Massive Amplitudes and Theories from Consistency Conditions

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1. Introduction

Traditional View of QFT

- Lagrangian --> Quantization&LSZ -> S-matrix
- Feynman rules --> amplitudes

Problem:
The concept of field is often redundant, especially for gauge theory

Modern View

- 1. Starting from S-matrix/Amplitudes Directly
- 2. Lorentz symmetry and unitarity
- 3. Fix and compute amplitudes.
- 4. Extract the underlying theory

Massless: Achievements

- "Classic" (Weinberg): S. Weinberg, Phys.Rev. 135 (1964) B1049-B1056

  - On-shell gauge symmetry: $k^\mu \mathcal{M}_\mu = 0$
  - Charge conservation(equivalent principle for spin-2)
  - No non-trivial interactions for spin > 2

- Modern(spinor formalism&on-shell):

  - Lorentz symm -> 3-point amplitudes
  - *Unitairty -- > Consistent factorization* --> 4-point

Vector boson Interactions:Yang-Mills theory

Key: Lorentz sym&Unitarity&locality encoded in amplitudes

Modern View: Massive

After spinor formalism

- Classify all 3–point amplitudes match with massless limits(UV/IR)

3–point

E. Conde and A. Marzolla, 1601.08113
S. Y. Choi and J. H. Jeong, 2111.08236
N. Arkani–Hamed, T.–C. Huang, and Y.–t. Huang, 1709.04891

4–point

B. Bachu and etc. 2305.02502&1912.04334
D. Liu and Z. Yin, 2204.13119
H.–Y. Lai, D. Liu, and J. Terning, 2312.11621

- Construct 4–point amplitudes
  <u>apply perturbative unitarity</u>
  obtain gauge theories with Higgs mechanism

- Generally corresponds to the method with Lagrangian

J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Phys. Rev. Lett. 30, 1268 (1973)
J. M. Cornwall, D. N. Levin, and G. Tiktopoulos,Phys. Rev. D 10, 1145 (1974),
C.H. Llewellyn Smith, Phys.Lett.B 46 (1973) 233–236

Motivation for Massive Case

- Status quo: the main method is still perturbative unitarity, the same as in 50 years ago
  - ➢ 3-point amplitudes are relative complete
  - ➢ 4-point requires computation off-shell(energy increasing)

- Problem: the core of Higgs mechanism not taken into account:

  Goldstone eaten by gauge boson(at least for 4-point)

  - ➢ Why additional degree of freedom from massless?
  - ➢ Precise cancellation indicates symmetry, what symmetry?

Our Main Goal: integrate those aspects

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Consistent Conditions for Massive Amplitudes

- Condition-1: On-shell Gauge Symmetry(OGS)

$$k^\mu \mathcal{M}_\mu = \pm i m_V \mathcal{M}(\varphi)$$

Goldstone Boson

$\frac{E}{m_V} \to \infty$, reduce to Goldstone Equivalence Theorem

- Condition-2: Strong Massive-Massless Continuation

$$\lim_{m_i \to 0} \mathcal{M}(\{p_i, \epsilon_i, m_i\}) = \text{finite}$$

Selects Elementary particles

OGS from Lorentz Symmetry

- Discontinuity: degrees of freedom for vector boson

Massive: 3 d.o.f
Massless: 2 d.o.f
$\Longrightarrow$
Higgs Mechanism: Goldstone "eaten" by Gauge Field

- Continuity: Lorent symmetry(little group)

$m \neq 0$ ***SO(3)***

$m = 0$ ***ISO(2)***

$$[J_i, J_i] = i\epsilon_{ijk}J_k \xrightarrow[J_2 \to S_2 = \epsilon J_1]{J_1 \to S_1 = \epsilon J_2} \begin{array}{l} [S_1, S_2] = i\epsilon^2 J_3 \\ [S_2, J_3] = iS_1, \\ [J_3, S_1] = iS_2 \end{array}$$

$\epsilon \to 0$

Group Constraction

$$[S_1, S_2] = 0$$
$$[S_2, J_3] = iS_1$$
$$[J_3, S_2] = iS_1$$

OGS from Lorentz Symmetry

- Vector Rep. of Lorentz Group: 4 d.o.f.

  1 d.o.f is time-like, negative norm

- Introduce scalar φ

  Condition-1(OSG): $k_\mu \epsilon_s^\mu = i m_V \epsilon^\varphi$

Total degrees of freedom: 4+1-2=3

- Identify $|V_0\rangle + |\varphi\rangle$ as the state

$$(\langle V_0| + \langle\varphi|)(|V_0\rangle + |\varphi\rangle) = (-1)\frac{k^\mu}{m_V} \cdot \frac{k_\mu}{m_V} + 1 = -1 + 1 = 0,$$

Eliminate negative norm

- Condition-2: Pol. vec. invariant under

$$\epsilon^\mu \to \epsilon^\mu + \xi k^\mu / m_V, \quad \epsilon^\varphi \to \epsilon^\varphi \pm \xi i.$$

Consistent Condition-2: Elementary Particle

- It's crucial to differentiate Higgs mechanism, which particles are elementary, from Composite Higgs scenario.

- Elementary particle:couplings remain constant at high energy(tree)
- Composite particle: couplings blow up at high energy(in EFT)

- **Strong massive-massless continuation: the amplitude remains finite as one of the particle goes to massless.**

$$\lim_{m_i \to 0} \mathcal{M}(\{p_i, \epsilon_i, m_i\}) = \text{finite}$$

This selects amplitudes of elementary particles

5-component formalism for amplitudes

- Combine gauge and Goldstone components, define 5-components field/polarization vector:

$$V^M = (V^\mu, \varphi) \qquad k^M:\ k^M = (k^\mu, \pm i m_V), \qquad k^M k^*_M = 0$$

- Massive vector boson amplitudes written as:

$$\mathcal{M}(p_1, p_2, ...) = \epsilon_{s_1}^{M_1}(p_1)\epsilon_{s_2}^{M_2}(p_2)...\mathcal{M}_{M_1 M_2 ...}(p_1, p_2, ...)$$

- **On-shell gauge symmetry(OGS)**

gauge condition

$$k^M \mathcal{M}_M = 0 \qquad\qquad r\cdot\epsilon(k,r) = 0 \quad r^\mu = (1, -\frac{\vec{k}}{|\vec{k}|})$$

Methods of Constructing Amplitudes

- Construct massive amplitudes from massless limits

$$\mathcal{M}_s(V) = \epsilon^M_{\pm,L}\mathcal{M}_M(p,m_V) \xrightarrow{m_V\to 0} \begin{cases} \epsilon^\mu_s \mathcal{M}_\mu(p) & s=\pm \\ \mathcal{M}^\phi(p) & s=L \end{cases} \Rightarrow \mathcal{M}^V_s(p,m_V) = a\ \epsilon^\mu_s \mathcal{M}^V_\mu(p) + b\ \mathcal{M}^\phi(p)$$

- Fix parameters with on-shell gauge symmetry& strong massive-massless continuation

No complexification of momenta

Off-shell continuation

- 3-point amplitudes -> 4-point amplitudes

$$\mathcal{M}_4 \xrightarrow{k_a^2\to m_a^2} \sum_{s,a} \mathcal{M}^s_{3,a}\frac{i}{k_a^2-m_a^2}\mathcal{M}^{-s}_{3,a} \qquad \sum_{s=\pm,0}\epsilon^M_s\epsilon^{*N}_s = -g^{MN} + \frac{k_i^M n^N + n^M k_i^{*N}}{n\cdot k_i} \xrightarrow{\text{O.G.S}} -g^{MN}$$

- Finally $\displaystyle\sum_{s,a} \mathcal{M}^s_{3,a}\frac{i}{k_a^2-m_a^2}\mathcal{M}^{-s}_{3,a} \xrightarrow{\text{off-shell}} \mathcal{M}_4 = \sum_a \mathcal{M}^M_{3,a}\frac{-ig_{MN}}{k_a^2-m_a^2}\mathcal{M}^N_{3,a}$

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Construct 3–point: massless

- Masslss gauge symmetry $k^\mu \mathcal{M}_\mu = 0$
- Massless SSV

$$\mathcal{M}_0(S_1S_2V_3) = (a_1k_1 + a_2k_2)\cdot\epsilon_3$$

OSG –> $\mathcal{M}_0(S_1S_2V_3) = (k_1 - k_2)\cdot\epsilon_3$

Boson sym –>

$$\mathcal{M}_0(S_1^aS_2^bV_3) = T_{ab}\mathcal{M}_0(S_1S_2V_3)$$

$$T_{ab} = -T_{ba}$$

➢ Massless VVV

$$f^{abc}\left[(\epsilon_1\cdot\epsilon_2)\ (k_1 - k_2)\cdot\epsilon_3 + \text{cyclic}\right]$$

f^{abc} is a totally anti-symmetric factor,

➢ “Massless” SVV

$$\mathcal{M}_0(S_1V_2V_3) = m\epsilon_2\cdot\epsilon_3 \quad \dim(m) = 1$$

Doesn’t satisfy O.S.G

Construct Massive SSV

- Amplitude of SSV

$$\mathcal{M}_0(SSV) \qquad \mathcal{M}_0(SS\varphi)$$

$$\mathcal{M}(S_1S_2V_3) = (k_1 - k_2)\cdot\epsilon_3 - \lambda_{SS\phi}\epsilon_3^4$$

- O.S.G. $\mathcal{M}(S_1S_2V_3)|_{\epsilon_3^M \to k_3^M} = 0 \quad \Rightarrow \quad \lambda_{S_1S_2\phi} = -i\frac{(m_1^2 - m_2^2)}{m_V}$

- Massive–massless continuation

$$\lim_{m_i \to 0} \lambda_{SS\phi}(m_1, m_2, m_3) \neq \infty$$

$$m_1^2 = c^{(0)} + c_1^{(1)} m_V + \mathcal{O}(m_V^2) \qquad m_2^2 = c^{(0)} + c_2^{(1)} m_V + \mathcal{O}(m_V^2) \qquad c^{(0)} = 0 \text{ if } \lambda_{SS\phi} \to 0, \text{ as } m_V \to 0$$

Particle masses (partially) have the same origin

- Special case: $m_1 = m_2 \to \lambda_{SS\varphi} = 0$ No dynamic Goldstone(Stueckelberg theory)

Construct Massive SVV

- General amplitude of SVV

$$\mathcal{M}(S_3V_1V_2) = m(\epsilon_1 \cdot \epsilon_2) - a_1(k_3 - k_1)\cdot \epsilon_2\ \epsilon_1^4 - a_2(k_3 - k_2)\cdot \epsilon_1\ \epsilon_2^4 + a_3\epsilon_1^4\epsilon_2^4$$

- O.S.G gives

$$a_1 = -\frac{im}{2m_1} \qquad a_2 = -\frac{im}{2m_2} \qquad a_3 = \frac{mm_3^2}{2m_1m_2}$$

- Massive–massless continuation

$$m = c_{1m}m_1 + \mathcal{O}(m_1^2) \qquad m_2 = c_{12}m_1 + \mathcal{O}(m_1^2) \qquad m_3 = c_3^{(13)}m_1 + \mathcal{O}(m_1^2)$$

Construct Massive VVV

- $$\mathcal{M}(V_1V_2V_3) = [(\epsilon_1\cdot\epsilon_2 + a_{12}\epsilon_1^4\epsilon_2^4)\ (k_1-k_2)\cdot\epsilon_3 + \text{cyclic}] + [b_{12}m_3\epsilon_1\cdot\epsilon_2\epsilon_3^4 + \text{cyclic}] + c_{123}\epsilon_1^4\epsilon_2^4\epsilon_3^4$$

$$\mathcal{M}(V_1V_2V_3) \longrightarrow \begin{cases} \mathcal{M}_0(V_1^TV_2^TV_3^T) \\ \mathcal{M}_0(V_1^0V_2^TV_3^T) + \text{cyclic} \\ \mathcal{M}_0(V_1^0V_2^0V_3^T) + \text{cyclic} \\ \mathcal{M}_0(V_1^0V_2^0V_3^0) \end{cases}$$

- O.S.G. gives

1. If $m_1 \neq 0, m_2 \neq 0, m_3 \neq 0$:

$$b_{12} = -i\frac{m_1^2-m_2^2}{m_3^2} \qquad b_{23} = -i\frac{m_2^2-m_3^2}{m_1^2} \qquad b_{31} = -i\frac{m_3^2-m_1^2}{m_2^2}$$

$$a_{12} = -\frac{m_1^2+m_2^2-m_3^2}{2m_1m_2} \qquad a_{23} = -\frac{m_2^2+m_3^2-m_1^2}{2m_2m_3} \qquad a_{31} = -\frac{m_3^2+m_1^2-m_2^2}{2m_3m_1}$$

$$c_{123} = 0$$

2. If $m_1 = 0, m_2 \neq 0, m_3 \neq 0$:

$$b_{12} = i \qquad b_{23} = 0 \qquad b_{31} = -i$$

$$a_{12} = 0 \qquad a_{23} = -1 \qquad a_{31} = 0$$

$$c_{123} = 0$$

$m_2 = m_3$ New

3. If $m_1 = 0, m_2 = 0, m_3 \neq 0$, then there is no non-trivial solution.

Sepecial cases: $m_1 > m_3$, $\cos\theta_W \equiv \frac{m_3}{m_1}$

$m_1 \neq 0$ WWZ

$m_1 = 0$ WWA

Construct 4-point Amplitude

- Classify amplitudes by number of scalars and vector bosons

- $n_V \leq 2$
  - $n_V = 1, n_S \geq 2$: SSV
  - $n_V \leq 2$, $n_S = 1$: VVS
  - $n_V \leq 2$, $n_S \geq 2$: VVS, SSV
- $n_V \geq 3$
  - $n_V \geq 3, n_S = 0$: VVV
  - $n_V \geq 3, n_S = 1$: VVV, VVS
  - $n_V \geq 3, n_S \geq 2$: VVV, VVS and SSV

4-point

VVVV, VVSS;

VVVV,VVSS;

$\boldsymbol{n_V} \leq 2$: VVVV from VVS

- Set $\boldsymbol{n_V} = 1$, $\boldsymbol{n_S} = 1$

$$\mathcal{M}_{\text{tot}} = \mathcal{M}_s + \mathcal{M}_t + \mathcal{M}_u + \mathcal{M}_c$$

$$\mathcal{M}_s(V_1V_2V_3V_4) \xrightarrow{p_{12}^2 \to m_S^2} \mathcal{M}(V_1V_2S_{12})\frac{1}{p_{12}^2 - m_S^2}\mathcal{M}(-S_{12}V_3V_4)$$

$$\mathcal{M}_t(V_1V_2V_3V_4) \xrightarrow{p_{13}^2 \to m_V^2} \sum_{s_{13}=\pm,0} \mathcal{M}(V_1V_3S_{13})\frac{1}{p_{13}^2 - m_V^2}\mathcal{M}(-S_{13}V_2V_4)$$

$$\mathcal{M}_u(V_1V_2V_3V_4) \xrightarrow{p_{14}^2 \to m_V^2} \sum_{s_{14}=\pm,0} \mathcal{M}(V_1V_3S_{14})\frac{1}{p_{14}^2 - m_V^2}\mathcal{M}(-V_{14}V_2S_3)$$

$$\mathcal{M}_s(V_1V_2V_3V_4) = \mathcal{M}(V_1V_2S_{12})\frac{1}{p_{12}^2 - m_S^2}\mathcal{M}(-S_{12}V_3V_4)$$

$$\mathcal{M}_t(V_1V_2V_3V_4) = M(V_1V_3S_{13})\frac{1}{p_{13}^2 - m_V^2}\mathcal{M}(-S_{13}V_2V_4)$$

$$\mathcal{M}_u(V_1V_2S_3S_4) = \mathcal{M}(V_1V_3S_{14})\frac{1}{p_{14}^2 - m_V^2}\mathcal{M}(-S_{14}V_2V_3)$$

- OSG

$$a_1 = a_2 = a_3 = 0$$

$$b_{11} = b_{12} = b_{21} = b_{22} = b_{31} = b_{31} = g_{VVS}^2$$

$$c \equiv \lambda_{\phi^4} = \frac{3}{4} g_{VVS}^2 \frac{m_S^2}{m_V^2}$$

$$\begin{aligned}\mathcal{M}_c = {} & a_1\epsilon_1 \cdot \epsilon_2\ \epsilon_3 \cdot \epsilon_4 + a_2\epsilon_1 \cdot \epsilon_3\ \epsilon_2 \cdot \epsilon_4 + a_3\epsilon_1 \cdot \epsilon_4\ \epsilon_3 \cdot \epsilon_2 \\ & + b_{11}\epsilon_1 \cdot \epsilon_2\ \epsilon_3^4 \cdot \epsilon_4^4 + b_{12}\epsilon_1^4 \cdot \epsilon_2^4\ \epsilon_3 \cdot \epsilon_4 \\ & + b_{21}\epsilon_1 \cdot \epsilon_3\ \epsilon_2^4 \cdot \epsilon_4^4 + b_{22}\epsilon_1^4 \cdot \epsilon_3^4\ \epsilon_2 \cdot \epsilon_4 \\ & + b_{31}\epsilon_1 \cdot \epsilon_4\ \epsilon_2^4 \cdot \epsilon_3^4 + b_{32}\epsilon_1^4 \cdot \epsilon_4^4\ \epsilon_2 \cdot \epsilon_3 \\ & + c\ \epsilon_1^4\epsilon_2^4\epsilon_3^4\epsilon_4^4\end{aligned}$$

all couplings fixed by masses (up to an overall coupling)

$n_V \leq 2$: VVSS from VVS and SSS

- $$\mathcal{M}_{\text{tot}} = \mathcal{M}_s + \mathcal{M}_t + \mathcal{M}_u + \mathcal{M}_c$$

$$\mathcal{M}_s(S_1^a S_2^a V_3 V_4) = \sum_{i=a,b} M(S_1^a S_2^a S_{12}^i)\frac{1}{p_{12}^2 - m_i^2}\mathcal{M}(S_{12}^i V_3 V_4)$$

$$\mathcal{M}_c(S_1^a S_2^a V_3 V_4) = g_{S^a S^a VV}\ \epsilon_3 \cdot \epsilon_4 + \lambda_{SS\varphi^a\varphi^a}\ \epsilon_3^4 \epsilon_4^4$$

$$\mathcal{M}_t(S_1^a S_2^a V_3 V_4) = \mathcal{M}(S_1^a V_3 S_{13}^b)\frac{1}{p_{13}^2 - m_b^2}\mathcal{M}(S_{13}^b S_2^a V_4)$$
$$+\mathcal{M}(S_1^a V_3 V_{13})^{M_{13}}\frac{-g_{M_{13}N_{13}}}{p_{13}^2 - m_V^2}\mathcal{M}^{N_{13}}(V_{13} S_2^a V_4^{s_4})$$

$$\mathcal{M}_u(S_1^a S_2^a V_3 V_4) = \mathcal{M}(S_1^a V_4 S_{14}^i)\frac{1}{p_{14}^2 - m_i^2}\mathcal{M}(S_{14}^i S_2^a V_3)$$
$$+\mathcal{M}(S_1^a V_4 V_{14})^{M_{14}}\frac{-g_{M_{14}N_{14}}}{p_{14}^2 - m^2}\mathcal{M}^{N_{14}}(V_{14} S_2^a V_3^{s_3})$$

$$\mathcal{M}_{\text{tot}}^{\epsilon_3^M \to p_3^M} = 0$$

$$g_{S^a S^a VV} = -2\left(g_{S^a S^b V}^2 - \frac{g_{VVS^a}^2}{4}\right)$$

$$\frac{1}{2}(\lambda_{S^a S^a S^a} g_{VVS^a} + \lambda_{S^a S^a S^b} g_{VVS^b}) - \lambda_{S^a S^a \varphi\varphi} m_V = g_{S^a S^b V}^2 \frac{m_a^2 - m_b^2}{m_V} - \frac{m_a^2}{2m_V} g_{VVS^a}^2$$

All couplings fixed if **scalar self-couplings are input.**

$n_V \leq 2$: VVSS from VVS、SSV&SSS

- Limit-1: $g_{S^a S^b V} = 0$ and $\lambda_{S^a S^a S^b} = 0.$

obtain

$$g_{SSVV} = \frac{1}{2} g_{VVS}^2$$

$$\lambda_{\varphi\varphi SS} = g_{VVS} \frac{\lambda_{SSS}}{m_V} + \frac{m_S^2}{2m_V^2} g_{VVS}^2$$

$$\lim_{m_V \to 0} \lambda_{SSS} = 0$$

Scalar QED

scalar self-couplings not fixed

- Limit-2: $g_{VVS} = 0$ $\quad m_a = m_b.$

obtain $\quad g_{S^a S^a VV} = -2 g_{S^a S^b V}^2 \quad \lambda_{S^a S^a \varphi\varphi} = 0$

Stueckelberg theory

Combined with 3-point: $m_1 = m_2 \to \lambda_{SS\varphi} = 0$

- The general case: U(1) version of SHDM
  needs more conditions(e.g. mixing) to fix the parameters

$\boldsymbol{n_V > 2}$: VVVV

- VVSS is the same as $\boldsymbol{n_V \leq 2}$
- VVVV: choose all masses to be equal; construct from V only first

$$\mathcal{M}_{\rm tot}(V_1^a V_2^b V_3^c V_4^d) = \mathcal{M}_s + \mathcal{M}_t + \mathcal{M}_u + \mathcal{M}_c$$

- parameters: set g_{VVV}=1

$$\begin{aligned}\mathcal{M}_c = {} & a_{12,34}\ \epsilon_1\cdot\epsilon_2\ \epsilon_3\cdot\epsilon_4 + a_{13,24}\ \epsilon_1\cdot\epsilon_3\ \epsilon_2\cdot\epsilon_4 + a_{14,23}\ \epsilon_1\cdot\epsilon_4\ \epsilon_2\cdot\epsilon_3 \\ & + b_{12,34}\ \epsilon_1\cdot\epsilon_2\epsilon_3^4\epsilon_4^4 + b_{13,24}\ \epsilon_1\cdot\epsilon_3\ \epsilon_2^4\epsilon_4^4 + b_{14,23}\ \epsilon_1\cdot\epsilon_4\ \epsilon_2^4\epsilon_3^4 \\ & + b_{34,12}\ \epsilon_3\cdot\epsilon_4\epsilon_1^4\epsilon_2^4 + b_{24,13}\ \epsilon_2\cdot\epsilon_4\ \epsilon_1^4\epsilon_3^4 + b_{23,14}\ \epsilon_2\cdot\epsilon_3\ \epsilon_1^4\epsilon_4^4 \\ & + \lambda_{\phi^4}\epsilon_1^4\epsilon_2^4\epsilon_3^4\epsilon_4^4\end{aligned}$$

- O.S.G. on helicities of 1TTT:

$$f^{abe}f^{cde} + f^{ace}f^{dbe} + f^{ade}f^{bce} = 0$$

Massless Yang–Mills theory

$n_V > 2$: VVVV

- 1LLT: no–solution for O.S.G (vector boson only)
- Solution: add a scalar (and VVS amplitudes)

$$\mathcal{M}_{\text{tot}} = \mathcal{M}_s + \mathcal{M}_t + \mathcal{M}_u + \mathcal{M}_c + \mathcal{M}_s^S + \mathcal{M}_t^S + \mathcal{M}_u^S$$

- Then applying O.S.G on 1LTT

$$b_{34,12} = \frac{1}{2} g_{V^aV^bS} g_{V^cV^dS}$$

$b_{ij,kl}$ by permutation

In addition:

$$g_{V^bV^aS} = g_{V^aV^bS}.$$

for equal masses

- 1LLT:

$$f^{abe} f^{cde} = g_{V^aV^cS} g_{V^bV^dS} - g_{V^aV^dS} g_{V^bV^cS}$$

$$f^{ace} f^{dbe} = -g_{V^aV^bS} g_{V^cV^dS} + g_{V^aV^dS} g_{V^bV^cS}$$

Not derived before?

- 1LLL

$$\lambda_{\varphi^4} = \frac{1}{4} \frac{m_S^2}{m_V^2} \left(g_{V^aV^bS} g_{V^cV^dS} + g_{V^aV^cS} g_{V^dV^bS} + g_{V^aV^dS} g_{V^bV^cS} \right)$$

$n_V > 2$: VVVV

- Adding a (mild) condition:

$$g \equiv g_{V^aV^bS} = g_{V^cV^dS} = g_{V^aV^cS} = g_{V^bV^dS} = g_{V^aV^dS} = g_{V^bV^cS}.$$

- The only nontrivial solution

$$g_{V^aV^bS} = 0 \quad \text{or} \quad g_{V^cV^dS} = 0 \qquad f^{abe} f^{cde} = 0 \qquad g_{VVS}^2 = f^{ace} f^{dbe} = f^{ade} f^{bce}$$

$$g_{VVS} \equiv g_{V^aV^cS} = g_{V^bV^dS} = g_{V^aV^dS} = g_{V^bV^cS}$$

Not derived before?

has only s and t channels

- Example: SU(2), agree in basis $W^{\pm}$

Conclusion

- We propose two consistent conditions to construct amplitudes with elementary massive vector bosons and scalars.
  - On-shell gauge symmetry(from Lorentz symmetry)
  - strong massive-massless continuation

- We construct possible 3-point&4-point amplitudes, including Goldstone-related.
- All couplings can be fixed by particle masses, except for scalar self-couplings
- Underlying theories:
  - Gauge theory with S.S.B. (e.g. Higgs mechanism)
  - Stuckelberg mechanism: no dynamic Goldstone