Xiong-Hui Cao 曹雄辉

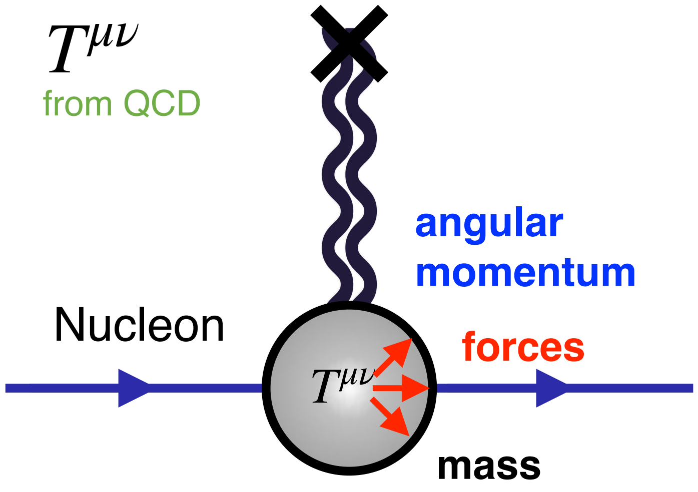
__XHC__, F.-K. Guo, Q.-Z. Li, D.-L. Yao, Nature Commun. 16 (2025) 6979;

__XHC__, F.-K. Guo, Q.-Z. Li, B.-W. Wu, D.-L. Yao, Eur. Phys. J. ST 235 (2026) 9, 2121

第十二届高能物理大会暨第十四届晨光杯评选

2026年7月15日，广州，华南师范大学

Gravitational structure of nucleons



- Gravity couples to matter through energy-momentum tensor (EMT) $T^{\mu\nu}$

study gravitational forces within hadrons

study **mechanical properties** of hadrons

- Definition:

Forces inside hadron, mechanical picture of quark confinement?

$$\left\langle N(p') \middle| T^{\mu\nu} \middle| N(p) \right\rangle = \frac{1}{4m_N}\bar{u}(p')\left[A(t)P^\mu P^\nu + J(t)\left(iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho\right) + D(t)\left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right)\right]u(p)$$

Kobzarev, Okun (1962)
Pagels (1966)

- **Mass** normalization:

$$m_N = \int d^3r T_{00}(r)$$

$$A(0) = 1$$

- **Spin** normalization:

$$J^i = \epsilon_{ijk}\int d^3r r_j T_{0k}(r)$$

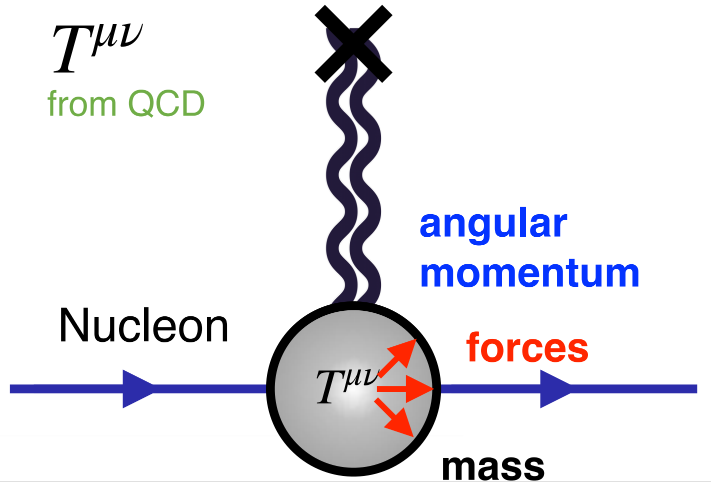
$$J(0) = 1/2$$

- **D-term**: $D \equiv D(0)$

$$D = -\frac{m_N}{2}\int d^3r\left(r_i r_j - \frac{1}{3}\delta_{ij}\right)T_{ij}(r)$$

$$D = ?$$

Gravitational structure of nucleons



D-term as the "last unknown global property"

Polyakov, and Schweitzer, (2018)

em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'\vert J^\mu_{\text{em}}\vert N\rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19}\text{C}$ $\mu = 2.792847356(23)\mu_N$
weak:	PCAC	$\langle N'\vert J^\mu_{\text{weak}}\vert N\rangle$	$\longrightarrow$	$g_A = 1.2694(28)$ $g_p = 8.06(55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N'\vert T^{\mu\nu}_{\text{grav}}\vert N\rangle$	$\longrightarrow$	$m = 938.272013(23)\,\text{MeV}/c^2$ $J = \frac{1}{2}$ $D = ?$

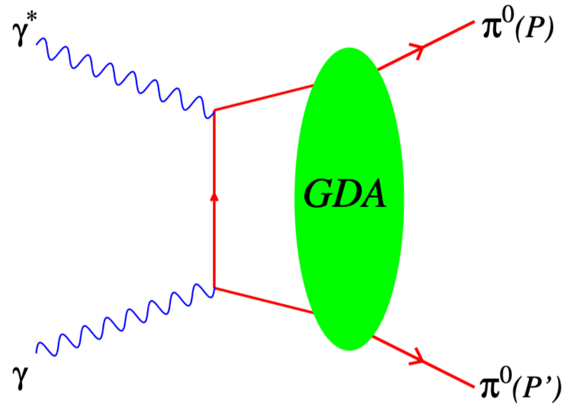
D-term form factor from experiments

- All GFFs can be measured from DVCS through sum rules derived for GPDs Ji, PRL (1997); Ji, Melnitchouk and Song, PRD (1997)
- π^0: $\gamma^*\gamma \to \pi^0\pi^0$ in e^+e^- Belle data: PRD 93, 032003 (2016)

  $D^Q_{\pi^0} \simeq -0.7$ at $\langle Q^2 \rangle = 16.6$ GeV^2

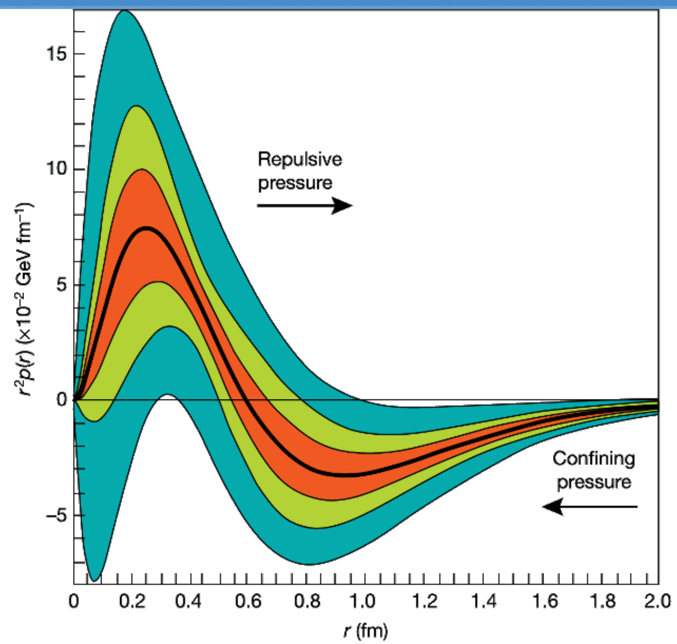
  Kumano, Song, Teryaev, PRD (2018)

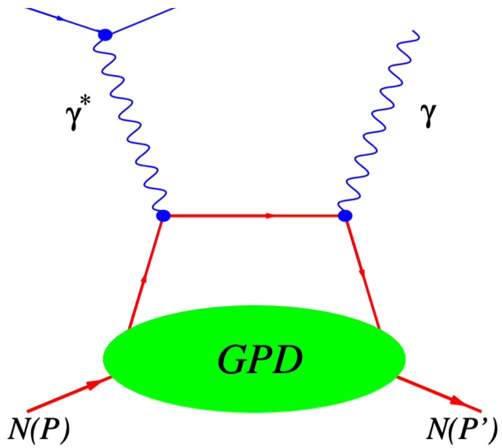
  Chiral symmetry: total $D_{\pi^0} \simeq -1$ (gluons contribute the rest)



- Proton: Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)

“Pressure” distribution inside proton using DVCS data





Subtraction constant in dispersion relations for DVCS amplitudes

- $C_{\mathscr{H}}(t, \mu^2) \to D^Q(t, \mu^2)$ scale-dependent

  model-dependent
- Explore t range at EIC & EicC

Scalar radius puzzle

- Trace FF:

$$\left\langle N(p')\middle| T^{\mu}_{\ \mu} \middle| N(p)\right\rangle = m_N \bar{u}(p')\left[A(t) - \frac{t}{4m_N^2}[A(t) - 2J(t) + 3D(t)]\right] u(p) \equiv \bar{u}(p')\Theta(t)u(p)$$

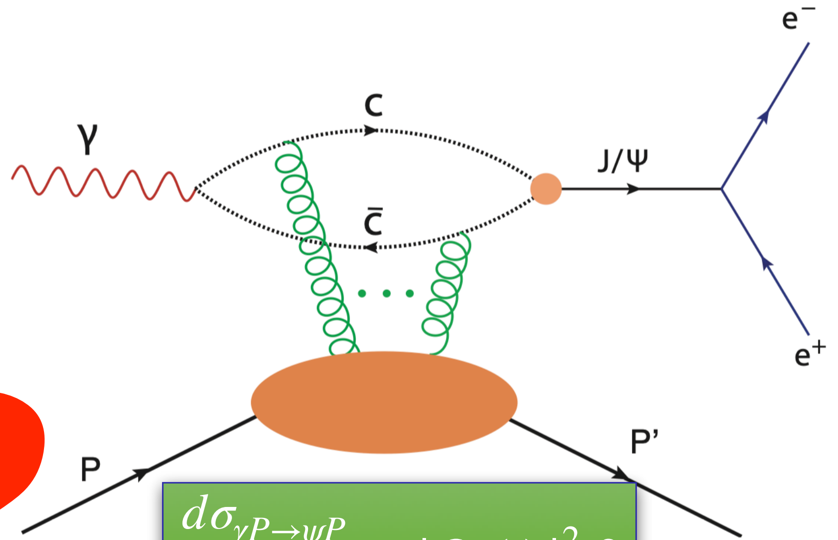
- Trace anomaly in QCD: $T^{\mu}_{\ \mu} = \frac{\beta(g)}{2g} F^{a,\mu\nu} F^{a,}{}_{\mu\nu} + (1+\gamma_m)\sum_q m_q \bar{\psi}_q \psi_q$

- Scalar radius: $\Theta(t) = m_N\left(1 + t\langle r_\Theta^2\rangle/6 + \dots\right)$

  - It could be extracted from the threshold photoproduction of the vector-meson, e.g., J/ψ, and the fit result is ~0.55 fm
    Kharzeev, PRD (2021)……

  - Recently, two LQCD calculations at near physical quark mass result in a large scalar radius, ~1 fm
    Hackett et al., PRL (2024); Wang et al. [χQCD], PRD (2024)

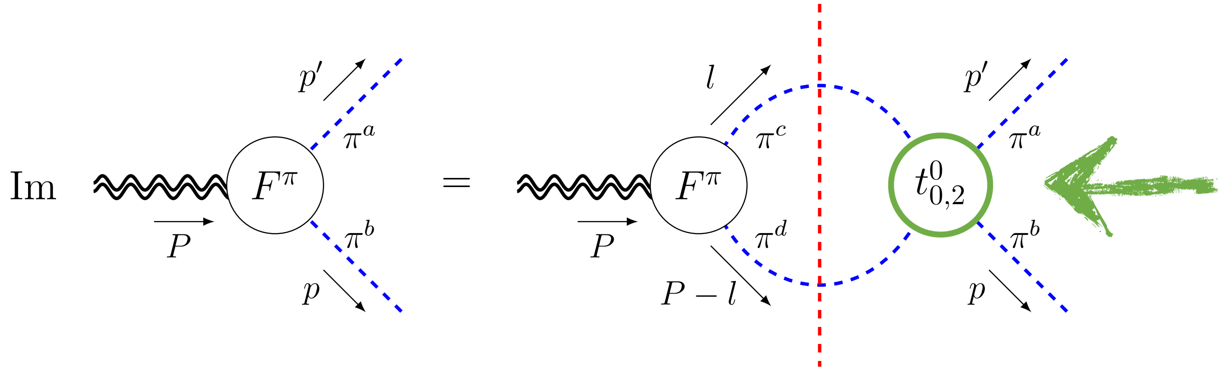
?



$$\frac{d\sigma_{\gamma P\to\psi P}}{dt} \propto |\Theta_g(t)|^2 \ ?$$

Unitarity relation for pion GFFs

- Partial-wave unitarity



$\pi\pi$ scattering

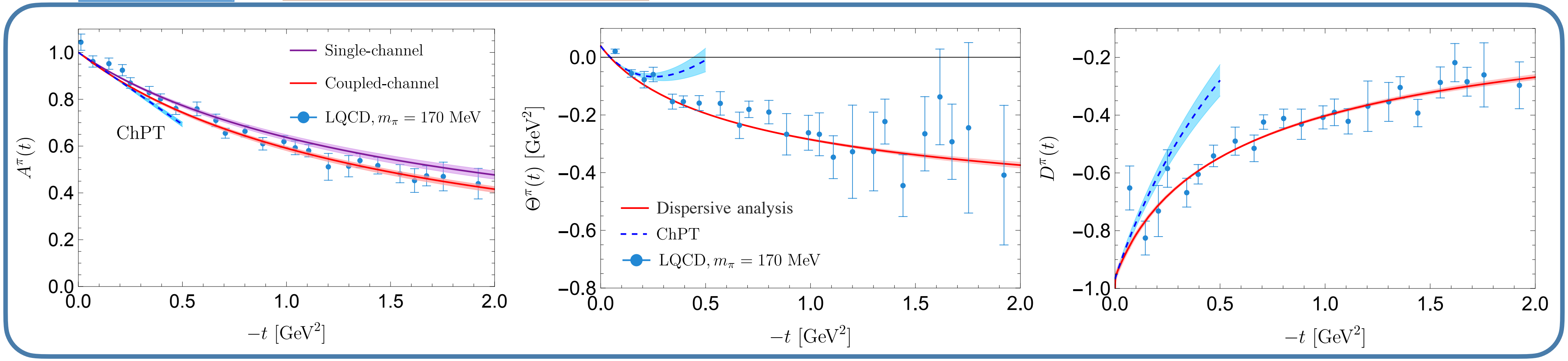
$\operatorname{disc} A^\pi(t) = 2iA^\pi(t)\sin\delta_2^0(t)e^{-i\delta_2^0(t)}\theta\left(t-t_\pi\right)$

Omnes problem

Pion

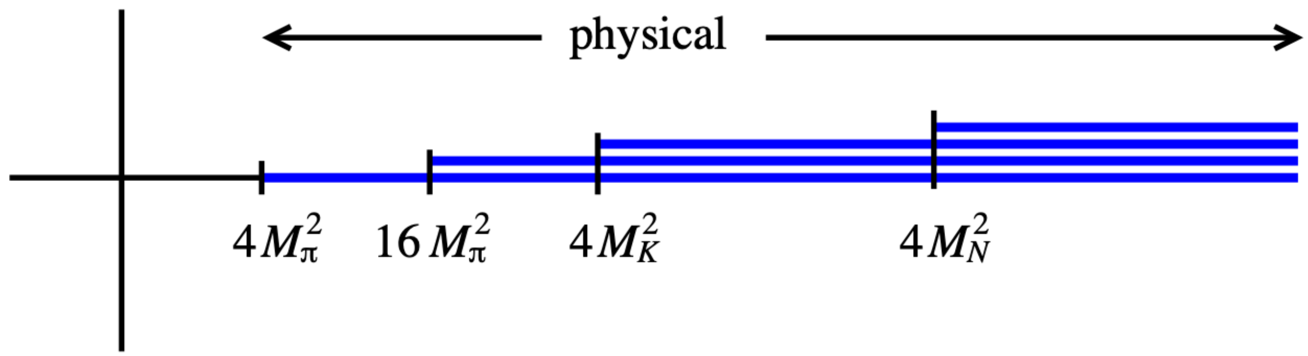
Prediction, Not fit

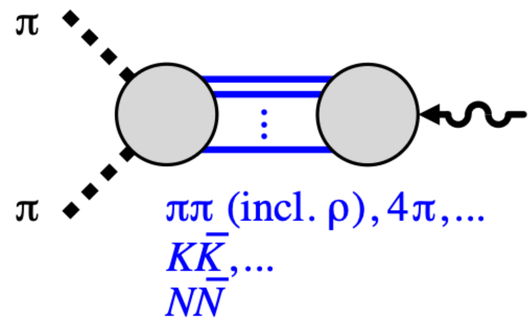
LQCD ($m_\pi \simeq 170$ MeV): Hackett et al., PRD (2023)



Unitarity relation for nucleon GFFs

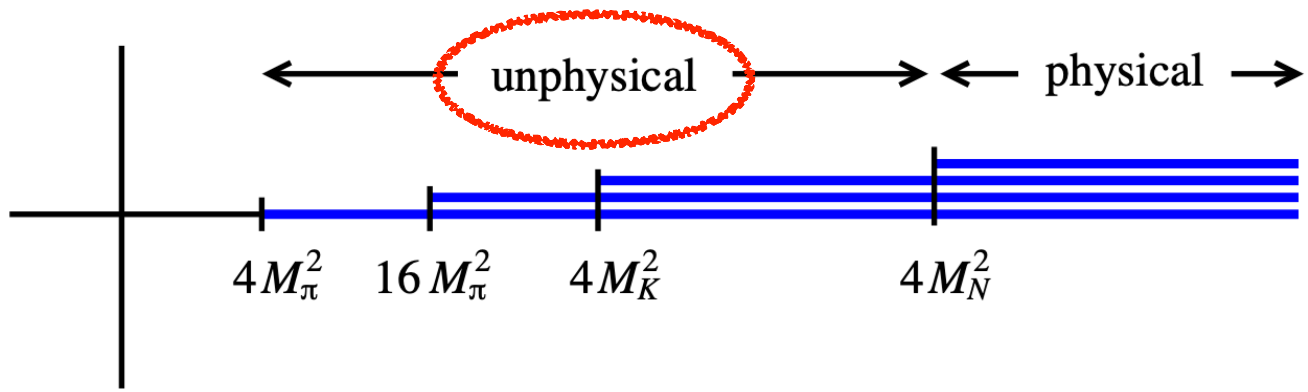
- Pion GFFs

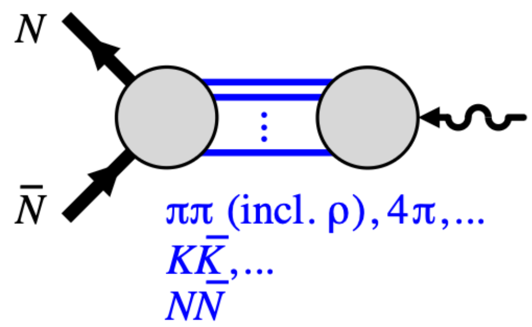




- Nucleon GFFs

Analytic continuation!

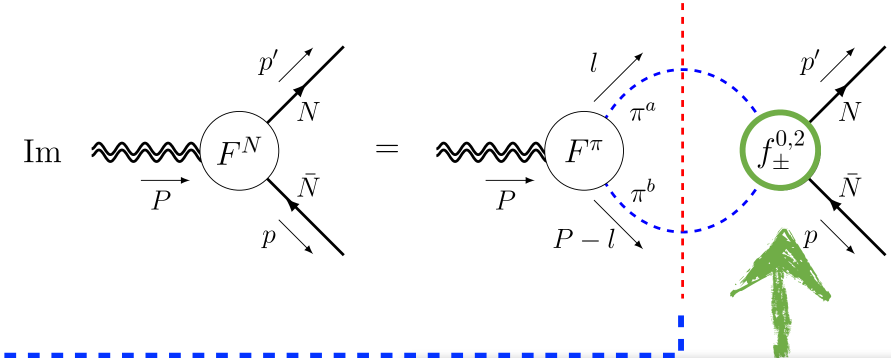




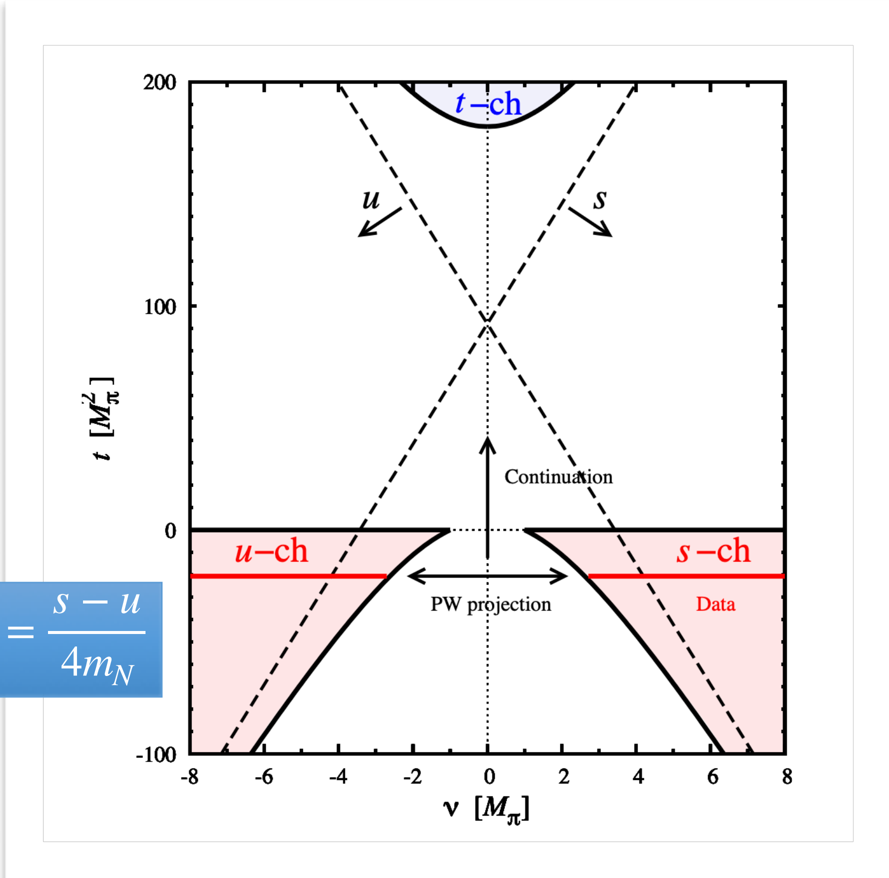
Unitarity relation for nucleon GFFs

- Discontinuity

$$\mathrm{Disc}\left\langle N(p')\,\bar N(p)\left|\hat T^{\mu\nu}(0)\right|0\right\rangle \propto \sum_n \langle N(p')\,\bar N(p)\,|\,n\rangle\langle n\,|\,\hat T^{\mu\nu}(0)\,|\,0\rangle^{*}\delta^4\left(p+p'-p_n\right)$$



$\pi\pi(K\bar K) \to N\bar N$ scattering



$$\nu = \frac{s-u}{4m_N}$$

- Idea: Construct $\pi\pi \to N\bar N$ PWAs $\Gamma_i(t)$ from $\pi N \to \pi N$ scattering data using amplitude analysis techniques

Analytically continue PWAs to region $t > 4M_\pi^2$ via **Roy-Steiner eq.**

Ditsche, et al., JHEP (2012);

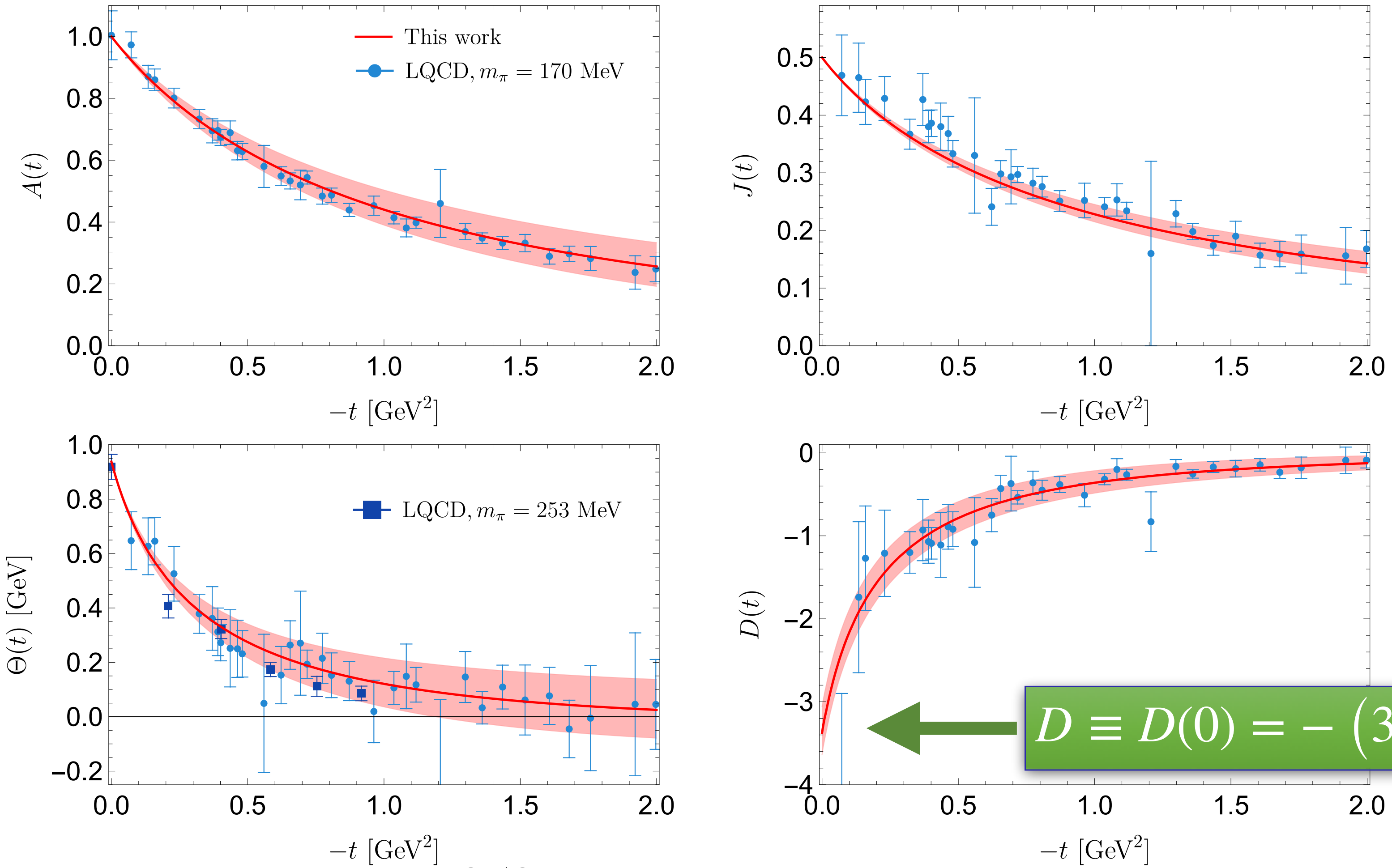
Hoferichter, et al., Phys. Rept. (2016);

XHC, Q.-Z. Li & H.-Q. Zheng, JHEP (2022)

Nucleon GFFs

Predictions：

LQCD ($m_\pi \simeq 170$ MeV): Hackett et al., PRL (2024)



$$D \equiv D(0) = -\left(3.38^{+0.34}_{-0.35}\right)$$

LQCD ($m_\pi \simeq 253$ MeV): Wang et al. [χQCD], PRD (2024)

☑ GFFs be model-independently predicted using data-driven dispersion relations + ChPT

☑ Not a fit!

Physics of the scalar (trace) radius

- The scalar (trace) radius is larger than the mass radius as long as $D < 0$

$\boxed{T_\mu^{\ \mu}}$ $\boxed{T^{00}}$

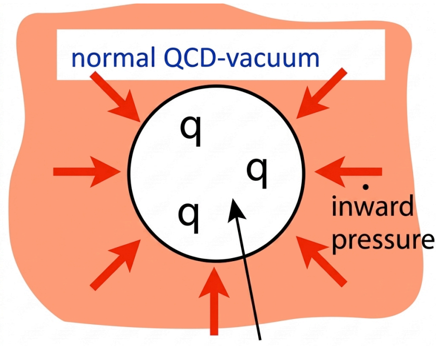
$$\sqrt{\langle r_\Theta^2 \rangle} = 0.97^{+0.03}_{-0.03}\text{fm} \quad > \quad \sqrt{\langle r_{C,p}^2 \rangle} = 0.84\text{fm} \quad > \quad \sqrt{\langle r_{\text{Mass}}^2 \rangle} = 0.70^{+0.03}_{-0.04}\text{fm}$$

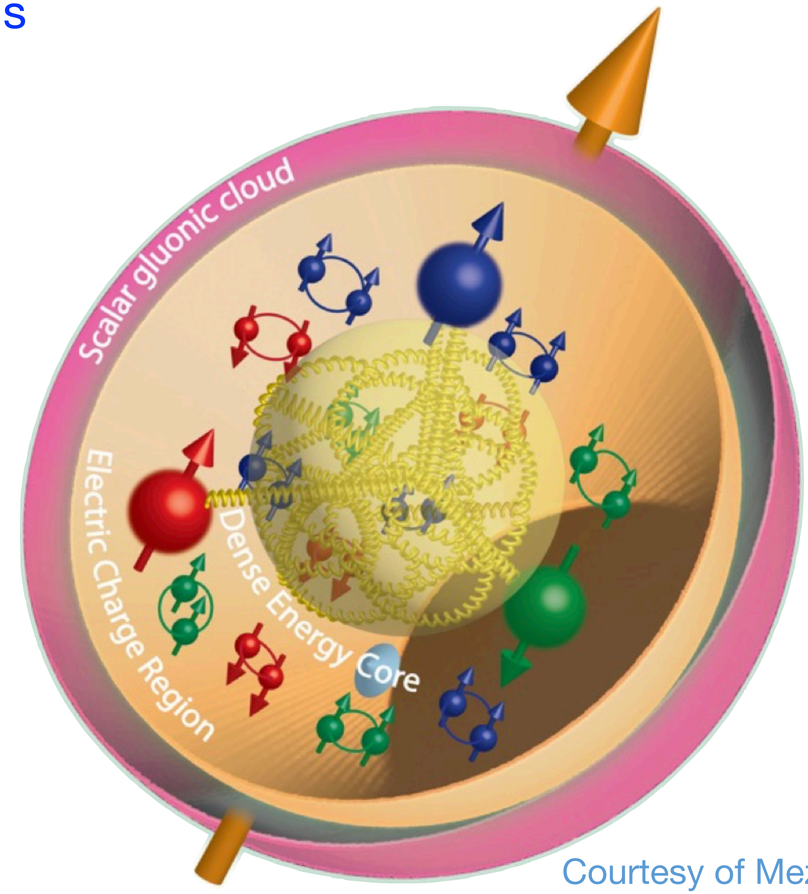
proton charge radius

"the largest radius in strong interaction"

- In the MIT bag model, scalar radius may be considered as the bag (confinement) radius ~1 fm (physical boundary of confinement)

Ji, Front.Phys.(Beijing) (2021)





Courtesy of Meziani

Summary

- The pion, kaon and nucleon GFFs are precisely determined using dispersive method with some inputs.

- Nucleon static properties:

$$D = -3.38^{+0.34}_{-0.35}, \quad \sqrt{\langle r^2_{\Theta} \rangle} = 0.97^{+0.03}_{-0.03}\mathrm{fm} \quad > \quad \sqrt{\langle r^2_{C,p} \rangle} = 0.84\mathrm{fm} \quad > \quad \sqrt{\langle r^2_{\mathrm{Mass}} \rangle} = 0.70^{+0.03}_{-0.04}\mathrm{fm}$$

proton charge radius

- Foundation for further analyses, interpretations, etc.

Thank you for your attention!

Nucleon GFFs: D-term & radii

- Various radii in the Breit frame
  - Radius of the scalar trace density

$$\langle r^2_\Theta \rangle = \frac{6\dot{\Theta}(0)}{m_N} = 6\dot{A}(0) - \frac{9D}{2m_N^2}$$

  - Radius of the mass (energy) density

$$\left\langle r^2_{\text{Mass}} \right\rangle = 6\dot{A}(0) - \frac{3D}{2m_N^2}$$

  - Mechanical radius

Polyakov, PLB (2003)
Polyakov, & Schweitzer, IJMPA (2018)

$$\left\langle r^2_{\text{Mech}} \right\rangle = \frac{6D}{\int_{-\infty}^{0} \mathrm{d}t D(t)}$$

  - Radius of the angular momentum (spin) density $J(t) + \frac{2}{3} t \frac{\mathrm{d}J(t)}{\mathrm{d}t}$

$$\langle r^2_J \rangle = 20 J'(0)$$

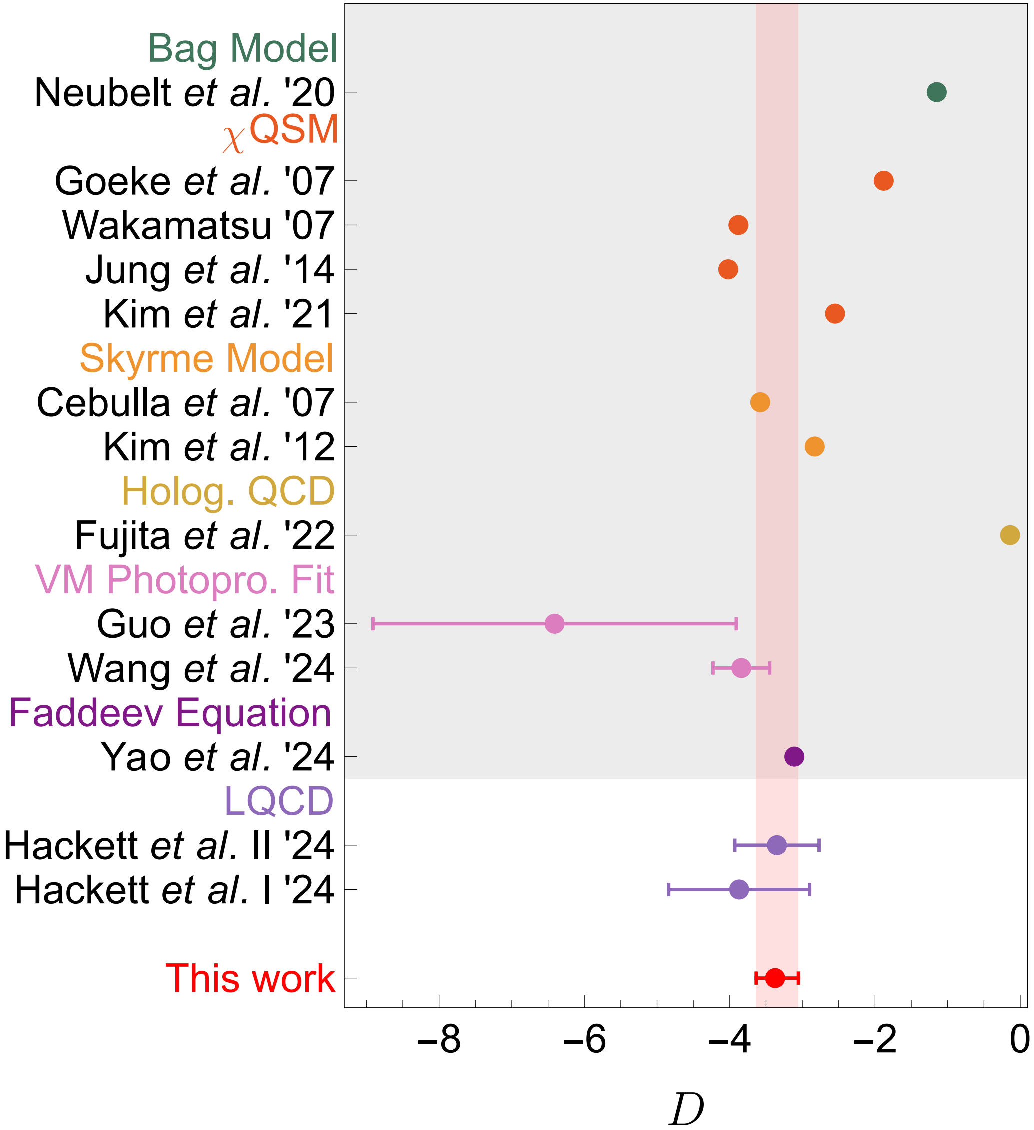
Polyakov, PLB (2003)
Lorcé et al., PLB (2018)

Quantity	Result	Error budget
D-term	$-3.38^{+0.34}_{-0.35}$	$+(0.18)_{\text{ChPT}}(0.12)_{\text{PWA}}(0.26)_{\text{eff}}$
		$-(0.16)_{\text{ChPT}}(0.12)_{\text{PWA}}(0.29)_{\text{eff}}$
$\sqrt{\langle r^2_\Theta \rangle}$	$0.97^{+0.03}_{-0.03}$	$+(0.01)_{\text{ChPT}}(0.01)_{\text{PWA}}(0.03)_{\text{eff}}$
		$-(0.02)_{\text{ChPT}}(0.01)_{\text{PWA}}(0.26)_{\text{eff}}$
$\sqrt{\langle r^2_{\text{Mass}} \rangle}$	$0.70^{+0.03}_{-0.04}$	$+(0.02)_{\text{ChPT}}(0.01)_{\text{PWA}}(0.02)_{\text{eff}}$
		$+(0.02)_{\text{ChPT}}(0.01)_{\text{PWA}}(0.03)_{\text{eff}}$
$\sqrt{\langle r^2_{\text{Mech}} \rangle}$	$0.72^{+0.09}_{-0.08}$	$+(0.02)_{\text{ChPT}}(0.00)_{\text{PWA}}(0.09)_{\text{eff}}$
		$-(0.03)_{\text{ChPT}}(0.01)_{\text{PWA}}(0.07)_{\text{eff}}$
$\sqrt{\langle r^2_J \rangle}$	$0.70^{+0.02}_{-0.02}$	$+(0.01)_{\text{ChPT}}(0.01)_{\text{PWA}}(0.01)_{\text{eff}}$
		$-(0.01)_{\text{ChPT}}(0.00)_{\text{PWA}}(0.02)_{\text{eff}}$

ChPT: NLO ChPT inputs
pwa: $\pi\pi/K\bar{K} \to N\bar{N}$
eff: effective poles m_S, m_D

Nucleon D-term

- Nucleon D-term $D \equiv D(0)$: $D = -\left(3.38^{+0.34}_{-0.35}\right)$



- Positivity bound:

$$D < -0.2$$

Gegelia & Polyakov, PLB (2021)

"The stability condition"

normal force $\frac{2}{3}s(r) + p(r) \geq 0$

$$\widetilde{D}(r) = \int \frac{d^3\mathbf{\Delta}}{(2\pi)^3} e^{-i\mathbf{\Delta r}} D\left(-\mathbf{\Delta}^2\right) < 0$$

Comparison with other works