Hypergeometric function representation of Feynman integral

费曼积分的超几何函数表示

Tai-Fu Feng，Hai-Bin Zhang (speaker) et al.
Hebei University, Baoding

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01

Research Background:
The Dilemma of Feynman Loop Calculations

Feynman Loop Diagrams: A Core Tool in Theoretical Physics

Core Mathematical Tools

Feynman loop diagrams are the core mathematical tools for calculating the interaction processes of elementary particles in quantum field theory.

Experimentally Driven Precision

With the improvement of experimental measurement precision, the requirements for the precision of theoretical predictions have become stringent.

High-Precision Calculation Needs

To obtain high-precision theoretical predictions, it is essential to perform accurate analytic or numerical calculations of multi-loop Feynman diagrams.

Key Research Directions

Therefore, developing efficient and accurate methods for Feynman loop calculations is a key issue in the field of theoretical physics.

Quantum Field Theory

Bottlenecks of Traditional Methods: Exponential Growth of Computational Complexity

Difficulties in Analytic Calculation

Traditional parametrization methods lead to extremely complex multiple integrals when dealing with multi-loop diagrams, making it difficult to obtain analytic solutions.

Inefficiency in Numerical Calculation

Although numerical calculation methods are universal, they are slow and inefficient when dealing with multi-loop and multi-scale problems, and have high requirements for CPU and memory.

Slow Research Progress

Despite urgent needs, the development of analytic calculation methods for multi-loop Feynman scalar integrals has been slow over the past four decades, becoming a bottleneck in this field.

02

Core Thought:
Hypergeometric Function Representation

New Path: Hypergeometric Function Representation

Core Idea: Transform complex Feynman integrals into analytic expressions represented by hypergeometric functions through a series of mathematical transformations.
(Such as: univariate Gauss function, bivariate Apell function)

Analyticity

Within convergence intervals, precise analytic solutions can be obtained, which are concise and elegant in form and easy to verify.

Efficiency

Computational programs based on analytic expressions are much faster than traditional numerical algorithms.

Universality

The method demonstrates strong adaptability and can handle various complex Feynman diagrams from one loop to multi-loop.

This method provides a completely new mathematical approach for the analytical calculation of Feynman diagrams.

03

Theoretical Discovery:

Analytical Solutions from one/two Loops to Multi-Loops

Phase 1: Basic Series Solutions for one/two Loops (2018-2019)

Research Objects

one-loop B0, C0 functions, two-loop vacuum diagrams, and sunset diagrams.

Main Achievements

- Derived the hypergeometric function representations of these Feynman integrals.
- Obtained precise analytic series solutions within the convergence interval around the zero point.
- Derived the homogeneous linear partial differential equations satisfied by these solutions.

Published in

Nuclear Physics B (2018, 2019)

Phase 2: Analytic Continuation and Hybrid Algorithm (2020-2023)

Core Challenge

How to analytically continue the solution from the neighborhood of zero to other regions? This is the key connecting local solutions to global solutions.

Innovative Method

Proposed a algorithm of Mellin-Barnes representation and Miller transformation, breaking through traditional continuation limits and achieving efficient computation.

Key Achievements

Established the GKZ PDEs on the ordinary manifold; solved the fundamental solution systems in the neighborhoods of zero and infinity, covering the core parameter space.

Published in: Nuclear Physics B (2020), European Physical Journal C (2023)

Phase 3: Introduction of Grassmannian Manifold (2022)

Core Challenge

How to solve the fundamental solution systems in the neighborhoods of all singularities?

Innovative Breakthrough

Introduced the geometric theory of Grassmannian manifold into the Feynman integral representation.

Key Achievements

Constructed GKZ hypergeometric systems on Grassmannian manifolds. Enabled fundamental solution systems in the neighborhoods of all singularities.

Published in: *Physical Review D (2022)*

Phase 4: Global Analytic Continuation (2025)

Core Goal

How to efficiently extend the analytic solution to the entire domain of definition?

Innovative Method

Proposed a method to complete analytic continuation by extending the three types of Gauss relations of hypergeometric functions.

Key Achievements

obtained the Laurent series in the neighborhood of spacetime dimension D=4, which is crucial for physical applications.

Published in

Physical Review D (2025)

Through the clever application of the Gauss relations of hypergeometric functions, we achieved the leap from local solutions to global solutions.

Phase 5: Breakthrough in Multi-Loop Problems (2023-2025)

Core Verification

Is the Hypergeometric Function Method applicable to multi-loop Feynman diagrams?

Research Objects

Three-loop and four-loop vacuum diagrams.

Key Achievements

Derived their GKZ hypergeometric equations and fundamental solution systems on the ordinary manifold.

Published in

Journal of High Energy Physics

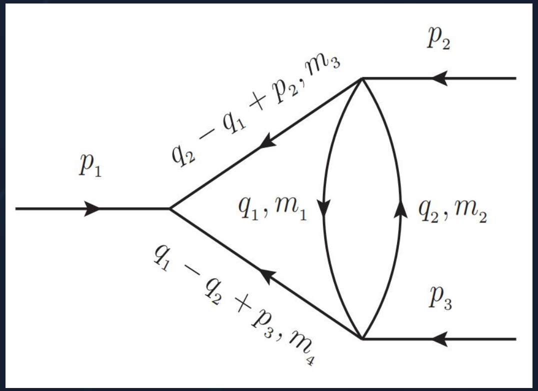
(2023, 2025)

This indicates the applicability of the hypergeometric function representation in the multi-loop calculations.

Present: two-Loop Dune diagram (2025-2026)

Research Objects

The massive two-loop Dune diagram.



Main Achievements

- Feynman integral of the 2-loop massive Dune diagram is embedded in Grassmannian $G_{5,8}$
- Through geometric configurations, the hypergeometric function solutions are attributed to ten types.
- The hypergeometric functions can be transformed into each other by the various Gauss relations.

04

Achievements & Outlook: Efficiency Verification

Computational Efficiency: Analytic vs. Numerical

Comparison Object

Analytic program based on the Hypergeometric Function representation vs. traditional numerical program FIESTA.

Conclusion

The analytic method shows anoverwhelming advantage in computational efficiency, providing a powerful tool for large-scale, high-precision physical calculations.

Multiplication factor of computing speed (Speedup)

Series of Representative Achievements: 8 Papers

- **Evaluating Feynman integrals by the hypergeometry**
  Nuclear Physics B 927 (2018)
- **The system of partial differential equations for the C0 function**
  Nuclear Physics B 940 (2019)
- **GKZ-hypergeometric systems for Feynman integrals**
  Nuclear Physics B 953 (2020)
- **Feynman integrals of Grassmannians**
  Physical Review D 106 (2022)
- **GKZ-system of the 2-loop self energy with 4 propagators**
  European Physical Journal C 83 (2023)
- **GKZ hypergeometric systems of the three-loop vacuum Feynman integrals**
  Journal of High Energy Physics 05 (2023)
- **Gauss Relations in Feynman Integrals**
  Physical Review D 111 (2025)
- **GKZ hypergeometric systems of the four-loop vacuum Feynman integrals**
  Journal of High Energy Physics 03 (2025)

Impact: The series covers from method proposal to multi-loop applications, published in JHEP, PRD, EPJC, NP B, etc.

Future Outlook

Deepen Multi-Loop Multi-Scale Research

Continue to conduct in-depth research on the analytic calculation of more complex multi-loop multi-scale Feynman diagrams.

Develop Efficient Programs

Based on existing results, develop more complete and efficient computational program packages to serve high-energy physics research.

第七届粒子物理前沿研讨会

时间：2026年7月10日至15日(10日报道，15日离开)

地点：河北省张家口市，兴垣维景国际酒店

网址：https://indico.ihep.ac.cn/event/29191/

会议主题围绕粒子物理相关最新进展展开，包括但不限于超越标准模型的新物理、Higgs物理、暗物质物理、中微子物理、粒子天体物理和宇宙学等方面的理论和实验的最新进展。

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THANKS