Reviving primordial black hole formation in slow first-order phase transitions

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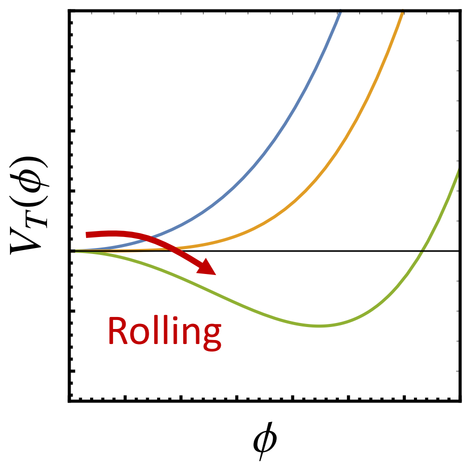
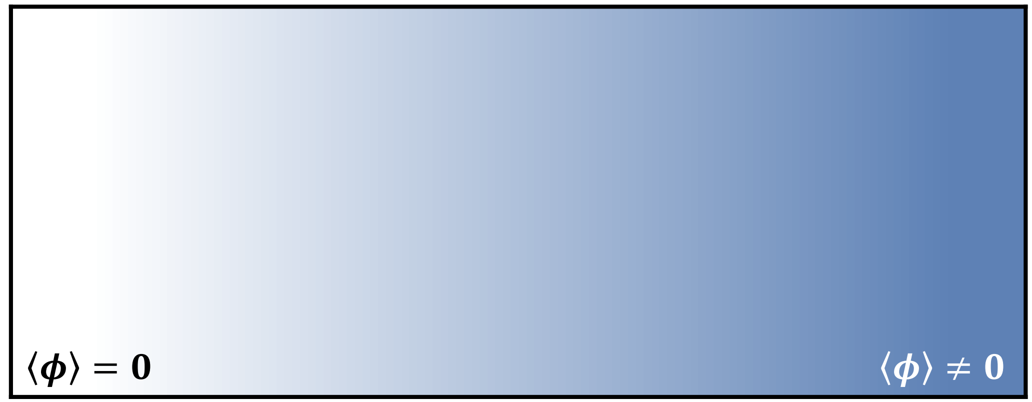
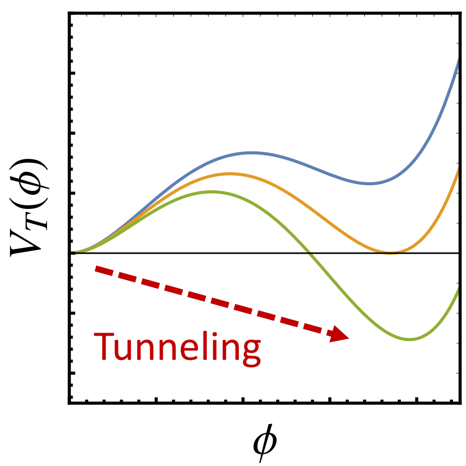
With Wen-Yuan Ai, arXiv: 2605.11332

Patterns of symmetry breaking

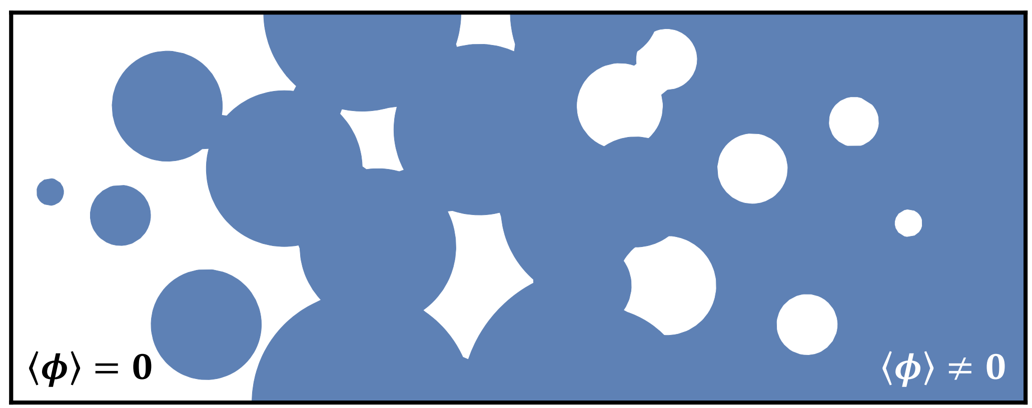


Second-order phase transition / Crossover



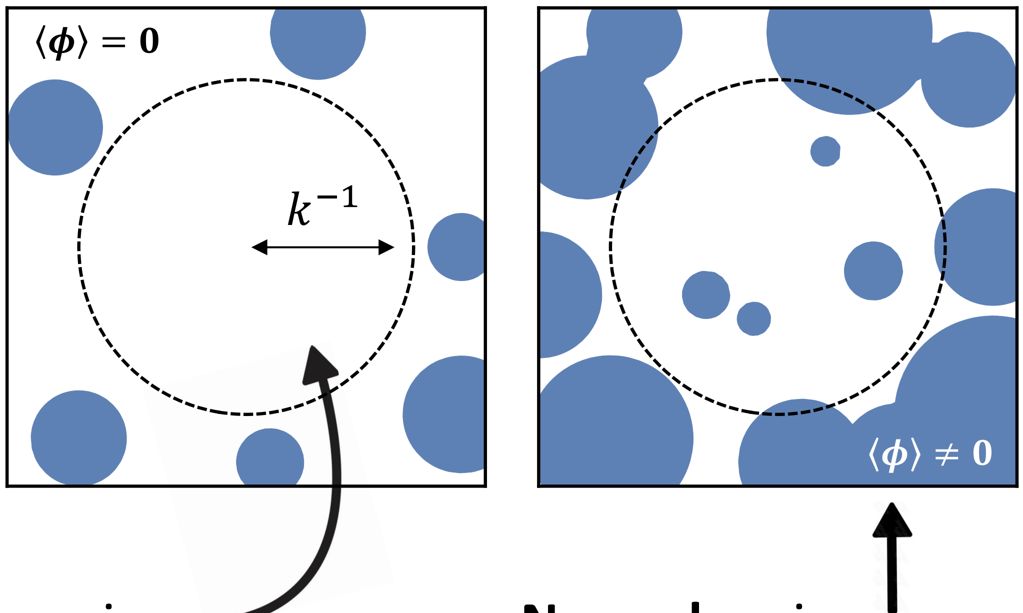


First-order phase transition



Generation of overdensities

Randomness of bubble nucleation



Delayed-decay region

More vacuum energy, $\rho_V \propto a^0$

Normal region

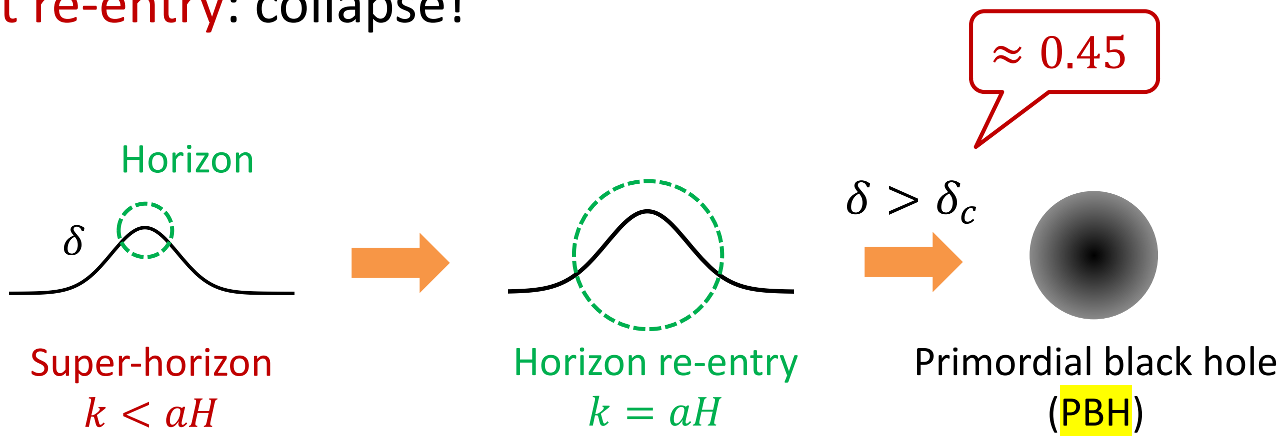
More radiation energy, $\rho_r \propto a^{-4}$

Energy density contrast (at scale k^{-1})

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} > 0$$

Black hole formation

At generation: super-horizon ($k < aH$), δ frozen
Horizon re-entry ($k = aH$), δ evolves
If $\delta > \delta_c$ at re-entry: collapse!



Tracing back to Kodama *et al*, Prog. Theor. Phys. 68 (1982) 1979
Recent **big progress** in Liu, Bian, Cai, Guo, and Wang, Phys.Rev.D 105 (2022) 2, L021303

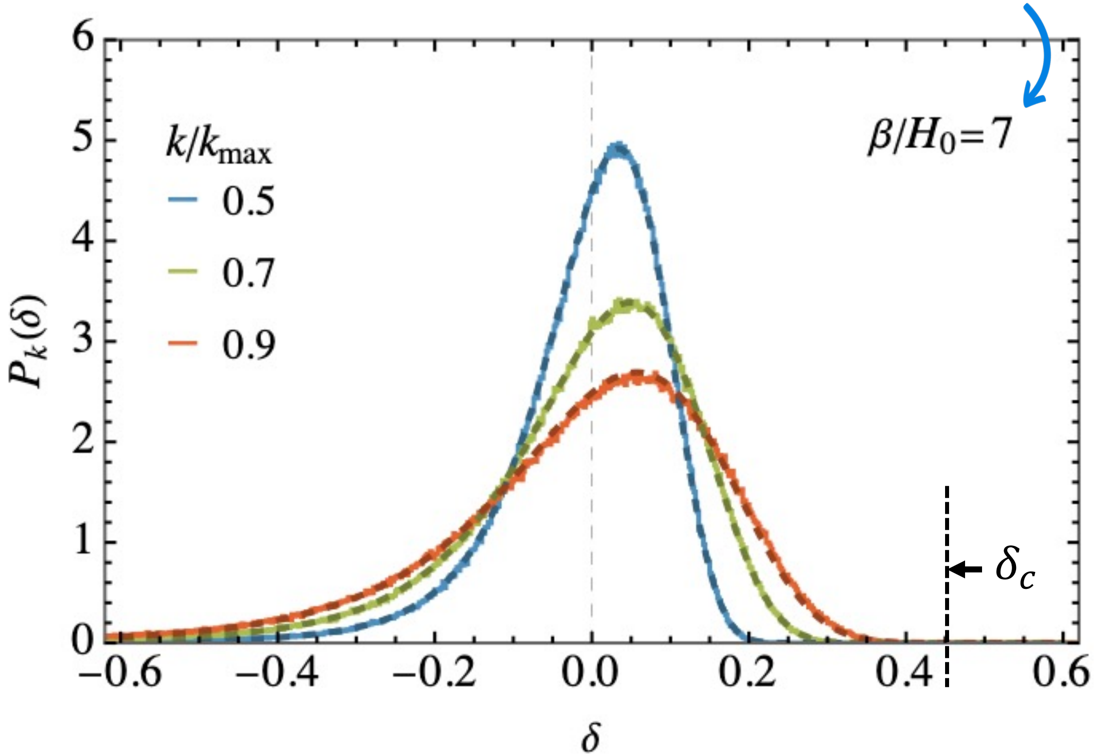
A promising and popular mechanism!
Hashino *et al*, PLB 838 (2023) 137688; Lewicki *et al*, JHEP 09 (2023) 092; He *et al*, SCPMA 67 (2024) 4, 240411; Cai *et al*, SCPMA 67 (2024) 9, 290411; Flores *et al*, PRD 110 (2024) 1, 015005; Kanemura, Tanaka, **KPX**, JHEP 06 (2024) 036; Kawana *et al*, PRD 108 (2023) 103531; Gouttenoire *et al*, PRD 110 (2024) 4, 4; Banerjee *et al*, JCAP 07 (2025) 007; Huang *et al*, PRD 113 (2026) 5, 055013; etc

Calculation framework

Bubble nucleation history simulation Lewicki *et al*, Phys.Rev.Lett. 133 (2024) 22, 221003

- Probability distribution of δ (non-Gaussian)

(Hubble time)/(Transition duration)



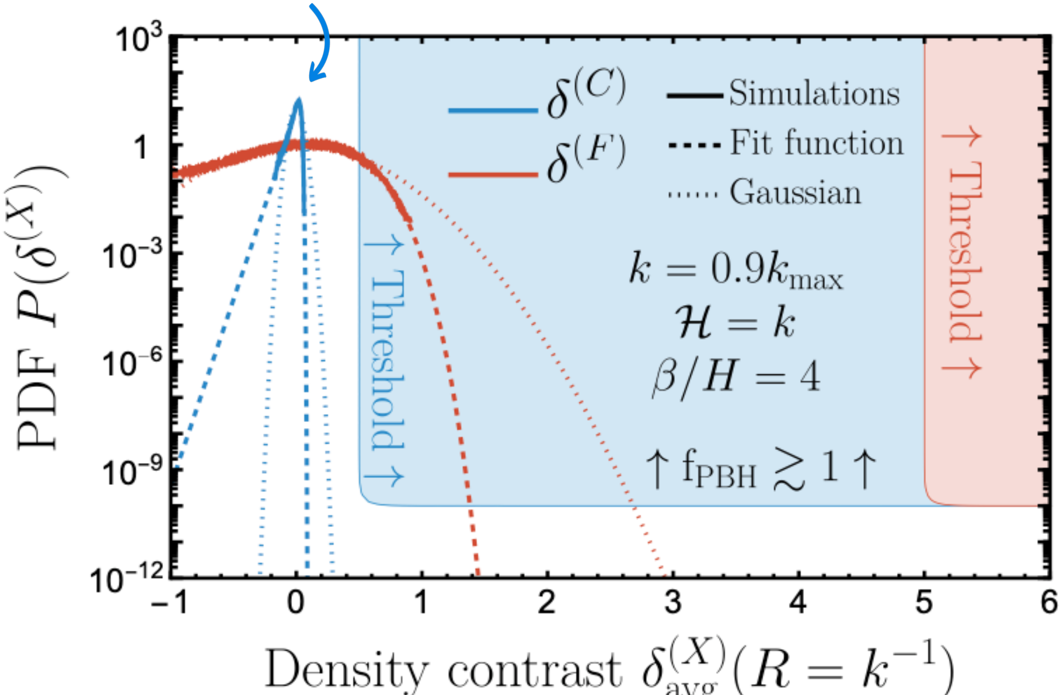
- PBH abundance obtained by integrating $P_k(\delta)$ over $\delta > \delta_c$

Slow transitions with $\beta/H_n \lesssim 10$ lead to abundant PBH formation

Debates

δ depends on coordinate choice Franciolini *et al*, Phys.Rev.Lett. 136 (2026) 17, 171404

- Bubble simulation → *flat-gauge* value $\delta^{(F)}$
- PBH threshold → *comoving-gauge* value $\delta^{(C)} \gtrsim 0.45$
- $\delta^{(C)} \sim 0.1\delta^{(F)}$ -- **too small** to form PBHs!



See the talk of Ran Ding on Monday

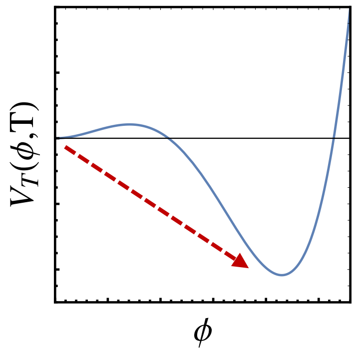
Similar result obtained independently in Wang, Balázs, Ding, and Tian, 2601.14412

Conclusion: **this mechanism is ruled out** *???*

Our insight

A *new* gauge-invariant approach to calculate $\delta^{(C)}$ [this work]

- Confirming $\delta^{(C)} \ll \delta_c \approx 0.45$
- But this does **not** preclude the formation of PBHs



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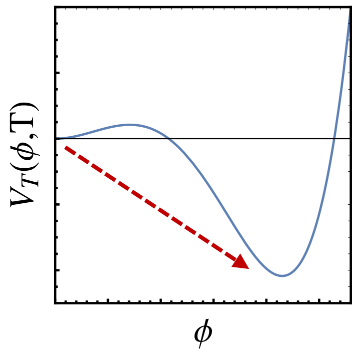
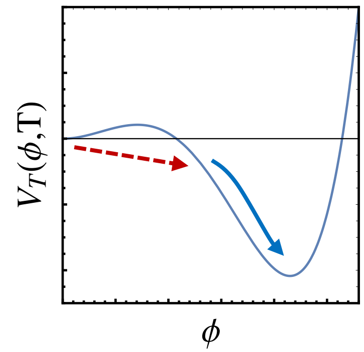
Fast reheating



$\Gamma_\phi / H_* > 1$

More accurately



Key variable:

$\dfrac{\Gamma_\phi}{H_*}$ — Γ_ϕ Decay width, H_* Hubble at transition

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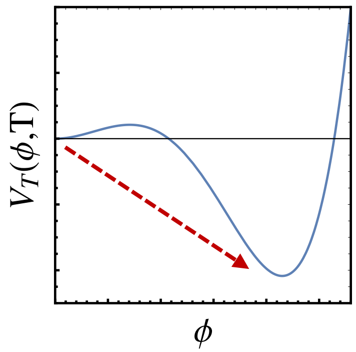

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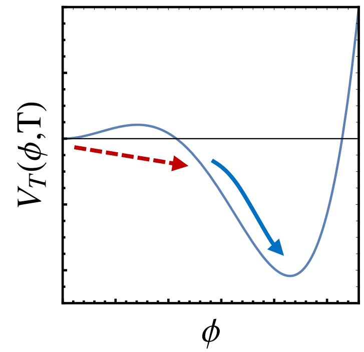
More accurately



Key variable:
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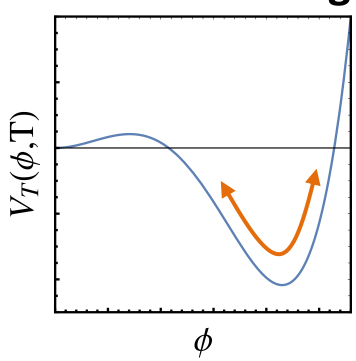
$\Gamma_\phi / H_* < 1$

Slow reheating



- ϕ coherently oscillates, decaying to SM particles, reheating the Universe
- **Early matter era** with $\rho_\phi \propto a^{-3}$, pressure ≈ 0
- Small $\delta^{(C)}$ grows and collapse, **forming PBHs** ✓

A unified treatment

	Condition	Energy transfer
Fast reheating	$\Gamma_\phi / H_* > 1$	Vacuum → Radiation
Slow reheating	$\Gamma_\phi / H_* < 1$	Vacuum → ϕ oscillation → Radiation

A unified treatment

	Condition	Energy transfer
Fast reheating	$\Gamma_\phi/H_* > 1$	Vacuum → Radiation
Slow reheating	$\Gamma_\phi/H_* < 1$	Vacuum → ϕ oscillation → Radiation

δ generation

δ growth & collapse (discussed later)

Two-fluid system

1. Vacuum (V), $P_V = -\rho_V$
2. Species (f), $P_f = w_f \rho_f$, with $f = r$ (fast reheating, $w_r = 1/3$) or ϕ (slow reheating, $w_\phi = 0$)

Supercooled FOPT

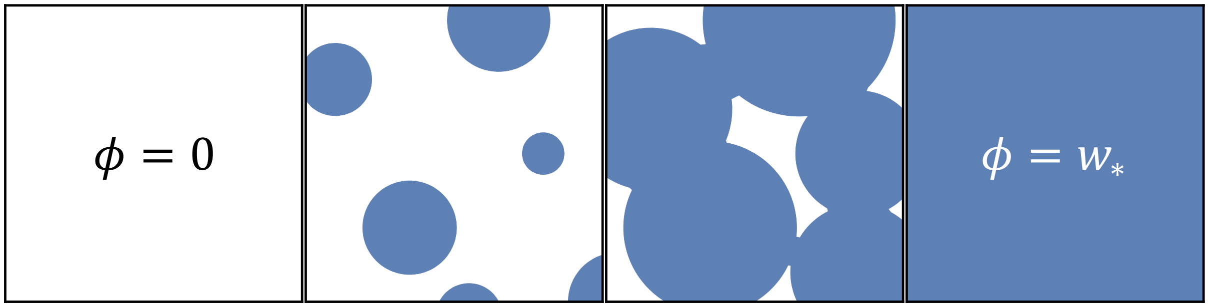
- Vacuum era before FOPT, comoving Hubble radius

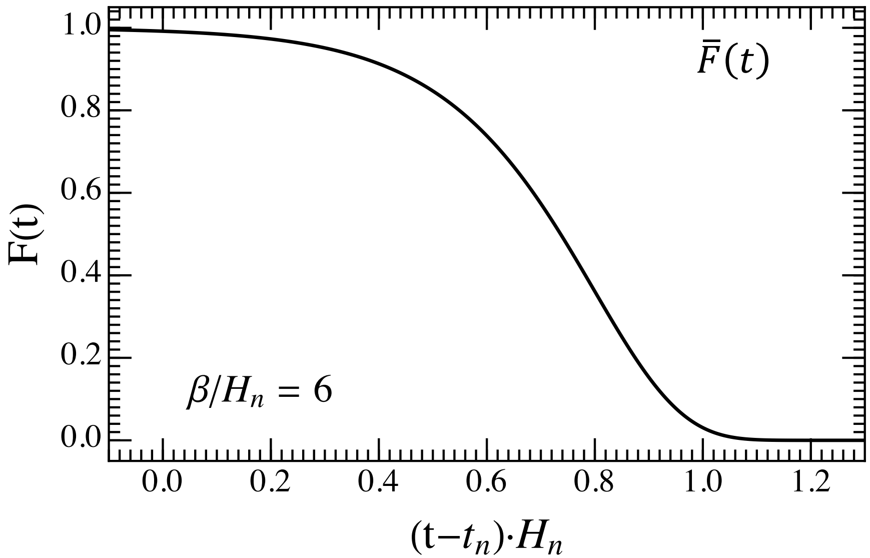
$$R = \frac{1}{aH}: \searrow\nearrow$$

- Any mode with $k < k_{\max} \equiv 1/R_{\min}$ will exit and **re-enter horizon**

Vacuum energy perturbation

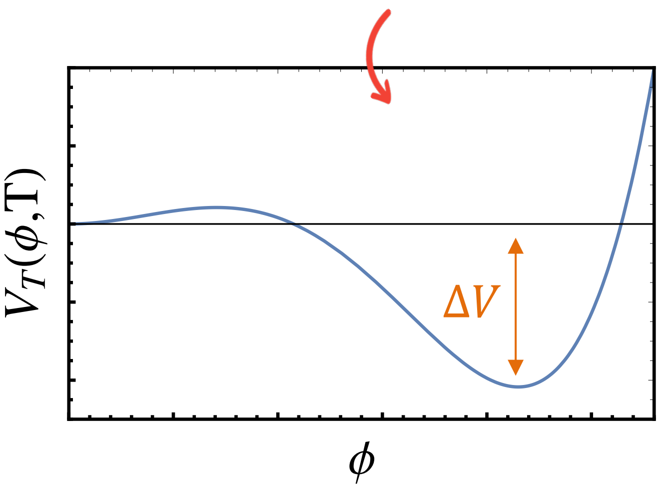
False vacuum volume fraction $F(t)$





Background vacuum energy

$$\bar{\rho}_V(t) = \Delta V \cdot \bar{F}(t)$$



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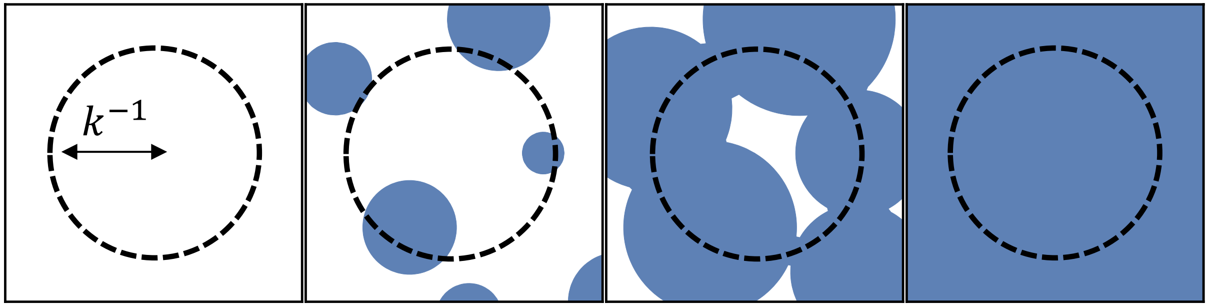

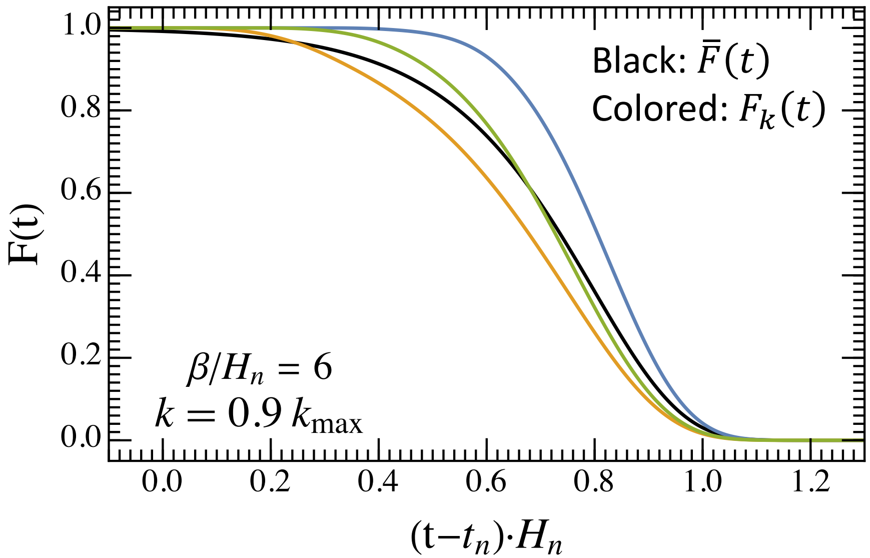

Background vacuum energy

$$\bar{\rho}_V(t) = \Delta V \cdot \bar{F}(t)$$

In a finite volume $4\pi k^{-3}/3$:

$$\rho_V(t) = \Delta V \cdot F_k(t)$$

Obtained from **bubble nucleation simulation**

Vacuum fluctuation generated: $\delta\rho_V = \rho_V - \bar{\rho}_V = \Delta V \cdot \left(F_k(t) - \bar{F}(t)\right)$

From $\delta\rho_V$ to δ

Relativistic perturbation theory

$$\rho(t,\mathbf{k}) = \bar{\rho}(t) + \delta\rho(t,\mathbf{k}); \qquad \delta = \frac{\delta\rho}{\bar{\rho}} = \frac{\delta\rho_V + \delta\rho_f}{\bar{\rho}_V + \bar{\rho}_f}$$

Metric perturbation

$$\mathrm{d}s^2 = (1+2A)\mathrm{d}t^2 - 2a(\partial_i B)\mathrm{d}t\mathrm{d}x^i$$
$$-a^2\left[\delta_{ij}(1+2C) + 2\partial_{\langle i}\partial_{j\rangle}E\right]\mathrm{d}x^i\mathrm{d}x^j$$

Solve δ (Einstein equations)
Evolve to horizon re-entry
Check $\boldsymbol{\delta > \delta_c} \rightarrow$ PBH?

From $\delta\rho_V$ to δ

Relativistic perturbation theory

$$\rho(t,\mathbf{k}) = \bar{\rho}(t) + \delta\rho(t,\mathbf{k}); \qquad \delta = \frac{\delta\rho}{\bar{\rho}} = \frac{\delta\rho_V + \delta\rho_f}{\bar{\rho}_V + \bar{\rho}_f}$$

Metric perturbation

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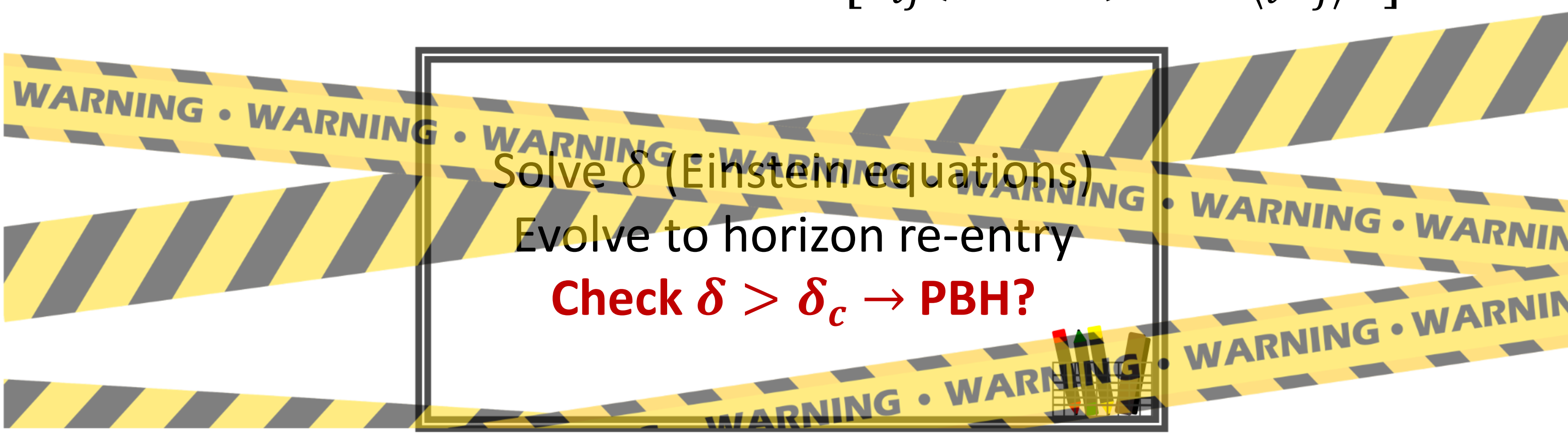


Solve δ (Einstein equations)

Evolve to horizon re-entry

Check $\boldsymbol{\delta > \delta_c}$ → PBH?

δ is gauge-dependent

Bubbles simulation → **flat gauge** ($C = E = 0$)

PBH criterion $\delta_c \approx 0.45$ → **comoving gauge** ($v + B = E = 0$)

From $\delta\rho_V$ to Δ

Gauge-invariant density contrast

$$\Delta = \frac{\delta\rho + a\dot{\bar{\rho}}(v+B)}{\bar{\rho}}$$

Evaluated in the **flat gauge** $[C = E = 0]$

$$\dot{\Delta} - 3H\frac{\bar{P}}{\bar{\rho}}\dot{\Delta} = \frac{k^2 Q}{a\bar{\rho}}$$

$$\dot{Q} + (4+3w_f)HQ = \frac{1+w_f}{a}\delta\rho_V - \left[\frac{w_f}{a}\bar{\rho} + \frac{aH}{2k^2}\left(\dot{\bar{\rho}} - 3(1+w_f)\dot{\bar{\rho}}_V\right)\right]\Delta$$

Gauge-invariant momentum perturbation

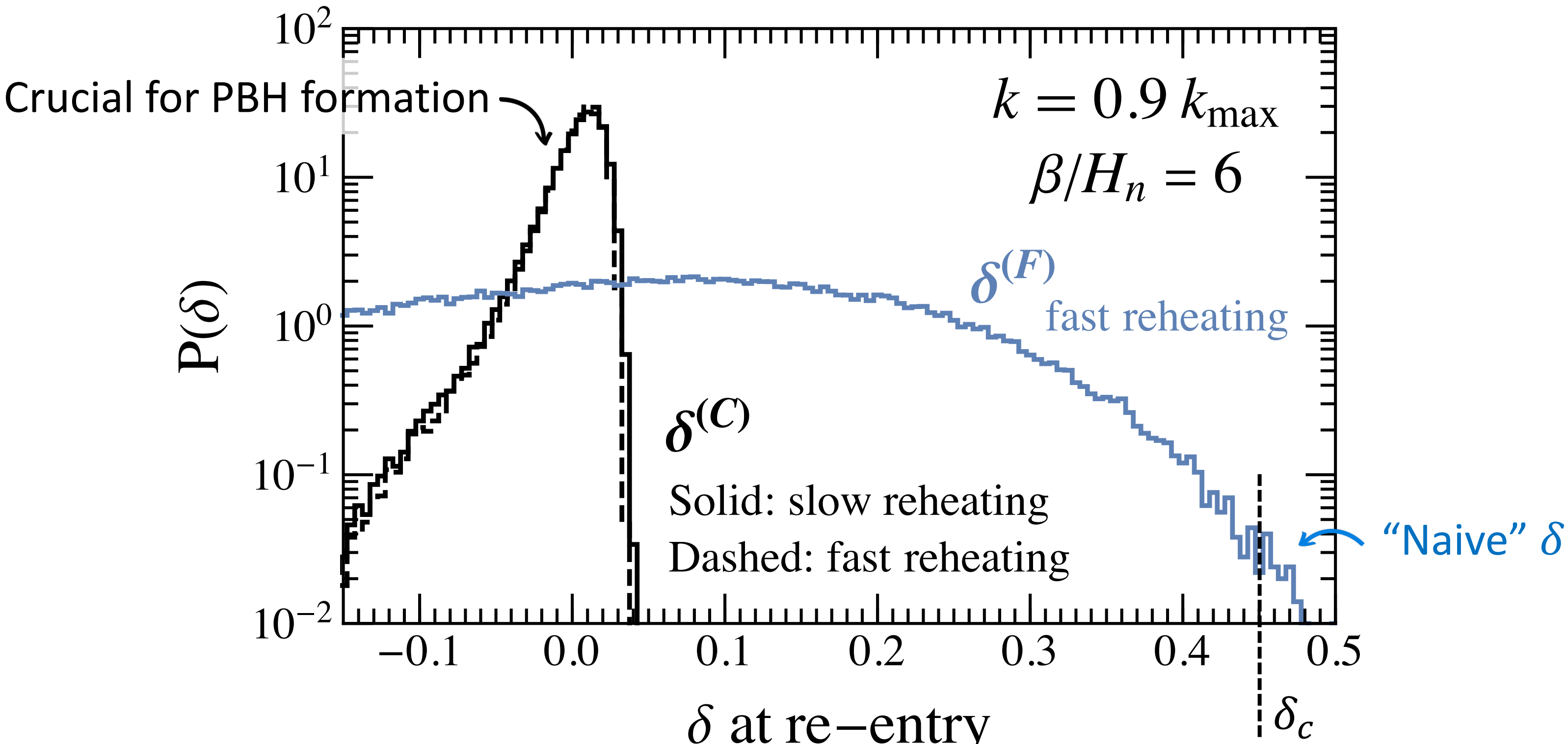
From bubble simulations

$\Delta \equiv \delta^{(C)}$ [**comoving gauge** $v + B = 0$], evolved to horizon re-entry

- Single realization → one $\delta^{(C)}$
- Multiple realizations → probability distribution of $\delta^{(C)}$

Distribution of $\delta^{(C)}$

δ very sensitive to gauge choice: $\delta^{(C)} \ll \delta^{(F)}$



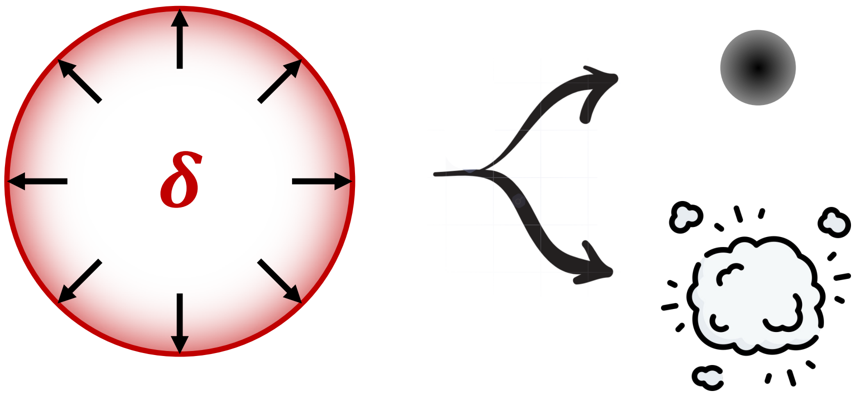
Fast reheating: consistent with Phys.Rev.Lett. 136 (2026) 17, 171404; 2601.14412

Slow reheating: new in our paper

Hereafter, δ solely denotes $\delta^{(C)}$

Fate of the density contrast

Fast reheating: pressure $P_r = \rho_r/3$ resists collapse



$\delta > \delta_c$: instantaneous collapse

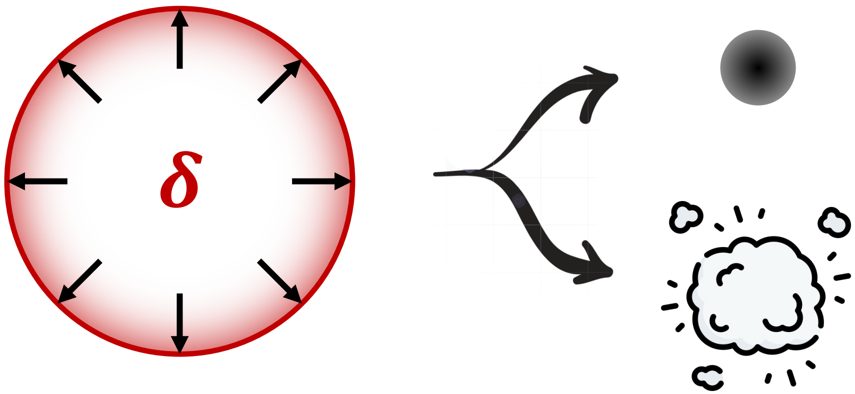
Schwarzschild black hole

$\delta < \delta_c$: no collapse

Overdensity dissipates

Fate of the density contrast

Fast reheating: pressure $P_r = \rho_r/3$ resists collapse



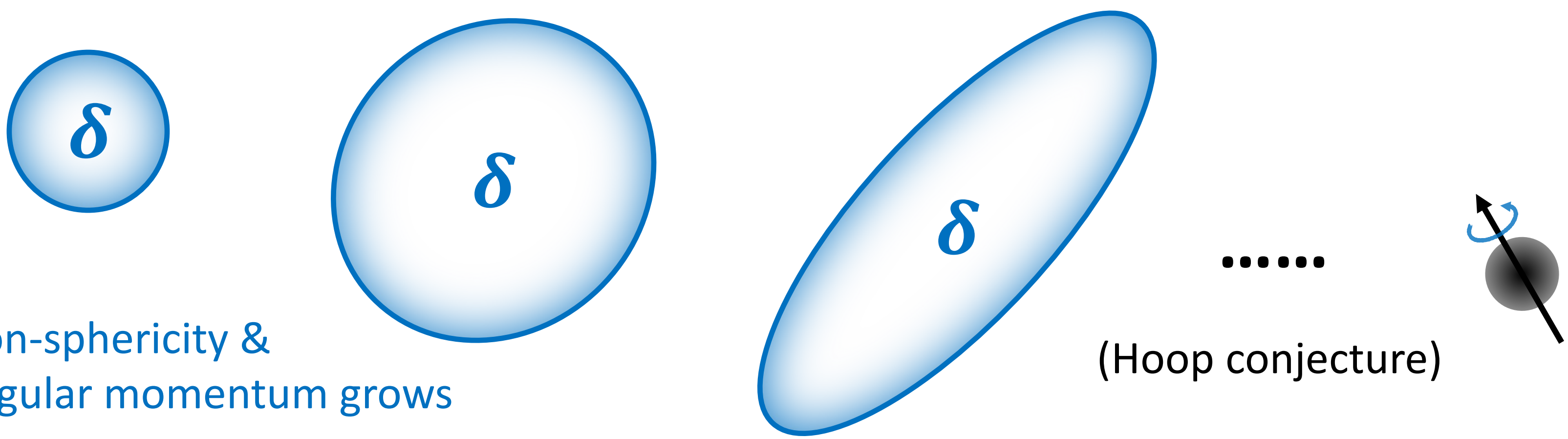
$\delta > \delta_c$: instantaneous collapse

Schwarzschild black hole

$\delta < \delta_c$: no collapse

Overdensity dissipates

Slow reheating: $P_\phi \approx 0$, perturbation δ increases with time



Non-sphericity &
angular momentum grows

(Hoop conjecture)

“Pancake collapse” at late time, high-spin **Kerr black hole**

PBH formation in the matter era

Perturbation shape describe by the **deformation tensor** $\boldsymbol{\mathcal{D}_{3\times 3}}$

Our crucial assumption

$$P(\mathcal{D}) = P(\delta) \times P_{\widetilde{\mathcal{D}}}\left(\widetilde{\mathcal{D}}_{ij}\right)$$

From our simulation (Non-Gaussian) ⟵ $\delta = -\operatorname{tr}\mathcal{D}$ Trace part

$\widetilde{\mathcal{D}}_{ij} = \mathcal{D}_{ij} + \frac{\delta_{ij}}{3}\delta$ ⟶ Assumed to be Gaussian

Traceless part

(Calling for future detailed simulations!)

Improving Harada *et al*, Astrophys. J. 833 (2016) no. 1, 61, Phys. Rev. D 96 (2017) no. 8, 083517

PBH formation criteria:

1. Sufficiently long matter-dominated duration, $\Gamma_\phi/H_* \lesssim 10^{-2}$
2. Kerr parameter

Angular momentum

$$\chi \equiv 8\pi \cdot J_{\rm pbh}\left(\frac{m_{\rm Pl}}{M_{\rm pbh}}\right)^2 < 1$$

(equivalent to $\delta > \delta_{\rm th} \sim 0.04$) $\ll \delta_c \approx 0.45$ in radiation era

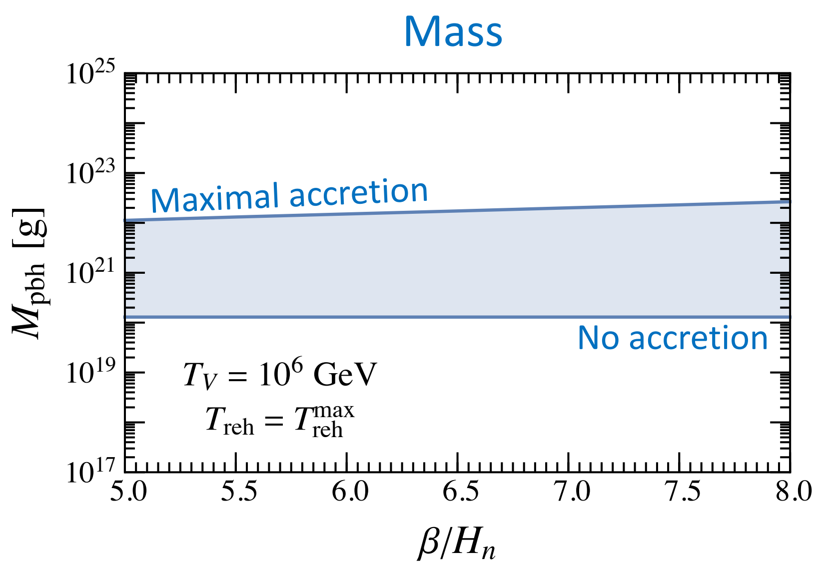
Integrating over $P(\delta)$ & $P_{\widetilde{\mathcal{D}}}\left(\widetilde{\mathcal{D}}_{ij}\right) \to$ PBH profile

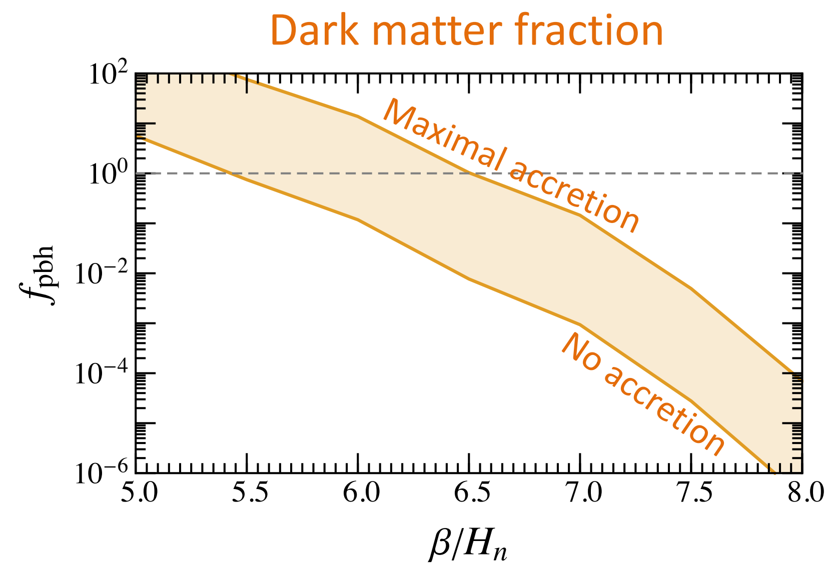
A benchmark

Profile most sensitive to $\boldsymbol{\beta/H_n}$ = (Hubble time)/(Transition duration)

Just one benchmark:

- New physics scale $\Delta V^{1/4} \approx T_V = 10^6\ \mathrm{GeV}$
- Fix $T_{\rm reh} \approx T_V \cdot \left(\Gamma_\phi/H_*\right)^{1/2}$ as $T_{\rm reh}^{\rm max}$ (equivalent to $\Gamma_\phi/H_* \approx 10^{-2}$)

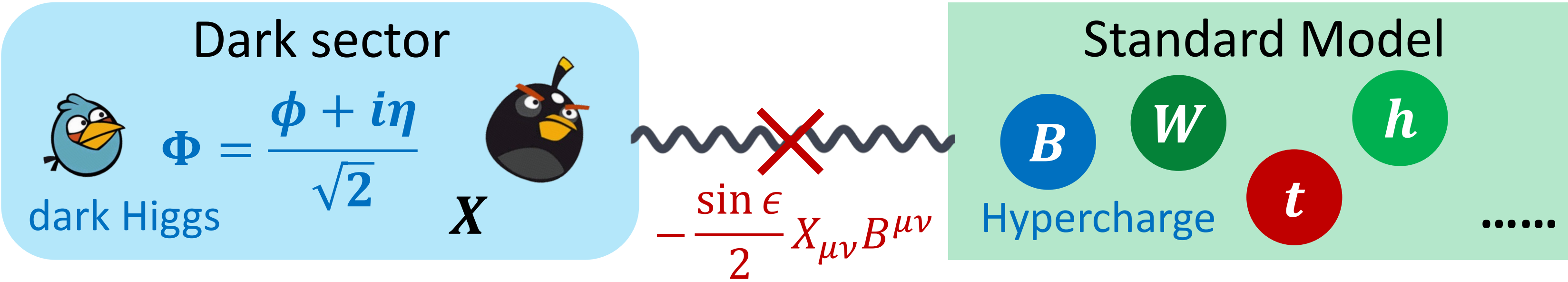


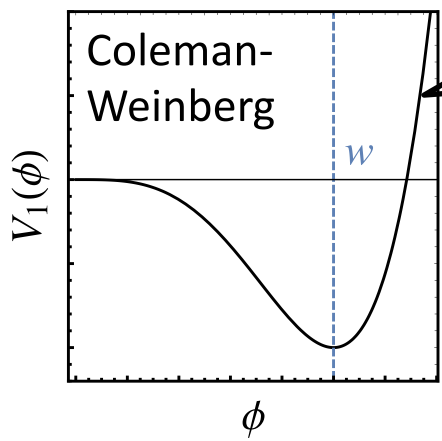


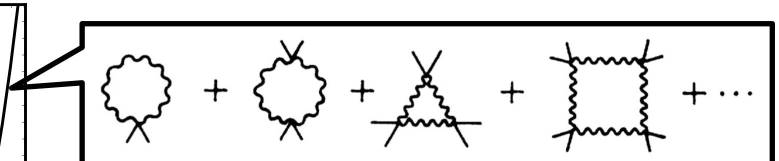
- Asteroid-mass ($10^{20}\ \mathrm{g} - 10^{22}\ \mathrm{g}$), allowing for 100% **dark matter**; detectable in the future (e.g., femto-lensing)

An illustrative model

Classically conformal gauge $U(1)_X$ model with **kinetic mixing**







$$m_X \approx g_X w; \quad m_\phi \approx \sqrt{\frac{3}{2} \frac{g_X^2 w}{2\pi}}$$

$\phi \to X^* X^* \to f \bar{f} f' \bar{f}', \phi \to ZZ, \ldots$ with $\Gamma_\phi \propto \epsilon^4$

- Setup: $m_X = 10^6$ GeV, $g_X = 0.58$, $\epsilon = 0.3$
- FOPT parameters $T_* \approx 4$ GeV, $\beta/H_n \approx 6.5$, and $\Gamma_\phi/H_* \approx 10^{-3}$

Supercooling Slow transition Slow reheating

Conclusion

This work Ai and **KPX**, arXiv: 2605.11332

- Gauge-invariant derivation of $\delta^{(C)}$ from FOPT without super-horizon approximations
- **PBH formation revived** via slow reheating
- High-spin **Kerr black holes** naturally produced

Outlook

- Full simulations of $\mathcal{D}_{3\times3}$ & exact GR collapse
- Accretion & spin-down effects
- PBH-FOPT GW correlation & early matter-domination imprint
- Non-spherical collapse GWs, particle signals …

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"Problems worthy of attack
prove their worth by fighting back."
— Piet Hein

值得攻克的难题
会以反击来证明自身的价值。

Thank you!

Backup: FOPT setup

Bubble nucleation rate

$$\Gamma(t) = H_n^4 e^{\beta(t-t_n)-\frac{\zeta^2}{2}(t-t_n)^2+\cdots}$$

t_n: nucleation time; $H_n \equiv H(t_n)$

Ratio of Hubble time to FOPT duration: β/H_n

Average false vacuum fraction

$$\bar{F}(t) = \exp\left[-\frac{4\pi}{3}\int_{-\infty}^{t} \mathrm{d}t'\Gamma(t')\left(\int_{t'}^{t} v_w \mathrm{d}t'' \frac{a(t')}{a(t'')}\right)^3\right]$$

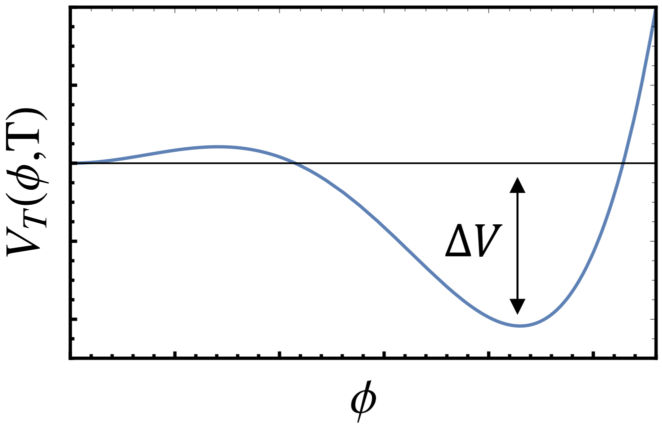
$v_w \approx 1$ for a supercooled FOPT

False vacuum fraction $F_k(t)$ in a comoving volume with radius k^{-1} obtained from simulations

Backup: evolution of comoving Hubble radius

Transition occurs at late time, with temperature $T_* \ll \Delta V^{1/4}$

Supercooled FOPT



Before transition → vacuum domination

- Hubble $H \sim \Delta V^{1/2}/m_{\rm Pl}^2$ [constant]
- Scale factor $a \sim e^{Ht}$
- (aH) increases

After transition → $\left\{ \begin{array}{l} \text{Fast reheating } H \propto a^{-2} \\ \\ \text{Slow reheating } H \sim a^{-3/2} \end{array} \right\}$ → (aH) decreases

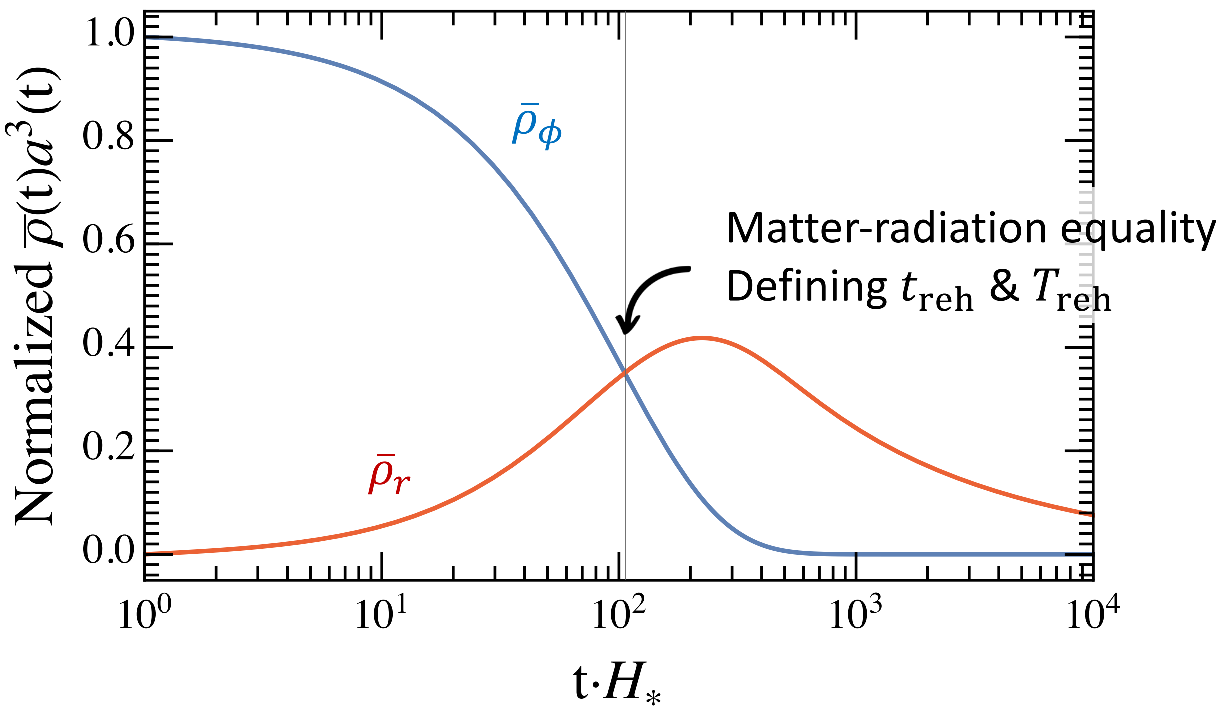
Important consequences

- Comoving Hubble radius $(aH)^{-1}$ ↘↗
- Any mode with $k < k_{\max} = (aH)_{\max}$ will exit and **re-enter horizon**

Backup: slow reheating

Slow reheating: matter-dominated era

Benchmark: $\Gamma_\phi / H_* = 10^{-2}$



$$\dot{\bar{\rho}}_\phi + 3H\bar{\rho}_\phi = -\Gamma_\phi \bar{\rho}_\phi; \quad \dot{\bar{\rho}}_r + 4H\bar{\rho}_r = \Gamma_\phi \bar{\rho}_\phi$$

Numerically, reheating temperature $T_{\mathrm{reh}} \approx 0.64\, T_V \cdot \left(\Gamma_\phi / H_*\right)^{1/2}$

Backup: PBH profile

PBH mass M_{pbh}

Current dark matter energy fraction

$$f_{\text{pbh}} = \Omega_{\text{pbh}}/\Omega_{\text{dm}}$$

Question: 

or not?

- If no accretion Ballesteros *et al*, JCAP 06 (2020) 014

$$\boxed{\begin{aligned} M_{\text{min}} &\sim 10^{20}\,\text{g} \times \left(\frac{10^6\,\text{GeV}}{T_V}\right)^2 \\ f_{\text{min}} &\sim 0.1 \times 10^{2.3\cdot(6-\beta/H_n)} \left(\frac{T_V}{10^6\,\text{GeV}}\right)\left(\frac{\Gamma_\phi/H_*}{10^{-2}}\right)^{1/2} \end{aligned}}$$

- If maximal accretion de Jong *et al*, JCAP 03 (2022) 03, 029

$$\boxed{\begin{aligned} M_{\text{max}} &\sim 10^{22}\,\text{g} \times \left(\frac{10^6\,\text{GeV}}{T_V}\right)^2 \left(\frac{10^{-2}}{\Gamma_\phi/H_*}\right) \\ f_{\text{max}} &\sim 10 \times 10^{2.3\cdot(6-\beta/H_n)} \left(\frac{T_V}{10^6\,\text{GeV}}\right)\left(\frac{10^{-2}}{\Gamma_\phi/H_*}\right)^{1/2} \end{aligned}}$$

$T_V \approx \Delta V^{1/4}$ set by particle physics model

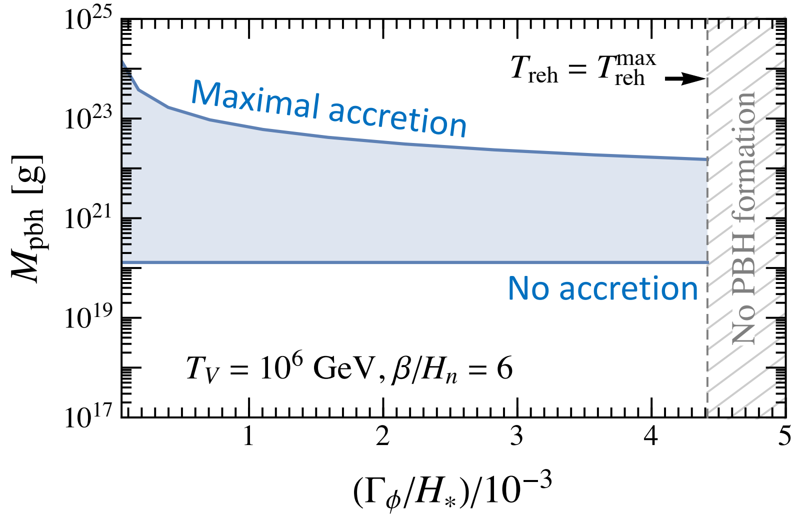
Backup: PBH mass & abundance

Dependence on $\boldsymbol{\Gamma_\phi/H_*}$ = (Hubble time)/(Reheating duration)

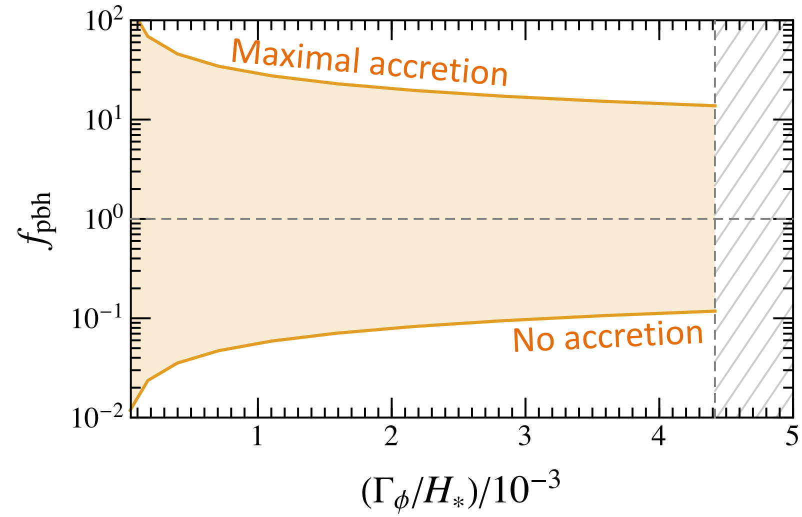
Just one benchmark:

- New physics scale $\Delta V^{1/4} \approx T_V = 10^6$ GeV
- $T_V \cdot \left(\Gamma_\phi/H_*\right)^{1/2} \approx T_{\rm reh} < T_{\rm reh}^{\rm max}$ (PBH formation condition)

Mass



Dark matter fraction



- Asteroid-mass (10^{20} g $-$ 10^{22} g), allowing for 100% **dark matter**; detectable in the future (e.g., femto-lensing)

Backup: details of the illustrative model

Classically conformal gauge $U(1)_X$ model, tree level: $V_0(\Phi) = \lambda_\phi |\Phi|^4$

Coleman-Weinberg potential

$$V_1(\phi) = \frac{3g_X^4}{32\pi^2}\phi^4\left(\log\frac{\phi}{w} - \frac{1}{4}\right)$$

Spectrum: $m_X = g_X w$ and $m_\phi = \sqrt{3/2}\, g_X^2 w/(2\pi)$

Kinetic mixing, $\mathcal{L}_{\rm int} = -(\sin\epsilon/2)X_{\mu\nu}B^{\mu\nu}$

Decay channels:

$$\left.\begin{array}{l} \bullet\ \text{4-body, } \phi \to X^*X^* \to f\bar{f}f'\bar{f}' \\ \bullet\ \text{3-body, } \phi \to X^*Z \to W^+W^-Z + f\bar{f}Z \\ \bullet\ \text{2-body, } \phi \to ZZ \end{array}\right\} \to \Gamma_\phi \propto \epsilon^4\text{, suppressed}$$

Finite-temperature effective potential

$$V_T(\phi, T) = V_1(\phi) + \frac{3T^4}{2\pi^2}J_B\left(\frac{g_X^2\phi^2}{T^2}\right) - \frac{T}{12\pi}g_X^3\left[\left(\phi^2 + \frac{T^2}{3}\right)^{3/2} - \phi^3\right]$$

Supercooled FOPT, see for example Ayazi *et al*, JHEP 03 (2019) 181