FRG approach to the QCD phase diagram

Wei-jie Fu

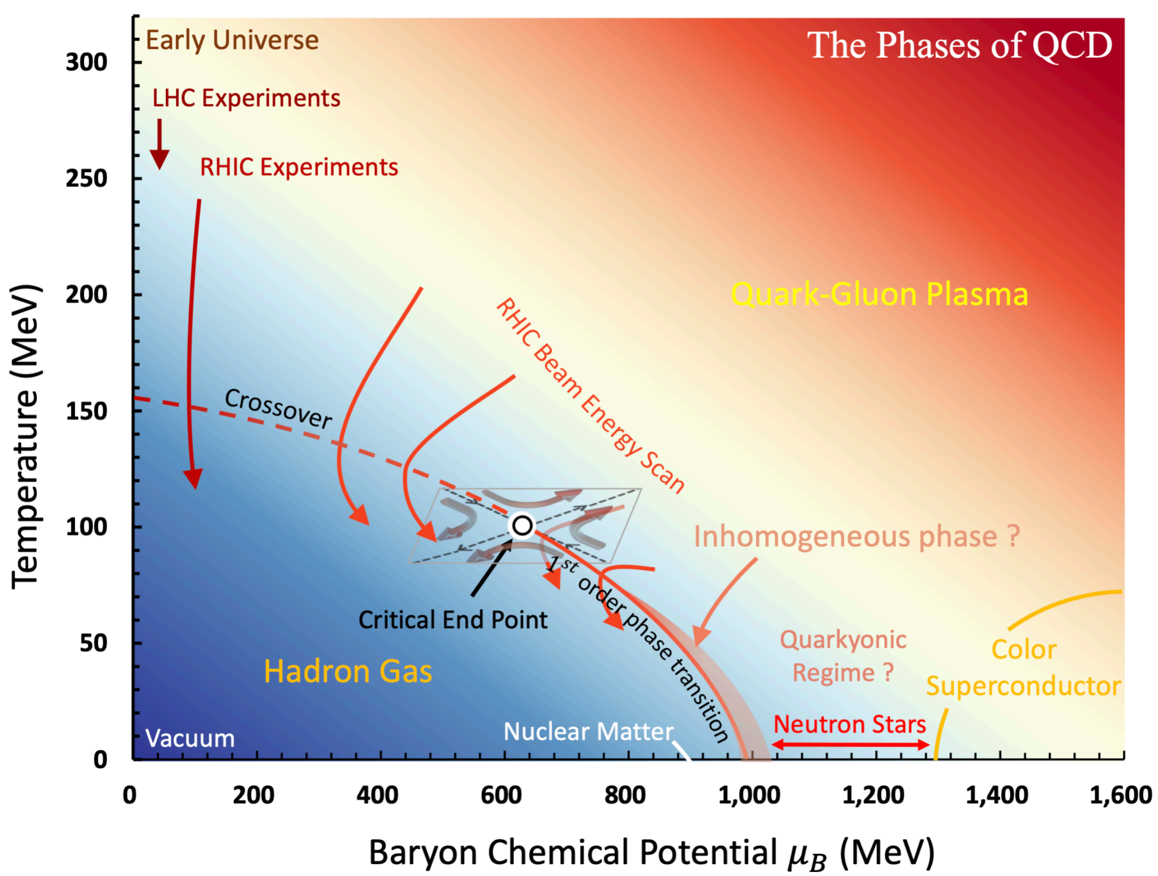
Dalian University of Technology

第十二届高能物理大会，广州，2026 年 7 月 13-19 日

Based on:
Zi-ning Wang, Li-jun Zhou, Chuang Huang, Rui Wen, Shi Yin, WF, arXiv:2607.07354.
WF, Chuang Huang, Jan M. Pawlowski, Fabian Rennecke, Rui Wen, Shi Yin, arXiv:2603.13455.
Rui-zhe Zha, Shi Yin, Shanjin Wu, Lipei Du, Xiaofeng Luo, WF, in preparation.

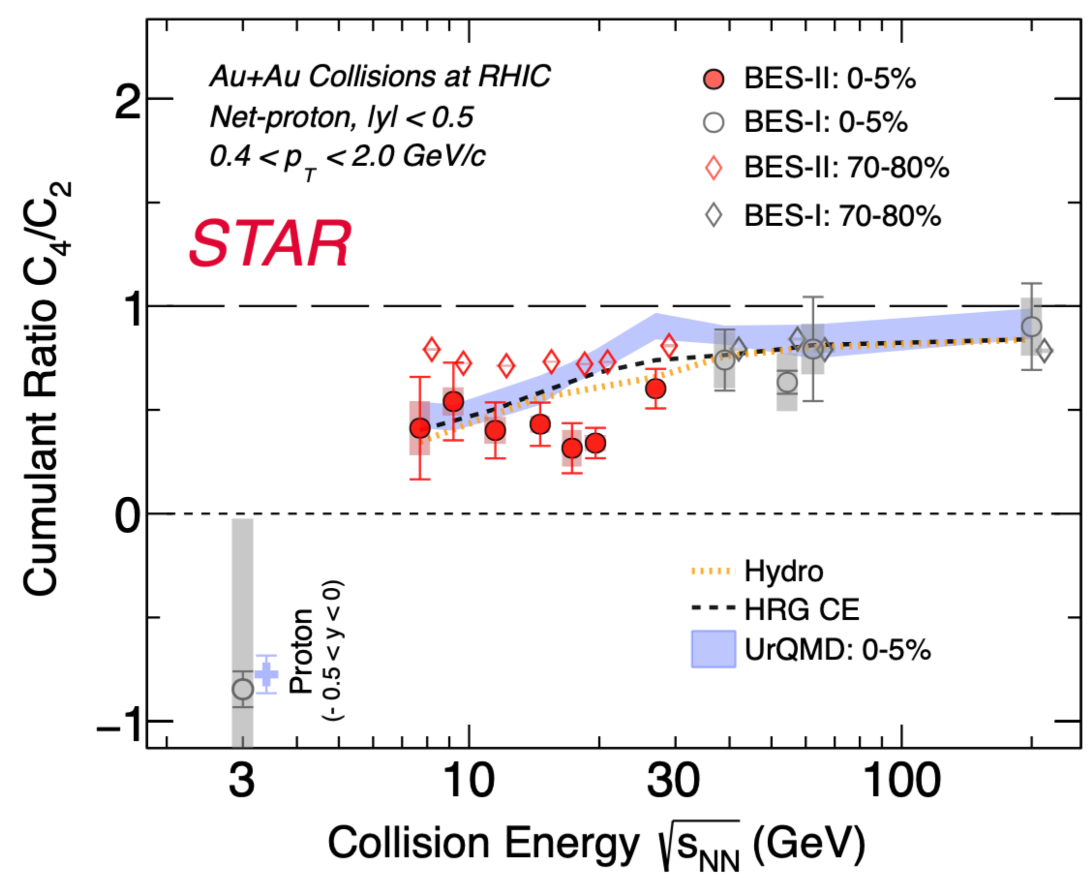
CEP in QCD phase diagram

QCD phase diagram



Non-monotonicity:
M. Stephanov, *PRL* 107 (2011) 052301

Fluctuations measured in BES-II

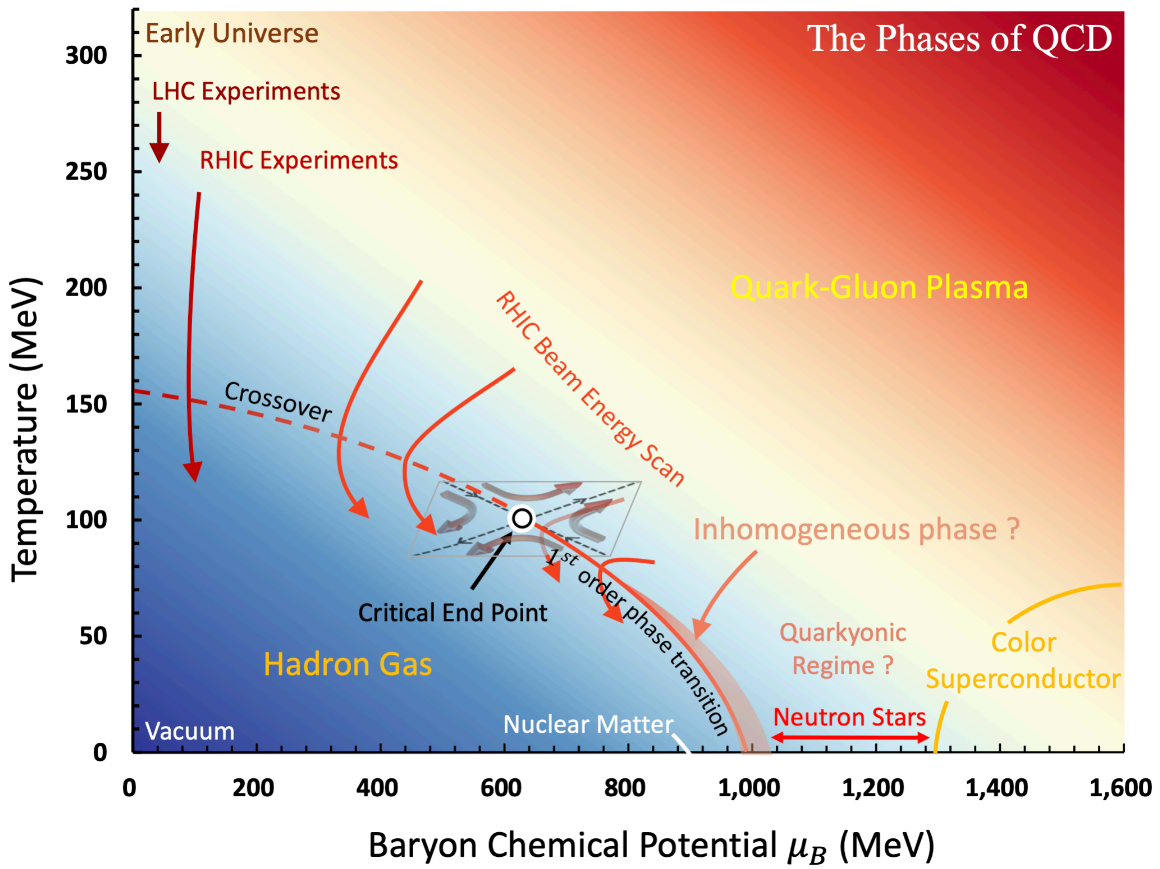


STAR Collaboration, *PRL* 135 (2025) 142301, arXiv:2504.00817

- Is there a "peak" structure serving as the smoking gun signal for the critical end point in the QCD phase diagram?

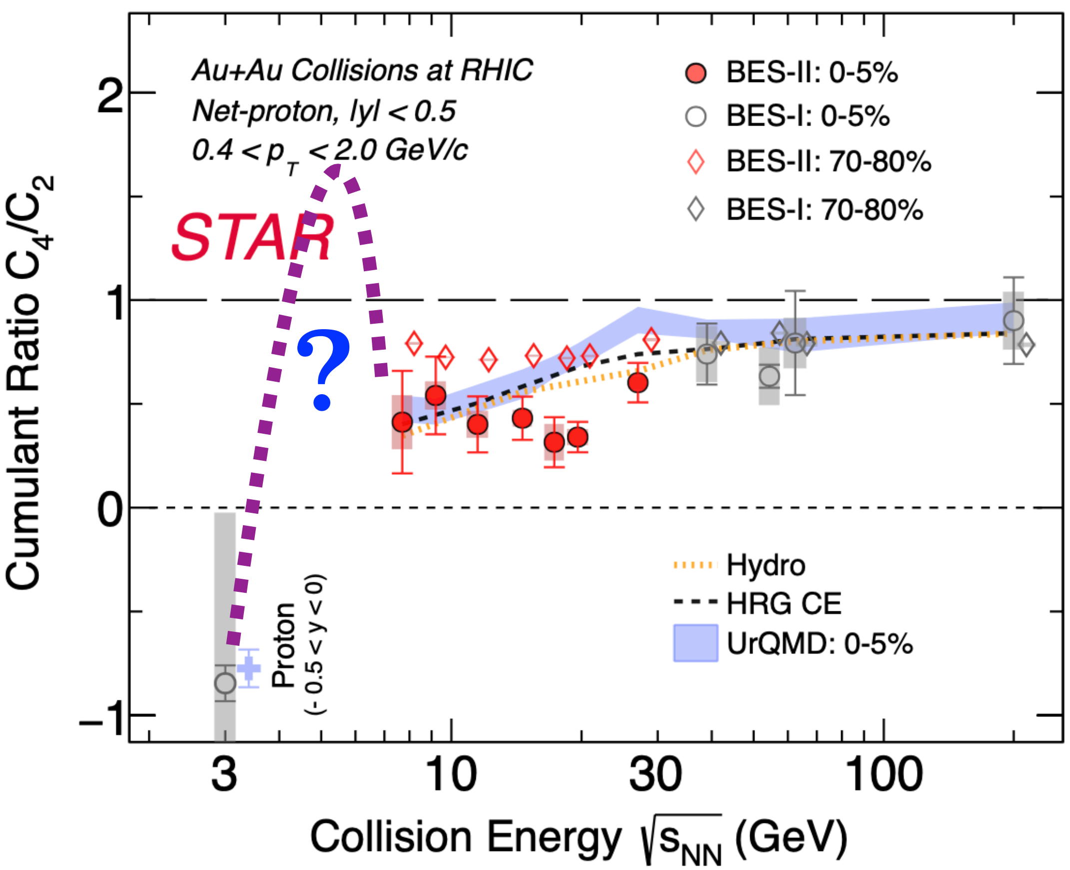
CEP in QCD phase diagram

QCD phase diagram



Non-monotonicity:
M. Stephanov, *PRL* 107 (2011) 052301

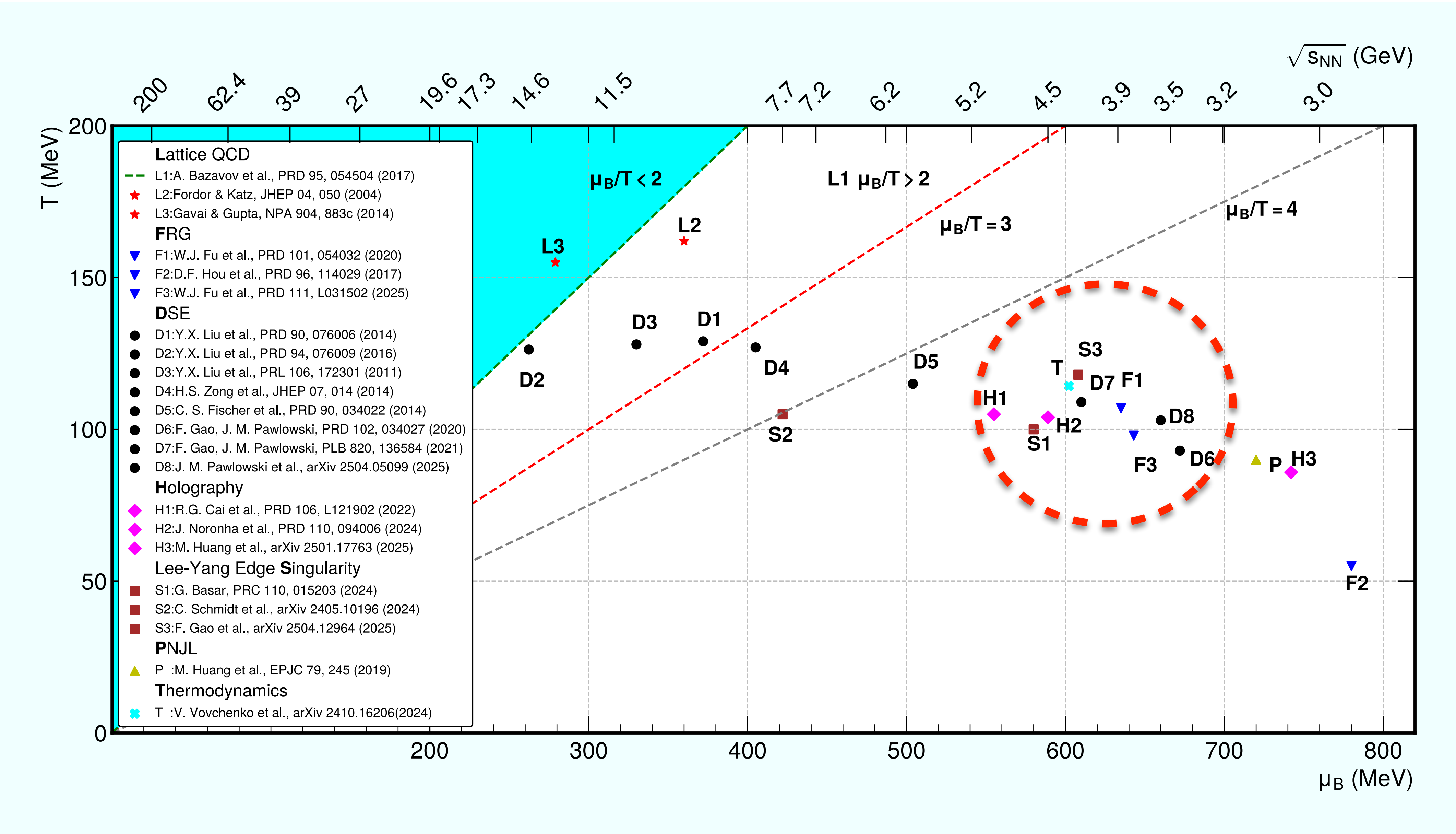
Fluctuations measured in BES-II



STAR Collaboration, *PRL* 135 (2025) 142301, arXiv:2504.00817

- Is there a "peak" structure serving as the smoking gun signal for the critical end point in the QCD phase diagram?

Theoretical predictions of CEP



By courtesy of Xiaofeng Luo

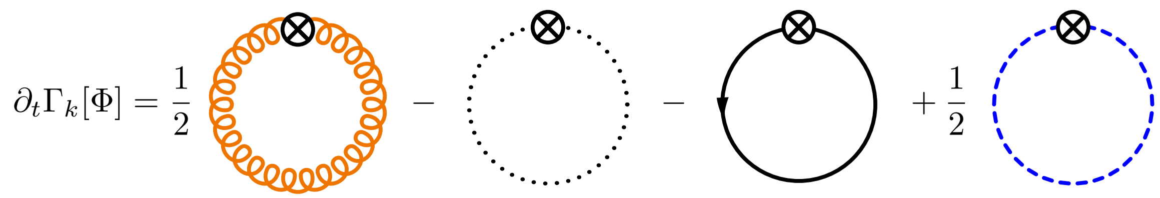
- There have been continuous efforts for more than two decades to predict the location of CEP from theoretical computations, and researchers in China have made significant contributions.

Outline

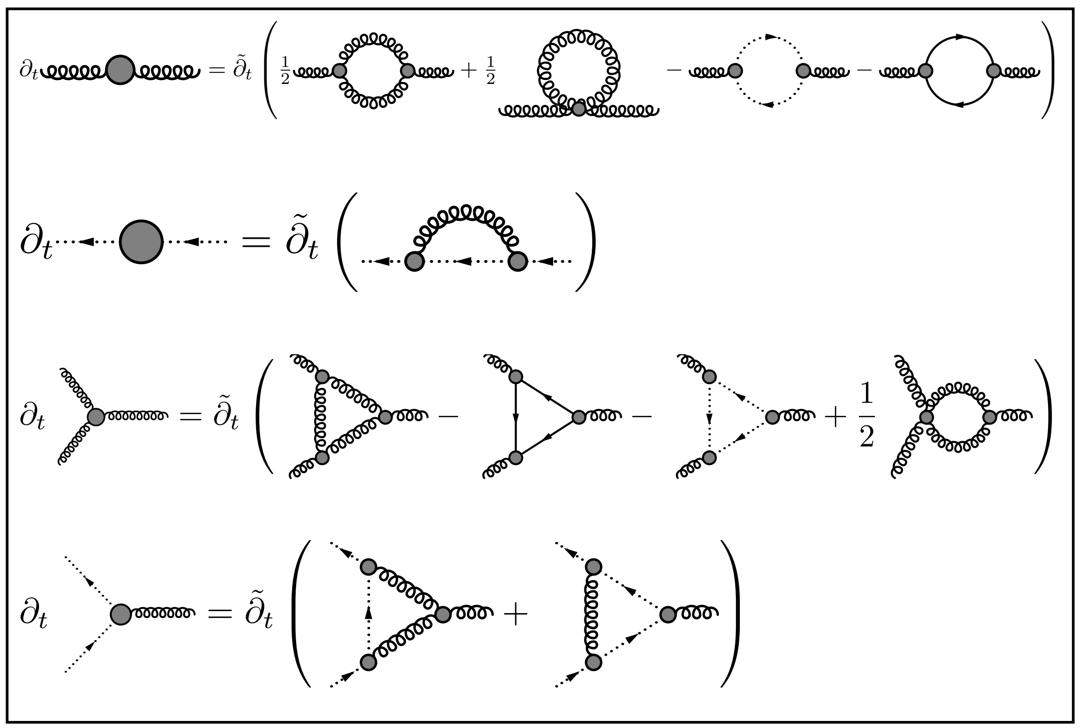
- Introduction
- Recent estimates of the location of CEP
- QCD phase diagram with Fierz-complete four-quark interactions
- Net-proton fluctuations with hydrodynamics
- Summary and outlook

QCD within fRG

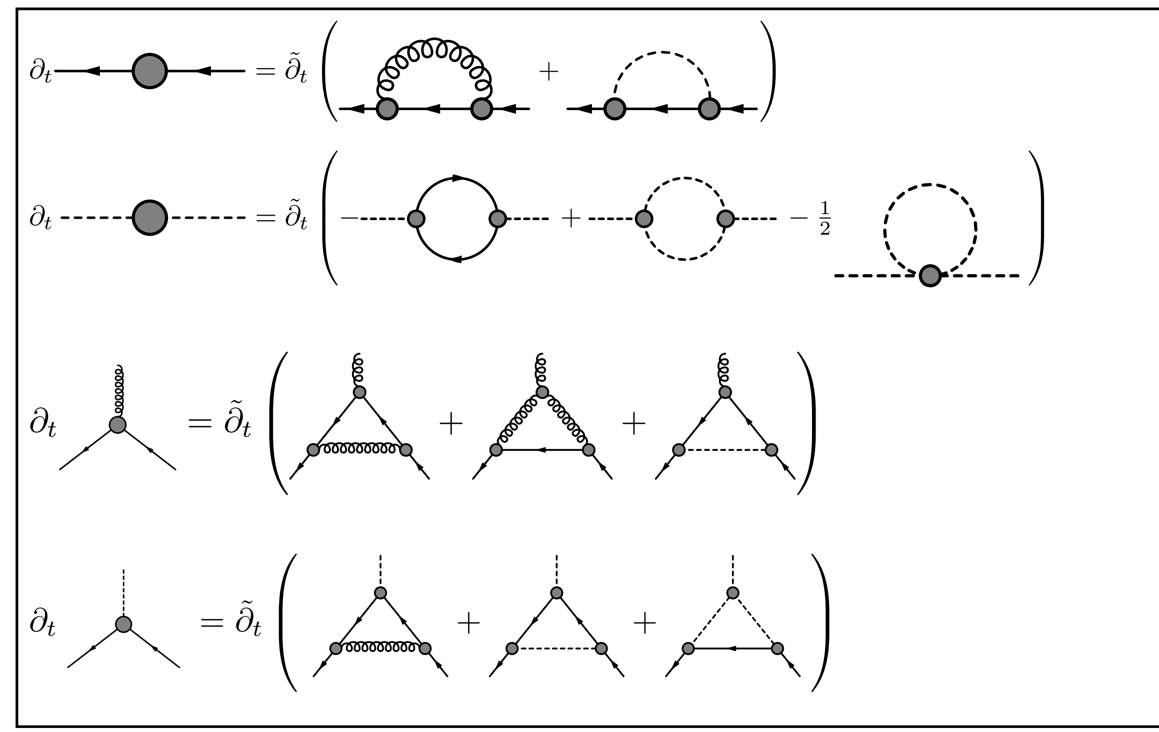
QCD flow equation:



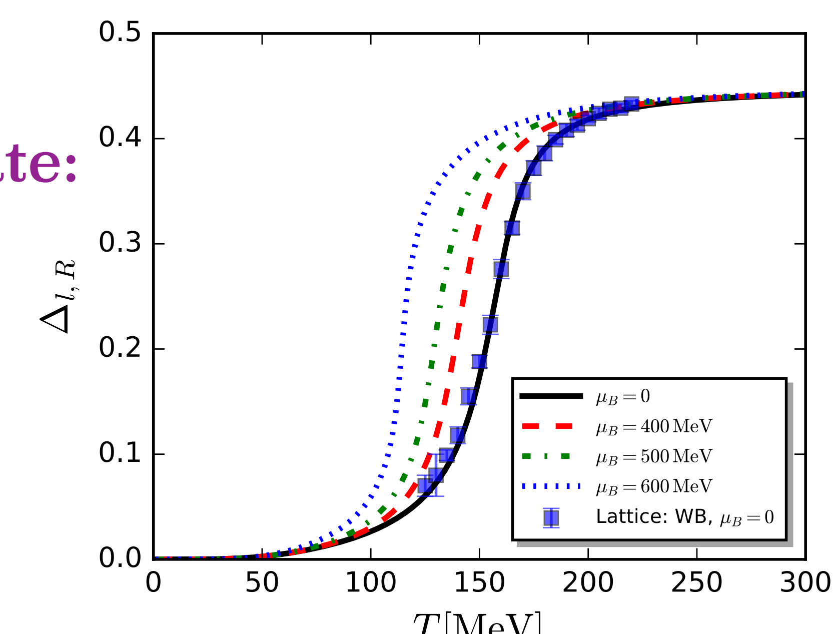
Glue sector:

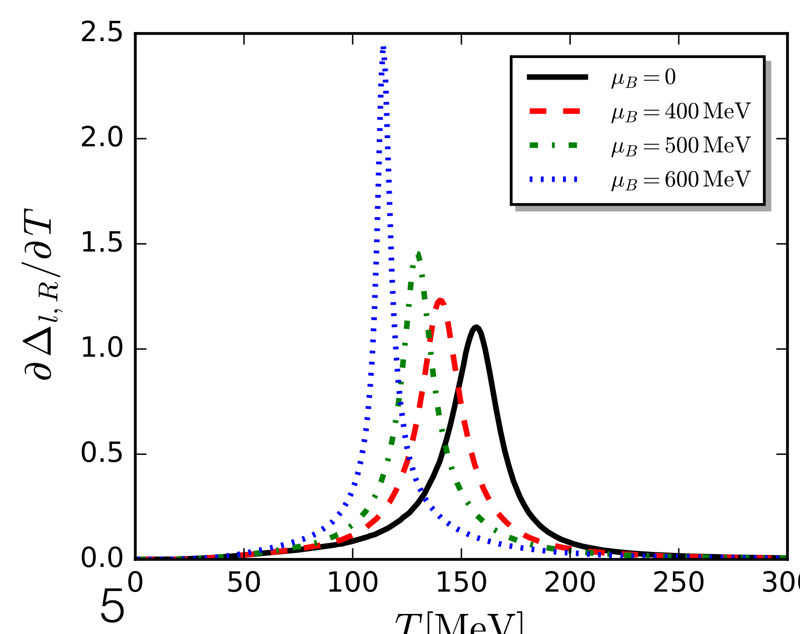


Matter sector:



Quark condensate:



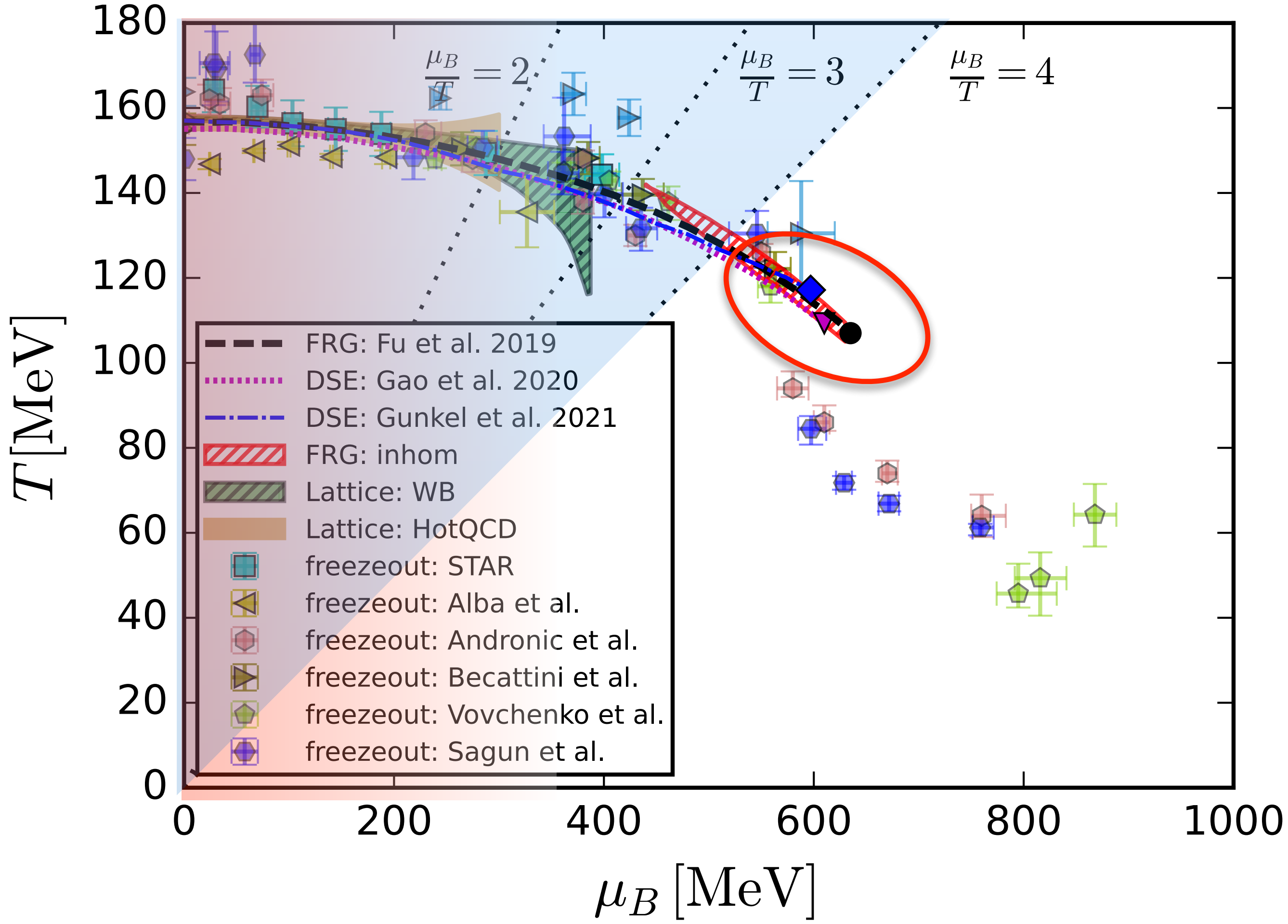


fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

Lattice: Borsanyi *et al.* (WB), *JHEP* 09 (2010) 073

Quantitative errors analysis in fRG: Ihssen, Pawlowski, Sattler, Wink, arXiv:2408.08413

CEP from functional QCD around the year 2020



Passing through strict benchmark tests in comparison to lattice QCD at vanishing and small μ_B.

Regime of quantitative reliability of functional QCD with $\mu_B/T \lesssim 4$.

Estimates of the location of CEP from first-principles functional QCD:

fRG:

● $(T, \mu_B)_{\mathrm{CEP}} = (107, 635)$ **MeV**

WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032, arXiv:1909.02991

DSE:

▼ $(T, \mu_B)_{\mathrm{CEP}} = (109, 610)$ **MeV**

Gao, Pawlowski, *PLB* 820 (2021) 136584, arXiv:2010.13705

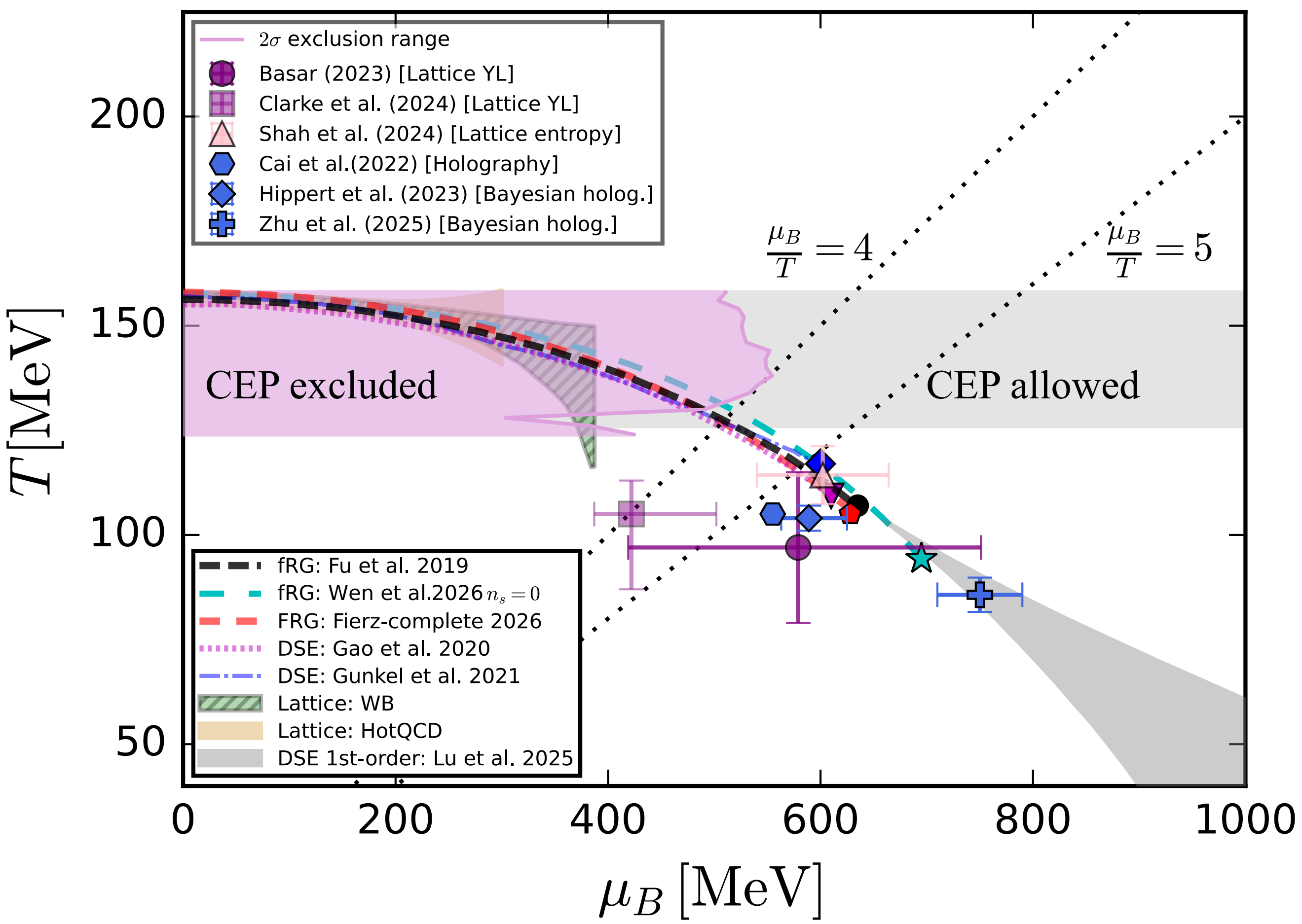
◆ $(T, \mu_B)_{\mathrm{CEP}} = (117, 600)$ **MeV**

Gunkel, Fischer, *PRD* 104 (2021) 054022, arXiv:2106.08356

- Recent studies of QCD phase structure from both fRG and DSE have shown convergent estimate for the location of CEP:

$$(T, \mu_B)_{\mathrm{CEP}} \approx (110 \pm 10, 630 \pm 30)\ \mathrm{MeV}$$

CEP from different approaches



Functional QCD (recent results):

fRG:
Pawlowski *et al.*, arXiv:2512.20510.
Fu *et al.*, arXiv:2603.13455.
DSE:
Lu *et al.*, *PRD* 113 (2026) 054019, arXiv:2504.05099.
Lu *et al.*, arXiv:2603.09336.

- **Combined results of different approaches indicate the location of CEP is not favored in the region $\mu_B/T \lesssim 4 \sim 5$.**

Lattice extrapolation:

Yang-Lee edge singularities

$$(T, \mu_B)_{\text{CEP}} = (97^{+18}_{-18}, 579^{+172}_{-160})\ \text{MeV}$$

Basar, *PRC* 110 (2024) 015203, arXiv:2312.06952.

$$(T, \mu_B)_{\text{CEP}} = (105^{+8}_{-18}, 422^{+80}_{-35})\ \text{MeV}$$

Clarke *et al.*, *PRD* 112 (2025) L091504, arXiv:2405.10196.

$$T_{\text{CEP}} \leq 103\ \text{MeV (84 \% level) or no CEP}$$

Adam *et al.*, arXiv:2507.13254.

Contours of constant entropy density

$$(T, \mu_B)_{\text{CEP}} = (114.3 \pm 6.9, 602.1 \pm 62.1)\ \text{MeV}$$

Shah *et al.*, *PRC* 113 (2026) L012201, arXiv:2410.16206.

No CEP at $\mu_{B_{\text{CEP}}} < 450$ MeV at the 2σ level

Borsanyi *et al.*, *PRD* 112 (2025) L111505, arXiv:2502.10267.

(Bayesian) holography:

$$(T, \mu_B)_{\text{CEP}} = (105, 555)\ \text{MeV}$$

Cai *et al.*, *PRD* 106 (2022) L121902, arXiv:2201.02004.

$$(T, \mu_B)_{\text{CEP}} = (104 \pm 3, 589^{+36}_{-26})\ \text{MeV (95\% level)}$$

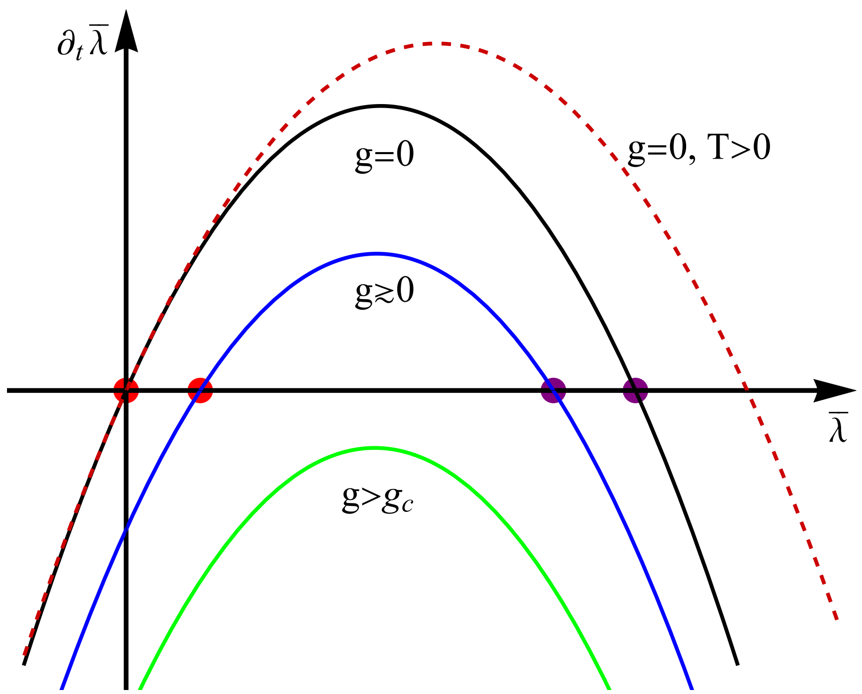
Hippert *et al.*, *PRD* 110 (2024) 094006, arXiv:2309.00579.

$$(T, \mu_B)_{\text{CEP}} = (81.6 - 89.8, 710 - 790)\ \text{MeV (95\% level)}$$

Zhu *et al.*, *PRD* 112 (2025) 026019, arXiv:2501.17763.

Four-quark interactions and chiral symmetry breaking

Beta function of four-quark couplings:



Braun, Gies, *JHEP* 06 (2006) 024.

$$\partial_t \bar\lambda = (d-2)\bar\lambda - a\bar\lambda^2 - b\bar\lambda g^2 - c g^4 \,,$$

Four-quark interactions

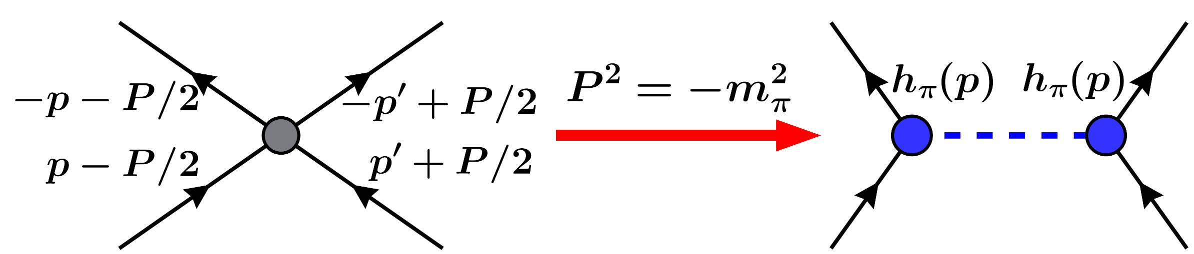
$$\Gamma_{4q} = -\int_x \sum_{\alpha\in\mathscr{F}} \lambda_\alpha \mathscr{T}^{(\alpha)}_{ijmn} \bar q_{l_i} q_{l_j} \bar q_{l_m} q_{l_n}$$

with Fierz-complete four-quark basis of ten tensors

$$\mathscr{F} = \Big\{ \sigma,\, \pi,\, a,\, \eta,\, (V\pm A)\,, (V-A)^{\rm adj}\,, \\ (S\pm P)^{\rm adj}_{-},\, (S+P)^{\rm adj}_{+} \Big\}$$

Yukawa couplings via dynamical hadronization

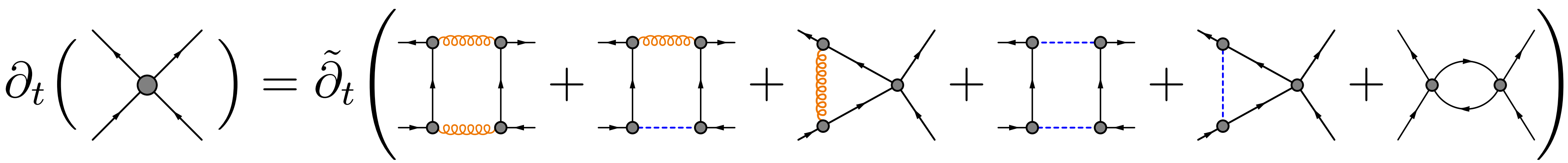
$$\Gamma_{\bar q q \phi} = h_\sigma \bar q_l T^0 \sigma q_l + h_\pi \bar q_l i\gamma_5 \vec{T}\cdot\vec{\pi} q_l$$

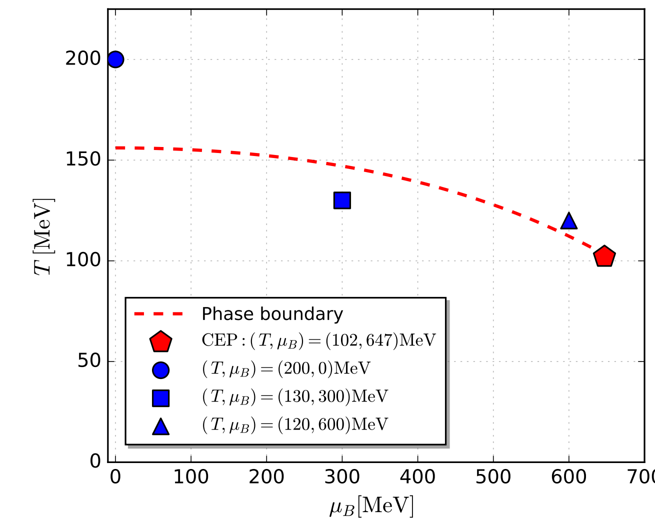


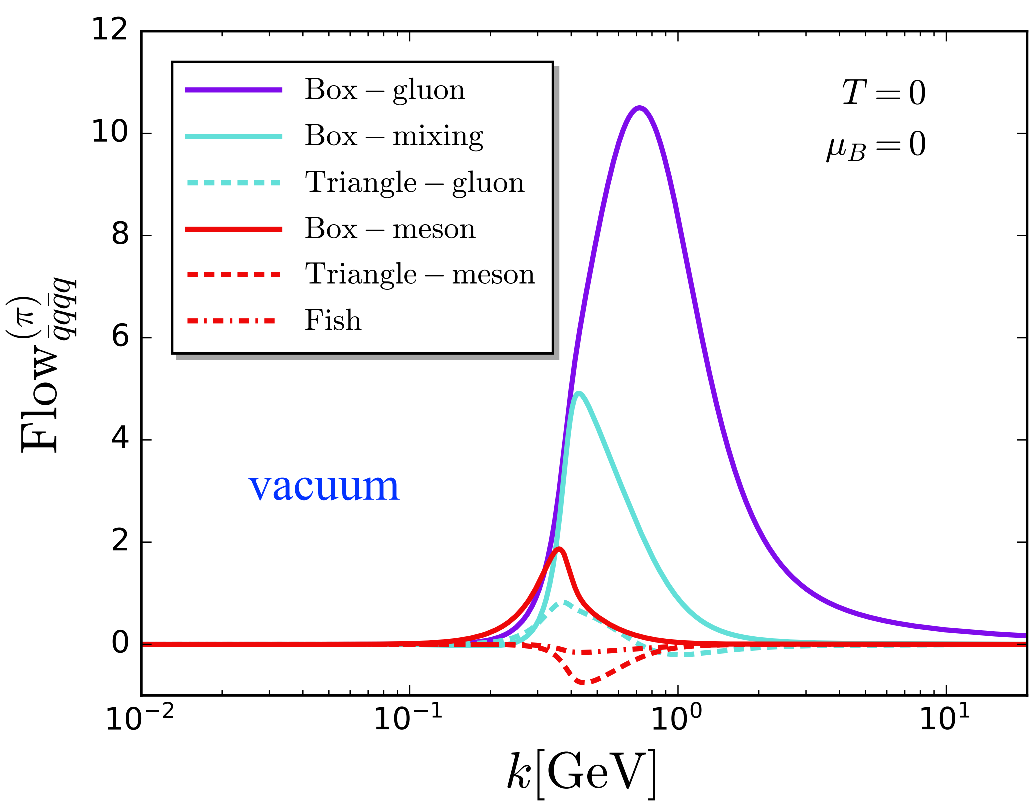
Wang, Zhou, Huang, Wen, Yin, WF, arXiv:2607.07354.

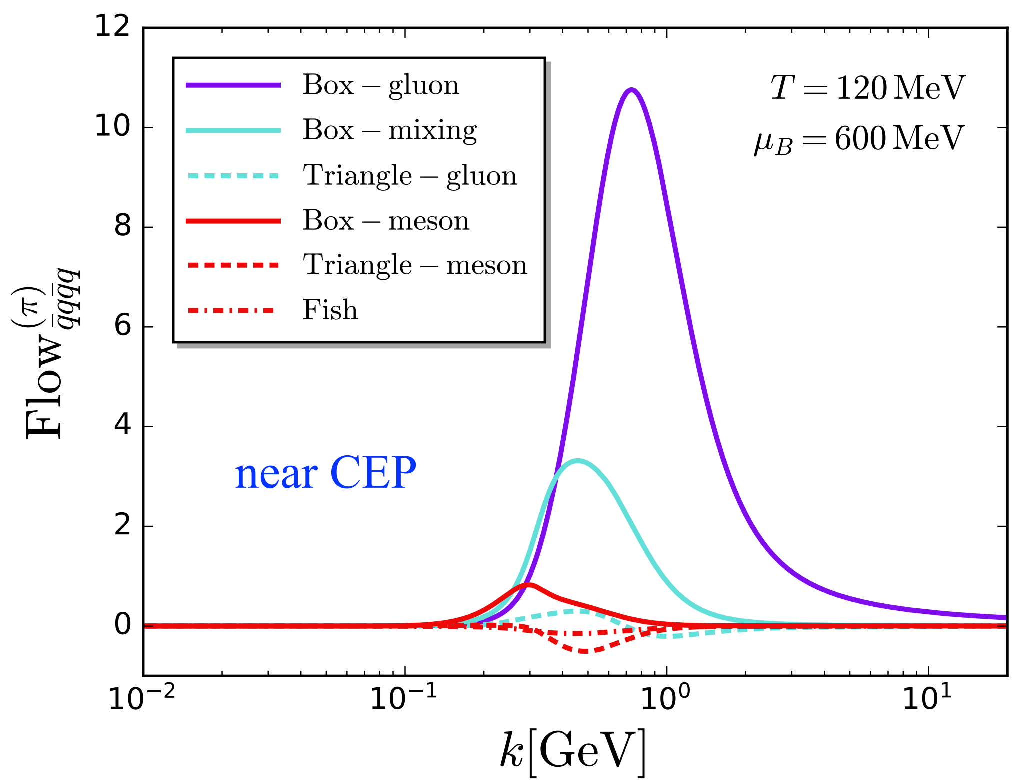
Four-quark flows

Flow equations of four-quark vertices:







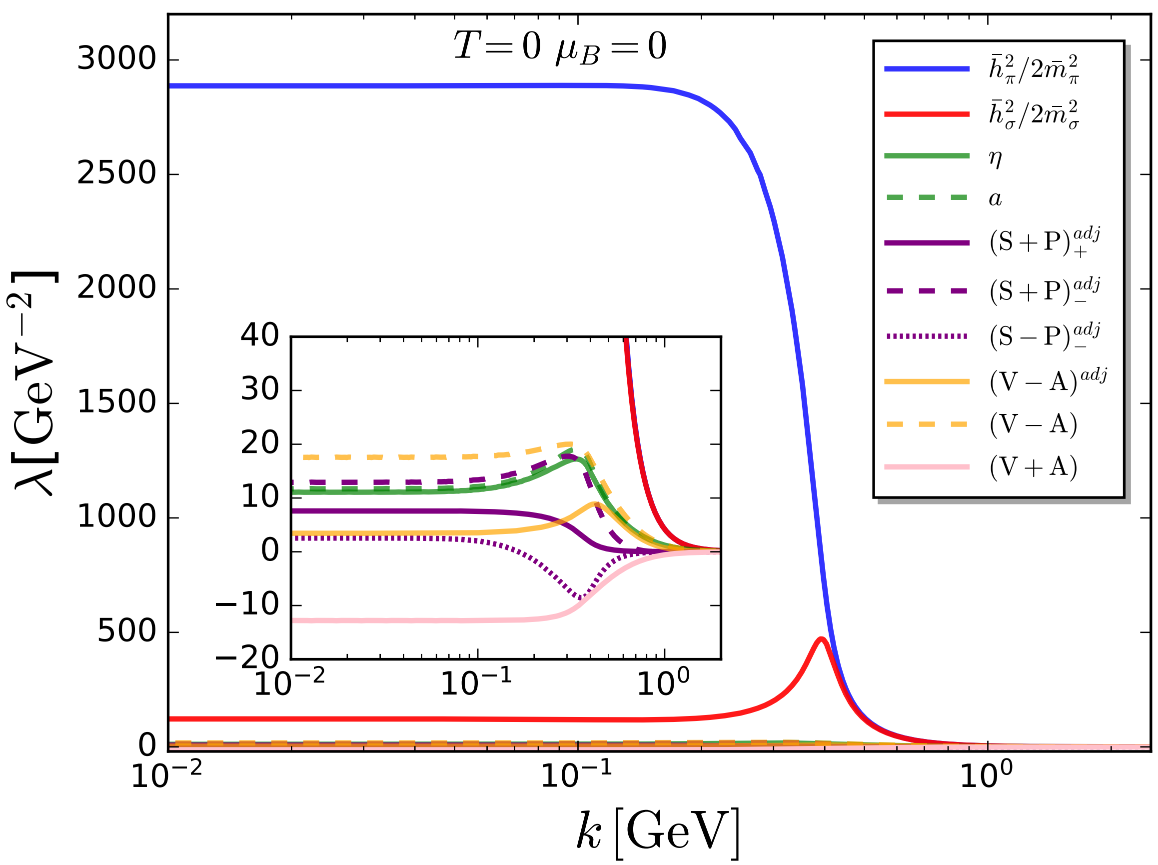


- **Diagrams of gluon and meson interactions show dominance at different RG scales.**

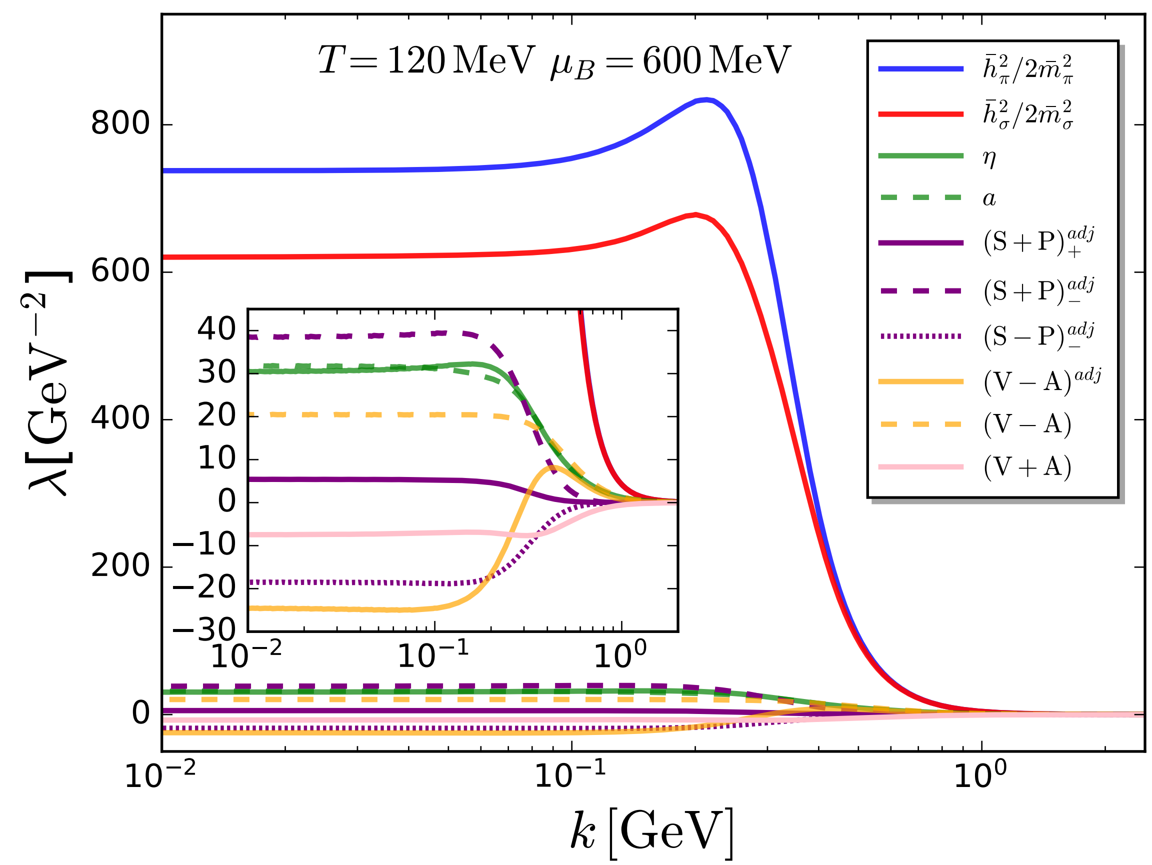
Wang, Zhou, Huang, Wen, Yin, WF, arXiv:2607.07354.

Four-quark couplings of different channels

Four-quark couplings in the vacuum:



Four-quark couplings near CEP:

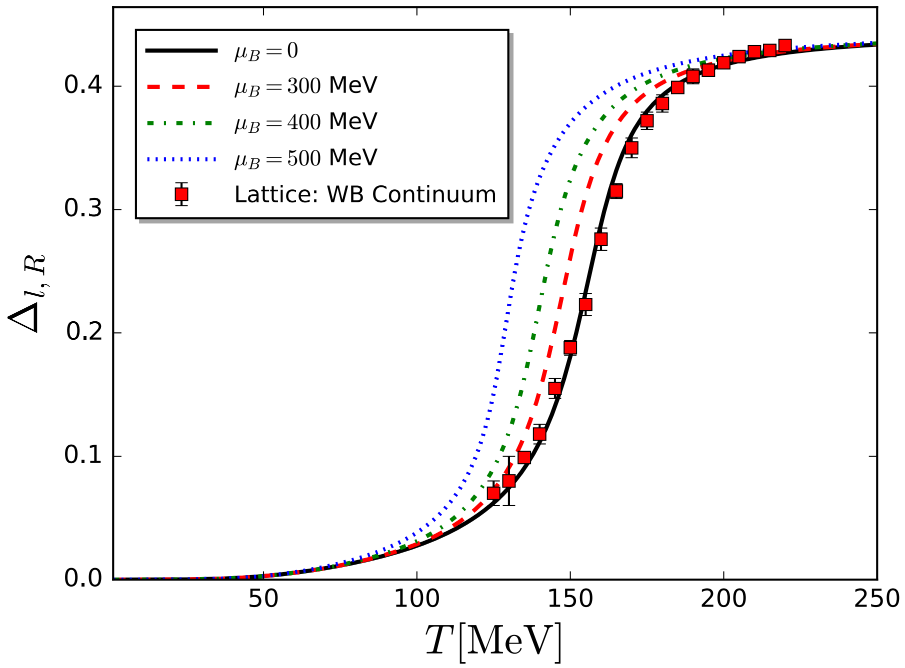


- **The pion and sigma channels play the dominant roles.**
- **Channels other than the scalar-pseudoscalar channels become more relevant towards CEP.**

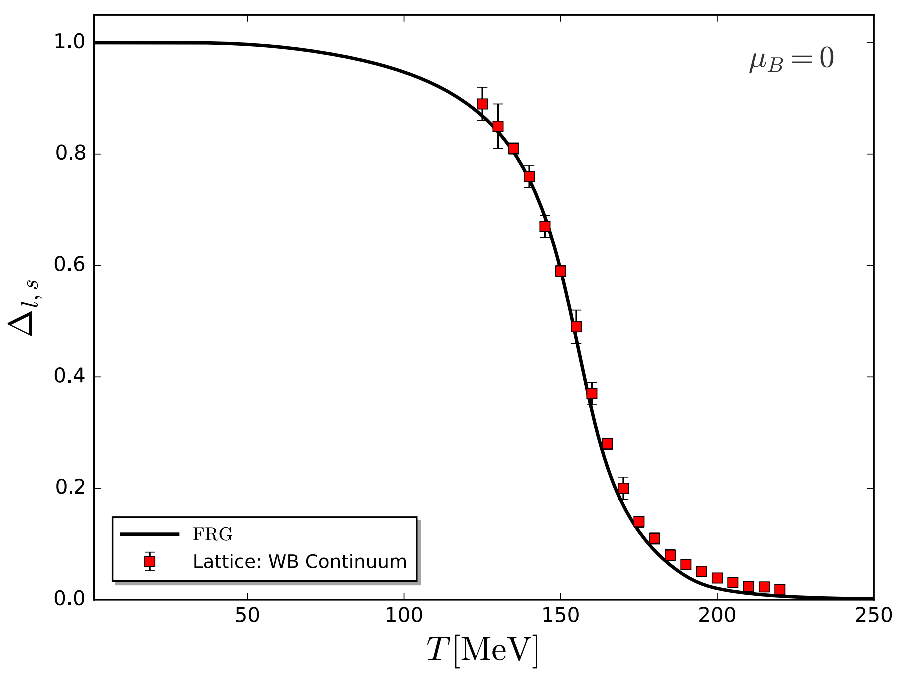
Wang, Zhou, Huang, Wen, Yin, WF, arXiv:2607.07354.

QCD phase diagram with Fierz-complete four-quark interactions

Light quark condensate:



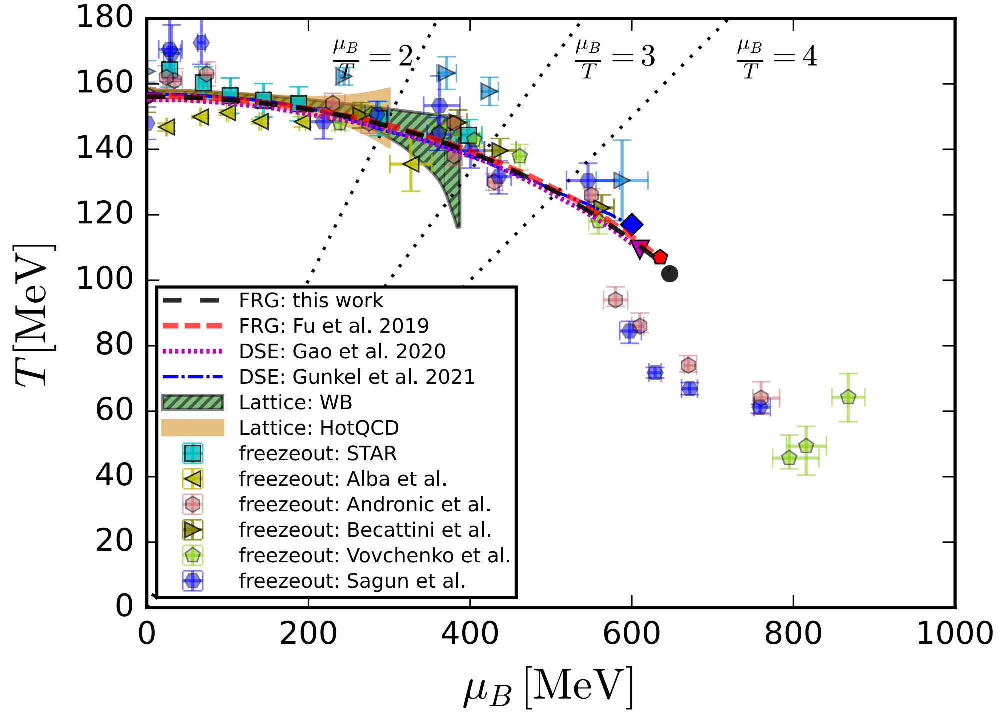
Reduced condensate:



fRG: Wang, Zhou, Huang, Wen, Yin, WF, arXiv:2607.07354.

Lattice: S. Borsanyi *et al.*, *JHEP* 09 (2010) 073.

QCD phase diagram:



Phase boundary:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \cdots$$

Curvature coefficients:

$$\kappa_2 = 0.0151(1)\,, \qquad \kappa_4 = 0.00023(2)$$

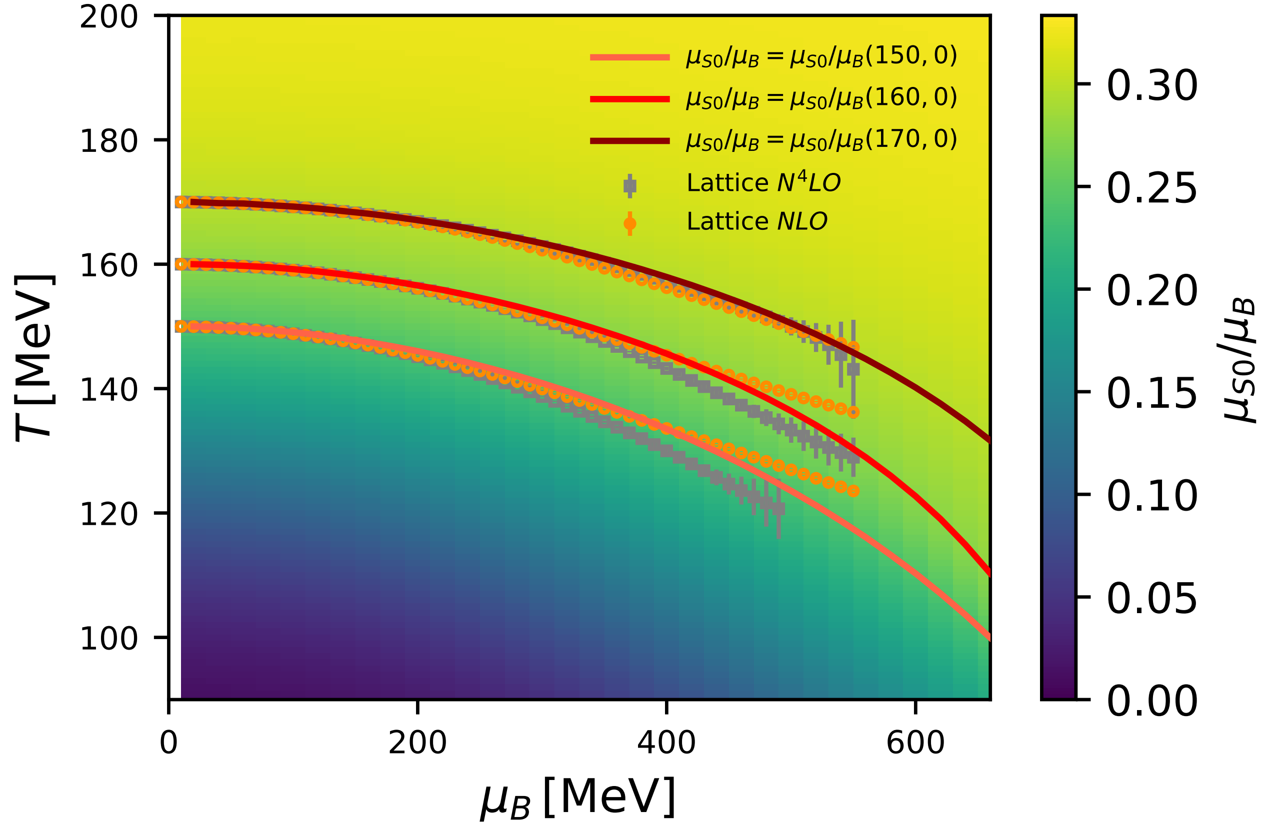
CEP:

$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (102{,}647)\,\mathrm{MeV}$$

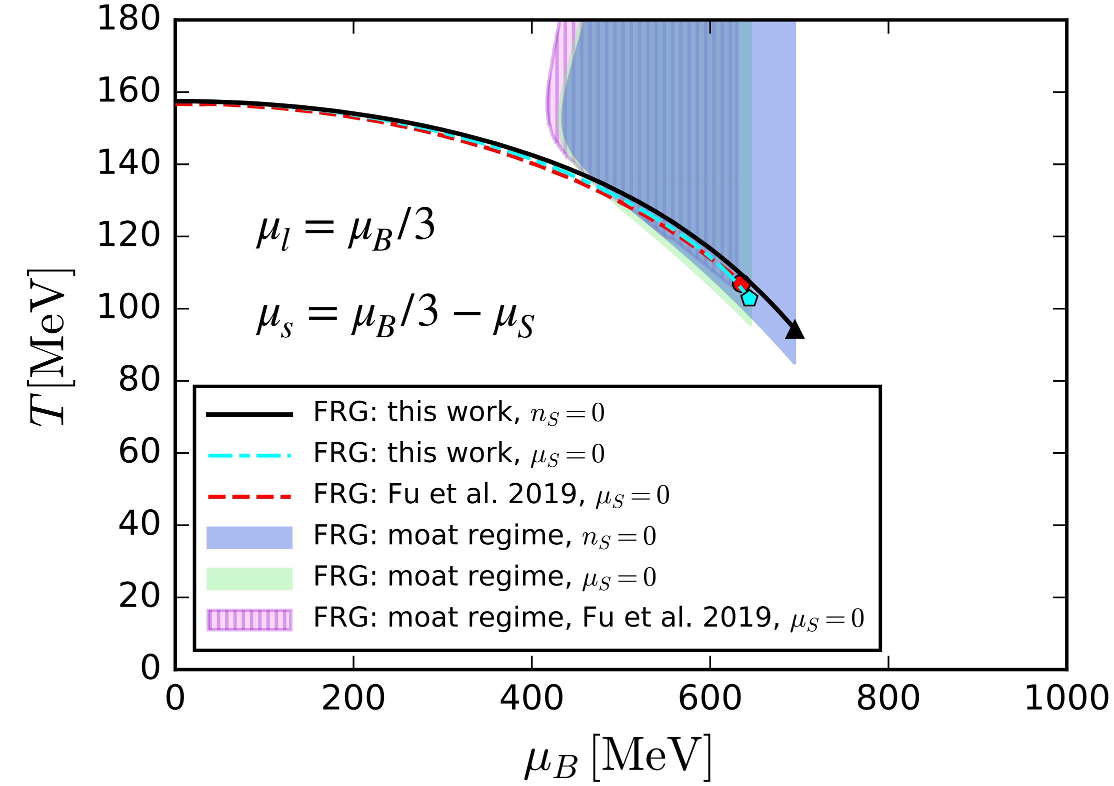
- **The dynamics of Fierz-complete four-quark interactions increases a bit the curvature of the phase boundary, and moves the CEP to location of larger baryon chemical potential and smaller temperature.**

CEP and strangeness neutrality

μ_S with constraint $n_S = 0$



Phase diagram with $n_S = 0$



Curvature of the phase boundary:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \cdots$$

fRG: $\dfrac{\kappa_2^{n_S=0}}{\kappa_2^{\mu_S=0}} = 0.897(20)$ **Lattice:** $\dfrac{\kappa_{2,\mathrm{lat}}^{n_S=0}}{\kappa_{2,\mathrm{lat}}^{\mu_S=0}} = 0.893(35)$

- **Strangeness neutrality flattens a bit the phase boundary and moves CEP to larger μ_B.**

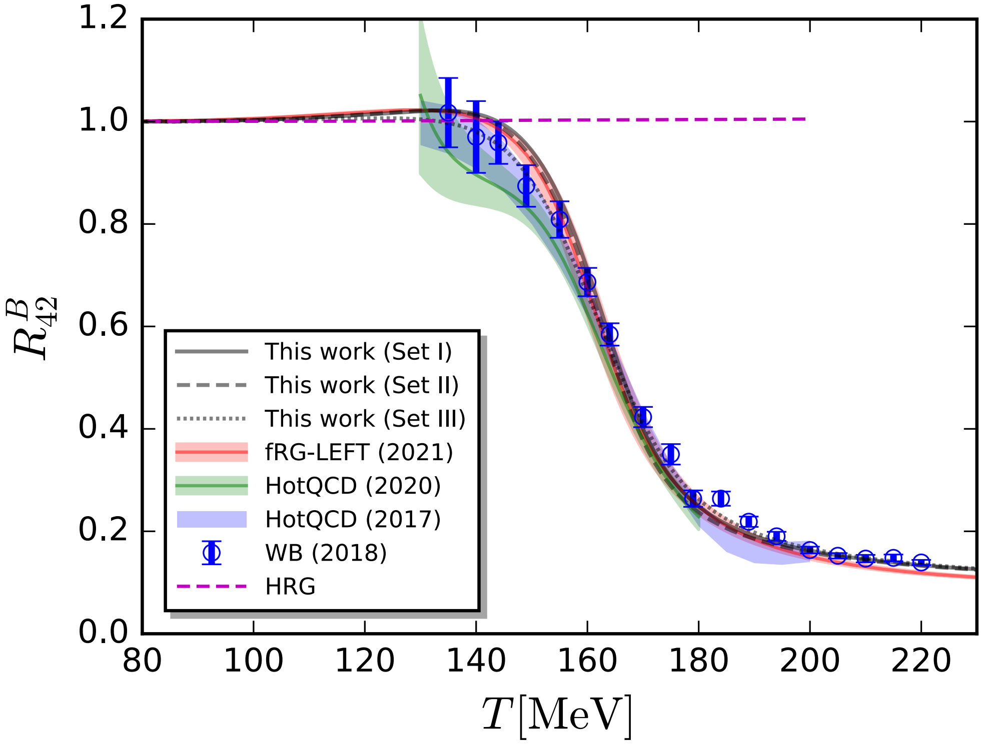
$\mu_S = 0$: $(T, \mu_B)_{\mathrm{CEP}} = (102, 644)$ MeV

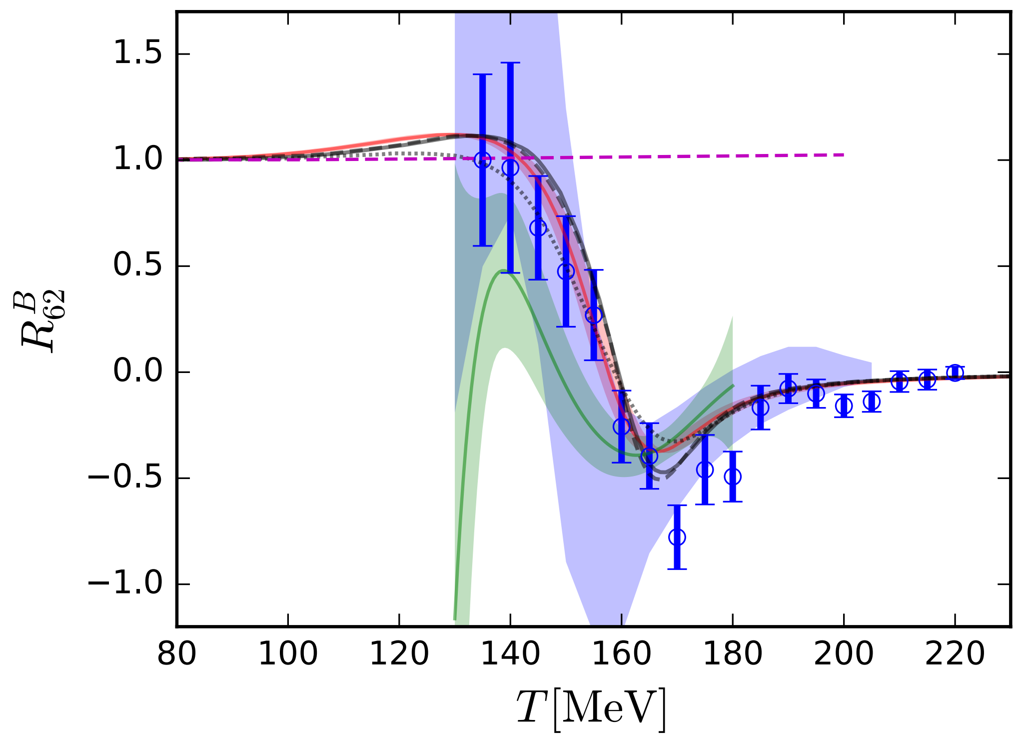
$n_S = 0$: $(T, \mu_B)_{\mathrm{CEP}} = (92, 696)$ MeV

fRG: WF, Huang, Pawlowski, Rennecke, Wen, Yin, arXiv:2603.13455.

Lattice: Ding *et al.*, *PRD* 109 (2024) 114516, arXiv:2403.09390.
Lattice: Borsanyi *et al.*, arXiv:2510.26455.

Baryon number fluctuations





$\mu_B = 0$

fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508; WF, Luo, Pawlowski, Rennecke, Wen, Yin, *PRD* 104 (2021) 094047.

HotQCD: A. Bazavov *et al.*, *PRD* 95 (2017), 054504; *PRD* 101 (2020), 074502.

WB: S. Borsanyi *et al.*, *JHEP* 10 (2018) 205.

Baryon number fluctuations

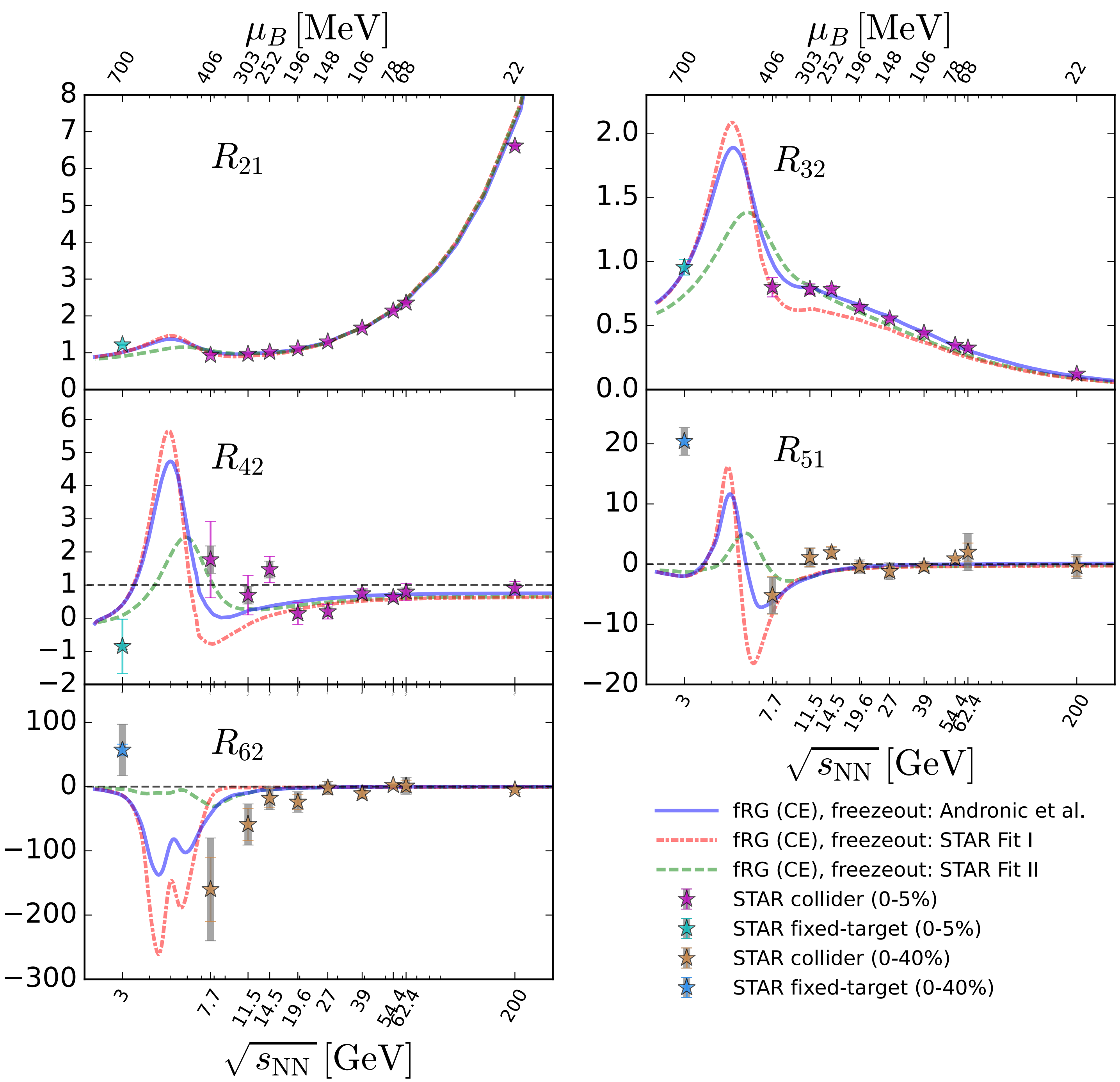
$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4} \qquad R_{nm}^B = \frac{\chi_n^B}{\chi_m^B}$$

Relation to the cumulants

$$\frac{M}{VT^3} = \chi_1^B\,,\ \frac{\sigma^2}{VT^3} = \chi_2^B\,,\ S = \frac{\chi_3^B}{\chi_2^B\sigma}\,,\ \kappa = \frac{\chi_4^B}{\chi_2^B\sigma^2}\,,$$

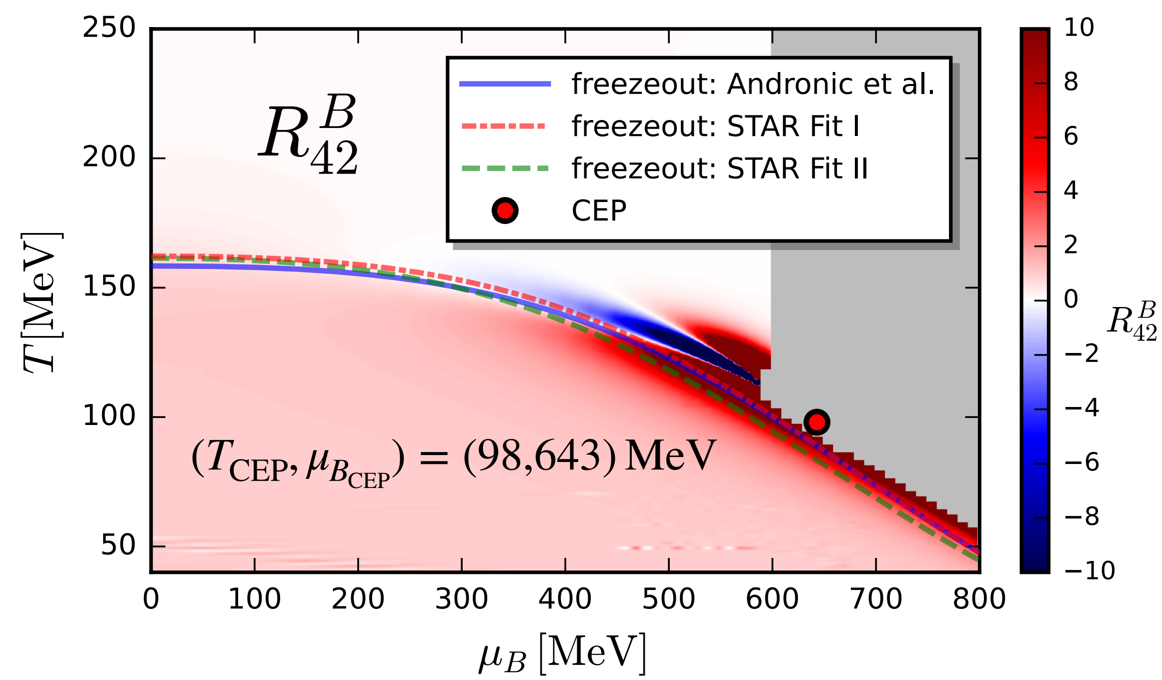
- **In comparison to lattice results and our former results, the improved results of baryon number fluctuations at vanishing chemical potential in the QCD-assisted LEFT are convergent and consistent.**

Fluctuations at the freeze-out curve



fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508.

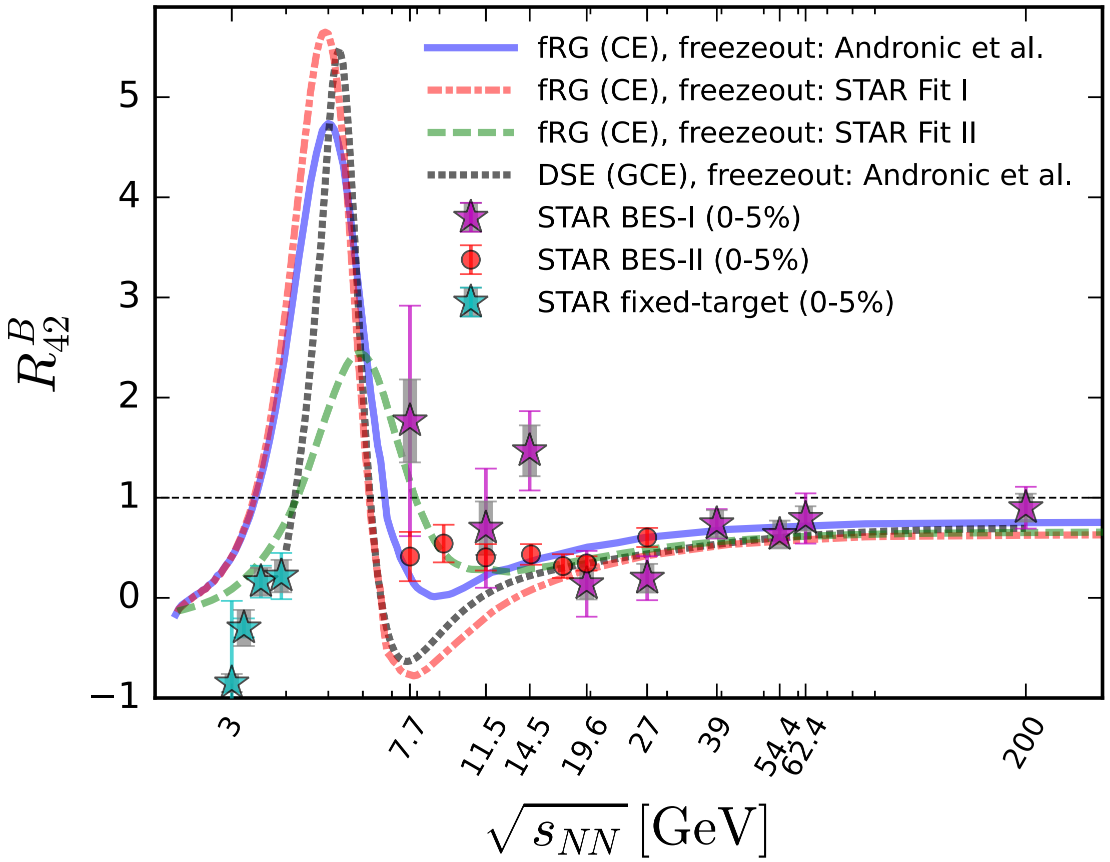
STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301; Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303; Aboona *et al.* (STAR), *PRL* 130 (2023) 082301.



- **Peak structure is found in 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV.**
- **Position of peak in R_{42} is $\mu_{B_{\rm peak}}$ = 536, 541 and 486 MeV for the three freeze-out curves, significantly smaller than $\mu_{B_{\rm CEP}}$ = 643 MeV.**

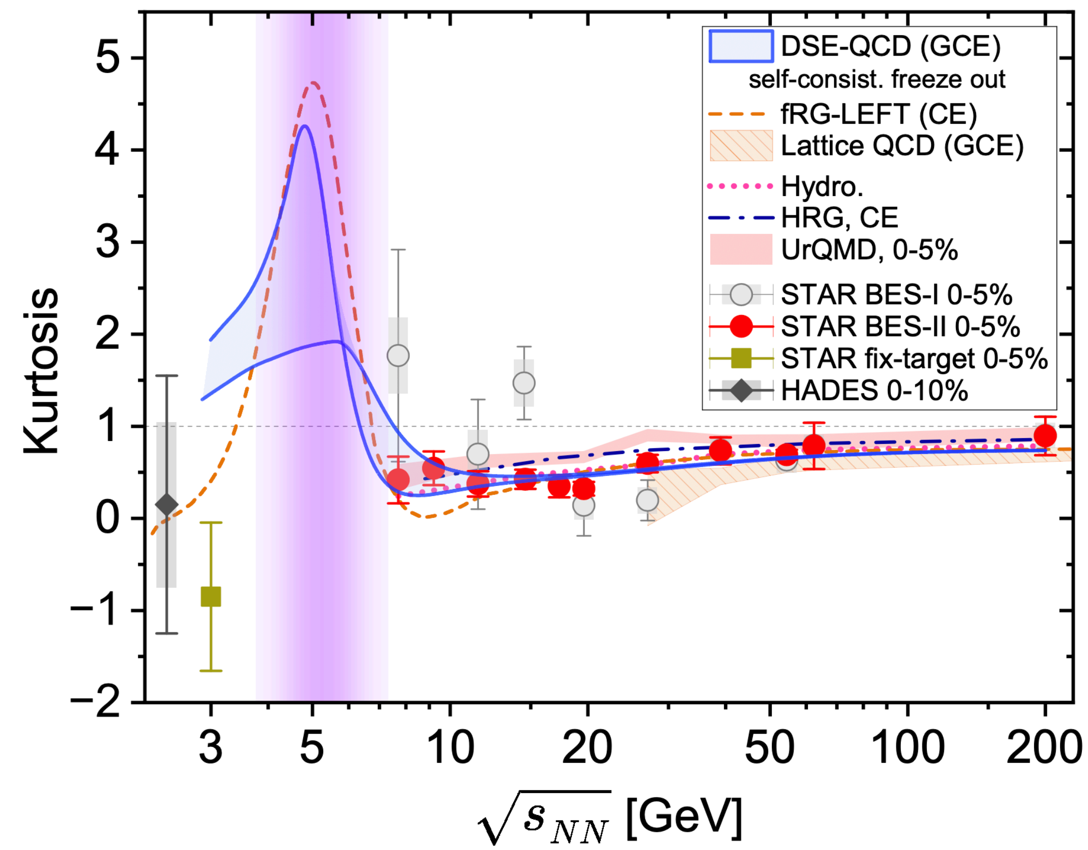
C4/C2: Comparison to STAR data

Net baryon (proton) number kurtosis:



STAR: *PRL* 135 (2025) 142301, arXiv:2504.00817.

fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508.

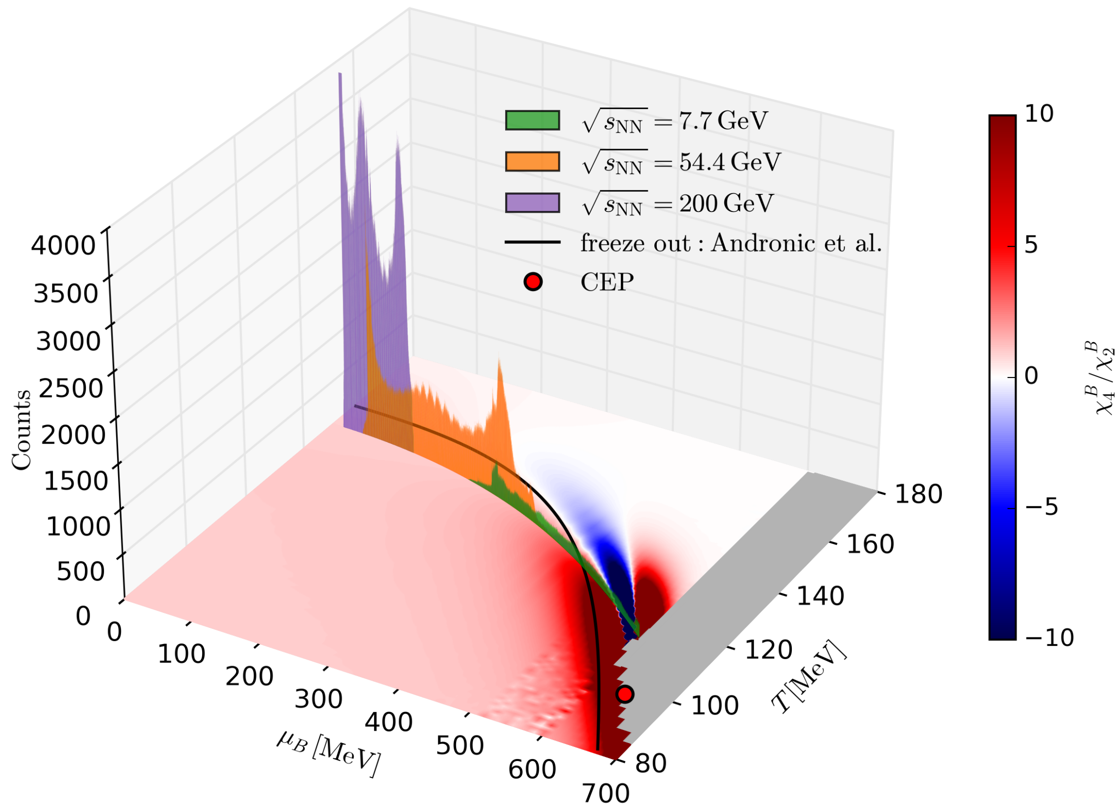


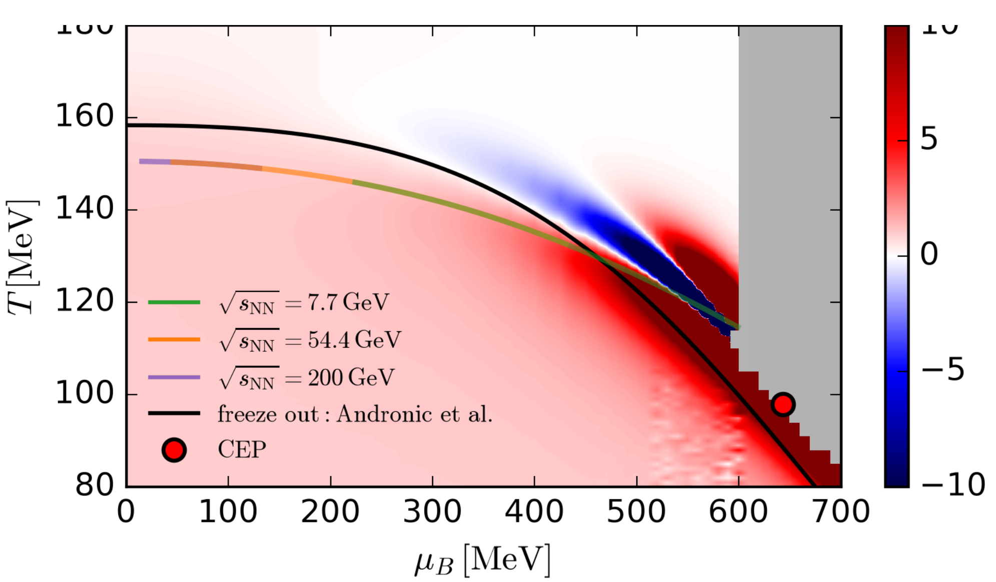
DSE: Lu, Fischer, Gao, Liu, Pawlowski, arXiv:2603.09336; Lu, Gao, Liu, Pawlowski, *PRD* 113 (2026) 054019, arXiv:2504.05099.

- **Theoretical prediction with critical fluctuations (fRG and DSE) is consistent with STAR data.**
- **A peak structure is predicted in the energy regime of fixed-target experiments, i.e. 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV. Experimental search of this peak is very important.**

Critical fluctuations + hydrodynamics

Fluid elements distributed on the phase diagram:





fRG+hydro: Zhao, Yin, Wu, Du, Luo, WF, in preparation.

Effects included:

- **Hypersurface elements** of different T and μ_B obtained from hydrodynamics simulations

$$\delta C_n^{B^\pm,gce}(x_i) = \delta V_i^{\mathrm{eff}} T^3(x_i)\chi_n^{B^\pm}(x_i).$$

- Implementing the baryon number fluctuations calculated from **fRG**

$$\chi_n^{B^\pm} = \frac{\chi_n^{B^\pm,HRG}}{\chi_n^{B,HRG}}\chi_n^{B,fRG}.$$

- Using the Cooper-Frye formula to determine the probability within an **acceptance window** of momenta and rapidity $p_{\mathrm{acc}}(x_i; \Delta p_{\mathrm{acc}})$.

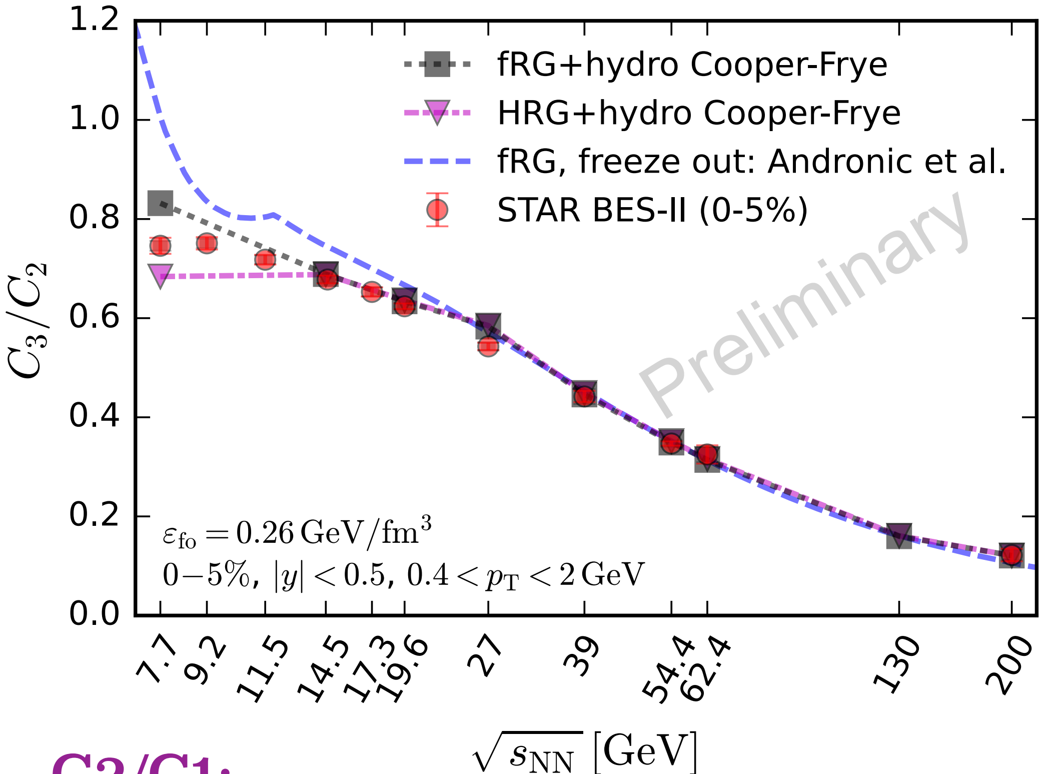
- Using the **isospin randomization** to calculate the (anti)proton cumulants.

- **Global baryon number conservation** via the subensemble acceptance method (SAM).

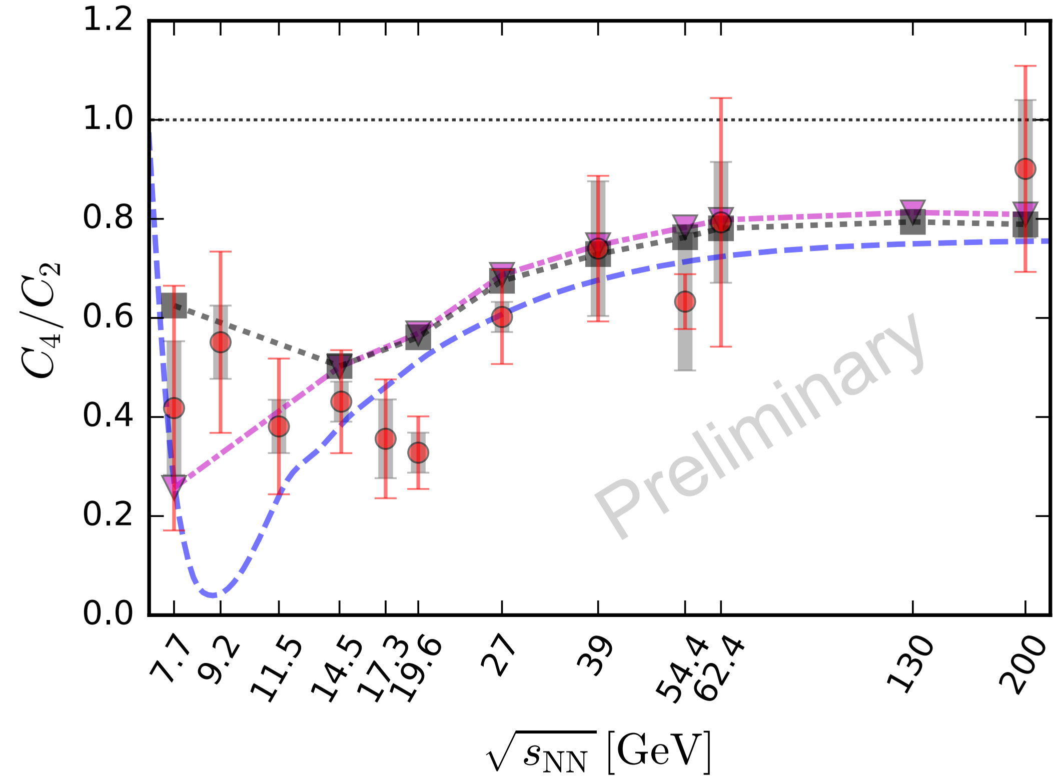
HRG+hydro: Vovchenko, Koch, Shen, *PRC* 105 (2022) 014904, arXiv:2107.00163.
hydro: Du, Gao, Jeon, Gale, *PRC* 109 (2024) 014907, arXiv:2302.13852.
SAM: Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, *PLB* 811 (2020) 135868.

Critical net-proton fluctuations with hydrodynamics

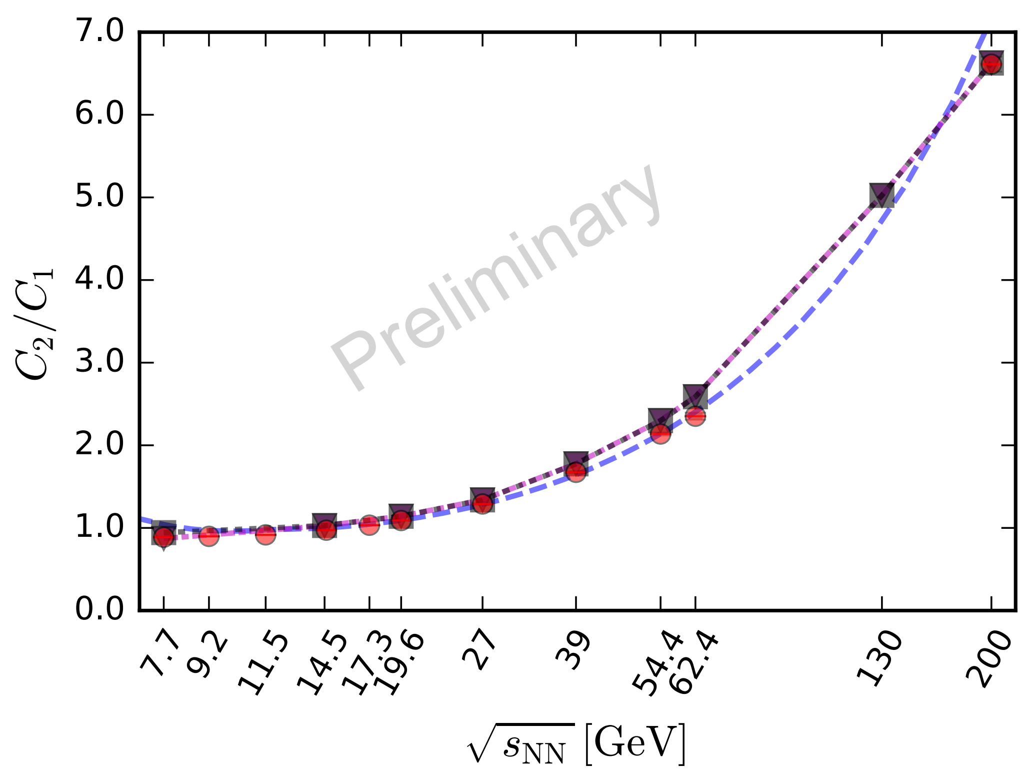
C3/C2:



C4/C2:



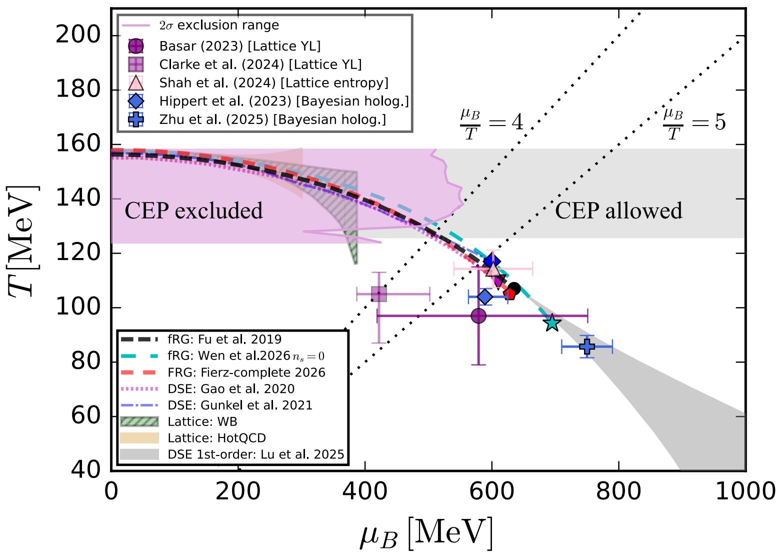
C2/C1:

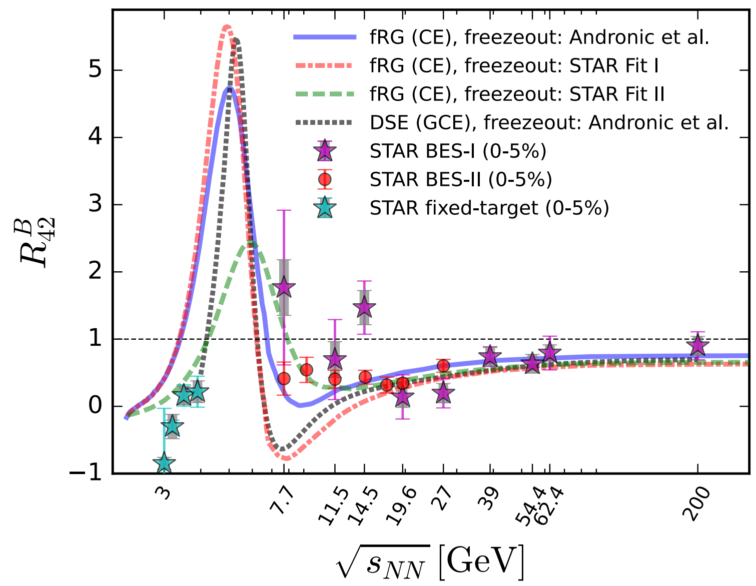


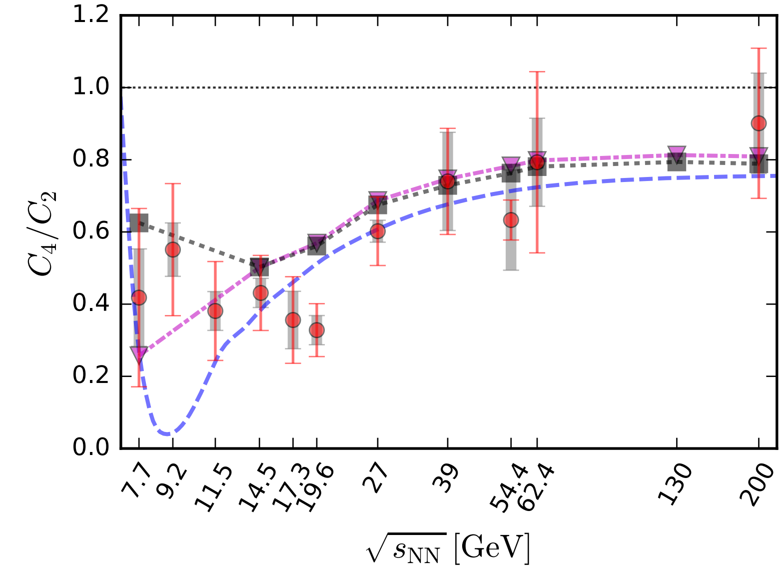
- **Non-monotonic dependence is observed in fRG+hydro, in comparison to the HRG+hydro.**
- **In comparison to the critical fluctuations computed on the parametrized freeze-out curve, fRG+hydro shows a better agreement with the experimental data.**

fRG+hydro: Zhao, Yin, Wu, Du, Luo, WF, in preparation.

Summary and outlook





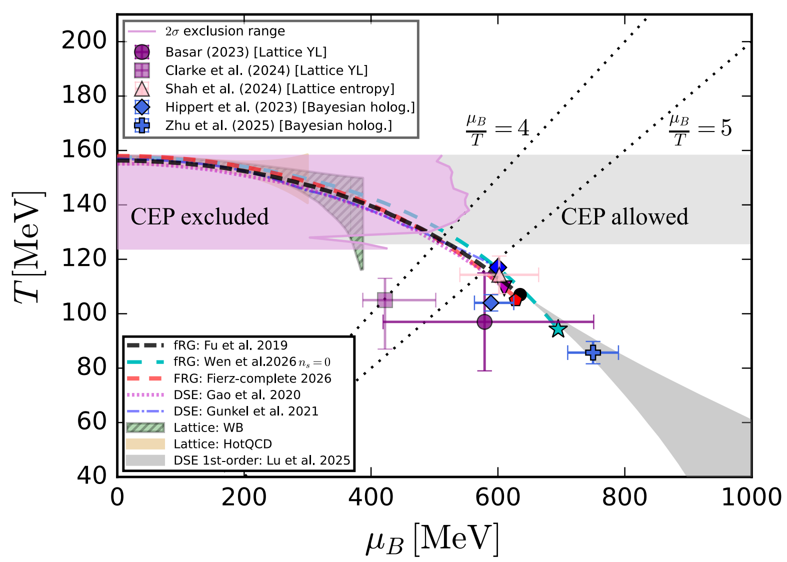


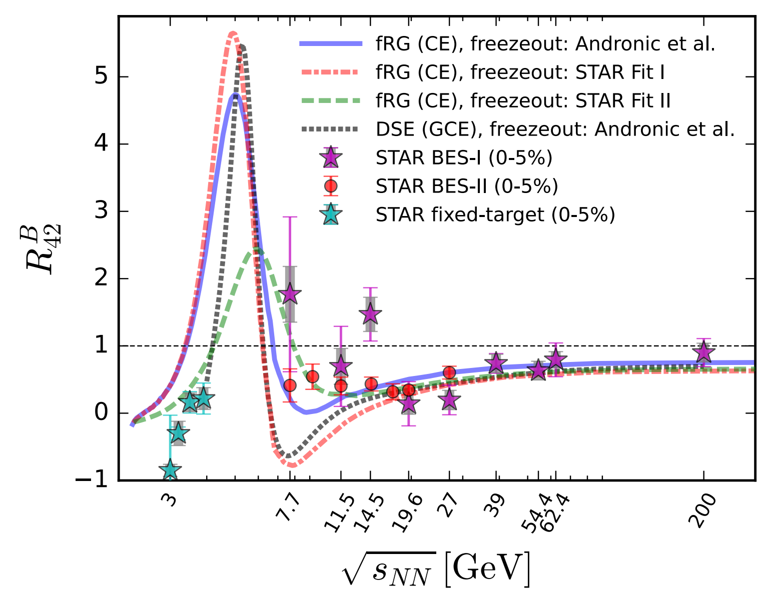
★ Combined results of different approaches, including functional QCD, lattice, holography, indicate that the location of CEP is not favored in the region $\mu_B/T \lesssim 4 \sim 5$.

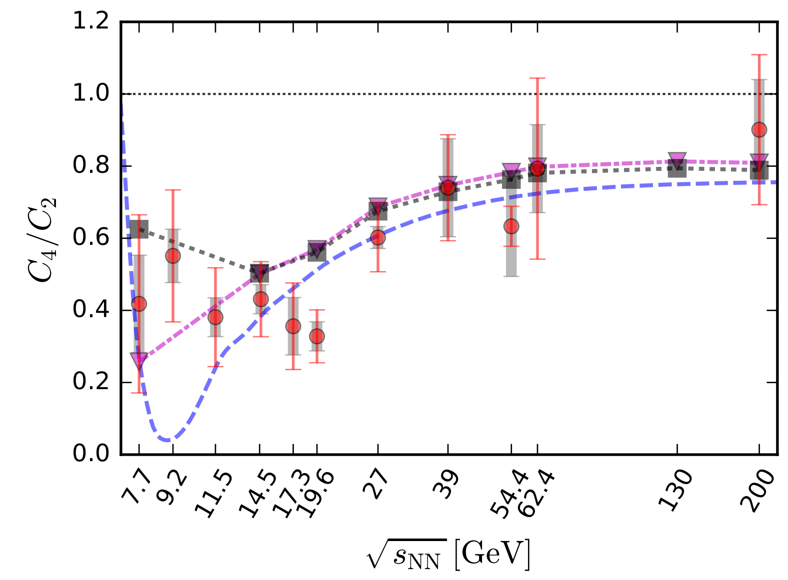
★ A prominent peak structure is predicted in the baryon number fluctuations in the fixed-target energy of 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV, which need to be confirmed in experiments.

★ A non-monotonic dependence on the collision energy is observed in net-proton fluctuations calculated with fRG and hydrodynamics.

Summary and outlook







- ★ Combined results of different approaches, including functional QCD, lattice, holography, indicate that the location of CEP is not favored in the region $\mu_B/T \lesssim 4 \sim 5$.

- ★ A prominent peak structure is predicted in the baryon number fluctuations in the fixed-target energy of 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV, which need to be confirmed in experiments.

- ★ A non-monotonic dependence on the collision energy is observed in net-proton fluctuations calculated with fRG and hydrodynamics.

Thank you very much for your attentions!

The 3rd International Workshop on Physics at High Baryon Density (PHD2026, 第三届高重子密度物理国际研讨会)

Nov 10 – 13, 2026
Crowne Plaza Xinghai, Dalian, China
Asia/Shanghai timezone

https://indico.ihep.ac.cn/event/29146/

Enter your search term

Physics Topics :

1) QCD Phase Structure at High Baryon Density

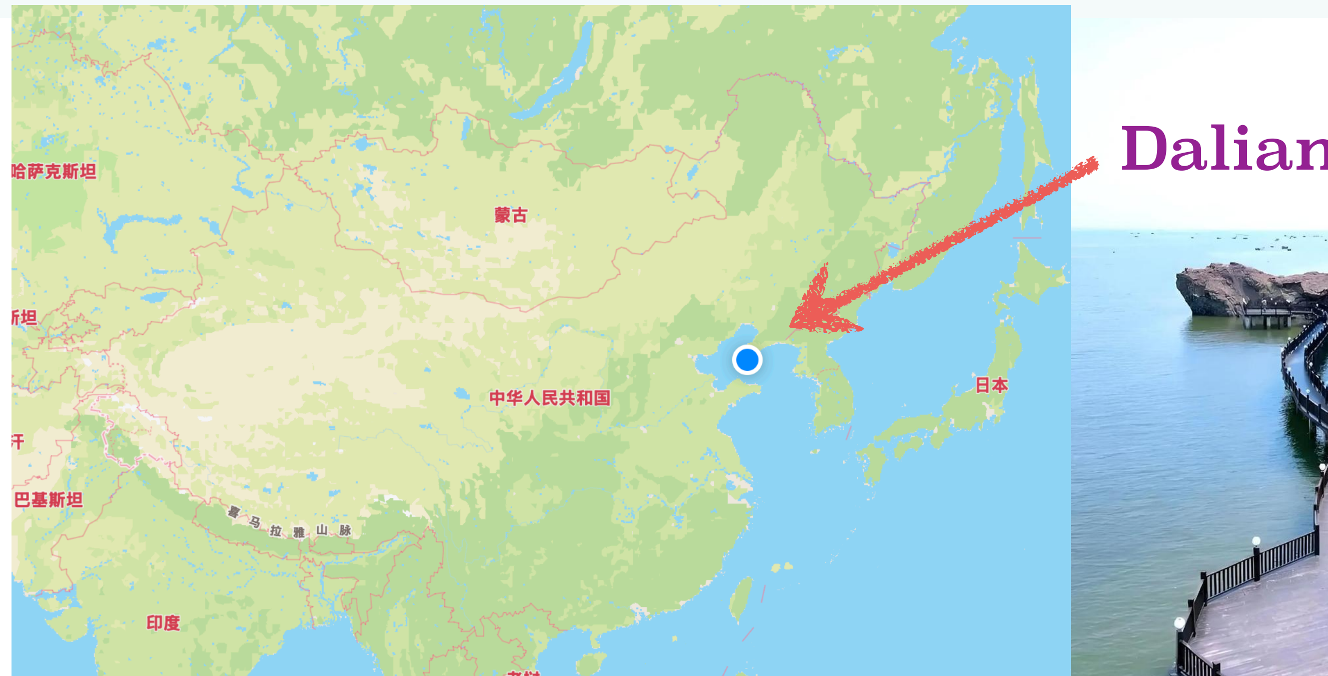
2) Nuclear Matter at High Density and Equation of State

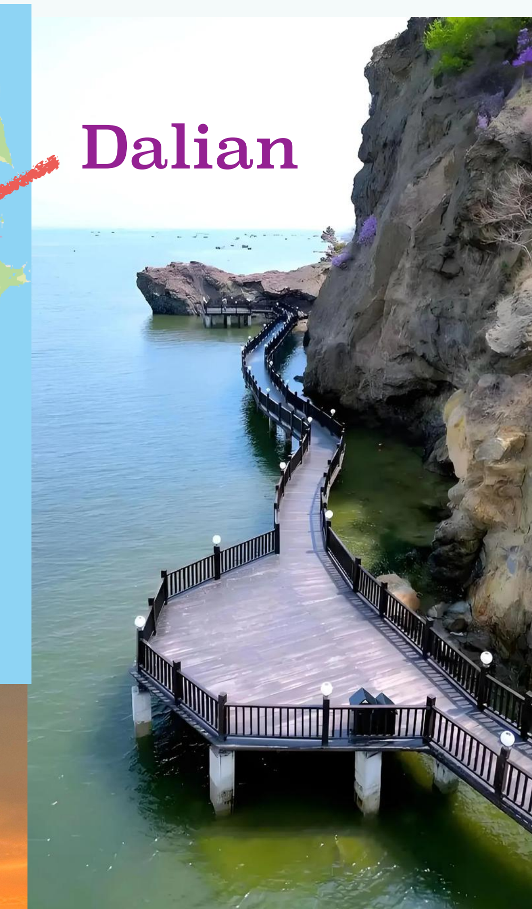
3) Dynamical Evolution of Heavy-ion Collisions

4) Nuclear Matter Under Extreme External Fields

5) Hadron Properties in Nuclear Medium

6) Nuclear Physics in Compact Stars





Local Organizing Committee:

Heng-Tong Ding (Central China Normal University)

Weijie Fu (Dalian University of Technology, co-Chair)

Defu Hou (Central China Normal University)

Sophia Han (T.D. Lee Institute, Shanghai Jiao Tong University)

Xiaofeng Luo (Central China Normal University, co-Chair)

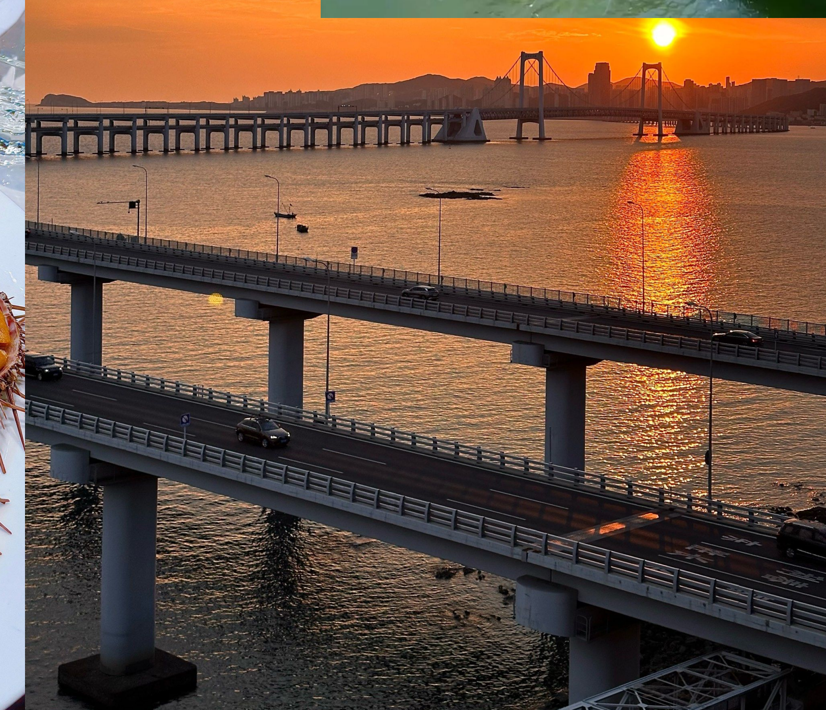
Guoliang Ma (Fudan University)

Zebo Tang (University of Science and Technology of China)

Chi Yang (Shandong University)

Pengfei Zhuang (Tsinghua University, co-Chair)

Yapeng Zhang (Institute of Modern Physics, CAS)



Welcome to PHD2026 in Dalian!

Backup

Functional renormalization group

Functional integral with an IR regulator

$$Z_k[J] = \int (\mathscr{D}\hat{\Phi}) \exp\left\{ -S[\hat{\Phi}] - \Delta S_k[\hat{\Phi}] + J^a \hat{\Phi}_a \right\}$$

$$W_k[J] = \ln Z_k[J]$$

regulator:

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$

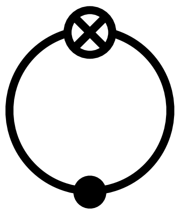
flow of the Schwinger function:

$$\partial_t W_k[J] = -\frac{1}{2} \mathrm{STr}\Big[\big(\partial_t R_k\big) G_k \Big] - \frac{1}{2} \Phi_a \partial_t R_k^{ab} \Phi_b$$

Legendre transformation:

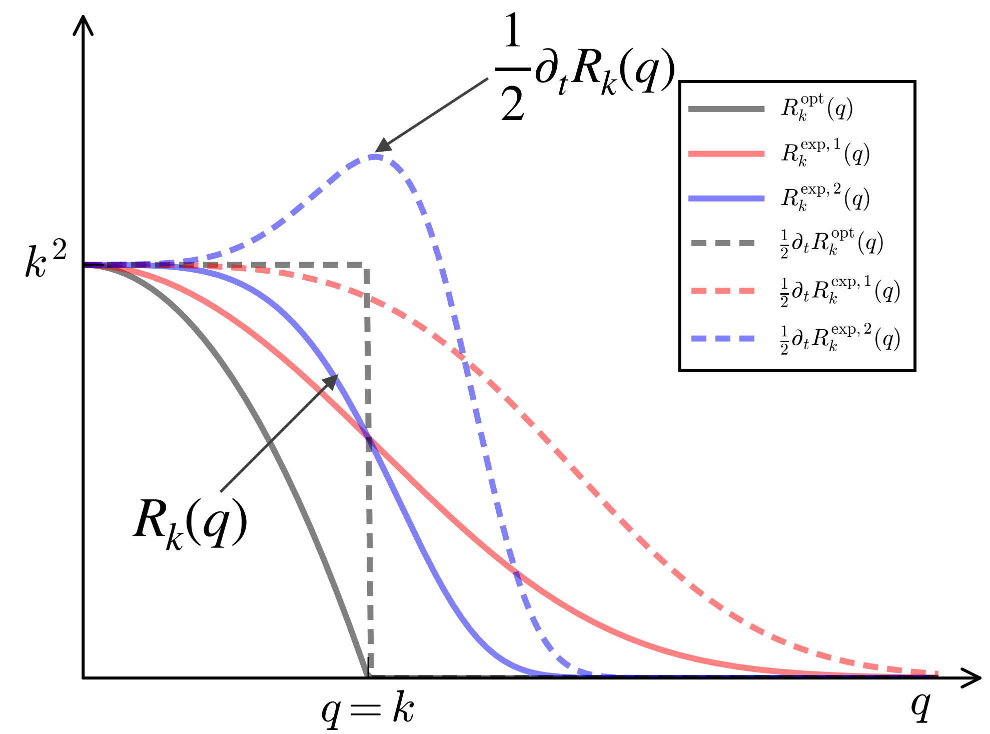
$$\Gamma_k[\Phi] = -W_k[J] + J^a \Phi_a - \Delta S_k[\Phi]$$

flow of the effective action:

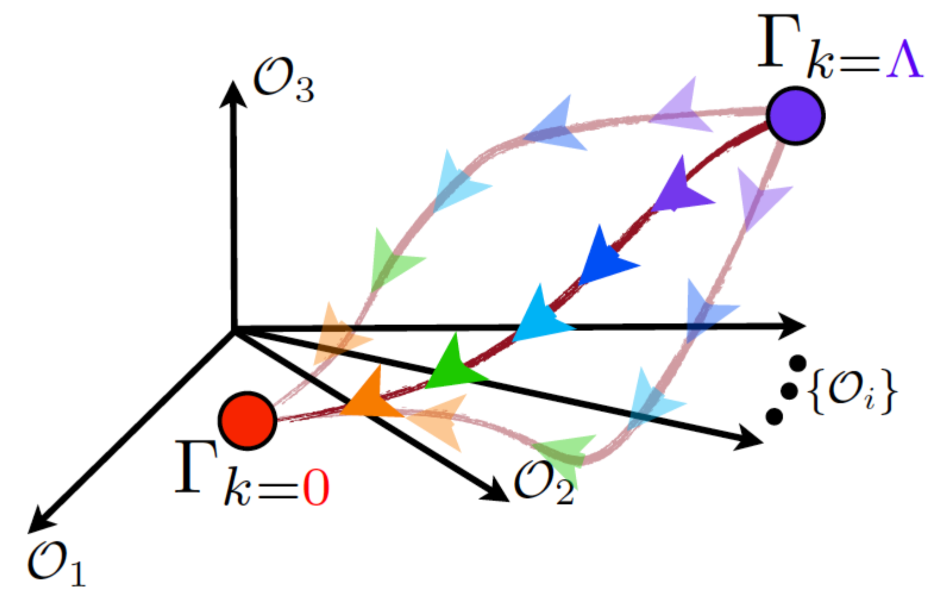
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{STr}\Big[\big(\partial_t R_k\big) G_k \Big] = \frac{1}{2}$$ 

Wetterich formula

C. Wetterich, *PLB*, 301 (1993) 90



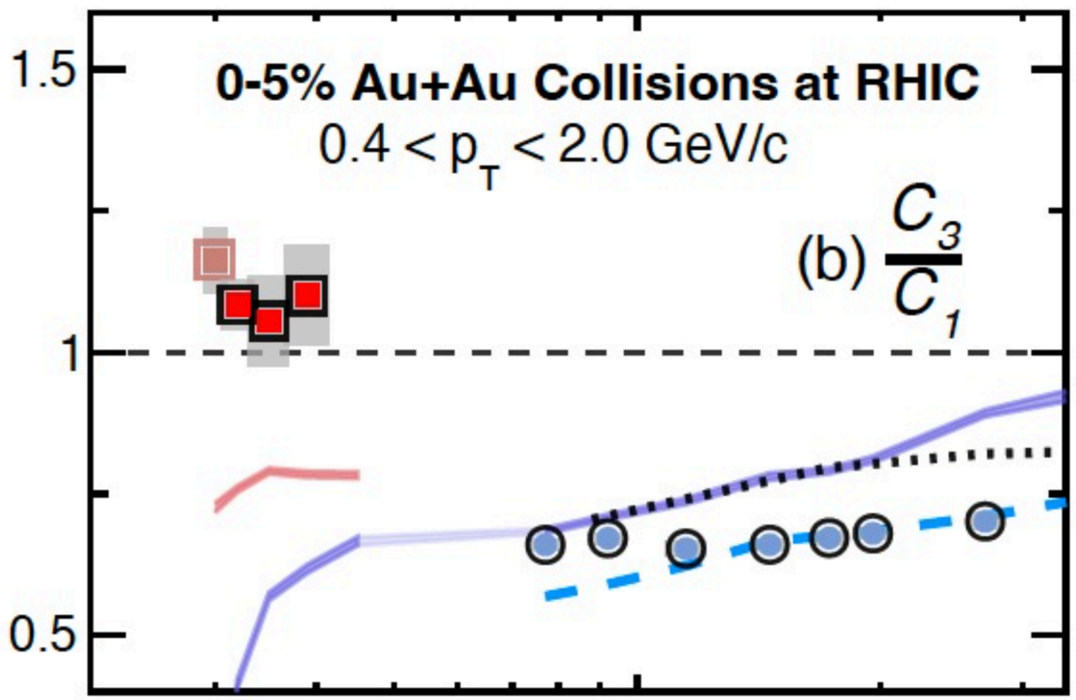
$$G_{k,ab} = \gamma^c{}_a \Big(\Gamma_k^{(2)}[\Phi] + \Delta S_k^{(2)}[\Phi] \Big)^{-1}_{cb},$$



Review: WF, *CTP* 74 (2022) 097304, arXiv: 2205.00468 [hep-ph]

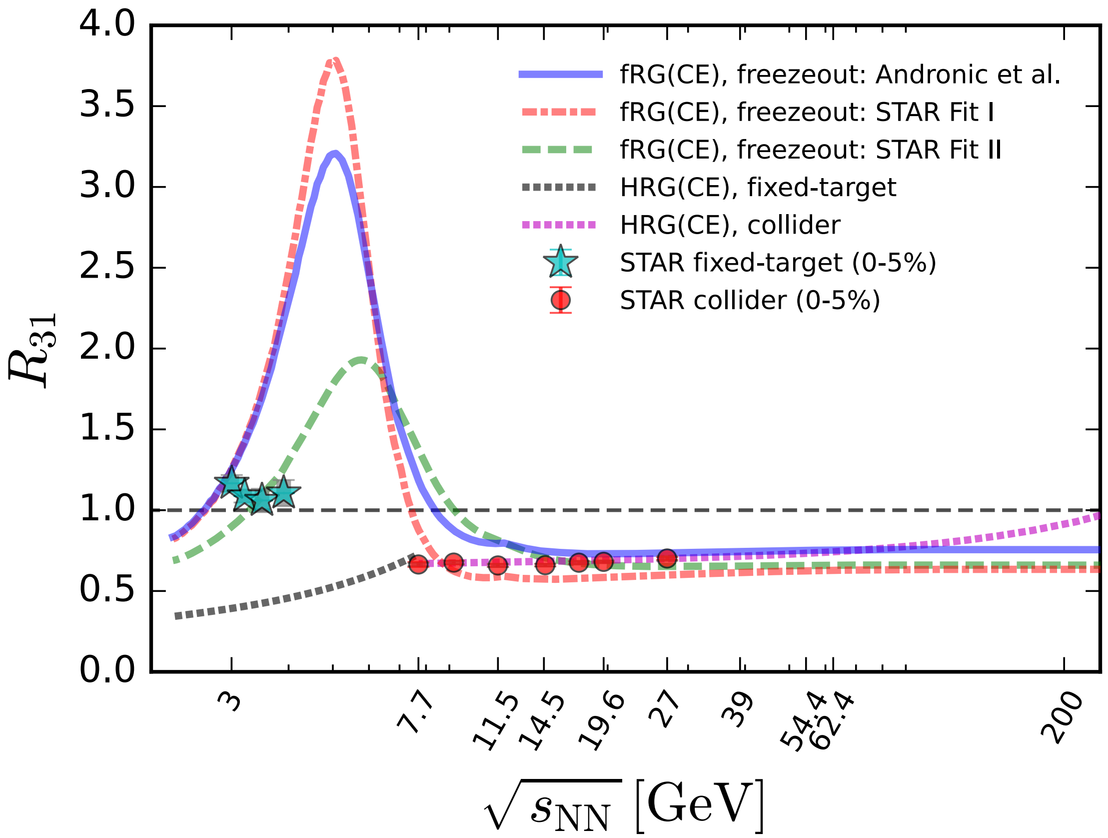
C_3/C_1: Comparison to STAR data

STAR data and UrQMD (baseline):



STAR: Z. Sweger, Quark Matter 2025
STAR: arXiv:2504.00817

fRG (critical) and HRG (baseline):



fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508; Zhao, Yin, WF, in preparation

- **Significant deviations from both the non-critical baseline results in UrQMD and HRG.**
- **fRG results are in accordance with data.**

Fierz-complete basis of four-quark interactions

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, $SU_A(N_f)$ and $U_A(1)$

$$\mathcal{T}^{(V-A)}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\gamma_\mu T^0 q)^2 - (\bar{q} i\gamma_\mu\gamma_5 T^0 q)^2 \,,$$

$$\mathcal{T}^{(V+A)}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\gamma_\mu T^0 q)^2 + (\bar{q} i\gamma_\mu\gamma_5 T^0 q)^2 \,,$$

$$\mathcal{T}^{(S-P)+}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\, T^0 q)^2 - (\bar{q}\,\gamma_5 T^0 q)^2 + (\bar{q}\, T^a q)^2 - (\bar{q}\,\gamma_5 T^a q)^2 \,,$$

$$\mathcal{T}^{(V-A)^{\mathrm{adj}}}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\gamma_\mu T^0 t^a q)^2 - (\bar{q} i\gamma_\mu\gamma_5 T^0 t^a q)^2 \,,$$

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, $SU_A(N_f)$, breaking $U_A(1)$

$$\mathcal{T}^{(S+P)-}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\, T^0 q)^2 + (\bar{q}\,\gamma_5 T^0 q)^2 - (\bar{q}\, T^a q)^2 - (\bar{q}\,\gamma_5 T^a q)^2 \,,$$

$$\mathcal{T}^{(S+P)^{\mathrm{adj}}_-}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\, T^0 t^a q)^2 + (\bar{q}\,\gamma_5 T^0 t^a q)^2 - (\bar{q}\, T^a t^b q)^2 - (\bar{q}\,\gamma_5 T^a t^b q)^2 \,,$$

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, $U_A(1)$, breaking $SU_A(N_f)$

$$\mathcal{T}^{(S-P)-}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\, T^0 q)^2 - (\bar{q}\,\gamma_5 T^0 q)^2 - (\bar{q}\, T^a q)^2 + (\bar{q}\,\gamma_5 T^a q)^2 \,,$$

$$\mathcal{T}^{(S-P)^{\mathrm{adj}}_-}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\, T^0 t^a q)^2 - (\bar{q}\,\gamma_5 T^0 t^a q)^2 - (\bar{q}\, T^a t^b q)^2 + (\bar{q}\,\gamma_5 T^a t^b q)^2 \,.$$

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, breaking $SU_A(N_f)$, $U_A(1)$

$$\mathcal{T}^{(S+P)+}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\, T^0 q)^2 + (\bar{q}\,\gamma_5 T^0 q)^2 + (\bar{q}\, T^a q)^2 + (\bar{q}\,\gamma_5 T^a q)^2 \,,$$

$$\mathcal{T}^{(S+P)^{\mathrm{adj}}_+}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\, T^0 t^a q)^2 + (\bar{q}\,\gamma_5 T^0 t^a q)^2 + (\bar{q}\, T^a t^b q)^2 + (\bar{q}\,\gamma_5 T^a t^b q)^2 \,.$$