陕西师範大學 SHAANXI NORMAL UNIVERSITY

中国物理学会高能物理分会
第十二届全国会员代表大会暨学术年会

QCD Phase Transition and Hydrodynamics in Non-extensive Statistics

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分会场 II：强子物理与味物理分会（二） 7月16日上午 2号楼二楼汕头厅		
9:05-9:20	J/ψ and exotic hadron productions in high energy nuclear collisions with the parton and hadron cascade model PACIAE	Wenchao Zhang

Outline

1. Background and motivations
2. Nonextensive Statistics
3. Result I: (3+1)-Dimensional Nonextensive Hydrodynamic Model
4. Result II: Nonextensive and Chemical Potential Effects on the QCD Phase Transition
5. Result III: QCD Equation of State under Finite Magnetic Fields
6. Summary

Background and motivations

- QCD predicts a transition from ordinary nuclear matter to a quark-gluon plasma (QGP) at high temperatures and energy densities.
- *Relativistic hydrodynamic* model provides a simple picture of the *space-time evolution* of QGP.
- It explains various observables: pseudorapidity distributions, p_T spectra, elliptic flow, etc.
- It indicates that the QGP behaves as a near-perfect fluid (very small η/s)

Background and motivations

- For the usual ideal hydrodynamic models, a *local thermal equilibrium* is assumed and the *BG statistics* is applied.
- The validity range is limited to p_T ~ 2–3 GeV/c.
- However, the *initial conditions fluctuate* from event to event and the fluctuations may *not be completely washed out* by subsequent interactions at either the QGP or the hadronic phase.
- Therefore, the occurrence of the *local thermal equilibrium* is *not expected* and some quantities such as the p_T spectra will develop *power-law tailed* rather than exp. distributions.

Nonextensive Statistics

- This phenomenon can be incorporated in terms of a non-extensive (Tsallis) statistics *without* going into *considerations* about *the source* of such fluctuations.

- Tsallis Entropy $S_q = k\frac{1-\sum_{R=1}^{W} p_R^q}{q-1}$ (p_R: the probability associated with the R-th state; q=1 recovers the BG entropy)

- Pseudo-additivity for composite systems:

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B)/k$$

- q-Exponential and q-Logarithm

$$\exp_q(x) = [1+(1-q)x]^{1/(1-q)}, \quad \ln_q x = \frac{x^{1-q}-1}{1-q}$$

q related to temperature fluctuations: $\boldsymbol{q = 1 + \langle(\delta T)^2\rangle/\langle T\rangle^2}$

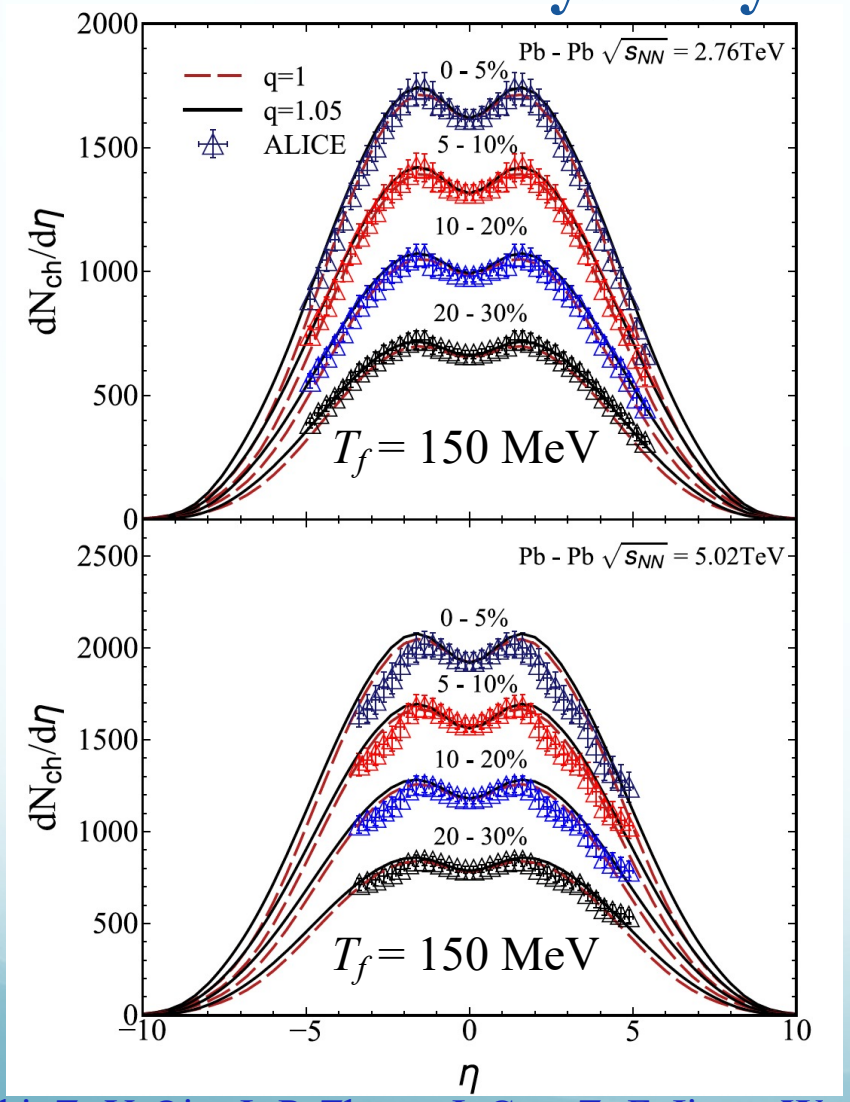
J. Statist. Phys. 52, 479 (1988) Eur. Phys. J. A 48, 160 (2012) J. Phys. G 39, 025006 (2012)

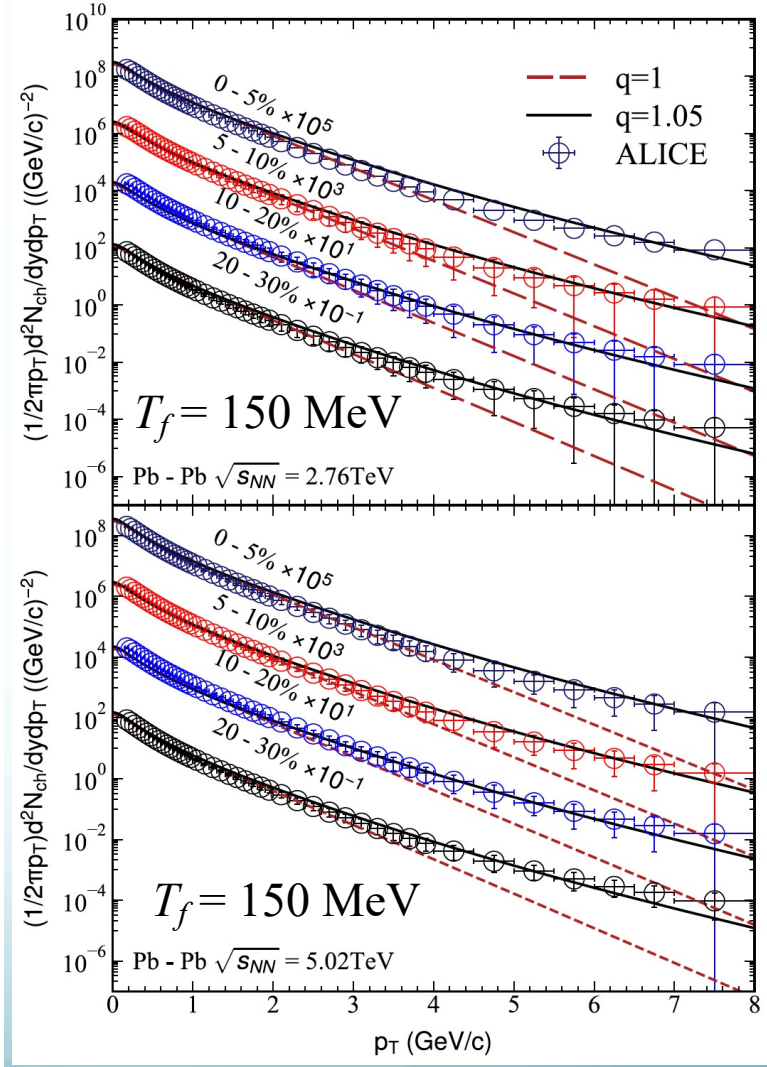
Result I: (3+1)-Dimensional Nonextensive Hydrodynamic Model

- In our first work, we develop a non-extensive (3+1)-*dimensional* hydrodynamic model, NEX-CLVisc, on the basis of the CLVisc framework.
- In order to elucidate the pure effect of non-extensive statistics, the *viscous corrections* have been *turned off*.
- In the model, the *q-corrections* are taken into account in the initial conditions (q corrections into longitudinal energy density profile), the EOS (q corrections into HRG + parton gas), and the kinetic freeze-out procedure (q-extended Cooper-Frye formula).
- In principle, the intrinsic fluctuations in different stages of the hydrodynamic evolution are different, thus the parameters q should be different.
- *For simplicity*, in this work we restrict ourselves to use the *same q values* at all stages of the collision process.

L.-G. Pang, H. Petersen, and X.-N. Wang, PRC 97, 064918 (2018)

Result I: (3+1)-Dimensional Nonextensive Hydrodynamic Model





J.-H. Shi, Z.-Y. Qin, J.-P. Zhang, J. Cao, Z.-F. Jiang, **W.-C. Zhang***, H. Zheng, **Phys. Rev. D 111**, 036010 (2025)

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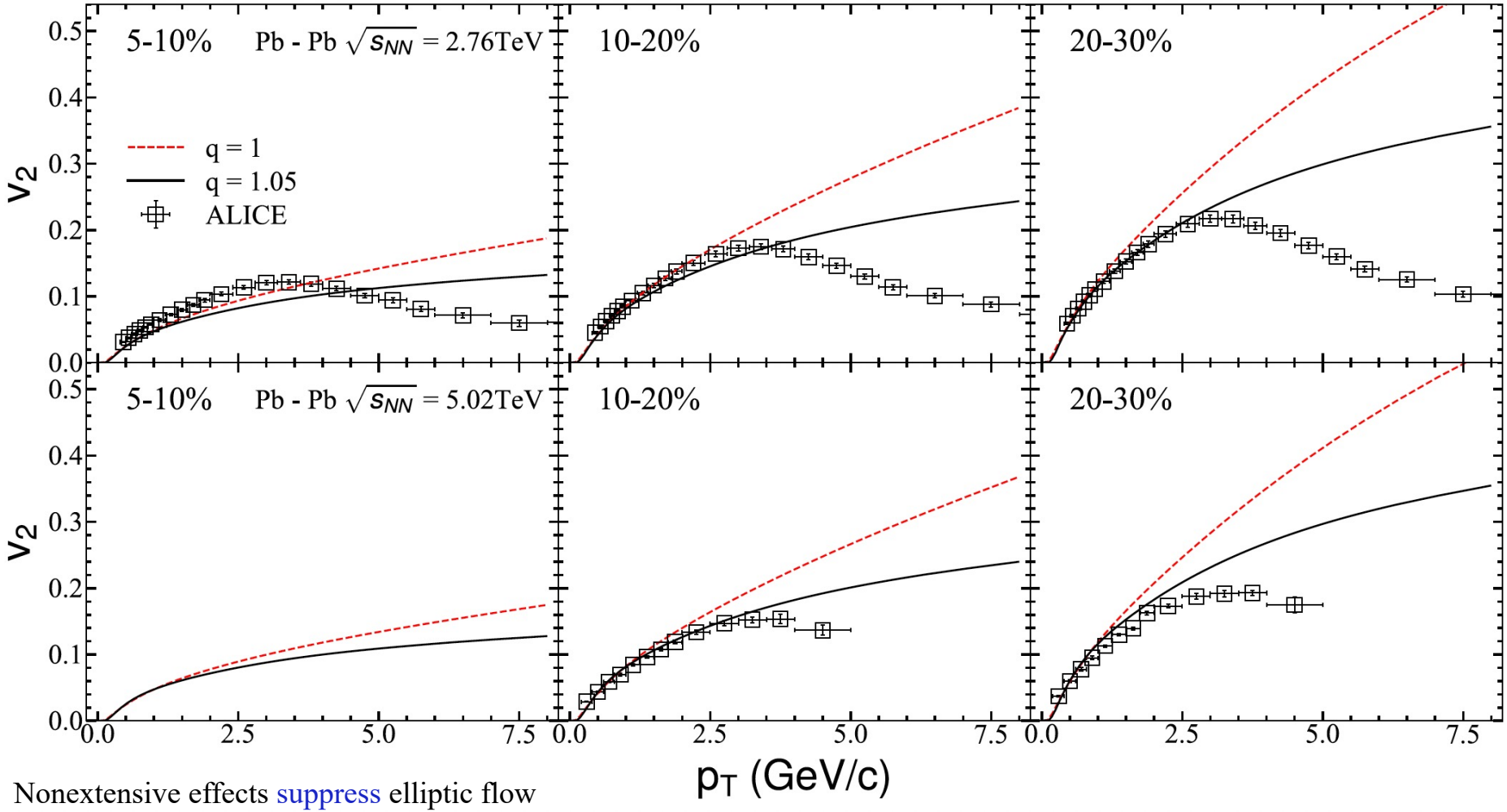


Nonextensive effects suppress elliptic flow (similar to shear viscosity).

J.-H. Shi, Z.-Y. Qin, J.-P. Zhang, J. Cao, Z.-F. Jiang, **W.-C. Zhang***, H. Zheng, **Phys. Rev. D 111**, 036010 (2025)

Result II: Nonextensive and Chemical Potential Effects on the QCD Phase Transition

- Nonextensive Partition Function: $Z_q = \left(\prod_k\right)_q \left[1 \pm \exp_q\left(-\frac{\epsilon_k - \mu}{T}\right)\right]^{\pm d}$

$\exp_q(x) \otimes_q \exp_q(y) = \exp_q(x+y)$

- Pressure: $P_q = T\int \frac{d^3k}{(2\pi)^3} \ln_q \left[1 \pm \exp_q\left(-\frac{\epsilon_k - \mu}{T}\right)\right]^{\pm d}$
- Entropy density: $s_q = \sum_i \partial P_q^i / \partial T$
- Smooth Crossover: $s_q(T) = f(T)s_q^{\mathrm{H}}(T) + [1 - f(T)]s_q^{\mathrm{QGP}}(T)$

$$f(T) = \frac{1}{2}[1 - \tanh((T - T_c)/\Gamma)], \quad \Gamma = 0.05T_c$$

$$P_q(T) = \int_0^T s_q(t)dt,$$

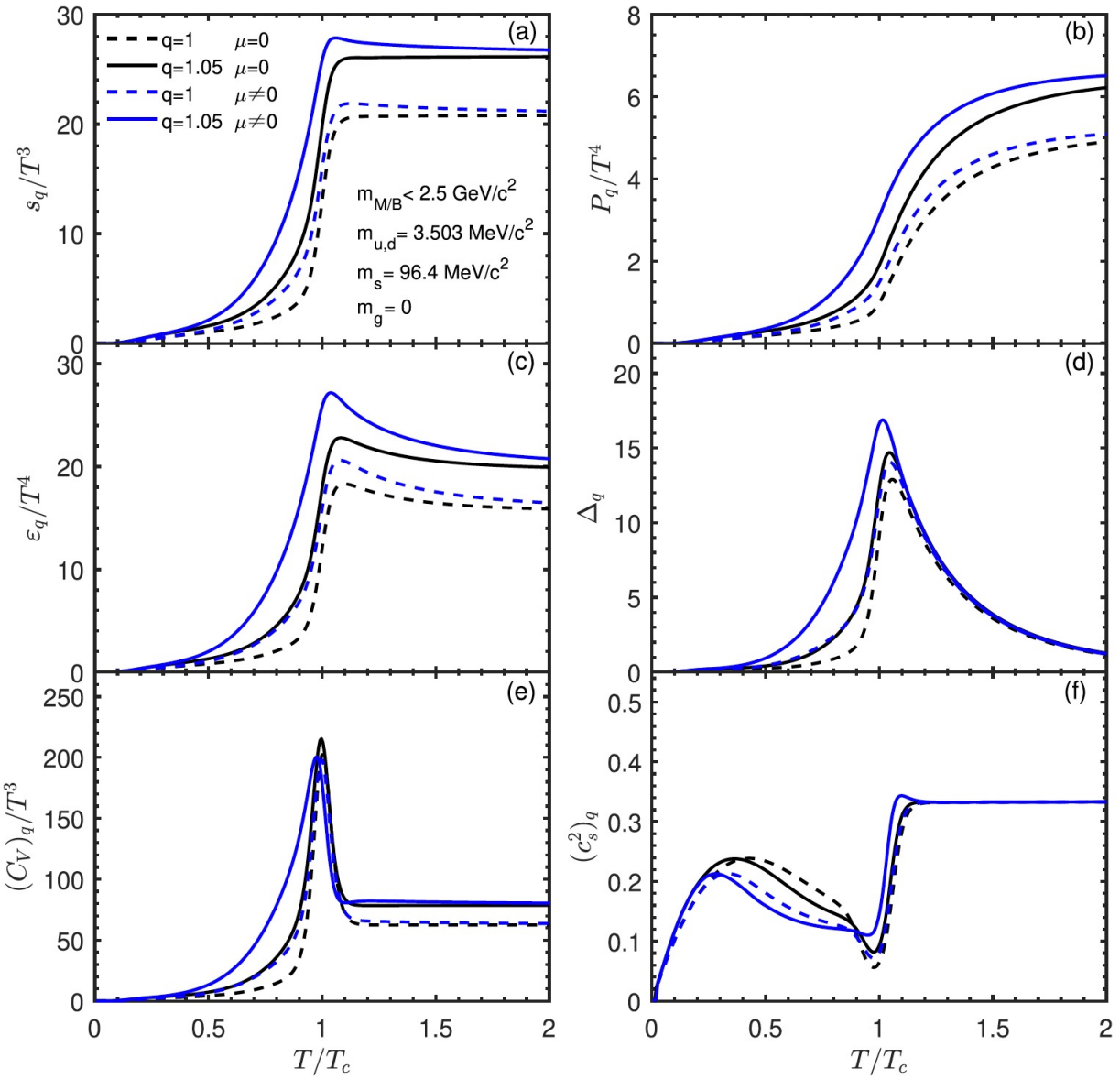
$$\varepsilon_q(T) = Ts_q(T) - P_q(T) + \sum_i \mu_i \int_0^T \frac{\partial s_q(t)}{\partial \mu_i} dt,$$

$$(C_V)_q(T) = \left.\frac{\partial \varepsilon_q(T)}{\partial T}\right|_V = T\frac{\partial s_q(T)}{\partial T} + \sum_i \mu_i \frac{\partial s_q(T)}{\partial \mu_i},$$

$$(c_s^2)_q(T) = \frac{\partial P_q(T)}{\partial \varepsilon_q(T)} = \frac{s_q(T)}{(C_V)_q(T)},$$

Z.-Y. Qin, J.-H. Shi, J.-P. Zhang, J. Cao, B. Feng, **W.-C. Zhang***, H. Zheng, S.-J. Mao, **Phys. Rev. D 112**, 096022 (2025)

Result II: Nonextensive and Chemical Potential Effects on the QCD Phase Transition



$(q = 1.05\ vs\ q = 1)$

$s_q/T^3, P_q/T^4, \varepsilon_q/T^4$: increase in both phases

$\Delta_q = (\varepsilon_q - 3P_q)/T^4$: peaks increase

· $(c_s^2)_q$ drops near T_c: ε_q rises faster than P_q → soften EoS

$(c_s^2)_q$: Nontrivial modifications near T_c and at low T

· $q > 1$ → larger particle yields,
→ thermodynamic quantities increase

Z.-Y. Qin, J.-H. Shi, J.-P. Zhang, J. Cao, B. Feng, **W.-C. Zhang***, H. Zheng, S.-J. Mao, **Phys. Rev. D 112**, 096022

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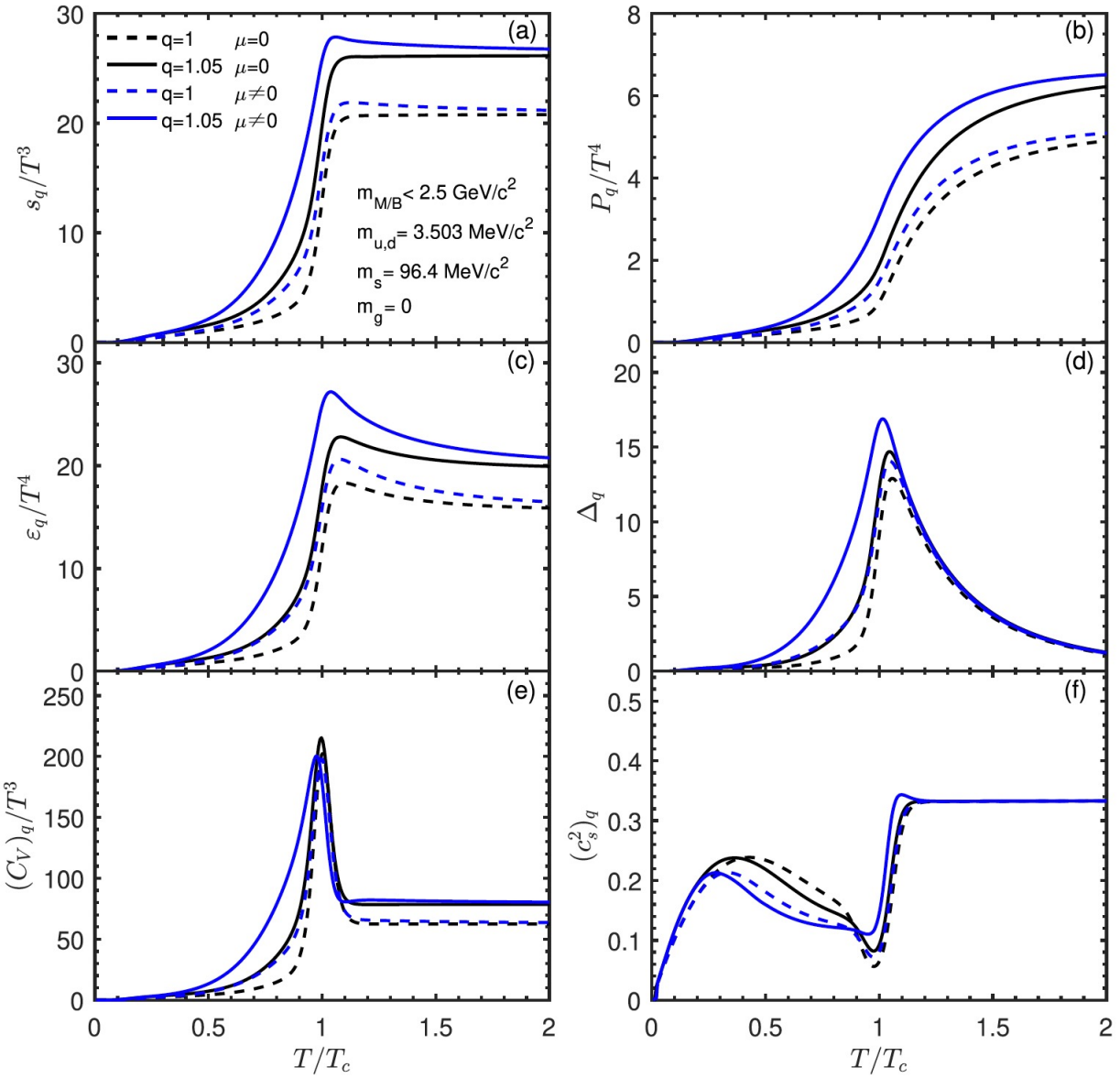

$(\mu \neq 0 \; vs \; \mu = 0)$

$\mu_B = 398.2$ MeV, $\mu_s = 89.5$ MeV, $\mu_Q = 0$

(corresponding to the chemical potentials in central Au-Au collisions at $\sqrt{s_{NN}} = 7.7$ GeV)

· $\mu \neq 0$ enhances $s_q/T^3, P_q/T^4, \varepsilon_q/T^4$ in both phases.

Reason: chemical potential lowers excitation energy for baryons/strange hadrons
→ enhances their contributions

· $(c_s^2)_q$ increases near T_c, decreases at low T.

Z.-Y. Qin, J.-H. Shi, J.-P. Zhang, J. Cao, B. Feng, **W.-C. Zhang***, H. Zheng, S.-J. Mao, **Phys. Rev. D 112**, 096022

Result III: QCD EoS under Finite Magnetic Fields

- Non-central heavy-ion collisions produce extremely strong magnetic fields ($eB \sim m_\pi^2$ to $10m_\pi^2$).
- Magnetic fields affect the QCD phase transition and thermodynamic properties.
- Low T: HRG model; High T: Ideal Parton Gas (IPG) model; Smooth crossover interpolation for the full EoS.
- Key Effects of the Magnetic Field:

  Pressure anisotropy $P_\parallel \neq P_\perp$;

  charged particle energies: $\epsilon_i^c = \sqrt{p_z^2 + m_i^2 + 2|q_i|eB(l + 1/2 - s_z)}$

  the allowed Landau levels

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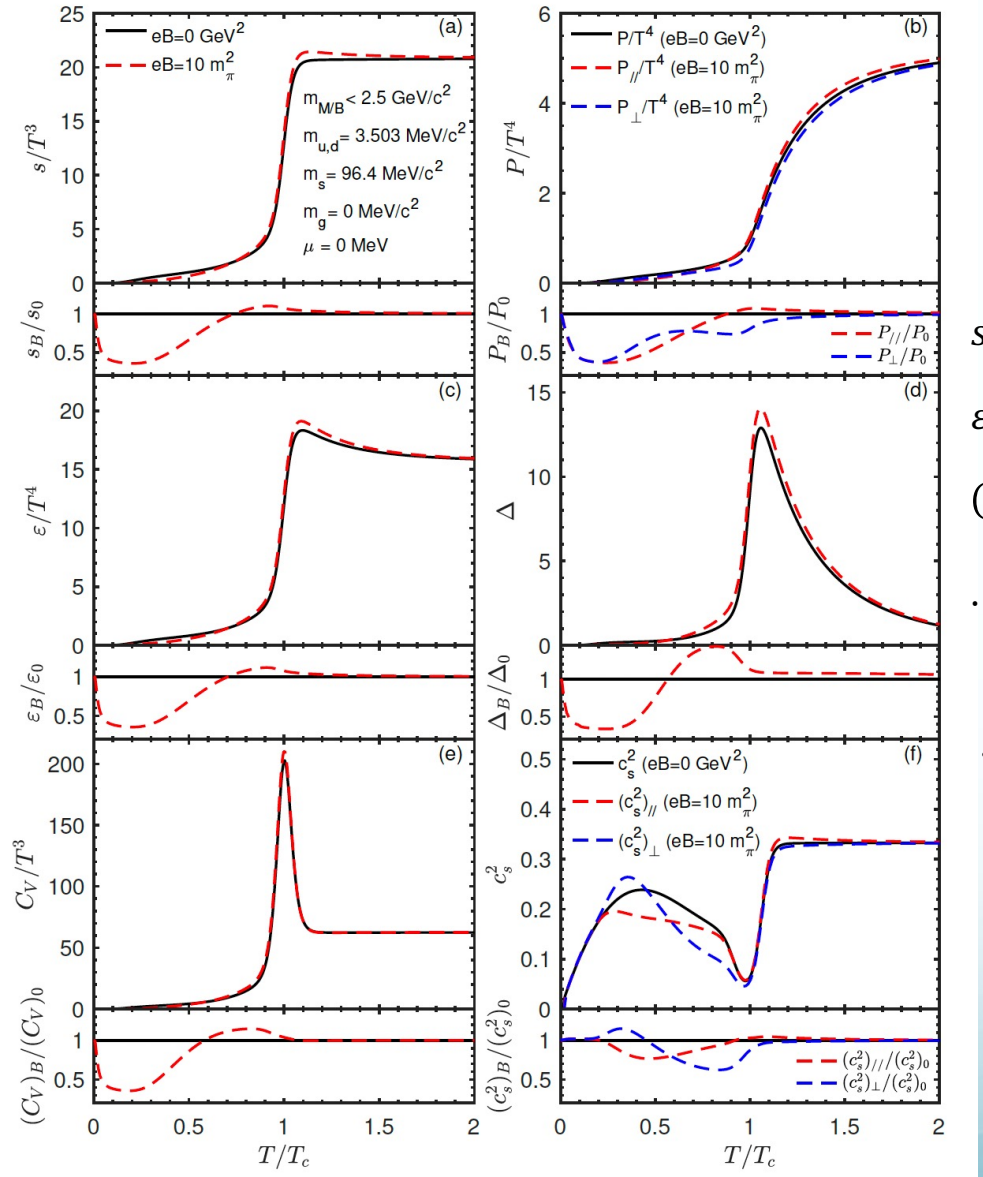
Result III: QCD EoS under Finite Magnetic Fields



$$eB \neq 0 \; vs \; eB = 0$$

s/T^3 and $P_\parallel/T^4$: suppressed at low T, enhanced at high T

$\varepsilon/T^4, \Delta$ and C_V/T^3: similar non-monotonic behavior

$(c_s^2)_\parallel$: suppressed below T_c and enhanced above T_c

· Low T: $P_\perp > P_\parallel$ (negative magnetization from charged scalar mesons)

· High T: $P_\perp < P_\parallel$ (positive magnetization from vector mesons)

Z.-Y. Qin, J.-H. Shi, J.-P. Zhang, J. Cao, B. Feng, **W.-C. Zhang***, H. Zheng, S.-J. Mao, arXiv:2602.11713 (**accepted** by **PRD**)

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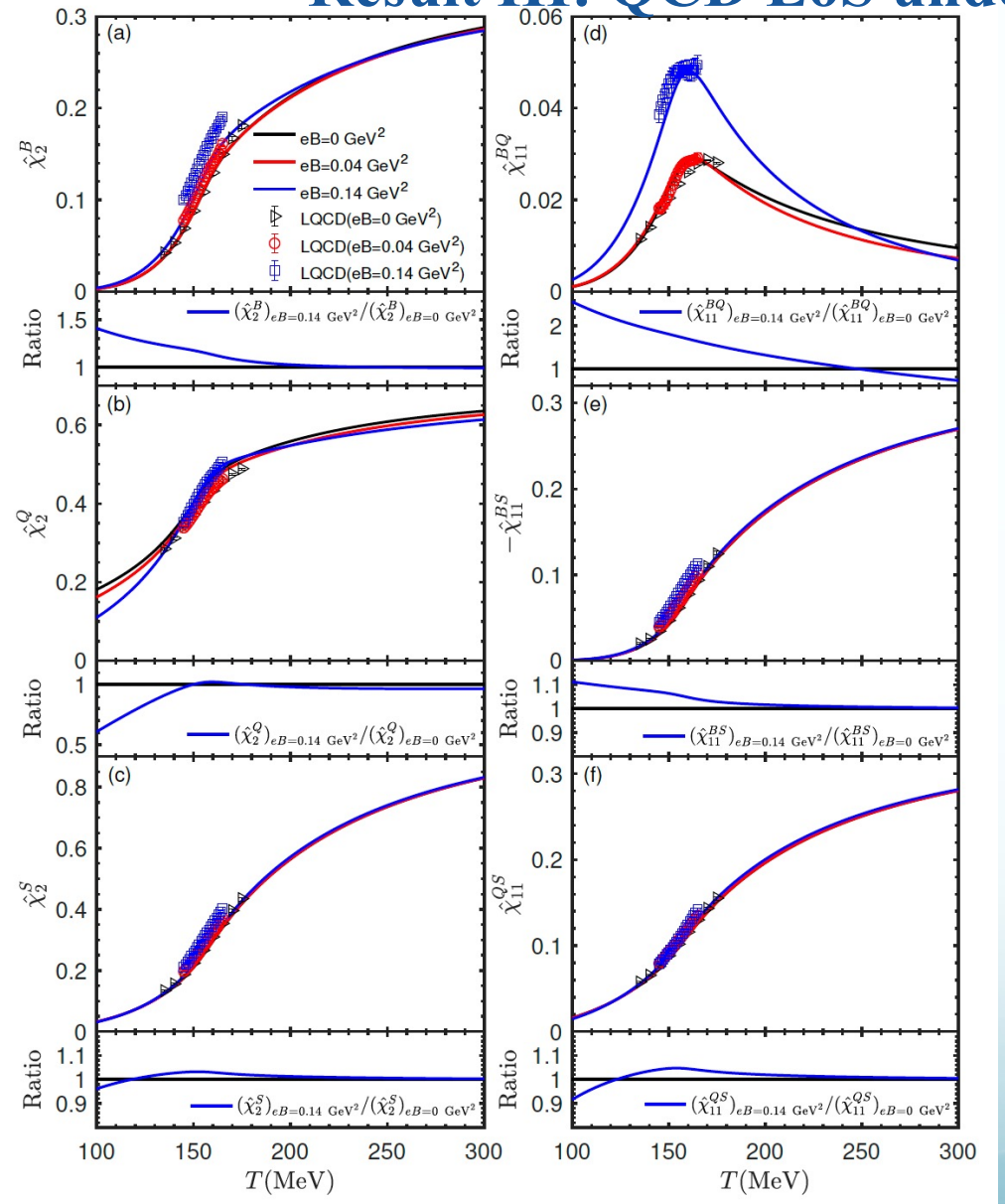


$\chi_2^B, \chi_2^Q, \chi_2^S$ increase with temperature

Magnetic field enhances χ_2^B, χ_{11}^{QS} and $-\chi_{11}^{BS}$, but suppresses χ_2^Q, χ_2^S and χ_{11}^{QS} at low T, and χ_2^B, χ_2^Q and χ_{11}^{BQ} at high T.

Comparison with LQCD:
agreement at eB=0 and 0.04 GeV^2;
underestimation at 0.14 GeV^2
(due to neglected AMM and interactions)

Z.-Y. Qin, J.-H. Shi, J.-P. Zhang, J. Cao, B. Feng, **W.-C. Zhang***, H. Zheng, S.-J. Mao, arXiv:2602.11713 (**accepted** by **PRD**)

Conclusions

- (3+1)-D nonextensive hydrodynamic model successfully describes p_T spectra up to 6–8 GeV/c.
- Nonextensive effects suppress elliptic flow (similar to shear viscosity).
- $q > 1$ enhances all thermodynamic quantities (in both phases).
- Chemical potential enhances thermodynamics, modifies speed of sound nontrivially.
- Magnetic field suppresses the thermodynamic quantities at low T, enhances them at high T.
- Conserved charge fluctuations are sensitive to magnetic field, partially agree with LQCD.

Backup

- The initial conditions are provided by the the *optical Glauber model*.
- The *non-extensive effect* is considered in the *longitudinal component* of the initial energy density.

longitudinal space-time rapidity

$$H_q(\eta_s) = \exp_q\left[-\frac{(|\eta_s| - \eta_w)^2}{2\sigma_\eta^2}\theta(|\eta_s| - \eta_w)\right]$$

the width of the Gaussian falloff at large rapidity

the width of the central rapidity plateau

- The *non-extensive* initial energy density at the hydrodynamic starting proper time τ_0 is written as

$$\varepsilon_q(x, y, \eta_s) = \varepsilon_0 W(x, y) H_q(\eta_s),$$

the maximum energy density determined by the particle yield in the most central AA collisions

the initial energy density profile in the transverse direction

Eur. Phys. J. C 80, 293 (2020)
PRC 104, 064903 (2021)
PRC 105, 034901 (2022)

Backup

- In the non-extensive statistics, the motion of an ideal QGP fluid with zero baryon density is determined by

$$\partial_\mu T_q^{\mu\nu} = 0$$

non-extensive energy-momentum tensor

- In the kinetic theory, $T_q^{\mu\nu}$ of the system in the hadronic or the QGP phase can be written as

$$T_q^{\mu\nu} = \sum_i \frac{g_i}{(2\pi)^3} \int \frac{\mathrm{d}^3 p}{E_i} p_i^\mu p_i^\nu f_i^q (E_i/T)$$

$$f(x) = \frac{\exp_q(x)}{1 \pm \exp_q(x)}$$

$$\varepsilon_q = T_q^{\mu\nu} u_\mu u_\nu = \sum_i \frac{g_i}{(2\pi)^3} \int \mathrm{d}^3 p E_i f_i^q$$

$$P_q = -\frac{1}{3} T_q^{\mu\nu} \Delta_{\mu\nu} = \sum_i \frac{g_i}{(2\pi)^3} \int \frac{\mathrm{d}^3 p}{3E_i} \boldsymbol{p}_i^2 f_i^q$$

$$s_q = \sum_i \frac{g_i}{(2\pi)^3} \int \mathrm{d}^3 p [f_i^q \ln_q f_i^q \pm (1 \mp f_i)^q \ln_q (1 \mp f_i)]$$

Backup

- For the baryon-free case, the usual thermodynamic relations still hold in the non-extensive statistics

$$Ts_q = \varepsilon_q + P_q; \qquad \frac{\partial P_q}{\partial T_q} = s_q$$

- The *EOS* in the hadronic and QGP phases are then, respectively, constructed by *embedding the q-correction* to the HRG model and parton resonance gas model.

- In order to *match* the pressure at the hadronic phase ($P_q^{\rm had}$) with that at the QGP phase ($P_q^{\rm QGP}$), the pressure in the the *mixed-phase* region is constructed as

$T \le T_f$ $\quad P_q^{\rm mix}(T) = P_q^{\rm had}(T)$

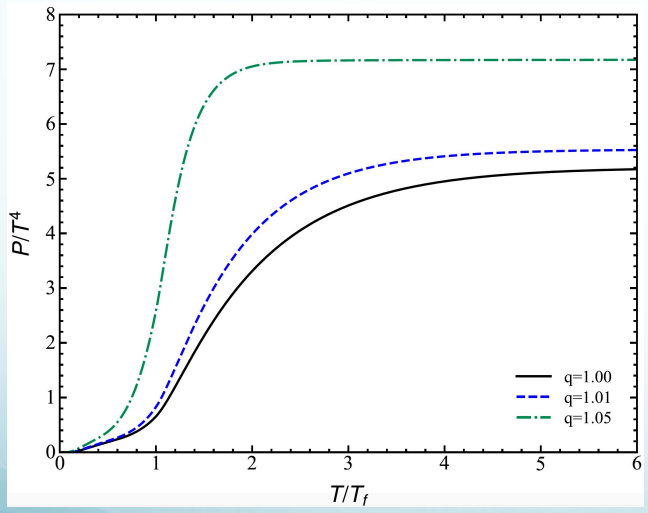
$T > T_f$ $\quad P_q^{\rm mix}(T) = S(T)P_q^{\rm had}(T_f) + (1 - S(T))P_q^{\rm QGP}(T)$

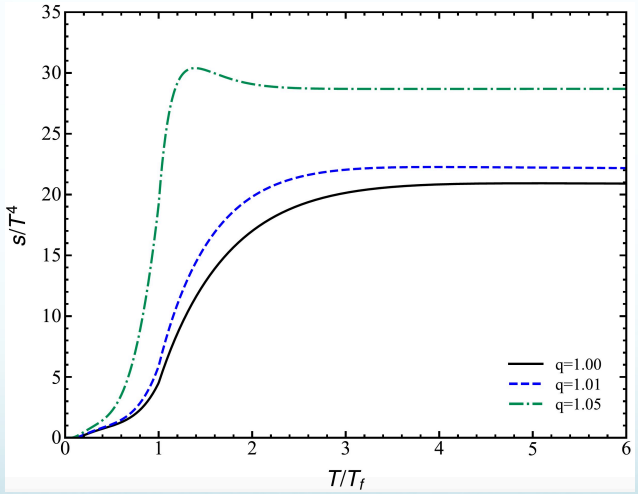
$$\frac{\partial P_q^{\rm mix}}{\partial T}(T_f) = \frac{\partial P_q^{\rm had}}{\partial T}(T_f),$$

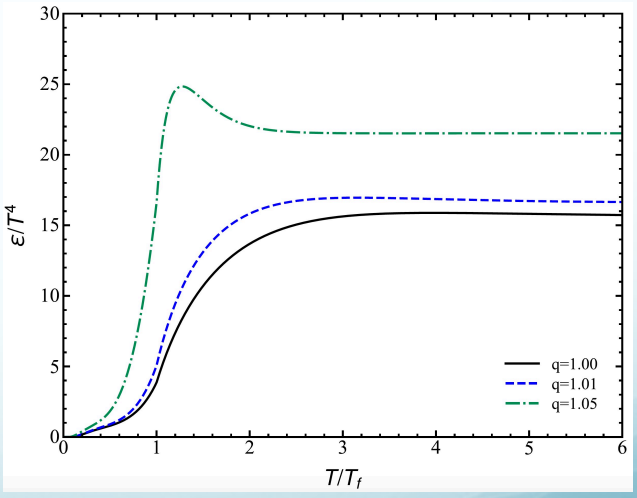
$$S(T) = \exp(-c(T - T_f))$$

Backup

- The QGP phase consists of *u*, *d*, and *s* quarks.
- In the hadronic phase, hadrons and hadron resonances composed of *light quarks* with masses below 2 GeV/c^2 are considered.



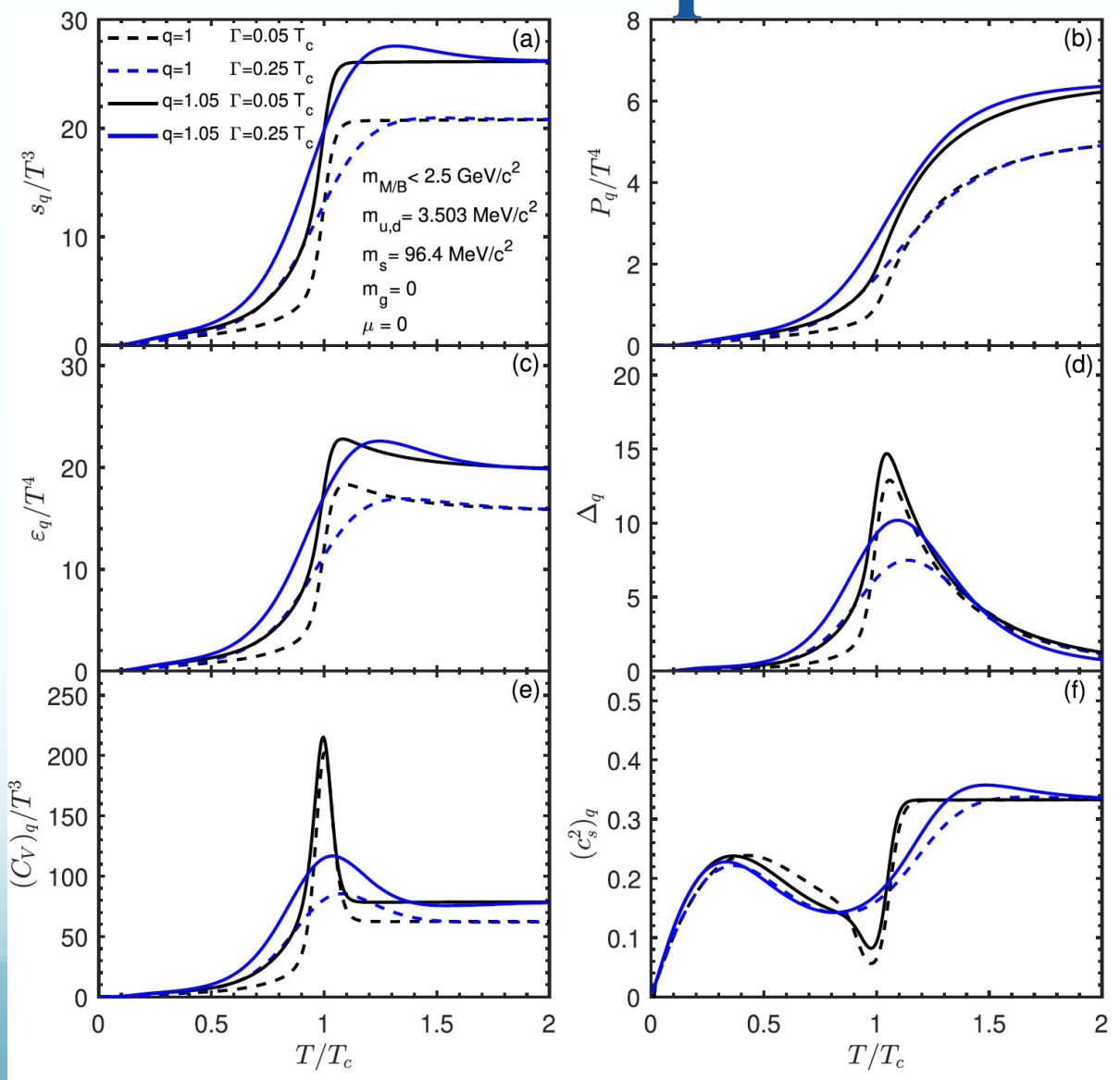




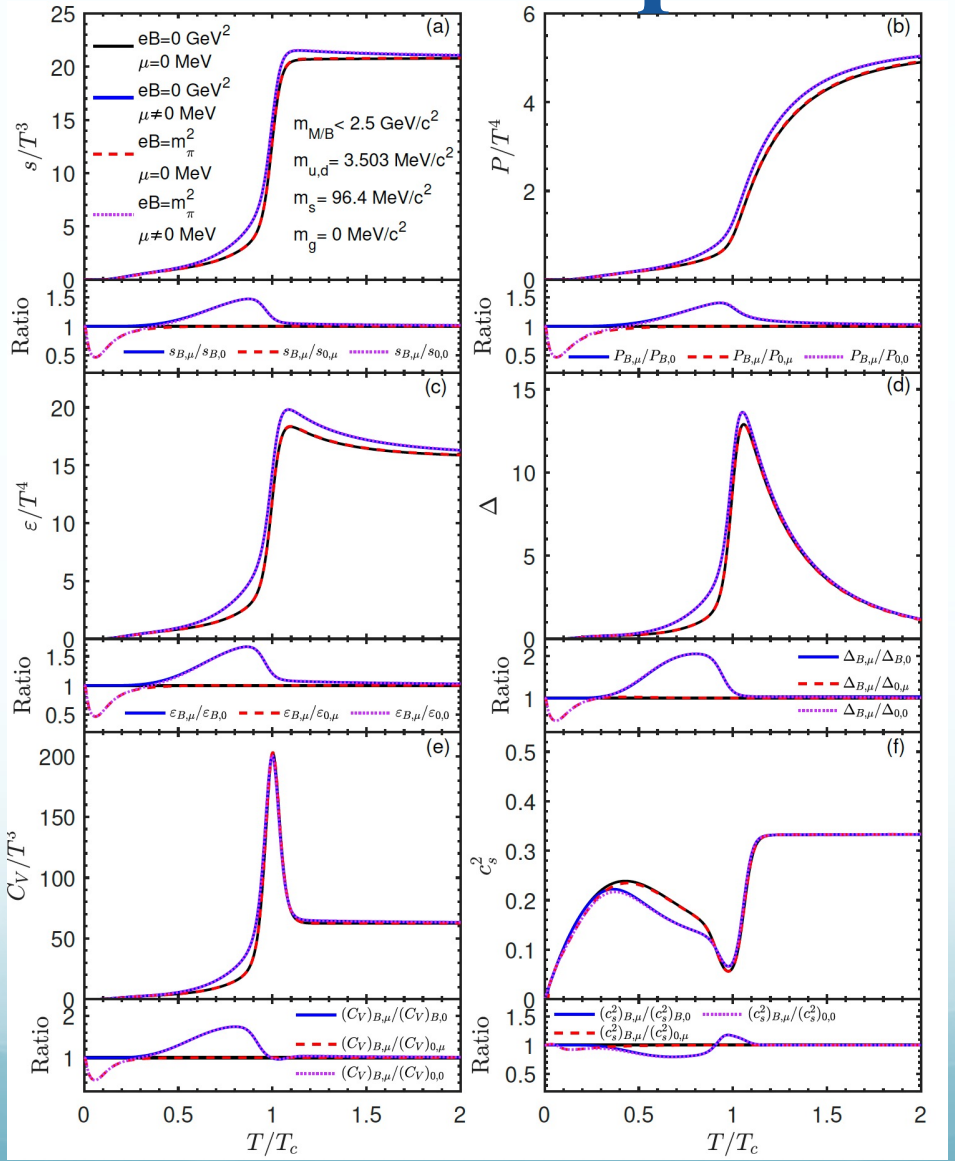
Backup

We have also explored the q-hydrodynamic description on the p_T spectra [76] and elliptic flow [77] of identified particles in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. It is found that the model can describe the spectra of charged pions and kaons up to 6 GeV/c while overestimates the proton spectra in the whole p_T region. Moreover, the model can reproduce the pion and kaon v_2 up to $p_T \sim 2$ GeV/c while fails to describe the proton v_2. The discrepancy in the proton spectra may indicate that the nonextensivity for protons is less than that for pions and kaons. In Refs. [41,72,78], the authors have shown that in proton-proton and AA collisions the nonextensivity for baryons is smaller than that for mesons. The decrease of q will lead to a reduction of the proton yield. The present work is our first step to develop the $(3 + 1)$-dimensional q-hydrodynamic model. We will consider the case that mesons and baryons have different q values in our next work.

Backup


q=1 Γ=0.05 T_c
q=1 Γ=0.25 T_c
q=1.05 Γ=0.05 T_c
q=1.05 Γ=0.25 T_c
m_M/B< 2.5 GeV/c^2
m_u,d= 3.503 MeV/c^2
m_s= 96.4 MeV/c^2
m_g= 0
μ = 0
(a)
(b)
(c)
(d)
(e)
(f)
s_q/T^3
P_q/T^4
ε_q/T^4
Δ_q
(C_V)_q/T^3
(c_s^2)_q
T/T_c
T/T_c

Backup


(a)
(b)
(c)
(d)
(e)
(f)
eB=0 GeV^2 μ=0 MeV
eB=0 GeV^2 μ≠0 MeV
eB=m$_\pi^2$ μ=0 MeV
eB=m$_\pi^2$ μ≠0 MeV
m$_{M/B}$< 2.5 GeV/c^2
m$_{u,d}$= 3.503 MeV/c^2
m$_s$= 96.4 MeV/c^2
m$_g$= 0 MeV/c^2
s/T^3
P/T^4
ε/T^4
Δ
C_V/T^3
c_s^2
Ratio
$s_{B,\mu}/s_{B,0}$ $s_{B,\mu}/s_{0,\mu}$ $s_{B,\mu}/s_{0,0}$
$P_{B,\mu}/P_{B,0}$ $P_{B,\mu}/P_{0,\mu}$ $P_{B,\mu}/P_{0,0}$
$\varepsilon_{B,\mu}/\varepsilon_{B,0}$ $\varepsilon_{B,\mu}/\varepsilon_{0,\mu}$ $\varepsilon_{B,\mu}/\varepsilon_{0,0}$
$\Delta_{B,\mu}/\Delta_{B,0}$ $\Delta_{B,\mu}/\Delta_{0,\mu}$ $\Delta_{B,\mu}/\Delta_{0,0}$
$(C_V)_{B,\mu}/(C_V)_{B,0}$ $(C_V)_{B,\mu}/(C_V)_{0,\mu}$ $(C_V)_{B,\mu}/(C_V)_{0,0}$
$(c_s^2)_{B,\mu}/(c_s^2)_{B,0}$ $(c_s^2)_{B,\mu}/(c_s^2)_{0,\mu}$ $(c_s^2)_{B,\mu}/(c_s^2)_{0,0}$
T/T_c