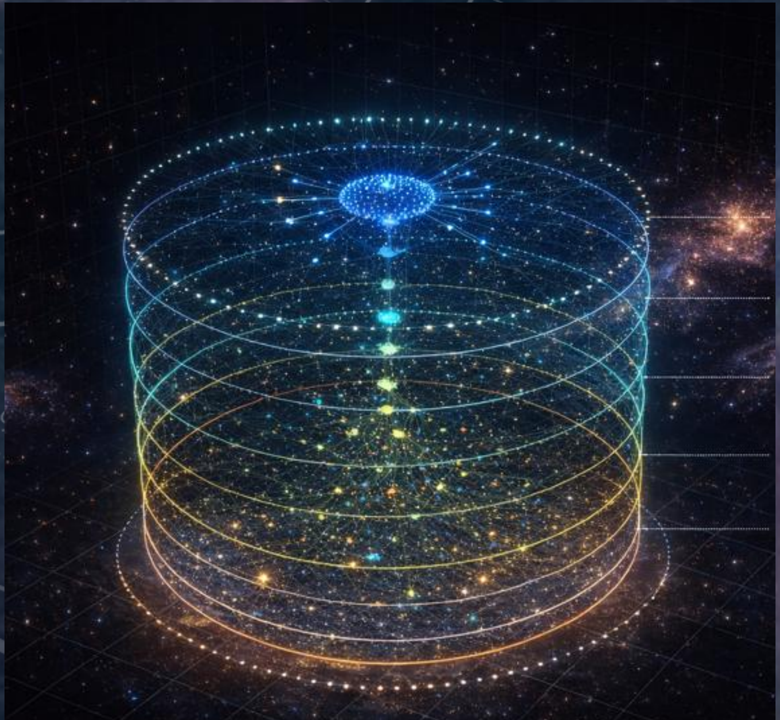
Mainly based on arXiv:2603.15128

新物理前沿与交叉学科研讨会
(NPhiS 2026)

A Path-Likelihood Formulation of Galaxy Formation on Halo Graphs (基于暗晕图的星系形成路径似然表述)

--- encoding new physics as controlled deformation on graph paths



https://github.com/DanengYang/GraphPathLikelihood (update soon)

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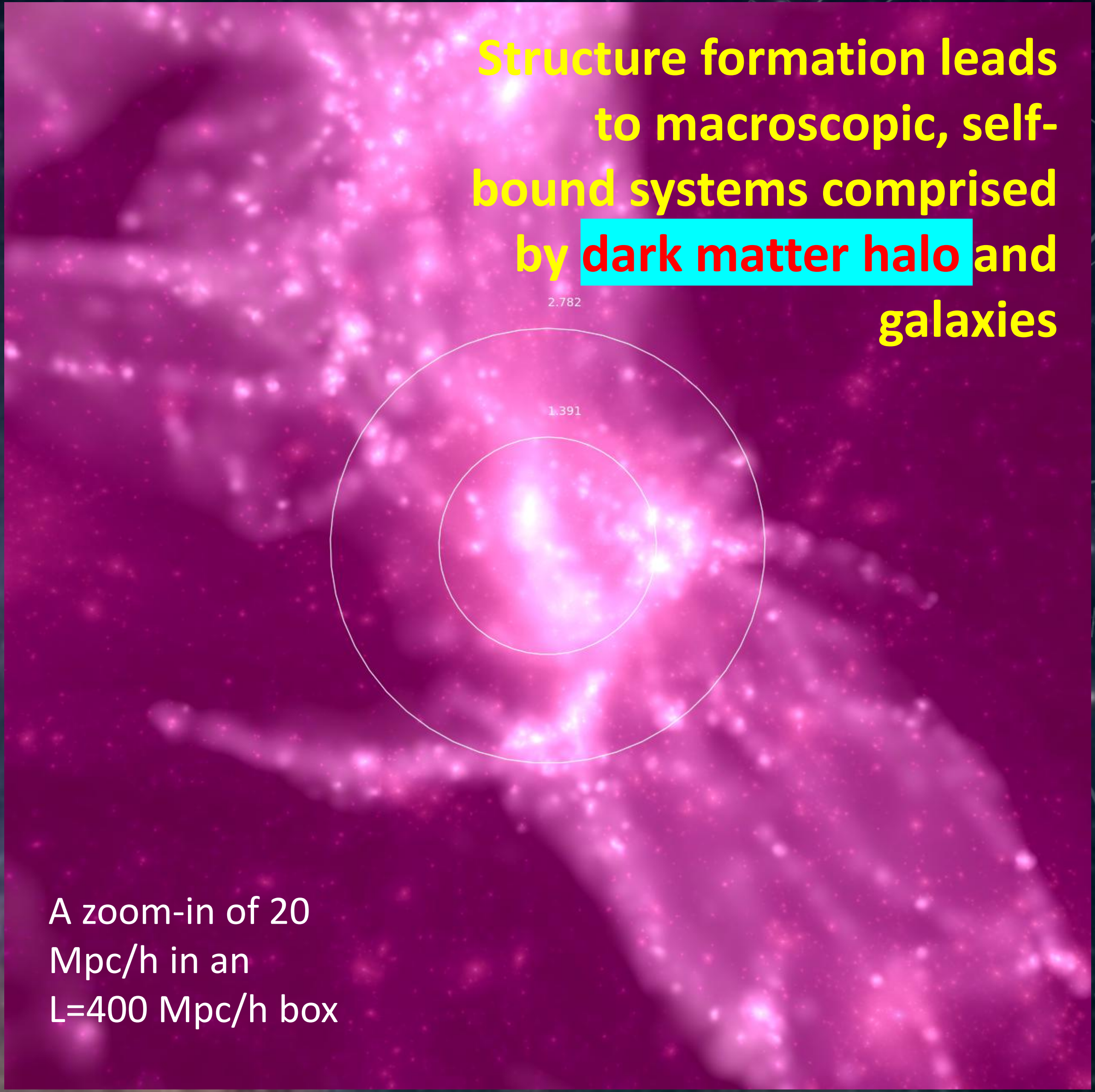
Outline

- From snapshot prediction to path probability
- Layered halo graphs
- A graph path likelihood model
- Example applications
- SIDM as controlled deformation on graph paths



Why do we need a path measure in galaxy formation?

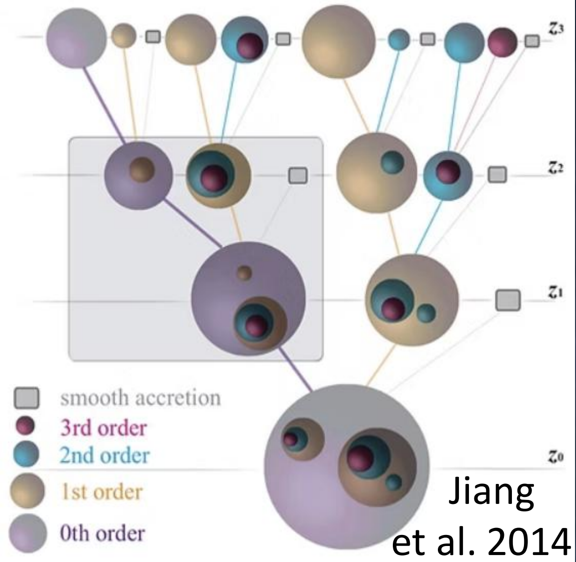
Goal: explore Self-Interacting Dark Matter (SIDM) through controlled deformation of galaxies

Galaxy formation in CDM → Galaxy formation in SIDM

A model for this mapping?

- **A deterministic backbone**
- **Uncertainty always present**
- **Both need to be modelled**

Yang+2305.16176



Jiang et al. 2014

Given halo merger history, existing models can predict galaxies in halos

Forward models ≠ path measures

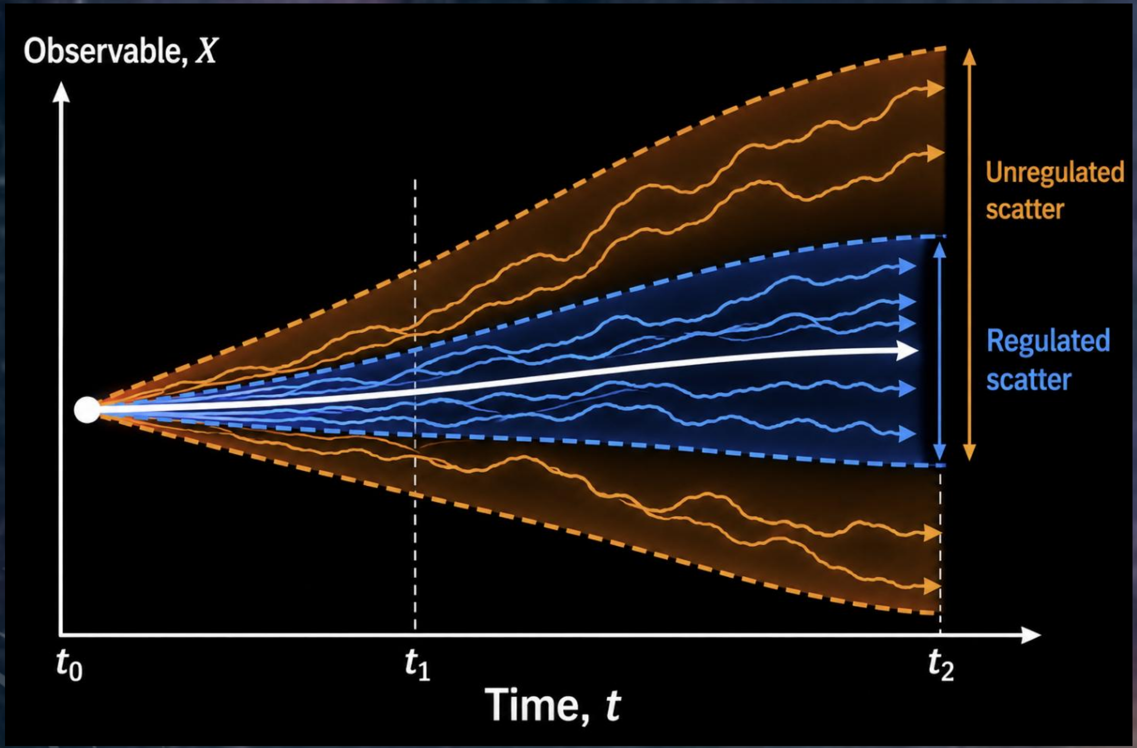
Question is: is the prediction deterministic?

A path measure model should offer a self-consistent way to capture the intrinsic scatter and is growth

Galaxy formation along halo assembly has intrinsic scatters

Full simulation data → **Snapshots** → **Merger trees & catalogs**

We lack full information for deterministic predictions, so coarse-graining is always needed



- Models use coarse-grained information or characteristics
- Unresolved differences grow dynamically over time
- Scatter remain time-correlated and graph-conditioned

Effective Scatter Is Dynamical, Not Just Instantaneous

Coarse-graining → dynamical effective scatter
→ stochastic effective dynamics
→ a population of paths and (endpoint) observables can be sampled, even when the accretion history is fixed

Observables and path distortions can be defined in a path integral formalism

$$\dot{x}(t) = b(x(t)) + \eta(t)$$

The Langevin equation

The Martin-Siggia-Rose-Janssen-De Dominicis (MSRJD) formalism

$$Z = \int \mathcal{D}\eta\, \mathcal{D}x\, \mathcal{D}\hat{x}\; e^{\int dt\, \hat{x}^T (\dot{x} - b - \eta)}\; e^{-\frac{1}{2}\int dt\, \eta^T D^{-1} \eta}$$

A path measure model

Hierarchical structure formation

Graph-trajectory measure

$$P(\mathbf{x}, G) = P(\mathbf{x} \mid G) P(G)$$

Graph-conditioned path measure

$$P(\mathbf{x} \mid G) \propto p_{\mathrm{attach}}(\mathbf{x} \mid G) \exp\left[-S(\mathbf{x}; G)\right]$$

Evolution in isolation

Evolution in environment

Future extension:

$$S_{\mathrm{tot}} = S_G[G_{0:K}] + S_x[\mathbf{x}_{0:K}; G_{0:K}] + S_{\mathrm{bnd}}[\mathbf{x}_{0:K}; G_{0:K}]$$

Probabilistic halo graph construction for clustering

Yang & Yu 2023 PR Research:

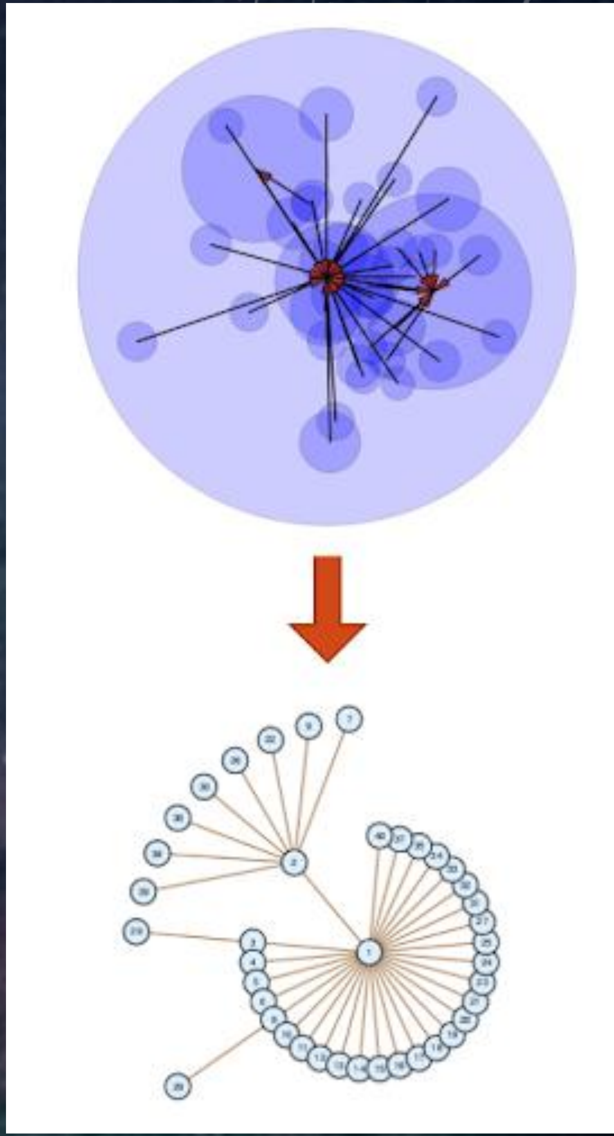
- Halo graphs can be constructed using **preferential attachment**
- They are power-law graphs

$$\text{Attachment Probability} = \frac{A_i}{\sum_j A_j}$$

Preferential attachment

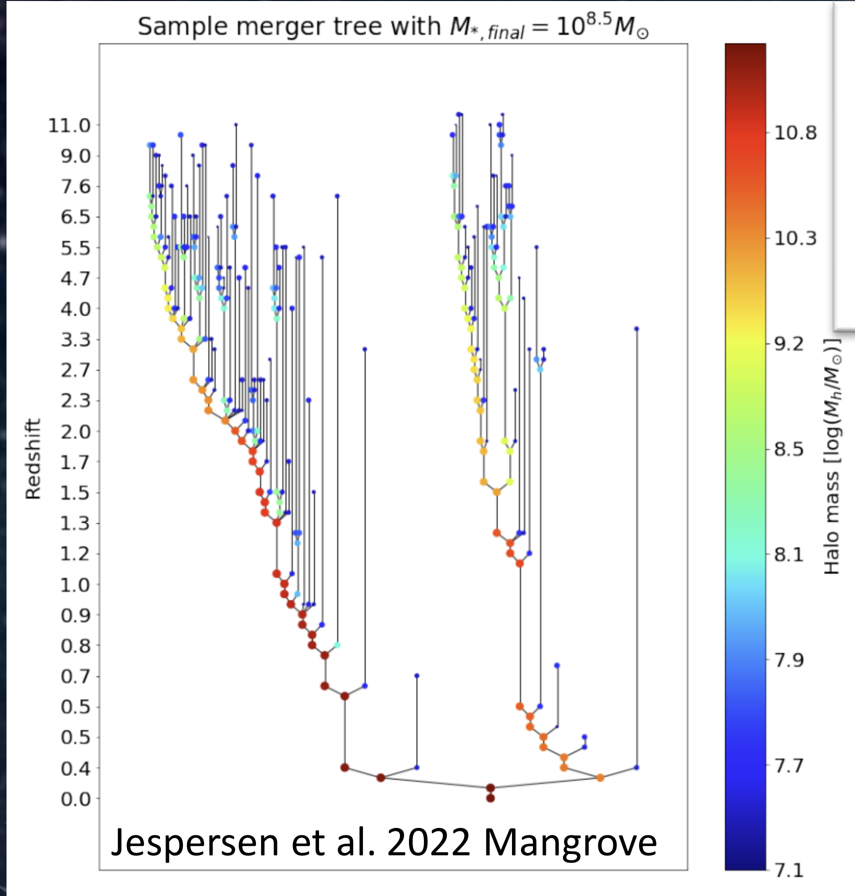
- Rich becomes richer
- Early attachment advantage

Self-similarity: Hierarchical structure naturally arise

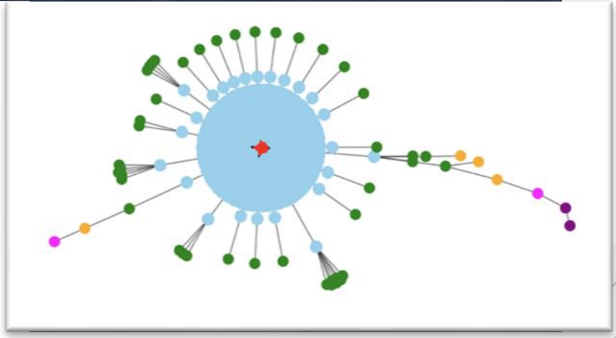


Encoding environmental features by layered halo graphs

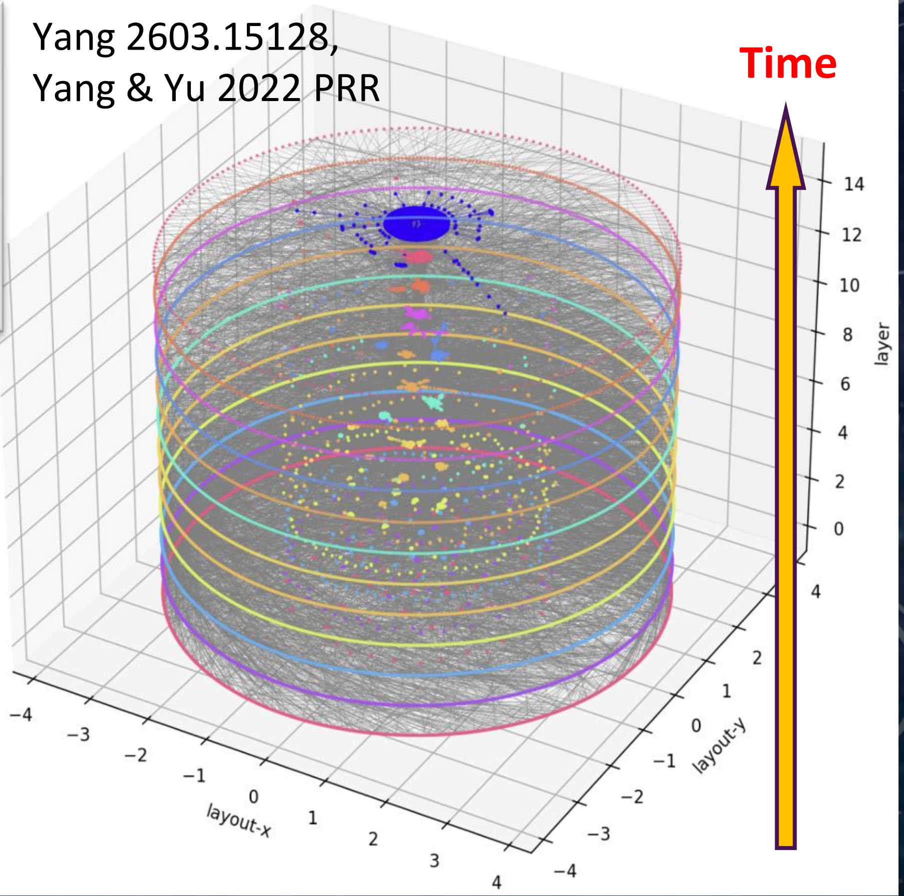
One halo at z=0 has one merger tree



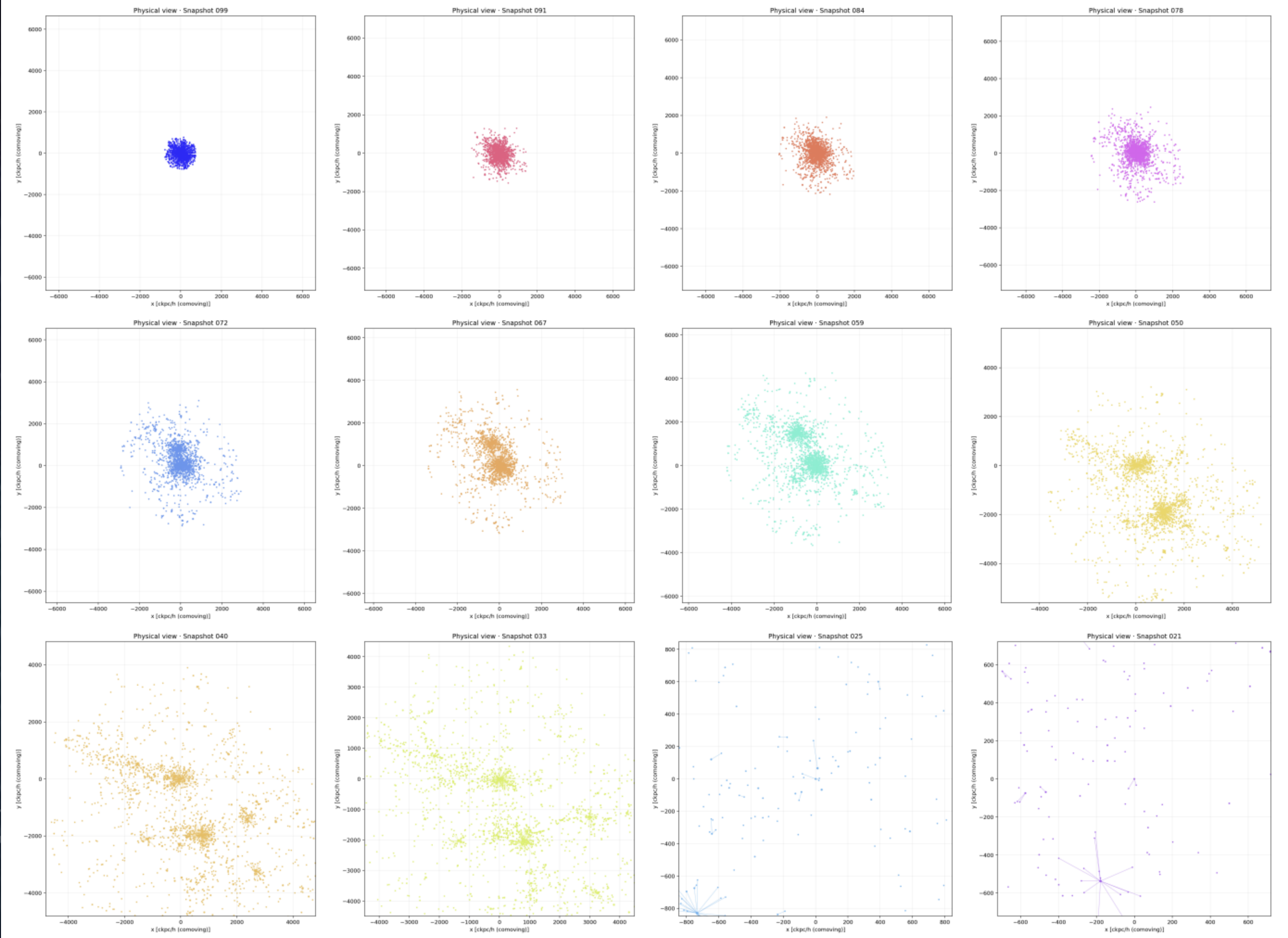
Grouped histories for main and sub halos



New: all halos belonging to an z=0 host halo has one layered halo graph

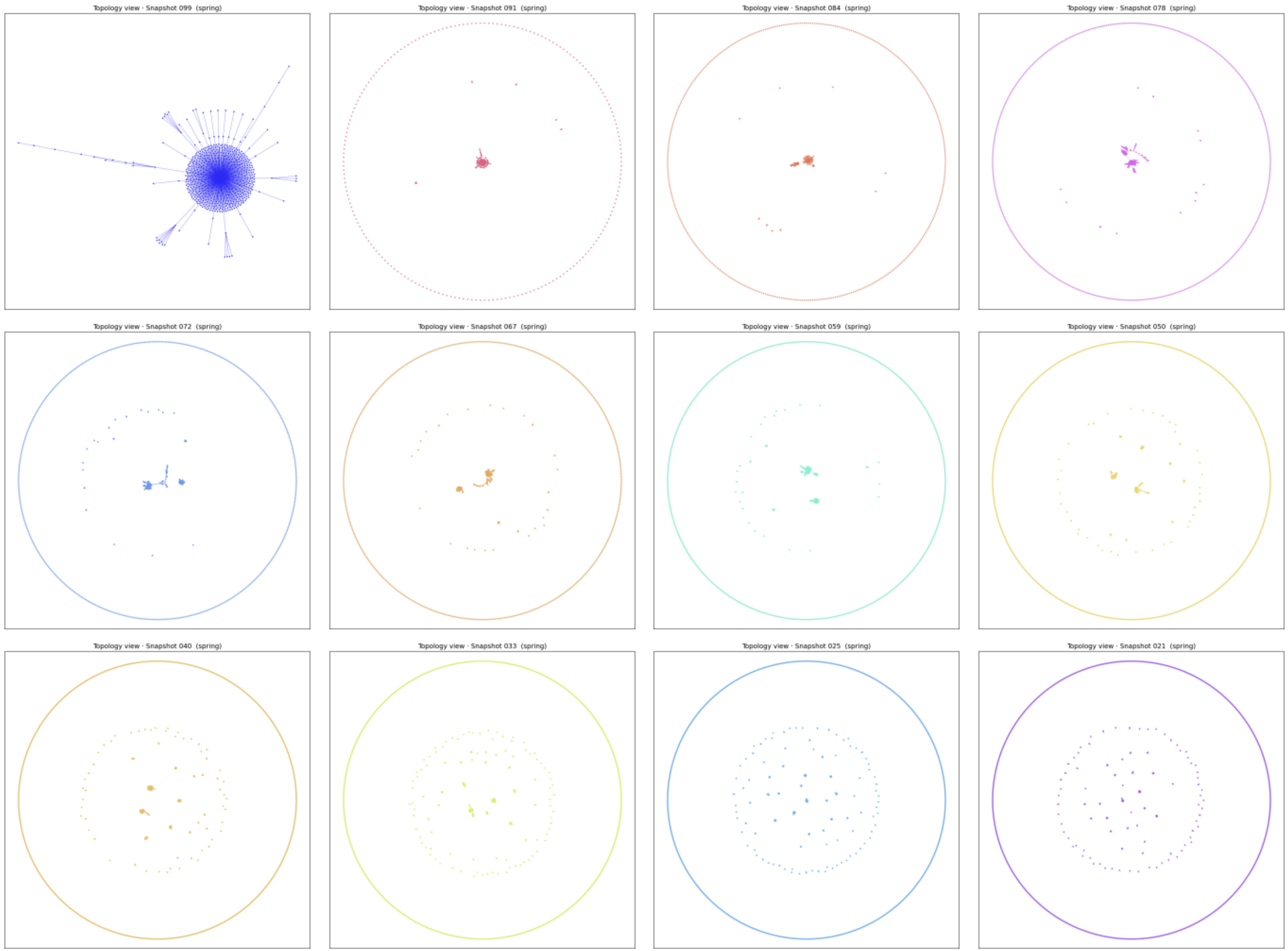


Probability-based construction naturally defines a graph measure P(G)



Physical (comoving-coordinate) view

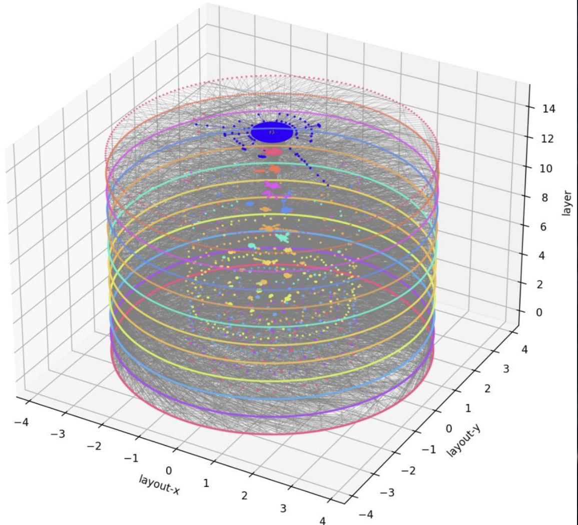
Hierarchical structures merge into a self-bound halo



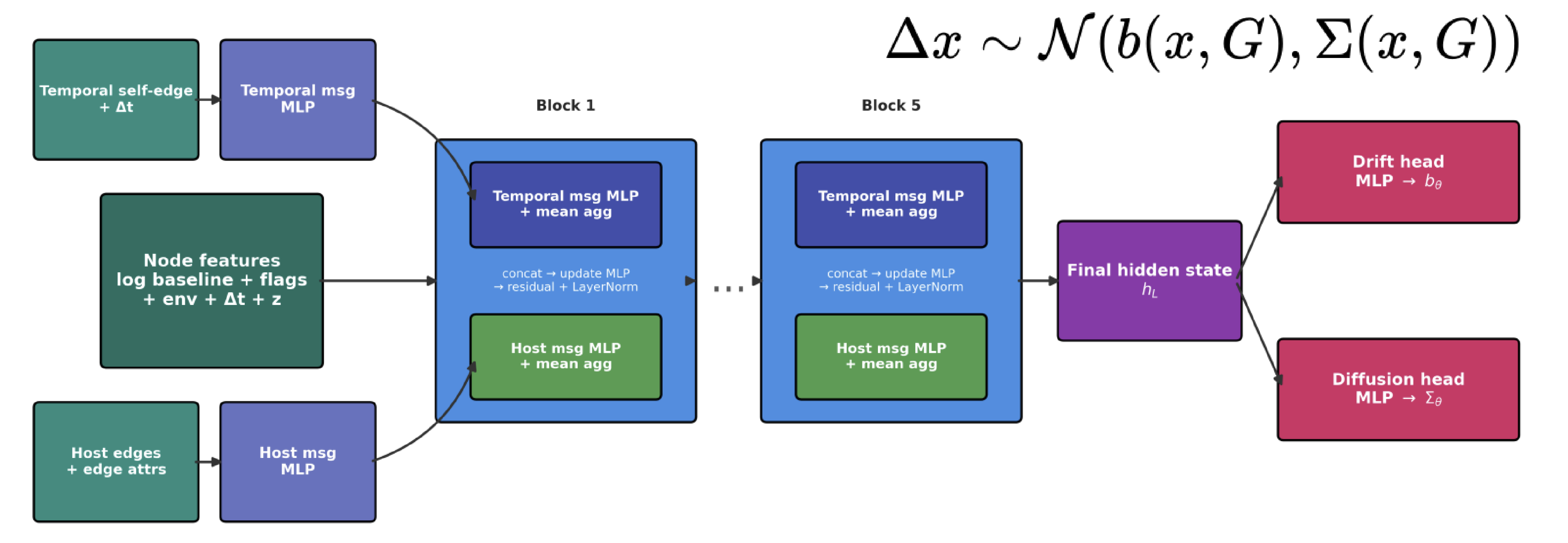
Topological view

- Point has no size
- Edge has no length

Stack the layers



A Graph Path Likelihood Model (GPLM) from machine learning

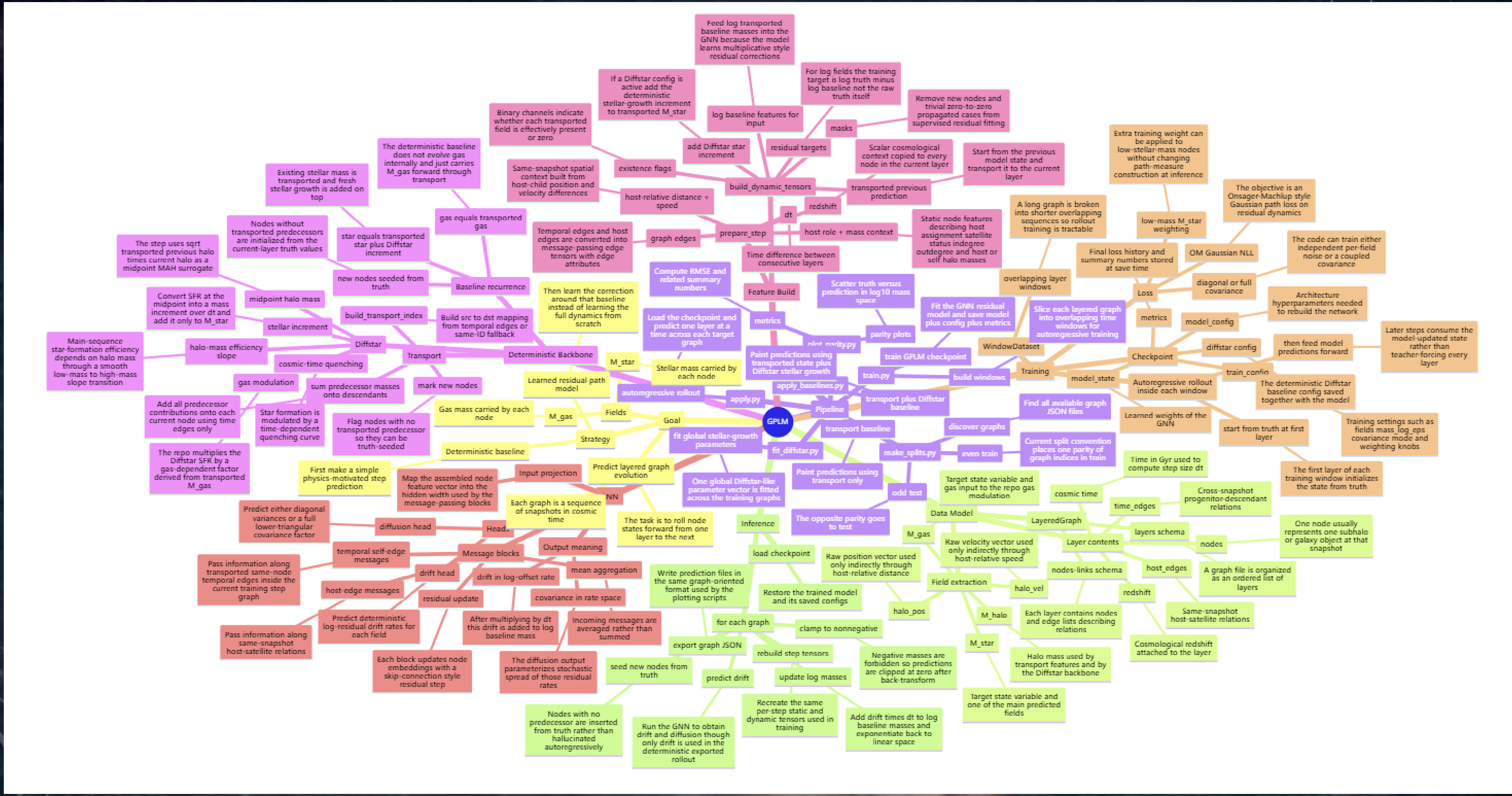


Graph encoding, conditioning

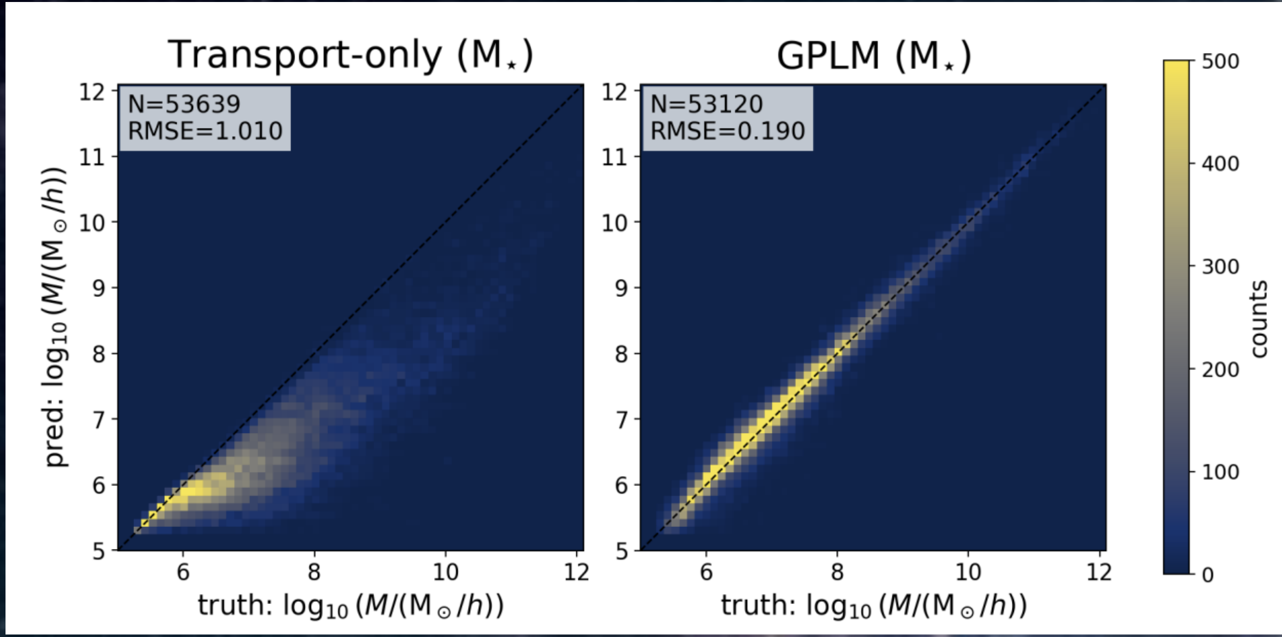
Graph Neural Network

- b: learned drift
- D: learned diffusion

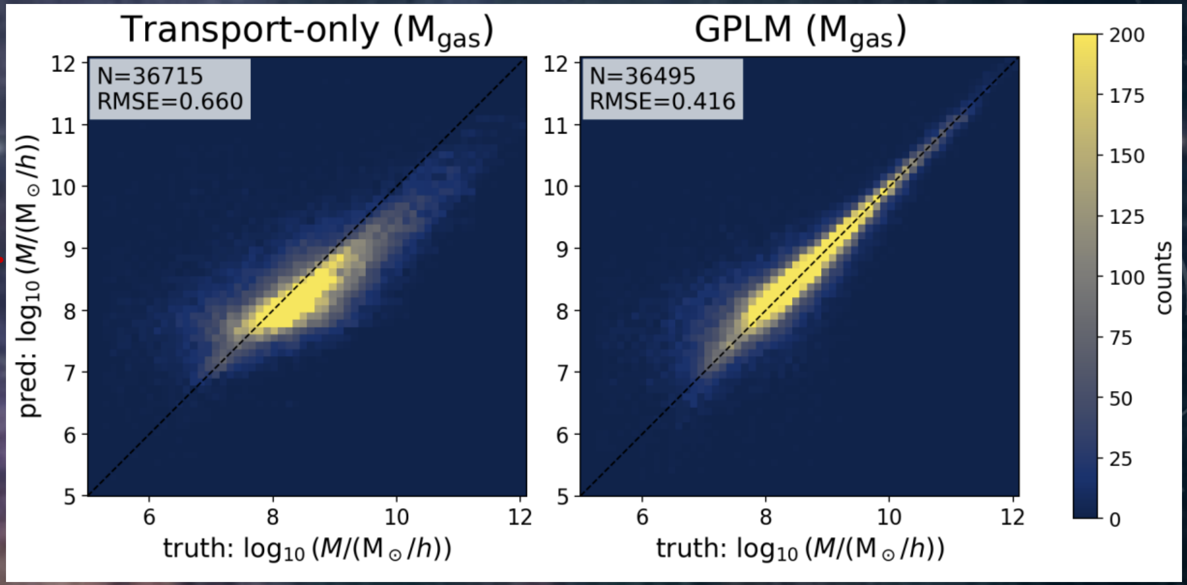
The GPLM code layout



Model performance

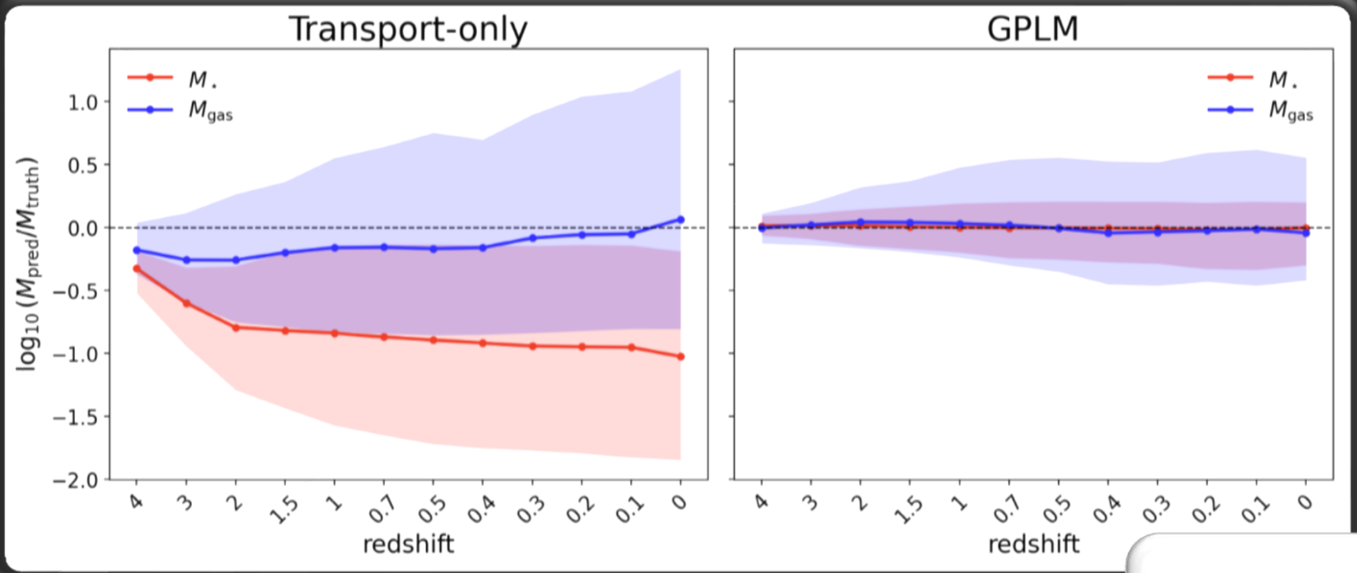


Prediction vs Truth in Stellar Mass



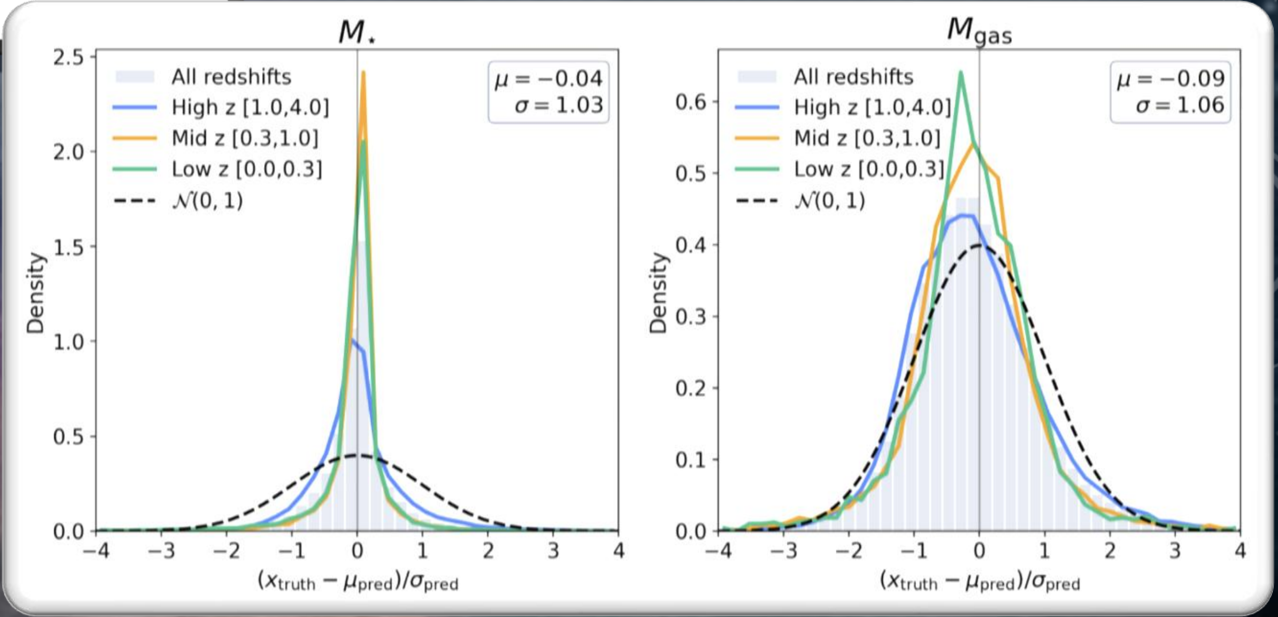
Prediction vs Truth in Gas Mass

Model performance



GPLM substantially reduces both the median bias and the spread of the residual distribution.

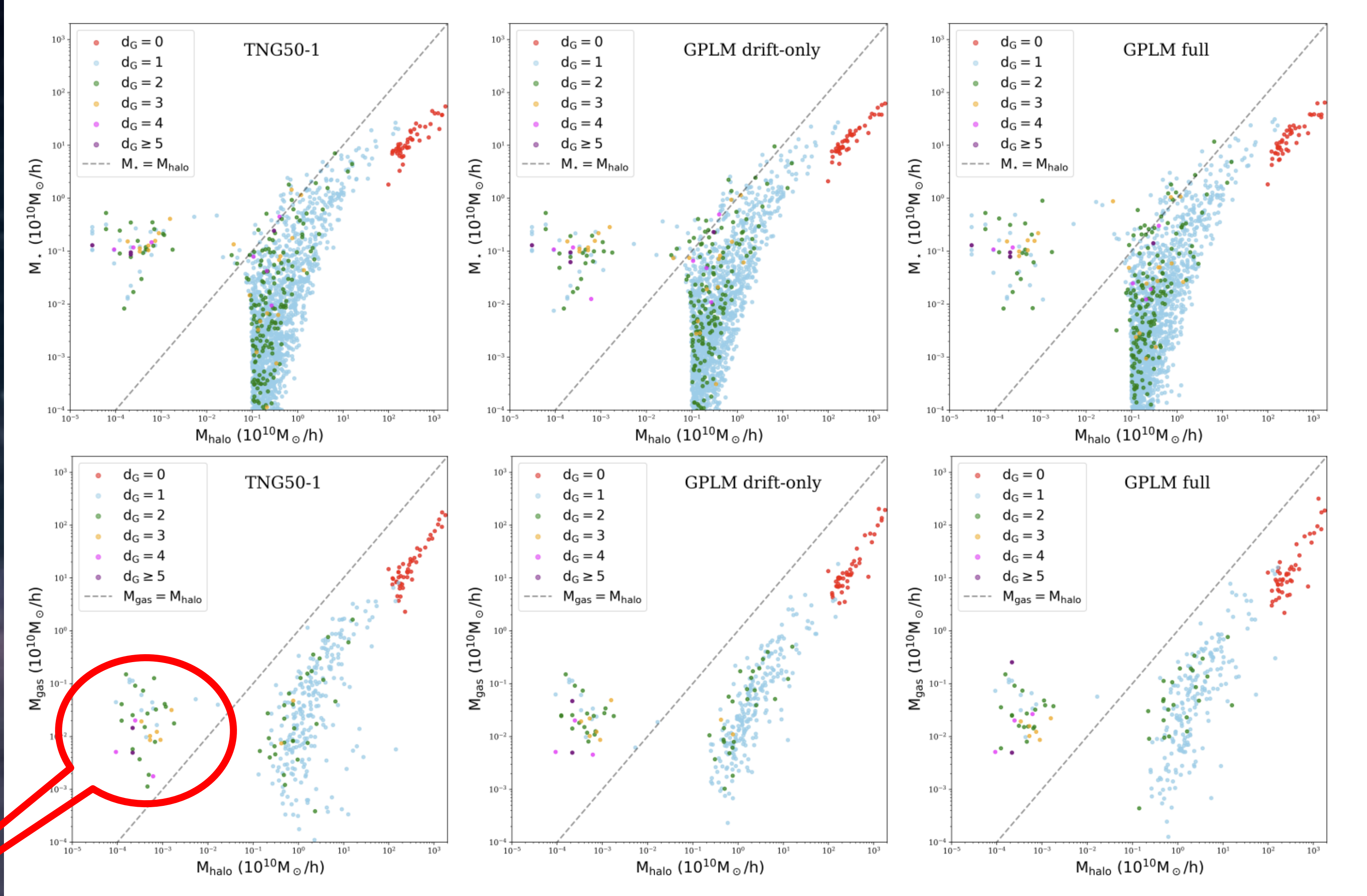
- ➢ Calibrated diffusions have expected mean and variance
- ➢ The Mstar channel suggests the need for non-Gaussianity



Model performance

The trained GPLM reliably reproduces the dG-split population structure observed in the simulation

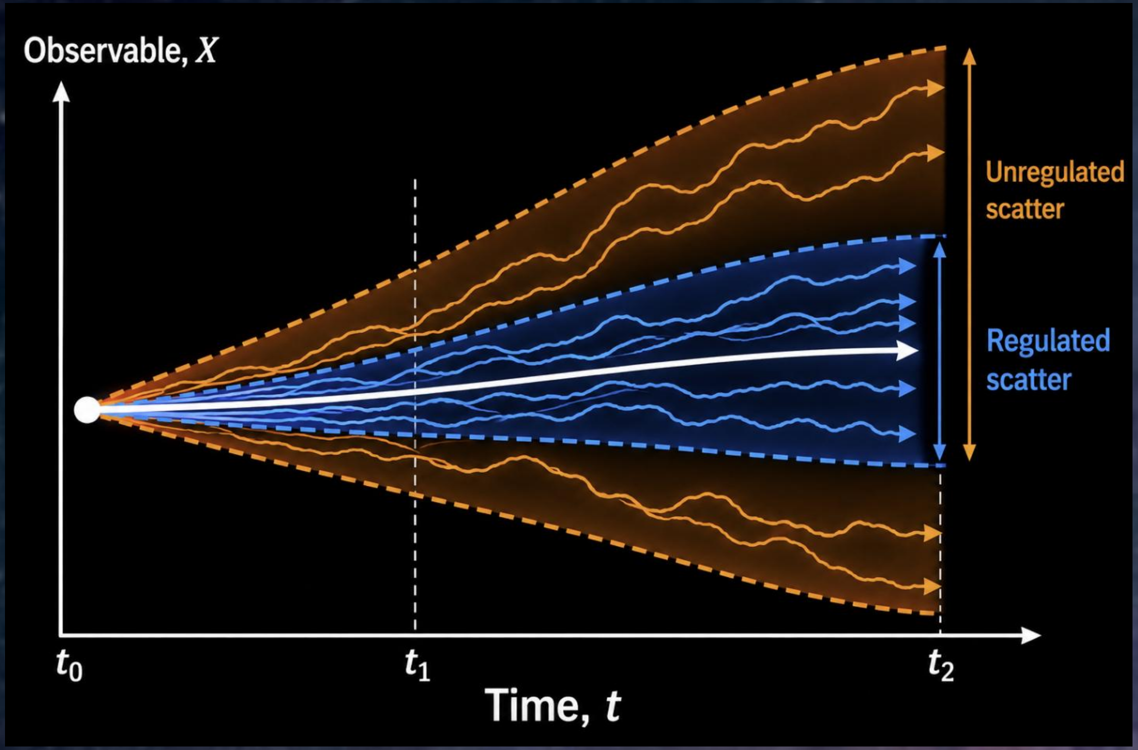
Dark Matter Deficient Galaxies



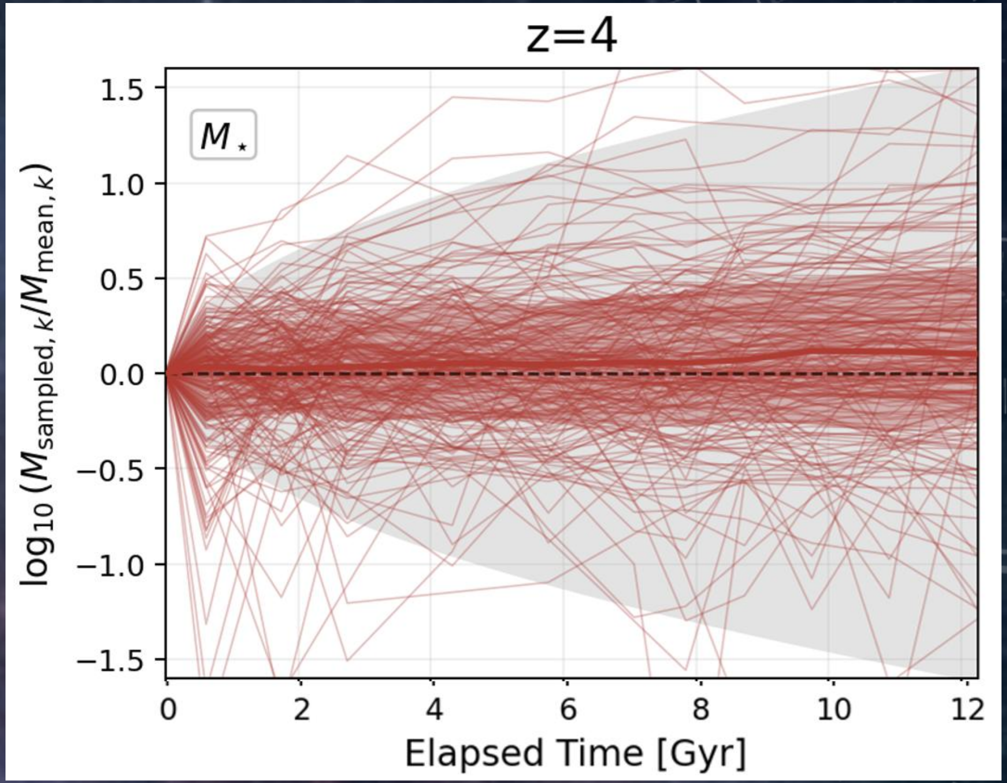
- *Different sampled path batches represent* ***distinct stochastic realizations*** *of the same learned effective dynamics.*
- *The endpoint observables retain* ***analogous stochastic structure*** *across these different realizations.*

Effective scatter is dynamical, with regulated growth

Earlier cartoon



In our trained model



MSRJD in GPLM: Discrete Realization of Stochastic Evolution

$$\Delta x_{i,k} \mid s_{i,k}, G \sim \mathcal{N}\left(\delta_{i,k}, \widehat{\Sigma}_{i,k}\right)$$

$$\delta_{i,k} \equiv b_{\theta,i,k} \Delta t_k \qquad \widehat{\Sigma}_{i,k} \equiv D_{\theta,i,k} \Delta t_k$$

$$\begin{cases} \Delta x_{i,k} = b_{\theta,i,k}(s_{i,k}, G_k)\, \Delta t_k + \xi_{i,k} \\ \langle \xi_{i,k}\, \xi^T_{j,k'} \rangle = D_{\theta,i,k}\, \Delta t_k\, \delta_{ij}\, \delta_{kk'} \end{cases}$$

$$S_{\mathrm{MSRJD}}[x, \hat{x}; G] = \int dt \left[\hat{x}^T \left(\dot{x}_{\mathrm{res}} - b_\theta(x^{\mathrm{tr}}, G) \right) - \frac{1}{2} \hat{x}^T D_\theta(x^{\mathrm{tr}}, G) \hat{x} \right]$$

enforces the stochastic dynamics

Gaussian Onsager-Machlup (OM) action

$$S_{\mathrm{OM}}[\mathbf{x}; G] = \frac{1}{2} \sum_{(i,k)\in\mathcal{V}_{\mathrm{sup}}} \left[(\Delta x_{i,k} - \delta_{i,k})^{\mathsf{T}} \widehat{\Sigma}^{-1}_{i,k} (\Delta x_{i,k} - \delta_{i,k}) + \log\det \widehat{\Sigma}_{i,k} \right]$$

non-Gaussianity could be handled by higher-order terms

Path Space Tools

Operator average	Controlled likelihood deformation	Likelihood Ratio; forward-backward asymmetry

Dark Matter Deficient Galaxy (DMDG) Probabilities

DMDG operator:

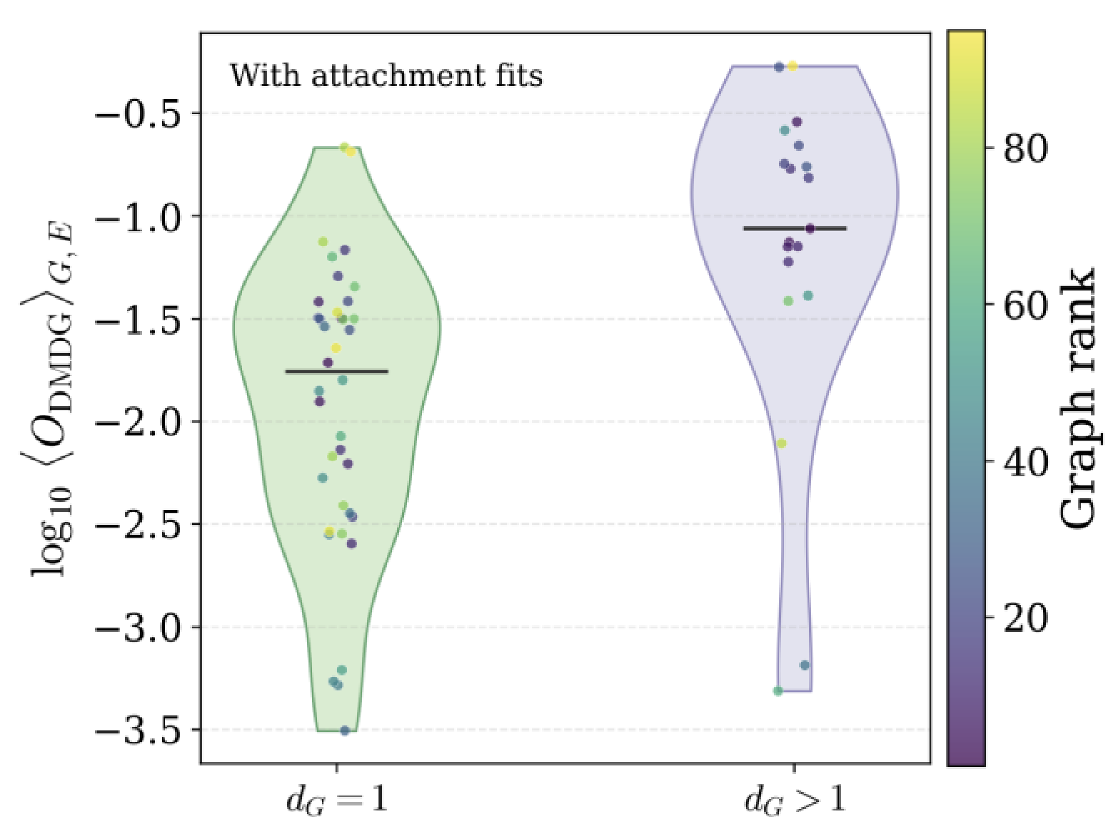
$$O^{(i)}_{\mathrm{DMDG}}(\mathbf{x}_i) = \Theta\left(\frac{M^{(i)}_\star(z=0) + M^{(i)}_{\mathrm{gas}}(z=0)}{M^{(i)}_{\mathrm{halo}}(z=0)} - 1\right)$$

Operator average

$$\langle O_{\mathrm{DMDG}}\rangle_{G,E} \equiv \int \mathcal{D}x \left[\frac{\sum_{i\in E} O^{(i)}_{\mathrm{DMDG}}[\mathbf{x}]}{N_E}\right] P_{\mathrm{tot}}(\mathbf{x} \mid G, x_0)$$

dG=1: satellites
dG>1: higher order satellites

Higher tidal stripping in dG>1 satellites leads to more DMDGs but with greater scatter

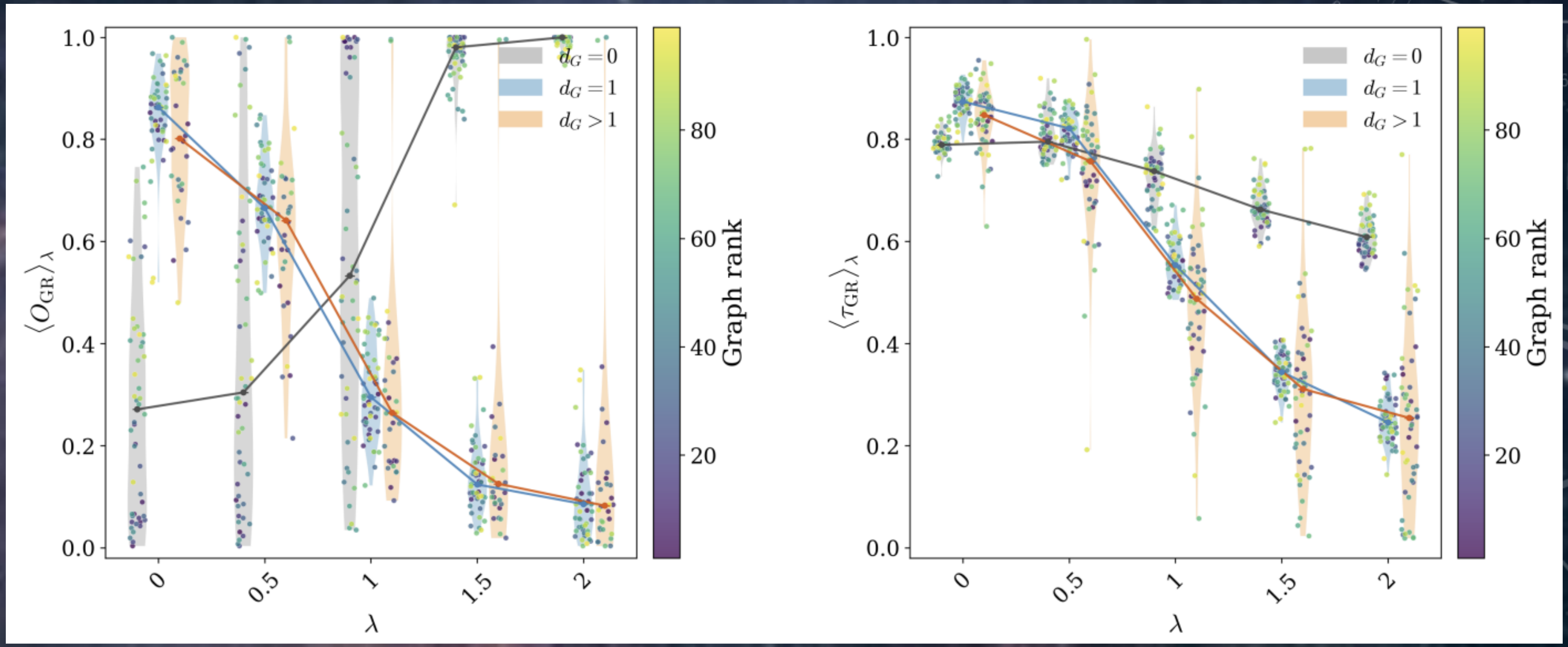


Gas-Rich response as a controlled likelihood deformation

Gas-Rich operator: $O_{\mathrm{GR}}(i) \equiv \Theta[M_{\mathrm{gas},i}(z{=}0) - M_{\star,i}(z{=}0)]$

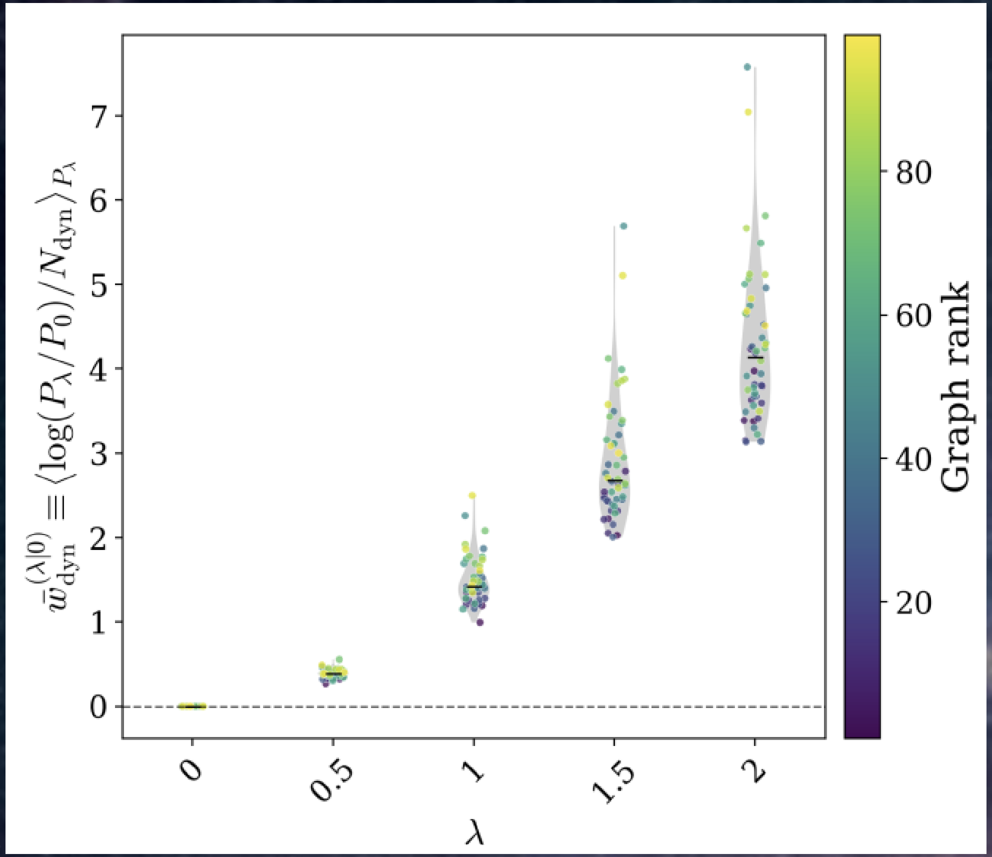
Controlled deformation: $b_{\mathrm{gas},\lambda}(x_k, G) = b_{\mathrm{gas,off}}(x_k, G) + \lambda[b_{\mathrm{gas,full}}(x_k, G) - b_{\mathrm{gas,off}}(x_k, G)]$

Ram-pressure stripping causes gas of satellites to feed into the host
dG=0: host
dG=1: satellites
dG>1: higher order satellites

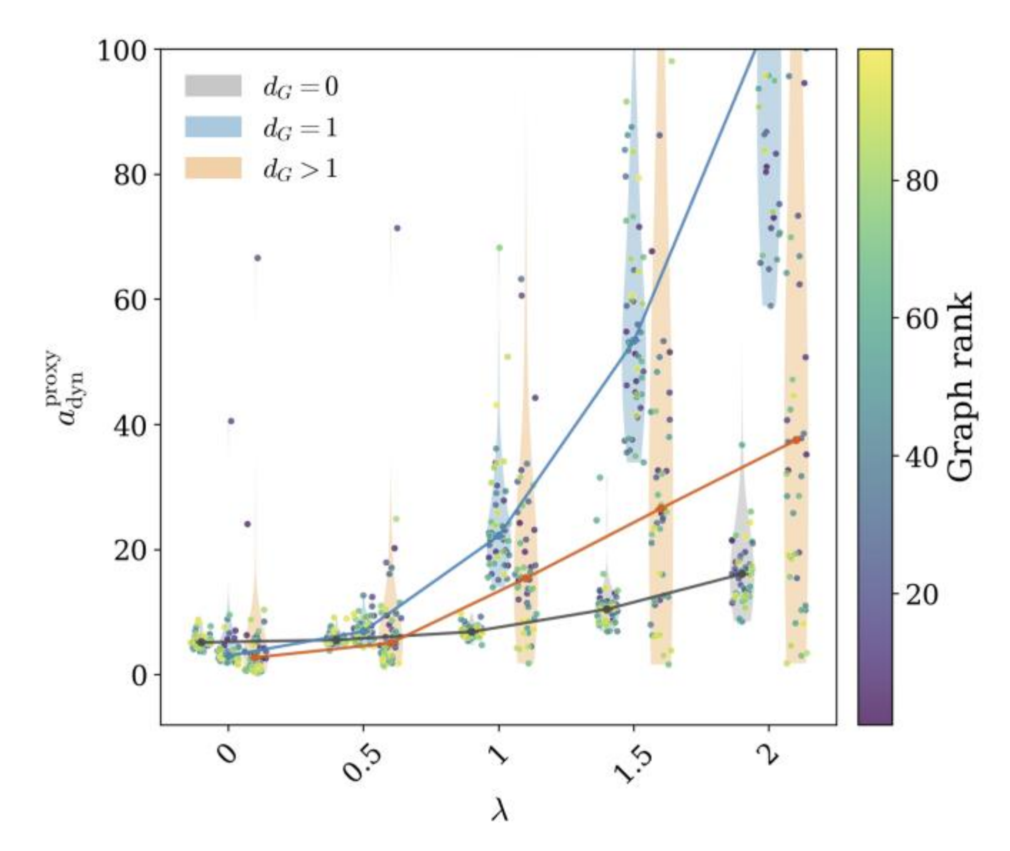


Path-Space likelihood diagnostics

A path-level KL-like distance



A local forward-reverse asymmetry



Larger λ drives a larger departure from λ=0 and broadens the graph-to-graph scatter

Larger λ makes the trajectories more irreversible
Higher-dG trajectories show broader spreads

Potential Theoretical Directions?

Non-Gaussian scatter

Geometric interpretation

Optimal Transport

The action can be rewrite as an energy functional of a curve in Riemannian geometry

$$S = \frac{1}{2}\int dt\, v^T g v \qquad v \equiv \dot{x} - b(x) \qquad \ddot{x}^i + \Gamma^i_{jk}\dot{x}^j \dot{x}^k = F^i(x, \dot{x})$$

Most probable path=geodesic in g=D^{-1} under an effective potential

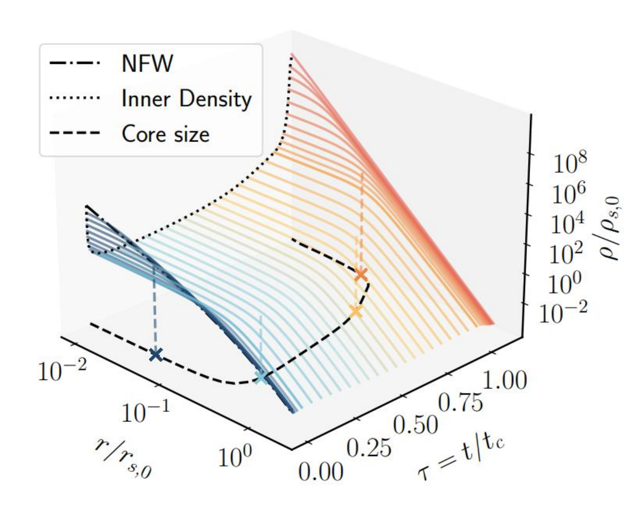
Galaxy evolution path=geodesic in environment-shaped metric+drift forcing

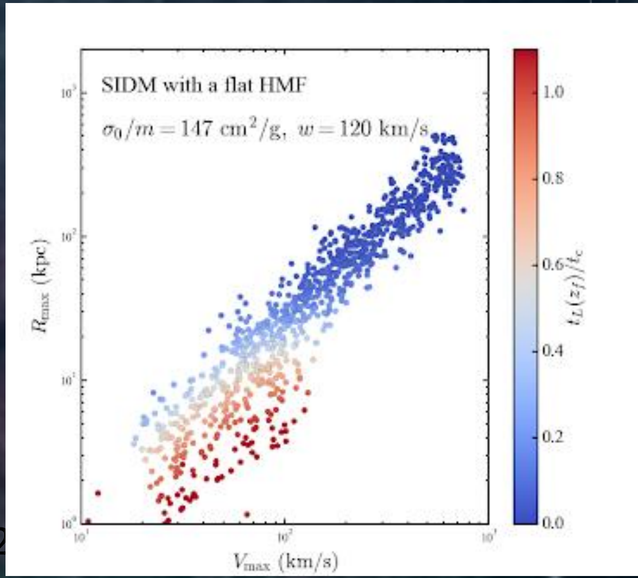
A program for analytic/parametric computation of SIDM effects

--- SIDM effects can be incorporated as controlled deformation of halos

- **2022** (JCAP): An **effective constant cross section** to map differential scatterings into a constant cross section (2205.03392)
- **2022** (PRR): A **graph model** for the clustering of dark matter halos (2206.05578)
- **2023** (JCAP 2024): A **kernel function** to map CDM halos into SIDM halos & An integral approach to incorporate **accretion history** (2305.16176)
- **2024** (PRD): Incorporating the **effect of baryons** (approximate) (2405.03787)
- **2025** (JCAP): A semi-analytic model to generate SIDM subhalos. Ando et al. (**SASHIMI-SIDM**, 2403.16633)
- **2026** (Science Bulletin): Extension: **2-component** SIDM with mass segregation (2506.14898)
- **2026** (arXiv:2603.15128): A **path-integral formalism** on layered halo graphs

+ many 3rd party applications

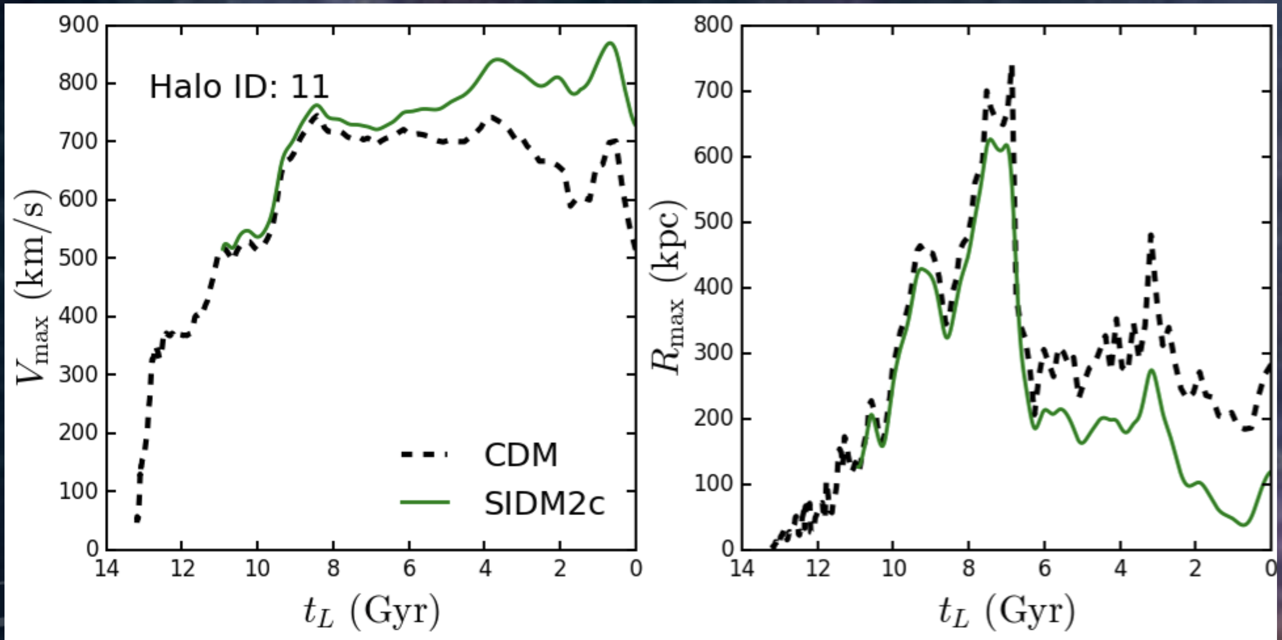


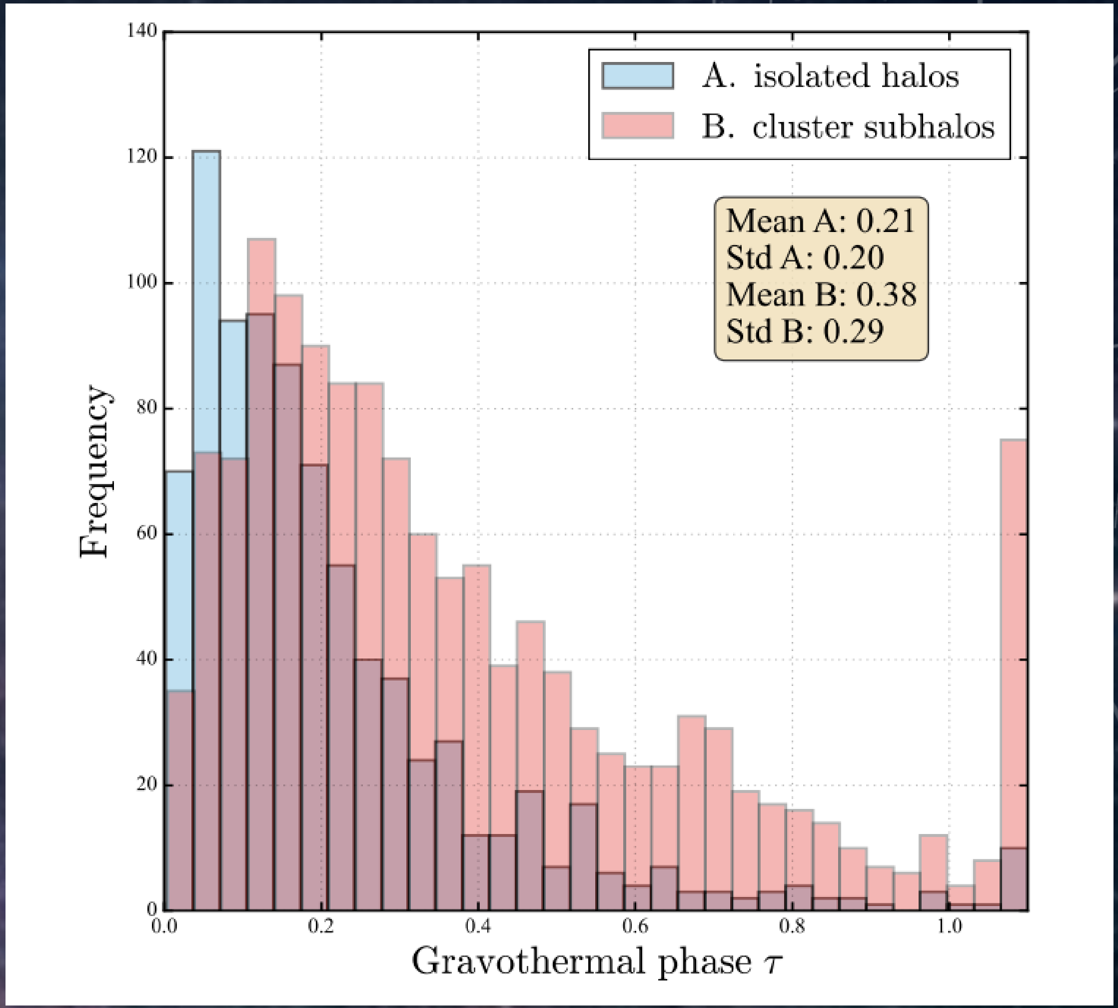


Path-conditioned predictions for 2-comp SIDM

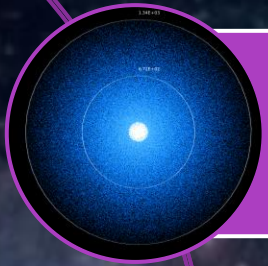
Yang, Fan, Hou, Tsai, Sci.Bull. 71 (2026)

- Isolated dwarfs are mostly core forming (tau<0.2)
- Dwarfs in clusters are mostly core collapsing (tau>0.2)

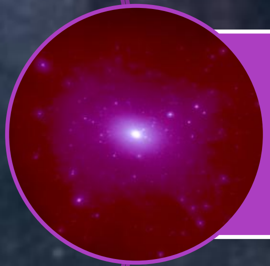




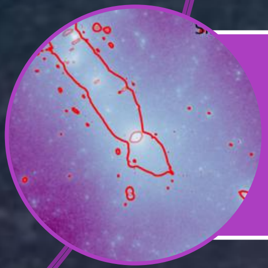
Crucial: dwarfs can have growing cores and densities simultaneously



Growing stellar cores in isolated dwarfs

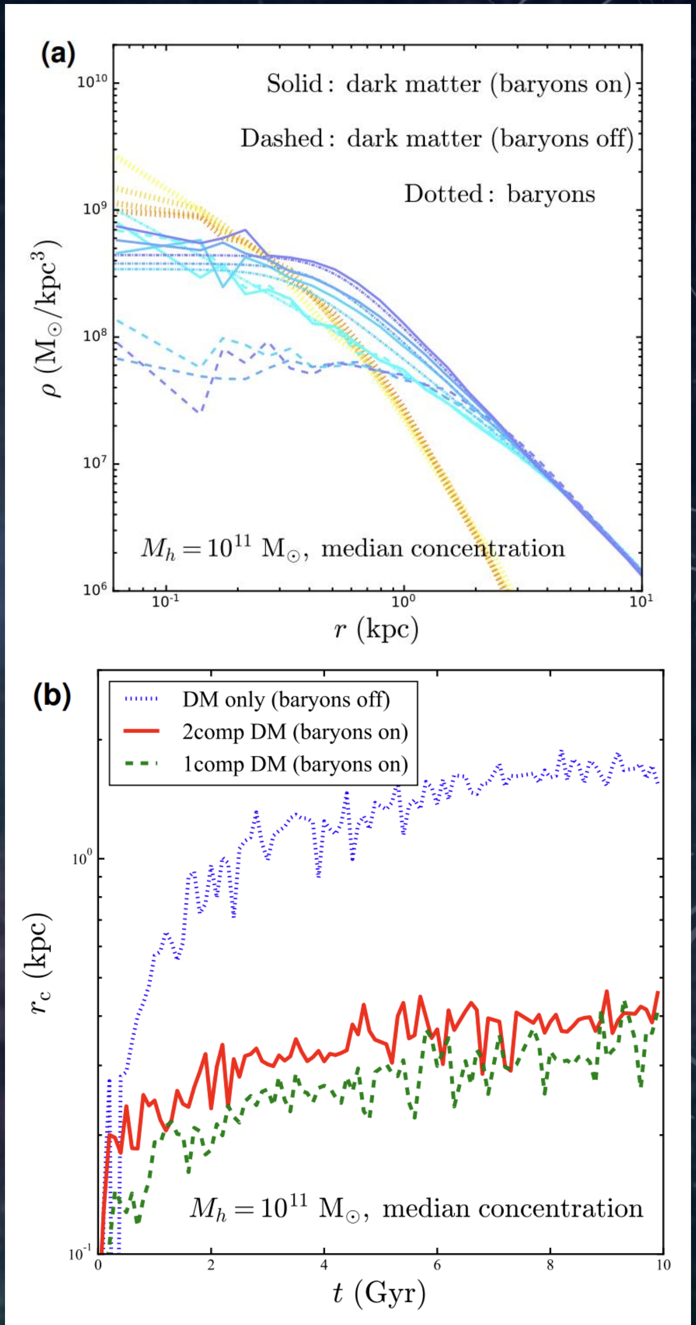


Boosted evolution explain strong lensing perturbers



Enhanced density explain GGSL

Yang, Fan, Hou, Tsai, Sci. Bull. 71 (2026)



SIDM as controlled deformation on graph paths

Analytic kernel → Neural Network Agent

- SIDM deforms (Vmax,Rmax) per time step
- Environmental Paths change endpoint observables

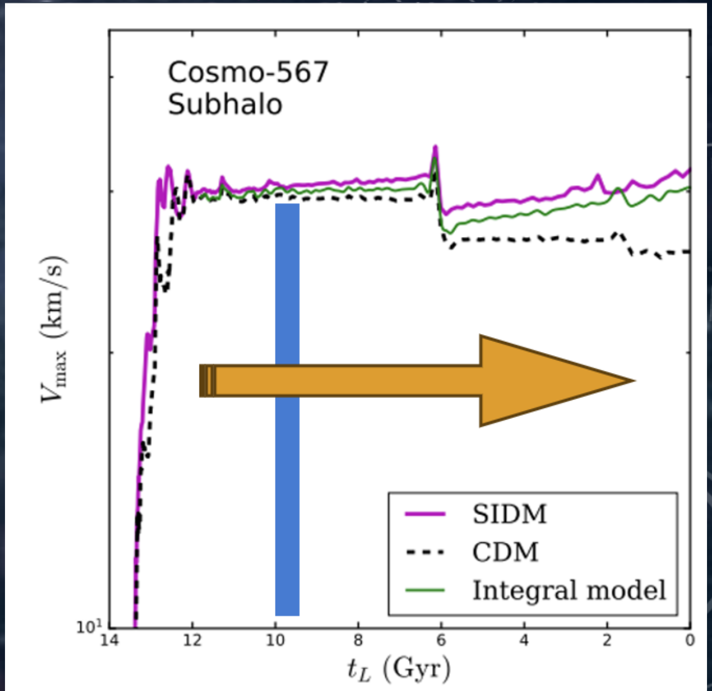
$$\mathbf{y}_k \equiv (V_{\max}, R_{\max})_k$$

$$\Delta \mathbf{y}_k = \Delta \mathbf{y}_k^{\mathrm{tr}} + \Delta\tau_k \mathbf{b}_{\mathrm{SIDM}}(s_k;\theta) + \boldsymbol{\epsilon}_k$$

(Non-linear) Transport

Residual Drift

Residual Scatter



Summary

- Galaxy formation after coarse-graining is a stochastic trajectory problem
- Halo assembly graphs provide the natural structure for those histories
- GPLM is a first step toward turning that idea into a usable statistical framework
- Baryonic response and dark-sector physics can be encoded as controlled deformations of graph-conditioned path ensembles

$$P(x|G) \propto p_{\text{attach}}(x|G)\, e^{-S(x;G)}$$

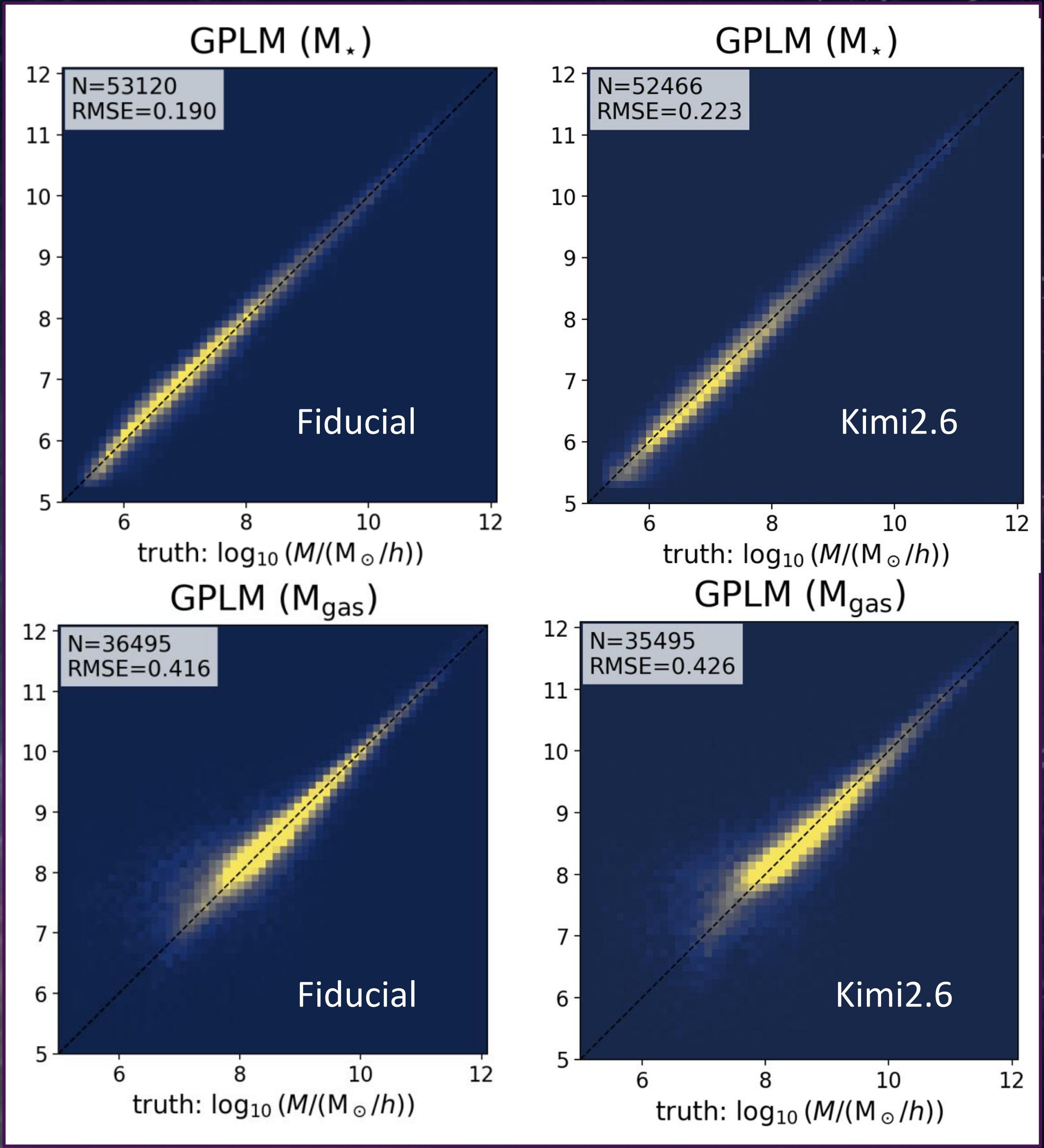
Thanks for your attention!

Back up

Agent-Mediated Reproductivity?

- Write what our program does in detail and an organized Markdown file (~1000 lines)
- Ask Kimi2.6 to construct this program based on the graph data and the markdown files.
- ~ 5 iterations of self-consistency checks

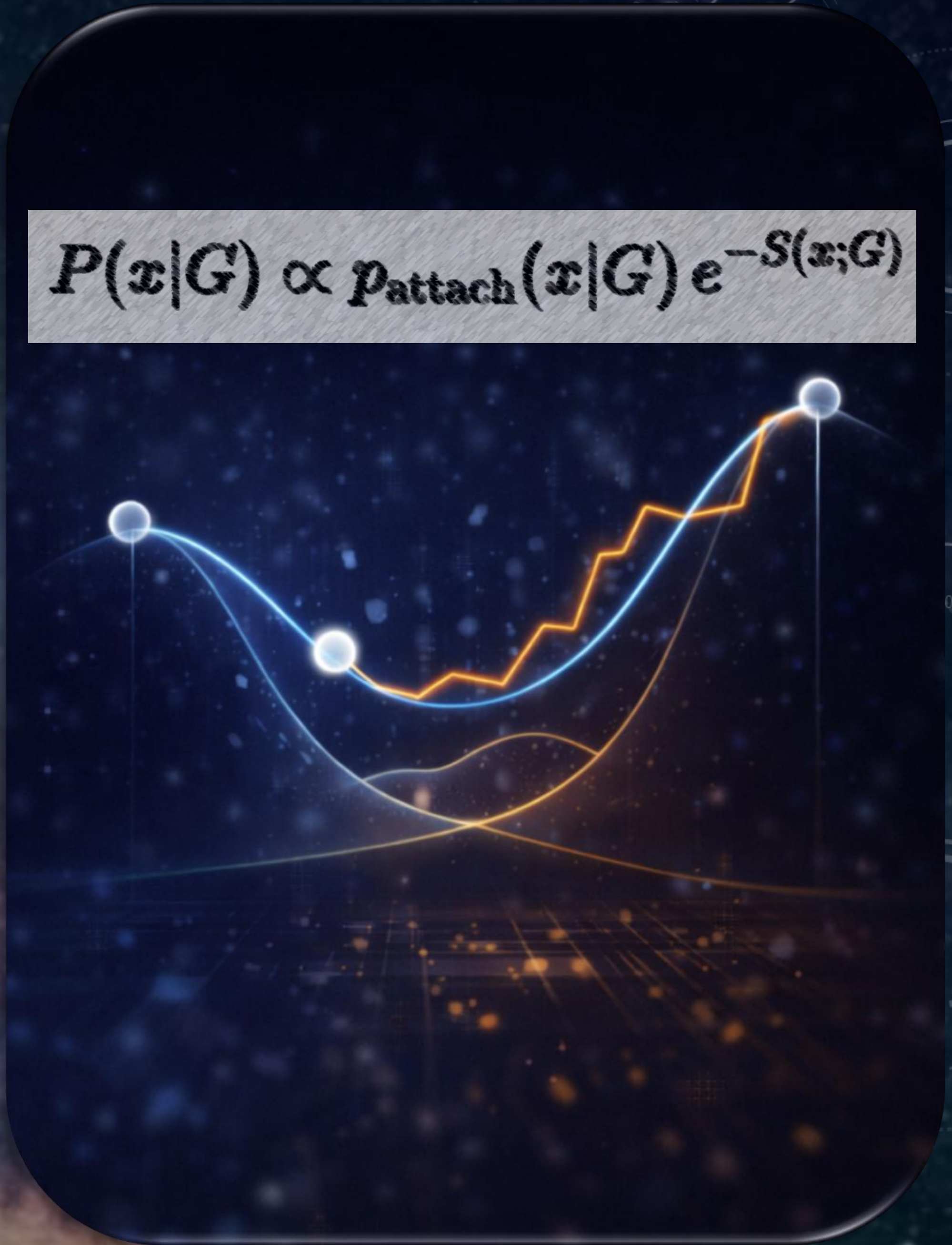
Almost identical performance obtained!



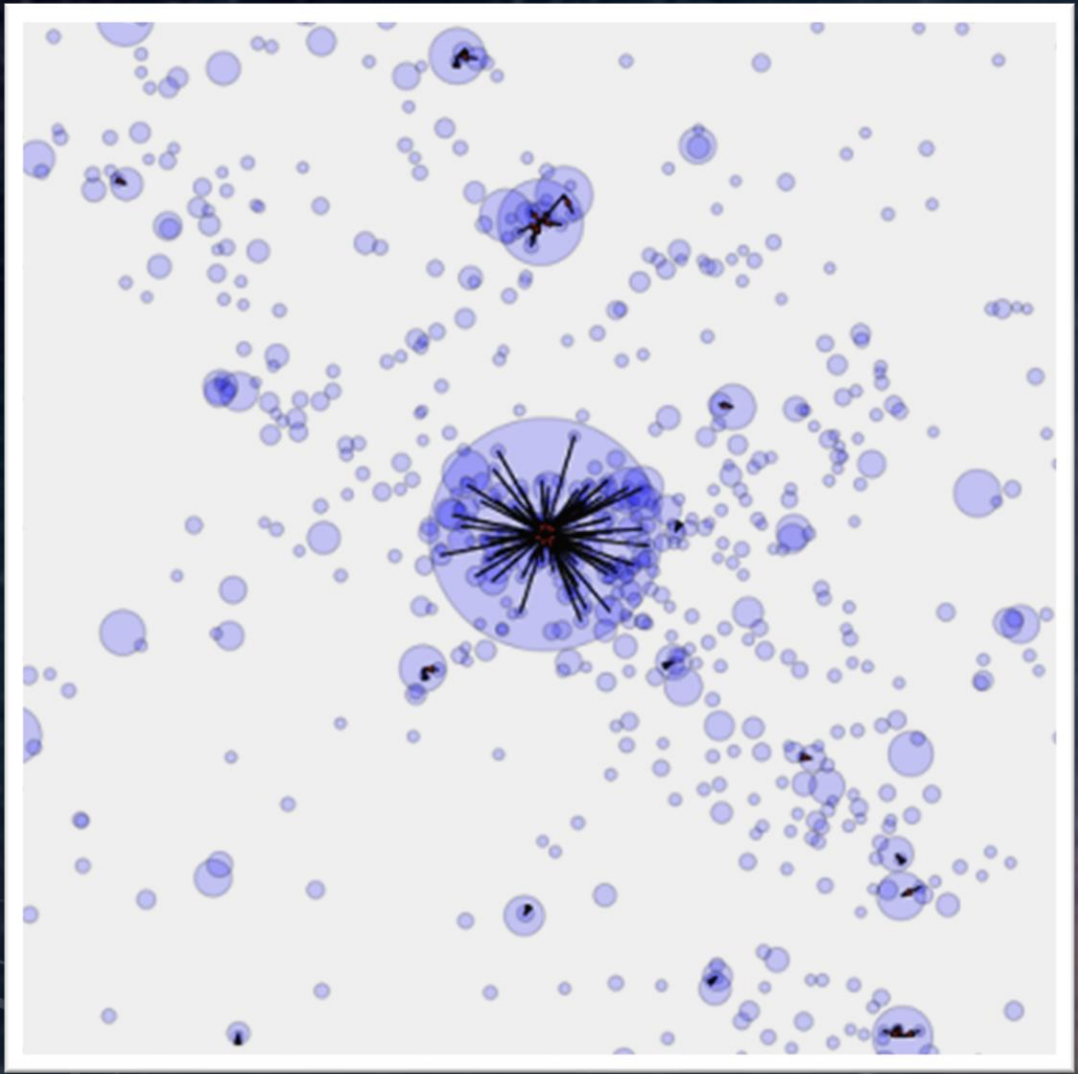
Codes (to be) available at
https://github.com/DanengYang/GraphPathLikelihood

Training data converted from TNG-50-1 simulation data available under the *dataGraphs* folder

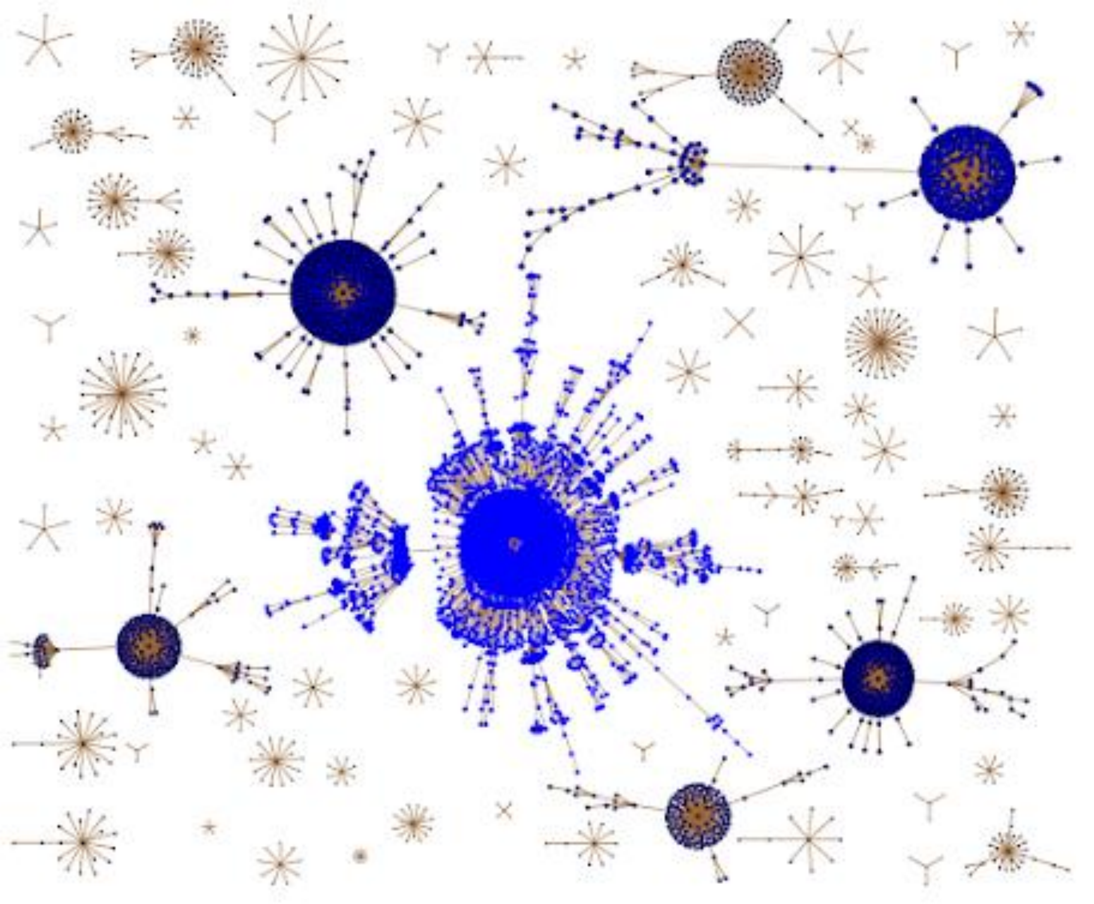
AI-drafted & AI-readable mark-downs facilitates further developments



Cosmological halo formation can be encoded by graphs



Yang & Yu, 2206.05578 [astro-ph.CO]
Phys. Rev. Research 5, 043187 (2023)



Stochastic field theory: noise-induced scatter

Aspect	QFT	SFT on layered graphs
Origin of scatter	Quantum fluctuations	Coarse-grained / environmental noise
Path integral weight	e^{iS}	e^{-S}
Nature	Coherent (phase)	Probabilistic
EOM	Deterministic + fluctuations	Intrinsically stochastic
Correlators	Propagators	Noise-driven correlations
Geometry	Spacetime	Noise structure
Mean of scatter	Quantum variance	Classical ensemble variance

MSRJD FROM NOISE MEASURE (1)

$$\dot{x}(t) = b(x(t)) + \eta(t)$$

$$Z = \int \mathcal{D}\eta \; P[\eta]$$

$$P[\eta] \propto \exp\left[-\frac{1}{2}\int dt\, \eta^T D^{-1} \eta\right]$$

$$1 = \int \mathcal{D}x \; \delta[\dot{x} - b(x) - \eta]$$

$$\delta[\dot{x} - b - \eta] = \int \mathcal{D}\hat{x} \;\; \exp\left[\int dt\, \hat{x}^T (\dot{x} - b - \eta)\right]$$

$$Z = \int \mathcal{D}\eta\, \mathcal{D}x\, \mathcal{D}\hat{x}\; e^{\int dt\, \hat{x}^T (\dot{x} - b - \eta)} \; e^{-\frac{1}{2}\int dt\, \eta^T D^{-1} \eta}$$

MSRJD FROM NOISE MEASURE (2)

$$\int \mathcal{D}\eta \, e^{-\frac{1}{2}\eta^T D^{-1}\eta - \hat{x}^T \eta} \;\propto\; \exp\left[-\frac{1}{2}\int dt \, \hat{x}^T D \hat{x}\right]$$

$$Z = \int \mathcal{D}x \, \mathcal{D}\hat{x} \; e^{-S_{\mathrm{MSRJD}}[x,\hat{x}]}$$

$$S_{\mathrm{MSRJD}} = \int dt \left[\hat{x}^T(\dot{x} - b(x)) - \frac{1}{2}\hat{x}^T D \hat{x}\right]$$

$$A \equiv (\dot{x}_{\mathrm{res}} - b_\theta)$$

$$-\frac{1}{2}\hat{x}^T D \hat{x} + \hat{x}^T A = -\frac{1}{2}(\hat{x} - D^{-1}A)^T D(\hat{x} - D^{-1}A) + \frac{1}{2}A^T D^{-1} A$$

$$\int \mathcal{D}\hat{x} \; \exp\left[-\frac{1}{2}(\hat{x} - \mu)^T D(\hat{x} - \mu)\right] \propto (\det D)^{-1/2}$$

$$Z \propto \int \mathcal{D}x \; \exp\left[-\frac{1}{2}\int dt \, A^T D^{-1} A\right] \times \exp\left[-\frac{1}{2}\int dt \, \log\det D_\theta\right]$$

MSRJD in GPLM

$$\dot{x}_{\text{res}}(t) = b_\theta(x^{\text{tr}}(t), G(t)) + \xi(t), \qquad \langle \xi(t)\xi(t')^T \rangle = D_\theta\, \delta(t - t').$$

$$S_{\text{MSRJD}}[x, \hat{x}; G] = \int dt \left[\hat{x}^T \left(\dot{x}_{\text{res}} - b_\theta(x^{\text{tr}}, G) \right) - \frac{1}{2} \hat{x}^T D_\theta(x^{\text{tr}}, G) \hat{x} \right]$$

$$S_{\text{OM}}[x; G] = \frac{1}{2} \int dt\, (\dot{x}_{\text{res}} - b_\theta)^T D_\theta^{-1} (\dot{x}_{\text{res}} - b_\theta) + \frac{1}{2} \int dt\, \log\det D_\theta(x^{\text{tr}}, G)$$

Discretization Matters: Same SDE, Different Path Measures

$$P[x] = \int \mathcal{D}\eta\, P[\eta]\, \delta[\dot{x} - b(x) - \eta]$$

$$P[x] = P[\eta = \dot{x} - b(x)] \times \det\left(\frac{\delta\eta}{\delta x}\right)$$

The definition of δ[ẋ−b(x)−η] is ambiguous without specifying discretization

$$x_{t+\Delta t} - x_t = b(x_\alpha)\,\Delta t + \sqrt{D(x_\alpha)}\,\Delta W$$

$$x_\alpha = (1-\alpha)x_t + \alpha x_{t+\Delta t}$$

- Itô: no Jacobian term
- Stratonovich: $+(1/2)\nabla\cdot b$
- General α: continuous family

$$P[x] \propto \exp\left[-\tfrac{1}{2}(\dot{x}-b)^T D^{-1}(\dot{x}-b) - \tfrac{1}{2}\log\det D - \alpha\nabla\cdot b\right]$$

The most probable path

The action can be rewrite as an energy functional of a curve in Riemannian geometry

$$S = \frac{1}{2}\int dt\, v^T g v$$

$$v \equiv \dot{x} - b(x)$$

$$\ddot{x}^i + \Gamma^i_{jk}\dot{x}^j\dot{x}^k = F^i(x, \dot{x})$$

Most probable path=geodesic in g=D^{-1} under an effective potential

Galaxy evolution path=geodesic in environment-shaped metric+drift forcing

The most probable path equation is a diffusion equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$$

$$\ddot{x} = (\nabla b)\dot{x} - D(\nabla b)^T D^{-1}(\dot{x} - b)$$

$$b(x) = -D\nabla\Phi(x)$$

Assuming constant D:

$$\ddot{x} = D^2\nabla\left[\tfrac{1}{2}|\nabla\Phi|^2\right]$$

Hamiltonian structure

$$\dot{x} = b(x, G, t) + \sqrt{D(x, G, t)}\,\eta$$

$$p \equiv D^{-1}(\dot{x} - b)$$

$$\dot{x} = b + Dp, \quad \dot{p} = -(\nabla_x b)^T p$$

p encodes deviation from deterministic drift

Non-Gaussianities

$$\dot{x}_{\mathrm{res},i}(t) = b_i\left(x^{\mathrm{tr}}(t), G(t)\right) + \xi_i(t),$$

$$\langle \xi_i(t)\xi_j(t')\rangle = D_{ij}\left(x^{\mathrm{tr}}(t), G(t)\right)\delta(t-t').$$

$$S[x,\hat{x};G] = \int dt\,\left[\hat{x}_i\left(\dot{x}_{\mathrm{res},i} - b_i(x^{\mathrm{tr}},G)\right) - \frac{1}{2}\hat{x}_i D_{ij}(x^{\mathrm{tr}},G)\hat{x}_j\right] + S_{\mathrm{NG}},$$

$$D_{ij}(x,G) = L_{ik}(G)L_{jk}(G) + \mathcal{O}(x)$$

$$S_{\mathrm{NG}} = \int dt\,\left[\frac{1}{3!}C^{(3)}_{ijk}(G)\,\hat{x}_i\hat{x}_j\hat{x}_k + \frac{1}{4!}C^{(4)}_{ijkl}(G)\,\hat{x}_i\hat{x}_j\hat{x}_k\hat{x}_l + \cdots\right]$$

$$b_i(x,G) = A_{ij}(G)x_j + \frac{1}{2}B_{ijk}(G)x_jx_k + \frac{1}{3!}C^{(x)}_{ijkl}(G)x_jx_kx_l + \cdots$$

Non-Gaussian stochastic dynamics = interacting field theory in (x,p)

Connects to optimal transport / Wasserstein geometry

ρ(x,t|G): the probability density (or empirical distribution) of galaxy states at layer t, conditioned on the graph/environment G

The same structure can be understood with optimal transport. In optimal transport (Benamou–Brenier):

$$\inf_{\rho,v} \int dt \int dx\, \rho(x,t)\, v^T M^{-1} v$$

Subject to (without drift):

$$\partial_t \rho + \nabla \cdot (\rho v) = 0$$

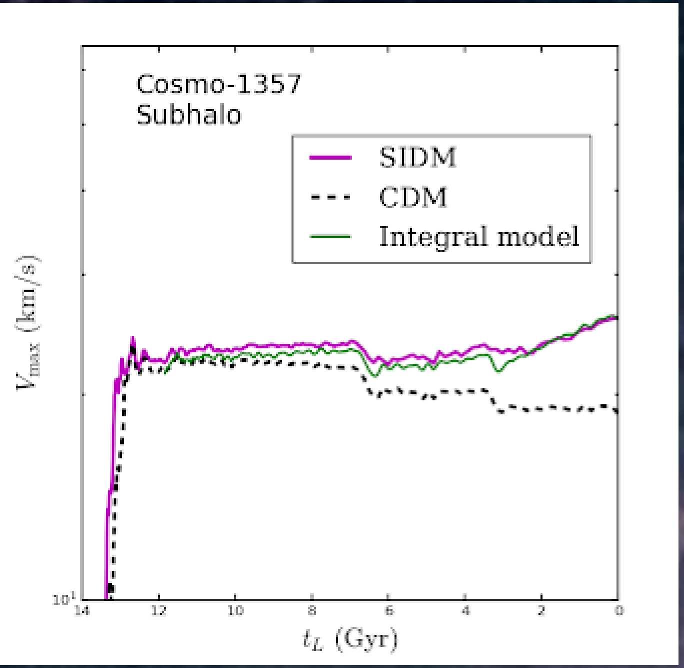
With drift:

$$\partial_t \rho = -\nabla \cdot (\rho b) - \nabla \cdot (\rho v)$$

Stochastic dynamics = gradient flow of free energy in Wasserstein metric

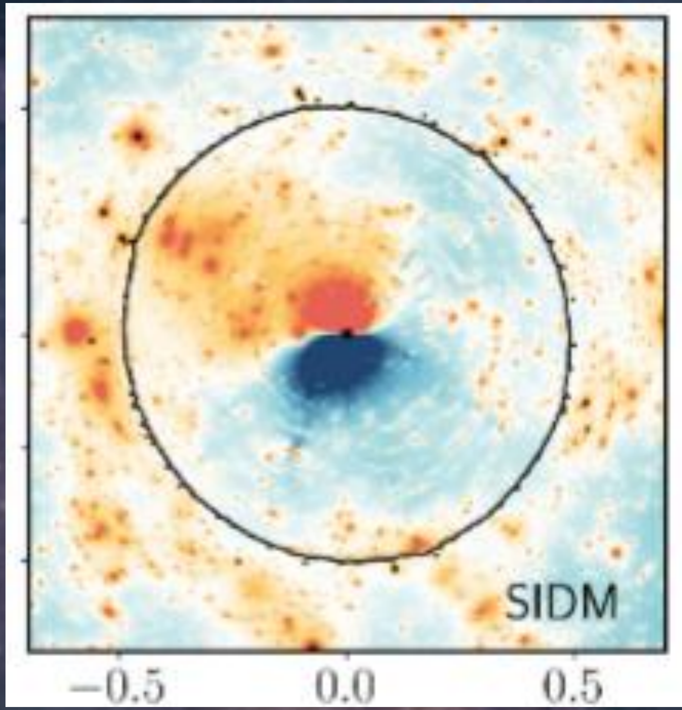
GPLM = graph-conditioned optimal transport

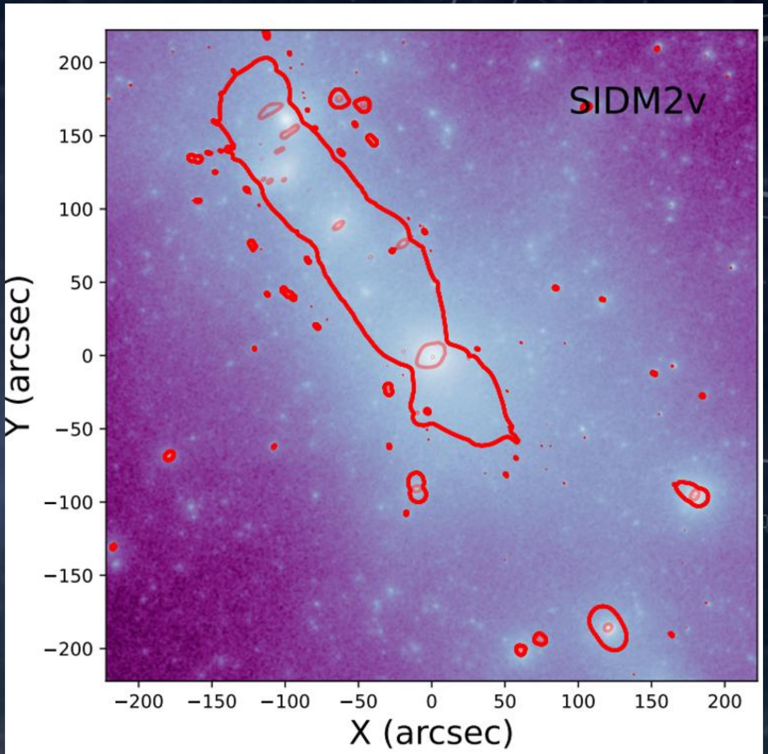
What we would like to achieve?



Predict the **evolution of a halo under SIDM** given its evolution in CDM based on a few **analytic** equations

Obtain the **lensing potential & deflection angle**, **analytically** by (a large number of) SIDM halos





& for **generic models with varying inner density slopes**, e.g., SIDM+mass segregation

Astrophysical probes of SIDM

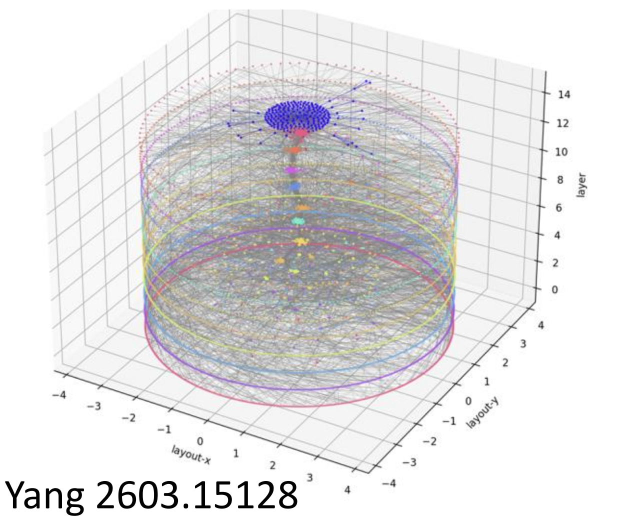
Traditional discussion focuses of the “small-scale crisis”

- Core–cusp problem
- Too big to fail
- Missing satellite galaxies
- Diversity problem

New small scale crisis	LCDM	SIDM
Dwarf clustering	✗	✓
Strong lensing perturber	✗	✓
GGSL	✗	✗
Diverse rotation curves	🤞	✓
Little red dots	✗	✓
BHB mergers	✗ (?)	✓
Stellar stream perturbers	(?)	✓

Our recent progress

A Graph Path Likelihood Model



Yang 2603.15128

Elastic scatterings
- Gravothermal evolution

Topological and statistical correlations
- Correlation functions and Graphs

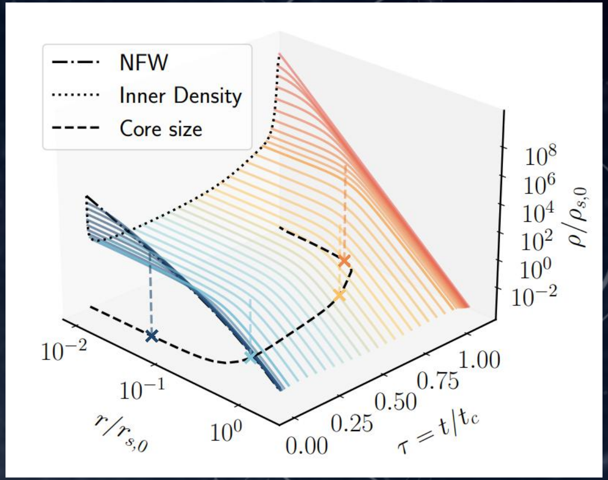
DM in a Path Action

Dissipative scatterings
- Boosted gravothermal evolution

Multi-components
- Mass segregation

Self-boosting scatterings
- Gravitational binding

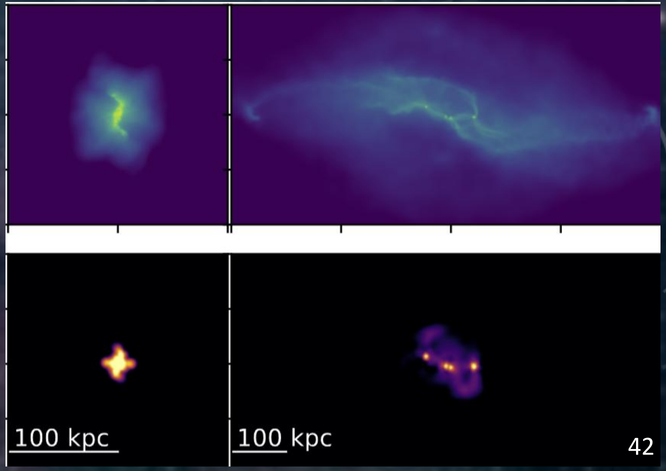
A parametric model for SIDM halos



https://github.com/DanengYang/parametricSIDM
Yang+2024 JCAP 02 (2024) 032
Yang 2024 PRD 110 (2024) 10, 103044
Yang+2025 PDU 47 (**2025**) 101807
Hou & Yang + JCAP08 (**2025**) 048

Yang, Fan, Hou, and Tsai, **Science Bulletin 2026** & PRD **2025** on mass segregation

Collisional Formation of Baryon Dominated Dwarf Galaxies



Wang & Yang et al. **ApJL 2026**