Gravitational Waves from Graviton Bremsstrahlung in Scalar Leptoquark Decays

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https://yzhxxzxy.github.io

Based on Qian-Jiu Wang, Hua Tong, ZHY, arXiv:2606.28699

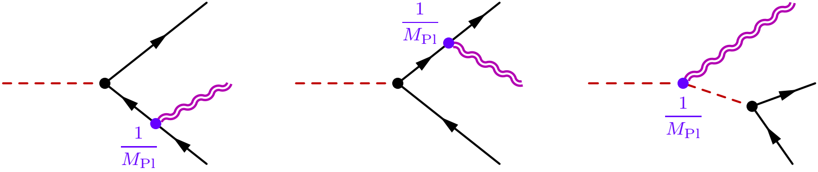
中国物理学会高能物理分会

第十二届全国会员代表大会暨学术年会

July 16, 2026, Guangzhou

Graviton Bremsstrahlung

- **Gravitational waves** (GW) now serve as a new cosmic messenger for exploring **new physics** beyond the **standard model** (SM)
- **New physics** typically predicts **heavy particles** produced in the early universe
- **Their decays** are expected to be accompanied by **graviton bremsstrahlung** via **quantum gravitational processes**, thereby inducing a potentially detectable **stochastic GW background** (SGWB) [Nakayama & Y Tang, 1810.04975, PLB]
- To produce an **appreciable SGWB** for GW observatories, the **parent particle** must be **sufficiently heavy** to **overcome** the **suppression** of the **Planck scale** $M_{\rm Pl} = 2.4 \times 10^{18}$ GeV



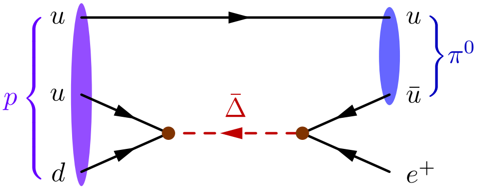
Scalar Leptoquarks

🎏 We focus on **scalar leptoquarks**, which are scalar bosons **simultaneously coupling** to a **quark** and a **lepton**, as predicted by **grand unified theories** (GUTs)

🔮 Interactions of **scalar leptoquarks violate** both **lepton** and **baryon numbers**, triggering **proton decay**

🪁 Stringent experimental limits on **proton decay** have pushed their **lower mass bounds** to $\mathcal{O}(10^{11})$–$\mathcal{O}(10^{13})$ GeV [Dorsner, Fajfer & Kosnik, 1204.0674, PRD]

🎃 Therefore, **graviton bremsstrahlung** from **decays** of such **superheavy scalar leptoquarks** offers a promising source for an **SGWB**



SU(5) Setup

As an illustrative case study, we consider the **scalar leptoquarks** Δ_a $(a = 1, 2, 3)$ from the $\mathbf{5}_\mathrm{H}$ **Higgs multiplet** in the $\mathrm{SU}(5)$ **GUT** [Georgi & Glashow, PRL **32** (1974) 438]:

$$(\mathbf{5}_\mathrm{H})^\alpha = (\Delta_1, \Delta_2, \Delta_3, H_1, H_2) = \Delta(\mathbf{3}, \mathbf{1}, -1/3) \oplus H(\mathbf{1}, \mathbf{2}, 1/2), \quad \alpha = 1, 2, 3, 4, 5$$

The vacuum expectation value (VEV) of a $\mathbf{24}_\mathrm{H}$ **Higgs multiplet** at the **GUT scale** $\sim \mathcal{O}(10^{15})$ **GeV** leads to symmetry breaking $\mathrm{SU}(5) \to \mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$

The VEVs the $\mathbf{5}_\mathrm{H}$ and $\mathbf{45}_\mathrm{H}$, v_5 and v_{45}, at the **electroweak scale** result in symmetry breaking $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \to \mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{EM}$

The **effective electroweak VEV** is given by $v \equiv \sqrt{v_5^2 + 24 v_{45}^2} = 246.22$ GeV

Three families of **SM fermions** live in

$$\begin{aligned}
(\mathbf{5}_{\mathrm{F},i})^\alpha &= (d_{1i}, d_{2i}, d_{3i}, \ell_i^\mathrm{C}, -\nu_i^\mathrm{C})_\mathrm{R} = d_{i\mathrm{R}}(\mathbf{3}, \mathbf{1}, -1/3) \oplus (L_{i\mathrm{L}})^\mathrm{C}(\mathbf{1}, \mathbf{2}, 1/2) \\
(\mathbf{10}_{\mathrm{F},i})^{\alpha\beta} &= (u_{i\mathrm{R}})^\mathrm{C}(\bar{\mathbf{3}}, \mathbf{1}, -2/3) \oplus Q_{i\mathrm{L}}(\mathbf{3}, \mathbf{2}, 1/6) \oplus (\ell_{i\mathrm{R}})^\mathrm{C}(\mathbf{1}, \mathbf{1}, 1), \quad i = 1, 2, 3
\end{aligned}$$

Three **right-handed neutrinos** $\mathbf{1}_{\mathrm{F},i} = N_{i\mathrm{R}}(\mathbf{1}, \mathbf{1}, 0)$ are introduced to realize the **type-I seesaw mechanism**

Yukawa couplings and Mass Origin

🌱 Renormalizable **Yukawa couplings** among $\mathbf{5}_{\mathrm{H}}$, $\mathbf{45}_{\mathrm{H}}$, $\mathbf{5}_{\mathrm{F},i}$, $\mathbf{10}_{\mathrm{F},i}$, and $\mathbf{1}_{\mathrm{F},i}$, together with the **Majorara mass terms** for $\mathbf{1}_{\mathrm{F},i}$, are given by

$$\begin{aligned}\mathcal{L} \supset & -Y^{10}_{ij}\varepsilon_{\alpha\beta\gamma\delta\epsilon}(\overline{\mathbf{10}^{\mathrm{C}}_{\mathrm{F},i}})^{\alpha\beta}(\mathbf{10}_{\mathrm{F},j})^{\gamma\delta}(\mathbf{5}_{\mathrm{H}})^{\epsilon} + Y^{5}_{ij}(\overline{\mathbf{5}_{\mathrm{F},j}})_{\alpha}(\mathbf{10}_{\mathrm{F},i})^{\alpha\beta}(\mathbf{5}^{*}_{\mathrm{H}})_{\beta}\\ & + Y^{45}_{ij}(\overline{\mathbf{5}_{\mathrm{F},j}})_{\gamma}(\mathbf{10}_{\mathrm{F},i})^{\alpha\beta}(\mathbf{45}^{*}_{\mathrm{H}})^{\gamma}_{\alpha\beta} + \tilde{Y}^{45}_{ij}\varepsilon_{\alpha\beta\gamma\delta\epsilon}(\overline{\mathbf{10}^{\mathrm{C}}_{\mathrm{F},i}})^{\alpha\beta}(\mathbf{10}_{\mathrm{F},j})^{\zeta\gamma}(\mathbf{45}_{\mathrm{H}})^{\delta\varepsilon}_{\zeta}\\ & + Y^{1}_{ij}(\overline{\mathbf{5}_{\mathrm{F},i}})_{\alpha}\mathbf{1}^{\mathrm{C}}_{\mathrm{F},j}(\mathbf{5}_{\mathrm{H}})^{\alpha} - M_{N,ij}\overline{\mathbf{1}^{\mathrm{C}}_{\mathrm{F},i}}\mathbf{1}_{\mathrm{F},j} + \mathrm{H.c.}\end{aligned}$$

🌾 The initially massless fermions acquire **masses** after $\mathbf{5}_{\mathrm{H}}$ and $\mathbf{45}_{\mathrm{H}}$ develop **VEVs**

🌽 We assume that the Yukawa coupling matrices Y^{5}, Y^{45}, and $\tilde{Y}^{45}$ are **symmetric**, and the fermionic mass matrices can be diagonalized by **unitary** transformations

$$\begin{aligned} & u^{m}_{i\mathrm{L}} \equiv (U^{\dagger}_{u})_{ij}u_{j\mathrm{L}}, \quad u^{m}_{i\mathrm{R}} \equiv (U^{\mathrm{T}}_{u})_{ij}u_{j\mathrm{R}}, \quad d^{m}_{i\mathrm{L}} \equiv (U^{\dagger}_{d})_{ij}d_{j\mathrm{L}}\\ & d^{m}_{i\mathrm{R}} \equiv (U^{\mathrm{T}}_{d})_{ij}d_{j\mathrm{R}}, \quad \ell^{m}_{i\mathrm{L}} \equiv (U^{\dagger}_{\ell})_{ij}\ell_{j\mathrm{L}}, \quad \ell^{m}_{i\mathrm{R}} \equiv (U^{\mathrm{T}}_{\ell})_{ij}\ell_{j\mathrm{R}}\\ & (\nu^{\mathrm{m}}_{\mathrm{L}})^{\mathrm{C}} \equiv U^{\mathrm{T}}_{\nu}(\nu_{\mathrm{L}})^{\mathrm{C}}, \quad N^{\mathrm{m}}_{\mathrm{R}} \equiv U^{\mathrm{T}}_{N}N_{\mathrm{R}}\end{aligned}$$

🍿 The **CKM** and **PMNS** matrices are defined as $V_{\mathrm{CKM}} \equiv U^{\dagger}_{u}U_{d}$ and $V_{\mathrm{PMNS}} \equiv U^{\dagger}_{\ell}U_{\nu}$

Proton Decay

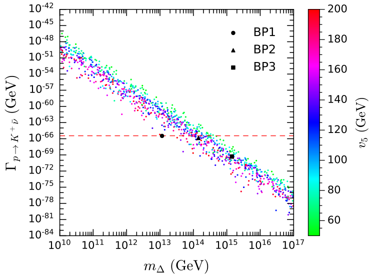
- The **Yukawa couplings** of the **scalar leptoquarks** Δ_a with **SM fermions** are

$$\mathcal{L} \supset -2(Y_{ij}^{10} + Y_{ji}^{10})\overline{\ell_{i\mathrm{R}}}(u_{aj\mathrm{R}})^{\mathrm{C}}\Delta_a + 2(Y_{ij}^{10} + Y_{ji}^{10})\varepsilon_{abc}\overline{(d_{ai\mathrm{L}})^{\mathrm{C}}}u_{bj\mathrm{L}}\Delta_c$$
$$+\frac{Y_{ij}^5}{\sqrt{2}}\varepsilon_{abc}\overline{u_{ai\mathrm{R}}}(d_{bj\mathrm{R}})^{\mathrm{C}}\Delta_c^* + \frac{Y_{ij}^5}{\sqrt{2}}\overline{(u_{ai\mathrm{L}})^{\mathrm{C}}}\ell_{j\mathrm{L}}\Delta_a^* - \frac{Y_{ij}^5}{\sqrt{2}}\overline{(d_{ai\mathrm{L}})^{\mathrm{C}}}\nu_{j\mathrm{L}}\Delta_a^* + \mathrm{H.c.}$$

with $Y^{10\dagger} + Y^{10*} = \frac{1}{\sqrt{2}v_5}U_u\tilde{M}_uU_u^{\mathrm{T}}$, $Y^{5*} = \frac{1}{2v_5}(3U_d\tilde{M}_dU_d^{\mathrm{T}} + U_\ell\tilde{M}_\ell U_\ell^{\mathrm{T}})$ and

$$\tilde{M}_u = \mathrm{diag}(m_u, m_c, m_t), \quad \tilde{M}_d = \mathrm{diag}(m_d, m_s, m_b), \quad \tilde{M}_\ell = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

- These couplings lead to **proton decay**
- Through a **random scan** in the parameter space, we find that the **strongest** constraint arises from the $p \to K^+\bar{\nu}$ channel, which **excludes** parameter points with the **scalar leptoquark mass** $m_\Delta \lesssim 10^{13}$ GeV



- Three **benchmark points** (BPs) with $m_\Delta = 1.15 \times 10^{13}, 1.41 \times 10^{14}, 1.42 \times 10^{15}$ GeV are selected for further study

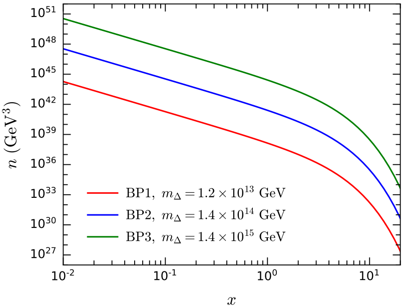
Number Density of Scalar Leptoquarks

🚗 The evolution of the **number density** of Δ_a **particles**, n_{Δ_a}, in the early universe is governed by the Boltzmann equation with **annihilation**, **coannihilation**, and **decay**:

$$\frac{\mathrm{d}n_{\Delta_a}}{\mathrm{d}t} + 3Hn_{\Delta_a} = -\left(\sum_b \langle\sigma_{\Delta_a\bar{\Delta}_b}v\rangle + \sum_{c\neq a}\langle\sigma_{\Delta_a\Delta_c}v\rangle\right)(n_{\Delta_a}^2 - n_{\Delta_a,\mathrm{eq}}^2) - \langle\Gamma_{\Delta_a}\rangle\,(n_{\Delta_a} - n_{\Delta_a,\mathrm{eq}})$$

🚌 We assume that after reheating, the scalar leptoquarks achieve **thermal equilibrium** at temperatures $T \gtrsim 10^2 m_\Delta$

🚚 The obtained **total number density** of scalar leptoquarks $n = \sum_{a=1}^{3}(n_{\Delta_a} + n_{\bar{\Delta}_a})$ closely follows its **equilibrium value**



$(x \equiv m_\Delta/T)$

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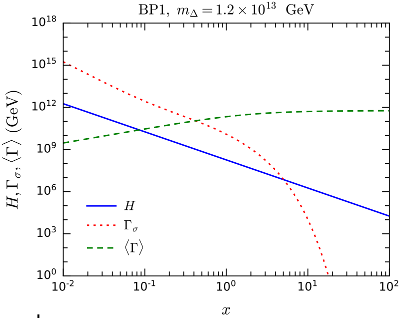
🚒 This is because either the **total annihilation and coannihilation rate** Γ_σ or the **total decay rate** $\langle\Gamma\rangle$ is **larger** than the **Hubble rate** H throughout the relevant epochs



$(x \equiv m_\Delta/T)$

Graviton Bremsstrahlung

Within the **effective quantum field theory** based on **Einstein gravity**, **differential decay rates** for **graviton bremsstrahlung** in **scalar leptoquark decays** are

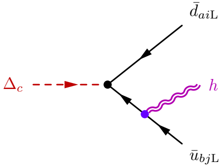
$$\frac{\mathrm{d}\Gamma_{\Delta_c\to\bar{d}_{ai\mathrm{L}}\bar{u}_{bj\mathrm{L}}h}}{\mathrm{d}E} = \frac{\varepsilon_{abc}^2|Y_{ij}^{10}+Y_{ji}^{10}|^2 m_\Delta^2}{96\pi^3 M_{\mathrm{Pl}}^2} f\left(\frac{E}{m_\Delta}\right)$$

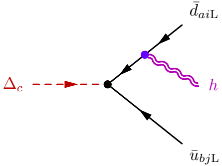
$$\frac{\mathrm{d}\Gamma_{\Delta_c\to\bar{d}_{bi\mathrm{R}}\bar{u}_{aj\mathrm{R}}h}}{\mathrm{d}E} = \frac{\varepsilon_{abc}^2|Y_{ij}^{5}|^2 m_\Delta^2}{768\pi^3 M_{\mathrm{Pl}}^2} f\left(\frac{E}{m_\Delta}\right)$$

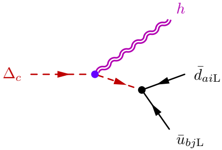
$$\frac{\mathrm{d}\Gamma_{\Delta_a\to\ell_{i\mathrm{R}}u_{aj\mathrm{R}}h}}{\mathrm{d}E} = \frac{|Y_{ij}^{10}+Y_{ji}^{10}|^2 m_\Delta^2}{96\pi^3 M_{\mathrm{Pl}}^2} f\left(\frac{E}{m_\Delta}\right)$$

$$\frac{\mathrm{d}\Gamma_{\Delta_a\to\ell_{i\mathrm{L}}u_{aj\mathrm{L}}h}}{\mathrm{d}E} = \frac{\mathrm{d}\Gamma_{\Delta_a\to\nu_{i\mathrm{L}}d_{aj\mathrm{L}}h}}{\mathrm{d}E} = \frac{|Y_{ij}^{5}|^2 m_\Delta^2}{768\pi^3 M_{\mathrm{Pl}}^2} f\left(\frac{E}{m_\Delta}\right)$$

$$f(x) \equiv \frac{1-2x}{x}(1-2x+2x^2)$$







E is the **energy** of the **bremsstrahlung graviton** h

SGWB Spectra

The incoherent superposition of **bremsstrahlung gravitons** emitted from **scalar leptoquark decays** forms an **SGWB**

The **differential SGWB energy density** with respect to the **present graviton energy** $E_0 = E(t)a(t)$ is estimated as

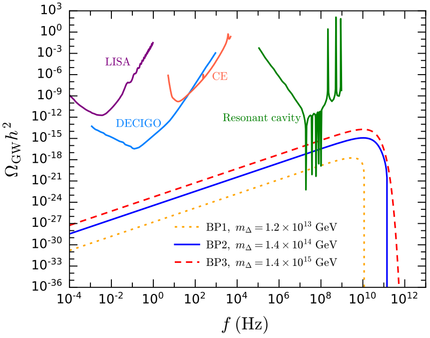
$$\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}E_0} = E_0 \int \mathrm{d}t\, n(t)\, a^2(t)\, \frac{\mathrm{d}\Gamma_h}{\mathrm{d}E}$$

$a(t)$ is the **cosmological scale factor**

$\mathrm{d}\Gamma_h/\mathrm{d}E$ is the **total differential decay rates** for **graviton bremsstrahlung**

The **dimensionless SGWB spectrum** is

$$\Omega_{\mathrm{GW}}(f) \equiv \frac{f}{\rho_{\mathrm{c}}} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f} = \frac{E_0}{\rho_{\mathrm{c}}} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}E_0}$$



High-frequency GW detectors employing **resonant cavity techniques** [Herman, Lehoucq & Fúzfa, 2203.15668, PRD] are **promising** for detecting the signals

Summary

- We study the **SGWB** originated from **graviton bremsstrahlung** in **decays** of **scalar leptoquarks** based on SU(5) **GUT**
- Stringent experimental bounds on **proton decay** force the **scalar leptoquark mass** $\gtrsim 10^{13}$ GeV, which in turn renders their **graviton bremsstrahlung**, induced by **quantum gravity effects**, **less suppressed**
- We find that **high-frequency GW detectors** employing **resonant cavity techniques** offer a **promising** means to probe the **SGWB signal**

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Thanks for your attention!

Fermion Masses

🍞 After $\mathbf{5}_\mathrm{H}$ and $\mathbf{45}_\mathrm{H}$ acquire their **VEVs**, the **mass terms** for **fermions** become

$$\begin{aligned}\mathcal{L} \supset\ & -M_{u,ij}\overline{u_{i\mathrm{L}}}u_{j\mathrm{R}} - M_{d,ij}\overline{d_{i\mathrm{L}}}d_{j\mathrm{R}} - M_{\ell,ij}\overline{\ell_{i\mathrm{L}}}\ell_{j\mathrm{R}} \\ & -\frac{1}{2}\begin{pmatrix}\overline{\nu_\mathrm{L}} & \overline{(N_\mathrm{R})^\mathrm{C}}\end{pmatrix}\begin{pmatrix}\mathbf{0} & M_\mathrm{D} \\ M_\mathrm{D}^\mathrm{T} & M_N\end{pmatrix}\begin{pmatrix}(\nu_\mathrm{L})^\mathrm{C} \\ N_\mathrm{R}\end{pmatrix} + \mathrm{H.c.}\end{aligned}$$

$$M_{u,ij} \equiv \sqrt{2}v_5(Y_{ji}^{10*} + Y_{ij}^{10*}) + 2\sqrt{2}v_{45}(\tilde{Y}_{ji}^{45} - \tilde{Y}_{ij}^{45}), \quad M_\mathrm{D} \equiv \frac{v_5}{\sqrt{2}}Y^{1*}$$

$$M_{d,ij} \equiv \frac{1}{2}Y_{ij}^{5*}v_5 + Y_{ij}^{45*}v_{45}, \quad M_{\ell,ij} \equiv \frac{1}{2}Y_{ji}^{5*}v_5 - 3Y_{ji}^{45*}v_{45}$$

🥖 The **diagonalization relations** are

$$M_u = U_u\tilde{M}_uU_u^\mathrm{T}, \quad M_d = U_d\tilde{M}_dU_d^\mathrm{T}, \quad M_\ell = U_\ell\tilde{M}_\ell U_\ell^\mathrm{T}$$

🥐 Assume the **seesaw hierarchy** $M_N \gg M_\mathrm{D}$, we have $M_\nu = -M_\mathrm{D}M_N^{-1}M_\mathrm{D}^\mathrm{T}$ and

$$U_\nu\tilde{M}_\nu U_\nu^\mathrm{T} = M_\nu, \quad U_N\tilde{M}_NU_N^\mathrm{T} = M_N$$

🥨 $\tilde{M}_\nu = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$ and $\tilde{M}_N = \mathrm{diag}(m_{N_1}, m_{N_2}, m_{N_3})$ are the diagonalized mass matrices for **light** and **heavy Majorana neutrinos**

Effective Operators for Proton Decay

In the framework of **effective field theory**, the **dimension-6 effective operators** involving **three quarks** and **one lepton**, induced by the **exchange** of Δ_a, are

$$\mathcal{L}_{\text{eff}} \supset a(d_{i\text{L}}, \ell_{j\text{L}})\varepsilon_{abc}\overline{(u^{\text{m}}_{a1\text{L}})^{\text{C}}}d^{\text{m}}_{bi\text{L}}\overline{(u^{\text{m}}_{c1\text{L}})^{\text{C}}}\ell^{\text{m}}_{j\text{L}} - a(d_{i\text{L}}, \ell_{j\text{R}})\varepsilon_{abc}\overline{(u^{\text{m}}_{a1\text{L}})^{\text{C}}}d^{\text{m}}_{bi\text{L}}\overline{(u^{\text{m}}_{c1\text{R}})^{\text{C}}}\ell^{\text{m}}_{j\text{R}}$$
$$- a(d_{i\text{R}}, \ell_{j\text{L}})\varepsilon_{abc}\overline{(d^{\text{m}}_{ai\text{R}})^{\text{C}}}u^{\text{m}}_{b1\text{R}}\overline{(u^{\text{m}}_{c1\text{L}})^{\text{C}}}\ell^{\text{m}}_{j\text{L}} + a(d_{i\text{R}}, \ell_{j\text{R}})\varepsilon_{abc}\overline{(d^{\text{m}}_{ai\text{R}})^{\text{C}}}u^{\text{m}}_{b1\text{R}}\overline{(\ell^{\text{m}}_{j\text{R}})^{\text{C}}}u^{\text{m}}_{c1\text{R}}$$
$$+ a(d_{i\text{L}}, d_{j\text{L}}, \nu_{k\text{L}})\varepsilon_{abc}\overline{(u^{\text{m}}_{a1\text{L}})^{\text{C}}}d^{\text{m}}_{bi\text{L}}\overline{(d^{\text{m}}_{cj\text{L}})^{\text{C}}}\nu^{\text{m}}_{k\text{L}} - a(d_{i\text{R}}, d_{j\text{L}}, \nu_{k\text{L}})\varepsilon_{abc}\overline{(d^{\text{m}}_{ai\text{R}})^{\text{C}}}u^{\text{m}}_{b1\text{R}}\overline{(d^{\text{m}}_{cj\text{L}})^{\text{C}}}\nu^{\text{m}}_{k\text{L}}$$

$$\begin{aligned}
a(d_{i\text{L}}, \ell_{j\text{L}}) &= -\frac{\sqrt{2}}{m_\Delta^2}[U_u^{\text{T}}(Y^{10} + Y^{10\text{T}})U_d]_{1i}(U_u^{\text{T}}Y^5U_\ell)_{1j} \\
a(d_{i\text{L}}, \ell_{j\text{R}}) &= -\frac{4}{m_\Delta^2}[U_u^{\text{T}}(Y^{10} + Y^{10\text{T}})U_d]_{1i}[U_\ell^\dagger(Y^{10\dagger} + Y^{10*})U_u^*]_{j1} \\
a(d_{i\text{R}}, \ell_{j\text{L}}) &= \frac{1}{2m_\Delta^2}(U_d^\dagger Y^{5\dagger}U_u^*)_{i1}(U_u^{\text{T}}Y^5U_\ell)_{1j} \\
a(d_{i\text{R}}, \ell_{j\text{R}}) &= \frac{\sqrt{2}}{m_\Delta^2}(U_d^\dagger Y^{5\dagger}U_u^*)_{i1}[U_\ell^\dagger(Y^{10\dagger} + Y^{10*})U_u^*]_{j1} \\
a(d_{i\text{L}}, d_{j\text{L}}, \nu_{k\text{L}}) &= \frac{\sqrt{2}}{m_\Delta^2}[U_u^{\text{T}}(Y^{10} + Y^{10\text{T}})U_d]_{1i}(U_d^{\text{T}}Y^5U_\nu)_{jk} \\
a(d_{i\text{R}}, d_{j\text{L}}, \nu_{k\text{L}}) &= -\frac{1}{2m_\Delta^2}(U_d^\dagger Y^{5\dagger}U_u^*)_{i1}(U_d^{\text{T}}Y^5U_\nu)_{jk}
\end{aligned}$$

Partial widths for Proton Decay Channels

Using the **chiral Lagrangian** techniques, the **partial widths** for **proton decay channels** $p \to \ell_i^+ \pi^0$ $(i = 1, 2)$, $p \to \pi^+ \bar{\nu}_i$ $(i = 1, 2, 3)$, and $p \to K^+ \bar{\nu}_i$ $(i = 1, 2, 3)$ are

$$\begin{aligned}
\Gamma_{p\to\ell_i^+\pi^0} &= \frac{(m_p^2 - m_{\pi^0}^2)^2}{64\pi f_\pi^2 m_p^3}(1 + D + F)^2 \\
&\quad \times \left[|\alpha a(d_{1\mathrm{L}}, \ell_{i\mathrm{L}}) + \beta a(d_{1\mathrm{R}}, \ell_{i\mathrm{L}})|^2 + |\alpha a(d_{1\mathrm{L}}, \ell_{i\mathrm{R}}) + \beta a(d_{1\mathrm{R}}, \ell_{i\mathrm{R}})|^2 \right] \\
\Gamma_{p\to\pi^+\bar{\nu}_i} &= \frac{(m_p^2 - m_{\pi^+}^2)^2}{32\pi f_\pi^2 m_p^3}(1 + D + F)^2 \, |\alpha a(d_{1\mathrm{R}}, d_{1\mathrm{L}}, \nu_{i\mathrm{L}}) + \beta a(d_{1\mathrm{L}}, d_{1\mathrm{L}}, \nu_{i\mathrm{L}})|^2 \\
\Gamma_{p\to K^+\bar{\nu}_i} &= \frac{(m_p^2 - m_{K^+}^2)^2}{32\pi f_\pi^2 m_p^3} \Big| \beta a(d_{1\mathrm{L}}, d_{2\mathrm{L}}, \nu_{i\mathrm{L}}) + \alpha a(d_{1\mathrm{R}}, d_{2\mathrm{L}}, \nu_{i\mathrm{L}}) \\
&\quad - \frac{m_p}{2m_{\Sigma^0}} [\beta a(d_{2\mathrm{L}}, d_{1\mathrm{L}}, \nu_{i\mathrm{L}}) + \alpha a(d_{2\mathrm{R}}, d_{1\mathrm{L}}, \nu_{i\mathrm{L}})](D - F) \\
&\quad + \frac{m_p}{6m_\Lambda} \{ \beta a(d_{2\mathrm{L}}, d_{1\mathrm{L}}, \nu_{i\mathrm{L}}) + \alpha a(d_{2\mathrm{R}}, d_{1\mathrm{L}}, \nu_{i\mathrm{L}}) \\
&\qquad + 2[\beta a(d_{1\mathrm{L}}, d_{2\mathrm{L}}, \nu_{i\mathrm{L}}) + \alpha a(d_{1\mathrm{R}}, d_{2\mathrm{L}}, \nu_{i\mathrm{L}})] \} (D + 3F) \Big|^2
\end{aligned}$$

$$\begin{aligned}
f_\pi &= 130.2(1.2)\ \text{MeV}, \quad D = 0.80(1), \quad F = 0.47(1) \\
\alpha &= -0.0112(25)\ \text{GeV}^3, \quad \beta = 0.0120(26)\ \text{GeV}^3
\end{aligned}$$

Parametrization

- Any 3×3 **unitary matrix** U can be **parametrized** as

$$U = K(\rho_1, \rho_2) R_{23}(\theta_{23}) R_{13}(\theta_{13}, \delta) R_{12}(\theta_{12}) D(\eta_1, \eta_2, \eta_3)$$

$$R_{23}(\theta_{23}) = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix}, \quad R_{13}(\theta_{13}, \delta) = \begin{pmatrix} c_{13} & & s_{13} e^{-i\delta} \\ & 1 & \\ -s_{13} e^{i\delta} & & c_{13} \end{pmatrix}$$

$$R_{12}(\theta_{12}) = \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}, \quad \begin{aligned} K(\rho_1, \rho_2) &= \mathrm{diag}(1, e^{i\rho_1}, e^{i\rho_2}) \\ D(\eta_1, \eta_2, \eta_3) &= \mathrm{diag}(e^{i\eta_1}, e^{i\eta_2}, e^{i\eta_3}) \\ c_{ij} \equiv \cos\,\theta_{ij}, &\quad s_{ij} \equiv \sin\theta_{ij} \end{aligned}$$

- The **mixing matrices** are expressed as

$$\begin{aligned}
V_{\mathrm{CKM}} &\equiv U_u^\dagger U_d = R_{23}(\theta_{23}^{\mathrm{CKM}}) R_{13}(\theta_{13}^{\mathrm{CKM}}, \delta^{\mathrm{CKM}}) R_{12}(\theta_{12}^{\mathrm{CKM}}) \\
V_{\mathrm{PMNS}} &\equiv U_\ell^\dagger U_\nu = R_{23}(\theta_{23}^{\mathrm{PMNS}}) R_{13}(\theta_{13}^{\mathrm{PMNS}}, \delta^{\mathrm{PMNS}}) R_{12}(\theta_{12}^{\mathrm{PMNS}}) D(\eta_1^{\mathrm{PMNS}}, \eta_2^{\mathrm{PMNS}}, 0) \\
U_u &= K(\rho_1^u, \rho_2^u) R_{23}(\theta_{23}^u) R_{13}(\theta_{13}^u, \delta^u) R_{12}(\theta_{12}^u) D(\eta_1^u, \eta_2^u, \eta_3^u) \\
U_\ell &= K(\rho_1^\ell, \rho_2^\ell) R_{23}(\theta_{23}^\ell) R_{13}(\theta_{13}^\ell, \delta^\ell) R_{12}(\theta_{12}^\ell) D(\eta_1^\ell, \eta_2^\ell, \eta_3^\ell)
\end{aligned}$$

Parameter Scan and Proton Decay Constraints

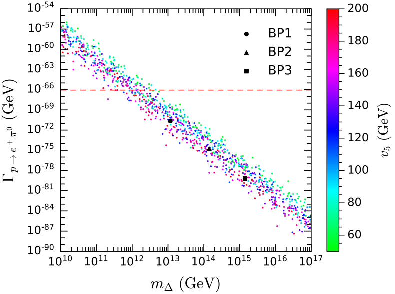
We carry out a **random scan** over the **free parameters** within the following ranges:

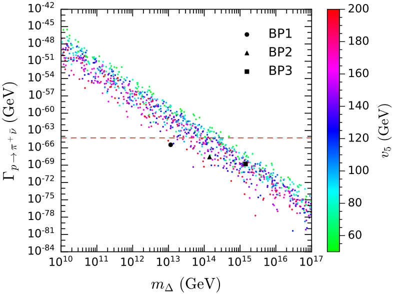
$$0 < \theta^u_{ij}, \theta^\ell_{ij} < \frac{\pi}{2}$$

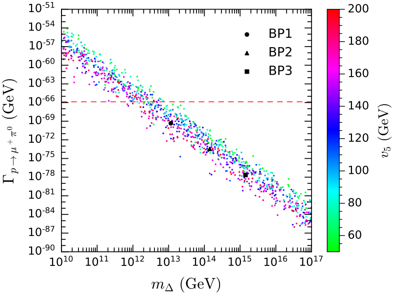
$$0 < \eta^{\rm PMNS}_i, \eta^u_i, \rho^u_i, \delta^u, \eta^\ell_i, \rho^\ell_i, \delta^\ell < 2\pi$$

$$50 \text{ GeV} < v_5 < 200 \text{ GeV}$$

$$10^{10} \text{ GeV} < m_\Delta < 10^{17} \text{ GeV}$$







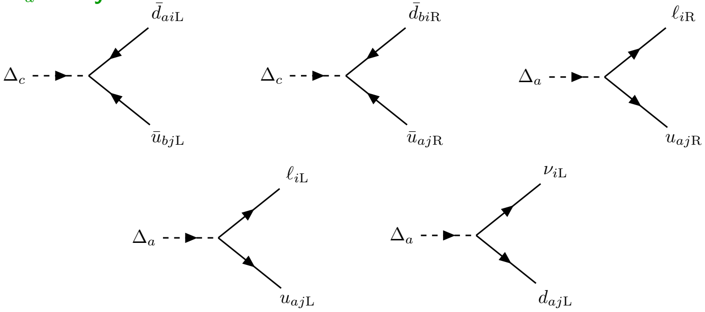
Benchmark Points

🥦 Among the parameter points that satisfy the experimental bounds on **proton decay**, we select **three benchmark points**, denoted as **BP1**, **BP2**, and **BP3**

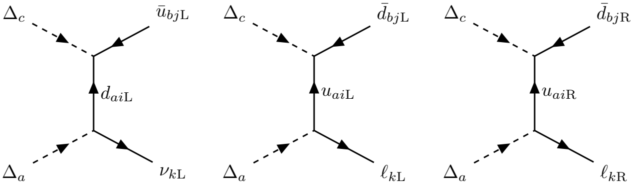
	● **BP1**	▲ **BP2**	■ **BP3**
m_Δ (GeV)	1.15×10^{13}	1.40×10^{14}	1.41×10^{15}
v_5 (GeV)	164.75	106.43	148.58
$\eta_1^{\rm PMNS}, \eta_2^{\rm PMNS}$	$4.76,\ 5.13$	$1.23,\ 4.47$	$6.20,\ 3.13$
$\theta_{12}^u, \theta_{13}^u, \theta_{23}^u$	$1.52,\ 0.25,\ 1.53$	$1.32,\ 0.21,\ 0.91$	$1.47,\ 0.64,\ 0.42$
$\delta^u, \rho_1^u, \rho_2^u$	$1.54,\ 2.75,\ 4.43$	$0.12,\ 0.33,\ 1.53$	$2.66,\ 1.50,\ 2.73$
$\eta_1^u, \eta_2^u, \eta_3^u$	$1.67,\ 3.21,\ 1.05$	$5.58,\ 0.26,\ 4.34$	$3.62,\ 5.91,\ 2.04$
$\theta_{12}^\ell, \theta_{13}^\ell, \theta_{23}^\ell$	$1.17,\ 0.25,\ 0.71$	$1.35,\ 1.35,\ 0.18$	$0.34,\ 0.10,\ 0.03$
$\delta^\ell, \rho_1^\ell, \rho_2^\ell$	$4.26,\ 3.66,\ 1.80$	$1.85,\ 0.26,\ 2.45$	$5.01,\ 1.47,\ 4.82$
$\eta_1^\ell, \eta_2^\ell, \eta_3^\ell$	$5.21,\ 5.67,\ 2.35$	$6.10,\ 0.78,\ 1.07$	$1.97,\ 1.48,\ 6.00$

Decay and Coannihilation of Scalar Leptoquarks

🚖 **2-body Δ_a decays**:

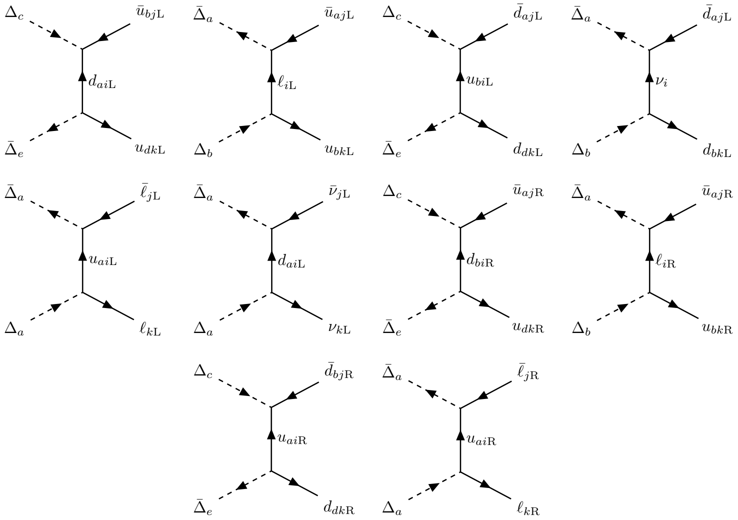


🚘 **$\Delta_a\Delta_c$ coannihilation** $(c \neq a)$ via **Yukawa couplings**:



Annihilation of Scalar Leptoquarks

🚔 $\Delta_a \bar{\Delta}_b$ **annihilation** via **Yukawa couplings**:



Quantum Gravity Treatments

🌲 Based on **Einstein's general relativity**, the **gravitational action** is given by

$$S = \int \mathrm{d}^4 x \sqrt{-\det(g_{\mu\nu})} \left(\frac{M_{\mathrm{Pl}}^2}{2} R + \mathcal{L} \right)$$

🌳 R is the Ricci scalar, and the Lagrangian $\mathcal{L}$ involves all fields in the $\mathrm{SU}(5)$ GUT

🌿 In **linearized gravity** for **weak gravitational field**, the **metric tensor** can be decomposed as $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ with **gravitational coupling constant** $\kappa \equiv \frac{2}{M_{\mathrm{Pl}}}$

🍒 The **symmetric tensor field** $h_{\mu\nu}$ is a **perturbation** around the **Minkowski metric** $\eta_{\mu\nu}$, and, upon **quantization**, corresponds to the **massless spin-2 graviton field**

🫐 The **gravitational interaction Lagrangian** can be constructed to any order in κ

[Choi, Shim & Song, hep-th/9411092, PRD]

🍎 **Feynman rules** for the **graviton couplings** to **spin-0**, **spin-1/2**, and **spin-1 particles** at $\mathcal{O}(\kappa)$ are given in Ref. [Barman, Bernal, Y Xu & Zapata, 2301.11345, JCAP]