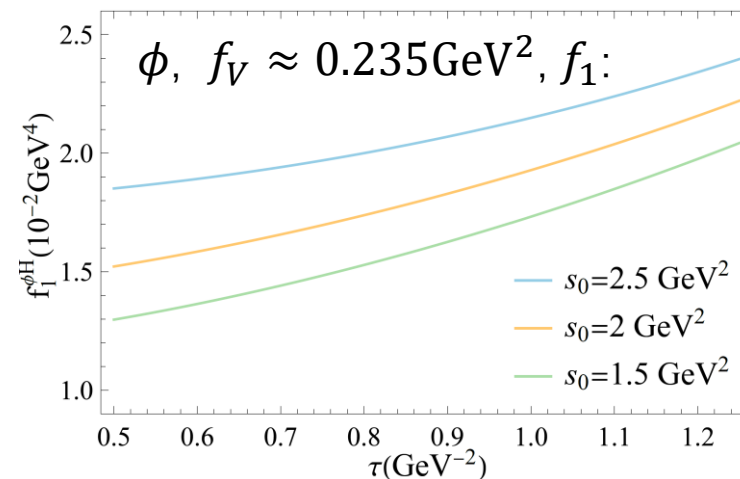
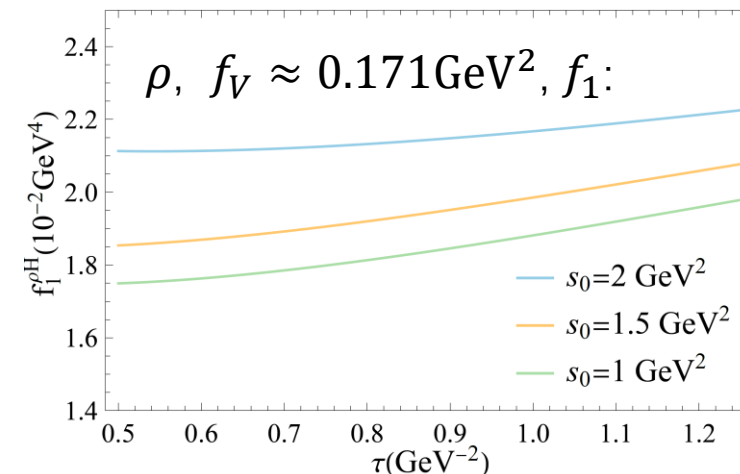
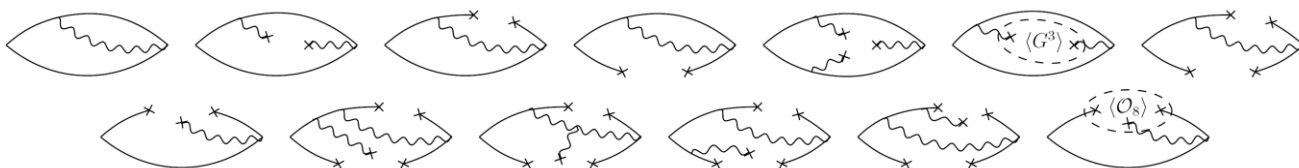
$$\langle \Omega | J^\mu | V(q) \rangle = f_1 \epsilon^\mu(q), \quad \langle \Omega | J^\mu | H(q) \rangle = f_2 \epsilon^\mu(q)$$

Vector meson $J_V^\mu = \bar{\Psi} \gamma^\mu \Psi$

$$\begin{aligned} \Pi^{\mu\nu}(q) &= i \int d^4x e^{-ipx} \langle \Omega | T J_V^\mu(x) J_H^\nu(0) | \Omega \rangle \\ &= \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) \Pi_v^{\text{VH}}(q^2) + \frac{q^\mu q^\nu}{q^2} \Pi_s^{\text{VH}}(q^2) \end{aligned}$$

$$\frac{1}{\pi} \text{Im} \Pi_v^{\text{VH}}(s) = f_V f_1 \delta(s - m_1^2) + \theta(s - s_0) \rho(s)$$

$$\langle \Omega | J_V^\mu | V(q) \rangle = f_V \epsilon^\mu(q)$$



$$f_1 \sim 0.02 \text{ GeV}^4$$

$$\rightarrow f_1/m_1^3 \sim 40 \text{ MeV}$$

$$\sim 20 - 35 \text{ MeV (P.L.B 175 (1986) 485)}$$

Gaussian Sum Rules

QCD side:

$$G(\tau, s, s_0) = \frac{1}{\sqrt{4\pi\tau}} \int_0^{s_0} dt \, e^{-\frac{(s-t)^2}{4\tau}} \frac{1}{\pi} \text{Im}\Pi_v(s)$$

Hadron side:

$$G_h(\tau, s) = \frac{1}{\sqrt{4\pi\tau}} \left(f_1^2 e^{-\frac{(s-m_1^2)^2}{4\tau}} + f_2^2 e^{-\frac{(s-m_2^2)^2}{4\tau}} \right)$$

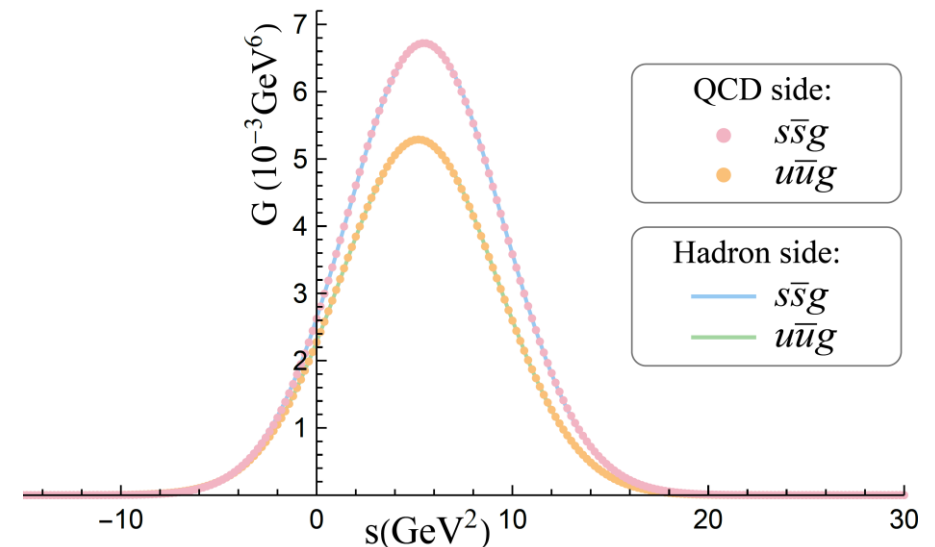
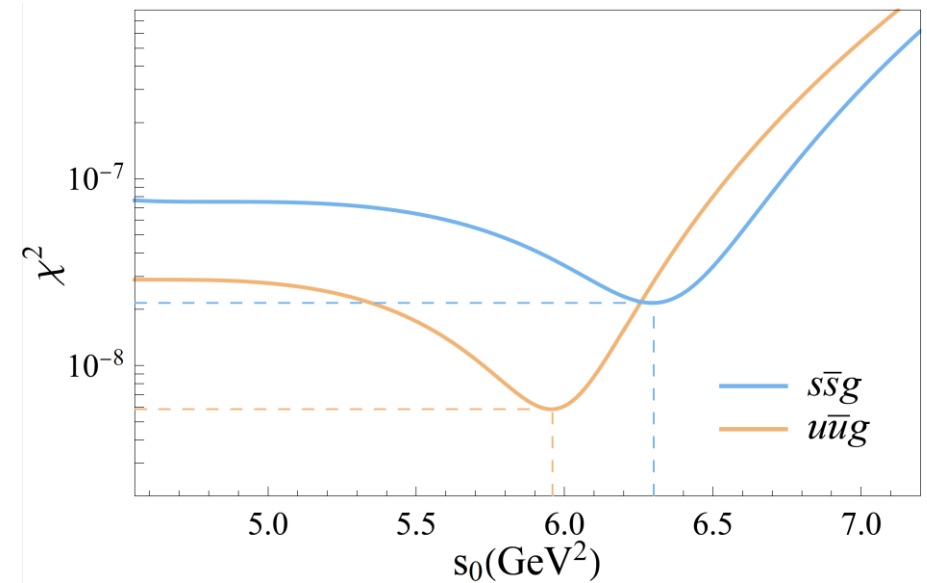
Fitting

$$\chi^2(\tau, s_0) = \sum_{i=0}^N [G(\tau, s_i, s_0) - G_h(\tau, s_i)]^2$$

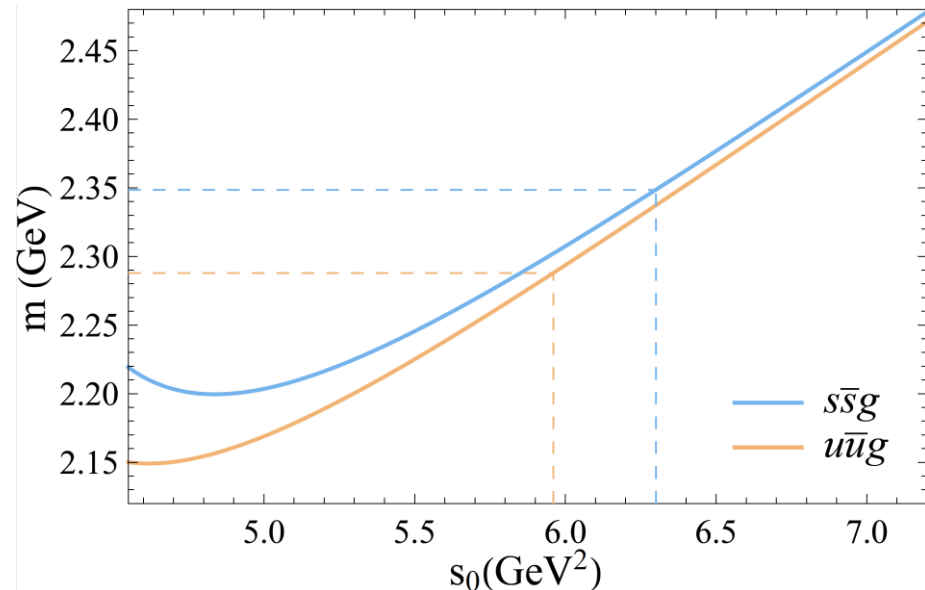
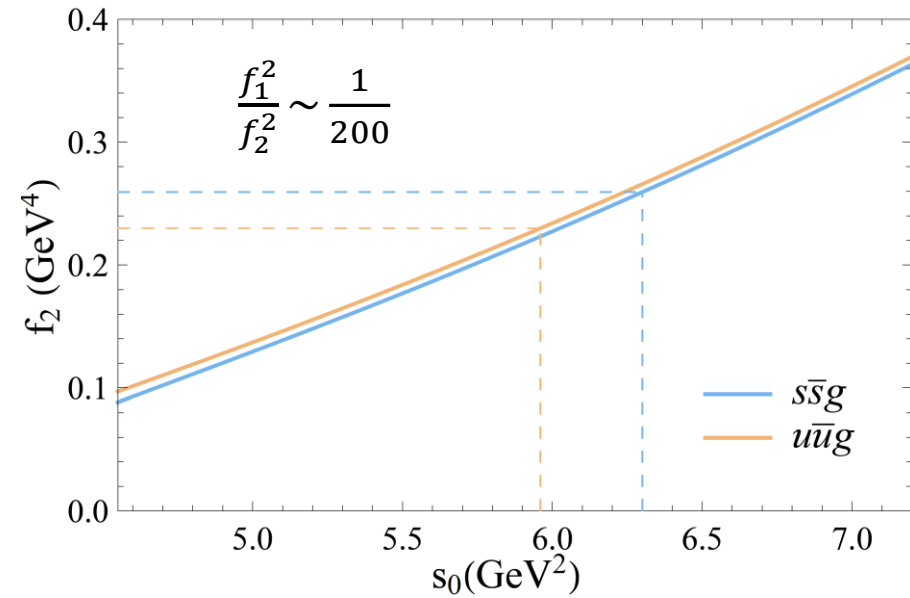
$$s_i = s_{\min} + i \, \delta s, \quad \delta s = 0.2 \text{ GeV}^2$$

$$s_{\min} = -15 \text{ GeV}^2, \quad s_{\max} = 30 \text{ GeV}^2$$

f_1, m_1 : fixed according to the vector meson



Gaussian Sum Rules



Normalized single resonance GSR:

$$\tilde{G}(\tau, s, s_0) = \frac{G(\tau, s, s_0)}{\int_{-\infty}^{\infty} ds G(\tau, s, s_0)}$$

$$\tilde{G}_h(\tau, s) = \frac{1}{\sqrt{4\pi\tau}} e^{-\frac{(s-m^2)^2}{4\tau}}$$

Parameters	NLO	
	$s\bar{s}g$	$u\bar{u}g$
$\overline{\chi^2_{\min}}$	1.17×10^{-5}	2.32×10^{-5}
$s_0(\text{GeV}^2)$	6.15 ± 1.21	5.85 ± 1.21
$m(\text{GeV})$	2.31 ± 0.23	2.25 ± 0.23

single resonance fitting with Monte Carlo uncertainty analysis (2000 random samples)

$$\langle GG \rangle = 0.07 \pm 0.02 \text{ GeV}^4$$

$$\langle \bar{q} G q \rangle = M_0^2 \langle \bar{q} q \rangle, \quad M_0^2 = 0.8 \pm 0.2 \text{ GeV}^2$$

10% error for $\langle \bar{q} q \rangle$

Robustness of Results

Modified LSR:

$$\mathcal{M}(\tau, s_0) = \frac{1}{\pi} \int_0^{s_0} ds e^{-\tau s} \text{Im}\Pi(s) = f_1^2 e^{-\tau m_1^2} + f_2^2 e^{-\tau m_2^2}$$

$$\tilde{\mathcal{M}}(\tau, s_0) \equiv \mathcal{M}(\tau, s_0) e^{\tau m_1^2} = f_1^2 + f_2^2 e^{\tau(m_1^2 - m_2^2)}$$

$$\frac{\partial_\tau^2 \tilde{\mathcal{M}}(\tau, s_0)}{\partial_\tau \tilde{\mathcal{M}}(\tau, s_0)} = m_1^2 - m_2^2$$



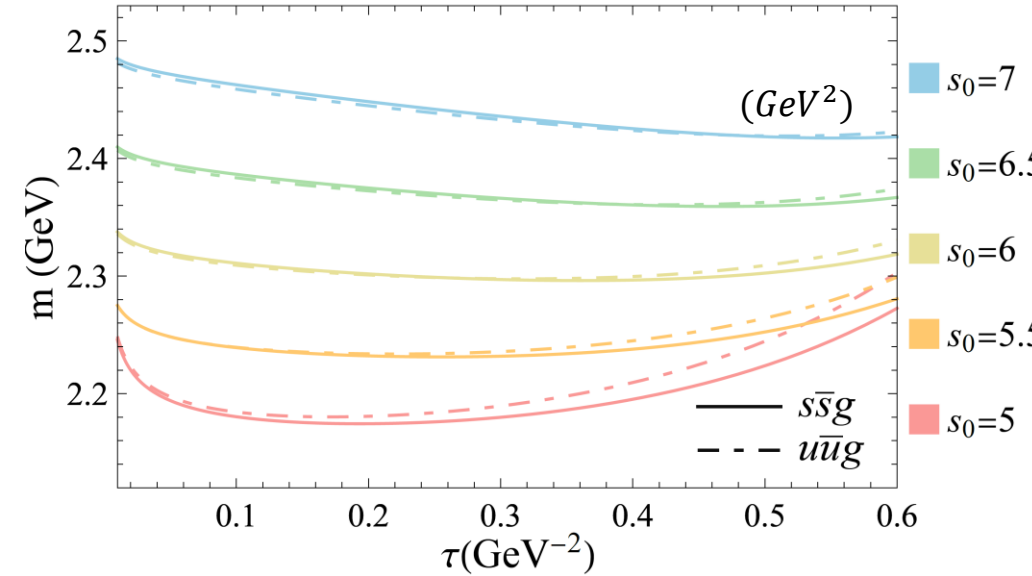
m_1 is vector meson mass

Factorization deviation

$$\begin{aligned} \langle \bar{q}q \rangle^2 &\rightarrow \kappa_6 \langle \bar{q}q \rangle^2 \\ \langle \bar{q}q \rangle \langle \bar{q}Gq \rangle &\rightarrow \kappa_8 \langle \bar{q}q \rangle \langle \bar{q}Gq \rangle \end{aligned}$$



Parameters	$\kappa_6 = 3, \kappa_8 = 2$		$\kappa_6 = 3, \kappa_8 = 5$	
	$s\bar{s}g$	$u\bar{u}g$	$s\bar{s}g$	$u\bar{u}g$
$\overline{\chi^2_{\min}}$	2.85×10^{-5}	7.28×10^{-5}	1.74×10^{-4}	4.30×10^{-4}
$s_0(\text{GeV}^2)$	6.03 ± 1.21	6.02 ± 1.31	6.40 ± 1.60	7.03 ± 2.16
$m(\text{GeV})$	2.29 ± 0.23	2.28 ± 0.24	2.34 ± 0.28	2.43 ± 0.37



Conclusion

NLO GSR:

$$s\bar{s}g: 2.31 \pm 0.23 \text{ GeV}$$
$$u\bar{u}g: 2.25 \pm 0.23 \text{ GeV}$$

- No conflict between Sum Rules and Lattice QCD
- Necessity of calculation beyond LO

Predicted mass agrees with $\phi(2170)$ in error

Future direction:

- Decay of 1^{--} hybrid

Lattice QCD:

$$2.2 - 2.3 \text{ GeV } (I = 1);$$

$$2.4 - 2.5 \text{ GeV } (I = 0)$$

Phys. Rev. D 84, 074023 (2011)

$$2.1 - 2.3 \text{ GeV } (s\bar{s}g)$$

Chinese Physics C 45, 013112 (2021)

$$\approx 2.1 \text{ GeV } (I = 1)$$

Phys. Rev. D 107, 074028 (2023)

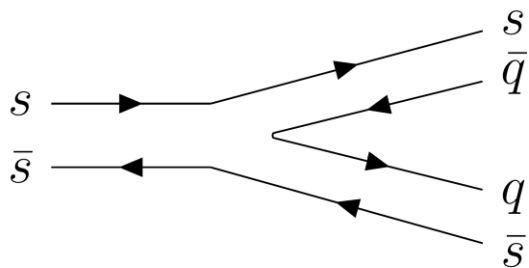
$$2.2 - 2.3 \text{ GeV } (\text{both } \rho\text{-like and } \phi\text{-like})$$

Phys. Rev. D 82, 034508 (2010)

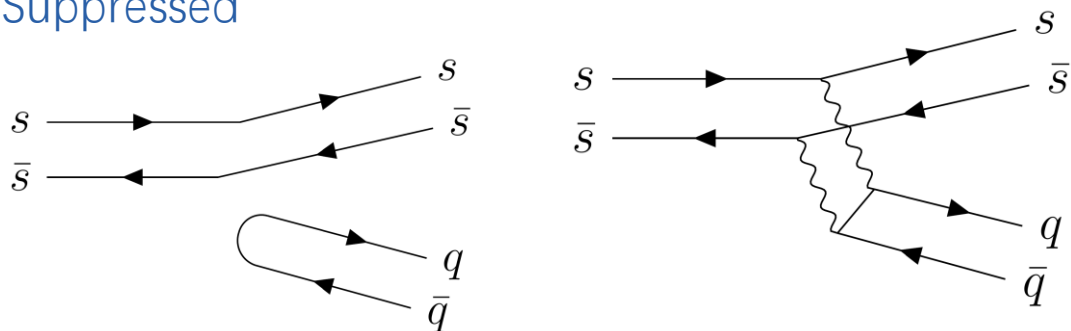
Thank You

Backup: Decay

Favored



Suppressed

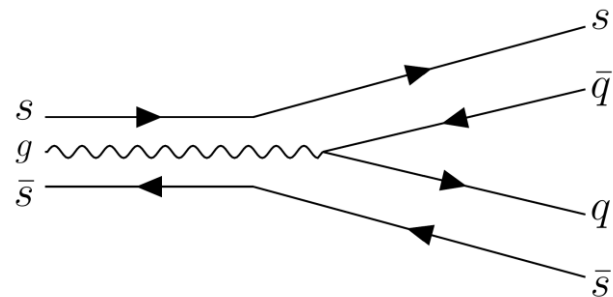


disturb the vacuum $\rightarrow q\bar{q}$ or two virtual gluons $\rightarrow q\bar{q}$
transfers momentum $|p| \sim m_{q\bar{q}}$

1^{--} strangeonium state ($2D$ or $3S$)

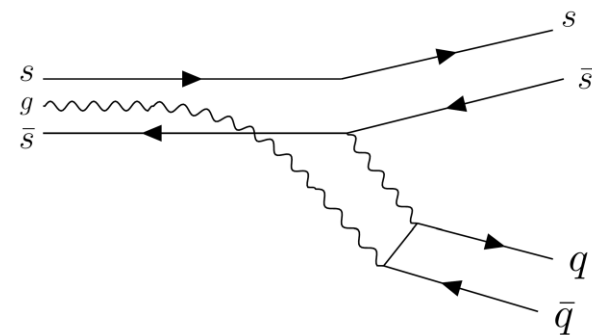
- decay into open strangeness states ($K\bar{K}^*$...)
- branching ratio to $\phi f_0(980)$ should be small
- broad width (large phase space)

Suppressed



$(1^{--} c\bar{c}g \rightarrow D^{(*)}\bar{D}^{(*)})$ is forbidden by symmetry
Physics Letters B 631 (2005) 164–169)

Allowed



$\phi(2170)$

- decay into closed strangeness states ($\phi f_0(980)$...)
- Narrower width ~ 100 MeV (selection rules)

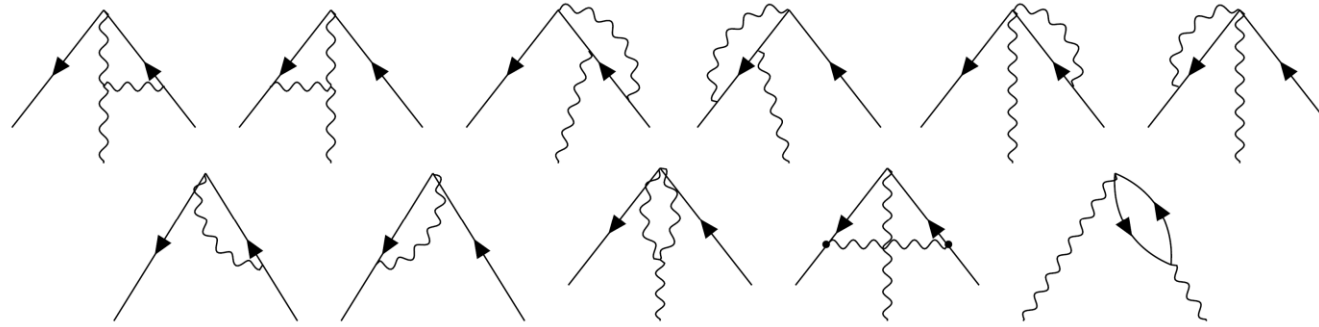
Backup: Renormalization of Generic Hybrid Current

$$J = g\bar{\Psi}\Gamma G\Psi$$

$$\langle T J(0) \Psi(q_1) A(q_2) \bar{\Psi}(q_3) \rangle$$

$$\langle T J(0) \Psi(q_1) \bar{\Psi}(q_2) \rangle$$

$$\langle T J(0) A(q_1) A(q_2) \rangle$$



Observation:

$$\text{[Four diagrams]} + \text{[Four diagrams]} = \text{[Feynman line]} \times \left\{ \text{[Four diagrams]} + \text{[Four diagrams]} + \text{[Four diagrams]} + \text{[Four diagrams]} \right\}$$

Corresponding counter terms $\sim \frac{1}{\varepsilon} \bar{\Psi} \Gamma \times \{\text{operators acting on } \Psi\}$

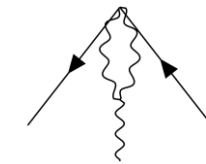
Counterterm bases:

$$\frac{1}{\varepsilon} \bar{\Psi} \Gamma \cdot \Gamma' G \Psi, \quad \frac{1}{\varepsilon} \bar{\Psi} \Gamma \cdot \Gamma' \nabla \nabla \Psi, \quad \frac{1}{\varepsilon} \bar{\Psi} \Gamma \cdot \Gamma' A (\gamma_\mu \nabla^\mu + i m) \Psi, \quad \frac{1}{\varepsilon} \bar{\Psi} \Gamma \cdot \Gamma' m^2 \Psi$$

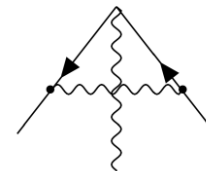


non-gauge-invariant counterterms must vanish under equation of motion

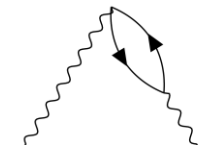
$$\Gamma' G \sim G^{\mu\nu}, \quad \sigma^{\mu\rho} G_\rho^\nu, \quad i \gamma^5 \epsilon^{\mu\nu\rho\eta} G_{\rho\eta}; \quad \Gamma' \nabla \nabla \sim \sigma^\nu_\rho \nabla^\mu \nabla^\rho, \quad \sigma^{\mu\nu} \nabla^2; \quad \dots$$



$$\bar{\Psi} \Gamma T^n \Psi \times \frac{1}{\varepsilon} G_{\mu\nu}^n$$



$$\frac{1}{\varepsilon} \bar{\Psi} \Gamma_1 \cdot \Gamma \cdot \Gamma_2 T^n \Psi \times G_{\mu\nu}^n$$



$$\frac{1}{\varepsilon} m G G, \quad \frac{1}{\varepsilon} G D G$$

Backup: Renormalization of Generic Hybrid Current

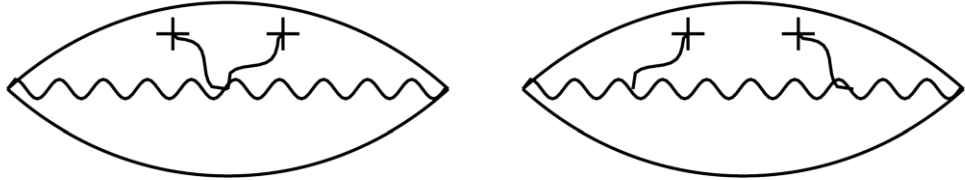
$$J = g\bar{\Psi}\Gamma G^{\mu\nu}\Psi$$

$$\begin{aligned}
 J_r = & (1 + C_0 + C_4)Z_2^{-1}Z_3^{-\frac{1}{2}}\bar{\Psi}_{f_a}g\Gamma G^{\mu\nu}\Psi_{f_b} \\
 & + C_1\left(\bar{\Psi}_{f_a}\left[(\overleftarrow{\nabla}^\mu\overleftarrow{\nabla}^\rho + \overleftarrow{\nabla}^\rho\overleftarrow{\nabla}^\mu)\sigma_\rho{}^\nu.\Gamma + \Gamma.\sigma^\nu{}_\rho(\nabla^\mu\nabla^\rho + \nabla^\rho\nabla^\mu)\right]\Psi_{f_b} - \{\mu \leftrightarrow \nu\}\right) \\
 & + C_2\left(\bar{\Psi}_{f_a}\left[(\overleftarrow{\nabla} - im_{f_a})\gamma^\nu.\Gamma A^\mu g + gA^\mu\Gamma.\gamma^\nu.(\nabla + im_{f_b})\right]\Psi_{f_b} - \{\mu \leftrightarrow \nu\}\right) \\
 & + C_3\bar{\Psi}_{f_a}(\overleftarrow{\nabla}^2\sigma^{\mu\nu}.\Gamma + \Gamma.\sigma^{\mu\nu}\nabla^2)\Psi_{f_b} \\
 & + C_5\bar{\Psi}_{f_a}(\gamma^5.\Gamma + \Gamma.\gamma^5)i\varepsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}\Psi_{f_b} \\
 & + C_6\left(\bar{\Psi}_{f_a}\left[i\sigma^{\rho\mu}.\Gamma + i\Gamma.\sigma^{\mu\rho}\right]G_\rho{}^\nu\Psi_{f_b} - \{\mu \leftrightarrow \nu\}\right) \\
 & + C_7\left(\bar{\Psi}_{f_a}\left[\overleftarrow{\nabla}^\mu\gamma^\nu.\Gamma m_{f_a} + m_{f_b}\Gamma.\gamma^\nu\nabla^\mu\right]\Psi_{f_b} - \{\mu \leftrightarrow \nu\}\right) \\
 & + C_8\bar{\Psi}_{f_a}(\sigma^{\mu\nu}.\Gamma m_{f_a}^2 + m_{f_b}^2\Gamma.\sigma^{\mu\nu})\Psi_{f_b} \\
 & + C_9\bar{\Psi}_{f_a}\gamma^\rho.\gamma^\sigma.\Gamma.\gamma_\sigma.\gamma_\rho G_{\mu\nu}\Psi_{f_b} \\
 & + C_{10}\delta_{f_af_b}\left(\text{Tr}[\Gamma.\sigma^{\alpha\beta}]m_{f_a}G^{n\mu\nu}G_{\alpha\beta}^n + \frac{2}{3}\text{Tr}[\Gamma.\gamma_\alpha]G^{n\mu\nu}D_\beta^{nm}G^{m\alpha\beta}\right).
 \end{aligned}$$

C_0	C_1	C_2	C_3	C_4	C_5
$\frac{3C_A g}{32\pi^2\varepsilon}$	$\frac{C_F g^2}{192\pi^2\varepsilon}$	$\frac{C_A g^2}{128\pi^2\varepsilon}$	$\frac{C_F g^2}{96\pi^2\varepsilon}$	$-\frac{C_A g}{16\pi^2\varepsilon}$	$\frac{C_A g^2}{128\pi^2\varepsilon}$
C_6	C_7	C_8	C_9	C_{10}	
$\frac{(3C_A^2+1)g^2}{128C_A\pi^2\varepsilon}$	$\frac{C_F g^2}{32\pi^2\varepsilon}$	$\frac{C_F g^2}{32\pi^2\varepsilon}$	$\frac{g^2}{128C_A\pi^2\varepsilon}$	$\frac{g^2}{64\pi^2\varepsilon}$	

In Feynman gauge

Backup: Infrared divergence



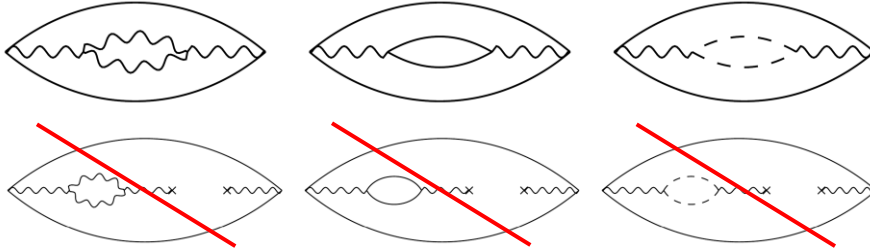
$$\langle T G^{\mu\nu}(k) G^{\alpha\beta}(k) \rangle \sim \frac{\langle GG \rangle}{k^4} (g^{\mu\alpha} g^{\nu\beta} + \dots)$$

$$\int d^d k \frac{1}{k^4}, \quad d = 4 - 2\epsilon, \quad \Rightarrow \sim \frac{1}{\epsilon} \text{ for } k \approx 0$$

$$J_r = (1 + \mathcal{C}_0 + \mathcal{C}_4) Z_2^{-1} \mathcal{Z}_3^{-1/2} g \bar{\Psi} \Gamma G^{\mu\nu} \Psi + \dots$$

$\mathcal{Z}_3$

$\leftrightarrow$

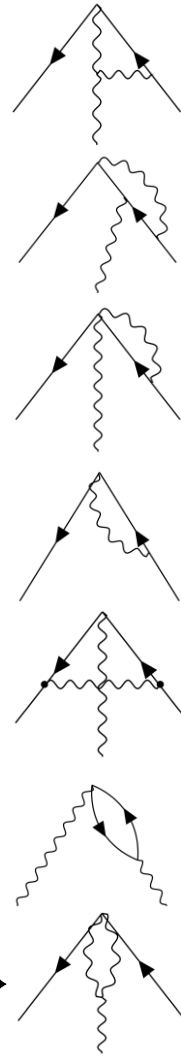


$$A_0^\mu = Z_3 A^\mu$$

$$\left(\mathcal{C}_0 + \mathcal{Z}_3^{-1/2} \mathcal{Z}_g^{-1} - 1 \right) g_0 \bar{\Psi} \Gamma G^{\mu\nu} \Psi$$

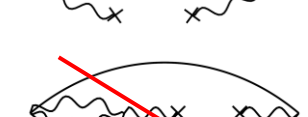
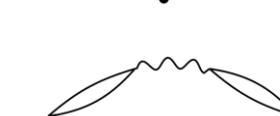
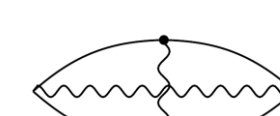
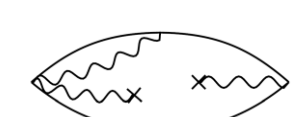
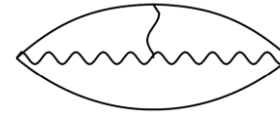
$$\mathcal{Z}_3 = 1 - \frac{g^2}{16\pi^2\epsilon} \left[-\frac{5}{3} C_A + \frac{2}{3} n_f \right], \quad \mathcal{Z}_g = 1 + \frac{g^2}{16\pi^2\epsilon} \left(\frac{11}{6} C_A - \frac{1}{3} n_f \right)$$

$$\mathcal{C}_0 g \bar{\Psi} \Gamma G \Psi \leftrightarrow$$



Perturbative

$\langle GG \rangle$



Backup: Error Estimation

$$\chi^2(\tau, s_0) = \sum_{i=0}^N [\tilde{G}(\tau, s_i, s_0) - \tilde{G}_h(\tau, s_i)]^2$$

$$s_i = s_{\min} + i \delta s, \quad \delta s = 0.2 \text{ GeV}^2$$

$$s_{\min} = -15 \text{ GeV}^2, \quad s_{\max} = 30 \text{ GeV}^2$$

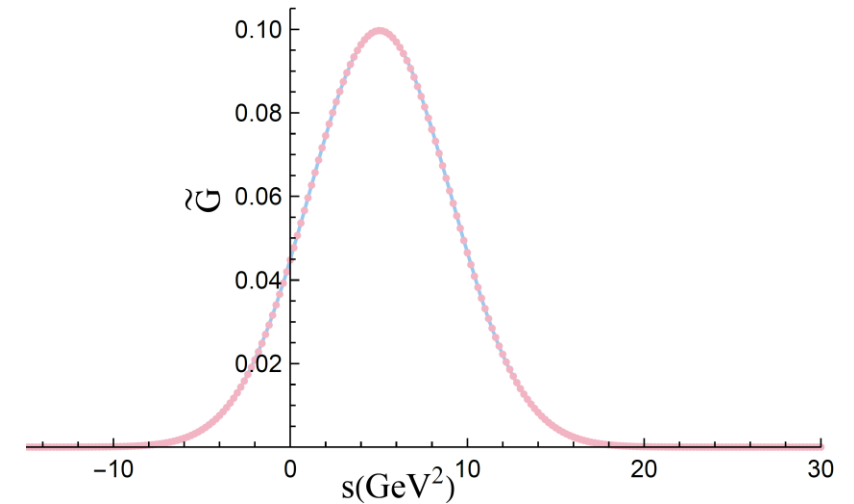
Error is defined by:

$$\sigma = \sqrt{\hat{\sigma}^2 + \Delta^2}$$

$\hat{\sigma}^2$: variance of 2000 fitted values $\sim$ statistic error

$$\Delta = \sqrt{2\chi_{\min}^2 H^{-1}(\chi_{\min}^2)} \quad \sim \text{systemtic error}$$

$$H(\chi^2) = \begin{bmatrix} \frac{\partial^2 \chi^2}{\partial s_0^2} & \frac{\partial^2 \chi^2}{\partial s_0 \partial m} \\ \frac{\partial^2 \chi^2}{\partial s_0 \partial m} & \frac{\partial^2 \chi^2}{\partial m^2} \end{bmatrix}$$



for small δs

$$\chi^2 \sim \frac{1}{\delta s}, \quad H^{-1} \sim \delta s \rightarrow \chi^2 H^{-1} \sim 1$$

for $s_{\min}$ and $s_{\max}$ away from the peak

$\tilde{G}(\tau, s_i, s_0), \tilde{G}_h(\tau, s_i) \rightarrow 0$, irrelevant to the specific $s_{\min}, s_{\max}$

(if average the Δ as $\Delta/(s_{\max} - s_{\min})$, choose large $s_{\max} - s_{\min}$ manually reduce the error)

$\Rightarrow$

Δ in the left is irrelevant to the choice of $\delta s, s_{\min}, s_{\max}$
reflect the intrinsic error of the fitting procedure