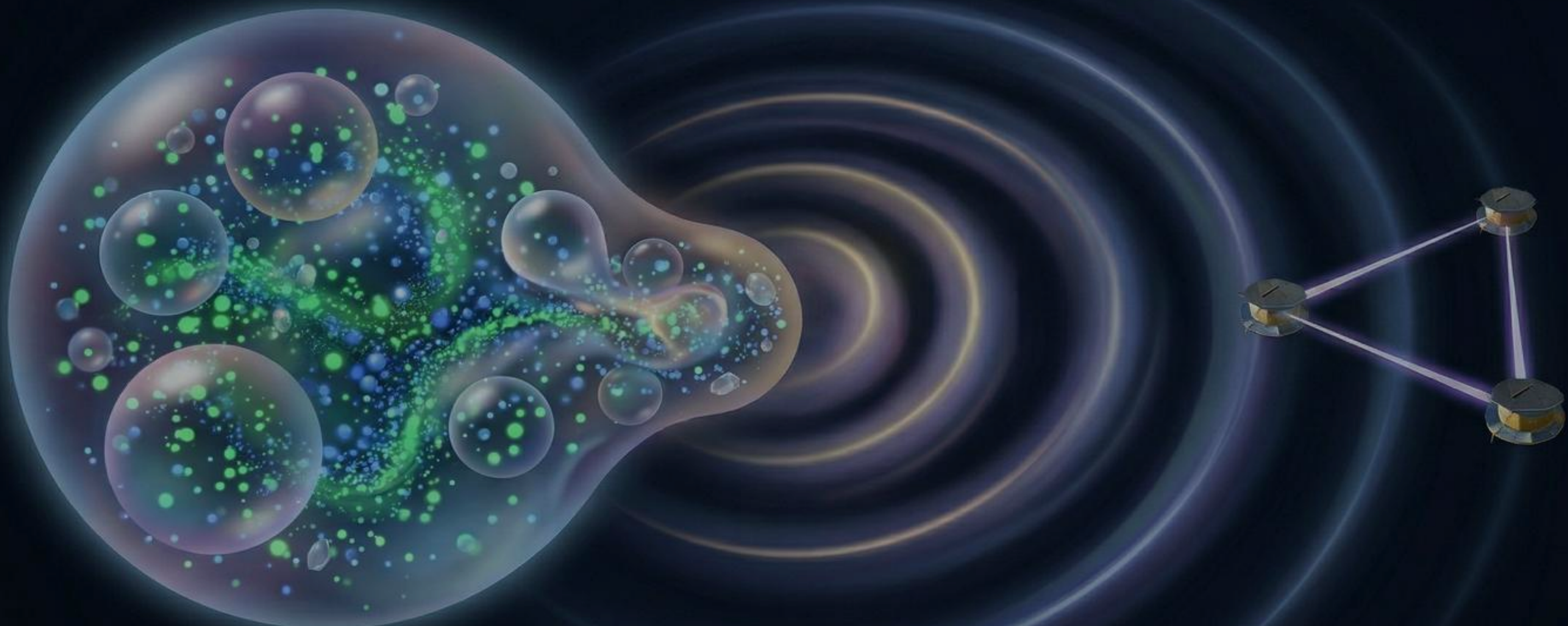
Dark matter in Classically Conformal Theories

WIMP and GW-detectable Supercooled Scenarios

Cheng-Hao Zhan (詹骋昊)
Beihang University



北京航空航天大學
BEIHANG UNIVERSITY

Based on **2603.09126(v2 is coming)**, with Ke-Pan Xie (谢柯盼)
2026.5.17 @ NPhiS2026, Nanjing

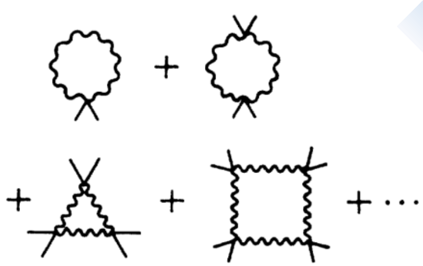
Outline

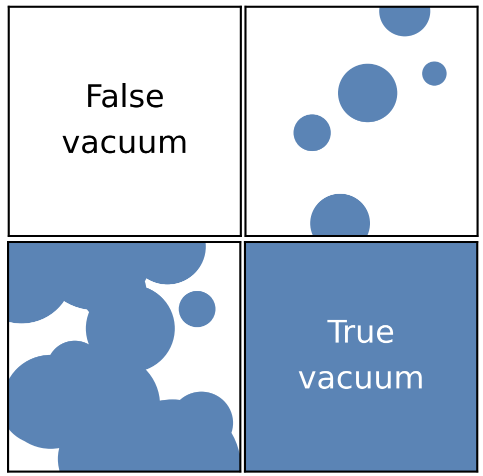
DM Problem

Hierarchy Problem

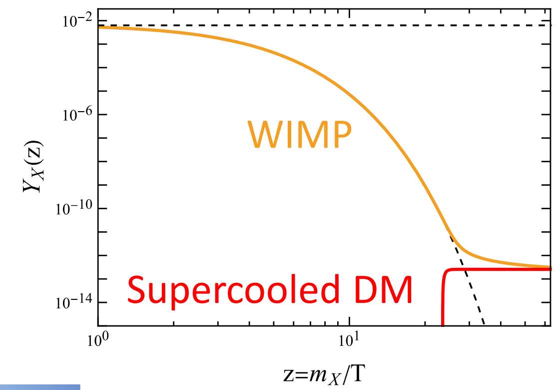
A possible solution

Classically Conformal





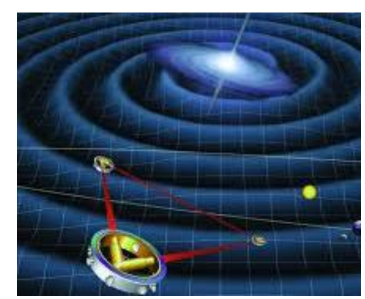
Cosmology
First Order Phase Transition



DM Landscape

Detectable

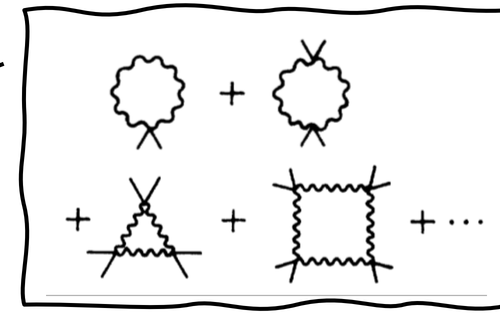
GW Observatories like LISA & BBO



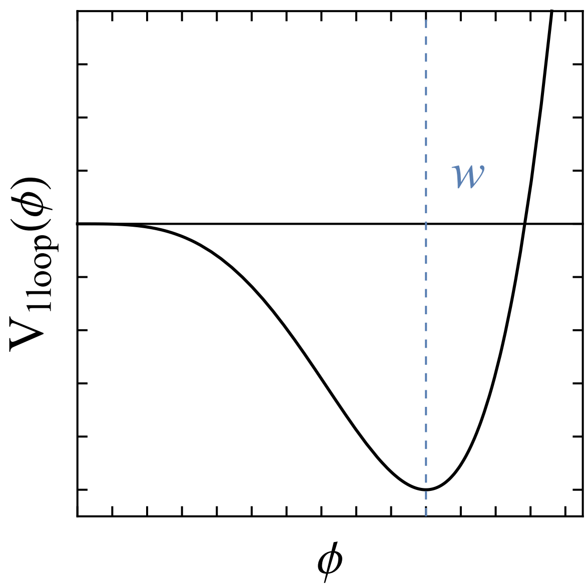
Classically Conformal Theory

$$V_{\mathrm{tree}}(\phi) = \frac{\lambda}{4}\phi^4$$

Radiative Symmetry Breaking $\Downarrow$



$$V_{\mathrm{1loop}}(\phi) = \frac{B}{4}\phi^4\left(\log\frac{\phi}{w} - \frac{1}{4}\right)$$



Conformal invariance in tree level (Classically)

S. Coleman & E. Weinberg PRD 7 (1973) 1888

No bare ϕ^2 term

Hopefully solving hierarchy problem

Bardeen, FERMILAB-CONF-95-391-T;

K. A. Meissner & H. Nicolai, PLB (2008) 660; etc

Classically Conformal Theory and Our Model

A realistic framework:

$$V_{\text{tree}}(h,s) = \frac{\lambda_s}{4}s^4 + \frac{\lambda_{sh}}{4}s^2h^2 + \frac{\lambda_h}{4}h^4$$

Keep classically conformal

Gauge choice for s: $U(1), SU(2)$, ...

Our choice: $SU(2)$ in **adjoint** representation

Ensures the stability of DM candidates (X^+/X^-)

0811.0172

$$\phi = \begin{pmatrix} G_1 \\ G_2 \\ s \end{pmatrix} \quad X^a$$

$$\Big\Downarrow \; SU(2)_X \rightarrow U(1)_X$$

$$s, X^+, X^-, A'$$

$$V_{\text{1loop}}(h,s) = \frac{B}{4}s^4\left(\log\frac{s}{w_0} - \frac{1}{4}\right) + \frac{\lambda_{sh}}{4}s^2h^2 + \frac{\lambda_h}{4}h^4$$

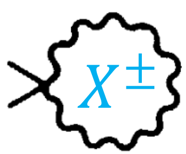
$$\langle s \rangle = w \approx w_0 \gg v$$

$$m_s = \frac{\sqrt{3}g_X^2}{2\pi}w$$

$$m_X = g_X w$$

$$m_{A'} = 0$$

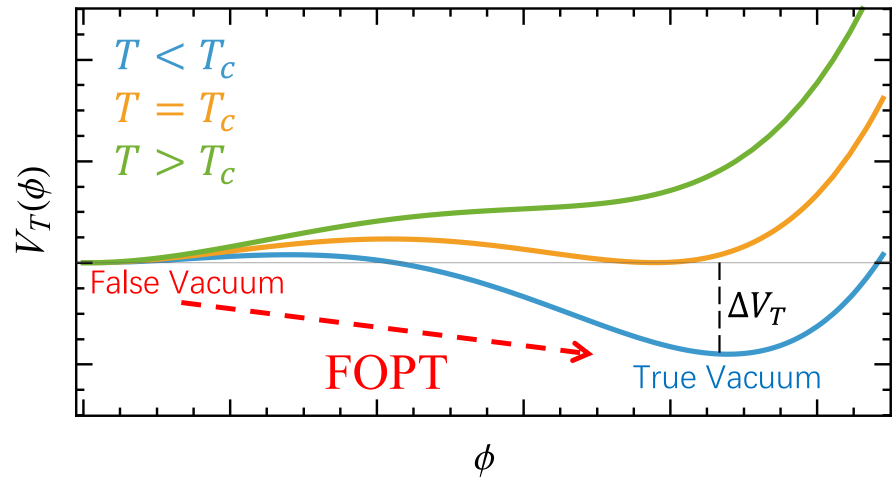
$$B = 2 \times \frac{3g_X^4}{8\pi^2}$$



First Order Phase Transition

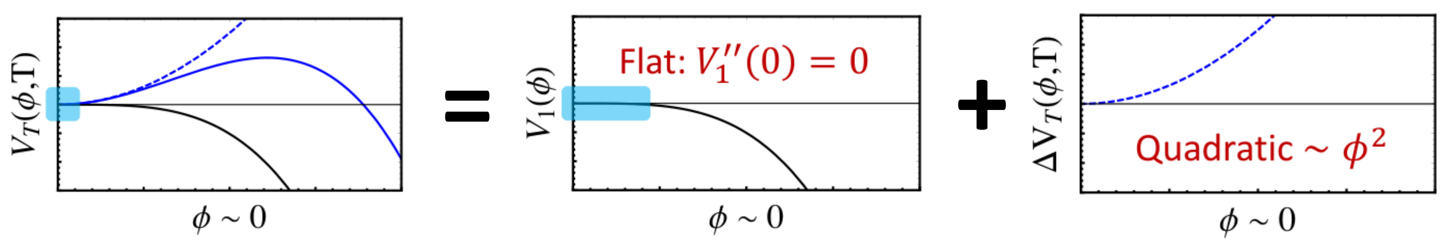
Baryogenesis:
- Strong non-equilibrium
- Satisfies Sakharov condition

Dark matter:
- New kinetics via bubble expansion
- **Dilution**, Filtration, etc.
  1805.01473, 1912.02830, …



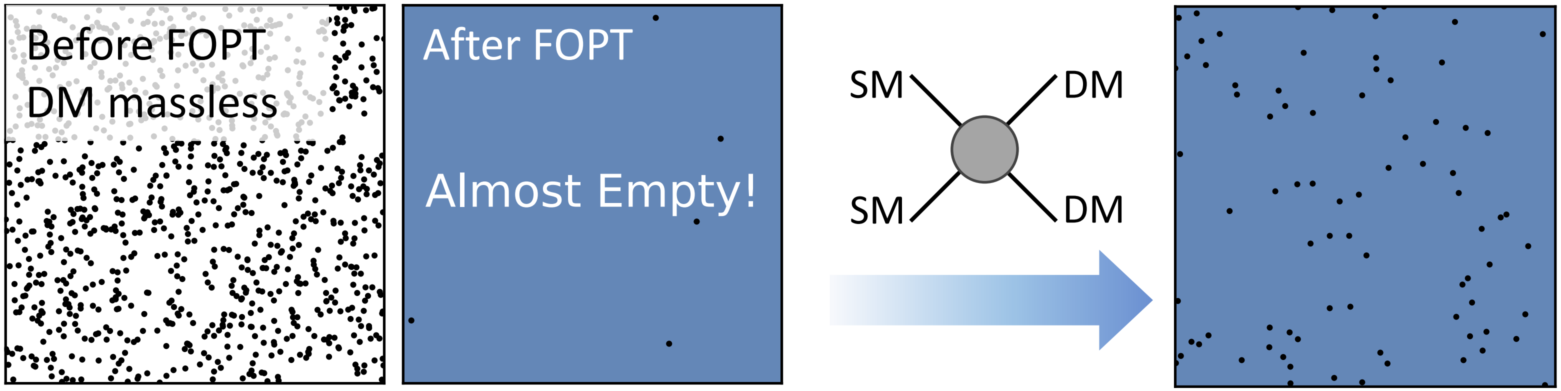
Classically Conformal can provide an FOPT

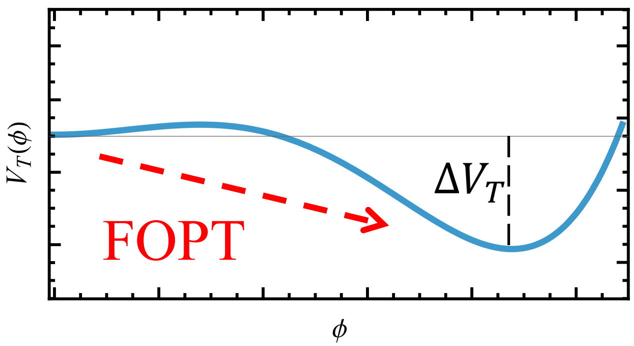
$$V_T(h, s, T) = V_{1\text{loop}}(h, s) + \frac{c_s T^2}{2} s^2 + \frac{c_h T^2}{2} h^2$$

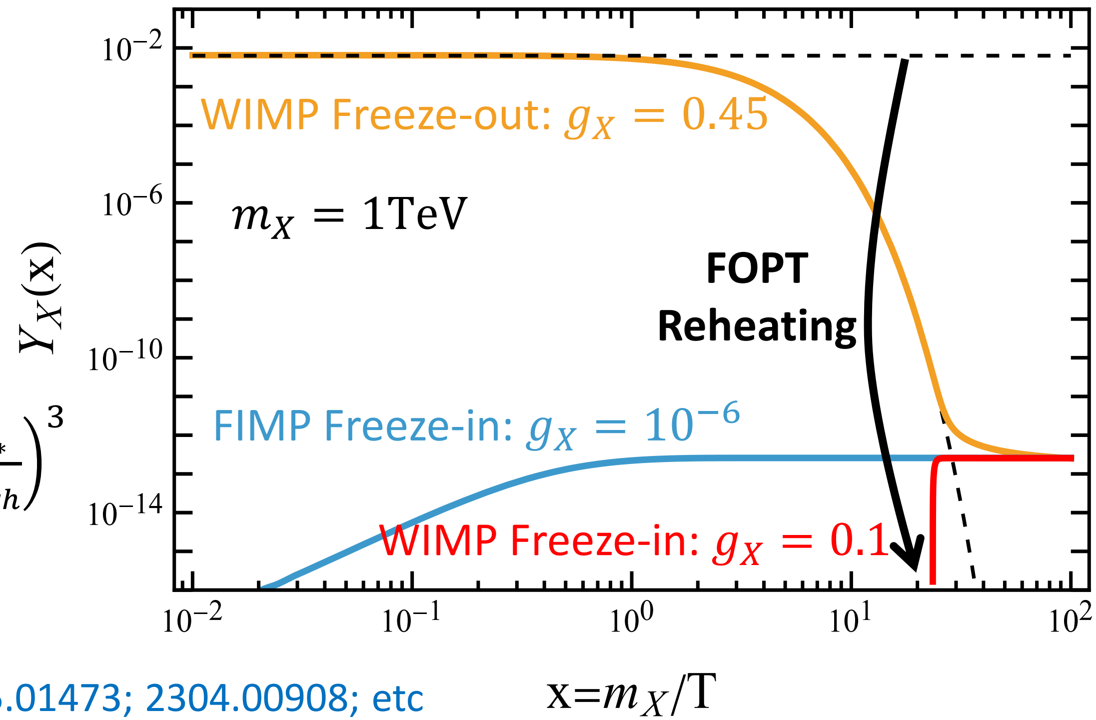


Flatter than polynomial at $\phi \sim 0$ → **Supercooled** FOPT ($T_* \ll w$)

Dark Matter in a Supercooled Universe







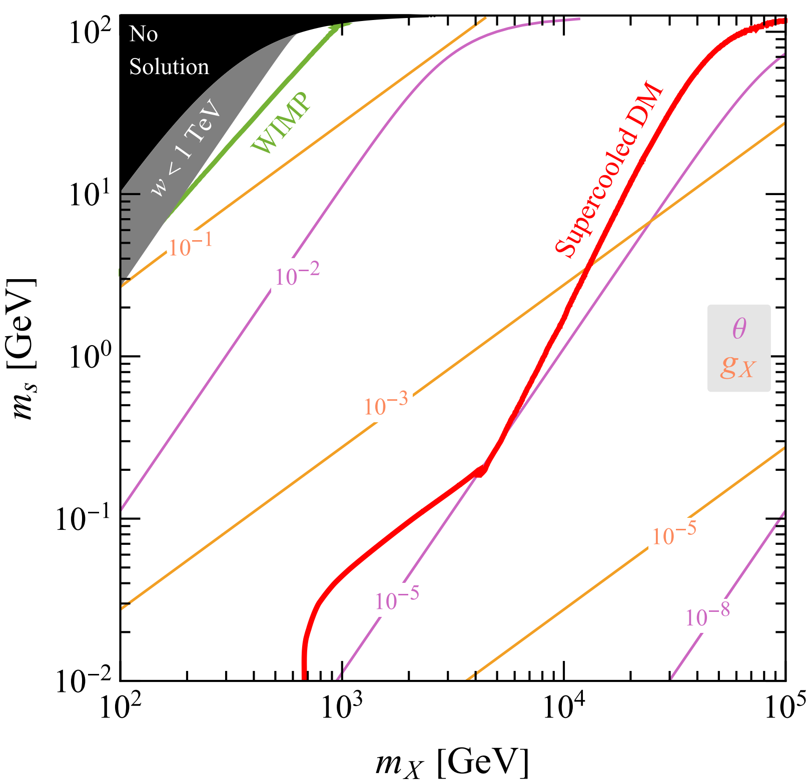
Vacuum energy ΔV_T release:

Temp: $T_* \to T_{rh}$; Entropy: $s \to s\left(\frac{T_*}{T_{rh}}\right)^3$

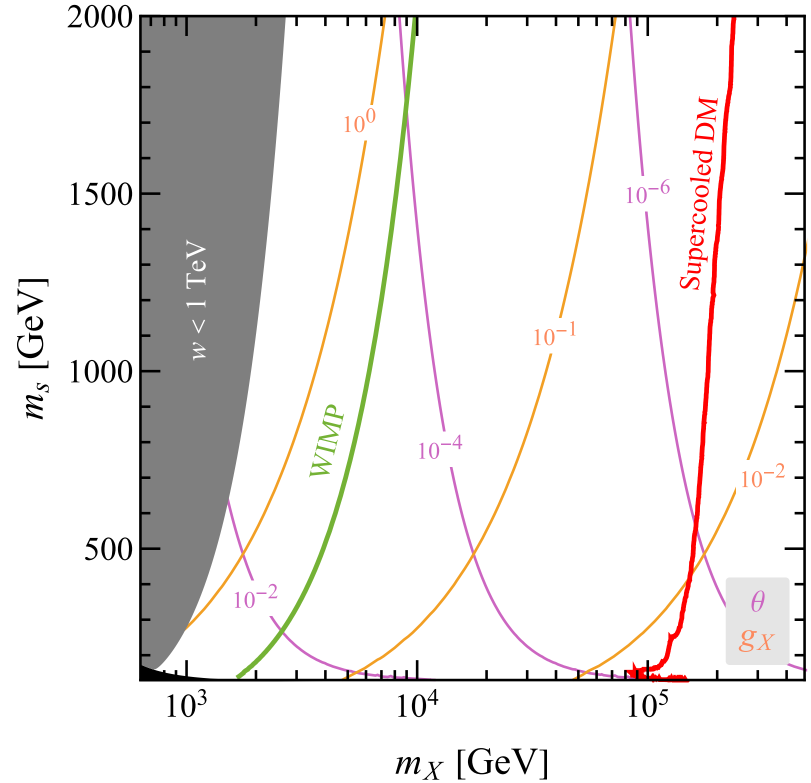
Dilution of DM: $Y_{rh} \sim Y_{eq}\left(\frac{T_*}{T_{rh}}\right)^3$

Also see 1805.01473; 2304.00908; etc

Dark Matter in a Supercooled Universe



$m_s < m_h$



$m_s > m_h$

WIMP: $g_X \sim \mathcal{O}(1 - 10^{-1})$, $m_X \sim \mathcal{O}(0.1 - 10)$ TeV

Supercooled DM: $g_X \sim \mathcal{O}(10^{-1} - 10^{-4})$, $m_X \sim \mathcal{O}(1 - 10^{2})$ TeV

Monopole cannot account for the observed relic abundance, also see 2509.21924

FOPT and GW

Novel dynamics ⇒ Novel detect way

4(5) key parameters:

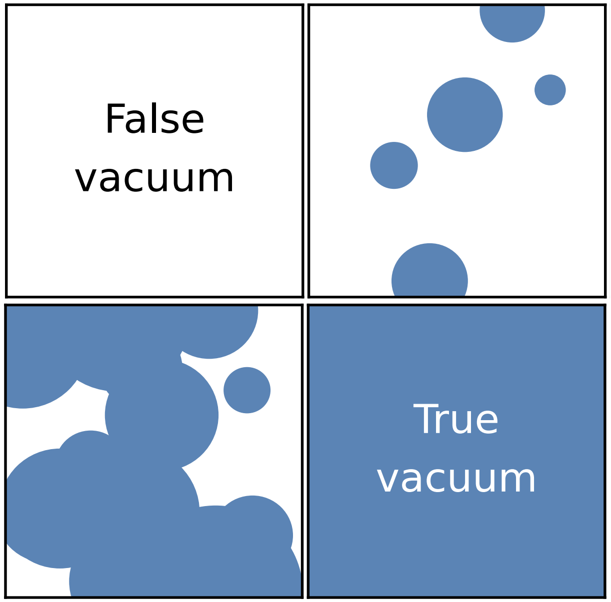
T_{rh}: Temperature after FOPT reheating

α: Strength of FOPT

$R_* H_*(\beta/H)$: scale(speed) of FOPT

v_w: Speed of bubble expand

$$\Omega_{GW}(T_*, \alpha, R_*, v_w) = \Omega_{sw} + \Omega_{col} + \Omega_{turb}$$



1512.06239; 1910.13125; 2305.02357; etc

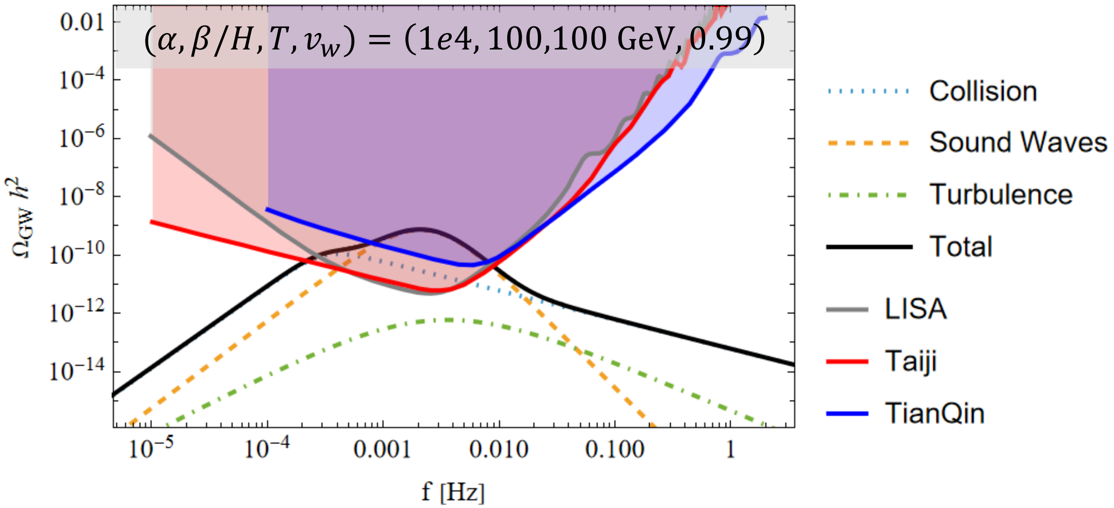
Supercooled FOPT:

$$\alpha \gg 1$$

$$v_w \sim 1$$

LISA Detectable:

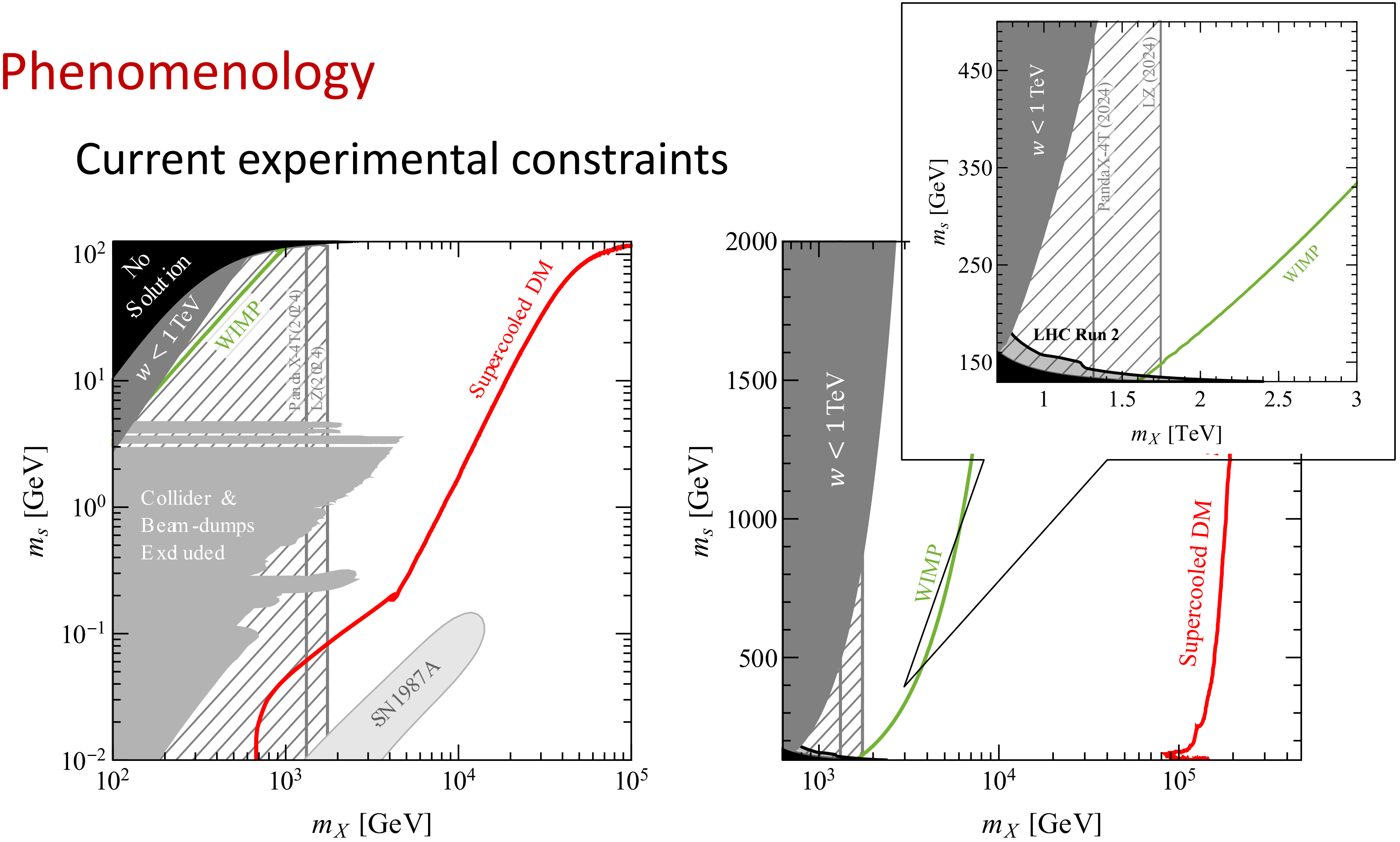
$$\beta/H_* \sim 10^2$$



Details in backup

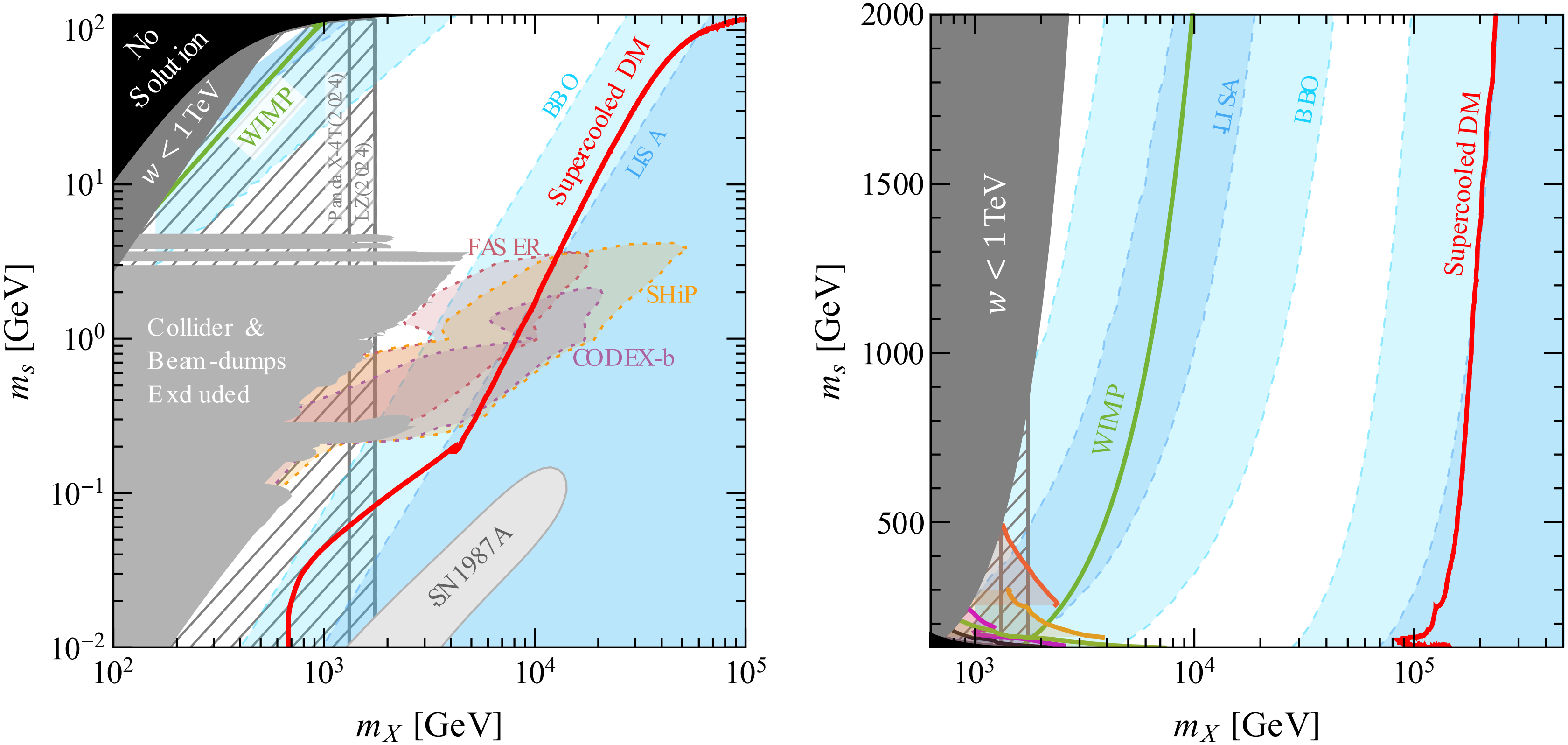
Phenomenology

Current experimental constraints



Phenomenology

Projected sensitivities of future experiment



WIMP & Supercooled DM can be probed via GW by **LISA & BBO**

Conclusion

Model

- **Classically Conformal**: hopeful way to solve hierarchy problem
- $SU(2)$ **triplet**: ensures the stability of vector DM (X^+/X^-)

Mechanics

Supercooled FOPT:

- Dilution and reproduction of DM
- Can be probe via GW

LISA & BBO detectable DM Landscape

Tips:

- Take initial bubble radius R_0 into account in extreme supercooled FOPT

See following backup for details

Thank you!

Backup: Monopole

At T_*, monopole density $\quad n_{mp}^* = p n_b^* \qquad p \sim 10^{-1}$

$$n_b = R_*^{-3}$$

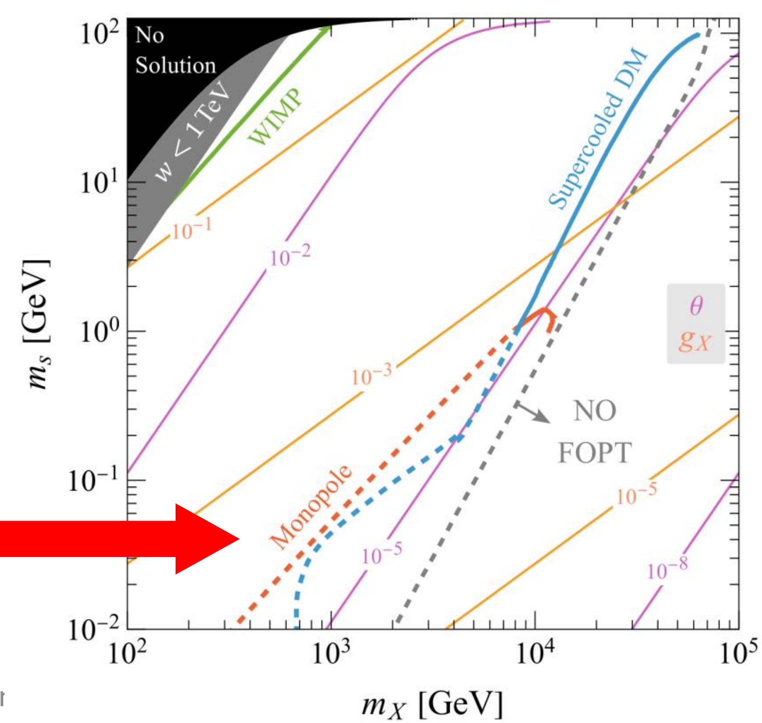
$$\Omega_{mp} h^2 \sim \frac{n_{mp}^*}{s_*} \left(\frac{T_*}{T_{rh}}\right)^3 s_0 M_{mp} \frac{8\pi}{3M_{pl}^2} \left(\frac{h}{H_0}\right)^2$$

$$\propto n_{mp}^* \propto R_*^{-3}$$

Typically, $R_* \sim \frac{(8\pi)^{1/3}}{\beta}$

For $\beta \gg 1$ (e.g.: $\beta = 10^8$), $n_{mp}^* \gg 1$

$\Omega_{mp} h^2 \sim 0.12$? **Monopole as DM?**



Backup: Monopole

At T_*, monopole density $\quad n_{mp}^* = p n_b^* \qquad p \sim 10^{-1}$

$$n_b = R_*^{-3}$$

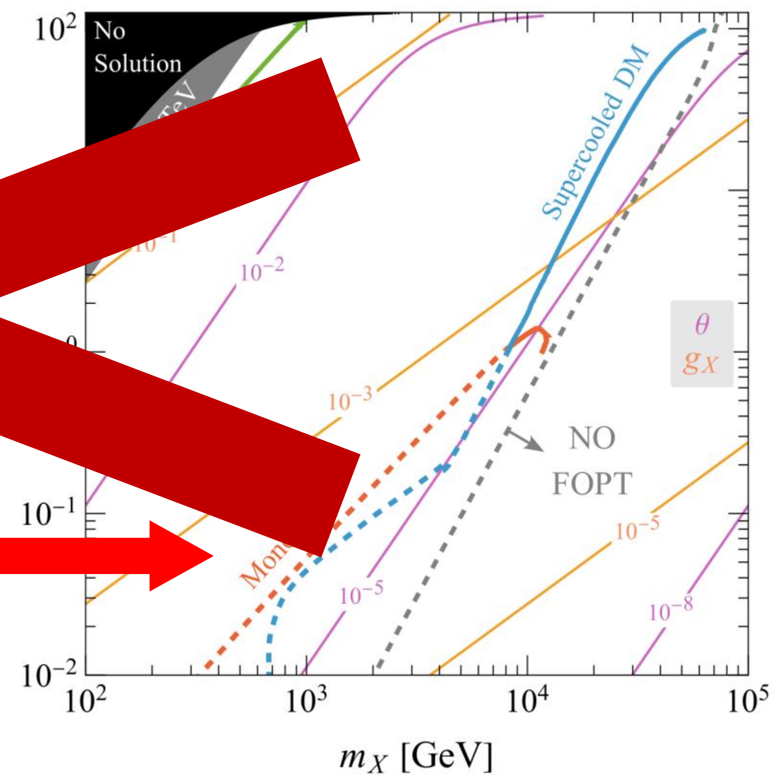
$$\Omega_{mp} h^2 \sim \frac{n_{mp}^*}{s_*} \left(\frac{T_*}{T_{rh}}\right)^3 s_0 M_{mp} \frac{8\pi}{3M_{pl}^2} \left(\frac{h}{H_0}\right)^2$$

$\propto n_{mp}^* \propto R_*^{-3}$

Typically, $R_* \sim \frac{[illegible]}{\beta}$

For $\beta \gg 1$ (e.g.: $\beta = 10^{8}$ [illegible]

$\Omega_{mp} h^2 \sim 0.$ [illegible] onopole as DM?



Backup: R_0 & R_*

$R_* \sim \frac{(8\pi)^{1/3} v_w}{\beta}$: Assuming fast FOPT & $R_0 = 0$

2404.00646

In this appendix, we derive the analytical results of the FOPT dynamics under the exponential approximation $\Gamma(t) \approx \Gamma_* e^{\beta(t-t_*)}$. Assume the FOPT is fast that $a(t)$ almost does not change during the transition, then the false vacuum fraction can be explicitly integrated out as

$$F(t) \approx \exp\left\{-\frac{4\pi}{3}\int_{t_c}^{t} \mathrm{d}t'\Gamma(t')\left(\int_{t'}^{t}\mathrm{d}t'' v_w\right)^3\right\} \approx \exp\left\{-\frac{8\pi v_w^3}{3\beta^4/\Gamma_*} e^{\beta(t-t_*)}\right\}, \qquad \text{(A.1)}$$

NO R_0

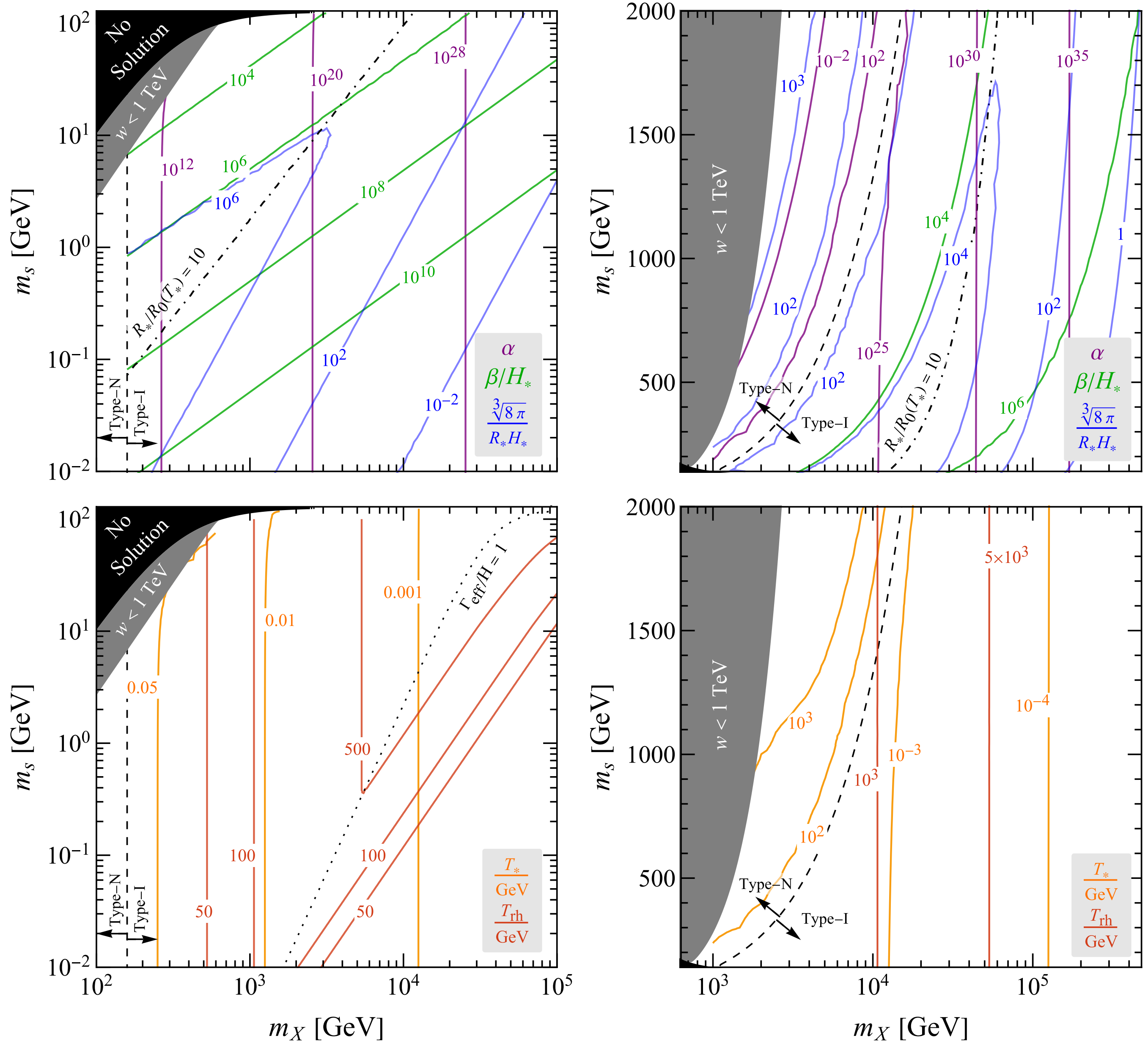
Therefore, in the $t \to \infty$ limit we obtain $\bar{R}/[(8\pi)^{1/3} v_w] \to \beta^{-1}$, and hence $(8\pi)^{1/3} v_w/(H_* \bar{R})$ approaches β/H_* [118].

In Fact: $R_0 \sim \frac{1}{g_X T_{roll}} \sim \frac{w}{m_h v_{QCD}} \propto w$ More supercooling, More large R_0

In extreme supercooling, $\frac{(8\pi)^{1/3} v_w}{\beta}$ can be **smaller** than R_0

Must consider R_0 in extreme supercooling

FOPT and GW: parameter space



FOPT and GW: details

$$\Gamma(T) \simeq T^4 \left(\frac{S_3}{2\pi T}\right)^{3/2} \exp\left(-\frac{S_3}{T}\right)$$

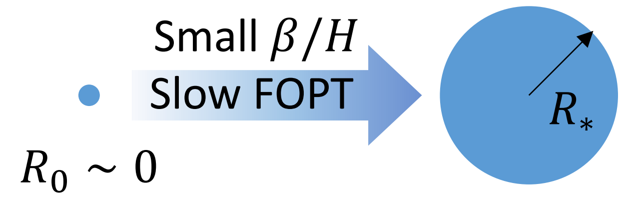
$$P(t) = \exp\left(-\frac{4\pi}{3}\int_{t_c}^{t} \mathrm{d}\,t' \Gamma(t') a^3(t') r(t-t')^3\right)$$

$$r(t-t') = R_0 + \int_{t'}^{t} \frac{v_w}{a(\tilde{t})}\mathrm{d}\tilde{t}$$

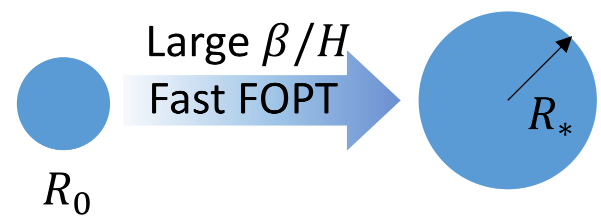
Must consider in extreme supercooling

$$n_b(T) = T^3 \int_{T}^{T_c} dT' \frac{\Gamma(T')}{T'^4 H} P(T')$$

$$R_* = n_b^{-1/3}$$



Normal supercooling



Extreme supercooling