Improved Formula for Spin Polarization at Local Thermodynamic Equilibirum

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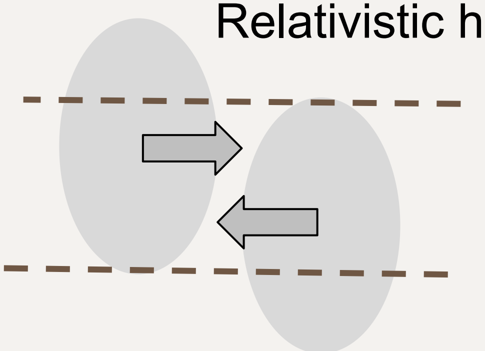
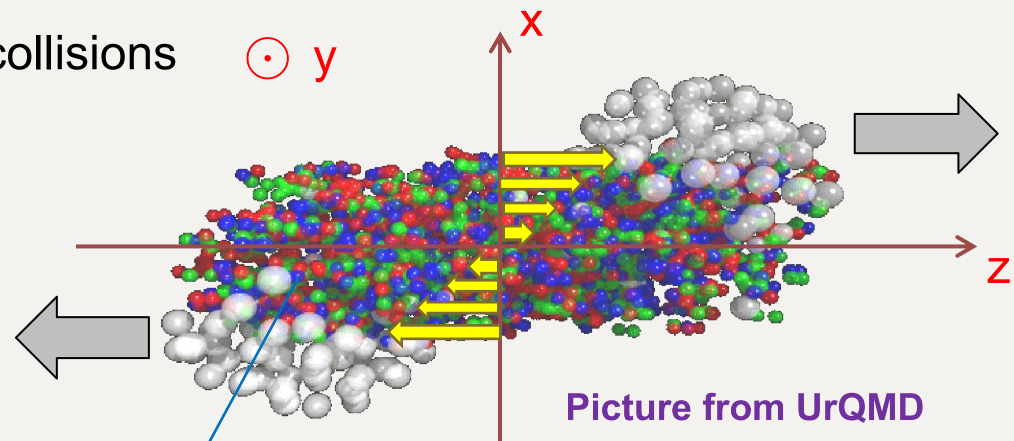
Voticity in QGP

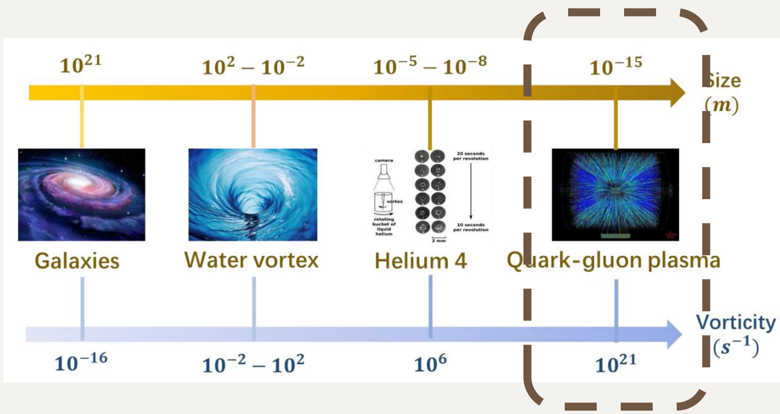
Relativistic heavy-ion collisions



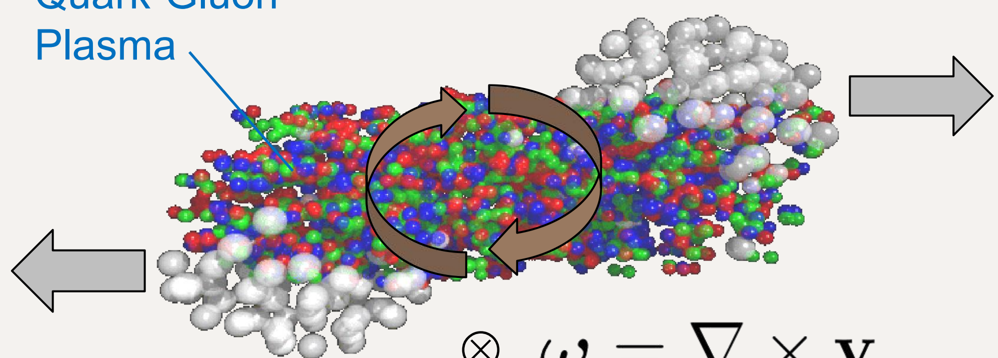


Picture from UrQMD

Quark-Gluon Plasma



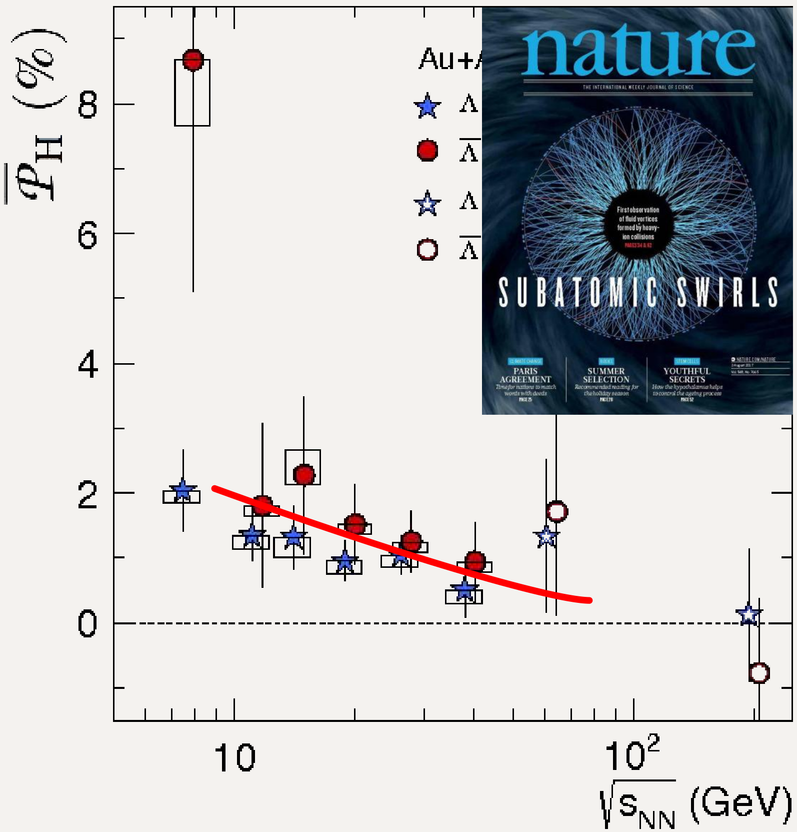
Most vortical field human ever made

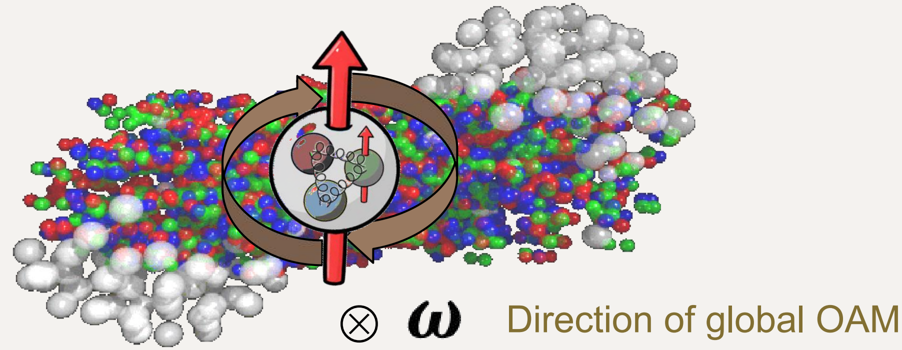


$\otimes\quad \boldsymbol{\omega} = \nabla \times \mathbf{v}$

Vorticity in QGP

Heavy-ion collisions





- Vorticity-induced polarization

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma d\Sigma_\lambda p^\lambda \varpi_{\rho\sigma} n_F(1-n_F)}{\int_\Sigma d\Sigma_\lambda p^\lambda n_F}$$

$$\varpi_{\mu\nu} \equiv \frac{1}{2}\left(\partial_\nu \beta_\mu - \partial_\mu \beta_\nu\right)$$

Evidence for vorticity field!

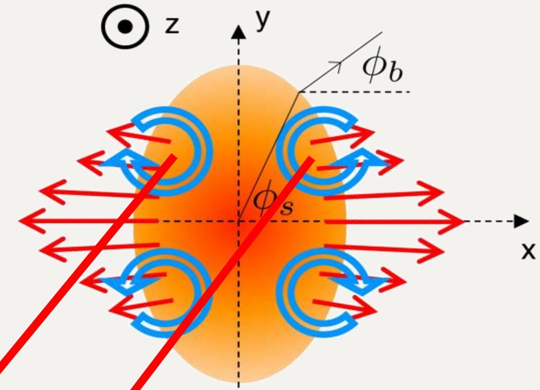
Related to:
Shi Pu's talk, July 18, 14:00
Ze-Fang Jiang's talk, July 18, 14:25
Yi-Liang Yin's talk, July 18, 15:10

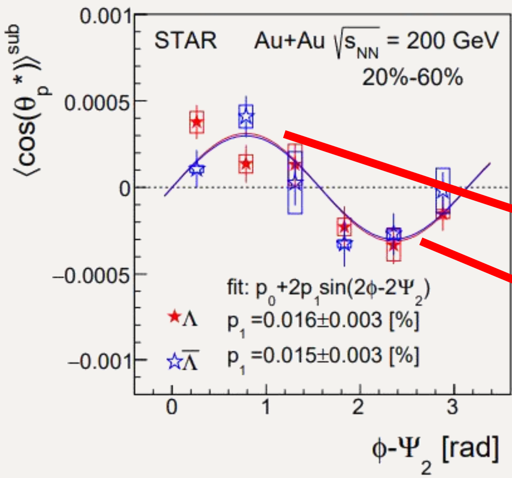
L. Adamczyk, et al. (STAR), Nature 548 (2017) 62.
S. A. Voloshin, nucl-th/0410089
Z.-T. Liang, X.-N. Wang, PRL 94, 102301 (2005) ; PLB 629, 20 (2005)
F. Becattini, F. Piccinini, J. Rizzo, PRC 76, 044901 (2007)
F. Becattini, M. Buzzegoli, T. Niida, S. Pu, A.-H. Tang, Int. J. Mod. Phys. E 33, 2430006 (2024)

Local spin polarization

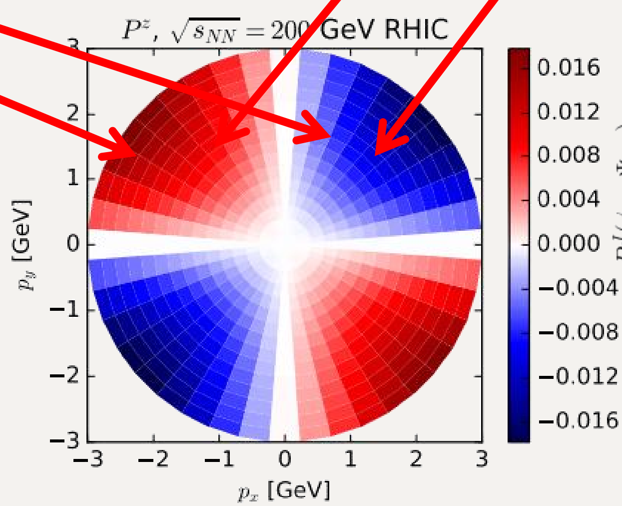
- Local polarization

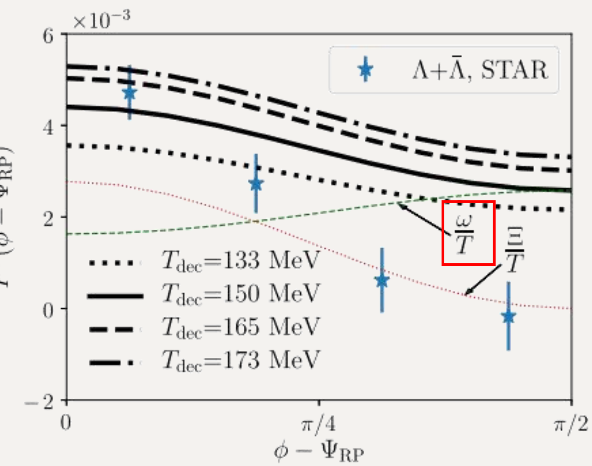
Momentum dependence of polarization
in beam or global OAM directions





Spin sign
puzzle!





Polarization in beam direction: experiments vs hydro simulations

Azimuthal-angle dep. of polarization in global OAM direction

STAR, Nucl. Phys. A 982, 511 (2019); Phys.Rev.Lett. 123 (2019) 13, 132301.
F. Becattini, I. Karpenko, Phys.Rev.Lett. 120 (2018) 1, 012302
F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, and A. Palermo, Phys. Rev. Lett. 127, 272302 (2021).
F. Becattini, M. Buzzegoli, T. Niida, S. Pu, A.-H. Tang, Int. J. Mod. Phys. E 33, 2430006 (2024)

Local spin polarization

- Shear-induced polarization $\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu\right)$

$$S^\mu_{\text{shear}}(p) \begin{cases} -\dfrac{1}{4mN_p}\epsilon^{\mu\rho\sigma\tau} p_\tau \displaystyle\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F) \dfrac{\hat{t}_\rho}{\hat{t}\cdot p} \xi_{\sigma\lambda} p^\lambda \\ -\dfrac{1}{4mN_p}\epsilon^{\mu\rho\sigma\tau} p_\tau \displaystyle\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F) \dfrac{u_\rho}{u\cdot p} \xi_{\sigma\lambda} p^\lambda \\ -\dfrac{1}{4mN_p}\epsilon^{\mu\rho\sigma\tau} p_\tau \displaystyle\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F) \dfrac{n_\rho}{n\cdot p} \xi_{\sigma\lambda} p^\lambda \end{cases}$$

F. Becattini, M. Buzzegoli, A. Palermo, Phys.Lett.B 820 (2021) 136519

S. Liu, Y. Yin, JHEP 07 (2021) 188

Y.-C. Liu, X.-G. Huang, Sci.China Phys.Mech.Astron. 65 (2022) 7, 272011;

- Unit vector in time direction, fluid velocity, or arbitrary timelike vector?
- Additional assumptions:
  Isothermal local equilibrium - neglecting *T*-gradient
  “Strange memory” scenario - using quark mass instead of hyperon mass
- Non-dissipative or dissipative?
  J.-R. Wang, S. Fang, D.-L. Yang, S. Pu, Phys.Rev.D 113 (2026) 11, 114026

Related to:
Jia-Rong Wang’s poster 3-14

- In this talk:
  Normal vector of hypersurface, automatical vanishing *T*-gradient

Zubarev approach

- Mean value at LTE

$$O_{\mathrm{LE}}(x) \equiv \mathrm{Tr}\left(\widehat{\rho}\, \widehat{O}(x)\right) \qquad \widehat{\rho} = \frac{1}{Z} \exp\left[-\int_{\Sigma_{\mathrm{D}}} \mathrm{d}\Sigma_\mu(y)\, \widehat{T}^{\mu\nu}(y)\beta_\nu(y)\right]$$

- Expansion around global equilibrium

Keeps all orders in gradients

$$\beta_\nu(y) = \beta_\nu(x) + \Delta\beta_\nu(y,x) \qquad \Delta\beta_\nu \approx (y-x)^\lambda \partial_\lambda \beta_\nu(x) + \cdots$$

$$O_{\mathrm{LE}}(x) = \frac{1}{Z}\mathrm{Tr}\left(\exp\left[-\beta(x)\cdot\widehat{P} - \int_{\Sigma_{\mathrm{D}}} \mathrm{d}\Sigma_\mu(y)\, \widehat{T}^{\mu\nu}(y)\Delta\beta_\nu(y)\right] \widehat{O}(x)\right)$$

Hydrodynamic limit: β is slowly varing

$\Delta\beta \ll \beta$

$$O_{\mathrm{GE}}(x) \equiv \frac{\mathrm{Tr}[e^{-\beta(x)\cdot\widehat{P}}\widehat{O}(x)]}{\mathrm{Tr}[e^{-\beta(x)\cdot\widehat{P}}]}$$

$$\Delta O_{\mathrm{LE}}(x) = -\int_{\Sigma_{\mathrm{D}}} \mathrm{d}\Sigma_\mu(y) \int_0^1 \mathrm{d}z\, \Delta\beta_\nu(y,x) \times \left\langle \widehat{O}(x),\, \mathrm{e}^{-z\beta(x)\cdot\widehat{P}}\widehat{T}^{\mu\nu}(y)\mathrm{e}^{z\beta(x)\cdot\widehat{P}} \right\rangle_{c,\mathrm{GE}}$$

GTE mean value

Linear response to $\Delta\beta$

D. N. Zubarev, Soviet Physics Doklady 10, 850 (1966).
D. N. Zubarev, A. V. Prozorkevich, and S. A. Smolyanskii, Theor. Math. Phys. 40, 821 (1979).
D. N. Zubarev and M. V. Tokarchuk, Teor. Mat. Fiz. 88N2, 286 (1991).
C. van Weert, Annals of Physics 140, 133 (1982).
F. Becattini, M. Buzzegoli, and E. Grossi, Particles 2, 197 (2019).
XLS, F. Becattini, D. Roselli, arXiv: 2509.14301

Wigner function

- Wigner operator for spin-1/2 fermions

$$\widehat{W}^{+}_{ab}(x,p) \equiv \theta(p^0)\int \frac{\mathrm{d}^4 y}{(2\pi)^4}\mathrm{e}^{-ip\cdot y}\overline{\Psi}_b\left(x+\frac{y}{2}\right)\Psi_a\left(x-\frac{y}{2}\right)$$

- Average spin polarization

$$S^\mu(p) = \frac{\int_{\Sigma_\mathrm{D}}\mathrm{d}\Sigma\cdot p\,\mathrm{tr}\left[\gamma^\mu\gamma^5 W^+(x,p)\right]}{2\int_{\Sigma_\mathrm{D}}\mathrm{d}\Sigma\cdot p\,\mathrm{tr}\left[W^+(x,p)\right]}$$

- Linear response to $\Delta\beta$

$$\begin{aligned}\Delta W^{+}_{\mathrm{LE}}(x,p) =& \frac{1}{(2\pi)^3}\int \mathrm{d}^4 q\,\delta(p\cdot q)\,G^{\mu\nu}(q)\,F_{\mu\nu}(q)\\ =& \frac{1}{(2\pi)^3}\int \mathrm{d}^4 q\,\delta(p\cdot q)\,\frac{1}{2}\delta\left(p^2+\frac{q^2}{4}-m^2\right)\theta(p^0_+)\theta(p^0_-)\\ &\times\frac{1-e^{\beta(x)\cdot q}}{\beta(x)\cdot q}\sum_{r,s}u_s(p_-)\bar{u}_r(p_+)\left\langle \widehat{a}^\dagger_r(p_+)\widehat{a}_s(p_-),\,\widehat{T}^{\mu\nu}\right\rangle_{c,\mathrm{GE}}\\ &\times\int_{\Sigma_\mathrm{D}}\mathrm{d}\Sigma_\mu(y)\,e^{-iq\cdot(y-x)}\Delta\beta_\nu(y,x)\end{aligned}$$

In hydrodynamic limit: $\Delta\beta$ is slowly varing, $F_{\mu\nu}$ is peaked at $q=0$

- Small-q expansion

$$G^{\mu\nu}(q) = \sum_{N=0}^{\infty}\frac{1}{N!}\left[\partial^q_{\nu_1}\partial^q_{\nu_2}\cdots\partial^q_{\alpha_N}G^{\mu\nu}(q)\right]\Big|_{q^\mu=0}q^{\nu_1}q^{\nu_2}\cdots q^{\nu_N}$$

$$\int \mathrm{d}^4 q\,\delta(p\cdot q)\mathrm{e}^{-iq\cdot(y-x)}q^{\nu_1}\cdots q^{\nu_N} = (2\pi)^3\frac{(-i)^N}{|p^0|}\partial_x^{\nu_1}\cdots\partial_x^{\nu_N}\delta^3\left(\mathbf{y}-\mathbf{x}-\frac{\mathbf{p}}{p^0}(y^0-x^0)\right)$$

XLS, F. Becattini, D. Roselli, arXiv: 2509.14301

Wigner function

- Linear response of LTE Wigner function

$$\Delta W^{+}_{\rm LE}(x,p) \;=\; \sum_{N=0}^{\infty}\sum_{\bar{y}(x,p)} \frac{1}{N!} D_y^N(\bar{y}) \left[G^{\mu\nu}(q) \frac{n_\mu(y)}{|p\cdot n(y)|} \Delta\beta_\nu(y,x) \right] \Bigg|_{q=0,y=\bar{y}(x,p)}$$

order in gradient

② ①

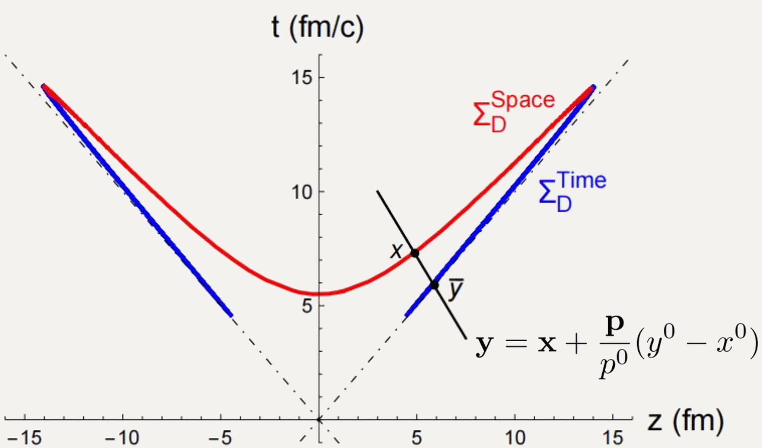
$$D_y(\bar{y}) \equiv -i\Delta^{\nu\rho}(\bar{y})\partial^y_\rho \partial^q_\nu \qquad \Delta^{\nu\rho}(\bar{y}) = g^{\nu\rho} - \frac{n^\nu(\bar{y})p^\rho}{p\cdot n(\bar{y})}$$

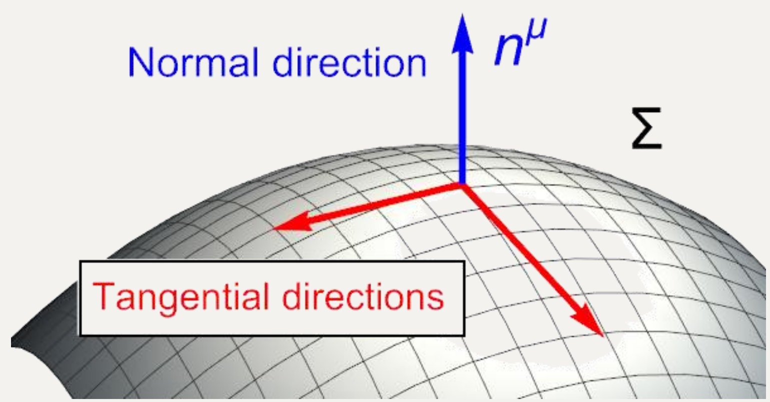
Derivatives in tangential directions

- Convergence conditions

$$l_c/L_H \ll 1, \quad l_c/L_G \ll 1\,.$$

- Sum over all intersections





Spin polarization

- Average spin polarization at leading order in gradient

$$S^\mu(p) \simeq -\frac{1}{8mN_p}\epsilon^{\mu\nu\rho\lambda}p_\nu \int_{\Sigma_D} d\Sigma(x)\cdot p\, n_F(x,p)[1-n_F(x,p)]$$

$$\times \sum_{\bar{y}(x,p)} \mathrm{sgn}[p\cdot n(\bar{y})] \left[\varpi_{\rho\lambda}(\bar{y}) + 2\frac{n_\rho(\bar{y})}{p\cdot n(\bar{y})}\xi_{\lambda\alpha}(\bar{y})p^\alpha\right]$$

① ② thermal vorticity ③ thermal shear

- Differences with previous results in literature

① Long-distance correlation

$$\Delta W^+_{\mathrm{LE}}(x,p) = \frac{1}{(2\pi)^3}\int_{\Sigma_D} d\Sigma_\mu(y)\Delta\beta_\nu(y,x)\Big[\underbrace{\int d^4q\, \delta(p\cdot q)e^{iq\cdot(x-y)}G^{\mu\nu}(q)}_{\int_0^1 dz\, \langle \widehat{W}^+(x,k), e^{z\widehat{A}}\widehat{T}^{\mu\nu}(y)e^{-z\widehat{A}}\rangle_{c,\mathrm{GE}}}\Big]$$

Large correlation between *x* and any point on $\mathbf{y} = \mathbf{x} - \frac{\mathbf{p}}{p^0}(y^0 - x^0)$

② Additional sign factor $\mathrm{sgn}[p\cdot n(\bar{y})]$

No difference if hypersurface is purely spacelike

XLS, F. Becattini, D. Roselli, arXiv: 2509.14301

Spin polarization

- Average spin polarization at leading order in gradient

$$S^\mu(p) \simeq -\frac{1}{8mN_p}\epsilon^{\mu\nu\rho\lambda}p_\nu \int_{\Sigma_{\rm D}} \mathrm{d}\Sigma(x)\cdot p\, n_F(x,p)[1-n_F(x,p)]$$

$$\times \underbrace{\sum_{\bar{y}(x,p)}}_{①} \underbrace{\mathrm{sgn}[p\cdot n(\bar{y})]}_{②} \left[\underbrace{\varpi_{\rho\lambda}(\bar{y})}_{\text{thermal vorticity}} + \underbrace{2\frac{n_\rho(\bar{y})}{p\cdot n(\bar{y})}}_{③}\underbrace{\xi_{\lambda\alpha}(\bar{y})p^\alpha}_{\text{thermal shear}}\right]$$

- Differences with previous results in literature

③ Normal vector of hypersurface n

XLS, F. Becattini, D. Roselli, arXiv: 2509.14301 (this talk)

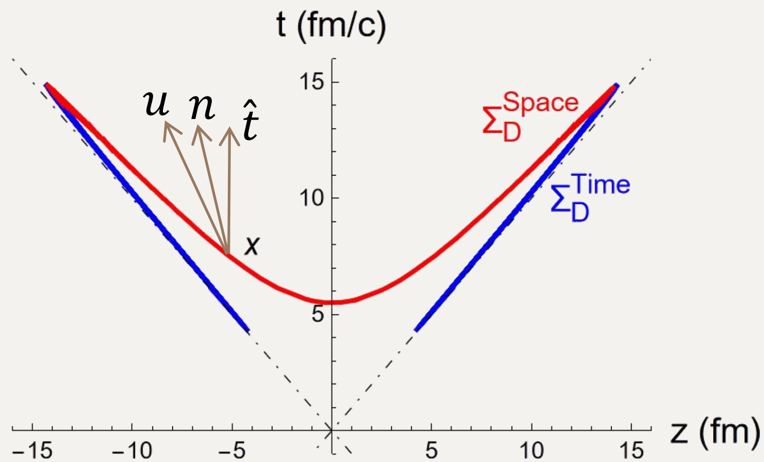
Unit vector in time direction $\hat{t}$

F. Becattini, M. Buzzegoli, A. Palermo, Phys.Lett.B 820 (2021) 136519

Fluid velocity u

S. Liu, Y. Yin, JHEP 07 (2021) 188

$n{\sim}\hat{t}{\sim}u$ at center region



Temperature gradient

- T-gradient in thermal vorticity/shear tensor

$$\varpi^{\mu\nu} = -\frac{1}{2T}(\partial^\mu u^\nu - \partial^\nu u^\mu) + \frac{1}{2T^2}(u^\nu \partial^\mu T - u^\mu \partial^\nu T)$$
$$\xi^{\mu\nu} = \frac{1}{2T}(\partial^\mu u^\nu + \partial^\nu u^\mu) - \frac{1}{2T^2}(u^\nu \partial^\mu T + u^\mu \partial^\nu T)\,.$$

- Spin polarization induced by T-gradient

$$S^\mu_{\text{T-grad}}(p) = -\frac{1}{8mN_p}\int_{\Sigma_D} \mathrm{d}\Sigma(x)\cdot p\, n_F(x,p)[1-n_F(x,p)]\epsilon^{\mu\nu\rho\lambda}p_\nu$$
$$\times \sum_{\bar{y}(x,p)} \mathrm{sgn}[p\cdot n]\frac{1}{T^2}\left\{u_\lambda\left[\partial_\rho T - n_\rho \frac{p\cdot\partial T}{p\cdot n}\right] - \frac{p\cdot u}{p\cdot n} n_\rho \partial_\lambda T]\right\}\Bigg|_{\bar{y}(x,p)}$$

- For hypersurface with T=const.

$$\partial_\mu T \propto n_\mu \Longrightarrow S^\mu_{\text{T-grad}}(p) = 0$$

★ A more general conclusion: Wigner function does not receive any contribution from T-gradient

"Iso-thermal approximation"

LTE density matrix does not contain any information beyond the hypersurface

F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, Phys.Rev.Lett. 127 (2021) 27, 272302

Conclusions

- An improved formula for Wigner function and spin polarization at local thermodynamic equilibrium

- Gradients of normal vector of decoupling hypersurface (curvature, gradient of curvature, ...)

  For spin polarization, such terms vanish at leading order

$$S^\mu(p) \simeq -\frac{1}{8mN_p}\int_{\Sigma_D} d\Sigma(x)\cdot p\,\mathrm{sgn}[p\cdot n(x)]n_F(1-n_F)$$
$$\times\epsilon^{\mu\nu\rho\lambda}p_\nu\left\{\varpi_{\rho\lambda}(x)+\frac{2n_\rho(x)}{p\cdot n(x)}\left[\xi_{\lambda\alpha}(x)p^\alpha-\partial_\lambda^x\zeta(x)\right]\right\}$$

- For shear-induced polarization, vector n instead of $\hat{t}$ or u, automatically excludes T-gradient

  Needs numerical tests in hydro simulations

Thanks for your attention!