NuclearScience
Computing CenteratCCNU

QCD in strong magnetic fields from Lattice QCD

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Work with Jin-Biao Gu, Arpith Kumar, Sheng-Tai Li, Jia Ni, Rishabh Thakkar & Dan Zhang based on

arXiv:2503.18467, arXiv:2508.07532, arXiv:2601.18354, arXiv:2606.30164, arXiv:2607.11625

中国物理学会高能物理分会第十二届全国会员代表大会暨学术年会

June 16@Guangzhou

Symmetries of QCD in vacuum

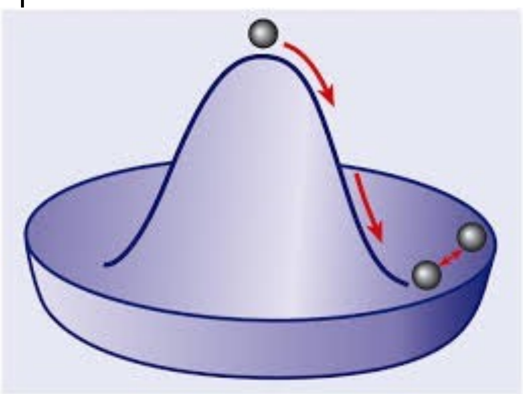
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\Big[i\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) - m_q\Big]q$$

Classical QCD symmetry (m_q=0)

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

Quantum QCD vacuum (m_q=0)

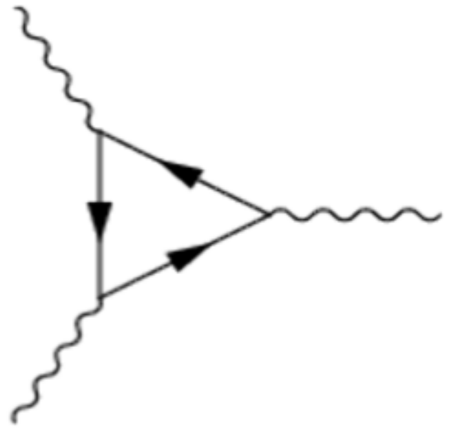
Chiral condensate:
spontaneous mass generation



$$\langle \bar{q}_R q_L \rangle \neq 0$$

Axial anomaly:
quantum violation of $U(1)_A$

$$\partial_{\mu} j_5^{\mu} = \frac{g^2 N_f}{16\pi^2} tr(\tilde{F}_{\mu\nu}F^{\mu\nu})$$



$$SU(N_f)_V \times U(1)_V$$

QCD symmetries at nonzero magnetic fields

- Magnetic field couples through electric charge $\mathcal{Q} = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} = \frac{1}{6}\mathbf{1} + \frac{1}{2}\tau^3,$
  - $[\tau^3, \mathcal{Q}] = 0$: neutral direction survives
  - $[\tau^{1,2}, \mathcal{Q}] \neq 0$: charged directions are explicitly broken by B

Symmetry condition:
$[T, \mathcal{Q}] = 0$

- Chiral symmetry: $SU(2)_L \times SU(2)_R$:

Isospin symmetry: $SU(2)_V \longrightarrow U(1)_V^{(3)},$

Remaining chiral symmetry: $SU(2)_A \longrightarrow U(1)_A^{(3)}$

- Divergence of axial current: $\partial_\mu J^\mu_{5,T} = i\bar{\psi}\{M_q, T\}\gamma_5\psi + 2\,\mathrm{tr}_f(T)\,q(x) + \dfrac{e^2}{8\pi^2}N_c \mathrm{tr}_f(TQ^2)F^{\mathrm{em}}_{\mu\nu}\widetilde{F}^{\mathrm{em}\,\mu\nu}$

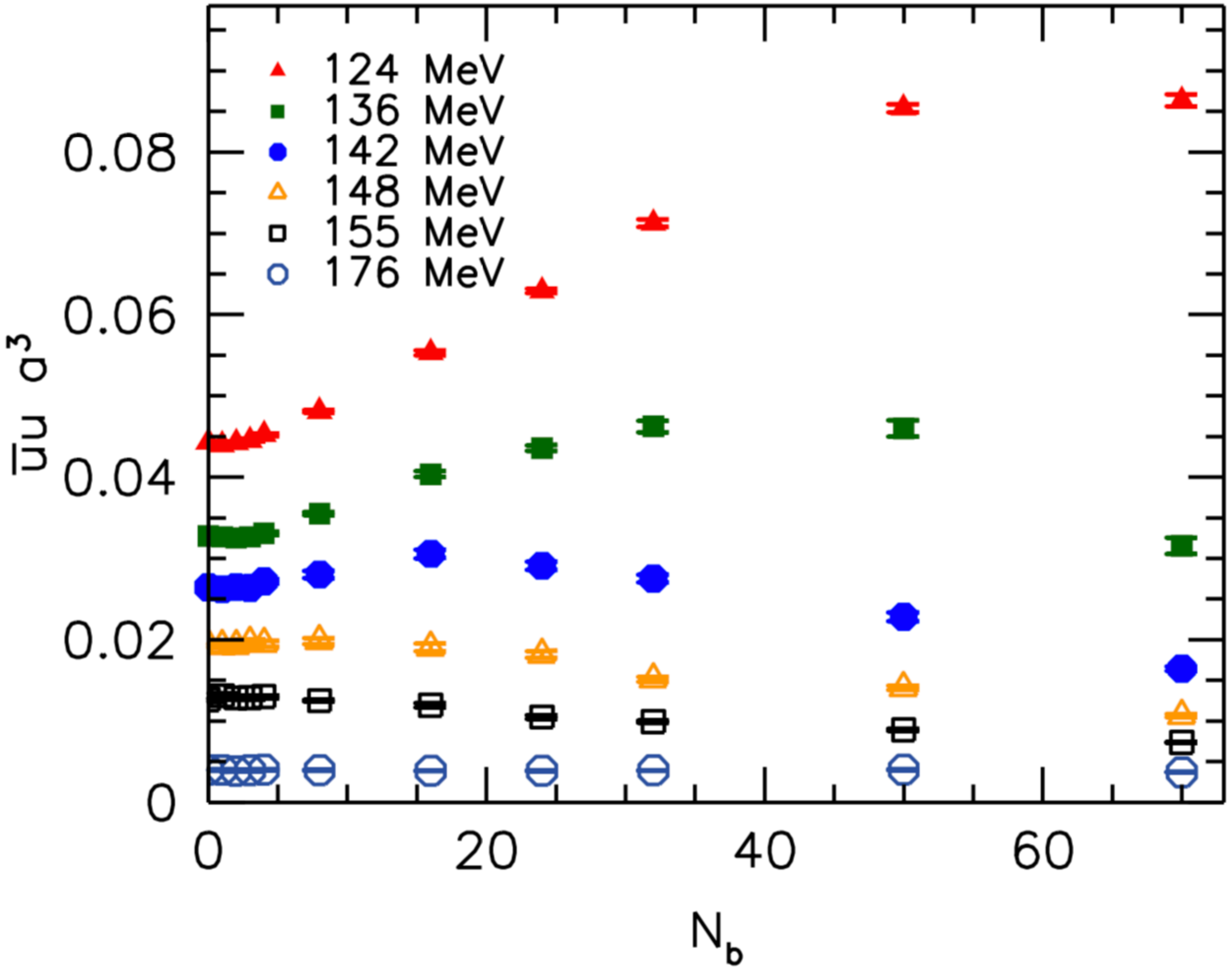
In pure magnetic fields $F^{\mathrm{em}}_{\mu\nu}\widetilde{F}^{\mathrm{em}\,\mu\nu} \propto \mathbf{E}\cdot\mathbf{B} = 0$

Inverse magnetic catalyses

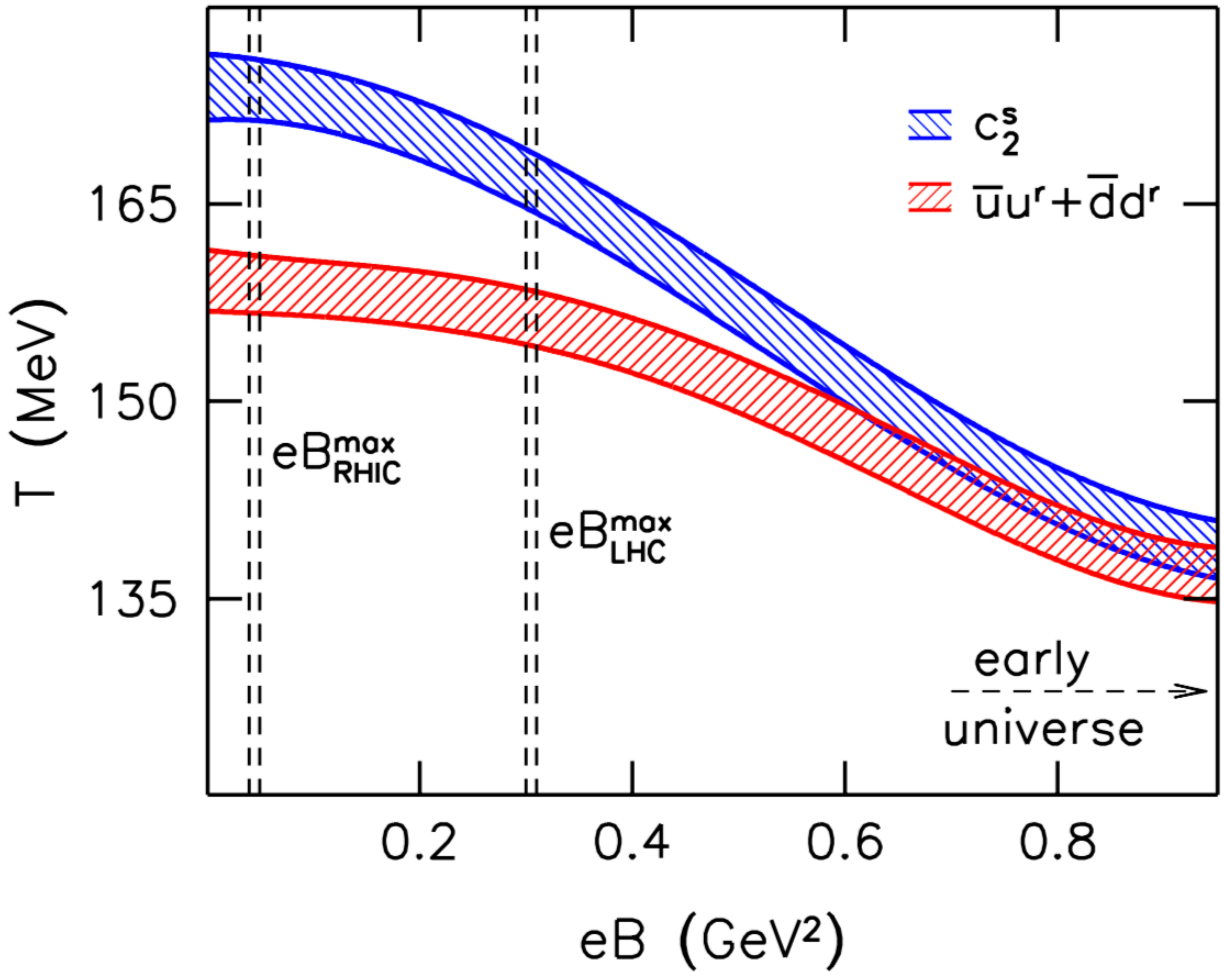
Continuum extrapolated lattice QCD results with physical pion mass

Bali et al., JHEP02(2012)044

Inverse magnetic catalysis (IMC)



$eB\uparrow\ T_{pc}\downarrow$



Signatures of symmetry restorations at eB=0

- Susceptibilities defined as integrated two point correlation functions of the local operators, e.g. $\chi_\pi = \int \mathrm{d}^4x \langle \pi^i(x) \pi^i(0) \rangle$ with $\pi^i(x) = \mathrm{i}\bar{\psi}_l(x)\gamma_5\tau^i\psi_l(x)$

$$\begin{tikzcd}
\chi_{5,\mathrm{con}} \quad \pi:\ \bar{q}\gamma_5\frac{\tau}{2}q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & \sigma:\ \bar{q}q \quad \chi_{\mathrm{con}} + \chi_{\mathrm{disc}} \arrow[d, leftrightarrow, "U(1)_A"] \\
\chi_{\mathrm{con}} \quad \delta:\ \bar{q}\frac{\tau}{2}q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"'] & \eta':\ \bar{q}\gamma_5 q \quad \chi_{5,\mathrm{con}} - \chi_{5,\mathrm{disc}}
\end{tikzcd}$$

$$\chi_{\mathrm{disc}} = \frac{T}{V} \int \mathrm{d}^4x \left\langle \left[\bar{\psi}(x)\psi(x) - \langle \bar{\psi}(x)\psi(x) \rangle \right]^2 \right\rangle$$

Restoration of $SU(2)_L \times SU(2)_R$:

$\chi_\pi - \chi_\sigma = 0$

$\chi_\delta - \chi_\eta = 0$

$\Rightarrow \chi_\pi - \chi_\delta = \chi_{\mathrm{disc}} = \chi_{5,\mathrm{disc}}$

Effective restoration of $U(1)_A$:

$\chi_\pi - \chi_\delta = 0$

$\chi_\sigma - \chi_\eta = 0$

$\Rightarrow \chi_\pi - \chi_\delta = \chi_{\mathrm{disc}} = \chi_{5,\mathrm{disc}} = 0$

Signatures of symmetry restorations at $eB \neq 0$

Chiral symmetry: $SU(2)_V \to U(1)_A^{(3)}$; Axial $U(1)_A$ symmetry: $U(1)_A \to U(1)_A^{\rm singlet}$

$$\begin{array}{ccc}
\chi_{5,\rm con} \; \boxed{-\chi_{5,\rm disc}^{(-)}} & & \chi_{\rm con} + \chi_{\rm disc}^{(+)} \\
\boxed{\pi^0} : \; i\bar{q}\gamma_5 \frac{\tau^3}{\sqrt{2}} q & \xleftrightarrow{\boxed{U(1)_A^{(3)}}} & \sigma : \; \bar{q}\frac{\mathbf{1}}{\sqrt{2}} q \\
\Big\updownarrow U(1)_A^{\rm singlet} & \boxed{B \neq 0} & \Big\updownarrow U(1)_A^{\rm singlet} \\
\boxed{\delta^0} : \; \bar{q}\frac{\tau^3}{\sqrt{2}} q & \xleftrightarrow[\boxed{U(1)_A^{(3)}}]{} & \eta : \; i\bar{q}\gamma_5 \frac{\mathbf{1}}{\sqrt{2}} q \\
\chi_{\rm con} \; \boxed{+\chi_{\rm disc}^{(-)}} & & \chi_{5,\rm con} - \chi_{5,\rm disc}^{(+)}
\end{array}$$

- $U(1)_A^{(3)}$: $\pi^0 \leftrightarrow \sigma$, $\delta^0 \leftrightarrow \eta$

  Probe: $O_\chi \propto \chi_{\pi^0} - \chi_\sigma$

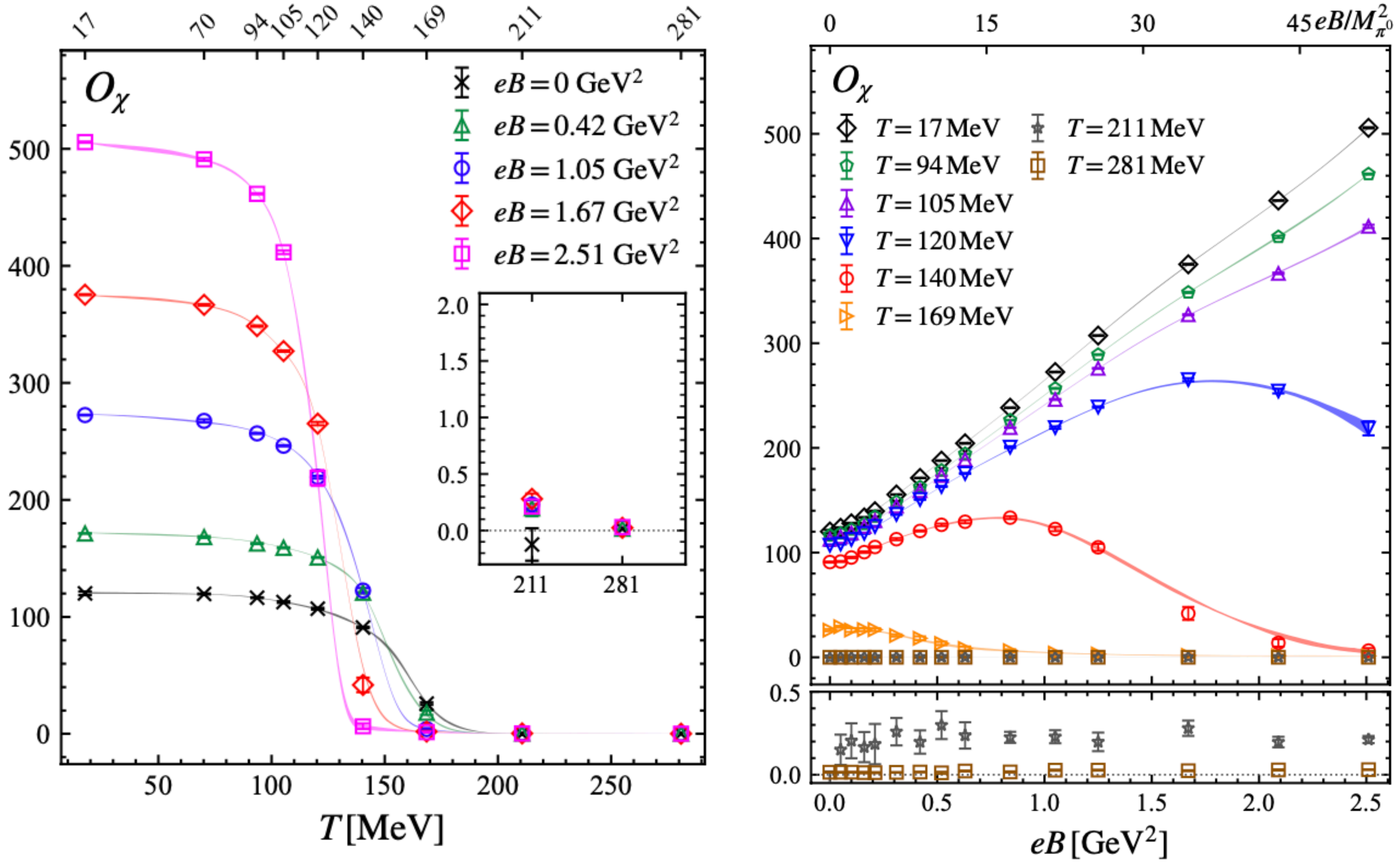
- $U(1)_A^{\rm singlet}$: $\pi^0 \leftrightarrow \delta^0$, $\sigma \leftrightarrow \eta$

  Probe: $O_A \propto \chi_{\pi^0} - \chi_{\delta^0}$

HTD, José Javier Hernández Hernández & Dan Zhang, arXiv:2607.11625

Measure of chiral symmetry breaking

$N_{f=2+1}$ QCD, $M_\pi(eB=0) \approx 220$ MeV,
$32^3 \times 96$ lattices with $a^{-1} \approx 1.7$ GeV and HISQ action



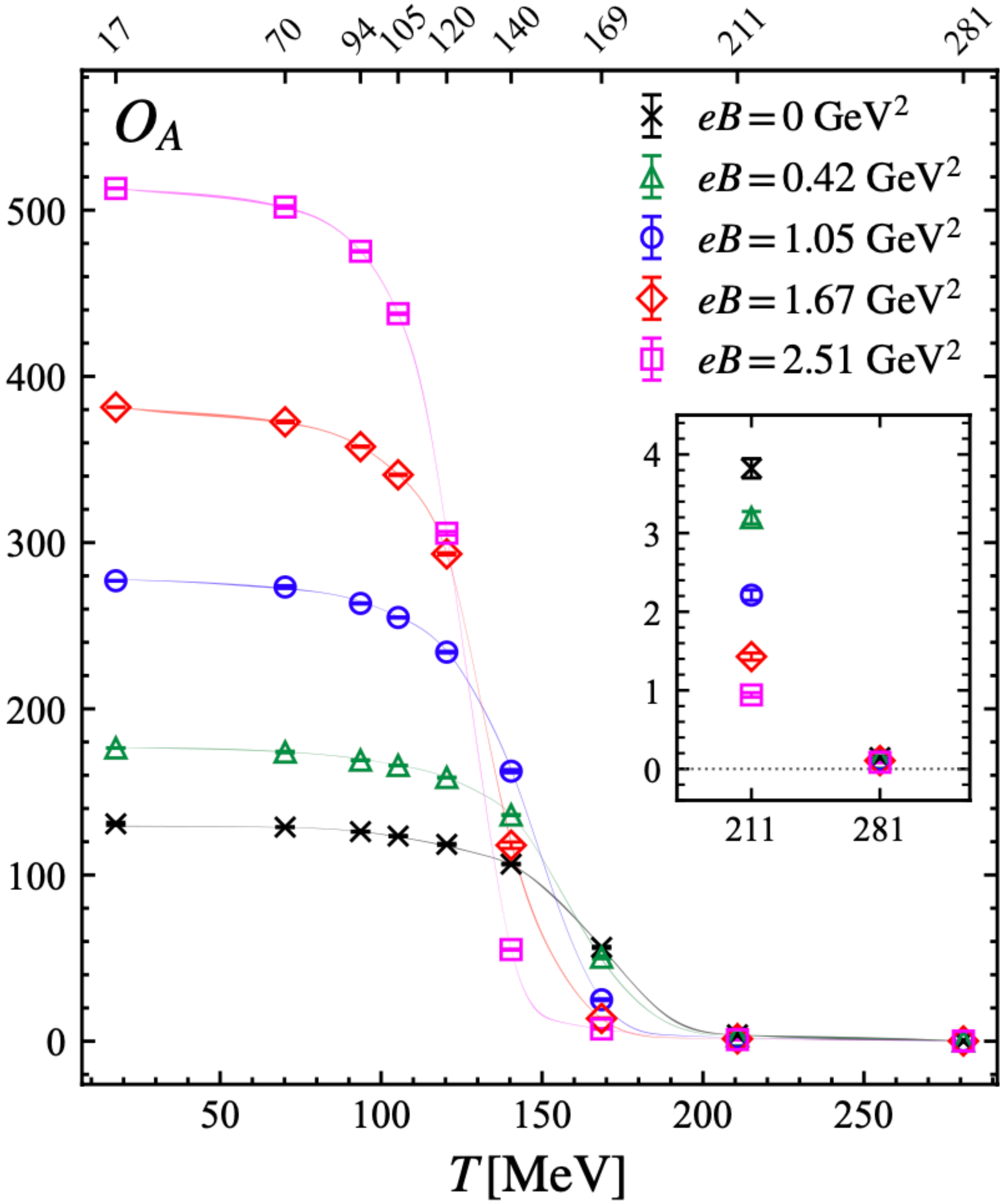
$$O_\chi(B,T) \equiv \frac{m_s^2}{f_K^4}\left[\chi_{\pi^0}(B,T) - \chi_\sigma(B,T)\right]$$

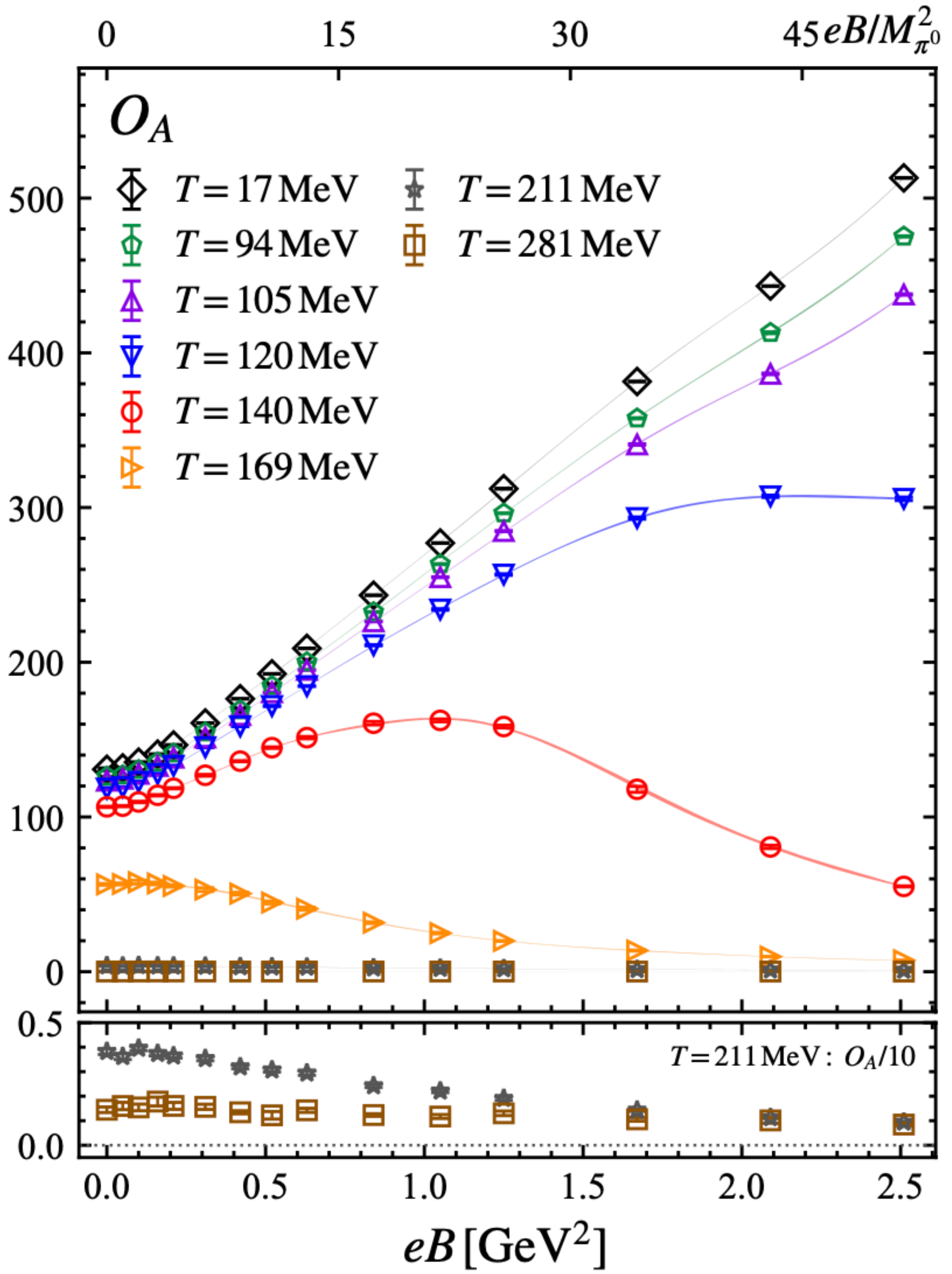
- $eB \uparrow$:
  - At low T $O_\chi \uparrow$
  - At high T $O_\chi \downarrow$
- counter part of IMC & MC

HTD, José Javier Hernández Hernández & Dan Zhang, arXiv:2607.11625

Measure of singlet $U(1)_A$ anomaly

N_f=2+1 QCD, $M_\pi(eB=0) \approx 220$ MeV,
$32^3 \times 96$ lattices with $a^{-1} \approx 1.7$ GeV and HISQ action





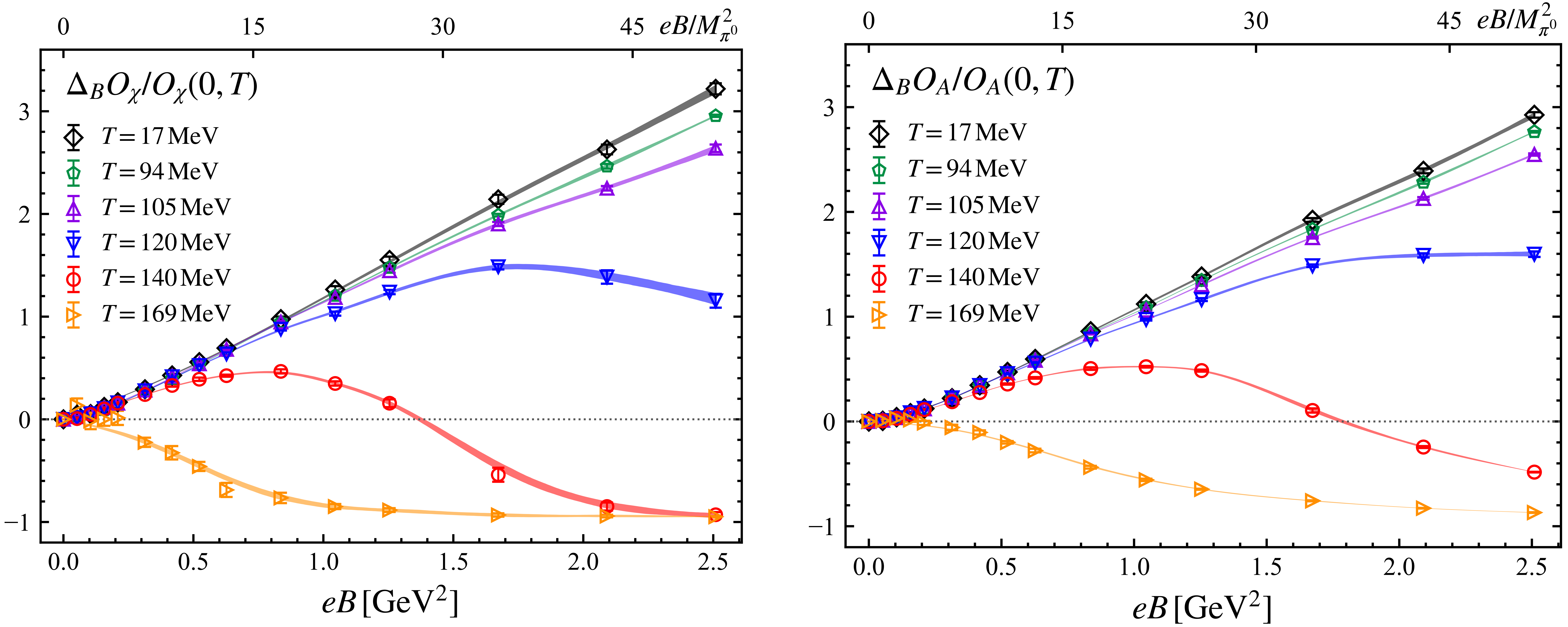
$$O_A(B,T) \equiv \frac{m_s^2}{f_K^4}\left[\chi_{\pi^0}(B,T) - \chi_\delta(B,T)\right]$$

- $eB \uparrow$:
  - At low T $O_A \uparrow$
  - At high T $O_A \downarrow$
- Similar to O_χ

HTD, José Javier Hernández Hernández & Dan Zhang, arXiv:2607.11625

Relative magnetic response of non-singlet chiral partner and singlet $U(1)_A$ splittings

$$R_{O_i}(eB,T) \equiv \frac{\Delta_B O_i(eB,T)}{O_i(0,T)} = \frac{O_i(eB,T) - O_i(0,T)}{O_i(0,T)}\,.$$

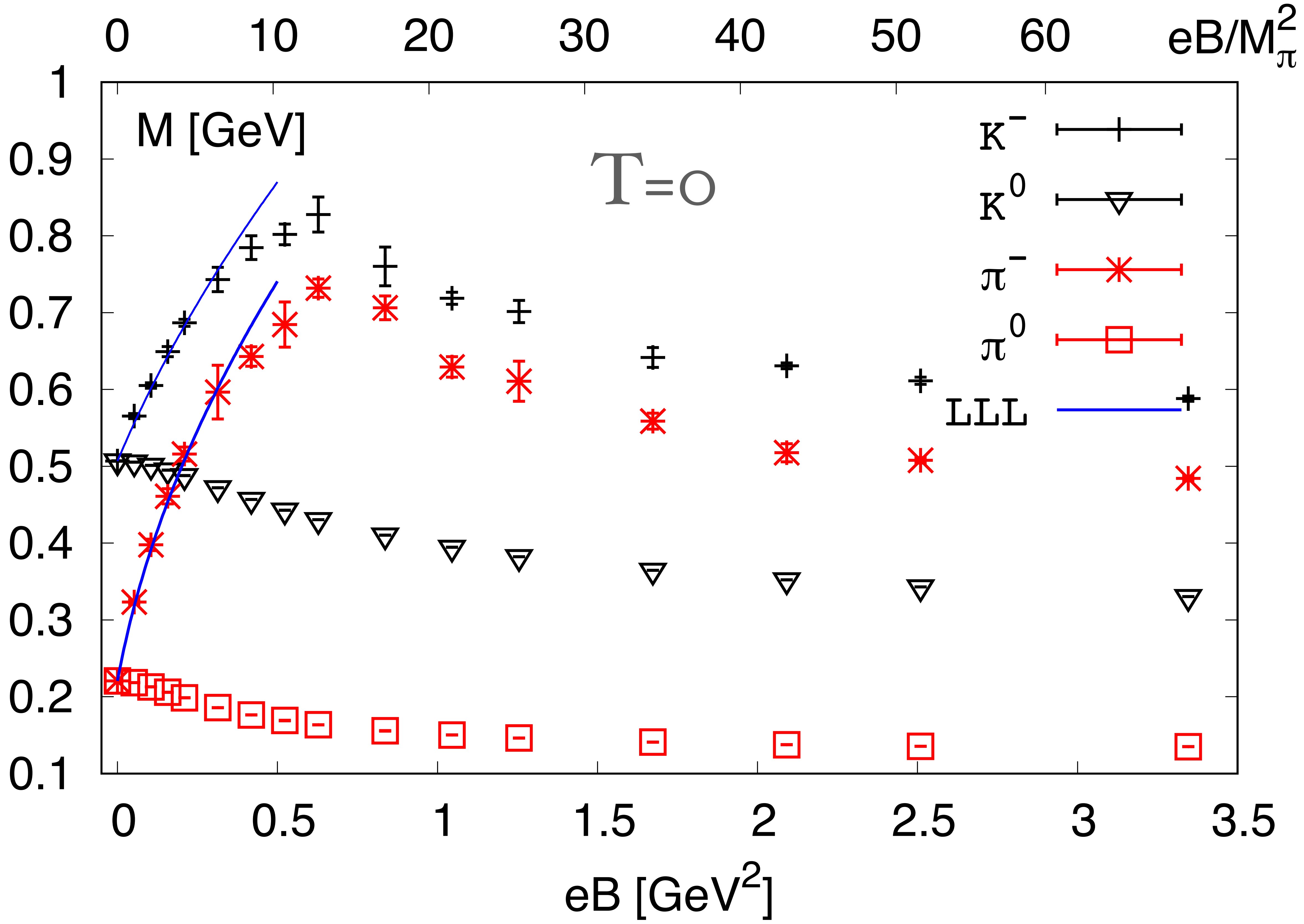


O_A: similar as O_χ, but suppression setting in at larger eB and remaining milder

HTD, José Javier Hernández Hernández & Dan Zhang, arXiv:2607.11625

Masses of $\pi^{0,\pm}$ and $K^{0,\pm}$ and energy density at larger than physical pion mass

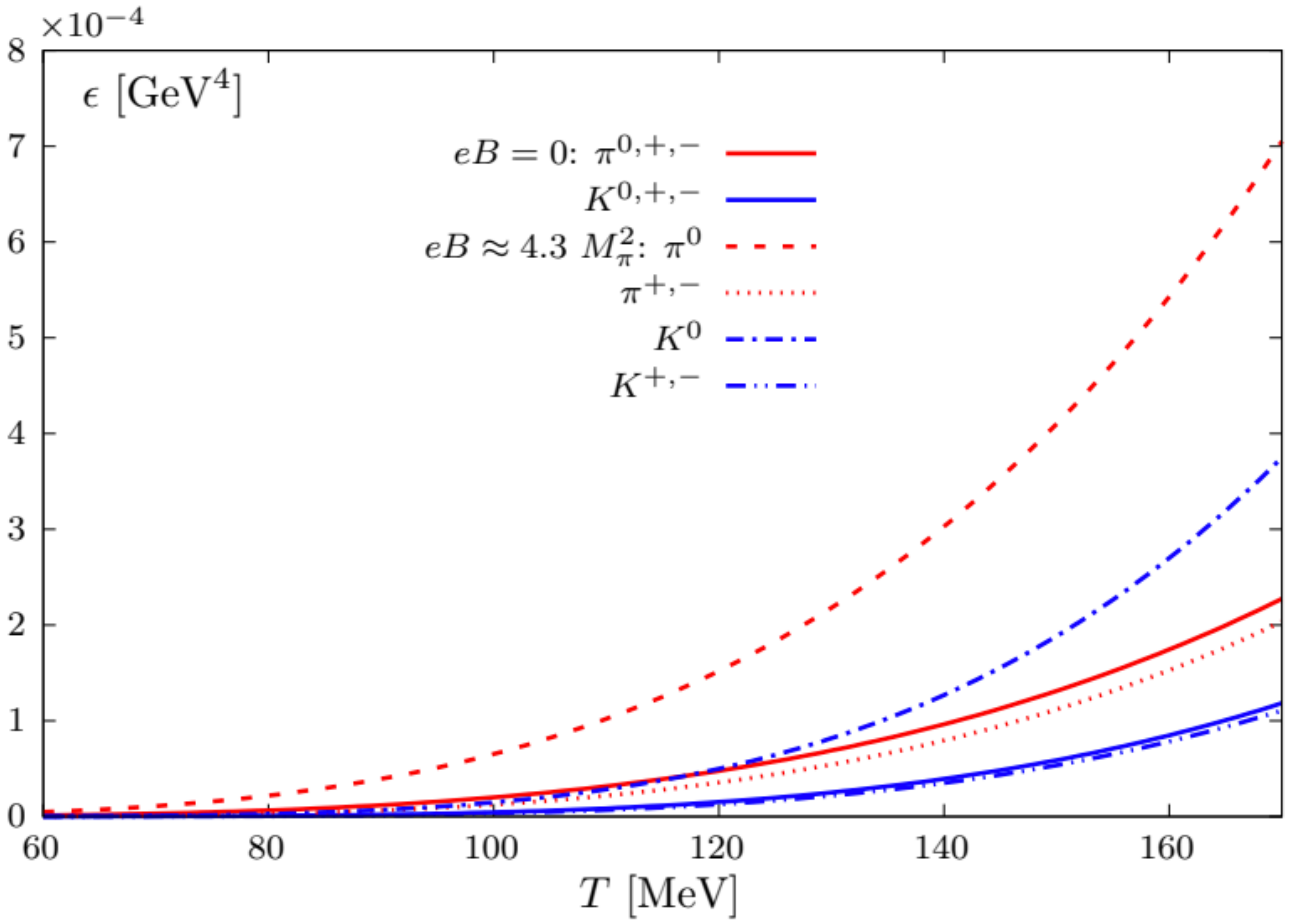
$N_{f=2+1}$ QCD, $M_\pi(eB=0) \approx 220$ MeV,
$32^3 \times 96$ lattices with $a^{-1} \approx 1.7$ GeV and HISQ action



HTD, S.-T. Li, A. Tomiya, X.-D. Wang, Y. Zhang, PRD 126 (2021) 082001

See quenched LQCD results in Bali et al., PRD 97 (2018) 034505, Luschevskaya et al., NPB 898 (2015) 627

Energy density in Hadron resonance gas model

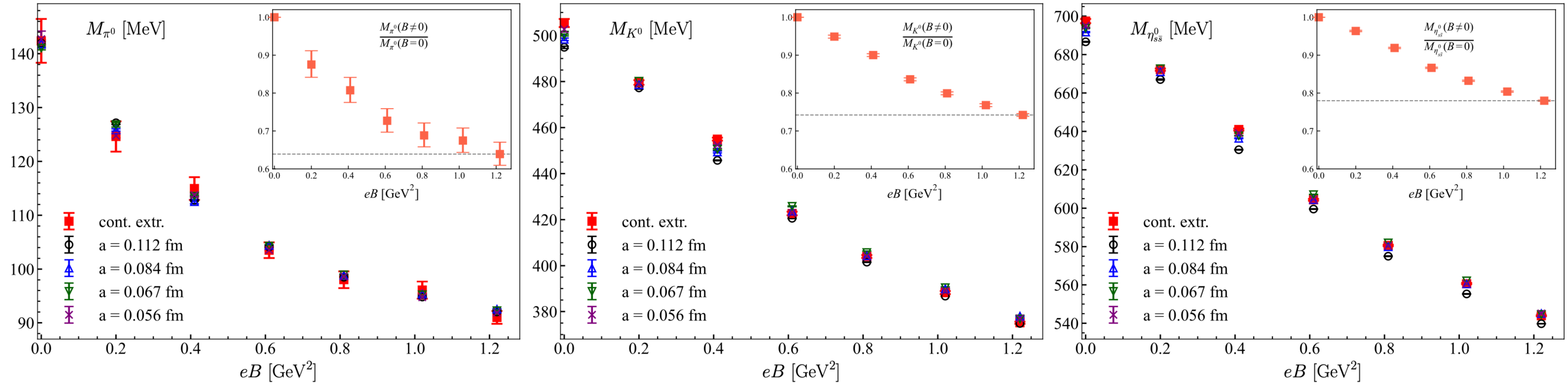


HTD, S.-T. Li, Q. Shi, A. Tomiya, X.-D. Wang, Y. Zhang, arXiv: 2011.04870

Updates on Masses of neutral PS mesons in the continuum limit with physical pion mass

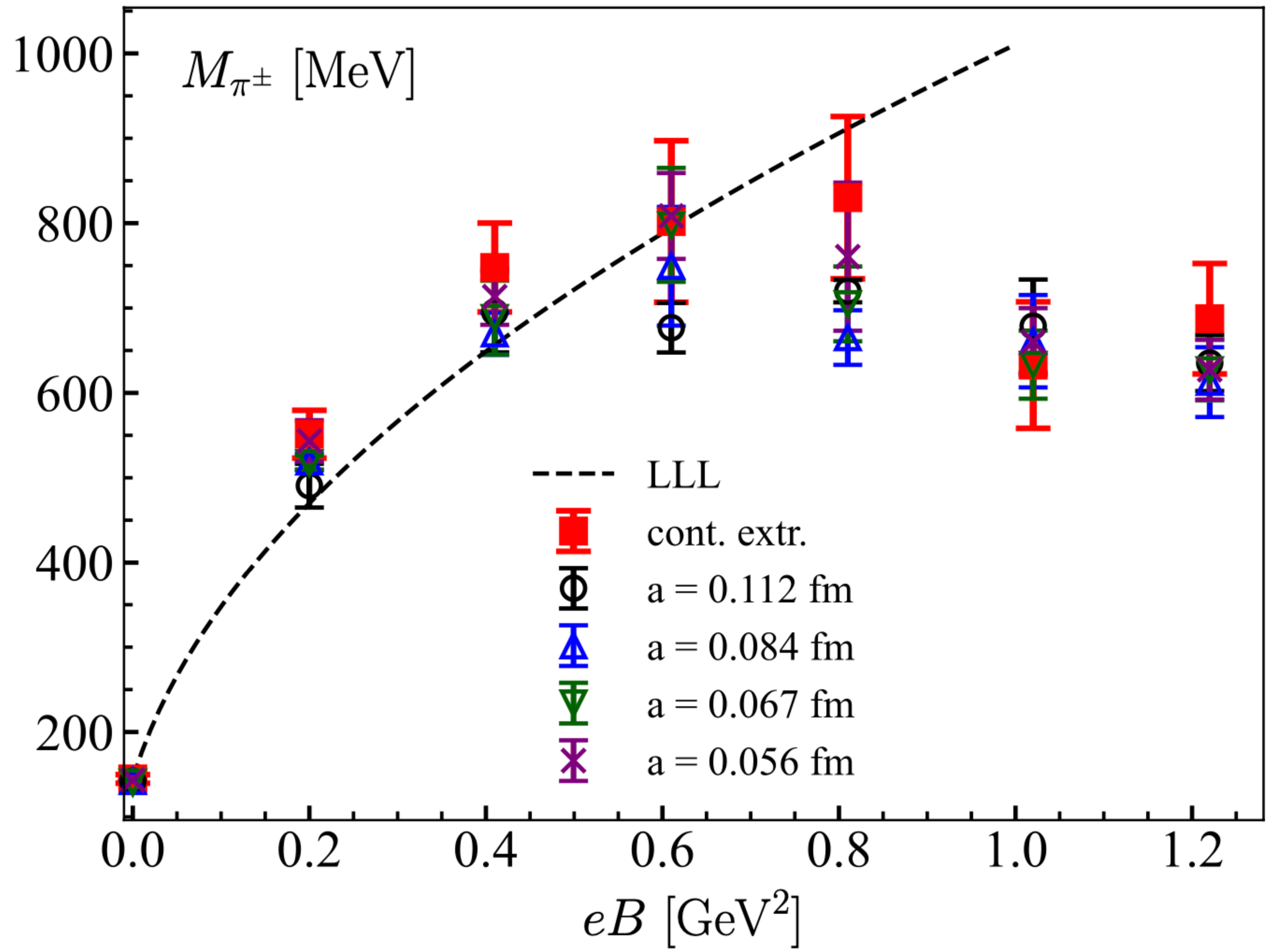
N_f=2+1 QCD,
$M_\pi(eB=0) \approx 140$ MeV

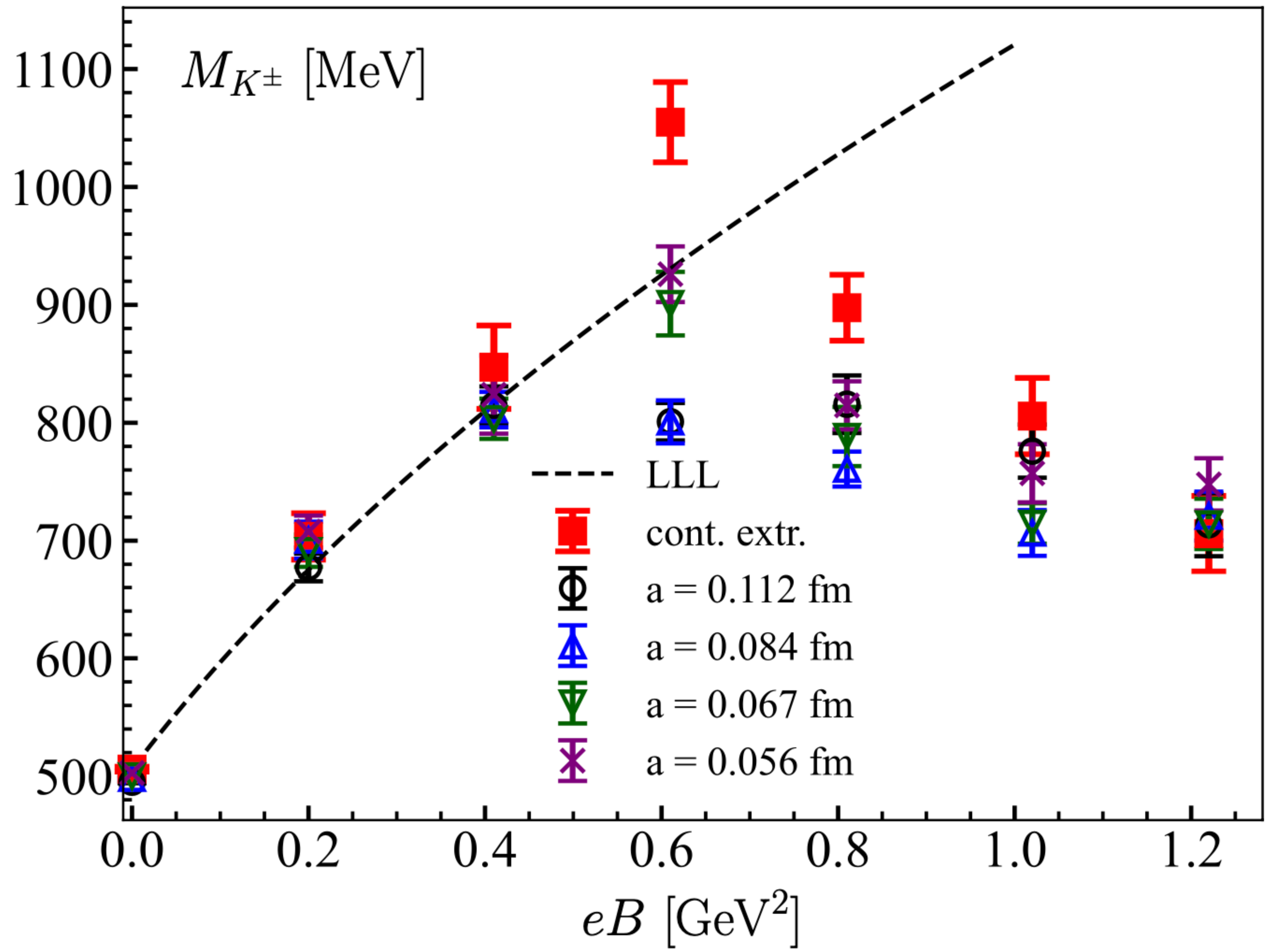
$N_\sigma^3 \times N_\tau$	a [fm]	β	am_s	am_l	# of conf at eB [GeV^2]						
					0.0	0.2	0.41	0.61	0.81	1.02	1.22
$24^3 \times 48$	0.112	6.722	0.0484	0.001793	600	701	1401	1350	1350	1350	1350
$32^3 \times 64$	0.084	7.016	0.0353	0.001307	600	800	800	800	800	800	800
$40^3 \times 80$	0.067	7.254	0.0274	0.001015	475	555	555	555	555	321	316
$48^3 \times 96$	0.056	7.455	0.0222	0.000822	150	266	250	250	220	306	215



HTD, Dan Zhang, arXiv:2601.18354, PRD 26'

Masses of charged PS mesons in the vacuum

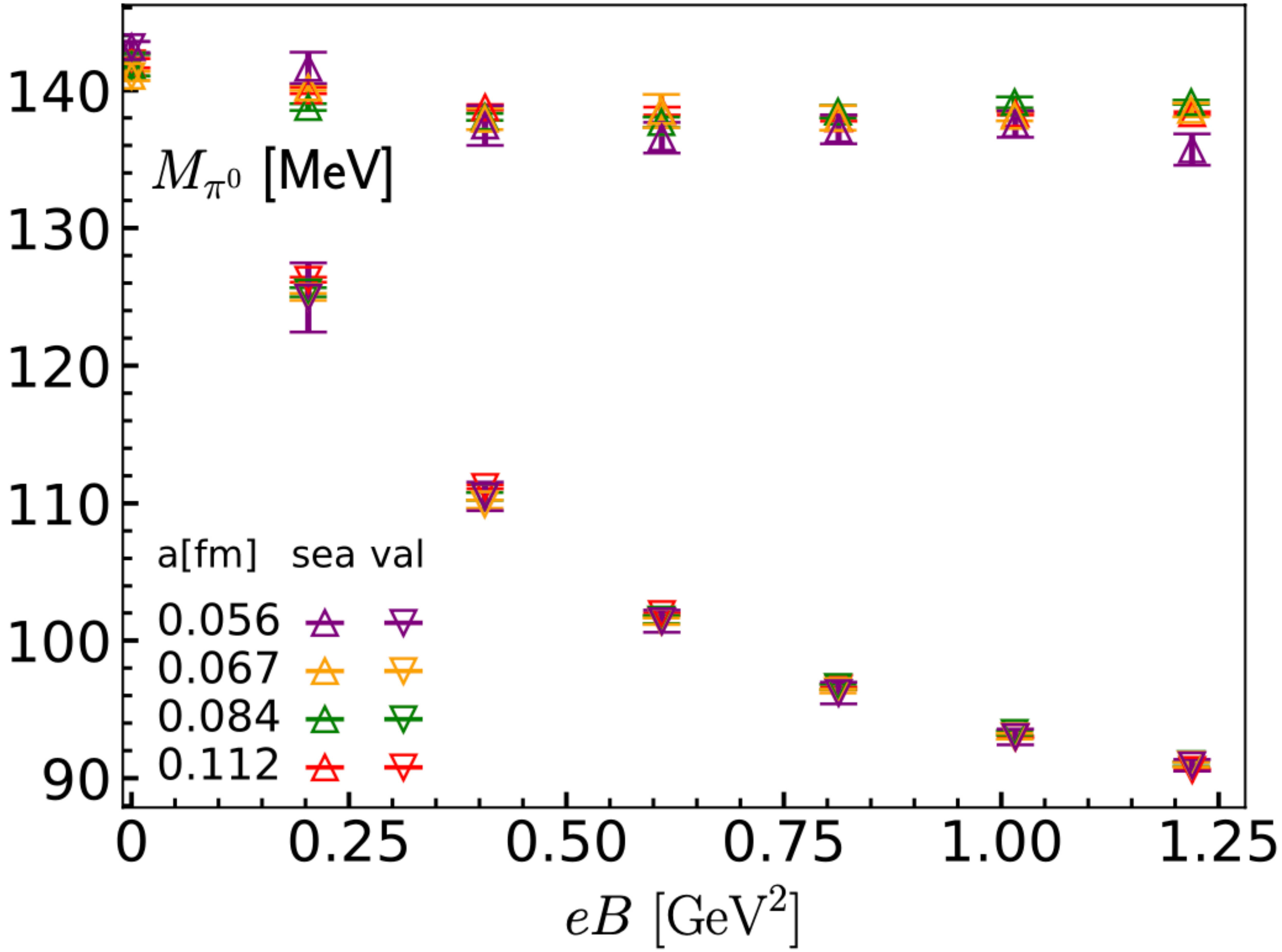




Non-monotonic behavior persists in the continuum limit

HTD, Dan Zhang, arXiv:2601.18354, PRD 26'

Masses From Sea and Valence Quark Contributions



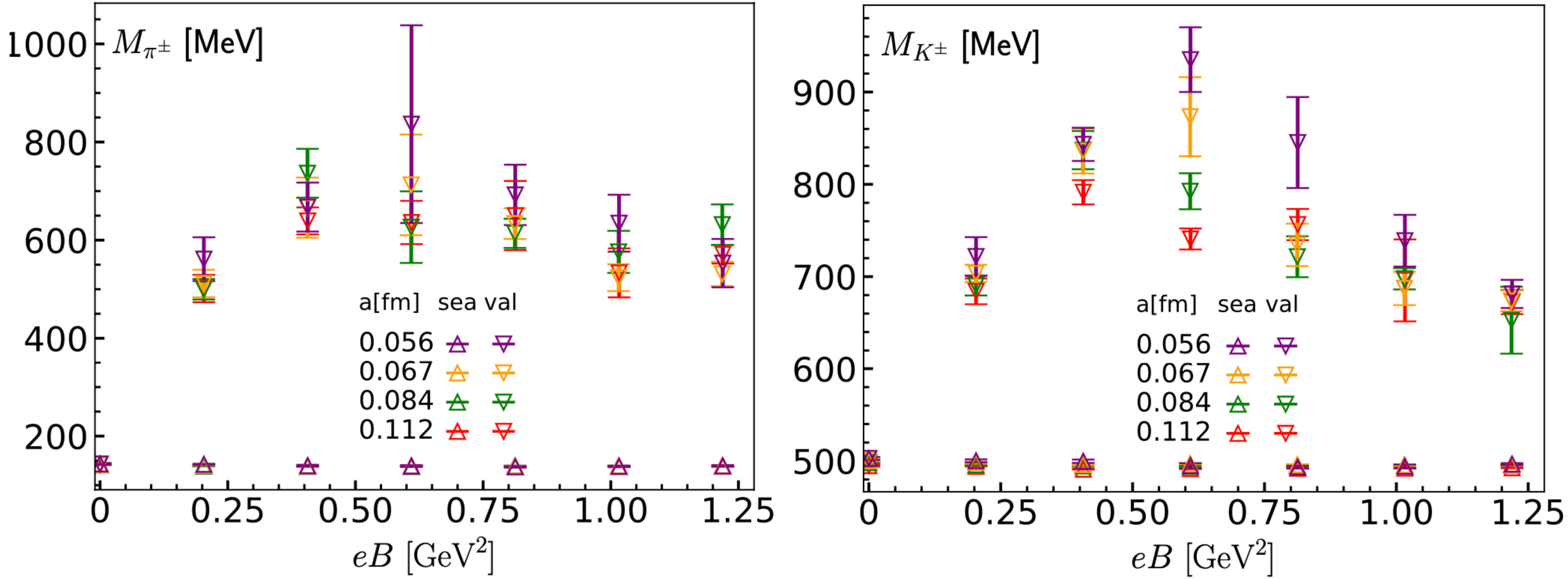
$$\langle O \rangle_{sea}(eB) = \frac{\int DUe^{-S_G}\det M(eB)\ O(eB=0)}{Z(eB)}$$

$$Z(eB) = \int DUe^{-S_G}\ \det M(eB)$$

$$\langle O \rangle_{valence}(eB) = \frac{\int DUe^{-S_G}\ \det M(eB=0)\ O(eB)}{Z(eB=0)}$$

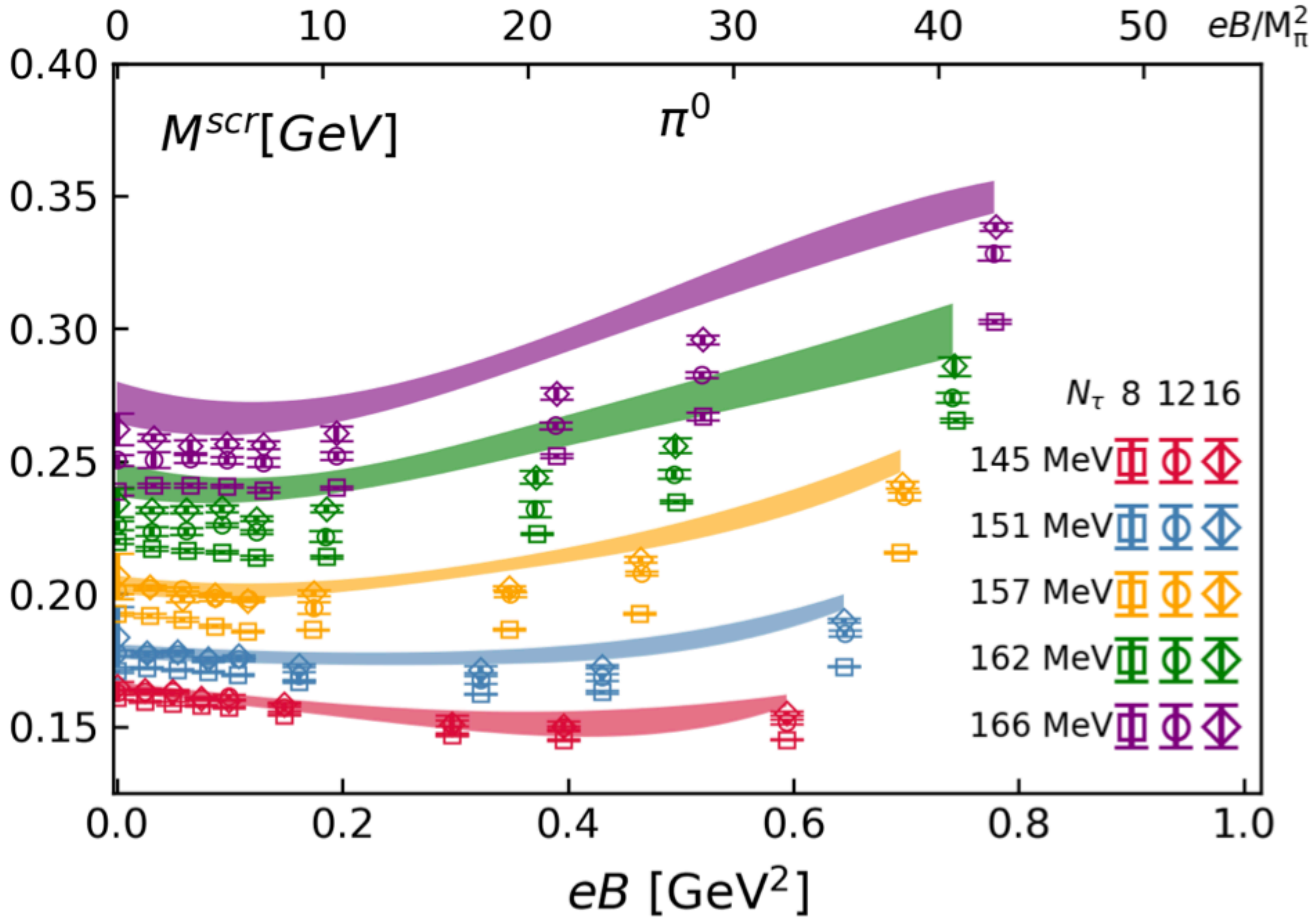
HTD, Dan Zhang, arXiv:2601.18354, PRD 26'

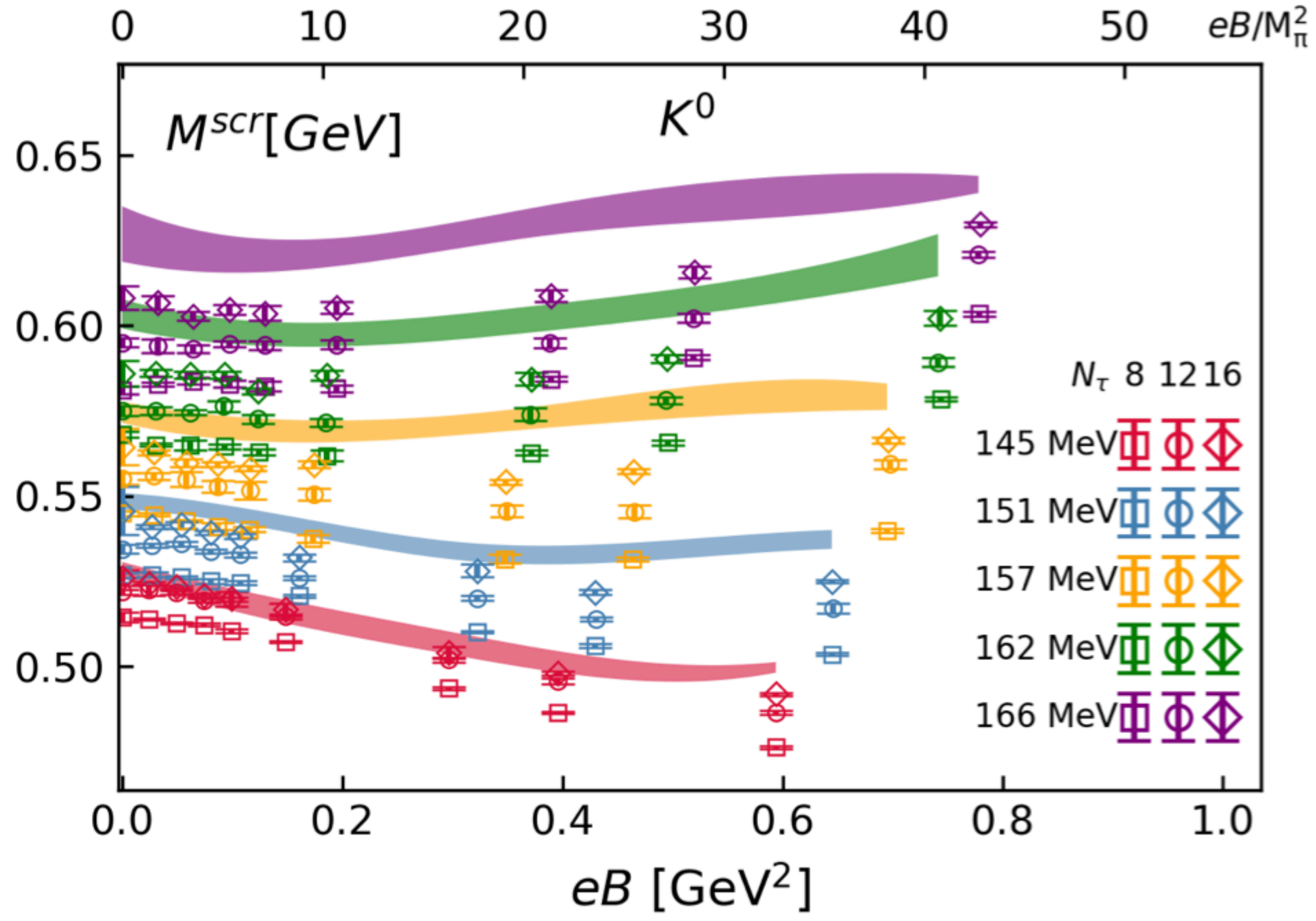
Masses From Sea and Valence Quark Contributions

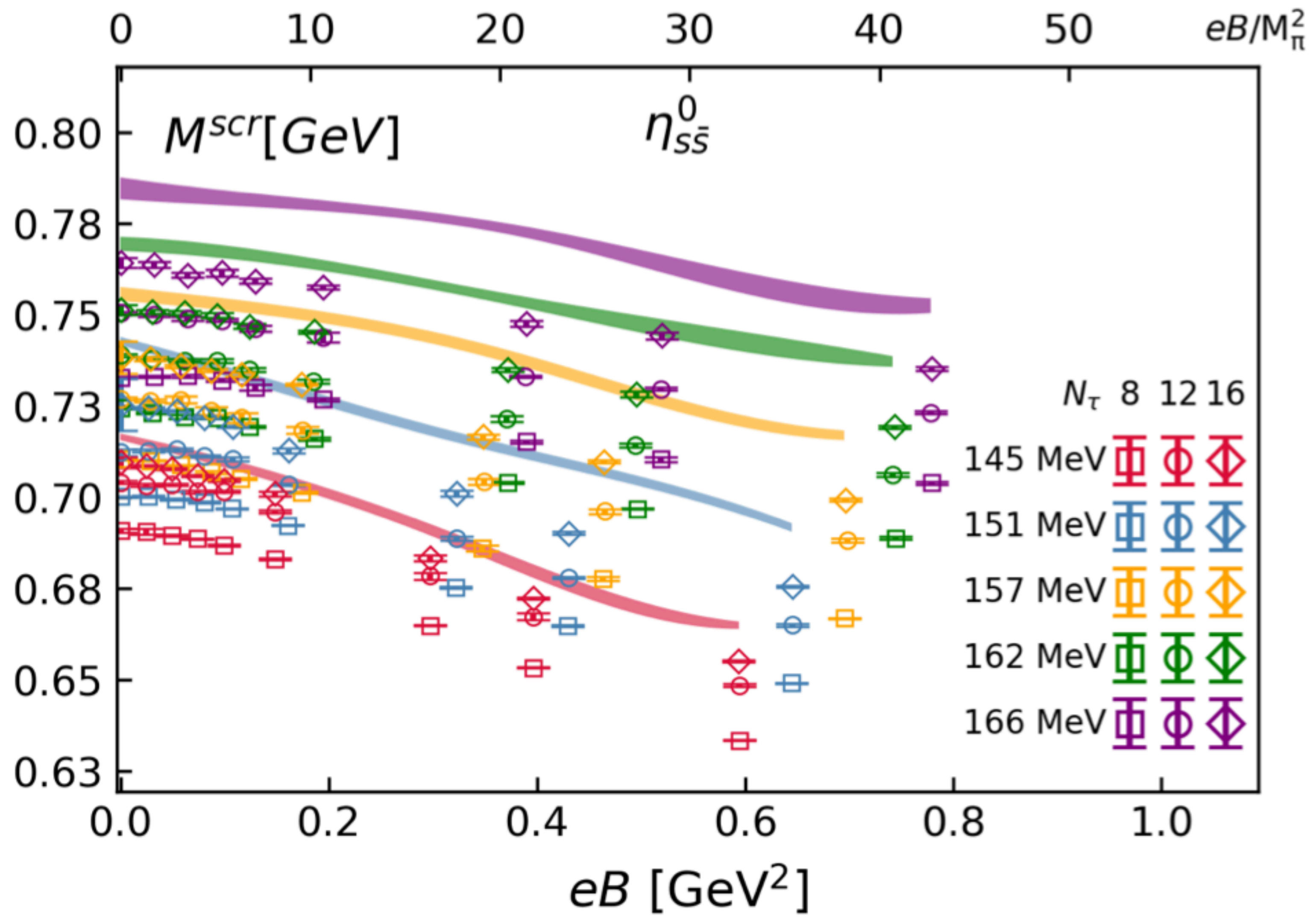


HTD, Dan Zhang, arXiv:2601.18354, PRD 26’

Screening mass of Pseudo-scalar mesons at the physical point



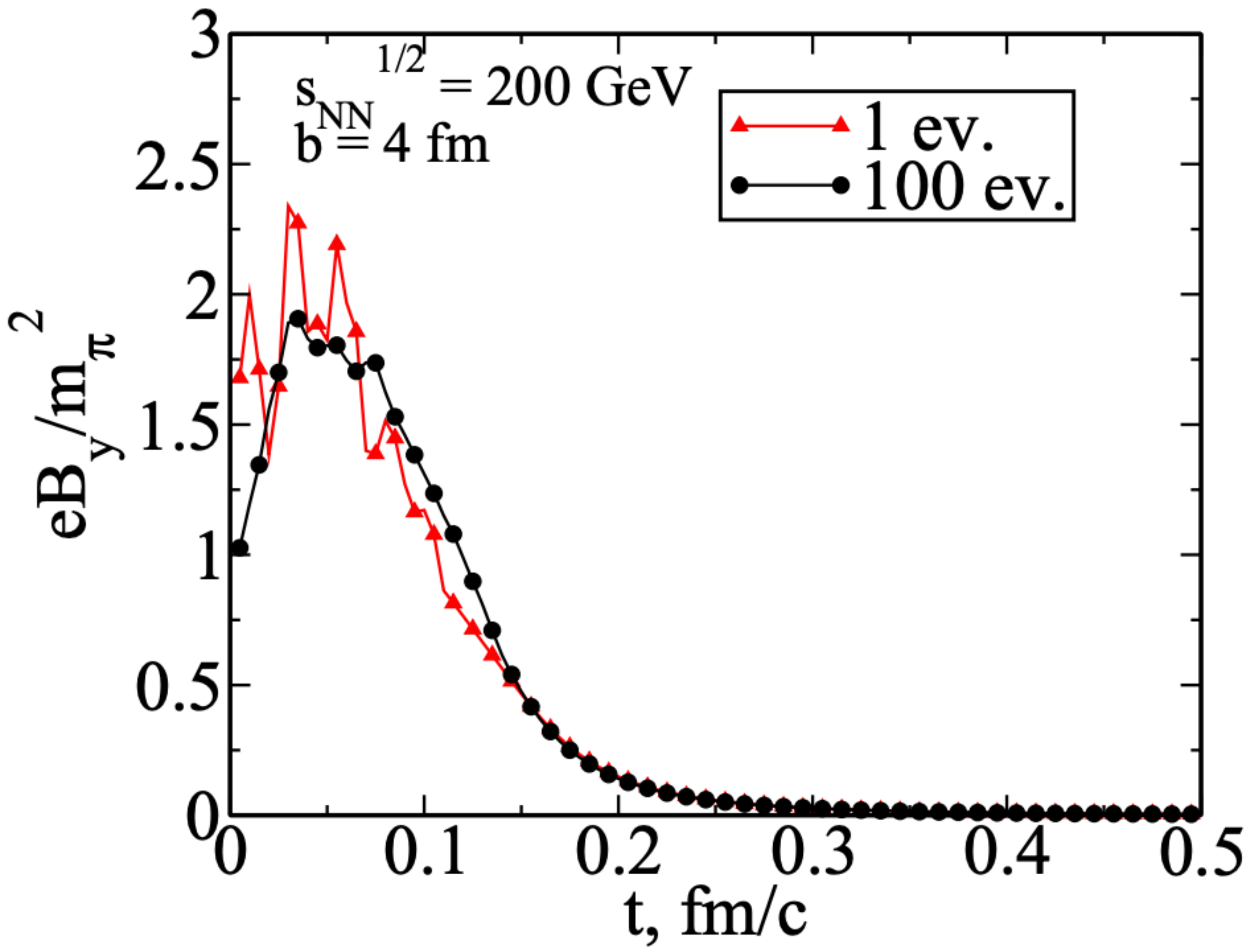




Lighter pseudoscalar channels exhibit
the stronger and more non-trivial magnetic-field response

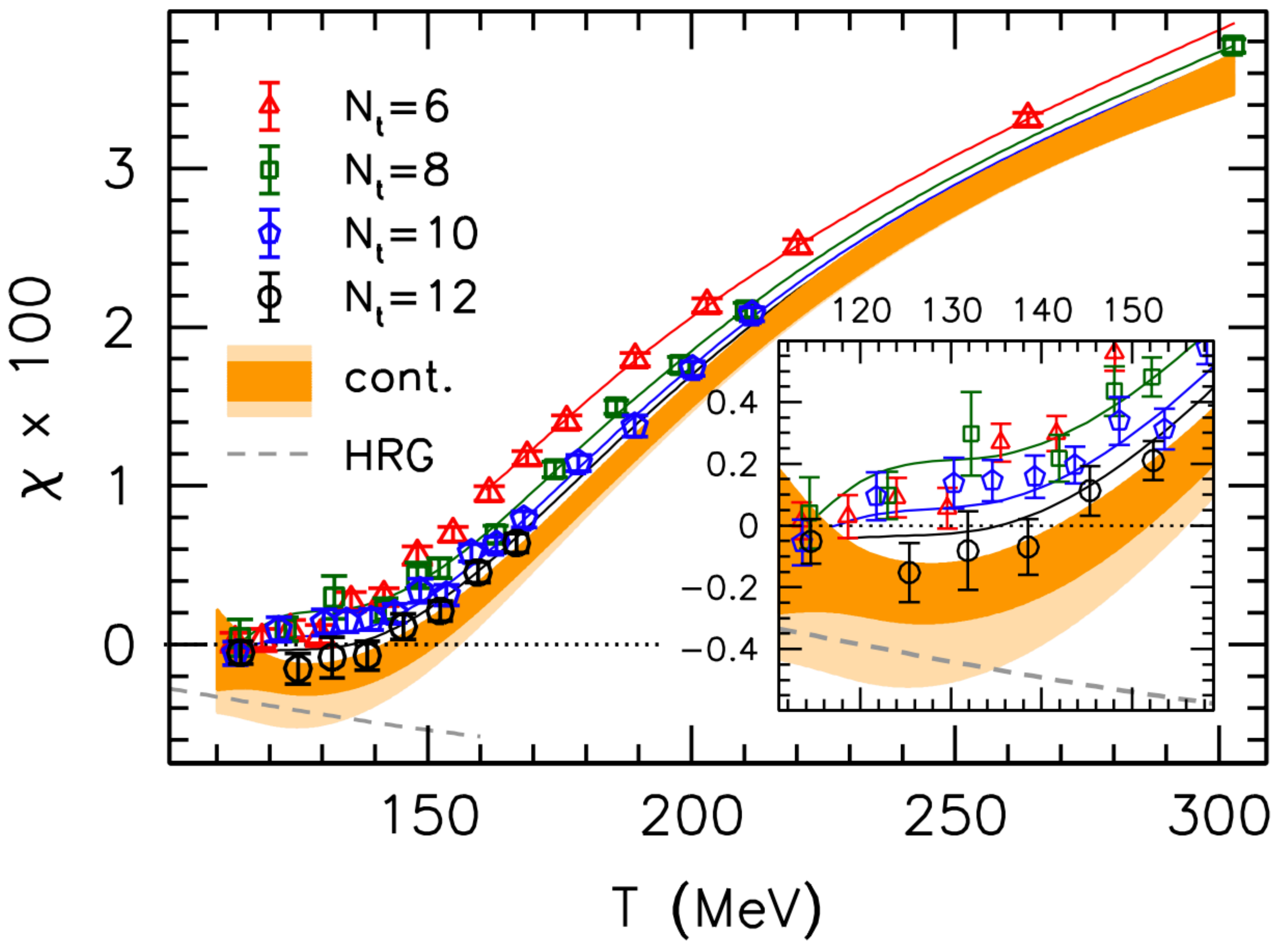
HTD, J.-B. Gu, S.-T. Li, R. Thakkar, Phys.Rev.D 111 (2025) 7, 074513

Magnetic field created in the early stage of HIC



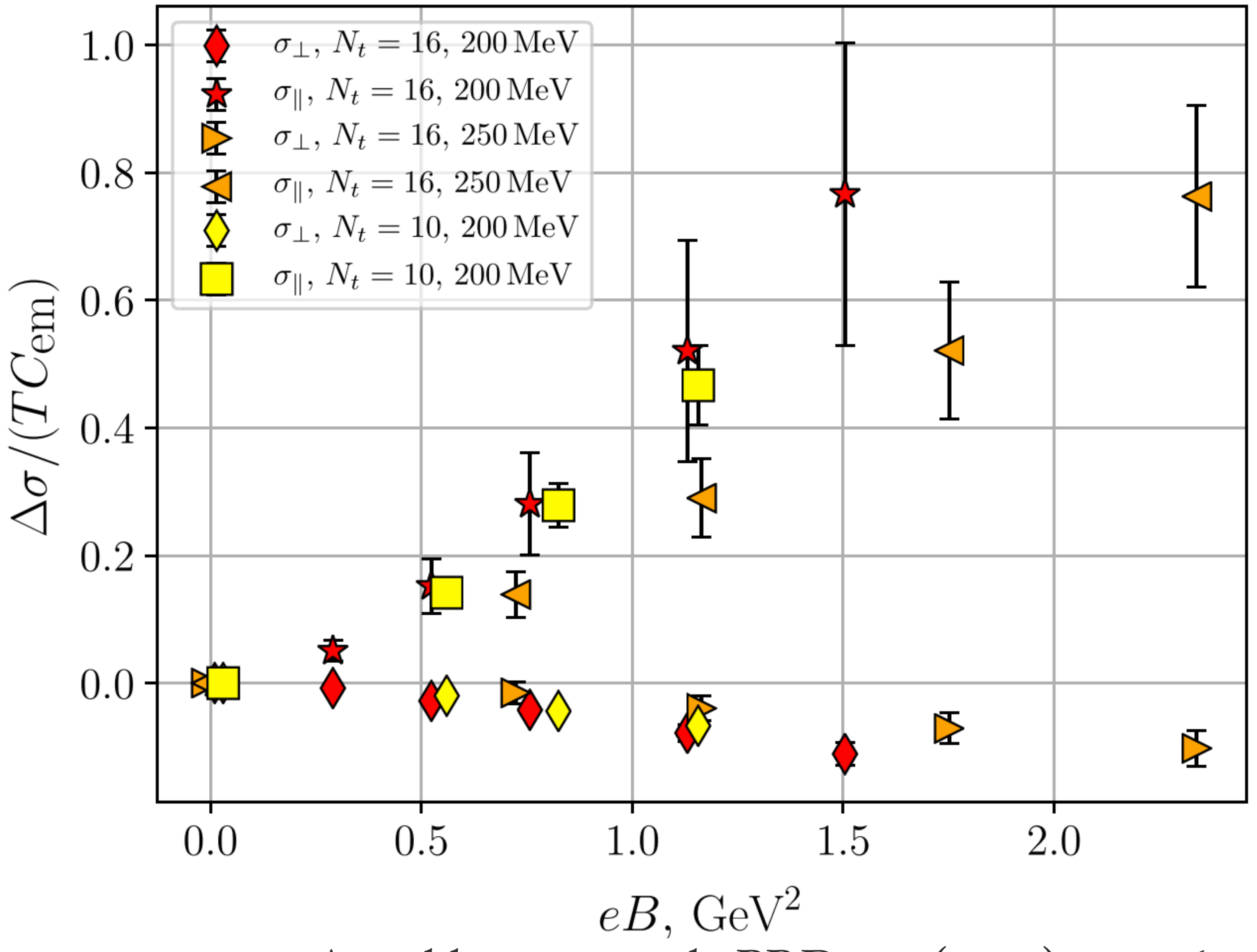
Skokov, Illarionov and Toneev, IJMPA 24 (2009) 5925

LQCD results on Magnetic susceptibility



Bali, Endrodi, Piemonte, JHEP 07 (2020) 183

LQCD results on difference to electromagnetic conductivity at *eB*=0



Astrakhantsev et al., PRD 102 (2020) 054516

t=0: RHIC: $eB \sim 5M_\pi^2$

LHC: $eB \sim 70M_\pi^2$

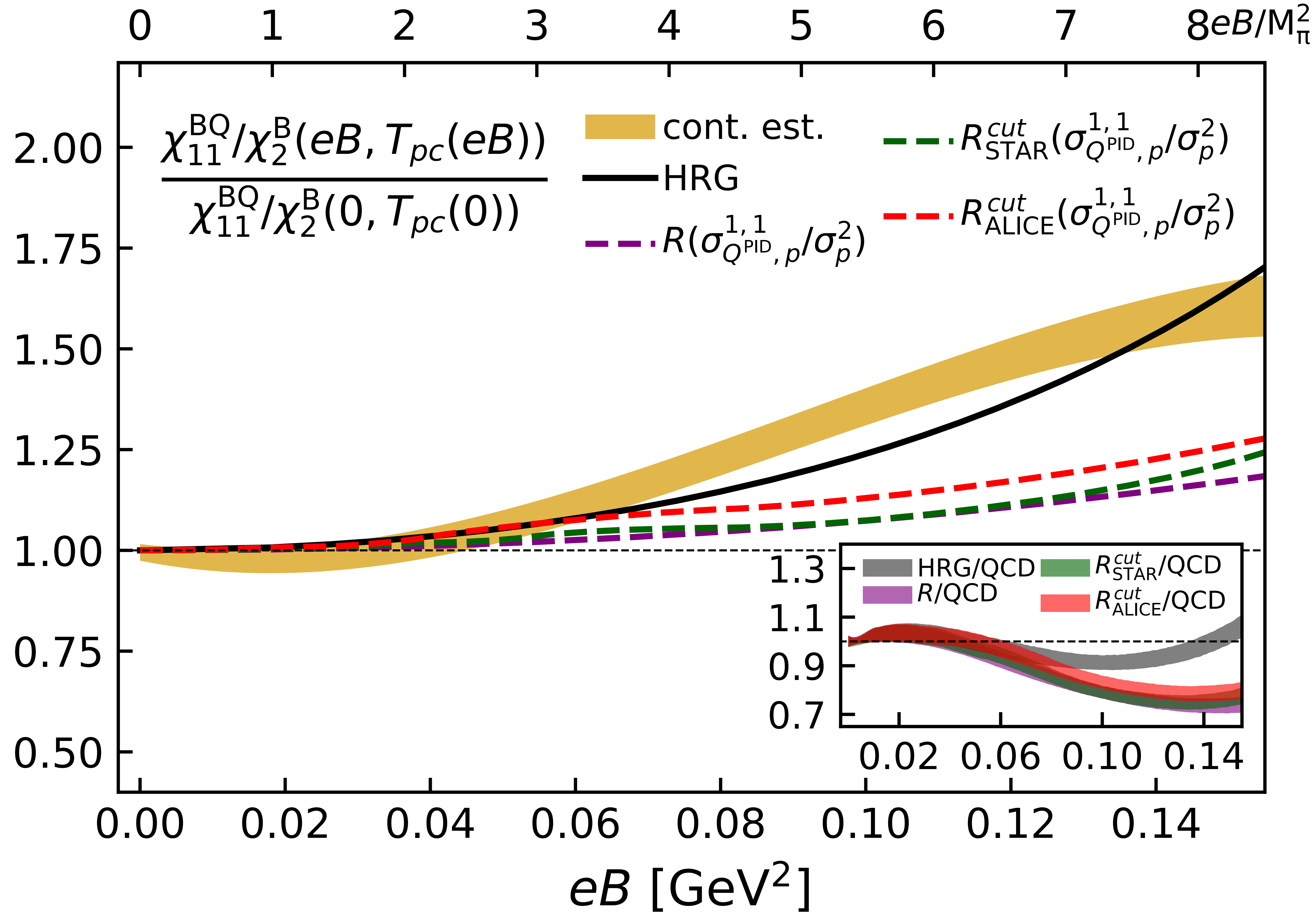
T> 155 MeV: Paramagnetic

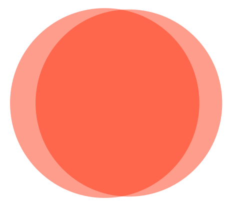
T<155 MeV: Diamagnetic

$eB \uparrow$ Parallel: $\uparrow$

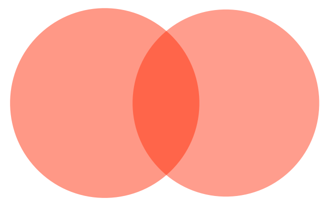
Transverse: $\downarrow$

Double ratios $R(\chi_{11}^{\mathrm{BQ}}/\mathcal{O}_1) \equiv \dfrac{\chi_{11}^{\mathrm{BQ}}/\mathcal{O}_1(eB, T_{pc}(eB))}{\chi_{11}^{\mathrm{BQ}}/\mathcal{O}_1(eB=0, T_{pc}(eB=0))}$, $\mathcal{O}_1 \equiv \{\boxed{\chi_2^{\mathrm{B}},}\ \chi_2^{\mathrm{Q}}, \chi_{11}^{\mathrm{QS}}\}$





Central collision



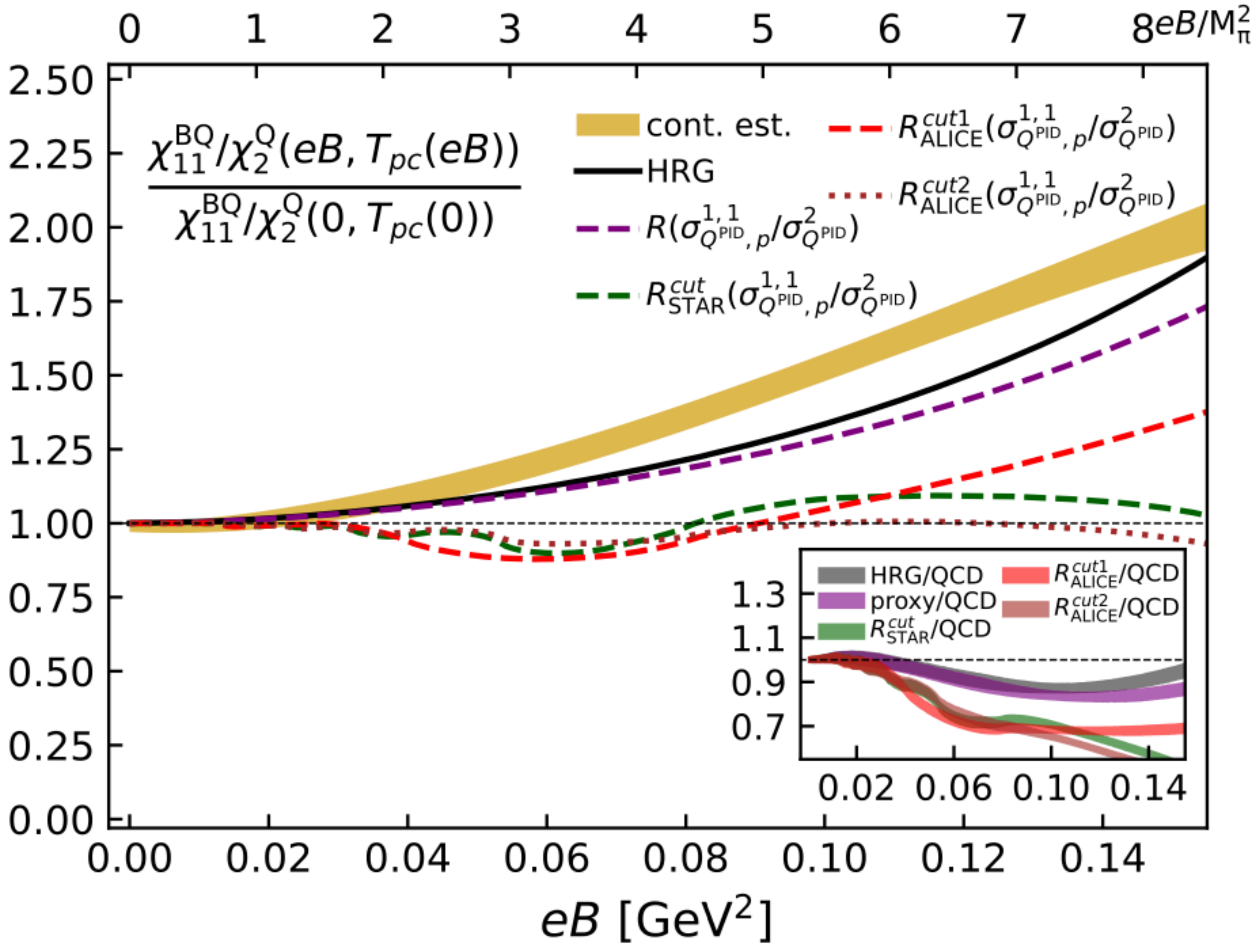
Peripheral collision

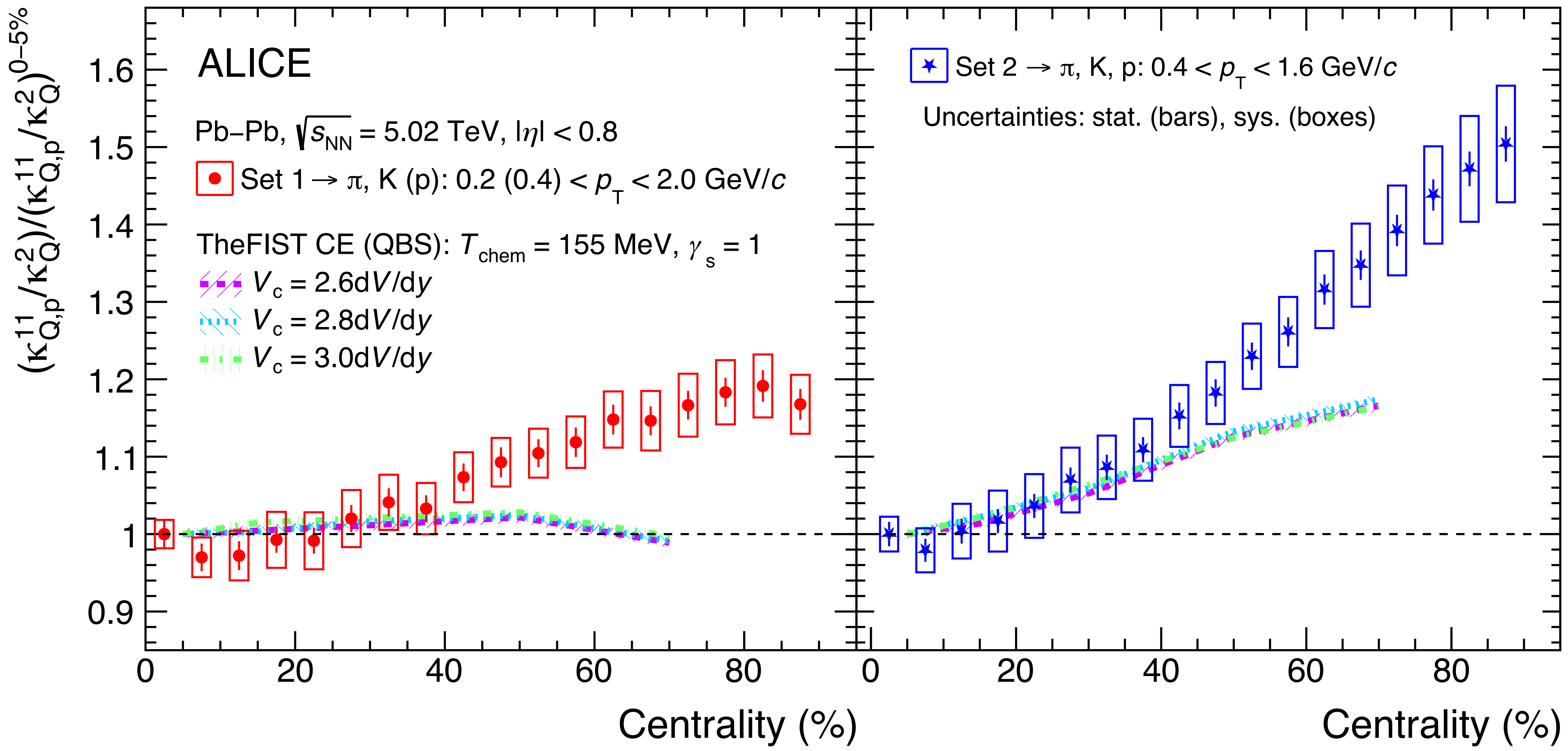
- At $eB \simeq 8M_\pi^2$, LQCD ~1.6
- expected weaker response: χ_2^{B} most sensitive in eB after χ_{11}^{BQ}
- Proxies with cuts reaches to about 1.25

HTD, J.-B. Gu, A. Kumar, S.-T. Li, PRD 25', arXiv:2503.18467

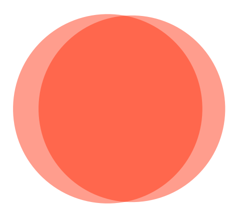
Double ratios

$$R(\chi_{11}^{BQ}/\mathcal{O}_1) \equiv \frac{\chi_{11}^{BQ}/\mathcal{O}_1(eB, T_{pc}(eB))}{\chi_{11}^{BQ}/\mathcal{O}_1(eB=0, T_{pc}(eB=0))}, \quad \mathcal{O}_1 \equiv \{\, \chi_2^{B}, \boxed{\chi_2^{Q}}, \chi_{11}^{QS} \}$$

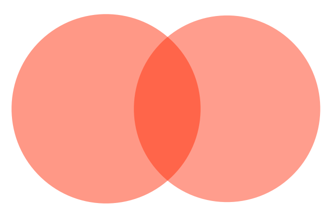




ALICE Collaboration, arXiv:2503.18743 [nucl-ex]



Central collision

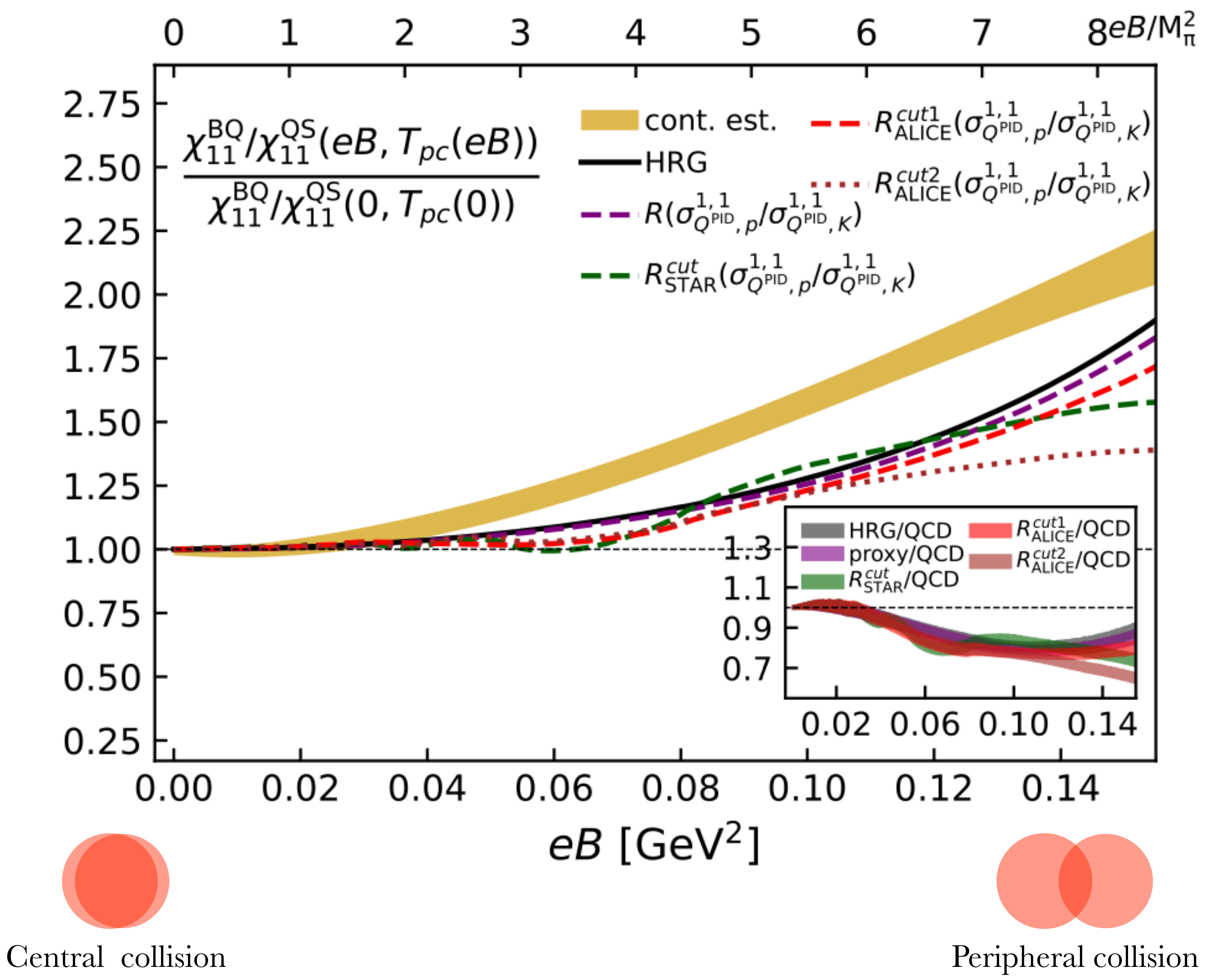


Peripheral collision

HTD, J.-B. Gu, A. Kumar, S.-T. Li, PRD 25', arXiv:2503.18467

Double ratios

$$R(\chi_{11}^{BQ}/\mathcal{O}_1) \equiv \frac{\chi_{11}^{BQ}/\mathcal{O}_1(eB, T_{pc}(eB))}{\chi_{11}^{BQ}/\mathcal{O}_1(eB=0, T_{pc}(eB=0))}, \quad \mathcal{O}_1 \equiv \{\chi_2^B, \ \chi_2^Q, \boxed{\chi_{11}^{QS}}\}$$



~ 2.2 Lattice QCD

$\sim 1.6 - 1.75$ Proxies

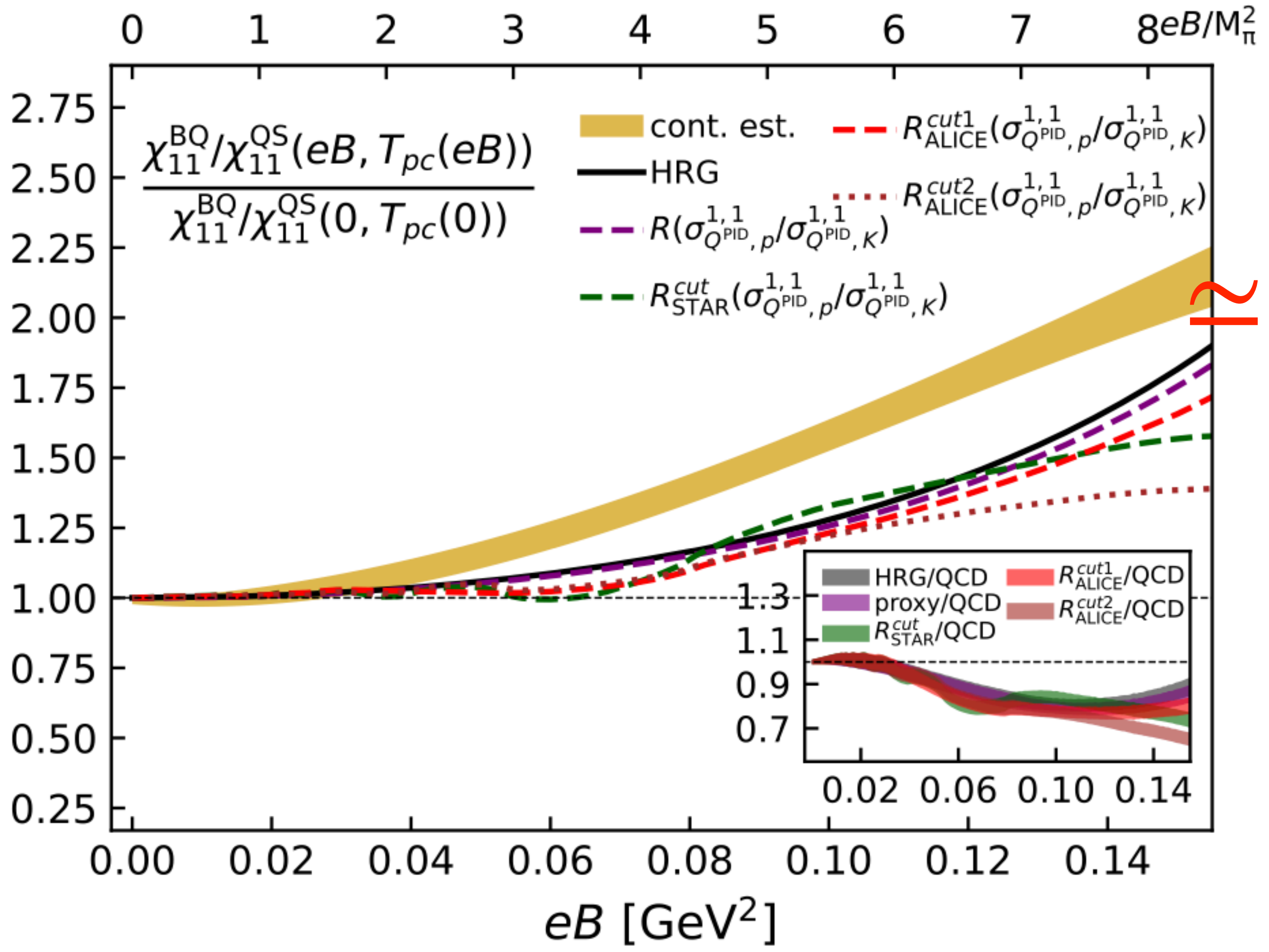
$\sim 75 - 84\,\%$ Proxies/LQCD

$R(\chi_{11}^{BQ}/\chi_{11}^{QS})$

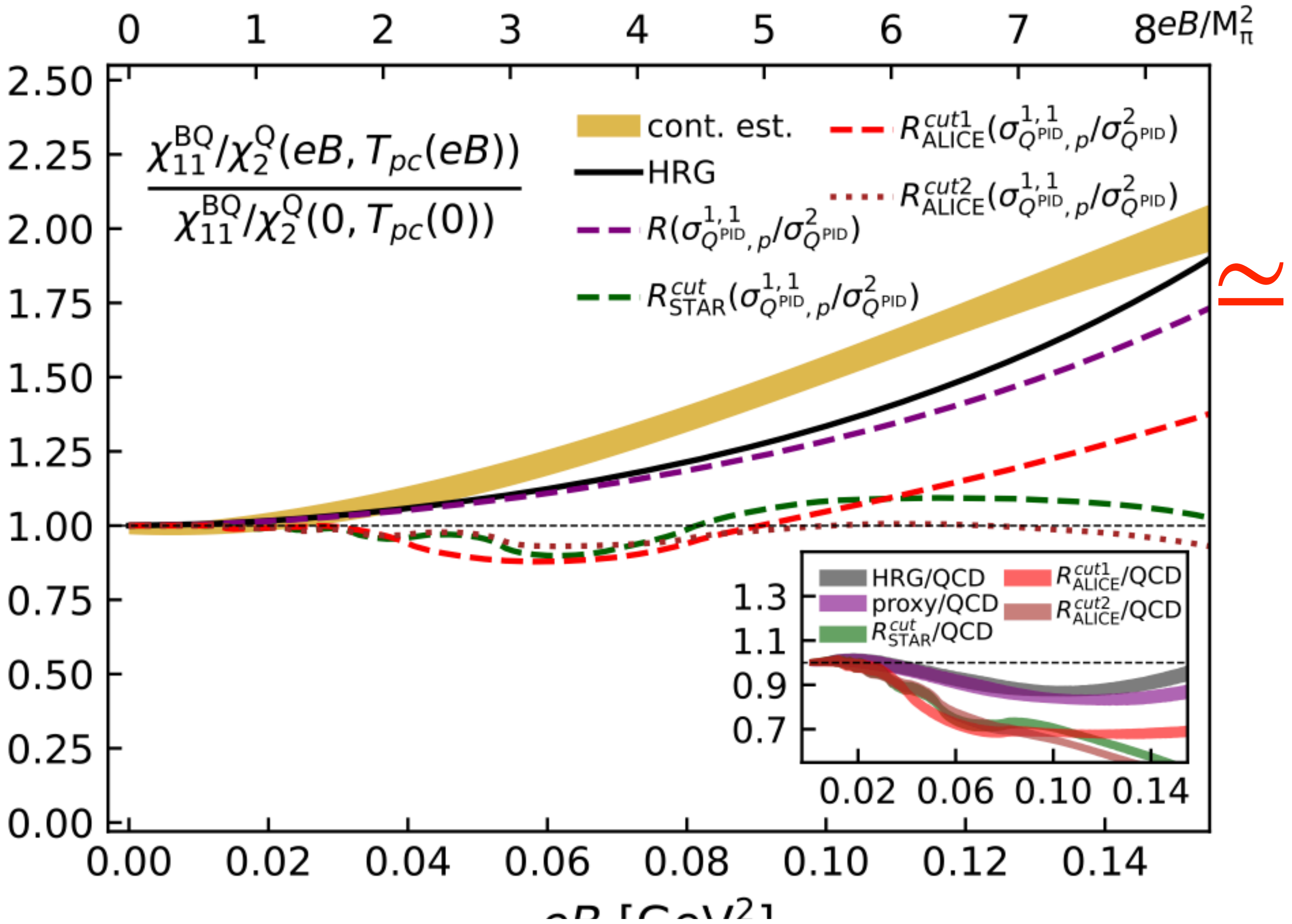
Even more effective probe in both ALICE & STAR!

HTD, J.-B. Gu, A. Kumar, S.-T. Li, PRD 25', arXiv:2503.18467

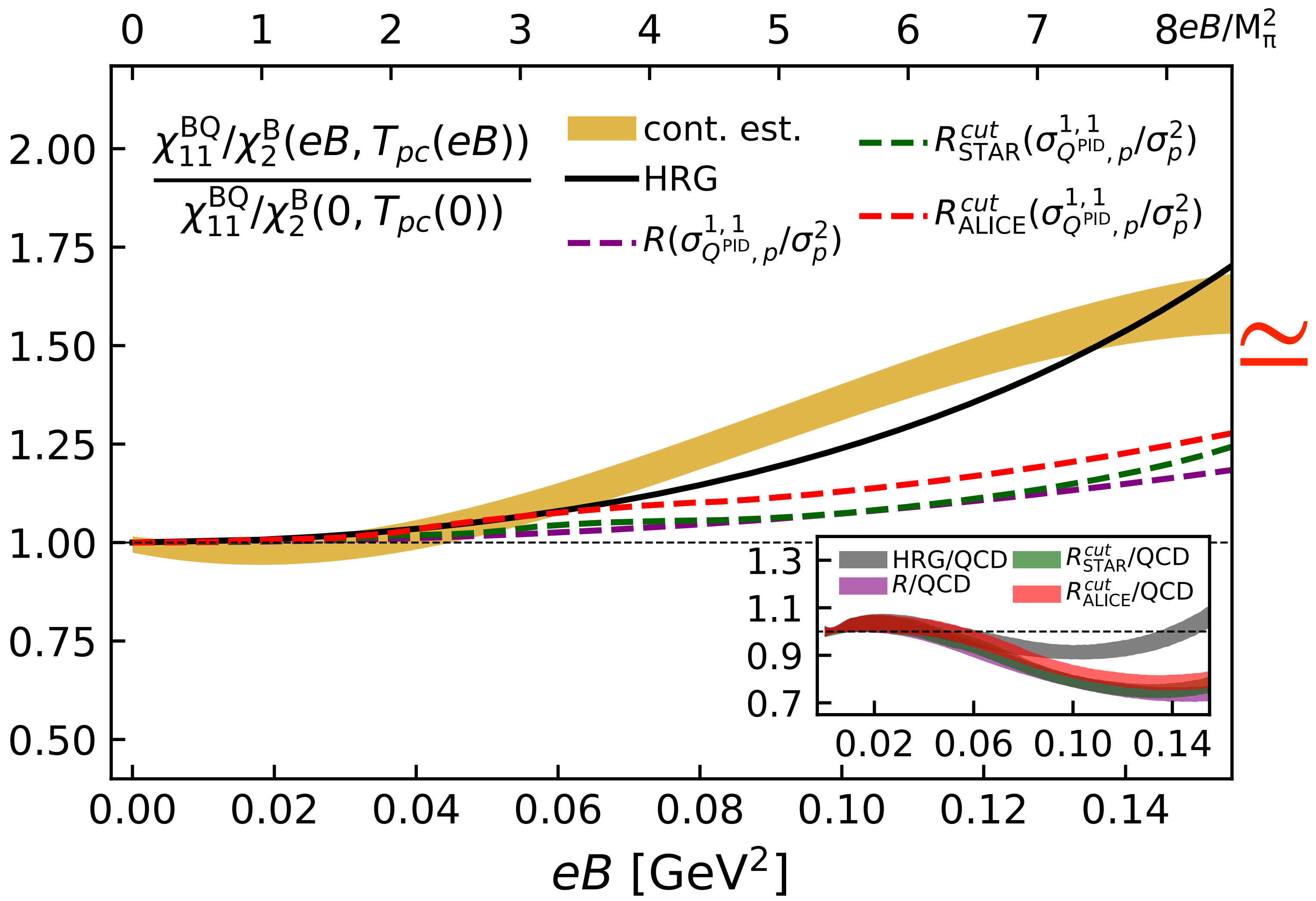
Ordering in magnetic responses of 2nd order conserved charge susceptibilities



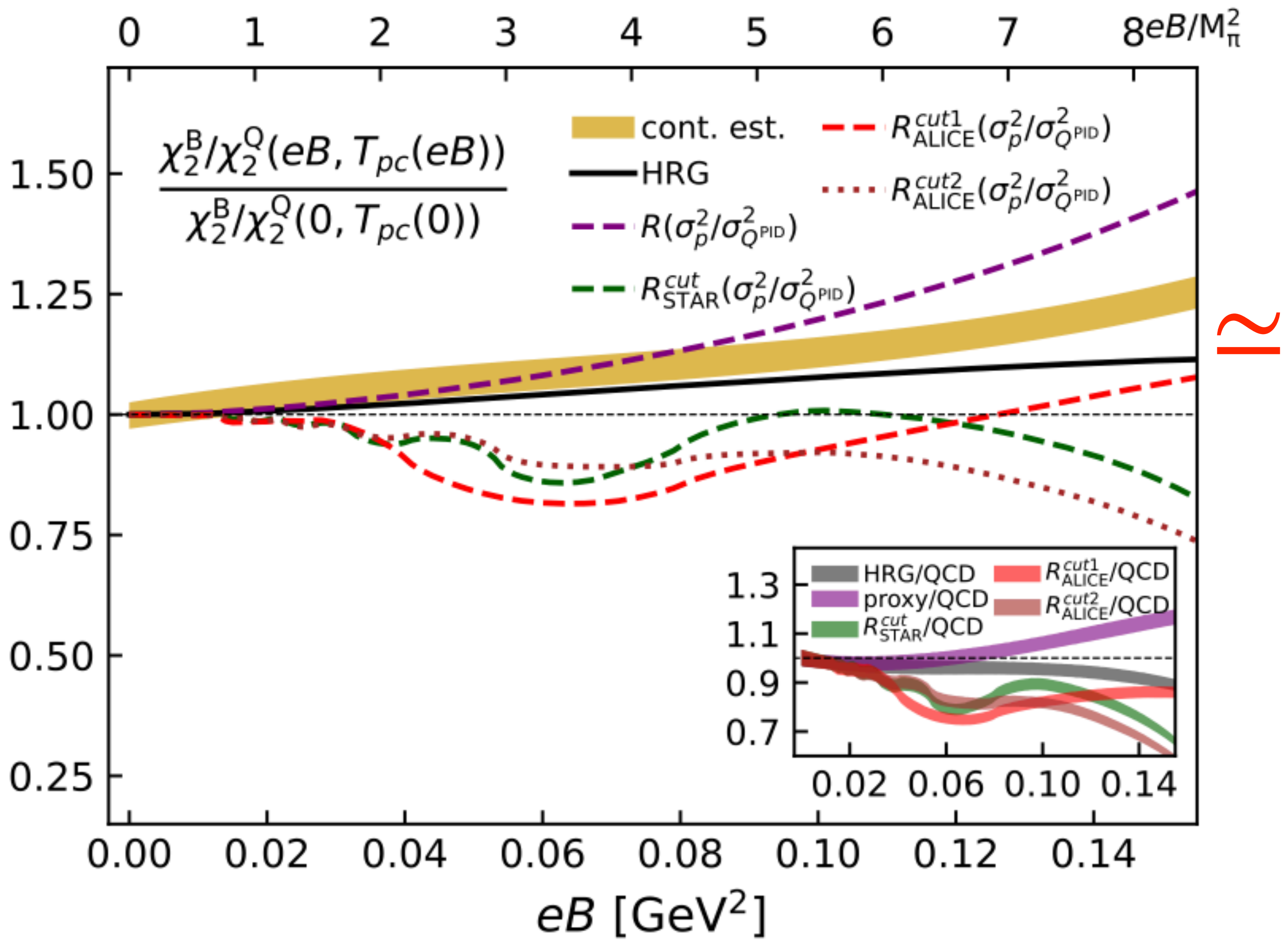
$\simeq 2.2$



$\simeq 2$



$\simeq 1.6$



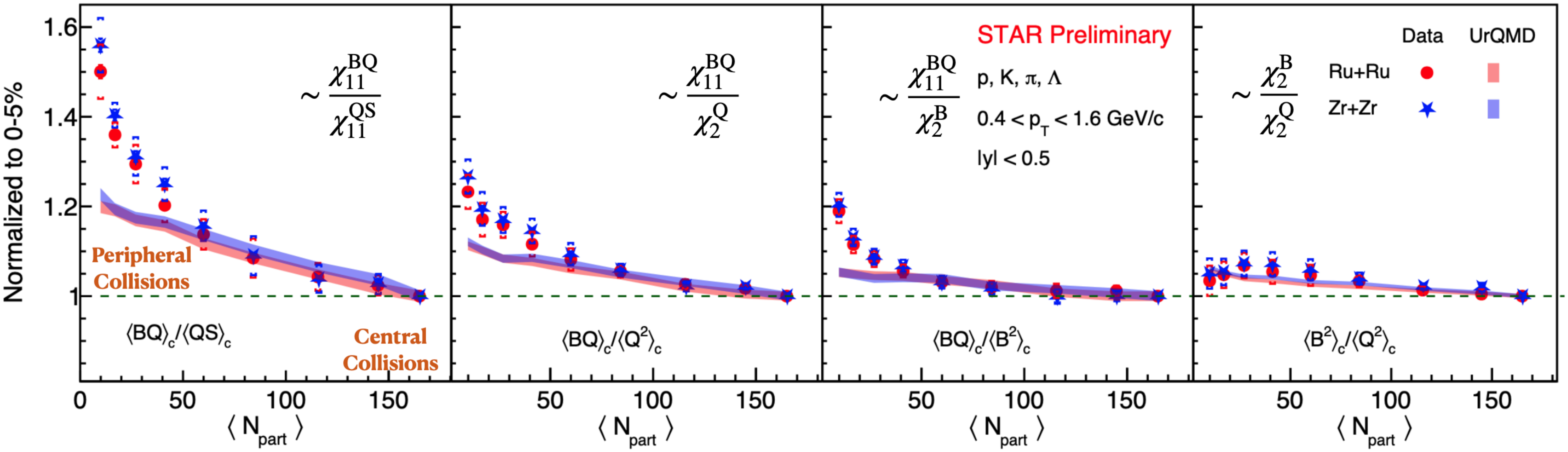
$\simeq 1.2$

Ordering from LQCD

$$\frac{\chi_{11}^{BQ}}{\chi_{11}^{QS}} > \frac{\chi_{11}^{BQ}}{\chi_{2}^{Q}} > \frac{\chi_{11}^{BQ}}{\chi_{2}^{B}} > \frac{\chi_{2}^{B}}{\chi_{2}^{Q}}$$

Ordering in magnetic responses of 2nd order conserved charge susceptibilities

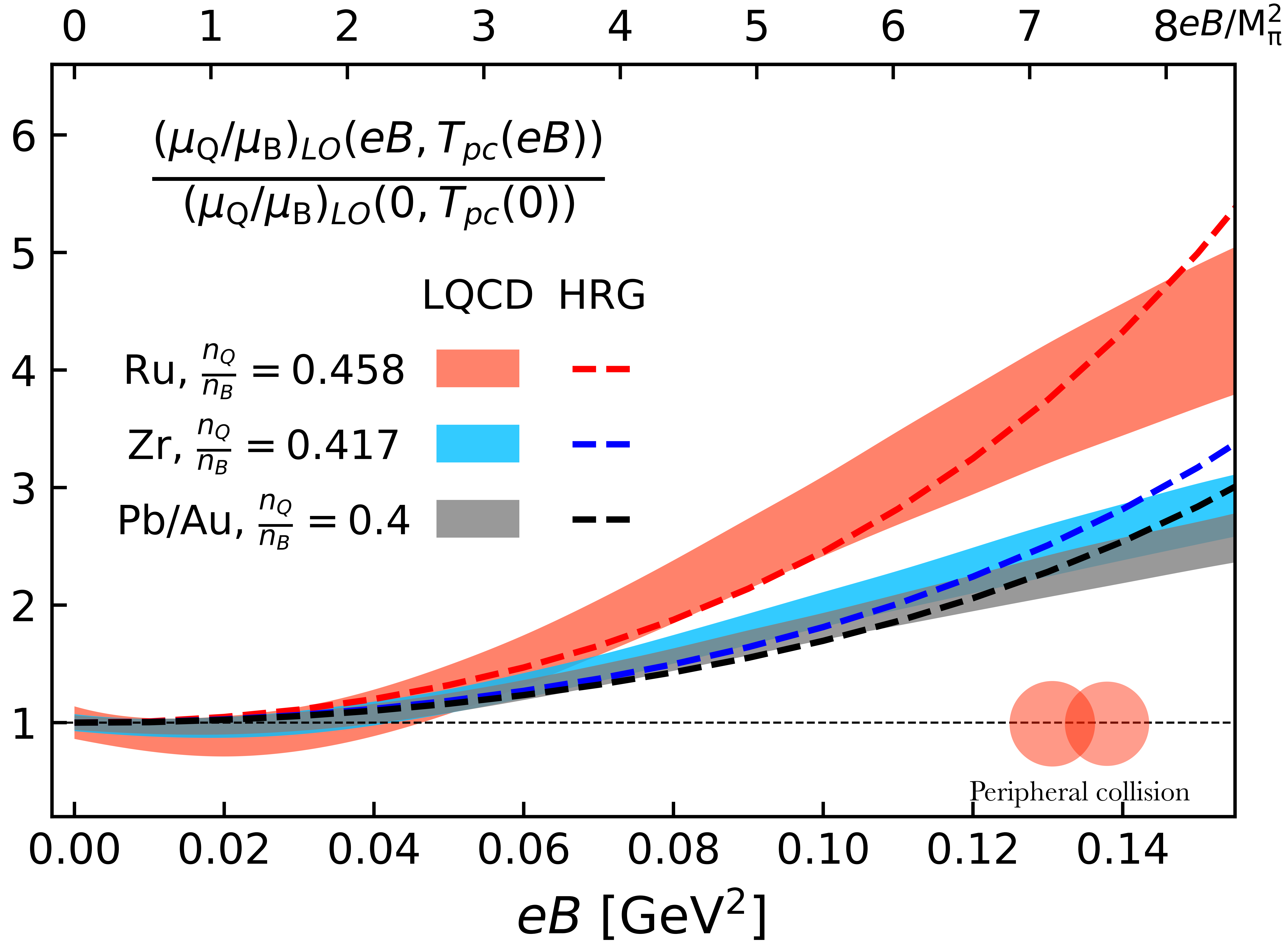
LQCD predicts: $\dfrac{\chi_{11}^{BQ}}{\chi_{11}^{QS}} > \dfrac{\chi_{11}^{BQ}}{\chi_{2}^{Q}} > \dfrac{\chi_{11}^{BQ}}{\chi_{2}^{B}} > \dfrac{\chi_{2}^{B}}{\chi_{2}^{Q}}$



Reported by Changfeng Li(STAR) @SQM 2026

STAR: similar ordering observed from the centrality dependence

$\mu_{\mathrm{Q}}/\mu_{\mathrm{B}}$ in different collision systems



$$\mu_{\mathrm{Q}}/\mu_{\mathrm{B}} = q_1 + q_3\mu_{\mathrm{B}}^2 + \mathcal{O}(\mu_B^4)$$

$$q_1 = \frac{r(\chi_2^{\mathrm{B}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{B}S}\chi_{11}^{\mathrm{B}S}) - (\chi_{11}^{\mathrm{B}Q}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{B}S}\chi_{11}^{\mathrm{Q}S})}{(\chi_2^{\mathrm{Q}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{Q}S}\chi_{11}^{\mathrm{Q}S}) - r(\chi_{11}^{\mathrm{B}Q}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{B}S}\chi_{11}^{\mathrm{Q}S})}$$

$$r = n_{\mathrm{Q}}/n_{\mathrm{B}}$$

${}^{96}_{44}\mathrm{Ru} + {}^{96}_{44}\mathrm{Ru}$: r=0.458

${}^{96}_{40}\mathrm{Zr} + {}^{96}_{40}\mathrm{Zr}$: r=0.417

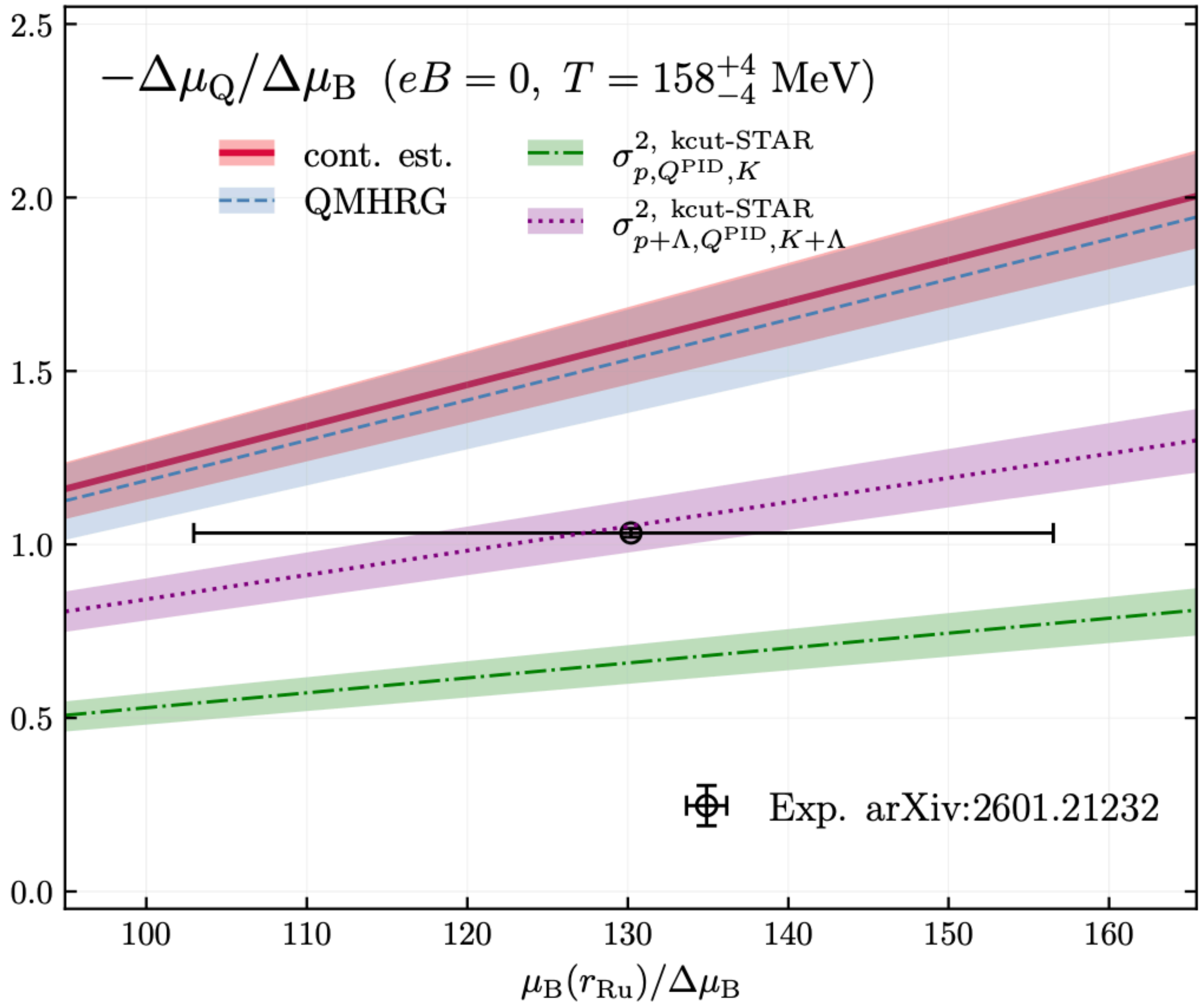
${}^{208}_{82}\mathrm{Pb} + {}^{208}_{82}\mathrm{Pb}$: r=0.4

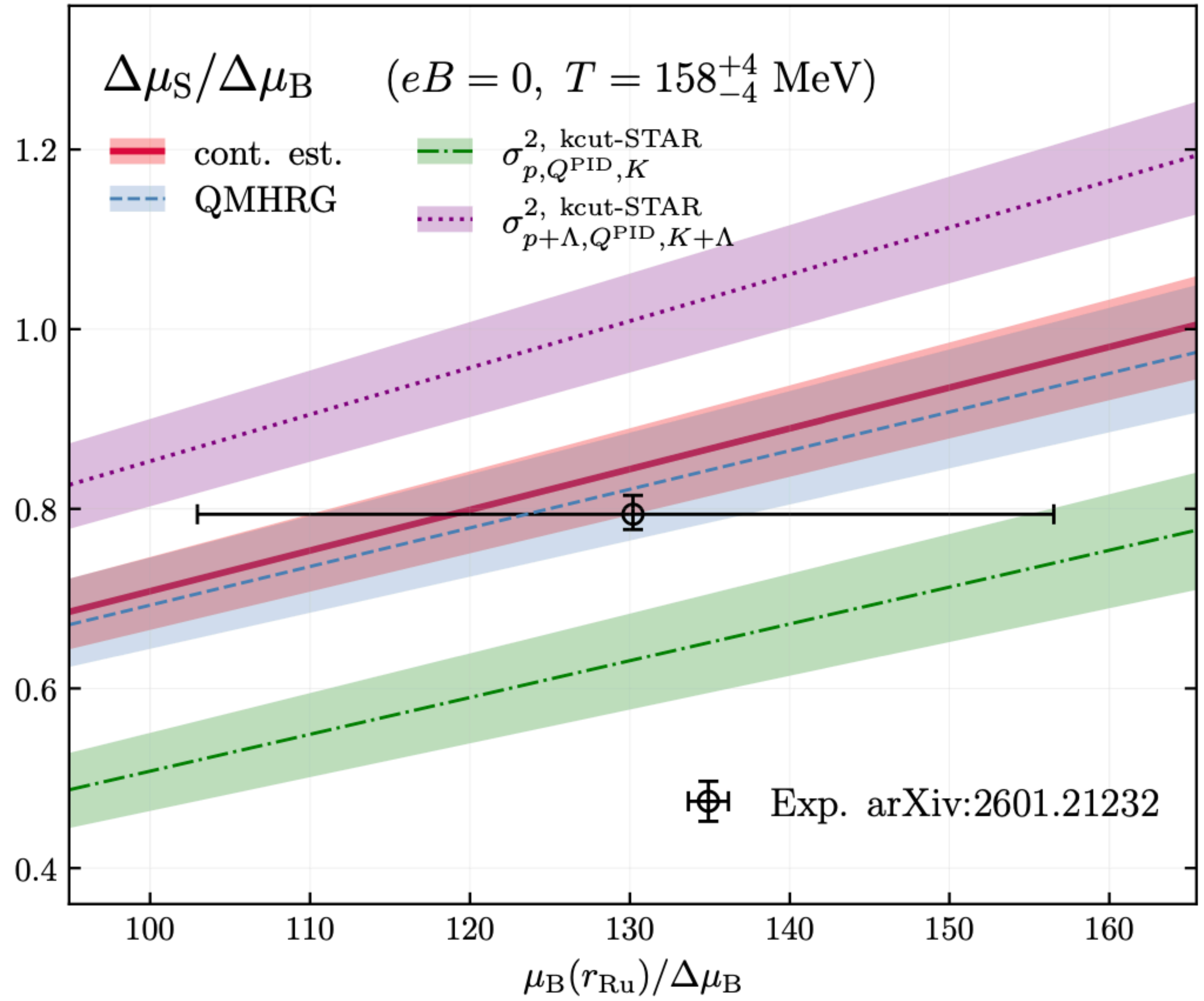
HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, PRL 132 (2024) 201903

Chemical potential splittings in isobar collisions in central collisions

$$\frac{\Delta\mu_Q}{\Delta\mu_B} \equiv q_1(r_{Zr}) + \left[q_1(r_{Zr}) - q_1(r_{Ru})\right]\frac{\mu_B(r_{Ru})}{\Delta\mu_B}$$

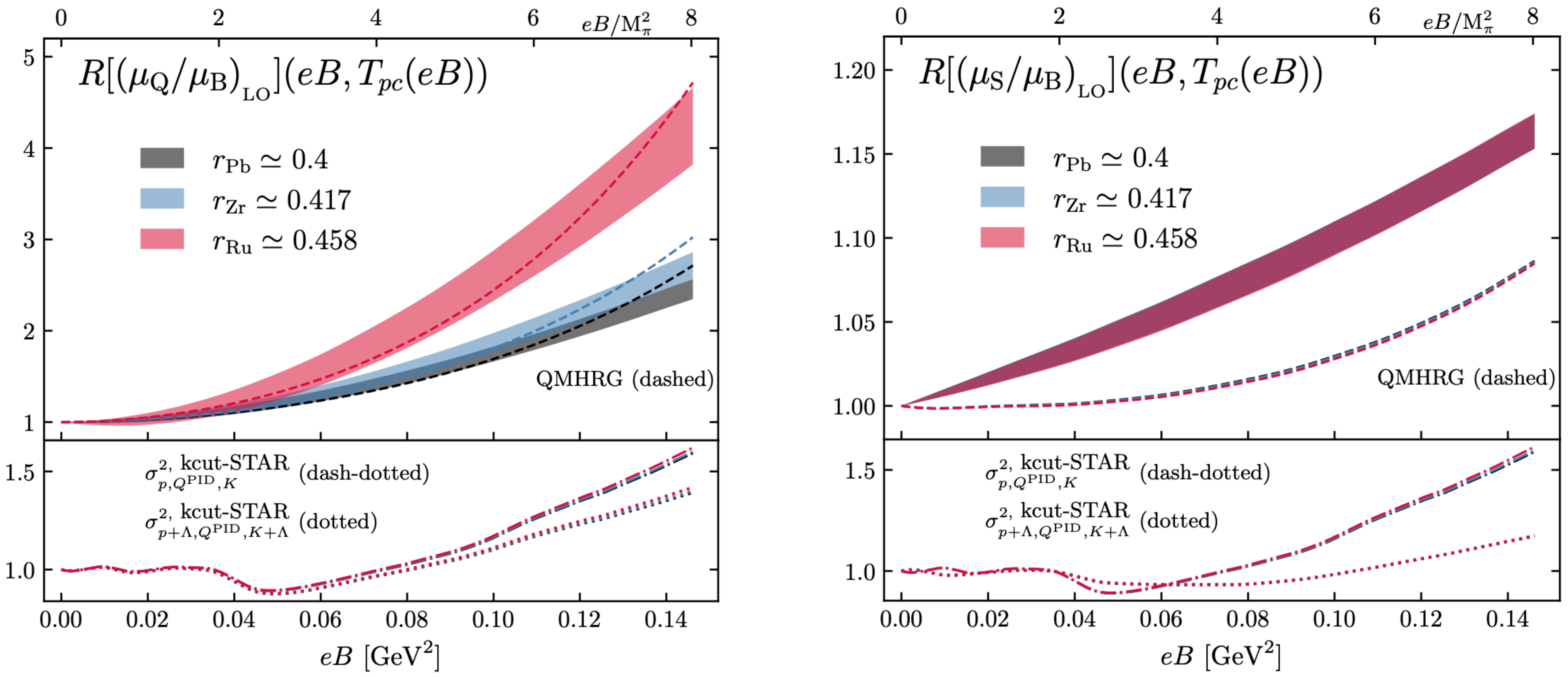
$$\frac{\Delta\mu_S}{\Delta\mu_B} \equiv s_1(r_{Zr}) + \left[s_1(r_{Zr}) - s_1(r_{Ru})\right]\frac{\mu_B(r_{Ru})}{\Delta\mu_B}$$





HTD, Jin-Biao Gu,Arpith Kumar, and Jia Ni, arXiv:2606.30164

Magnetic field responses in the normalized ratios



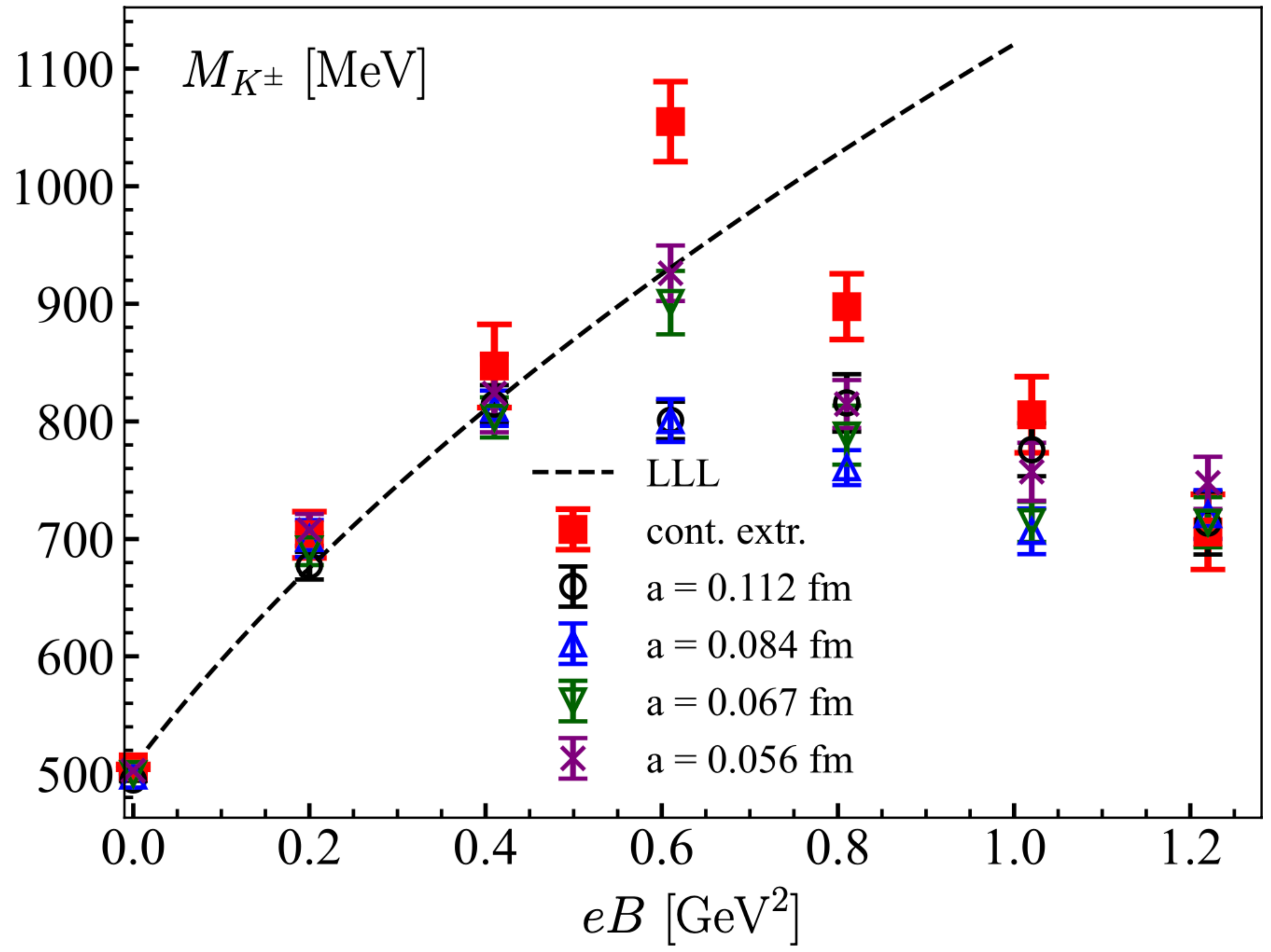
HTD, Jin-Biao Gu,Arpith Kumar, and Jia Ni, arXiv:2606.30164

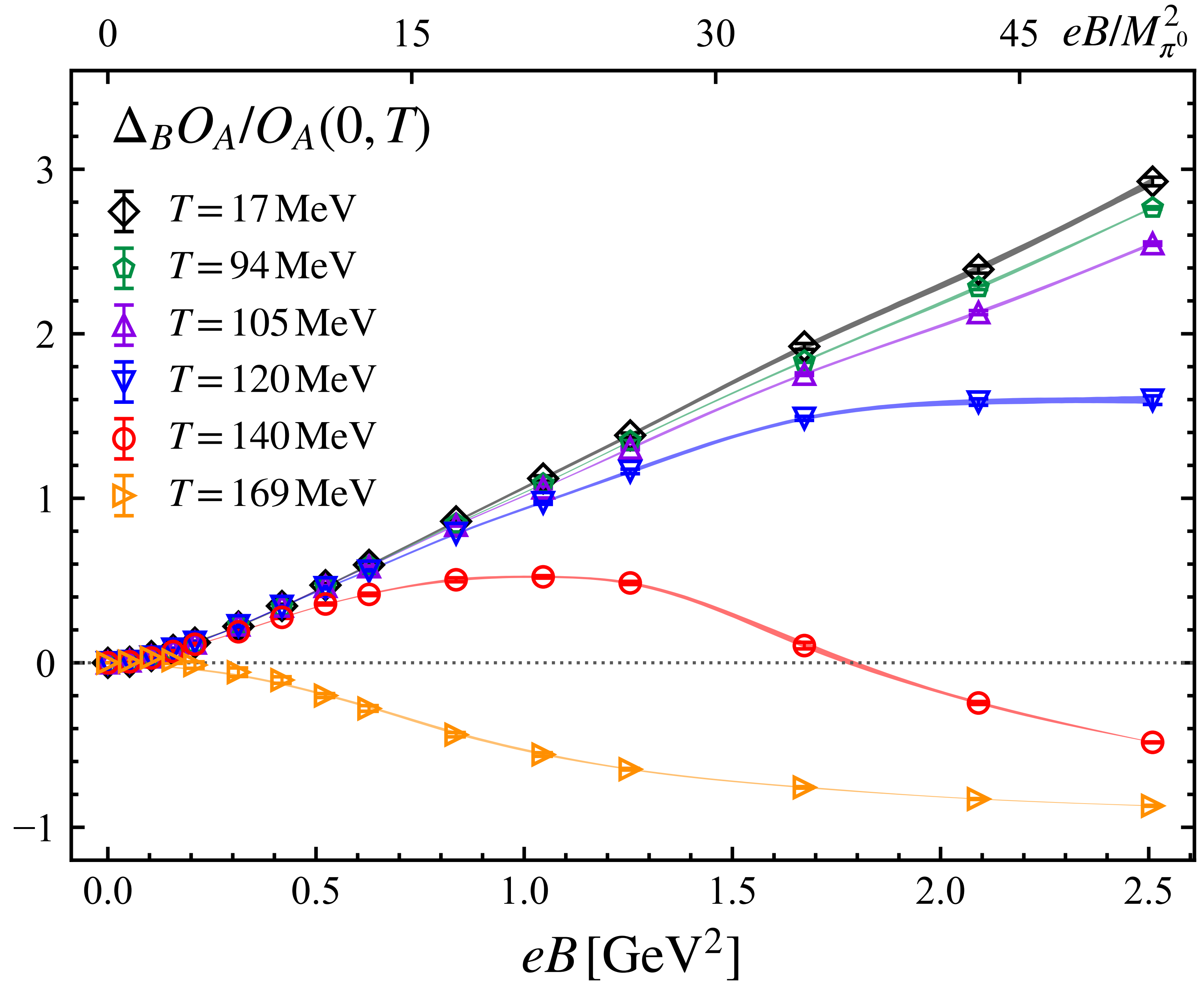
Summary

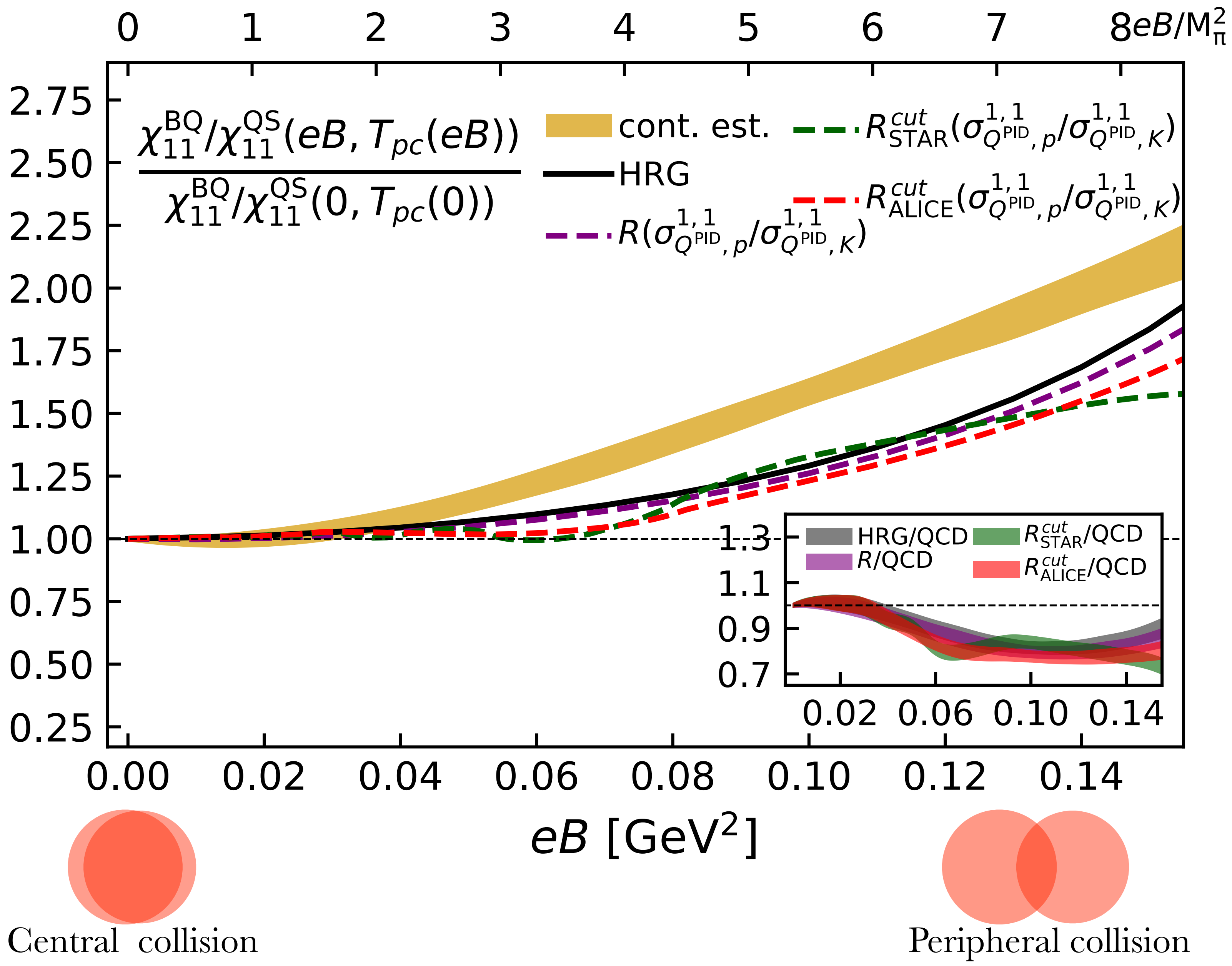
- Masses of charged PS meson: Non-montonic dependence on eB; Masses of neutral PS mesons: monotonic decreases as eB increases

- IMC and MC like behavior in the neutral susceptibilities measures of chiral symmetry and $U(1)_A$

- Probes to detect imprints of magnetic fields in HIC: χ_{11}^{BQ} measured from proxy and μ_Q/μ_B obtained from thermal fits to particle yields in HIC







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