Spin Entanglement in QCD

from Non-Equilibrium Scattering and Topological Fluctuations

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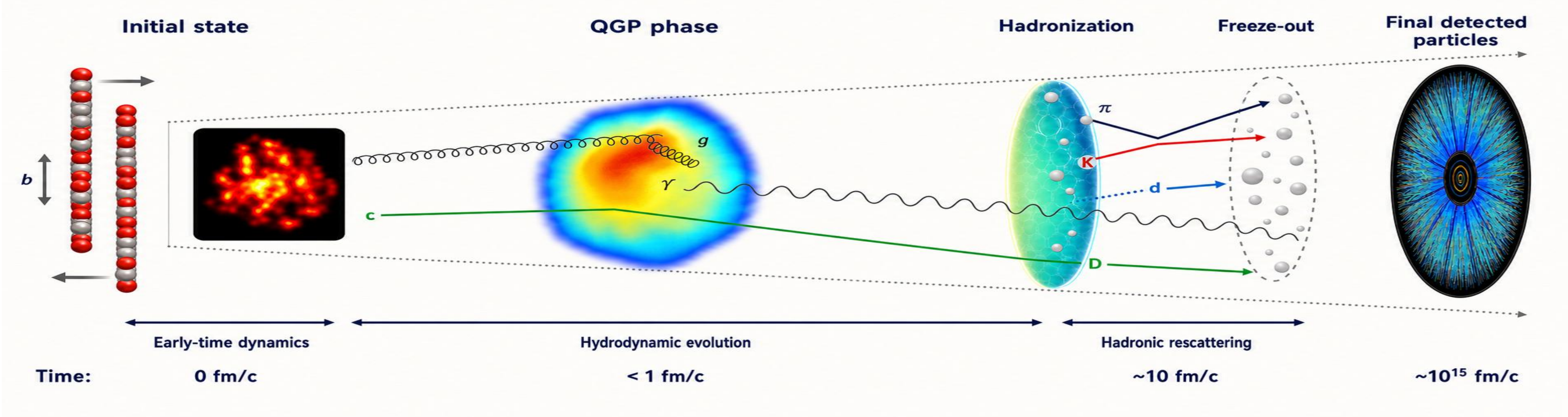
07.16 at Guanzhou

Entropy paradox in QCD matter



Melting nuclei
in relativistic heavy ion collision → ***new state of QCD matter*** → ***fast equilibrium~1 fm/c***

unitary evolution

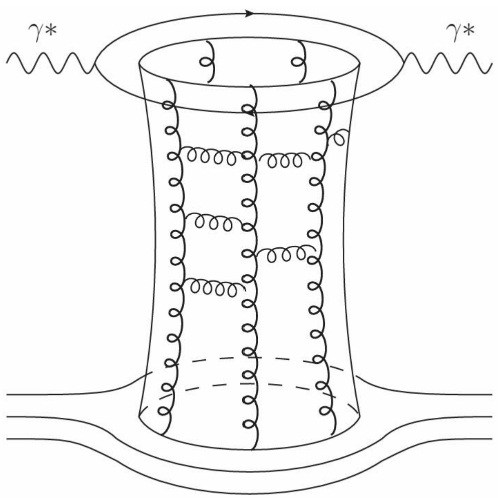
Quark or gloun state is pure ? ***locally thermalized final state***
$S= -Tr\, \rho \ln \rho = 0$ ***with HUGE entropy***

Lorentz boost

……

- **Quantum entropy is superficially contradictory for an isolated matter !**

Entropy Paradox in QCD Matter



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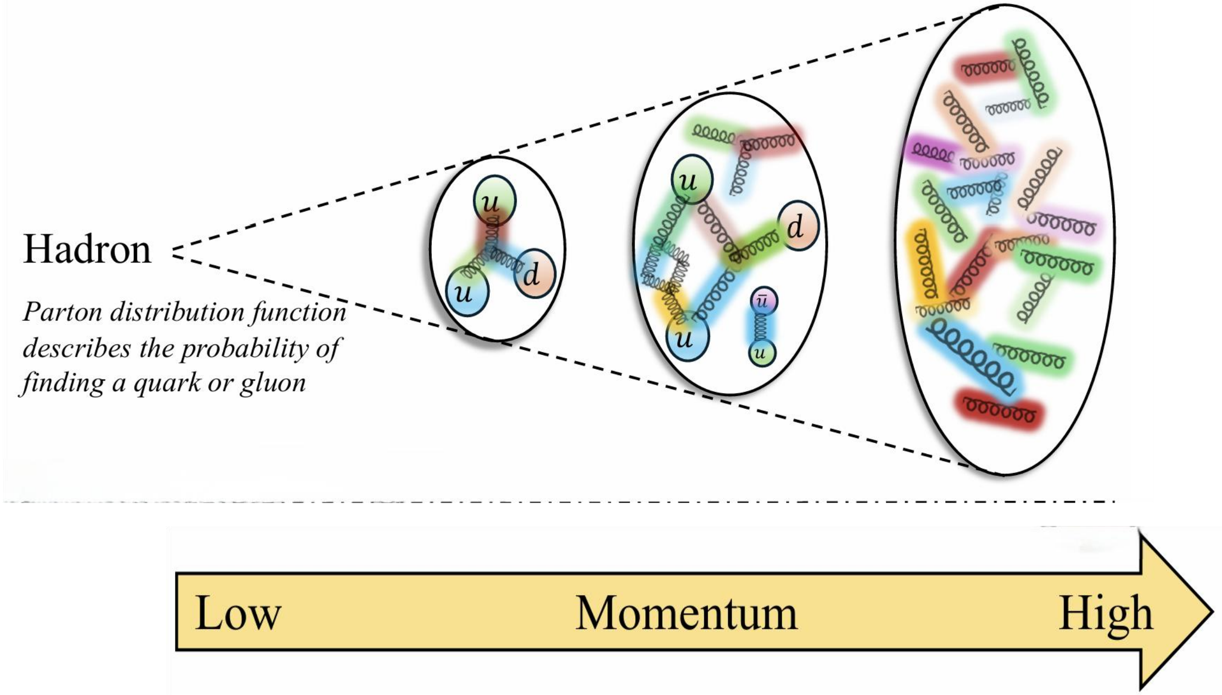


- Non-linear evolution of QCD gives maximum of entanglement entropy among partons, under limit of high momentum scale and small x.

- Scale-dependent entropy is natural in DIS picture.

$$S(x) = \ln[xG(x)] \simeq \Delta \ln[1/x] \simeq \Delta \ln \frac{L}{\epsilon}$$

CFT $\longleftrightarrow$

$$S_E = \frac{c}{3} \ln \frac{L}{\epsilon}$$

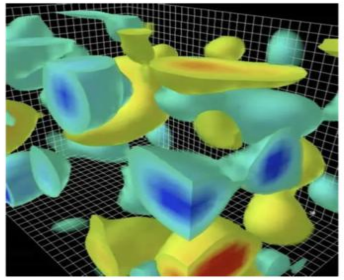
Entangle Entropy in CFT

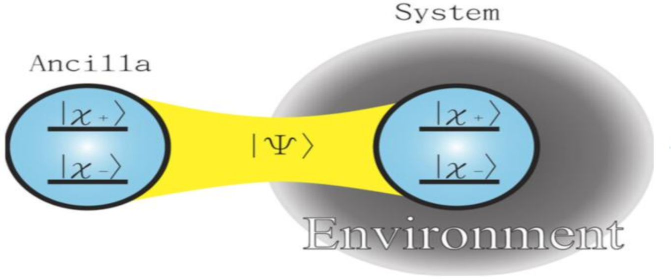
$$S_E \sim \frac{c}{3} \ln\left(\frac{1}{\pi T \epsilon}\right) \sinh(\pi T L).$$

finite temperature

- **Problems Not Solved**: Entanglement entropy $\neq$ thermal entropy
- **Entropy relation in quantum thermalization?**
- **Where does Entanglement among partons come from?**

Where does the entropy derive from?





Vacuum topology(instanton ensemble) → ***entanglement*** —

Prompt Quark Production by exploding Sphalerons

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(November 7, 2018)

hadron and nucleus-nucleus collisions at large $\sqrt{s}$. Simple arguments indicate that while in hadron-hadron scattering production via sphalerons, it is only a small part of the multiplicity, in nucleus-nucleus collisions it is not so and hundreds of sphalerons may be released, contributing substantially to the total entropy produced [16]. In view of that, the production of $2N_F$ chiral quarks per cluster may make the quark-gluon plasma produced in heavy-ion

total number of qq collision, the production of sphaleron-like:

$$N_{coll} = 8\pi\sigma_{qq}n_q^2\int_0^R dr_t r_t\left(R^2 - r_t^2\right)$$
$$= 3^{4/3}2^{-5/3}\pi\sigma_{qq}n_q^2\left(\frac{AN_q}{\pi n_q}\right)^{4/3},$$

at RHIC energies

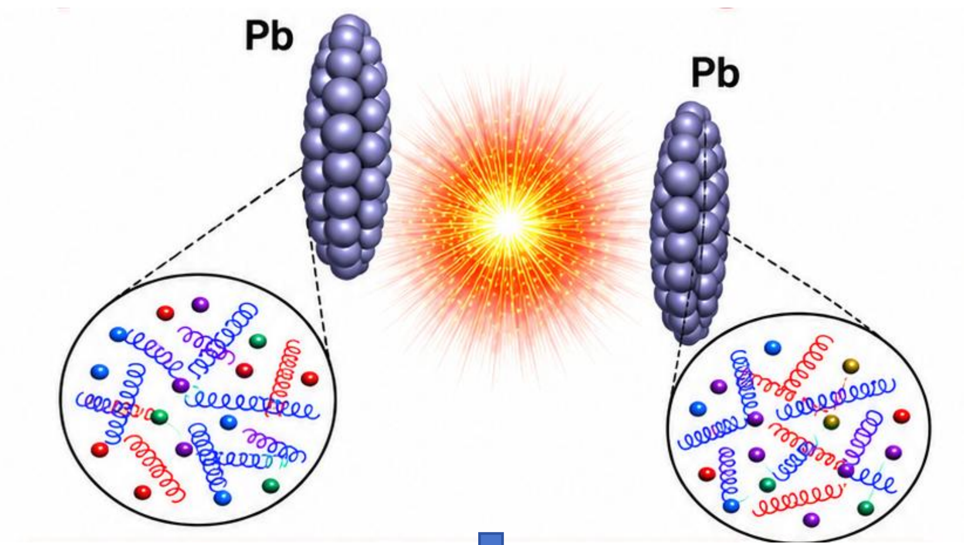
$$N_{objects} \approx 400\,,$$

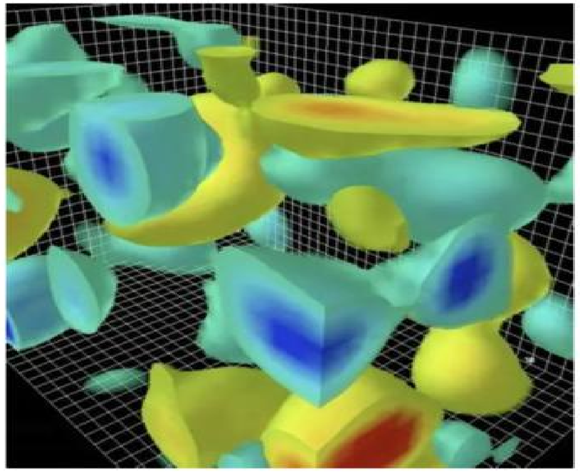
E. Shuryak. hep-ph/0205031
hep-ph/0206022

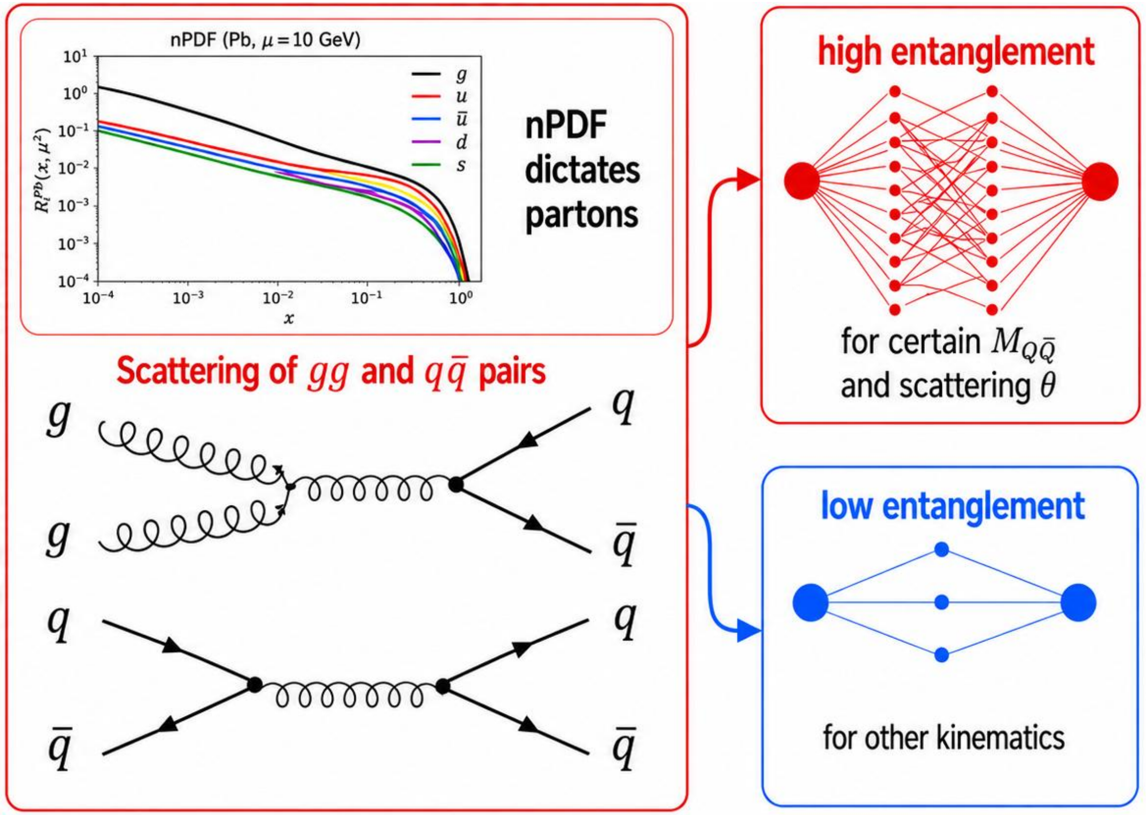
- Topological structure of QCD prompt parton release, potentially leads to entropy production

- Entanglement as a novel probe of early QCD matter
  - reflecting the non-comutative effects from topological field
  - providing entropy among partons

Entanglement From Parton Scattering







- Partons can come from two sources: **nuclear PDF** & **Sphaleron Burst**
- Partons scatter to form entanglement.
- **Nuclear Partons** contribtution to *heavy flavour entanglement* is dominant over **Sphaleron source** due to **low** production rate.

Amplitude to Concurrence WorkFlow

Initial (4 x 4)

$$\hat{\rho}_{in} = \frac{1}{N_a N_b} \sum_{s_a, s_b} |s_a, s_b\rangle\langle s_a, s_b|$$

evolve by scattering

$$\hat{R} = \mathcal{M}\hat{\rho}_{\text{in}}\mathcal{M}^\dagger$$

Final state Density Matrix

$$R^{(\eta_2,\zeta_2)}{}_{(\eta_1,\zeta_1)} = \frac{1}{N_a N_b} \sum_{s_a, s_b} \mathcal{M}^{s_a, s_b}_{\eta_1, \zeta_1} \mathcal{M}^{\dagger\eta_2\zeta_2}_{s_a s_b}\ , \quad \rho_{\text{out}} = \frac{R}{\text{Tr}R}$$

Decompose it with Correlation

$$\rho = \frac{1}{4}\left(\mathbb{1} \otimes \mathbb{1} + \widetilde{C}_{ij}\hat{\sigma}_i \otimes \hat{\sigma}_j\right)$$

$$\tilde{\rho} = (\sigma_2 \otimes \sigma_2)\,\rho\,(\sigma_2 \otimes \sigma_2)$$

$$\lambda_i \in \sqrt{\text{EigenV}(\text{R}_w := \rho\tilde{\rho})}$$

W.K. Wooters, Phys. Rev. Lett. **80**, 2245 (1998)

Concurrence

$$\mathcal{C} = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

$$= \frac{1}{2}\max(0,\ -C_{nn} + |C_{rr} + C_{kk}|)$$

- Concurrence is measurable

$$t\bar{t} \to (l^+\nu b)\left(l^-\bar{\nu}\bar{b}\right)$$

Nature **633**, 542-547 (2024) ATLAS

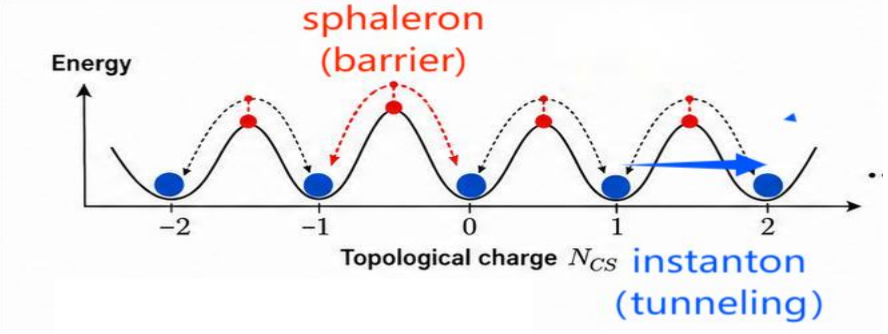
φ: Angles between two leptons

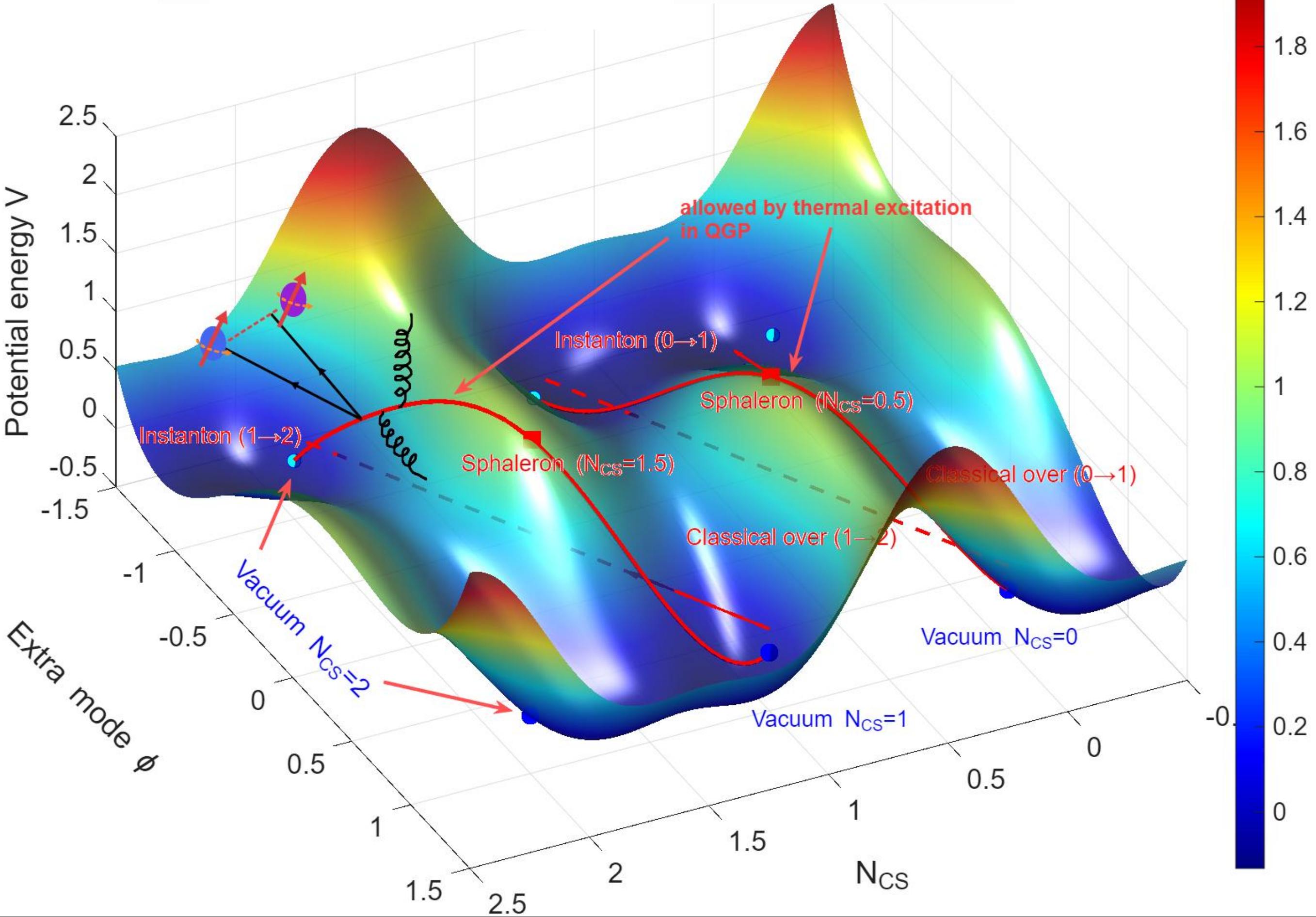
$$D = \langle\cos\varphi\rangle = -\frac{1 + 2\mathcal{C}[\rho]}{3}$$

- Entanglement of Formation (Free From Ambience & Projective to trivial knot)

$$E_F(\mathcal{C}(\rho)) := h\left(\frac{1 + \sqrt{1 - \mathcal{C}^2}}{2}\right), \quad h(x) = -x\log_2 x - (1 - x)\log_2(1 - x)$$

QCD Topo-induced Entanglement





- Pure gauge config has homotopy:

$$\pi_3(SU(N>1)) = Z$$

with **Chern-Simon number**:

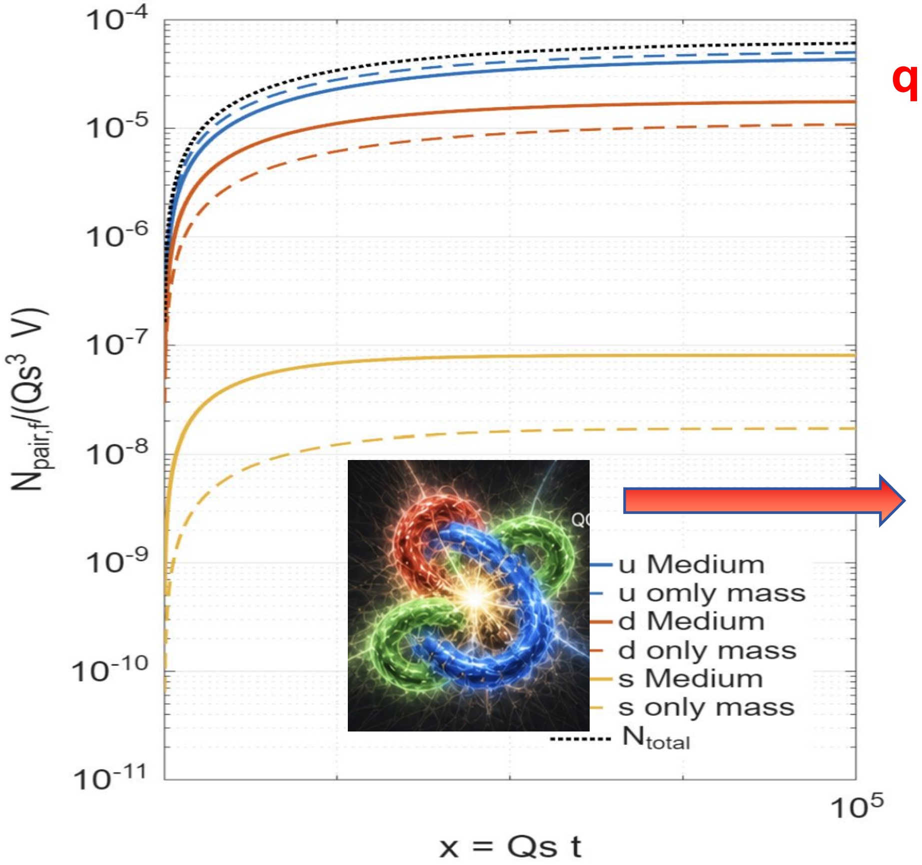
$$n = \frac{1}{24\pi^2}\int d^3x\,\epsilon^{ijk}\,\mathrm{Tr}\big[(U^\dagger\partial_i U)(U^\dagger\partial_j U)(U^\dagger\partial_k U)\big] \in \mathbb{Z}.$$

- Spharleron is a static classical solution A_μ^{cl} with non-trivial topo number in Path Integral

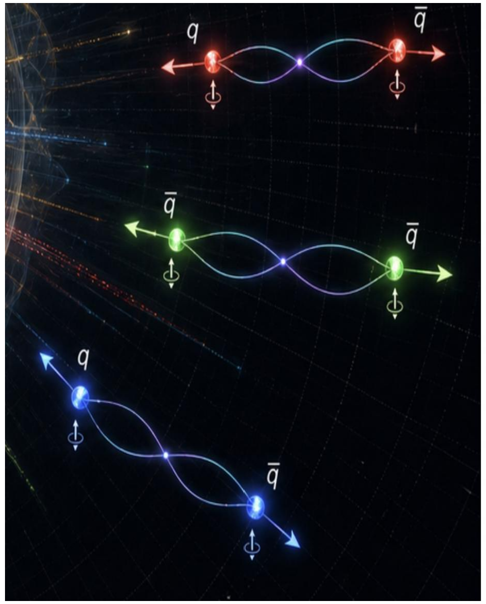
$$Z[A_\mu,\bar{q}_f,q_f] = \int \mathcal{D}A^\mu_{inst}\prod_f \mathcal{D}\bar{q}_f\mathcal{D}q_f exp\{-\int d^4x\left[\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \sum_{f=1}^{N_f}\bar{q}_f(\not{D})q_f\right]\}$$
$$+\int \mathcal{D}A^\mu_{excited}\prod_f \mathcal{D}\bar{q}_f\mathcal{D}q_f exp\{-\int d^4x\left[\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \sum_{f=1}^{N_f}\bar{q}_f(\not{D})q_f\right]\}$$

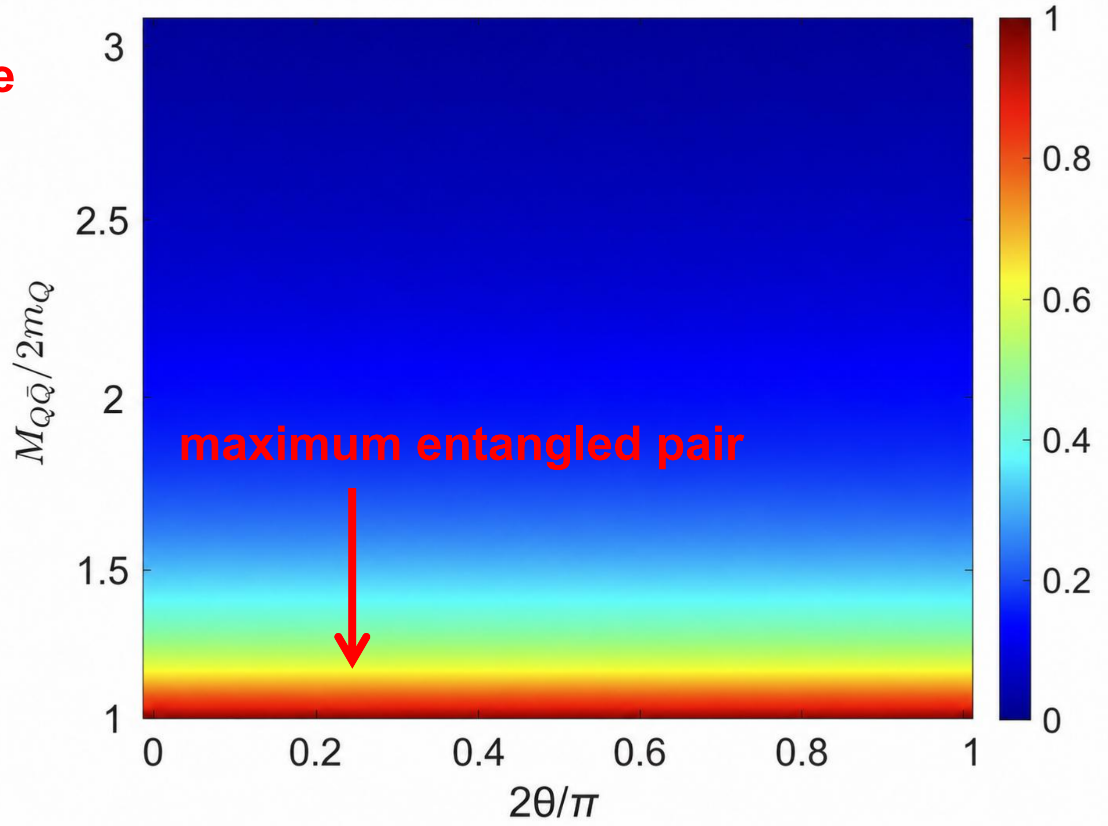
- **Fermi zero mode leads to divergence in second part of path integral, absorbed by external quark line,which gives rise to real quarks production.**

$Q\bar{Q}$ Entanglement from Sphaleron



quark pair concurrence





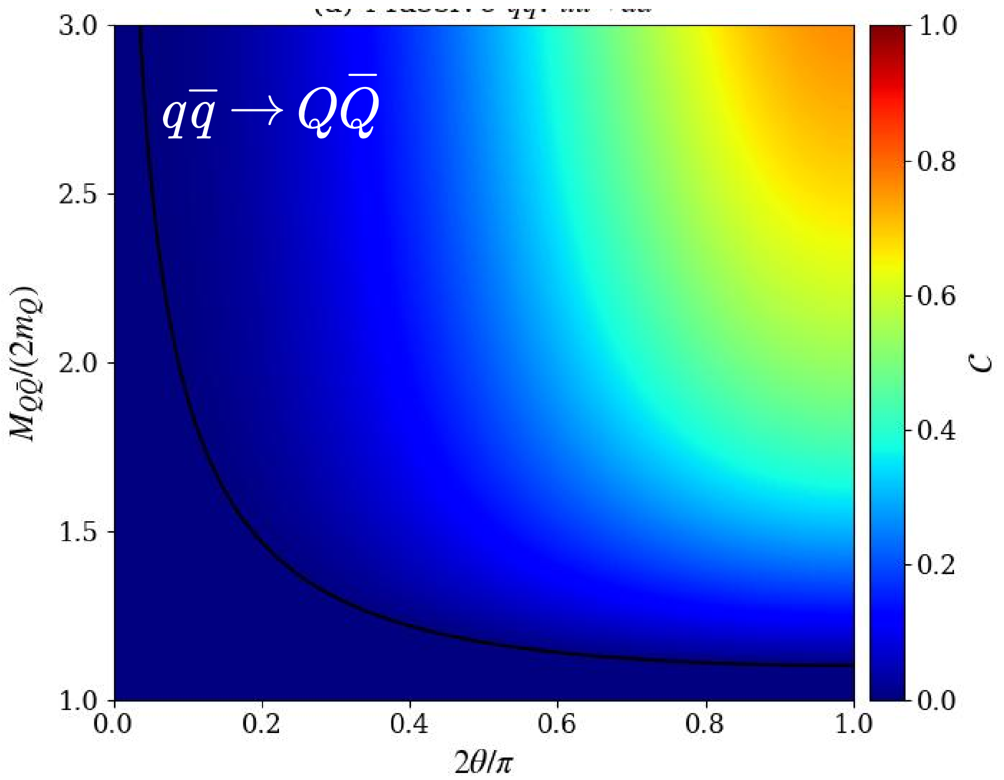
$$\mathcal{N}_f(t) = 2\int d\rho\ \delta(\rho - \rho_*(t)) \int d\Phi_2\ C_{\rm color} \left|S^{(f)}_{\rm spec}(\rho,t)\right|^2 \left(\frac{4\pi^2\rho^3}{N_c}\right)^2 \mathcal{F}_0^2(p\rho/2)\, \mathcal{F}_0^2(q\rho/2)\, 2p\cdot q.$$

- Spin entanglement can be extracted fromquark pair state, with amplitude **M** matched using spaleron rate.

$$\mathcal{M}_{Q,f;s\bar{s}} = e^{i\delta_Q(t)} \sqrt{\frac{r_f(t)\,\Gamma^{\rm neq}_{\rm sph}(t)}{\mathcal{N}_f(t)}}\, T_{Q,f;s\bar{s}}. \qquad dP_f = r_f(t)\,\Gamma^{\rm neq}_{\rm sph}(t)\, d^3X\, dt.$$

- Spectator flavor and Schwinger–Keldysh propagators are included in T matrix, non-equilibrium medium effect *enhances* the pair production.

Entanglement from n-Parton Scattering



- Start from Helicity Amplitude

$$\mathcal{M}_{\lambda_3\lambda_4;\lambda_1\lambda_2} = \begin{pmatrix} -(1+C) & r_i S & r_i S & -(1-C) \\ -r_f S & -r_i r_f C & -r_i r_f C & r_f S \\ -r_f S & -r_i r_f C & -r_i r_f C & r_f S \\ -(1-C) & -r_i S & -r_i S & -(1+C) \end{pmatrix}$$

$$r_{i/f} \equiv \frac{2m_{i/f}}{M_{\mathrm{inv}}},$$

$$C \equiv \cos\theta, \; S \equiv \sin\theta$$

- Under LO collinear approxiamtion, scattering angle can be intepreted as ***pseudorapidity*** $\eta = -\ln(\tan\theta/2)$

- One can obtain its entropy from correlation

$$C_{nn} = \frac{r_f^2(S^2 + r_i^2 C^2) - (1-r_i^2)S^2}{1 + C^2 + r_i^2 S^2 + r_f^2(S^2 + r_i^2 C^2)}, \quad C_{rr} + C_{kk} = \frac{2}{1 + C^2 + r_i^2 S^2 + r_f^2(S^2 + r_i^2 C^2)}$$

$$\mathcal{C}_{\mathrm{conc}} = \max\left[0, \frac{(1-r_i^2)\sin^2\theta - r_f^2(\sin^2\theta + r_i^2\cos^2\theta)}{1 + \cos^2\theta + r_i^2\sin^2\theta + r_f^2(\sin^2\theta + r_i^2\cos^2\theta)}\right]$$

- The threshold non-entangled behaviour for heavy flavours is dictated by Conservation of Angular Momentum
  - Non-vanishing amplitude $\quad r_i \to 0 \Longrightarrow$ Only $\mathcal{M}_{++} = \mathcal{M}_{--} \neq 0$
  - Conservation $\quad L = 0, \; S_z(q\bar{q}) = L + S_z(Q\bar{Q})$

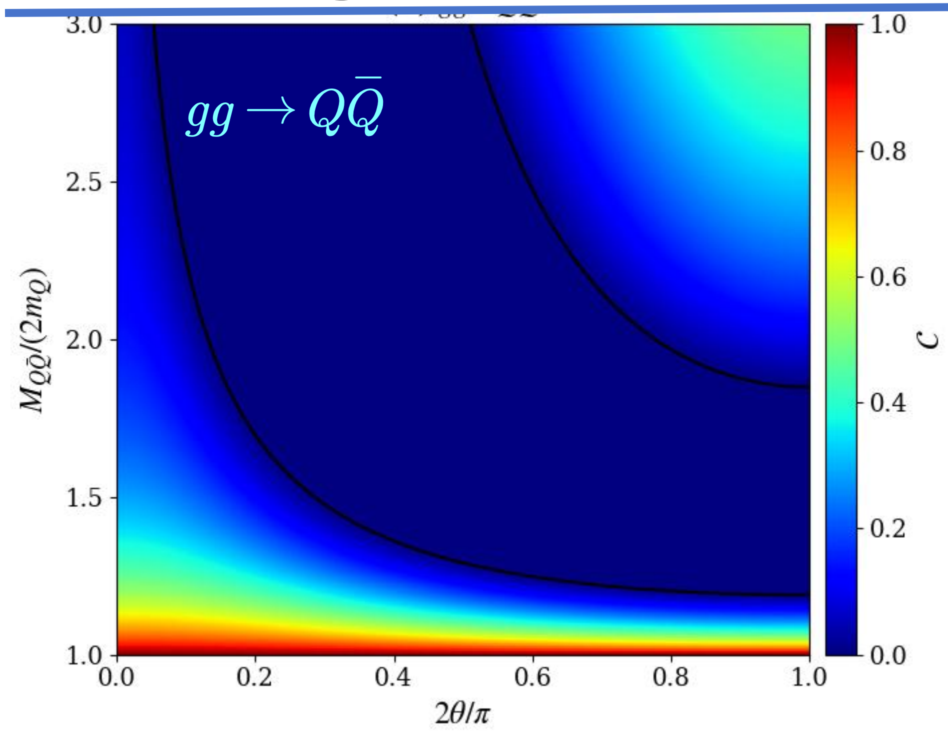
$$q_L\bar{q}_R : S_z = 1 \to |\Uparrow\rangle, \quad q_R\bar{q} : S_z = -1 \to |\Downarrow\rangle$$

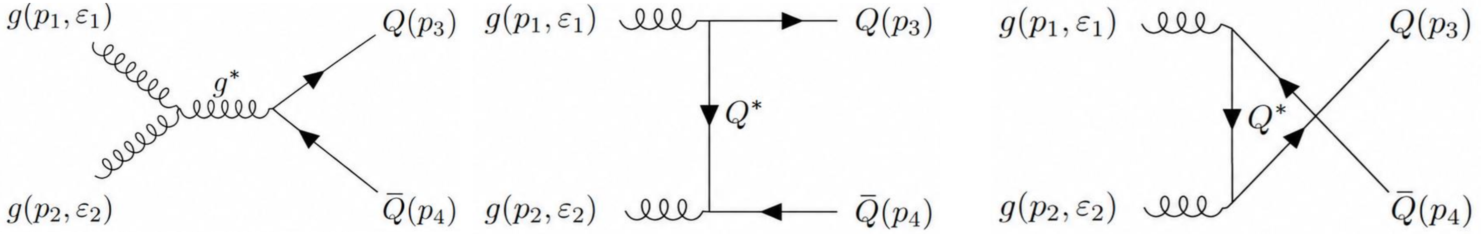
  - The final state is a classical mixture of two separable states.

- Max entanglement of parton occurs at high invariant mass ($r_f \to 0$) and $\boldsymbol{\theta = \pi/2}$ by producing spin-triplet

$$\mathcal{M}(q_R\bar{q}_L \to |\pm\pm\rangle) = \mathcal{M}(q_L\bar{q}_R \to |\pm\pm\rangle) \to \psi^{Q\bar{Q}} \propto |++\rangle + |--\rangle$$

Entanglement from n-Parton scattering





$$\mathcal{M} = g_s^2\left[T^aT^b\,M_- + T^bT^a\,M_+\right]$$

	$\uparrow\uparrow$	$\uparrow\downarrow$	$\downarrow\uparrow$	$\downarrow\downarrow$
RR	$(0,0)$	$\left(\frac{r(1+\beta)}{D_-},\frac{r(1+\beta)}{D_+}\right)$	$\left(-\frac{r(1-\beta)}{D_-},-\frac{r(1-\beta)}{D_+}\right)$	$(0,0)$
LL	$(0,0)$	$\left(-\frac{r(1-\beta)}{D_-},-\frac{r(1-\beta)}{D_+}\right)$	$\left(\frac{r(1+\beta)}{D_-},\frac{r(1+\beta)}{D_+}\right)$	$(0,0)$
LR	$\left(\frac{\boxed{\beta}s(1+c)}{D_-},\frac{\beta s(1+c)}{D_+}\right)$	$\left(\frac{\boxed{\beta}rs^2}{D_-},\frac{\beta rs^2}{D_+}\right)$	$\left(\frac{\boxed{\beta}rs^2}{D_-},\frac{\beta rs^2}{D_+}\right)$	$\left(\frac{\boxed{\beta}s(1-c)}{D_-},\frac{\beta s(1-c)}{D_+}\right)$
RL	$\left(-\frac{\boxed{\beta}s(1-c)}{D_-},-\frac{\beta s(1-c)}{D_+}\right)$	$\left(\frac{\boxed{\beta}rs^2}{D_-},\frac{\beta rs^2}{D_+}\right)$	$\left(\frac{\boxed{\beta}rs^2}{D_-},\frac{\beta rs^2}{D_+}\right)$	$\left(-\frac{\boxed{\beta}s(1+c)}{D_-},-\frac{\beta s(1+c)}{D_+}\right)$

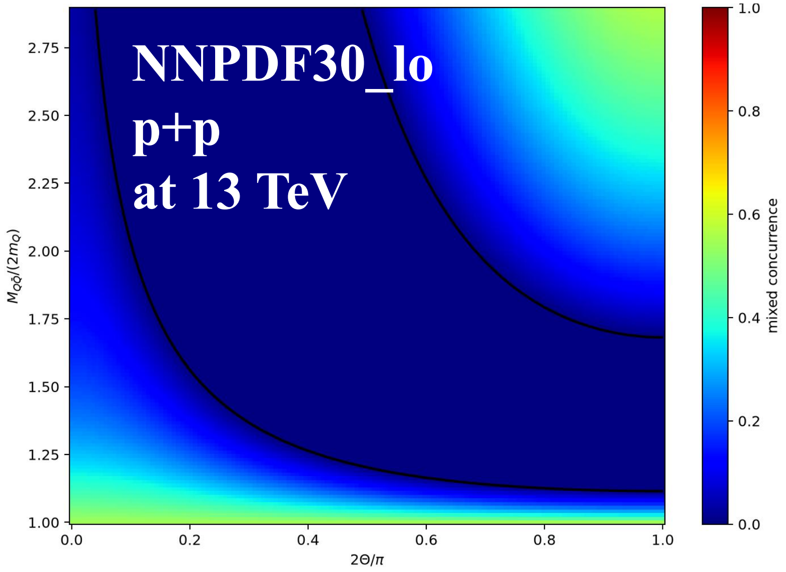
- **Max entanglement at the threshold**
  - Initial gluons have angular momentum projections $g_1^{L/R}g_2^{L/R}:\ J_z=0$ $\quad \cancel{g_1^{R/L}g_2^{L/R}:\ J_z=\pm 2}$
  - Threshold **$\beta \to 0$ (L=0)**, **conservation of momentum** and **Parity-Proof** wavefunction only allows for spin-singlet from *unlike-heliciy* fushion

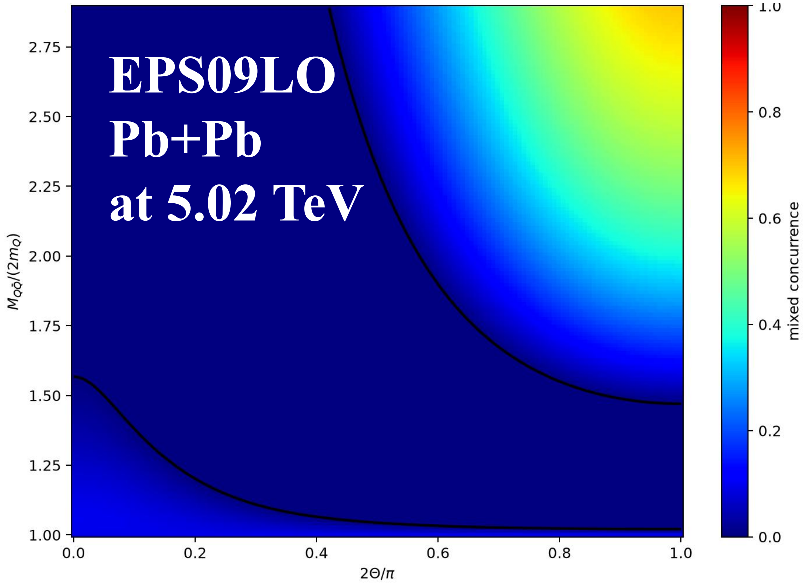
$$Q\bar{Q}:\ J=0,J_z=0,L=0 \Longrightarrow S=0$$

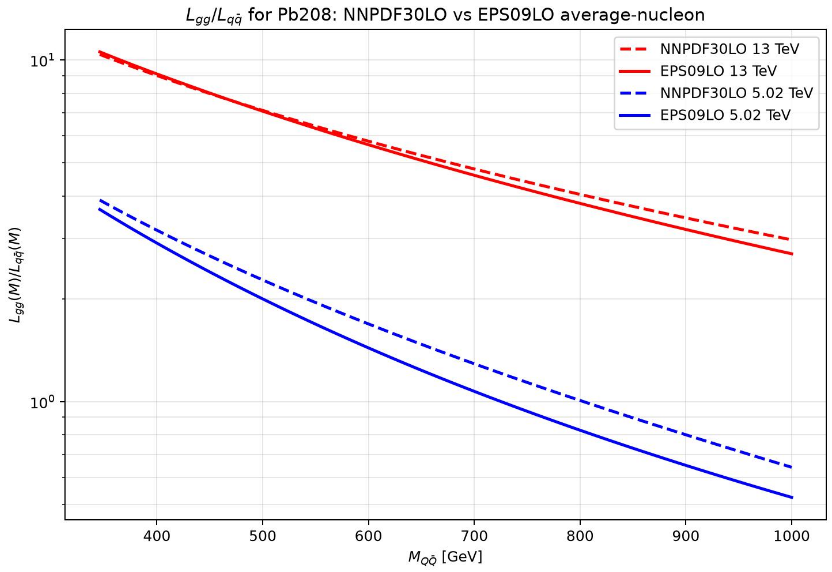
$$\Longrightarrow |\psi(Q\bar{Q})\rangle \propto |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle:\ \text{Max Entangled Bell State}$$

- **Max entanglement ALSO at high energy limit,** similarly seen from the helicity amplitude.

Entanglement from combined scattering process







➤ **We use PDF & nPDF to compute luminosities for weighing each process, mimicking *pp* and *Pb-Pb* collision.**

- Different behaviour of entanglement in each collision is manifested in the vicinity of threshold
- More dominance of ***gg*** process leads to **larger entanglement** at the threshold for ***pp*** collisions
- We tend to believe that the stark difference derives from the energy scale of each collision instead of inner PDF, supporting the picture in DIS parton entanglement at high collision scale or small *x*.

Entropy Relation in qauntum thermalization

Where will the generated entanglement go?

What is the quantitative relation of entropy in thermalization?

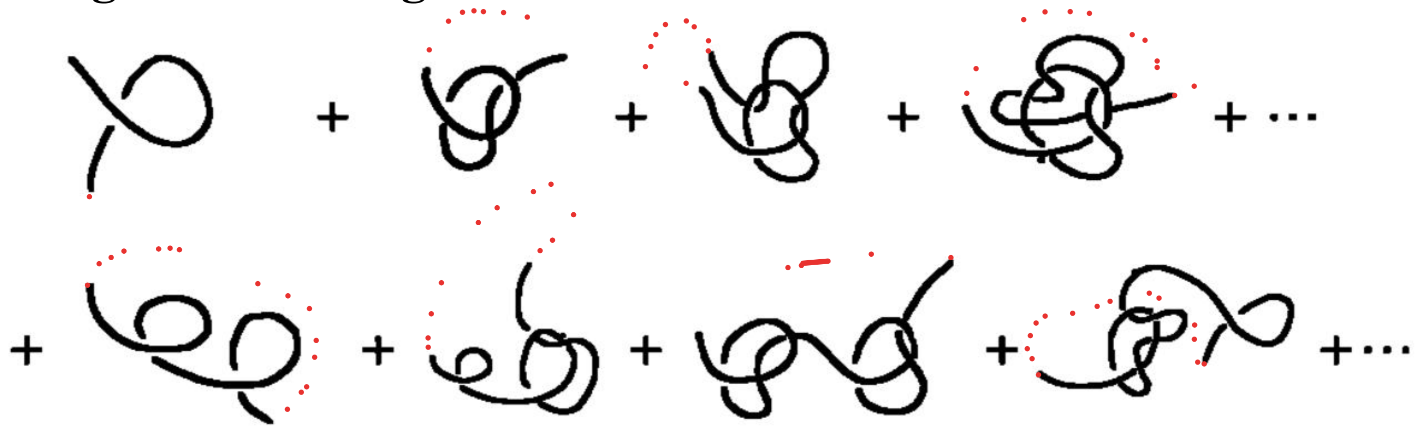
Is the entanglement entropy partial or all of the thermal entropy?

Thermalization $\xleftrightarrow[\textit{scramble}]{\textit{ETH}}$ **entropy propagation**

Entropy Relation in Quantum Thermalization

➤ **R generating rules for general entanglement**

$E^{irr} =$



$$d_K = |\Delta_K(-1)|$$

$$a_J(K) = \left\|V_K(t) - V_K(t^{-1})\right\|_1$$

$$r_{\mathrm{Kh}}(K) = \sum_{i,j} \operatorname{rank} \mathrm{Kh}^{i,j}(K)$$

$$\tau_{\mathrm{Kh}}(K) = \sum_{i,j} \operatorname{rank} \operatorname{Tor} \mathrm{Kh}^{i,j}(K)$$

$$\mathcal{R} : \mathfrak{M} \to \mathrm{Hyp}(P)$$

$$E_k(\rho_P; K) = \sum_{e \in E_K[k]} \omega_{K,e} \mathcal{Q}_k(\rho_e).$$

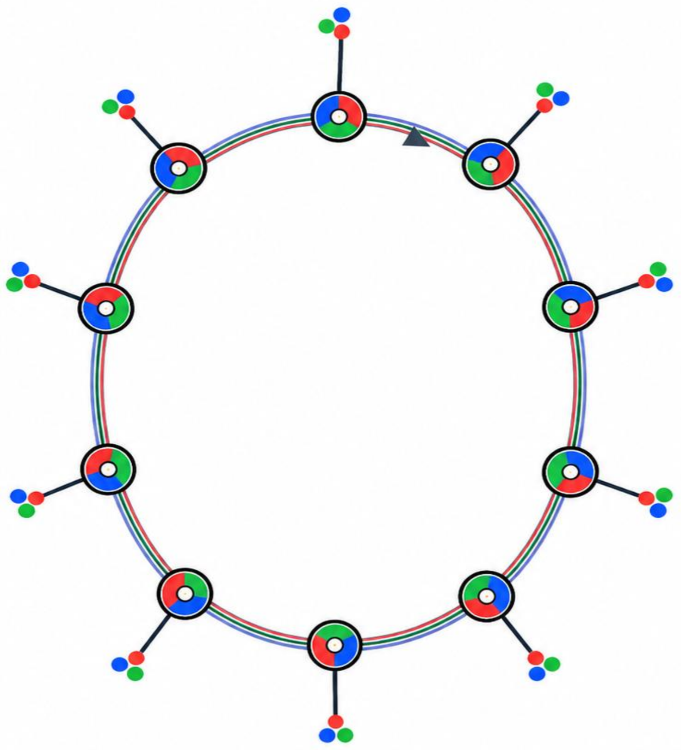
$$\omega_{K,e} = \frac{|\mathcal{Z}_1(K,e)|^2}{|\mathcal{Z}_0(e)|^2 + |\mathcal{Z}_1(K,e)|^2}$$

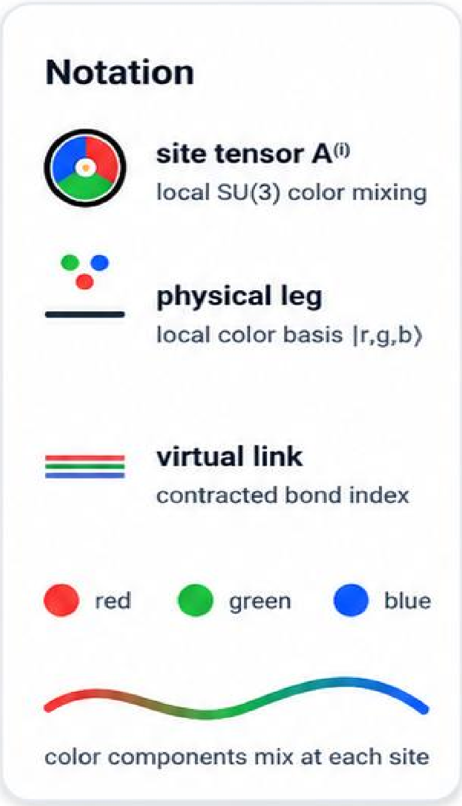
- **Thermalization rigorous condition in terms of entanglement:**

$$\rho_A(t) \xrightarrow{\|\cdot\|_1} \tau_A \quad \Longleftrightarrow \quad \mathfrak{L}_{R,A}(\rho_A(t)\|\tau_A) \longrightarrow 0$$

$$\left|S(\rho_A(t)) - E_R^{\mathrm{irr}}(A; \rho(t))\right| \le |\mathcal{C}_R(A;\tau) - \mathcal{I}_R(A;\tau)| + \delta_A(t)\log(d_A - 1) + h_2(\delta_A(t)) + \sum_a g_a(\delta_{e_a}(t)).$$

SU(3) link Quantum Simulation





Kogut–Susskind-type Hamiltonian has the structure

$$H_{\mathrm{KS}} = m\sum_i (-1)^i \psi_i^\dagger \psi_i - \kappa \sum_i \left(\psi_i^\dagger U_{i,i+1}\psi_{i+1} \mathrm{h.c.}\right)$$

$$\frac{g^2}{2}\sum_i \boldsymbol{E}_i^2 \frac{1}{2g^2}\sum_{\square}\left(2N_c - \mathrm{Tr}\, U_\square - \mathrm{Tr}\, U_\square^\dagger\right).$$

effective model:

$$H_{\mathrm{eff}} = m\sum_{i=0}^{N-1}(-1)^i n_i - w\sum_{i,c}\left(c_{i,c}^\dagger c_{i+1,c} + c_{i+1,c}^\dagger c_{i,c}\right) + J\sum_{i=0}^{N-2}\boldsymbol{Q}_i\cdot\boldsymbol{Q}_{i+1}$$

TEBD evolution gate： $\widetilde{\Theta}_{\alpha\gamma}^{s_i' s_{i+1}'} = \sum_{s_i, s_{i+1}} G_{s_i s_{i+1}}^{s_i' s_{i+1}'} \Theta_{\alpha\gamma}^{s_i s_{i+1}}$

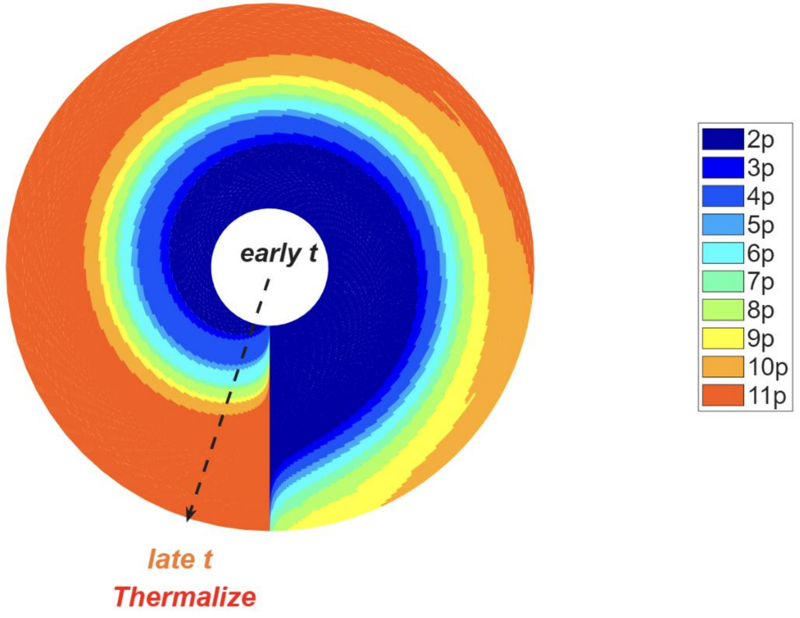
MPS writes the wavefunction as

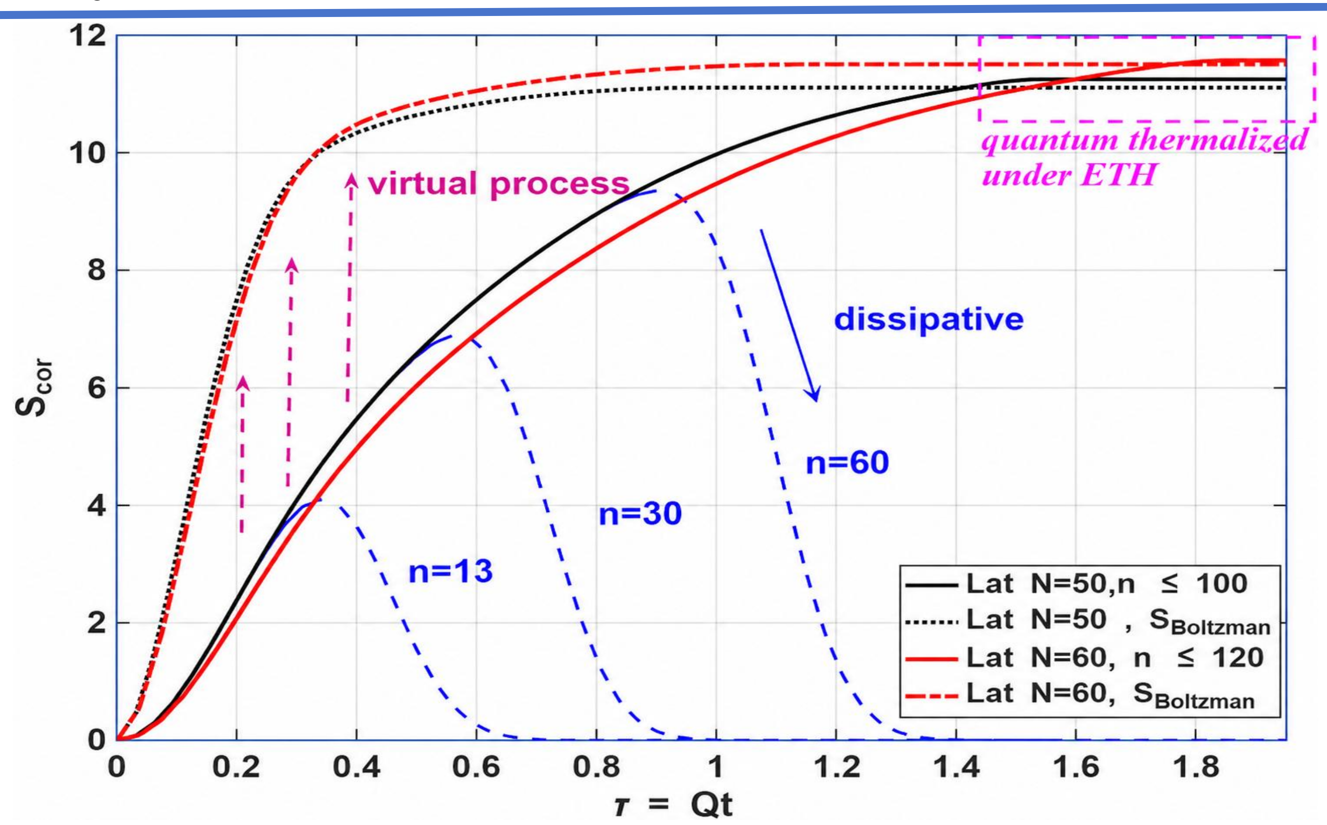
$$|\Psi\rangle = \sum_{s_0,\dots,s_{N-1}} A^{[0]s_0} A^{[1]s_1} \cdots A^{[N-1]s_{N-1}} |s_0 s_1 \cdots s_{N-1}\rangle \longrightarrow |\Psi'(+\Delta t)\rangle$$

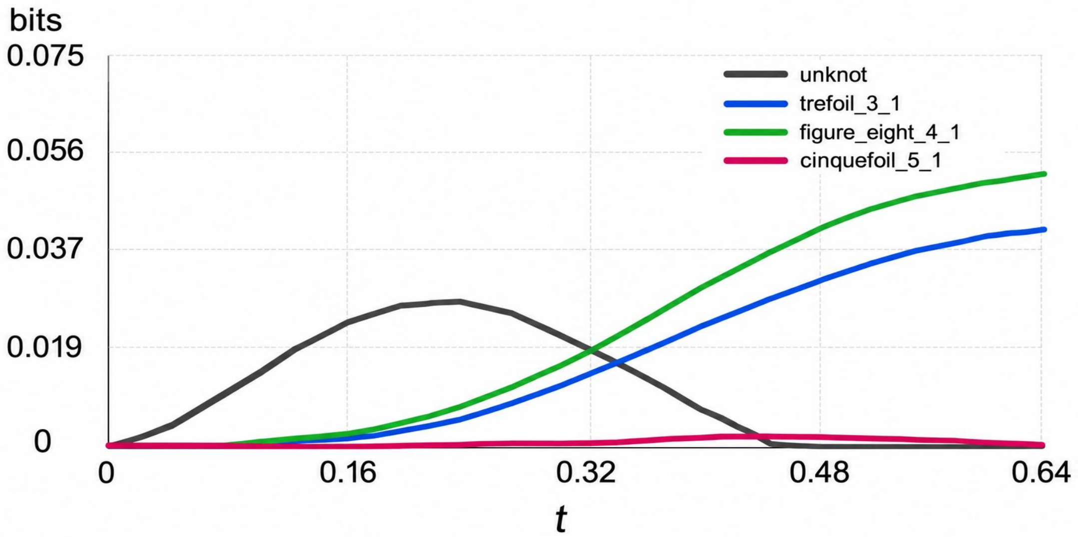
second-order Trotter step is

$$e^{-iH\Delta t} = e^{-iH_m \Delta t/2} e^{-iH_{\mathrm{even}}\Delta t} e^{-iH_{\mathrm{odd}}\Delta t} e^{-iH_m\Delta t/2} + O(\Delta t^3).$$

SU(3) link quantum simulation







- **Thermalization can be studied in quantum scrambling process, in terms of entropy evolution**.
- **The large energy and smaller physical mass, with shorter 1/Qs, corresponds to the further outer layers of the evolving pie chart.**

Summary

- **We Propose that *entanglement entropy* can provide a new perspective for the thermlization time problem in HIC.**

- Entanglement entropy can come from QCD topological sphaleron effects and parton scattering at the early stage of collisions.

- By quantum simulation, we find that quantum entanglement entropy can evolve to thermal entropy by quantum scrambling.

- **The time to fulfill scrambling is equivalent to the thermalization time, making it an effective tool to study thermalization.**