Puzzles in semileptonic B decays and a possible solution

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Overview

I: 半轻衰变的核心问题

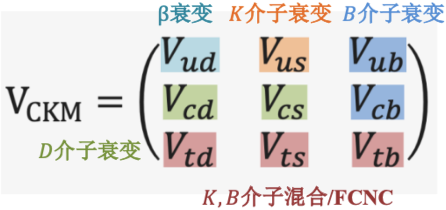
II: 一些最新进展

i: $|V_{cb}|$ from $\bar{B} \to X_c l \bar{\nu}_l$

ii: $|V_{ub}|$ from $B \to \pi\pi l^+ \nu_l$

III: 总结和展望

- rotation between the weak interaction eigenstates and the mass eigenstates



- $VV^\dagger = V^\dagger V = I_3$ in the Standard Model
- $VV^\dagger \neq V^\dagger V \neq I_3$ New Physics
- $|V_{ub}|/|V_{cb}|$ contributes to *CPV* measurement in B decays

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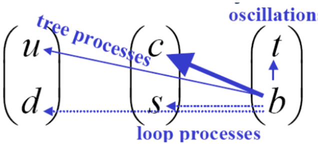
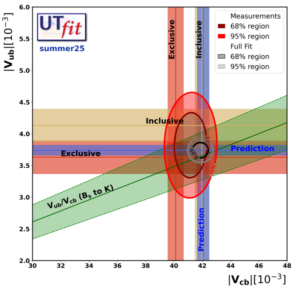
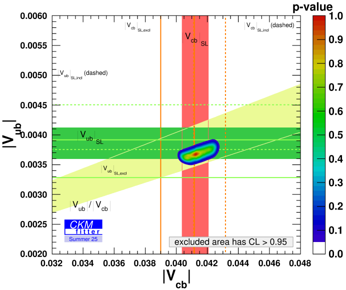
- CKM matrix elements are mainly determined via **the charged current processes**, i.e, $b \to u l^- \bar{\nu}, b \to c l^- \bar{\nu}, c \to s l^+ \nu$
- **Flavor changing neutral current precesses** are sensitive to new physical contributions, i.e, $b \to s l^+ l^-, b \to d l^+ l^-$



核心问题: $|V_{cb}|$, $|V_{ub}|$ 疑难





- $|V_{cb}|$ tension $\quad |V_{cb}| = (41.1 \pm 1.2) \times 10^{-3}$ [PDG 2024, 25 update]

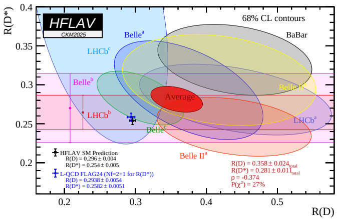
$*$ 2.5σ tension between $(42.2 \pm 0.5) \times 10^{-3}$ and $(39.8 \pm 0.6) \times 10^{-3}$ determined via the $B \to X_c l^- \bar{\nu}$ and $B \to D^{(*)} l^- \bar{\nu}$ processes, respectively

- $|V_{ub}|$ tension $\quad |V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ [PDG 2024, 25 update]

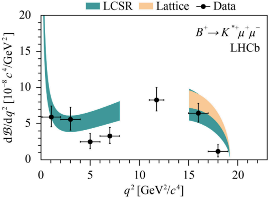
$*$ 2.5σ tension between $(4.13 \pm 0.25) \times 10^{-3}$ and $(3.67 \pm 0.15) \times 10^{-3}$ determined via the $B \to X_u l^- \bar{\nu}$ and $B \to \pi l^- \bar{\nu}$ processes, respectively

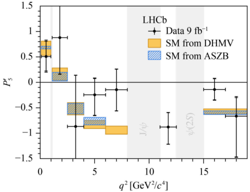
核心问题: 反常现象

- LFU $\quad R_{D^{(*)}} = \dfrac{\mathcal{B}(B \to D^{(*)}\tau^-\bar{\nu})}{\mathcal{B}(B \to D^{(*)}\mu^-\bar{\nu})}$
  - $*$ exceed the SM predictions given above, by 2.5σ and 2.3σ respectively [HFCPV 2025]
  - $*$ would make the CKM determinations more complicated if confirmed



- Anomalies in FCNC processes $B \to K^*\mu^+\mu^-$
  - $*$ 3.6σ derivation from SM of $d\mathcal{B}(B \to K^*\mu^+\mu^-)/dq^2$ in $q^2 \in [1, 6]$ GeV^2
  - $*$ 1.9σ derivation from SM of $p_5' = S_5/\sqrt{F_L(1 - F_L)}$ in $q^2 \in [4, 8]$ GeV^2





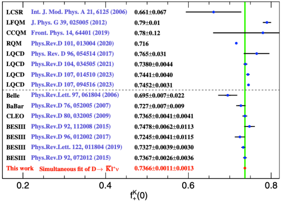
[Heavy Flavour Physics and CP Violation at LHCb: a Ten-Year Review, Front. Phys.18 (2023) 44601]

解决方案 在传统过程继续奋斗

- 更精确的测量和格点计算

  $|V_{cs}| = 0.975 \pm 0.006$ [PDG 2024, 25 update]

∗ 0.972 ± 0.007 and 0.984 ± 0.012 measured via the $D \to K l \nu$ and $D_s \to \mu^+ \nu_\mu$ processes, respectively $\sim 3\sigma \to \sim 1.5\sigma$

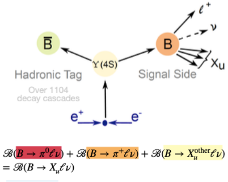


[BESIII 2023,24, Fermi Lattice and MILC 23]

- 更全面更系统的物理分析方法

  $|V_{ub}|$ result from Belle collaboration with Simultaneous Determination in excl. and incl. processes [Belle PRL131 (2023) 211801]

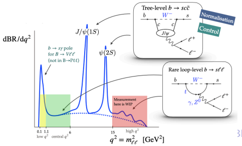
∗ $(3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3}$ and $(3.88 \pm 0.20 \pm 0.31 \pm 0.09) \times 10^{-3}$



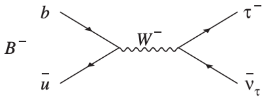
- 精细结构、丰富的 QCD 效应

  high order QCD corrections, more structures

  [AK, TM, YMW, JHEP 02 (2013) 010]
  [AK, TM, AAP, YMW, JHEP 09 (2010) 089]

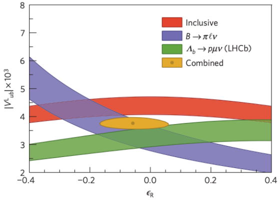


解决方案 寻找新的增长极



- $|V_{ub}|$ from **pure leptonic decay**
  $(4.11 \pm 0.39) \times 10^{-3}$ [FLAG, PRD 107 (2023) 052008]

* $|V_{ub}| f_B = 0.72 \pm 0.09$ MeV from Belle, 1.01 ± 0.14 MeV from BABAR, 0.77 ± 0.12 MeV average



- $|V_{ub}|$ from **baryon decay**
  [LHCb, Nature Physics 11 (2015) 743-747]

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{\mathcal{B}(\Lambda_b^0 \to p\mu^-\bar{\nu})}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+\mu^-\bar{\nu})} R_{\rm FF} = 0.68 \pm 0.07 \Downarrow$$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.079 \pm 0.006 \xrightarrow{|V_{cb}|} |V_{ub}| = (3.97 \pm 0.26) \times 10^{-3}$$

* consistent with the determinations in exclusive $B \to \pi l \bar{\nu}$ decay

- $\left.\frac{|V_{ub}|}{|V_{cb}|}\right|_{\rm high} = 0.098 \pm 0.008$ via $\frac{\mathcal{B}(B_s \to K^-\mu^+\nu)}{\mathcal{B}(B_s \to D_s^-\mu^+\nu)}$ [LHCb, PRL 126. 081804 (2021)]
- $|V_{ub}| = 3.58(9) \times 10^{-3}$ from $B_s \to K l \nu$ [PRD104. 114041 (2021)]
- $|V_{cb}| = (42.3 \pm 1.7) \times 10^{-3}$ from $B_s \to D_s \mu^+ \nu$ [LHCb, PRD 101 (2020) 072004]
- $d\mathcal{B}(B \to K^* \mu^+ \mu^-)/dq^2$ and p_5' in $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$ [LHCb, JHEP 06 (2015) 115]

一些最新进展

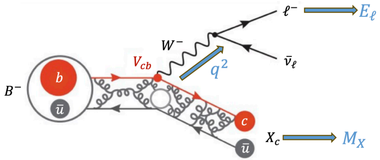
i 在单举过程 $\bar{B} \to X_c l \bar{\nu}$ 中确定 $|V_{cb}|$ 的最新精度

ii 在 B 介子四体半轻衰变中确定 $|V_{ub}|$ 的探索

I apologize for not covering many of the latest developments, such as the baryonic decays and the outstanding performance of lattice QCD.

$|V_{cb}|$ from $\bar{B} \to X_c l \bar{\nu}_l$

- $m_b \gg \Lambda_{\rm QCD} \;\Rightarrow\; p_b = m_b v + k$,
  expand in $k \sim i\mathcal{D}$ $\Downarrow$
  heavy quark expansion (HQE)
- Optical Theorem $\Downarrow$
  operator product expansion (OPE)



[picture from Matteo Fael (CERN)]

$$d\Gamma = \sum_i \frac{d\Gamma_i}{(m_b)^i}, \qquad d\Gamma_i = \sum_k C_i^{(k)} \langle B | O_i^{(k)} | B \rangle$$

non-perturbative matrix elements string of $(i\mathcal{D})^k$

perturbative Wilson coefficients $\Downarrow$

parameters μ_π^2, μ_G^2 and ρ_D^2, ρ_{LS}^2

- the inclusive semi-decay decay rate (SM)

$$\Gamma(\bar{B} \to X_c l^- \bar{\nu}_l) = |V_{cb}|^2 G_F^2 \frac{m_b^5}{16\pi^3} f(m_b, m_c, \mu_\pi^2, \mu_g^2, \rho_D^3, \rho_{LS}^3, \cdots)$$

$$f = f^{\rm LP} + f^{\rm NLO,\pi} \frac{\mu_\pi^2}{m_b^2} + f^{\rm NLO,G} \frac{\mu_G^2}{m_b^2} + f^{\rm NNLO,D} \frac{\rho_D^3}{m_b^3} + f^{\rm NNLO,LS} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^4}{m_B^4}\right)$$

- extract $|V_{cb}|$ provided we know the parameters μ_π^2, μ_G^2 and ρ_D^2, ρ_{LS}^2

$|V_{cb}|$ from $\bar{B} \to X_c l \bar{\nu}_l$

- the inclusive decay spectrum depends on 3 kinematical variables

$$\frac{d^3\Gamma}{dE_l dq^2 dm_X^2} = |V_{cb}|^2 G_F^2 \frac{m_b^5}{16\pi^3} \frac{d^3 f(m_b, m_c, \mu_\pi^2, \mu_g^2, \rho_D^3, \rho_{LS}^3, \cdots)}{dE_l dq^2 dm_X^2}$$

- define the spectral moment to confront the experimental data

$$M_{ijk} = \int dE_l dq^2 dm_X^2 \, [E_l]^{(i)} \left[q^2\right]^{(j)} \left[m_X^2\right]^{(k)} \frac{d^3\Gamma}{dE_l dq^2 dm_X^2}$$

- extract HQE parameters μ_π^2, μ_G^2 and ρ_D^2, ρ_{LS}^2 by global fitting of the **normalized moments** $\hat{M}_{ijk} \equiv \frac{M_{ijk}}{M_{000}}$ in (E_l, q^2, m_X^2) with $|V_{cb}|^2$ dropps out

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- i.e., q^2 moments are measured at several low cuts

$$M_n(q^2_{\rm cut}) = \int_{q^2_{\rm cut}}^{q^2_{\rm max}} dq^2 \left[q^2\right]^n \frac{d\Gamma}{dq^2}, \qquad \langle q^{2n} \rangle = \frac{M_n(q^2_{\rm cut})}{M_0(q^2_{\rm cut})}$$

- define central moments to reduce correlations

$$Q_1(q^2_{\rm cut}) = \langle q^2 \rangle, \qquad Q_2(q^2_{\rm cut}) = \langle \left(q^2 - \langle q^2 \rangle\right)^2 \rangle, \qquad Q_3(q^2_{\rm cut}) = \langle \left(q^2 - \langle q^2 \rangle\right)^3 \rangle$$

- measurements only available at Belle (2021) and Belle II (2022)

$|V_{cb}|$ from $\bar{B} \to X_c l \bar{\nu}_l$

state-of-the-art calculation

[picture from Gael Finauri (TUM)]

	dE_ℓ	dm_X^2	dq^2	Γ
1	α_s^2 [Melnikov 2008] [Pak, Czarnecki 2008]	α_s^2	α_s^2 NEW [Fael, Herren 2024]	α_s^3 [Fael, Schönwald, Steinhauser 2020]
$1/m_b^2$	α_s [Alberti, Ewerth, Gambino, Nandi 2012, 2013]	α_s	α_s	α_s
$1/m_b^3$	1 Need improvement! [Gremm, Kapustin 1997]	1	α_s [Mannel, Moreno Pivovarov 2021]	α_s [Mannel, Pivovarov 2019]
$1/m_b^{4,5}$ $1/(m_b^3 m_c^2)$	1 [Fael, Mannel, Vos 2018] [Mannel, Turczyk, Uraltsev 2010]	1	1 [Mannel, Milutin, Vos 2023]	1 [Mannel, Turczyk, Uraltsev 2010]

Proliferation of non-perturbative parameters
RPI can reduce them, but restricted to dq^2
[Finauri, 2025]

- take kinematical scheme to avoid renormalon ambiguities and badly converging perturbative series ($\mu_k = 1$ GeV, $\alpha_s^{(4)}(m_b) = 0.2185$)

- fit to (E_l, q^2, m_X^2) moments [2501.09090, 2310.20324]

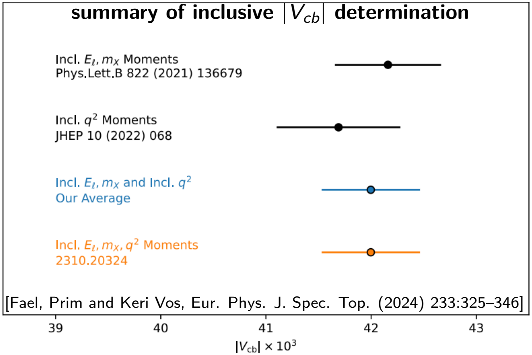
m_b^{kin}	$\overline{m}_c(2\,\text{GeV})$	μ_π^2	μ_G^2	ρ_D^3	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.572	1.090	0.430	0.282	0.161	-0.091	10.61	41.83
0.012	0.010	0.040	0.048	0.018	0.089	0.15	0.47

1.1% precision

[screenshot from Gael Finauri, "Challenges in Semileptonic B decays" 2024, Vienna]

- $\boxed{(42.2 \pm 0.5) \times 10^{-3}}$ and $(39.8 \pm 0.6) \times 10^{-3}$ via $B \to X_c l^- \bar{\nu}$ and $B \to D^{(*)} l^- \bar{\nu}$

$|V_{cb}|$ from $\bar{B} \to X_c l \bar{\nu}_l$



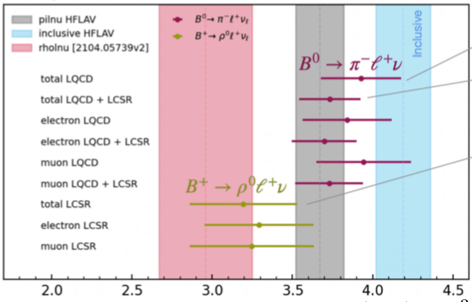
- open source Python package (kolya) for inclusive $|V_{cb}|$ determination [Fael, Milutin and Keri Vos, 2409.15007]
- recent advances in inclusive charm decay [see Qin Qin's talk at "HFCPV 2025", PKU]

$|V_{ub}|$ from $B \to \pi\pi l^+\nu$

- 含有激发态粒子的过程也可以提供独立的测量 $b \to u$ transition
- $B \to V$ transitions updated via B-meson LCSRs [Gao, et.al., PRD 101 (2020) 074035]

$$|V_{ub}| \times 10^3 \xrightarrow{B\to\rho l\nu} 3.05^{+1.34}_{-1.30}\big|_{\rm th}\ {}^{+0.19}_{-0.20}\big|_{\rm da}, \qquad \xrightarrow{B\to\omega l\nu} 2.54^{+1.09}_{-1.05}\big|_{\rm th}\ {}^{+0.18}_{-0.19}\big|_{\rm da}$$

- simultaneous measurements of $d\mathcal{B}/dq^2$ for $B \to \pi^- l^+\nu_l$ and $B \to \rho^0 l^+\nu_l$



$|V_{ub}| \times 10^3$

$|V_{ub}|_{B\to\pi\ell\nu_\ell} = (3.93 \pm 0.09 \pm 0.13 \pm 0.19) \times 10^{-3}$
LQCD[HFLAV 2023] stat syst theo

$|V_{ub}|_{B\to\pi\ell\nu_\ell} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16) \times 10^{-3}$
LQCD+LCSR [P. Bharucha, et.al., JHEP 08(2016)098]

$|V_{ub}|_{B\to\rho\ell\nu_\ell} = (3.19 \pm 0.12 \pm 0.17 \pm 0.26) \times 10^{-3}$
LCSR

- Leading systematic unc. are the modelling of continuum and non-resonant $B \to X_u\ell\nu$ decays
- Overall **theoretical** uncertainty dominating

Cao Lu's talk at Moriond EW 2024

[Belle-II PRD 111 (2025) 112009]

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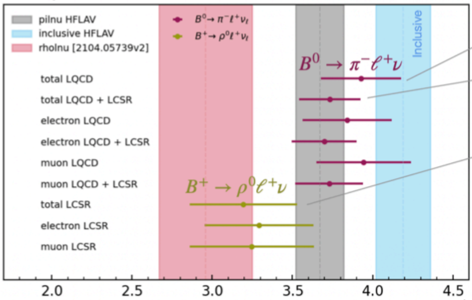

$|V_{ub}| \times 10^3$

Cao Lu's talk at Moriond EW 2024

[Belle-II PRD 111.112009(2025)]

$|V_{ub}|_{B\to\pi\ell\nu_\ell} = (3.93 \pm 0.09 \pm 0.13 \pm 0.19) \times 10^{-3}$ (stat, syst, theo)
LQCD[HFLAV 2023]

$|V_{ub}|_{B\to\pi\ell\nu_\ell} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16) \times 10^{-3}$
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- Leading systematic unc. are the modelling of continuum and non-resonant $B \to X_u \ell \nu$ decays
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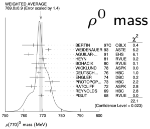
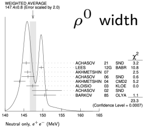
- different partial waves (P, D, S)
- different resonants in P-wave
- non-resonant QCD background

$|V_{ub}|$ from $B \to \pi\pi l^+ \nu$





[figures from PDG]

- the $\pi\pi$ spectra in $[0.554, 0.996]$GeV serves as the candidate region for ρ
- ρ is usually treated as a stable single particle in the QCD studies of $B \to \rho$ transitions (LQCD, LCSRs)

- the signal channel in a $B \to \rho l \nu$-type decay is $B \to \pi\pi l\nu$ (B_{l4})
- How to accurately describe the **width effects** of unstable intermediate particles, the contributions and **interference effects** of different partial waves, and the **QCD backgrounds** from non-resonant states
- Dipion LCDAs (2π**DAs**) provide a most general description of $\pi\pi$ spectral

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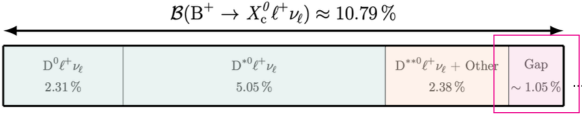
- opportunities for $|V_{cb}|$ from $B \to D^* l\nu, X_c l\bar{\nu}$ and $B \to K^* l^+ l^-$ anomalies



- careful study of $D\pi$, $D\pi\pi$ and $D\pi\pi\pi$ spectral

...or is it even bigger?

2πDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$

$$\langle \pi^a(k_1)\pi^b(k_2)|\overline{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(\mu, u, \zeta, k^2)$$

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\rho^-(p)\rangle = f_\rho m_\rho \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[p_\mu \frac{\epsilon^{(\lambda)}\cdot x}{p\cdot x}\left(\phi_\parallel(u,\mu) - \phi_\perp^3(u,\mu)\right) + \cdots\right]$$

- 2πDAs is decomposed in terms of $C_n^{3/2}(2u-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(u,\zeta,k^2,\mu) = 6u(1-u)\sum_{n=0,\text{even}}^{\infty}\sum_{l=1,\text{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)\, C_n^{3/2}(2u-1)\, C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(u,\zeta,k^2,\mu) = 6u(1-u)\sum_{n=1,\text{odd}}^{\infty}\sum_{l=0,\text{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)\, C_n^{3/2}(2u-1)\, C_\ell^{1/2}(2\zeta-1)$$

- **Evolution from $4m_\pi^2$ to large k^2** via the Watson theorem of $\pi\pi$ scattering amplitudes

$$B_{n\ell}^I(k^2) = B_{n\ell}^I(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B_{n\ell}^I(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\, \frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

△ **2πDAs in a wide range of energies is given by δ_ℓ^I and a few subtraction constants**

2πDAs

- The subtraction constants of $B_{n\ell}(k^2)$ at low k^2 (around the threshold)

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^{π}, a_2^{ρ} [**SC** 2019, 2023]

- 2πDAs were introduced at leading twist [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu\tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

- improved to twist-three level recently [**SC**, PRD 112 (2025) L111301]

$$\langle\pi(k_1)\pi(k_2)|\bar{q}(0)q(x)|0\rangle = \int du e^{i\bar{u}k\cdot x}\frac{ik^2(k\cdot x)}{2f_{2\pi}^{\perp}}\Phi_{\parallel}^{(s)},$$

$$\langle\pi(k_1)\pi(k_2)|\bar{q}(0)\sigma^{\mu\nu}q(x)|0\rangle = -\frac{i}{f_{2\pi}^{\perp}}\int du e^{i\bar{u}k\cdot x}\Big[\frac{k_\mu\bar{k}_\nu - k_\nu\bar{k}_\mu}{2\zeta-1}\Phi_{\perp} - k^2\frac{k_\mu x_\nu - k_\nu x_\mu}{k\cdot x}\Phi_{\parallel}^{(t)}\Big].$$

$B \to \pi\pi$ from factors

- **QCD dynamics of B_{l4} decays is incorporated in $B \to \pi\pi$ form factors**, instead of the $B \to \rho$ ffs followed by $\rho \to \pi\pi$ in the cascade decay
  [X.-W. Kang, et.,al, PRD 89 (2014) 053015, S. Faller, et.,al, PRD 89 (2014) 014015]

$$
\begin{aligned}
i\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle &= F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\, i\epsilon_{\nu\alpha\beta\gamma}\, q^\alpha\, k^\beta\, \bar{k}^\gamma \\
&+F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right) \\
&+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot \bar{k})}{\lambda_B}\,k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right)
\end{aligned}
$$

* $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function, $\quad q\cdot k = (m_B^2 - q^2 - k^2)/2$,
$q\cdot\bar{k} = \sqrt{\lambda}\beta_\pi(k^2)\cos\theta_\pi/2 = \sqrt{\lambda}\,(2\zeta-1)$, $\quad \beta_\pi(k^2) = \sqrt{1-4m_\pi^2/k^2}$

- **SU(3) flavor symmetry/breaking** with the intermediate resonance
  - [R.M. Wang, Y.G. Xu, J.H. Sheng, X.D. Cheng, et.al., 2301.00090, PRD 112 (2025) 033002]

- **QCDF** (QCD factorization) in the large two-pion mass
  - [P. Böer, T. Feldmann and D. van Dyk, JHEP02 (2017) 133] $\quad T_I \propto F_{B\to\pi} \otimes \phi_\pi$,

$B \to \pi\pi$ from factors

- **LQCD** (Lattice QCD) in the ρ resonance region with a simple BW model
  - [L. Leskovec, et.al, PRL 134 (2025) 161901, **Editors' Suggestion**]

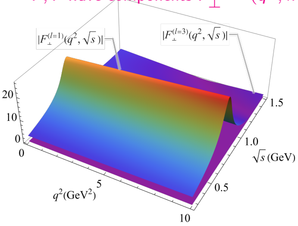
- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large q^2 by taking dispersive methods in terms of Omnés functions
  - [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89 (2014) 053015]

  in the full phase-space by a novel parameterization with unitarity
  - [F. Herren, B. Kubis and R. van Tonder, PRD 112 (2025) 014037, **Editors' Suggestion**]

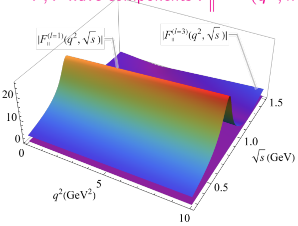
- **LCSRs** (Light-cone sum rules) in the small and intermediate q^2
  - [**SC**, A. Khodjamirian and J. Virto, JHEP 05 (2017)157] B-meson LCSRs , [S. Descotes-Genon, A. Khodjamirian, J. Virto and K.K. Vos, JHEP 12 (2019) 083, 06 (2023) 034] $B \to K\pi$
  - [C. Hambrock and A. Khodjamirian, NPB 905 (2016) 379-390] 2πDAs LCSRS of $F_{\parallel,\perp}$
  - [**SC**, A. Khodjamirian and J. Virto, PRD(R) 96 (2017) 051901] timelike-helicity FF F_t and F_0
  - [**SC**, PRD 99 (2019) 053005] 2πDAs updates and $B \to [\pi\pi]_{\rm S,P}$ FFs
  - [**SC** and J.M Shen, EPJC 6 (2020) 554, **SC** and S.L Zhang, EPJC 84 (2024) 379] Pheno
  - [**SC**, PRD 112 (2025) L111301] twist-three 2πDAs and $|V_{ub}|$ extraction this talk
  - [**SC**, L.Y. Dai, J.M. Shen and S.L. Zhang, PRD 113 (2026) L031901] $D_s \to [\pi\pi]_{\rm S}\, e\nu$, minor $q\bar{q}$ contribution

$B \to \pi\pi$ from factors

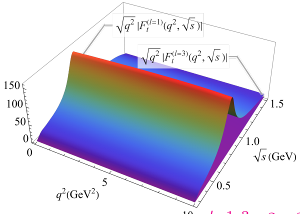
P, F-wave components $F_{\perp}^{l=1,3}(q^2, k^2)$



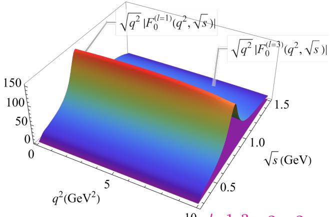
P, F-wave components $F_{\parallel}^{l=1,3}(q^2, k^2)$



[**SC**, PRD 112 (2025) L111301]

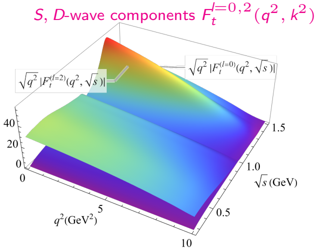


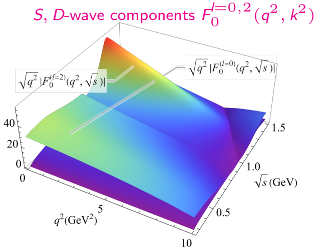
P, F-wave components $F_t^{l=1,3}(q^2, k^2)$



P, F-wave components $F_0^{l=1,3}(q^2, k^2)$

$B \to \pi\pi$ from factors





- twist-three contribution vanish in $F_{\perp,\parallel}^{(l=1,3,\cdots)}$
- persist in $F_{t,0}^{(l=0,1,\cdots)}$, give a significant correction $\sim 40\%$ to leading twist result

- ρ resonance is dominate in $F_{\perp,\parallel,t,0}^{(l=1)}(q^2, k^2)$, F-wave contributions are negligible
- S-wave dominates in the small k^2 regions, D-wave is sizable in the large k^2 and small q^2 regions

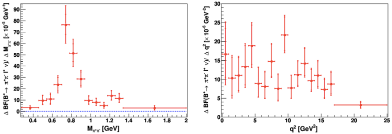
$|V_{ub}|$ from $B \to \pi\pi l^+ \nu$

- 2D partial differential decay width

$$\frac{d^2\Gamma}{dq^2\,dk^2} = G_F^2 |V_{ub}|^2 \frac{\beta_\pi \sqrt{\lambda} q^2}{3\,(4\pi)^5\, m_B^3} \left[\left(|F_0^{(S)}|^2 + |F_0^{(P)}|^2 \right) + \beta_\pi^2 \left(|F_\parallel^{(P)}|^2 + |F_\perp^{(P)}|^2 \right) + \cdots \right]$$

- 2D measurement of $\triangle\mathcal{B}^i$ (10^{-5}) in different bins

[Belle, PRD 103 (2021) 112001]



Bins	$\sqrt{s}$	q^2	$\triangle\mathcal{B}^i$	$\triangle\mathcal{B}^i$ [20]
1	$[4m_\pi^2, 0.6]$	[0, 8]	$0.27 \pm 0.03 \pm 0.06$	$0.84^{+0.39}_{-0.32} \pm 0.18$
2	(0.6, 0.9]	[0, 4]	$1.91 \pm 0.21 \pm 0.38$	$2.39^{+0.53}_{-0.47} \pm 0.32$
3	(0.6, 0.9]	(4, 8]	$1.54 \pm 0.17 \pm 0.27$	$2.16^{+0.47}_{-0.42} \pm 0.23$
4	(0.9, 1.2]	[0, 4]	$0.65 \pm 0.07 \pm 0.12$	$0.70^{+0.32}_{-0.25} \pm 0.20$
5	(0.9, 1.2]	(4, 8]	$0.41 \pm 0.04 \pm 0.08$	$0.64^{+0.28}_{-0.22} \pm 0.11$
6	(1.2, 1.5]	[0, 4]	$0.57 \pm 0.05 \pm 0.10$	$0.91^{+0.35}_{-0.28} \pm 0.12$
7	(1.2, 1.5]	(4, 8]	$0.16 \pm 0.02 \pm 0.02$	$0.64^{+0.32}_{-0.26} \pm 0.08$

take the PDG value: $|V_{ub}| \times 10^3 = 3.82 \pm 0.20 \Rightarrow$
a good consistent in ρ, f_0 regions

$|V_{ub}| \times 10^3 = 4.27 \pm 0.49|_{\rm Data} \pm 0.55|_{\rm LCSRs}$ in ρ region of B_{l4} decays

$|V_{ub}| \times 10^3 = 3.96 \pm 0.47|_{\rm Data} \pm 0.52|_{\rm LCSRs}$ in f_0 region of B_{l4} decays

总结和展望

- $|V_{cb}|, |V_{ub}|$ 疑难和 $R_{D^{(*)}}, p_5', \frac{d\Gamma}{dq^2}$ 等反常仍是半轻衰变研究的核心问题

- 上述问题的根本解决同时依赖于单举和遍举过程的独立研究
- 传统过程的精细研究和新过程的逐步探索是遍举研究的两条并行道路

- 四体半轻衰变（H_{l4}）提供了一个新方案　两介子系统光锥分布振幅

- 当前在 B_{l4} 过程抽取 $|V_{ub}|$ 的误差较大 (10%)，但是精度可以提高
  - * Belle-II 积分亮度未来五年有望达到 $3ab^{-1}$，实验精度将至少提高 3 倍
  - * LCSRs 和 LQCD 对形状因子的联合分析，至少可以将理论误差减小一半

- H_{l4} 过程的研究正在进入高精度时代　$\gamma^* \to \pi\pi\gamma$ and $\gamma^*\gamma \to \pi\pi$

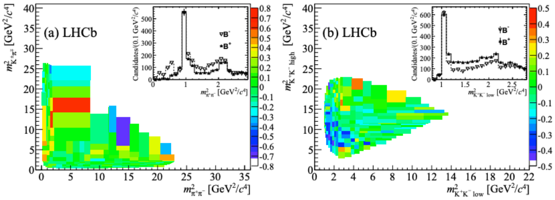
Thank you for your patience.

More opportunities/phenomena of 2πDAs

pion EMFF widely used in the three-body B decays studied from pQCD and QCDF is the asymptotic term of 2πDAs

[Chai, **SC** and Ma, PRD 105. 033003 (2022)]

normalized to unit as $\Gamma^{I=1}_{M_1M_2}(0) = 1$. When the invariant mass of dimeson system is small, the higher $\mathcal{O}(s)$ terms in the expansion of coefficient $B_{nl}(s,\mu)$ around the resonance pole can be safely neglected due to the large suppression $\mathcal{O}(s/m_b^2)$ in contrast to the energetic dimeson system in B decay, so the relation $B_{n1}(s,\mu) \to a_n(\mu)\Gamma^{I=1}_{M_1M_2}(s)$ can be obtained in the lowest partial wave approximation. This argument induces the basic assumption in PQCD that the energetic dimeson DAs can be deduced from the DAs of resonant meson by replacing the decay constant by the timelike form factor.



$B \to K\pi\pi, KKK$

[PRL 111 (2013) 101801 LHCb]

- Non-resonant contributions are small ($< 10\%$) in three-body D decays
- large/dominate in the penguin dominated three-body B decays, the theoretically unresolved problem