One Point Charge Correlator

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2605.15280 + work with Wanchen and Dingyu

Outline

- Hadron probe/Fragmentation
- Observables （Detectors）
  - Energy flow/Correlators
  - OPCC and Transverse Charge Distribution
- Phenomenology
- Conclusion

Hadron probe/Fragmentation

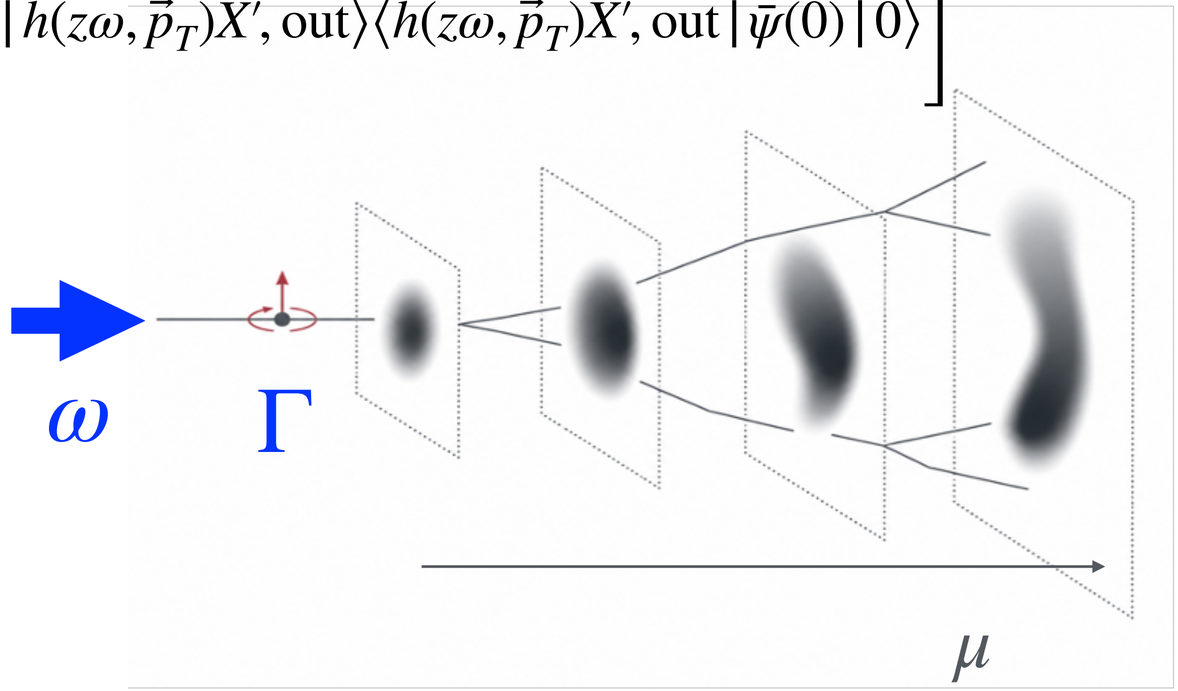
- A portal to hadronization
- An important probe
- Fragmentation functions (FFs)

e.g.,

$$d_{h/i}(z,\vec{p}_T) = \int \frac{d\xi d^2\vec{x}_T}{2z(2\pi)^3} e^{i\omega\xi}\,\mathrm{Tr}\left[\frac{\Gamma}{2}\langle 0|\psi(\xi,x_T)\sum\!\!\!\!\!\!\int_{X'}|h(z\omega,\vec{p}_T)X',\mathrm{out}\rangle\langle h(z\omega,\vec{p}_T)X',\mathrm{out}|\bar{\psi}(0)|0\rangle\right]$$

hadron distribution of fragmentation

$$|0\rangle \xrightarrow{\bar{\psi}} |hX'\rangle$$



Hadron probe/Fragmentation

- A portal to hadronization
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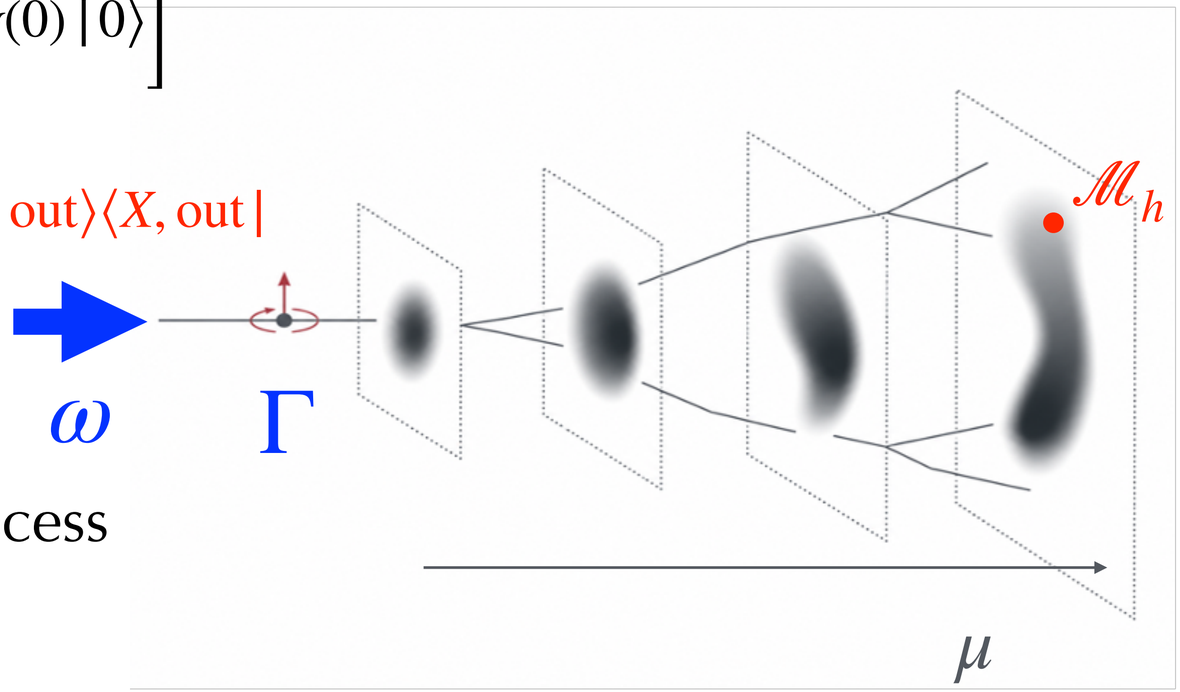
Fragmentation process

Detector that probes fragmentation

e.g.,

$$d_{h/i}(z,\vec{p}_T) = \int d\xi d^2\vec{x}_T e^{i\omega\xi}\,\mathrm{Tr}\left[\frac{\Gamma}{2}\langle 0|\psi(\xi,x_T)\,\mathcal{M}_h\,\bar{\psi}(0)|0\rangle\right]$$

$$\mathcal{M}_h(z,\vec{p}_T) = \sum\!\!\!\!\!\!\int_X \sum_{a\in X} \delta_{ah}\delta(z-\frac{E_a}{\omega})\delta^2(\vec{p}_T-\vec{p}_{a,T})\,|X,\mathrm{out}\rangle\langle X,\mathrm{out}|$$



Measure a hadron in a fragmentation process to probe fragmentation

Hadron probe/Fragmentation

- A portal to hadronization
- An important probe
- Fragmentation functions (FFs)

Fragmentation process

e.g.,

Detector that probes fragmentation

$$d_{h/i}(z,\vec{p}_T) = \int d\xi d^2\vec{x}_T e^{i\omega\xi}\,\mathrm{Tr}\left[\frac{\Gamma}{2}\langle 0|\psi(\xi,x_T)\,\mathcal{M}_h\,\bar{\psi}(0)|0\rangle\right]$$

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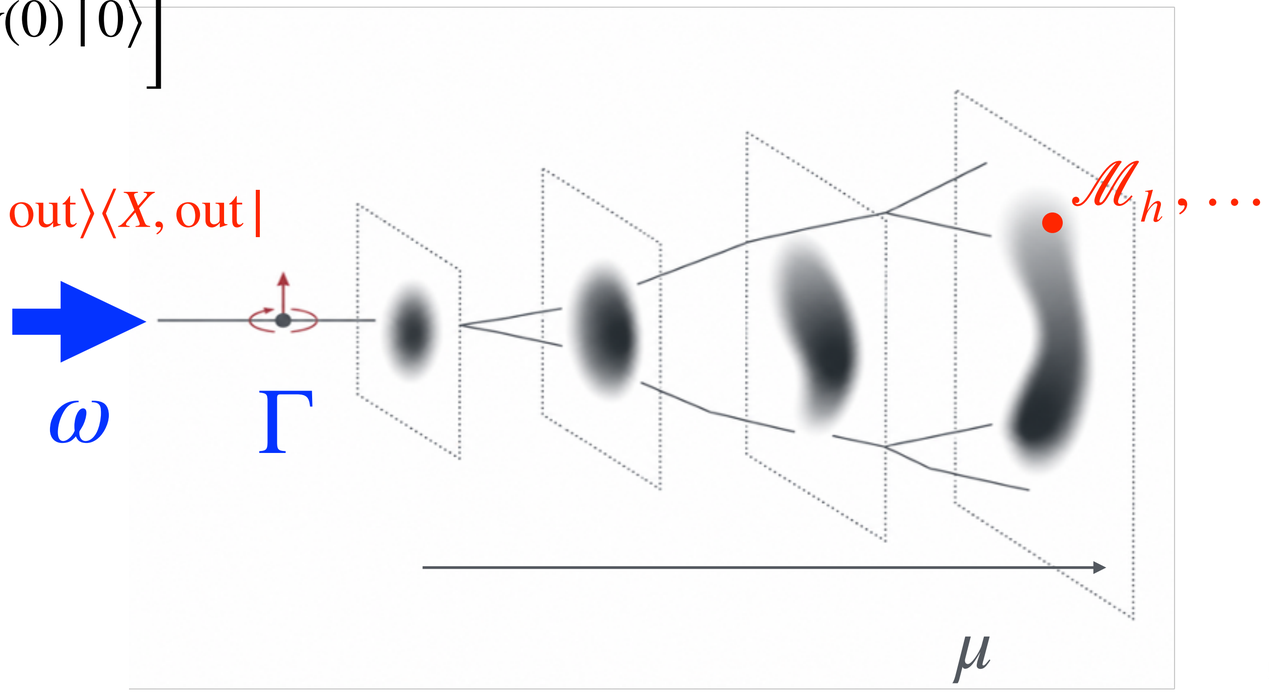
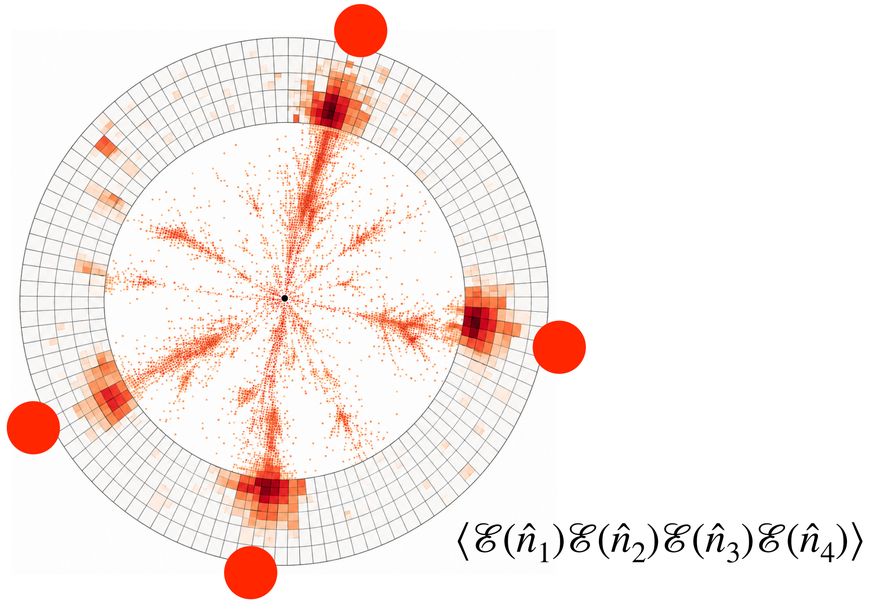


- Hadron detectors (FFs) are not unique for fragmentation
- **We can have other detectors,** theoretical/experimentally more friendly

Detectors: Energy flow/Energy Correlator

Energy flow detector/energy correlators



$$\lim_{E_3 \to 0} E_1 + E_2 + E_3 = E_1 + E_2$$

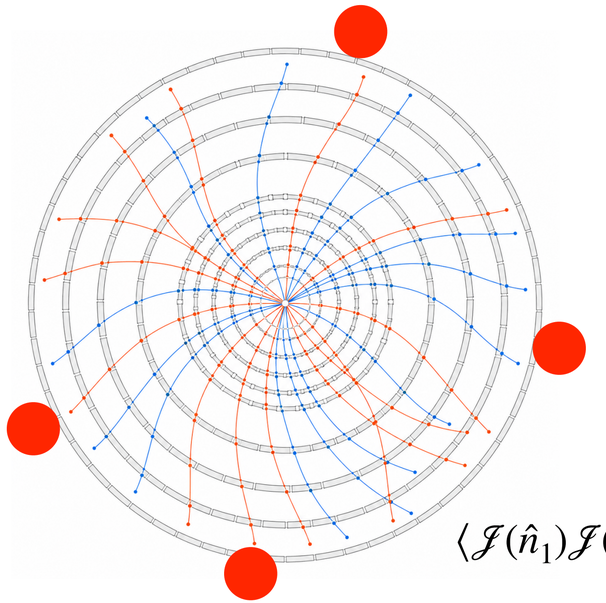
$$\mathcal{E}(\hat{n}) = \lim_{r \to \infty} r^2 \int dt\, \hat{n}_i\, T^{0i}(t, r\hat{n})$$

- Measure energy and angle, more inclusive than hadron detector
- Infra-red safe ⇒ pQCD calculable
- Calorimeter information + PID?

Mout, Zhu, 2506.09119 for review

Detectors: Track

Charge flow detector/charge correlators??

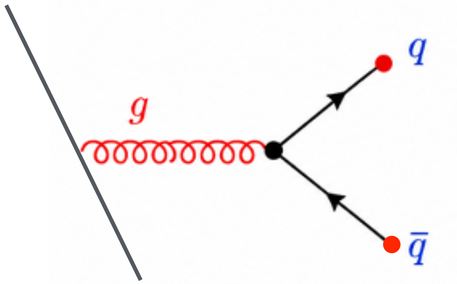


$$\langle \mathcal{J}(\hat{n}_1)\mathcal{J}(\hat{n}_2)\mathcal{J}(\hat{n}_3)\mathcal{J}(\hat{n}_4)\rangle$$

$$\lim_{E_3\to 0} Q_1 + Q_2 + Q_3 = Q_1 + Q_2 + Q_3$$

$$\mathcal{Q}(\hat{n}) = \lim_{r\to\infty} r^2 \int dt\, \hat{n}_i\, \mathcal{J}^{0i}(t, r\hat{n})$$

- Measure charge sign and angle, more inclusive than hadron detector
- Track information, better resolution
- Infra-red safe??

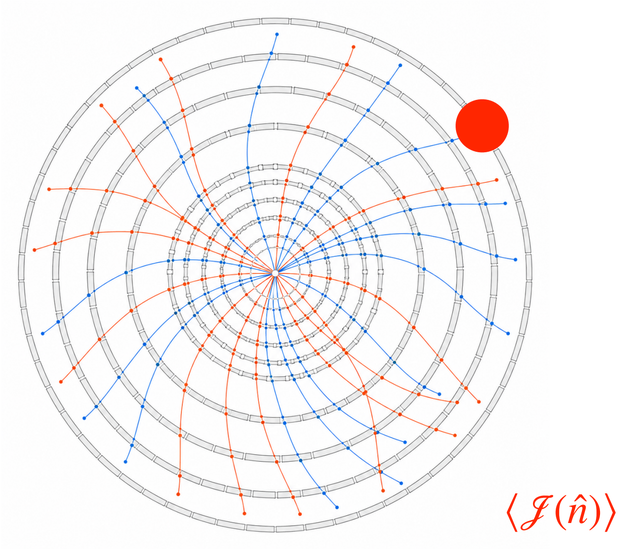


$$\sim \frac{\alpha_s^2}{\epsilon} \times Q\bar{Q}$$ event-by-event

NO virtual to cancel the pole

One Point Charge Correlator

Charge flow detector/One Point Charge Correlator

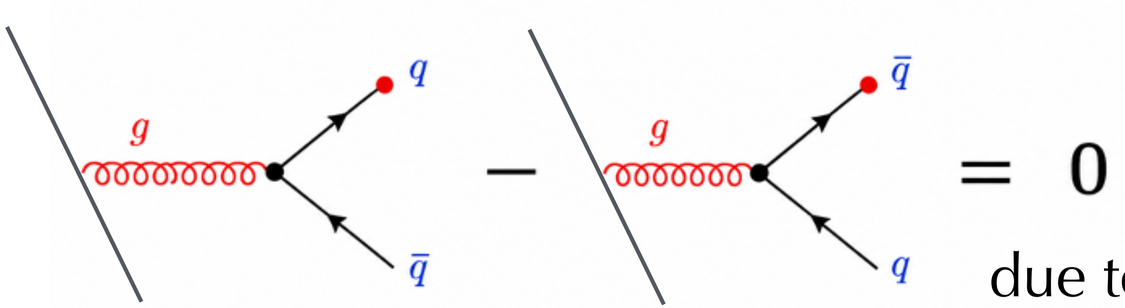


$$\mathcal{Q}(\hat{n}) = \lim_{r\to\infty} r^2 \int dt\, \hat{n}_i\, \mathcal{J}^{0i}(t, r\hat{n})$$

- Measure charge sign and angle, more inclusive than hadron detector
- Track information, better resolution
- Infra-red safe for OPCC !

(Also true for b2b QCC, see 2508.00977)

$$\lim_{E_3\to 0} Q_1 + Q_2 + Q_3 = Q_1 + Q_2 + Q_3$$



When averaging over events,

cancel among real emissions,

due to C symmetry of soft QCD and C-odd of OPCC

OPCC and transverse charge distribution

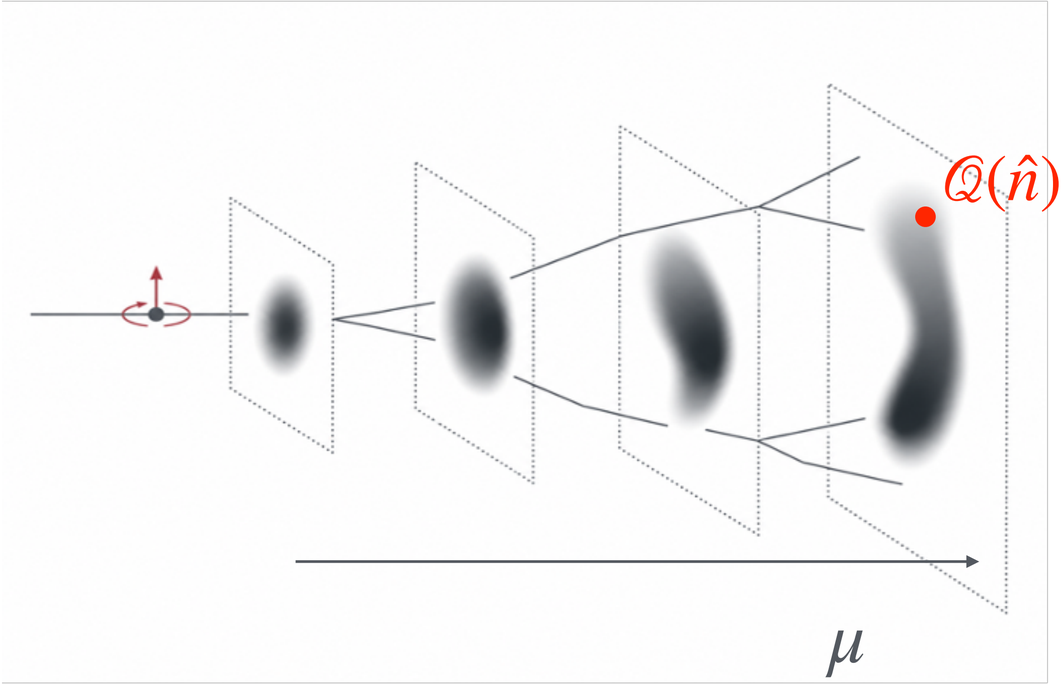
- A new portal to hadronization
- **Transverse charge distribution** Li et al.

$$\mathcal{J}_{\omega,\Gamma}(\vec{n}_T) = \int d\xi d^2\vec{x}_T e^{i\omega\xi}\,\mathrm{Tr}\left[\frac{\Gamma}{2}\langle 0|\psi(\xi,x_T)\,\mathcal{Q}(\hat{n})\,\bar{\psi}(0)|0\rangle\right]$$

$$\mathcal{Q}(\hat{n}) = \lim_{r\to\infty} r^2 \int dt\,\hat{n}_i\,\mathcal{J}^{0i}(t,r\hat{n})$$

$$\hat{n} = (\vec{n}_T, \cos\theta) = (\sin(\theta)(\cos\phi, \sin\phi), \cos\theta) \approx (\vec{n}_T, 1)$$

Measures how charge is (re)-distributed by hadronization



OPCC and transverse charge distribution

Properties (collinear limit)

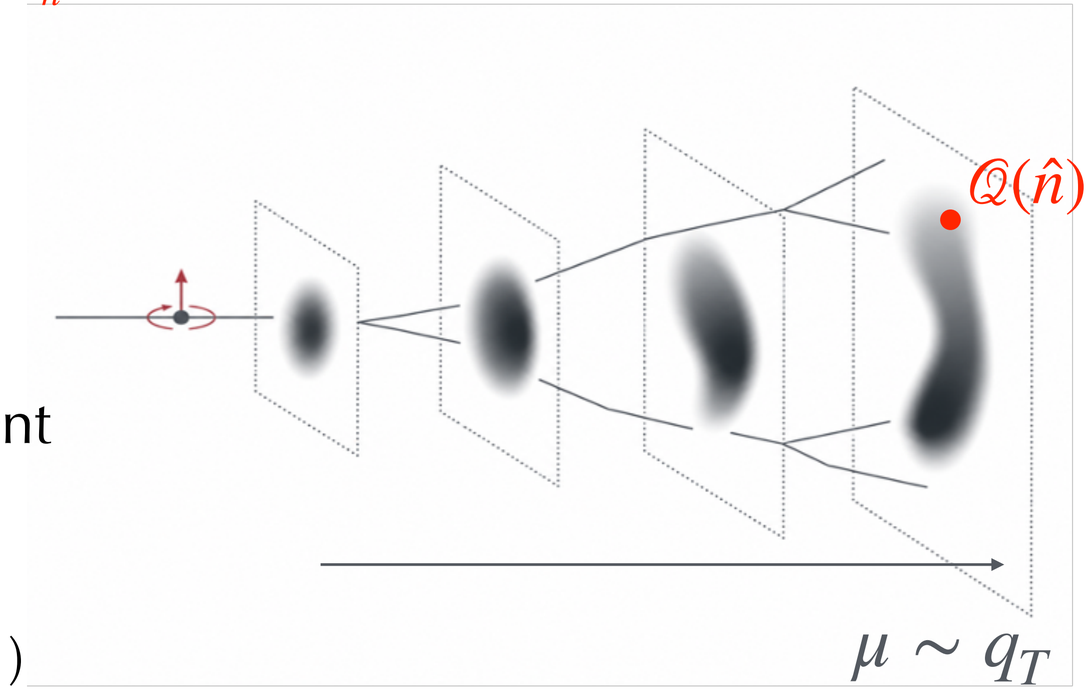
$$\mathcal{J}_{\omega,\Gamma}(\vec{n}_T) = \int d\xi d^2\vec{x}_T e^{i\omega\xi}\,\mathrm{Tr}\left[\frac{\Gamma}{2}\langle 0|\psi(\xi,x_T)\,\mathcal{Q}(\hat{n})\,\bar{\psi}(0)|0\rangle\right]$$

- $\Gamma = \gamma^+$ unpol, $\Gamma = i\sigma^{\rho+}\gamma_5$ transversely pol. quark fragmentation
- $\mathcal{J}_{\omega,\Gamma}(\vec{n}_T) = \mathcal{J}_{\omega,\Gamma}(\omega\vec{n}_T) = \mathcal{J}_{\omega,\Gamma}(\vec{q}_T)$ due to boost invariance $\omega \to \omega e^Y$, $\vec{n}_T \to \vec{n}_T e^{-Y}$
- Connection to the TMD FF $\quad \mathcal{J}_{\omega\Gamma}(\vec{q}_T) = \int dz z^2 \sum_h Q_h\, d_{h/i,\Gamma}(z, z\vec{q}_T)$

$$\mathcal{Q}(\hat{n}) \propto \sum\!\!\!\!\!\!\int_X \sum_{a\in X} Q_a \delta(\vec{q}_T - \vec{p}_{aT}/z_a)\,|X,\mathrm{out}\rangle\langle X,\mathrm{out}|$$

$$\mathcal{M}_h(z,p_T) = \sum\!\!\!\!\!\!\int_X \sum_{a\in X} \delta_{ah}\delta(z - \frac{p_a^+}{2\omega})\delta^2(p_T - q_{T,a})\,|X,\mathrm{out}\rangle\langle X,\mathrm{out}|$$

- The connection is not a must-be, $\mathcal{Q}$ is an observable independent of TMD FF (different measurement, no energy …)
- Allows for recycling known results in TMD physics (factorization, ope, calculations …)



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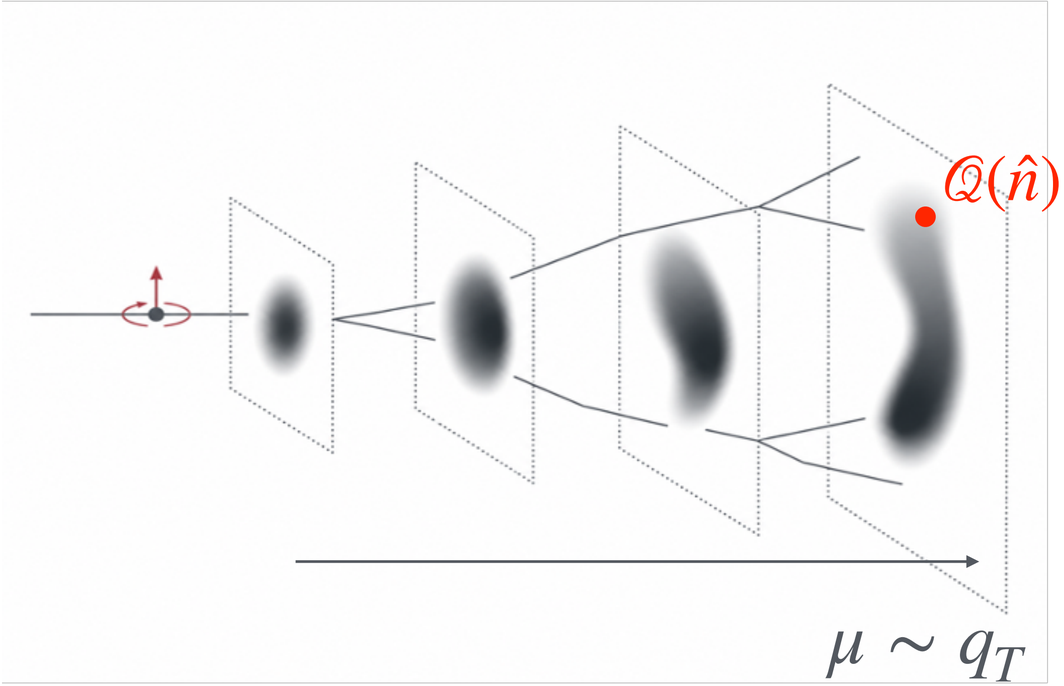
- Fourier transformation

$$\mathcal{J}_{\omega,\Gamma}(\vec{b}_T) \equiv \int d^2\vec{q}_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\mathcal{J}_{\omega,\Gamma}(\vec{q}_T)$$

acts only on the detector in the parton frame

$$\int d^2\vec{q}_T\, e^{i\vec{b}_T\cdot\vec{q}_T}\,\mathcal{Q}(\vec{q}_T)$$



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$$\mathcal{J}_{\omega,\Gamma}(\vec{n}_T) = \int d\xi d^2\vec{x}_T e^{i\omega\xi}\,\mathrm{Tr}\left[\frac{\Gamma}{2}\langle 0|\psi(\xi,x_T)\,\mathcal{Q}(\hat{n})\,\bar{\psi}(0)|0\rangle\right]$$

- OPE for $b_T\Lambda_{\mathrm{QCD}} \to 0$

$$\int d^2\vec{q}_T\, e^{i\vec{b}_T\cdot\vec{q}_T}\,\mathcal{Q}(\vec{q}_T) \quad = \int d^2\vec{q}_T\,(1 + i\vec{b}_T\cdot\vec{q}_T + \ldots)\,\mathcal{Q}(\vec{q}_T)$$

$$= \hat{Q}_{\mathrm{tot}} + i\vec{b}\cdot\int d^2\vec{q}_T\,\vec{q}_T\,\hat{\mathcal{Q}}(\vec{q}_T) + \ldots$$

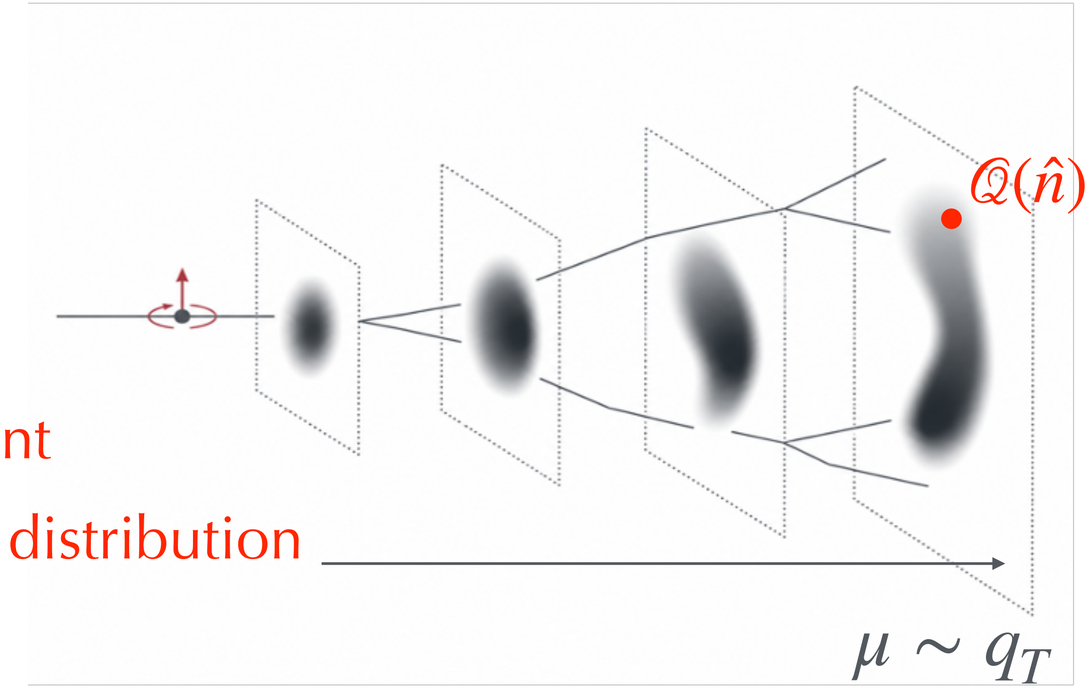
Measures the total charge of fragmentation

$$[\hat{Q}_{\mathrm{tot}},\bar{\psi}] = Q_q\bar{\psi}$$

Measures the dipole moment

sideways distortion of the net-charge distribution

OPE of $\mathcal{J}_{\omega,\Gamma}$ ~ multiple expansion of the fragmentation charge distribution



OPCC and transverse charge distribution

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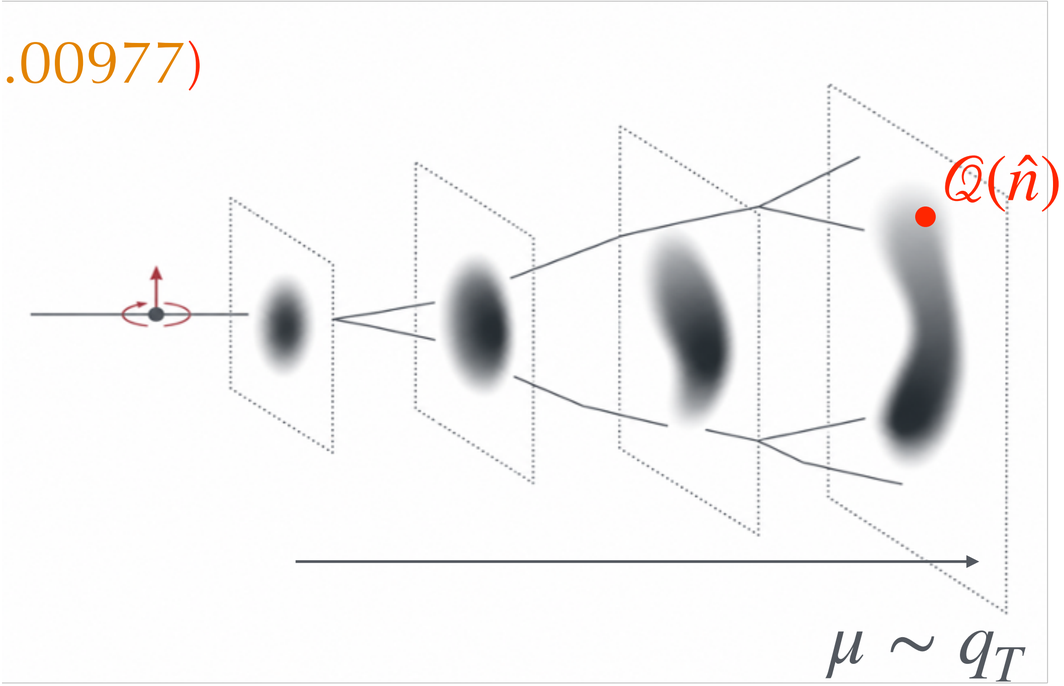
$$\mathcal{J}_{\omega,\Gamma}(\vec{n}_T) = \int d\xi d^2\vec{x}_T e^{i\omega\xi}\,\mathrm{Tr}\left[\frac{\Gamma}{2}\langle 0|\psi(\xi,x_T)\,\mathcal{Q}(\hat{n})\,\bar{\psi}(0)|0\rangle\right]$$

- OPE for $b_T\Lambda_{\mathrm{QCD}} \to 0$

$$\hat{Q}_{\mathrm{tot}}: \quad \mathcal{J}_{\gamma^+}(\vec{b}_T) \sim \sum_{j>0} \tilde{C}^D_{j/i}(b_T;\mu,\zeta)\,Q_j$$

Can be understood perturbatively (N3LO 2508.00977)

Good for a probe



OPCC and transverse charge distribution

Properties (collinear limit)

- OPE for $b_T \Lambda_{\rm QCD} \to 0$

$$\mathcal{J}_{\omega,\Gamma}(\vec{n}_T) = \int d\xi d^2\vec{x}_T e^{i\omega\xi}\, {\rm Tr}\left[\frac{\Gamma}{2}\langle 0|\psi(\xi, x_T)\, \mathcal{Q}(\hat{n})\, \bar{\psi}(0)|0\rangle\right]$$

$$\mathcal{Q}(\hat{n}) \propto \sum\!\!\!\!\!\!\int_X \sum_{a\in X} Q_a \delta(\vec{q}_T - \vec{p}_{aT}/z_a)|X, {\rm out}\rangle\langle X, {\rm out}|$$

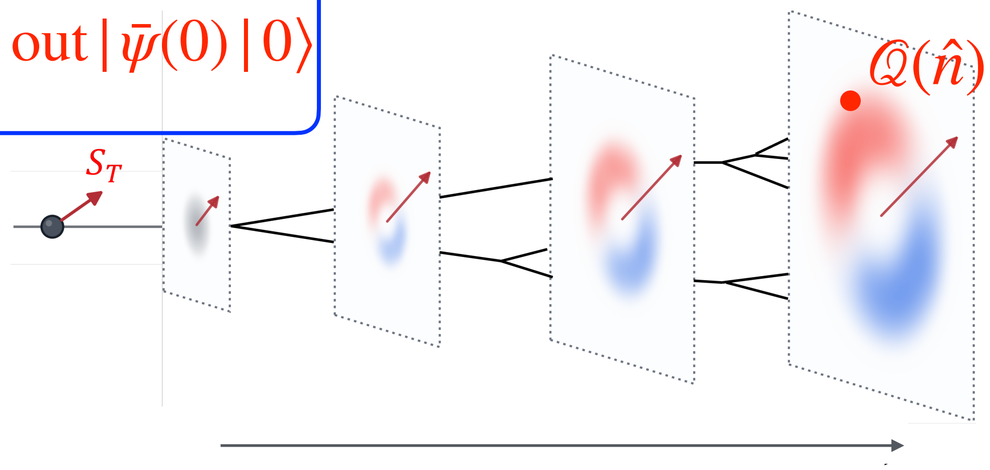
$$i b_T \cdot \hat{\overrightarrow{D}}_T = i\vec{b}\cdot \int d^2\vec{q}_T\, \vec{q}_T\, \hat{\mathcal{Q}}(\vec{q}_T) : \quad \mathcal{J}_{\sigma^{\rho+}\gamma_5}(\vec{b}_T) = i\sum_{j>0} C^H_{j/i}(b_T;\mu,\zeta)\, b_T^\alpha\, \mathcal{D}^{\rho\perp}_{\alpha,j}$$

$$\sum_h Q_h \int d\Phi_h \frac{p_{hT,\alpha}}{z_h} \boxed{\ldots \frac{i\sigma^\rho\gamma_5}{2}\langle 0|\psi(\xi,\vec{x}_T)\sum\!\!\!\!\!\!\int_{X'} |hX', {\rm out}\rangle\langle hX', {\rm out}|\bar{\psi}(0)|0\rangle}$$

1st moment of **Collins Function**

Collins effect ~ dipole moment

(a non-perturbative number, easier to fit)



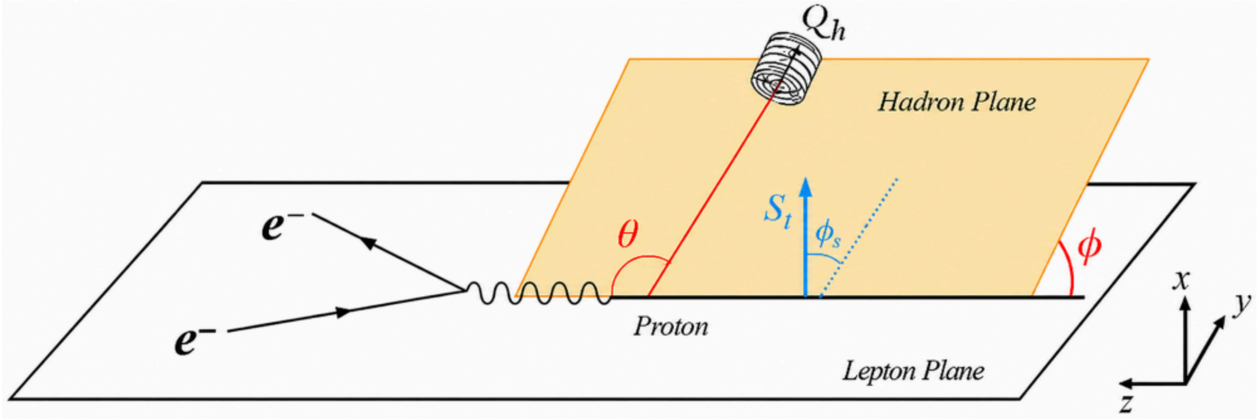
$\mu \sim q_T$

Phenomenology

- measurement w/ track + angular information only, but probe similar physics as the TMDFF

$$\Sigma_{\mathcal{Q}}(\vec{n}, x_B, Q^2) = \sum_h \int d\sigma_{ep^\uparrow \to eh+X}\, Q_h\, \delta(\vec{n} - \vec{n}_h)$$

$$= F_{UU} + |S_t| \sin(\phi - \phi_S)\, F_{UT} + \dots$$



$\theta \to \pi$ 2605.15280

- Factorization

$$F_{UU} = \sigma_0 H(Q^2, \mu) \frac{\bar{\theta} Q^2}{4} \int_0^\infty \frac{b\, db}{2\pi} J_0\left(b \frac{Q\bar{\theta}}{2}\right) \times \sum_f e_f^2 J_{f,\mathcal{Q}}(b, \mu, \nu) f_1^f(x_B, b, \mu, \nu) S(b, \mu, \nu)$$

$$F_{UT} = \sigma_0 H(Q^2, \mu) \frac{\bar{\theta} Q^2}{4} \int_0^\infty \frac{b^2\, db}{4\pi} J_1\left(b \frac{Q\bar{\theta}}{2}\right) \times \sum_f e_f^2 J_{f,\mathcal{Q}}(b, \mu, \nu) f_{1T}^{\perp, f}(x_B, b, \mu, \nu) S(b, \mu, \nu)$$

Overall vanishing contribution to the charge from soft radiation due to its charge conjugation symmetry

Exactly the TMD-like factorization with TMD FF replaced by the Transverse Charge Distribution

Phenomenology

- measurement w/ track + angular information only, but probe similar physics as the TMDFF

$$\Sigma_{\mathcal{Q}}(\vec{n}, x_B, Q^2) = \sum_h \int d\sigma_{ep^\uparrow \to eh+X}\, Q_h\, \delta(\vec{n} - \vec{n}_h)$$

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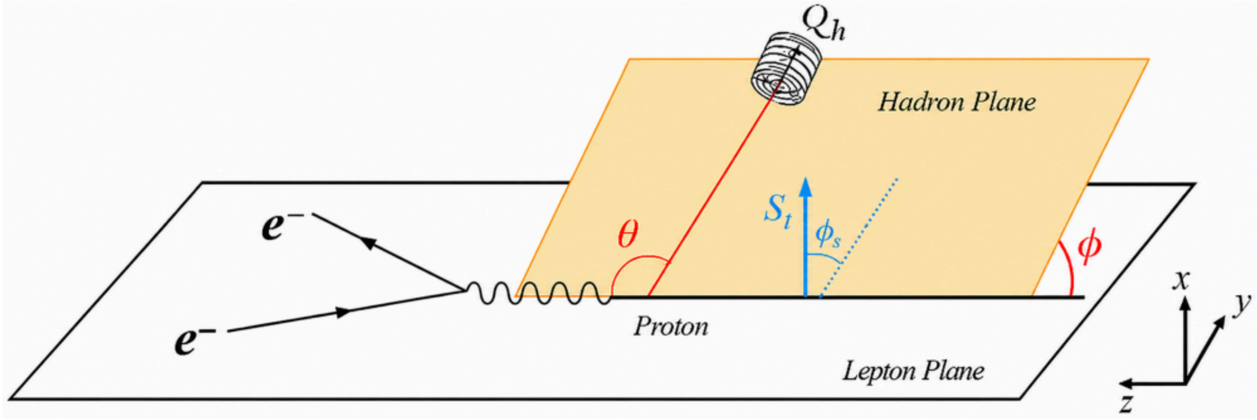

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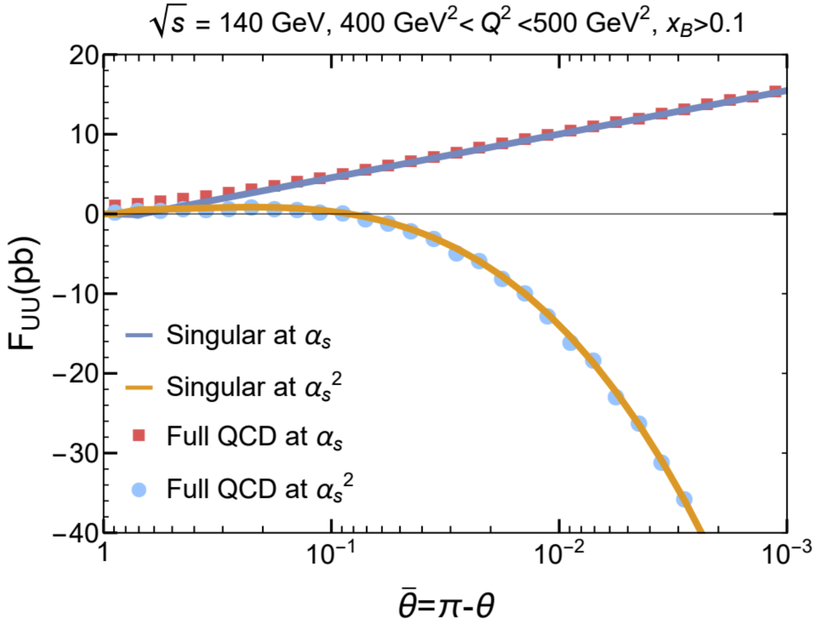

Test of the factorization

Good agreement between full QCD and results from the factorization

$\Rightarrow$

Singular behavior captured by the factorization

Phenomenology

- measurement w/ track + angular information only, but probe similar physics as the TMDFF

$$\Sigma_{\mathcal{Q}}(\vec{n}, x_B, Q^2) = \sum_h \int d\sigma_{ep^\uparrow \to eh+X}\, Q_h\, \delta(\vec{n} - \vec{n}_h)$$

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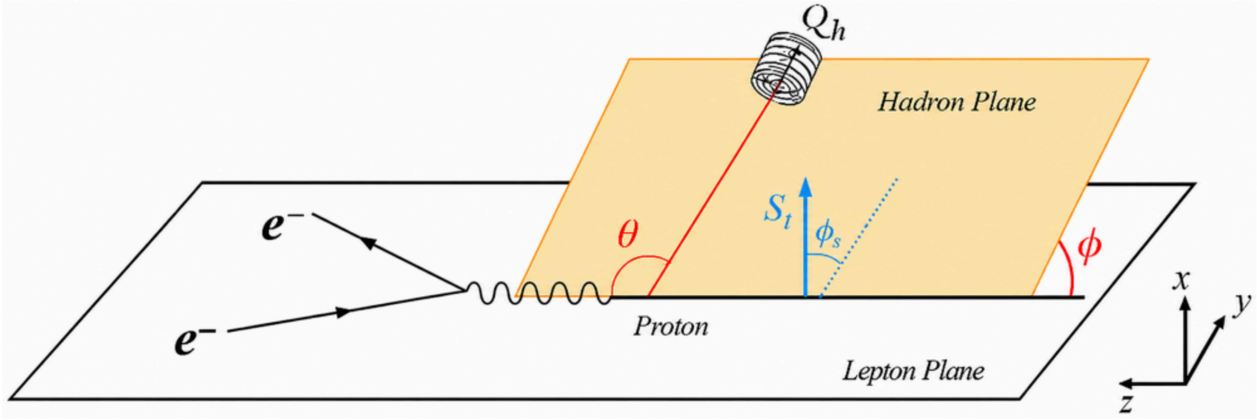
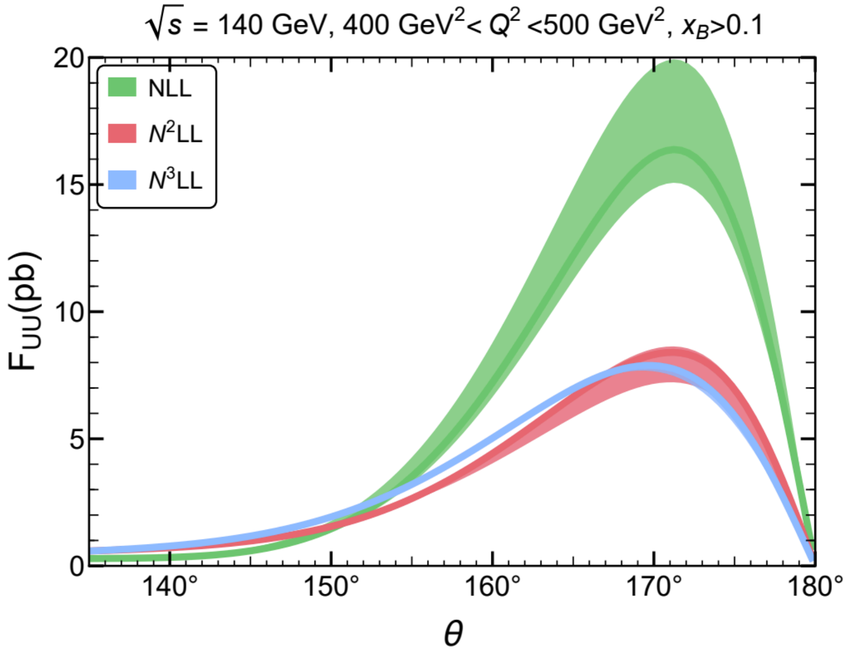
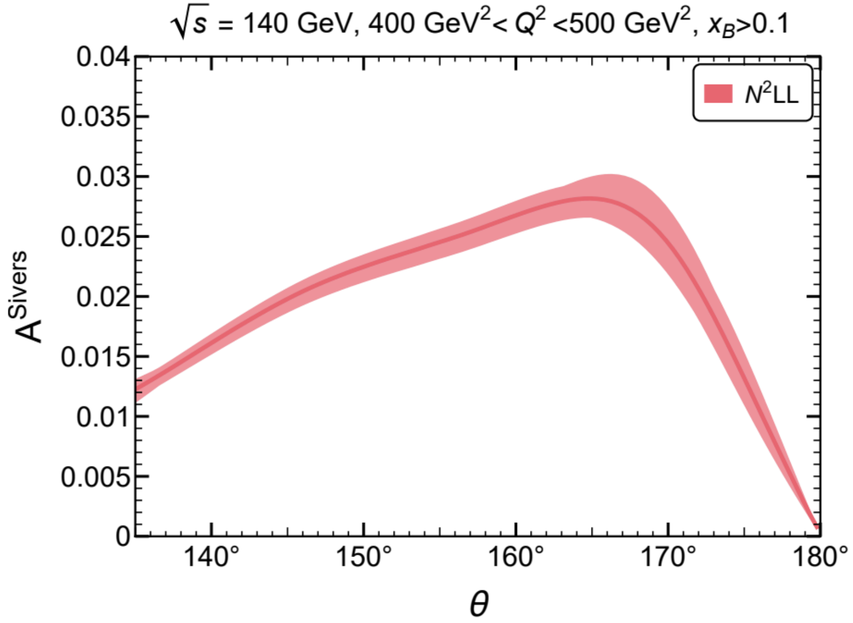


$\theta \to \pi$ 2605.15280





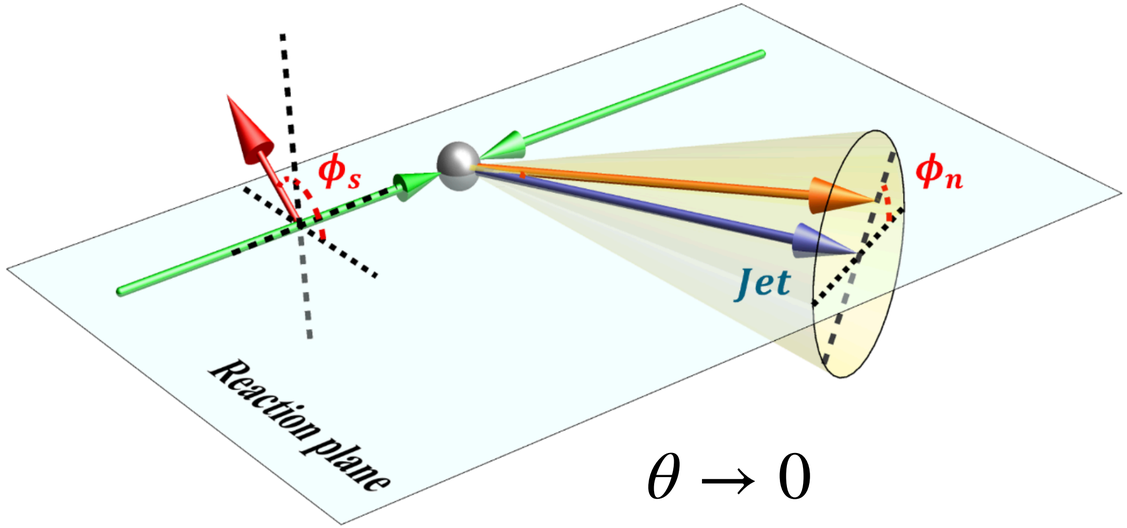
Phenomenology

- Probing transversity

$$\frac{d\Sigma_Q}{d\theta_n\, d\phi_n\, d\eta\, dp_T} = \sum_{h\in J} \int d^2\Omega_h\, Q_h\, \delta(\theta_n - \theta_h)$$

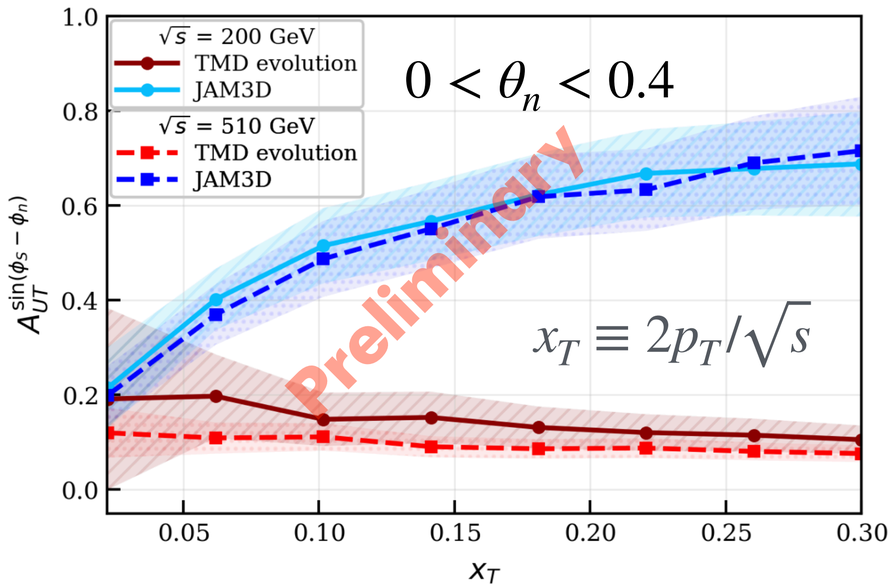
$$\times\, \delta(\phi_n - \phi_h) \frac{d\sigma}{d^2\Omega_h\, d\eta\, dp_T}$$

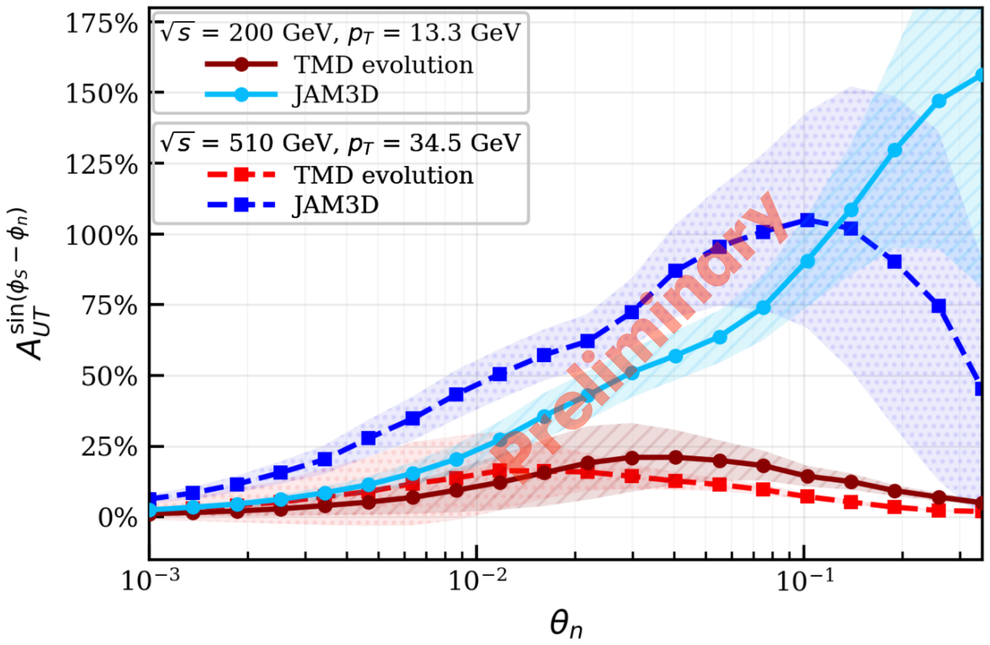
$$= Z^Q_{UU} + \sin(\phi_s - \phi_n)\, Z^Q_{UT}$$



$\theta \to 0$

Li et al. to appear





Conclusion

- One Point Correlator complements the hadronization studies
  - Theoretically clean, known with high precision
  - A probe requires only track and angular information
  - Improved sensitivity to C-odd signals
- Conclusion