New chiral structures for BNV nucleon decays and their applications to ALPs

Hao-Lin Wang (王昊琳)
South China Normal University

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Why interested in BNV nucleon decays?

- BNV is related to the **stability of matter** and existence of ourselves
- As one of the three Sahkarov conditions to explain **matter-antimatter asymmetry** in the Universe
- The fundamental importance → BNV has never been directly observed after years of searches
  - Past: IMB (1983), KamLAND (2015), SNO+ (2019), …
  - Future: DUNE, Hyper-K, JUNO, THEIA, …

See also Xiao-Dong's talk

- Large samples for nucleon decays
- All possible decay channels should be attempted, ***exotic*** as well as conventional
- If BNV exists → Origin at high scale → EFT-based framework for low energy pheno.

General $|\Delta B| = 1$ triple-quark interactions

- In EFT framework, the lowest-dim operators are generally dominant
- Nucleon has $B = 1 \rightarrow$ at least 3 light quarks q involved
- We find only **four** triple-quark operators **without** derivative in LEFT or xLEFT

$$\mathcal{O}_a^{yzw} = (\overline{\Psi_a} q^\alpha_{\mathrm{L},y})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z}} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_b^{yzw} = (\overline{\Psi_b} q^\alpha_{\mathrm{R},y})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z}} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_c^{yzw} = (\overline{\Psi_{c,\mu}} q^\alpha_{\mathrm{L},\{y})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z\}}} \gamma^\mu q^\gamma_{\mathrm{R},w})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_d^{yzw} = (\overline{\Psi_{d,\mu\nu}} q^\alpha_{\mathrm{L},\{y})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z}} \sigma^{\mu\nu} q^\gamma_{\mathrm{L},w\}})\epsilon_{\alpha\beta\gamma}$$

$+\mathrm{L} \leftrightarrow \mathrm{R}$

Chiral partners

Combination of non-QCD fields

Completeness and independence of them can be proved using Fierz identities:

$$(\overline{\Psi_{c,\mu}} q^\alpha_{\mathrm{L},[y})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z]}} \gamma^\mu q^\gamma_{\mathrm{R},w})\epsilon_{\alpha\beta\gamma} = \frac{1}{2}(\overline{\Psi_{c,\mu}} \gamma^\mu q^\alpha_{\mathrm{R},w})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z}} q^\gamma_{\mathrm{L},y})\epsilon_{\alpha\beta\gamma} \sim \mathcal{O}_b^{yzw}$$

$$(\overline{\Psi_{d,\mu\nu}} q^\alpha_{\mathrm{R},y})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z}} \sigma^{\mu\nu} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma} = -2i(\overline{\Psi_{d,\mu\nu}} \gamma^\mu q^\alpha_{\mathrm{L},\{z})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},w\}}} \gamma^\nu q^\gamma_{\mathrm{R},y})\epsilon_{\alpha\beta\gamma}$$
$$\sim \mathcal{O}_c^{yzw}$$

$$(\overline{\Psi_{d,\mu\nu}} q^\alpha_{\mathrm{L},[y})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z]}} \sigma^{\mu\nu} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma} = -\frac{1}{2}(\overline{\Psi_{d,\mu\nu}} \sigma^{\mu\nu} q^\alpha_{\mathrm{L},w})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},y}} q^\gamma_{\mathrm{L},z})\epsilon_{\alpha\beta\gamma} \sim \mathcal{O}_a^{yzw}$$

$$\boxed{\begin{aligned} &y, z, w = 1,2,3 \\ &q_{1,2,3} = u, d, s \end{aligned}}$$

$$A_{\{y}B_{z\}} \equiv \frac{1}{2}(A_y B_z + A_z B_y)$$

$$A_{\{y}B_z C_{w\}} \equiv \frac{1}{6}[A_y B_z C_w + 5 \text{ permutations of } (y, z, w)]$$

$$A_{[y}B_{z]} \equiv \frac{1}{2}(A_y B_z - A_z B_y)$$

*Y. Liao, X.-D. Ma, and **HLW**, PRL 135 (2025)*

General $|\Delta B| = 1$ triple-quark interactions

- We find only **four** triple-quark operators **without** derivative in LEFT or xLEFT

$$\mathcal{O}_a^{yzw} = (\overline{\Psi}_a q^\alpha_{\mathrm{L},y})(\overline{q^{\beta C}_{\mathrm{L},z}} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_b^{yzw} = (\overline{\Psi}_b q^\alpha_{\mathrm{R},y})(\overline{q^{\beta C}_{\mathrm{L},z}} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma}$$

$+\mathrm{L} \leftrightarrow \mathrm{R}$

$$\mathcal{O}_c^{yzw} = (\overline{\Psi}_{c,\mu} q^\alpha_{\mathrm{L},\{y})(\overline{q^{\beta C}_{\mathrm{L},z\}}} \gamma^\mu q^\gamma_{\mathrm{R},w})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_d^{yzw} = (\overline{\Psi}_{d,\mu\nu} q^\alpha_{\mathrm{L},\{y})(\overline{q^{\beta C}_{\mathrm{L},z}} \sigma^{\mu\nu} q^\gamma_{\mathrm{L},w\}})\epsilon_{\alpha\beta\gamma}$$

Combination of non-QCD fields

- The first two were well-known in 1980s in the form of dim-6 LEFT operators:

F. Wilczek and A. Zee, PRL 43 (1979)
J. R. Ellis, M. k. Gaillard, and D. V. Nanopoulos, PLB 88 (1979)
S.Weinberg, PRL 43 (1979) & PRD 22 (1980)
L. F. Abbott and M. B. Wise, PRD 22 (1980)
O. Kaymakcalan, C.-H. Lo, and K. C. Wali, PRD 29 (1984)
M. Claudson, M. B. Wise, and L. J. Hall, NPB 195 (1982)

$$\overline{\Psi}_{a.b} = \overline{\ell},\ \bar{\nu}$$

- The operators corresponding to $\mathcal{O}_{c,d}^{yzw}$ vanish for dim-6 LEFT operators

$$\overline{\Psi}_{c,\mu} = \overline{\ell}\gamma_\mu$$

$$\overline{\Psi}_{d,\mu\nu} = \overline{\ell}\sigma_{\mu\nu}$$

$$\boxed{\begin{aligned} \gamma^\mu P_\pm \otimes \gamma_\mu P_\mp &= -2P_\mp \odot P_\pm \\ \sigma^{\mu\nu} P_\pm \otimes \sigma_{\mu\nu} P_\pm &= -4P_\pm \otimes P_\pm - 8P_\pm \odot P_\pm \end{aligned}}$$

When $\mathscr{O}_c$ and $\mathscr{O}_d$ enter?

*Y. Liao, X.-D. Ma, and **HLW**, PRL 135 (2025)*

$$\mathscr{O}_c^{yzw} = (\overline{\Psi_{c,\mu}} q^{\alpha}_{\mathtt{L},\{y}})(\overline{q^{\beta\mathtt{C}}_{\mathtt{L},z\}}} \gamma^{\mu} q^{\gamma}_{\mathtt{R},\mathtt{w}})\epsilon_{\alpha\beta\gamma}$$

$$\mathscr{O}_d^{yzw} = (\overline{\Psi_{d,\mu\nu}} q^{\alpha}_{\mathtt{L},\{y}})(\overline{q^{\beta\mathtt{C}}_{\mathtt{L},z}} \sigma^{\mu\nu} q^{\gamma}_{\mathtt{L},\mathtt{w}\}})\epsilon_{\alpha\beta\gamma}$$

$+\mathtt{L} \leftrightarrow \mathtt{R}$

- **Identified for the first time**
- **When will they become relevant or even important?**

- Important for ***exotic*** nucleon decays:
    - *Forbidden processes at dim-6*: e.g. $\Delta I = 3/2$ BNV nucleon decays, $n \to \ell^- \pi^+$
    - BNV nucleon decays through *higher dimensional operators*
    - BNV nucleon decays involving *new light particles*, e.g. aLEFT, DSEFT et al.
- We will show the new structures have nontrivial chiral realizations in ChPT

Chiral realization

- Explore the chiral realization within ChPT: symmetries (***chiral***, C/P/T, ***Lorentz***)
- Decompose each interaction into non-quark and quark factors

$$\mathcal{O}_a^{yzw} = (\overline{\Psi}_a q^\alpha_{\mathrm{L},y})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z}} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_b^{yzw} = (\overline{\Psi}_b q^\alpha_{\mathrm{R},y})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z}} q^\gamma_{\mathrm{L},w})\epsilon_{\alpha\beta\gamma} \qquad +\mathrm{L} \leftrightarrow \mathrm{R}$$

$$\mathcal{O}_c^{yzw} = (\overline{\Psi}_{c,\mu} q^\alpha_{\mathrm{L},\{y})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z\}}} \gamma^\mu q^\gamma_{\mathrm{R},\mathrm{w}})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{O}_d^{yzw} = (\overline{\Psi}_{d,\mu\nu} q^\alpha_{\mathrm{L},\{y})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L},z}} \sigma^{\mu\nu} q^\gamma_{\mathrm{L},\mathrm{w}\}})\epsilon_{\alpha\beta\gamma}$$

$$\mathcal{L}^{\cancel{B}}_{q^3} = \sum_i C_i^{yzw} \mathcal{O}_i^{yzw}$$

Non-quark factor	**Quark factor**
$\boxed{C_i^{yzw}\overline{\Psi}}$	$\boxed{\Gamma_1 q(\overline{q^{\mathrm{C}}}\Gamma_2 q)}$
$\equiv \mathcal{P}^i_{yzw}$	$\equiv \mathcal{N}^i_{yzw}$

$$\mathcal{L}^{\cancel{B}}_{q^3} = \Big\{ \Big[(\mathcal{P}^{\mathrm{LL}}_{yzw}\, q^\alpha_{\mathrm{L,y}})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L,z}}} q^\gamma_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma} + (\mathcal{P}^{\mathrm{RL}}_{yzw}\, q^\alpha_{\mathrm{R,y}})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L,z}}} q^\gamma_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma}$$
$$+ (\mathcal{P}^{\mathrm{LR},\mu}_{yzw}\, q^\alpha_{\mathrm{L},\{\mathrm{y}})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L,z}\}}} \gamma_\mu q^\gamma_{\mathrm{R,w}})\epsilon_{\alpha\beta\gamma} + (\mathcal{P}^{\mathrm{LL},\mu\nu}_{yzw}\, q^\alpha_{\mathrm{L},\{\mathrm{y}})(\overline{q^{\beta\mathrm{C}}_{\mathrm{L,z}}} \sigma_{\mu\nu} q^\gamma_{\mathrm{L,w}\}})\epsilon_{\alpha\beta\gamma} \Big] + \mathrm{L} \leftrightarrow \mathrm{R} \Big\} + \mathrm{h.c.}$$

Chiral properties

Triple-quark sector:

Chiral group: $SU(3)_\mathrm{L} \otimes SU(3)_\mathrm{R}$

- Under the chiral group, each structure belongs an irreducible representation

$$\mathcal{N}^{\mathrm{LL}}_{yzw} \equiv q^{\alpha}_{\mathrm{L,y}}(\overline{q^{\beta C}_{\mathrm{L,z}}}q^{\gamma}_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma} \in \mathbf{8}_\mathrm{L} \otimes \mathbf{1}_\mathrm{R}$$

$$\mathcal{N}^{\mathrm{RL}}_{yzw} \equiv q^{\alpha}_{\mathrm{R,y}}(\overline{q^{\beta C}_{\mathrm{L,z}}}q^{\gamma}_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma} \in \bar{\mathbf{3}}_\mathrm{L} \otimes \mathbf{3}_\mathrm{R}$$

$$\boxed{\mathcal{N}_{\mathbf{8}_\mathrm{L}\otimes\mathbf{1}_\mathrm{R}} \xrightarrow{G} \hat{L}\mathcal{N}_{\mathbf{8}_\mathrm{L}\otimes\mathbf{1}_\mathrm{R}}\hat{L}^\dagger \qquad \mathcal{N}_{\bar{\mathbf{3}}_\mathrm{L}\otimes\mathbf{3}_\mathrm{R}} \xrightarrow{G} \hat{R}\mathcal{N}_{\bar{\mathbf{3}}_\mathrm{L}\otimes\mathbf{3}_\mathrm{R}}\hat{L}^\dagger}$$

Can be organized in the matrix form

$$\mathcal{N}_{\mathbf{8}_\mathrm{L}\otimes\mathbf{1}_\mathrm{R}} = \begin{pmatrix} \mathcal{N}^{\mathrm{LL}}_{uds}\mathcal{N}^{\mathrm{LL}}_{usu}\mathcal{N}^{\mathrm{LL}}_{uud} \\ \mathcal{N}^{\mathrm{LL}}_{dds}\mathcal{N}^{\mathrm{LL}}_{dsu}\mathcal{N}^{\mathrm{LL}}_{dud} \\ \mathcal{N}^{\mathrm{LL}}_{sds}\mathcal{N}^{\mathrm{LL}}_{ssu}\mathcal{N}^{\mathrm{LL}}_{sud} \end{pmatrix} \qquad \mathcal{N}_{\bar{\mathbf{3}}_\mathrm{L}\otimes\mathbf{3}_\mathrm{R}} = \begin{pmatrix} \mathcal{N}^{\mathrm{RL}}_{uds}\mathcal{N}^{\mathrm{RL}}_{usu}\mathcal{N}^{\mathrm{RL}}_{uud} \\ \mathcal{N}^{\mathrm{RL}}_{dds}\mathcal{N}^{\mathrm{RL}}_{dsu}\mathcal{N}^{\mathrm{RL}}_{dud} \\ \mathcal{N}^{\mathrm{RL}}_{sds}\mathcal{N}^{\mathrm{RL}}_{ssu}\mathcal{N}^{\mathrm{RL}}_{sud} \end{pmatrix}$$

$$\mathcal{N}^{\mathrm{LR},\mu}_{yzw} \equiv q^{\alpha}_{\mathrm{L,\{y}}(\overline{q^{\beta C}_{\mathrm{L,z\}}}}\gamma^\mu q^{\gamma}_{\mathrm{R,w}})\epsilon_{\alpha\beta\gamma} \in \mathbf{6}_\mathrm{L} \otimes \mathbf{3}_\mathrm{R}$$

$$\mathcal{N}^{\mathrm{LL},\mu\nu}_{yzw} \equiv q^{\alpha}_{\mathrm{L,\{y}}(\overline{q^{\beta C}_{\mathrm{L,z}}}\sigma^{\mu\nu} q^{\gamma}_{\mathrm{L,w\}}})\epsilon_{\alpha\beta\gamma} \in \mathbf{10}_\mathrm{L} \otimes \mathbf{1}_\mathrm{R}$$

$$\boxed{\begin{aligned} \mathcal{N}^{\mathrm{LR},\mu}_{yzw} &\xrightarrow{G} \hat{L}_{yy'}\hat{L}_{zz'}\hat{R}_{ww'}\mathcal{N}^{\{y'z'\}w',\mu}_{\mathbf{6}_\mathrm{L}\otimes\mathbf{3}_\mathrm{R}} \\ \mathcal{N}^{\mathrm{LL},\mu\nu}_{yzw} &\xrightarrow{G} \hat{L}_{yy'}\hat{L}_{zz'}\hat{L}_{ww'}\mathcal{N}^{\{y'z'w'\},\mu\nu}_{\mathbf{10}_\mathrm{L}\otimes\mathbf{1}_\mathrm{R}} \end{aligned}}$$

Chiral properties

Non-quark sector:

Chiral group: $SU(3)_{\rm L}\otimes SU(3)_{\rm R}$

- Treat non-quark factors as a spurion field
- Assign transformation rules to the spurion fields so that the interactions are invariant

$$\mathscr{P}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}} \to \hat{L}\mathscr{P}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}}\hat{L}^\dagger \qquad \mathscr{P}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}} \to \hat{L}\mathscr{P}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}}\hat{R}^\dagger \qquad \mathscr{P}^{\rm LR,\mu}_{yzw} \to \hat{L}^*_{yy'}\hat{L}^*_{zz'}\hat{R}^*_{ww'}\mathscr{P}^{\rm LR,\mu}_{y'z'w'} \qquad \mathscr{P}^{\rm LL,\mu\nu}_{yzw} \to \hat{L}^*_{yy'}\hat{L}^*_{zz'}\hat{L}^*_{ww'}\mathscr{P}^{\rm LL,\mu\nu}_{y'z'w'}$$

- Organize $\mathscr{P}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}}$ and $\mathscr{P}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}}$ in matrix form similarly

$$\mathscr{P}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}} = \begin{pmatrix} \boxed{0} & \mathscr{P}^{\rm LL}_{dds} & \mathscr{P}^{\rm LL}_{sds} \\ \mathscr{P}^{\rm LL}_{usu} & \mathscr{P}^{\rm LL}_{dsu} & \mathscr{P}^{\rm LL}_{ssu} \\ \mathscr{P}^{\rm LL}_{uud} & \mathscr{P}^{\rm LL}_{dud} & \mathscr{P}^{\rm LL}_{sud} \end{pmatrix} \qquad \mathscr{P}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}} = \begin{pmatrix} \mathscr{P}^{\rm RL}_{uds} & \mathscr{P}^{\rm RL}_{dds} & \mathscr{P}^{\rm RL}_{sds} \\ \mathscr{P}^{\rm RL}_{usu} & \mathscr{P}^{\rm RL}_{dsu} & \mathscr{P}^{\rm RL}_{ssu} \\ \mathscr{P}^{\rm RL}_{uud} & \mathscr{P}^{\rm RL}_{dud} & \mathscr{P}^{\rm RL}_{sud} \end{pmatrix} \qquad \mathscr{P}^{\rm LL,\mu\nu}_{yzw} \qquad \mathscr{P}^{\rm LR,\mu}_{yzw}$$

$$\mathscr{L}^{\not B}_{q^3} = {\rm Tr}\left[\mathscr{P}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}}\mathscr{N}_{\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}} + \mathscr{P}_{\mathbf{1}_{\rm L}\otimes\mathbf{8}_{\rm R}}\mathscr{N}_{\mathbf{1}_{\rm L}\otimes\mathbf{8}_{\rm R}}\right] + {\rm Tr}\left[\mathscr{P}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}}\mathscr{N}_{\bar{\mathbf{3}}_{\rm L}\otimes\mathbf{3}_{\rm R}} + \mathscr{P}_{\bar{\mathbf{3}}_{\rm L}\otimes\mathbf{3}_{\rm R}}\mathscr{N}_{\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}}\right]$$
$$+\left[\mathscr{P}^{\{yz\}w,\mu}_{\bar{\mathbf{6}}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}}\mathscr{N}^{\{yz\}w}_{\mathbf{6}_{\rm L}\otimes\mathbf{3}_{\rm R},\mu} + \mathscr{P}^{\{yz\}w,\mu}_{\bar{\mathbf{3}}_{\rm L}\otimes\bar{\mathbf{6}}_{\rm R}}\mathscr{N}^{\{yz\}w}_{\mathbf{3}_{\rm L}\otimes\mathbf{6}_{\rm R},\mu}\right] + \left[\mathscr{P}^{\{yzw\},\mu\nu}_{\overline{\mathbf{10}}_{\rm L}\otimes\mathbf{1}_{\rm R}}\mathscr{N}^{\{yzw\}}_{\mathbf{10}_{\rm L}\otimes\mathbf{1}_{\rm R},\mu\nu} + \mathscr{P}^{\{yzw\},\mu\nu}_{\mathbf{1}_{\rm L}\otimes\overline{\mathbf{10}}_{\rm R}}\mathscr{N}^{\{yzw\}}_{\mathbf{1}_{\rm L}\otimes\mathbf{10}_{\rm R},\mu\nu}\right] + {\rm h.c.}$$

Lorentz properties

Triple-quark sector:

Lorentz algebra $\mathfrak{su}_l(2) \oplus \mathfrak{su}_r(2)$

- It important to incorporate the correct Lorentz properties for chiral matching

$$\mathcal{N}^{\mathrm{LL}}_{yzw} \equiv q^{\alpha}_{\mathrm{L,y}}(\overline{q^{\beta\mathrm{C}}_{\mathrm{L,z}}} q^{\gamma}_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma} \in \mathbf{8}_{\mathrm{L}} \otimes \mathbf{1}_{\mathrm{R}}$$

$$\mathcal{N}^{\mathrm{RL}}_{yzw} \equiv q^{\alpha}_{\mathrm{R,y}}(\overline{q^{\beta\mathrm{C}}_{\mathrm{L,z}}} q^{\gamma}_{\mathrm{L,w}})\epsilon_{\alpha\beta\gamma} \in \bar{\mathbf{3}}_{\mathrm{L}} \otimes \mathbf{3}_{\mathrm{R}}$$

$$\mathcal{N}_{\mathbf{8}_{\mathrm{L}}\otimes\mathbf{1}_{\mathrm{R}}} \in (\frac{1}{2},0) \qquad \mathcal{N}_{\bar{\mathbf{3}}_{\mathrm{L}}\otimes\mathbf{3}_{\mathrm{R}}} \in (0,\frac{1}{2})$$

$$\mathcal{N}^{\mathrm{LR},\mu}_{yzw} \equiv q^{\alpha}_{\mathrm{L},\{\mathrm{y}}(\overline{q^{\beta\mathrm{C}}_{\mathrm{L,z}\}}} \gamma^{\mu} q^{\gamma}_{\mathrm{R,w}})\epsilon_{\alpha\beta\gamma} \in \mathbf{6}_{\mathrm{L}} \otimes \mathbf{3}_{\mathrm{R}}$$

$$\mathcal{N}^{\mathrm{LL},\mu\nu}_{yzw} \equiv q^{\alpha}_{\mathrm{L},\{\mathrm{y}}(\overline{q^{\beta\mathrm{C}}_{\mathrm{L,z}}} \sigma^{\mu\nu} q^{\gamma}_{\mathrm{L,w}\}})\epsilon_{\alpha\beta\gamma} \in \mathbf{10}_{\mathrm{L}} \otimes \mathbf{1}_{\mathrm{R}}$$

$$\mathcal{N}^{\mathrm{LR},\mu}_{yzw} \in \left(1,\frac{1}{2}\right) \qquad \mathcal{N}^{\mathrm{LL},\mu\nu}_{yzw} \in \left(\frac{3}{2},0\right)$$

These complicates the chiral matching

$$\gamma_\mu \mathcal{N}^{\mathrm{LR},\mu}_{yzw} = \gamma_\mu \mathcal{N}^{\mathrm{RL},\mu}_{yzw} = 0 \qquad \gamma_\mu \mathcal{N}^{\mathrm{LL},\mu\nu}_{yzw} = \gamma_\mu \mathcal{N}^{\mathrm{RR},\mu\nu}_{yzw} = 0$$

Chiral matching

- To build the interactions for baryons and pseudosaclars →

Building blocks in ChPT: *ChPT is a systematic and consistent approach*

$$\Sigma(x) = \xi^2(x) = \exp\left(\frac{i\sqrt{2}\Pi(x)}{F_0}\right) \quad \Pi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \quad B(x) = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix}$$

+ Spurion fields

Chiral group: $SU(3)_{\mathrm{L}} \otimes SU(3)_{\mathrm{R}}$

$$\Sigma \to \hat{L}\Sigma\hat{R}^\dagger, \ \ \xi \to \hat{L}\xi\hat{h}^\dagger = \hat{h}\xi\hat{R}^\dagger, \ \ B \to \hat{h}B\hat{h}^\dagger$$

- Chiral Lagrangian is ordered by the typical momentum transfer p

$$\boxed{\{\Sigma, \xi, B, D_\mu B\} \sim \mathcal{O}(p^0) \qquad D_\mu\Sigma \sim \mathcal{O}(p^1)}$$

The higher the chiral order, more suppressed the term

$$D_\mu\Sigma = \partial_\mu\Sigma - il_\mu\Sigma + i\Sigma r_\mu \qquad D_\mu B = \partial_\mu B + [\Gamma_\mu, B] \qquad \Gamma_\mu = \frac{1}{2}\left[\xi(\partial_\mu - ir_\mu)\xi^\dagger + \xi^\dagger(\partial_\mu - il_\mu)\xi\right]$$

Chiral matching

- Project the approciate irreducible Lorentz rep. from ***vector-spinor*** and ***tensor-spinor***:

$$\boxed{\Gamma^{\mathrm{L,R}}_{\mu\nu} \equiv (g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu)P_{\mathrm{L,R}}} \quad \left(1,\frac{1}{2}\right) \quad \left(\frac{1}{2},1\right)$$

$$\Gamma^{\mathrm{L,R}}_{\mu\rho}\Gamma^{\mathrm{L,R}\,\rho}_{\nu} = \Gamma^{\mathrm{L,R}}_{\mu\nu} \qquad \gamma^\mu\Gamma^{\mathrm{L,R}}_{\mu\nu} = 0$$

E. Delgado et al., EPJA 51 (2015)

$$\boxed{\hat{\Gamma}^{\mathrm{L,R}}_{\mu\nu\alpha\beta} \equiv \frac{1}{24}\Big(2[\sigma_{\mu\nu},\sigma_{\alpha\beta}] - \{\sigma_{\mu\nu},\sigma_{\alpha\beta}\}\Big)P_{\mathrm{L,R}}} \quad \left(\frac{3}{2},0\right) \quad \left(0,\frac{3}{2}\right)$$

$$\hat{\Gamma}^{\mathrm{L,R}}_{\mu\nu\rho\sigma}\hat{\Gamma}^{\mathrm{L,R}\,\rho\sigma}_{\alpha\beta} = \hat{\Gamma}^{\mathrm{L,R}}_{\mu\nu\alpha\beta} \qquad \gamma^\mu\hat{\Gamma}^{\mathrm{L,R}}_{\mu\nu\alpha\beta} = 0$$

- Construct the *LO* chiral-invariant Lagrangian:

$$\mathscr{L}^{\not{B},0}_{B} = c_1\mathrm{Tr}\left[\mathscr{P}_{\bar{\mathbf{3}}_\mathrm{L}\otimes\mathbf{3}_\mathrm{R}}\xi B_\mathrm{L}\xi - \mathscr{P}_{\mathbf{3}_\mathrm{L}\otimes\bar{\mathbf{3}}_\mathrm{R}}\xi^\dagger B_\mathrm{R}\xi^\dagger\right] + c_2\mathrm{Tr}\left[\mathscr{P}_{\mathbf{8}_\mathrm{L}\otimes\mathbf{1}_\mathrm{R}}\xi B_\mathrm{L}\xi^\dagger - \mathscr{P}_{\mathbf{1}_\mathrm{L}\otimes\mathbf{8}_\mathrm{R}}\xi^\dagger B_\mathrm{R}\xi\right]$$

$$+\frac{c_3}{\Lambda_\chi}\left[\mathscr{P}^{\mathrm{LR},\mu}_{yzi}\Gamma^{\mathrm{L}}_{\mu\nu}(\xi iD^\nu B_\mathrm{L}\xi)_{yj}\Sigma_{zk}\epsilon_{ijk} - \mathscr{P}^{\mathrm{RL},\mu}_{yzi}\Gamma^{\mathrm{R}}_{\mu\nu}(\xi^\dagger iD^\nu B_\mathrm{R}\xi^\dagger)_{yj}\Sigma^*_{kz}\epsilon_{ijk}\right] + \text{h.c.},$$

$$\mathscr{L}^{\not{B},1}_{B} = \frac{c_4}{\Lambda^2_\chi}\left[\mathscr{P}^{\mathrm{LL},\mu\nu}_{yzw}\hat{\Gamma}^{\mathrm{L}}_{\mu\nu\alpha\beta}(\xi D^\alpha B_\mathrm{L}\xi)_{yi}\Sigma_{zj}\boxed{(D^\beta\Sigma)_{wk}}\epsilon_{ijk} - \mathscr{P}^{\mathrm{RR},\mu\nu}_{yzw}\hat{\Gamma}^{\mathrm{R}}_{\mu\nu\alpha\beta}(\xi^\dagger D^\alpha B_\mathrm{R}\xi^\dagger)_{iy}\Sigma^*_{jz}(D^\beta\Sigma)^*_{kw}\epsilon_{ijk}\right] + \text{h.c.}\,.$$

Involve at least one pesudoscalar

*Y. Liao, X.-D. Ma, and **HLW**, PRL 135 (2025)*

Determination of the LECs

*Y. Liao, X.-D. Ma, and **HLW,** PRL 135 (2025)*

$$\mathscr{L}_B^{\not{B},0} = c_1 \mathrm{Tr}\left[\mathscr{P}_{\bar{\mathbf{3}}_\mathrm{L}\otimes\mathbf{3}_\mathrm{R}}\xi B_\mathrm{L} \xi - \mathscr{P}_{\mathbf{3}_\mathrm{L}\otimes\bar{\mathbf{3}}_\mathrm{R}}\xi^\dagger B_\mathrm{R} \xi^\dagger\right] + c_2 \mathrm{Tr}\left[\mathscr{P}_{\mathbf{8}_\mathrm{L}\otimes\mathbf{1}_\mathrm{R}}\xi B_\mathrm{L}\xi^\dagger - \mathscr{P}_{\mathbf{1}_\mathrm{L}\otimes\mathbf{8}_\mathrm{R}}\xi^\dagger B_\mathrm{R} \xi\right]$$

$$+\frac{c_3}{\Lambda_\chi}\left[\mathscr{P}^{\mathrm{LR},\mu}_{yzi}\Gamma^\mathrm{L}_{\mu\nu}(\xi i D^\nu B_\mathrm{L}\xi)_{yj}\Sigma_{zk}\epsilon_{ijk} - \mathscr{P}^{\mathrm{RL},\mu}_{yzi}\Gamma^\mathrm{R}_{\mu\nu}(\xi^\dagger i D^\nu B_\mathrm{R}\xi^\dagger)_{yj}\Sigma^*_{kz}\epsilon_{ijk}\right] + \mathrm{h.c.},$$

$$\mathscr{L}_B^{\not{B},1} = \frac{c_4}{\Lambda_\chi^2}\left[\mathscr{P}^{\mathrm{LL},\mu\nu}_{yzw}\hat{\Gamma}^\mathrm{L}_{\mu\nu\alpha\beta}(\xi D^\alpha B_\mathrm{L}\xi)_{yi}\Sigma_{zj}(D^\beta\Sigma)_{wk}\epsilon_{ijk} - \mathscr{P}^{\mathrm{RR},\mu\nu}_{yzw}\hat{\Gamma}^\mathrm{R}_{\mu\nu\alpha\beta}(\xi^\dagger D^\alpha B_\mathrm{R}\xi^\dagger)_{iy}\Sigma^*_{jz}(D^\beta\Sigma)^*_{kw}\epsilon_{ijk}\right] + \mathrm{h.c.}\,.$$

- LQCD results for c_1 & c_2:
  $$c_1 = \alpha = -0.01257(111)\,\mathrm{GeV}^3$$
  $$c_2 = \beta = 0.01269(107)\,\mathrm{GeV}^3$$
  J.-S. Yoo, et al., PRD 105 (2022)

- NDA for the unknown LECs:

S. Weinberg, PRL 63 (1989), A. Manohar and H. Georgi, NPB 234 (1984), B. M. Gavela et al. EPJC 76 (2016)

$$C_{q,\mathrm{had}} = g(4\pi)^{2-m}\Lambda_\chi^{D-4}$$

Reduced coupling

g: coupling constant D: dim. of op.

m: minimal # of physical fields

$$c_{1,2,3} \sim \Lambda_\chi^3/(4\pi)^2 \approx 0.011\,\mathrm{GeV}^3$$

$$c_4 \sim \Lambda_\chi^2 F_0/(4\pi\sqrt{2}) \approx 0.007\,\mathrm{GeV}^3$$

The reliability of NDA can be verified with $c_{1,2}$

- ***We expect NDA really works, but still we encourage calculations on the two new LECs c_3 and c_4 by LQCD community***

Phenomenological applications

- Offer a starting point to all *BNV octet baryon decays* into *SM particles* and *new light particles* collected in spurions

$$\boxed{\mathscr{L} \supset M^n(\mathscr{P}B)}$$

A. The $\Delta I = 3/2$ processes: $n \to \ell^- \pi^+$ with $\ell = e, \mu$:

leading contribution from irreps $\mathbf{6}_{\mathrm{L(R)}} \otimes \mathbf{3}_{\mathrm{R(L)}}$

$$\mathscr{L}_{\mathscr{P}\pi n} = \frac{i\sqrt{2}c_3}{(F_0\Lambda_\chi)}\pi^- \mathscr{P}^{\mathrm{LR},\mu}_{ddd}\, i\tilde{\partial}_\mu n_{\mathrm{L}} + \mathrm{L} \leftrightarrow \mathrm{R}$$

From the dim-7 SMEFT operator:

$$\mathcal{O}^{prst}_{DLddQ} = (\overline{iD_\mu L_p} d^\alpha_{\{r})(\overline{d^{\beta C}_{s\}}}\gamma^\mu Q^\gamma_t)\epsilon_{\alpha\beta\gamma}$$

$$\tilde{\partial}_\mu = \Gamma_{\mu\nu}\partial^\nu = \partial_\mu - \frac{1}{4}\gamma_\mu \not{\partial}$$

$$\mathscr{P}^{\mathrm{RL},\mu}_{ddd} = -0.91 C^{\ell 111}_{DLddQ}\, i\partial^\mu \overline{\ell_{\mathrm{L}}}$$

RGE effects $\sim \Lambda_\ell^{-3}$

$$\Gamma(n \to \ell^-\pi^+) \sim \left(\frac{7\times 10^9\,\mathrm{GeV}}{\Lambda_\ell}\right)^6 / (5\times 10^{31}\,\mathrm{yr})$$

B. The process: $p \to \ell^+_\alpha \ell^+_\alpha \ell^-_\beta$ with $\ell_\alpha = e, \quad \ell_\beta = \mu$ or vice versa

- Among the most experimentally constrained proton decay processes
- Can be only mediated by dim-9 and higher LEFT operators
- Associated with irreps $\mathbf{6}_{\mathrm{L(R)}} \otimes \mathbf{3}_{\mathrm{R(L)}}$

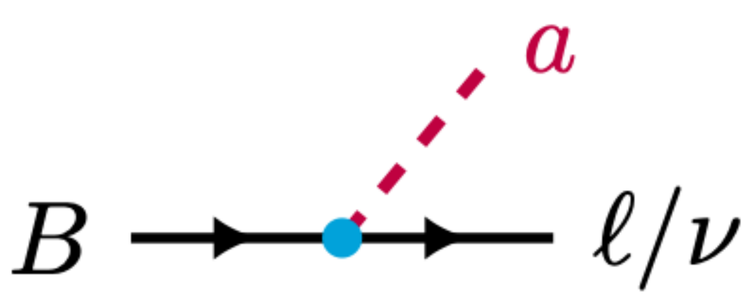
$$\mathcal{O}_{\ell\ell'} = (\overline{\ell^{\mathrm{C}}_{\mathrm{L}}}\gamma_\mu \ell_{\mathrm{R}})(\overline{\ell'_{\mathrm{R}}} u^\alpha_{\mathrm{L}})(\overline{u^{\beta C}_{\mathrm{L}}}\gamma^\mu d^\gamma_{\mathrm{R}})\epsilon_{\alpha\beta\gamma} \qquad \mathscr{P}^{\mathrm{LR},\mu}_{uud} = \Lambda^{-5}_{\ell\ell'}(\overline{\ell^{\mathrm{C}}_{\mathrm{L}}}\gamma^\mu \ell_{\mathrm{R}})\overline{\ell'_{\mathrm{R}}}$$

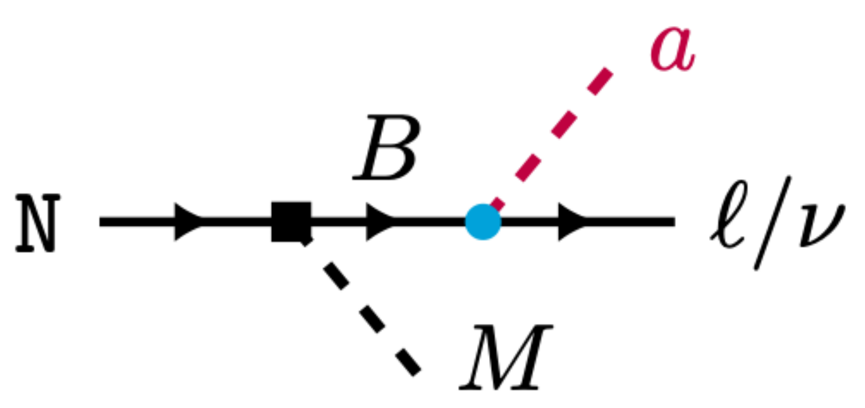
$$\Gamma(p \to e^+e^+\mu^-) \sim \Gamma(p \to \mu^+\mu^+e^-) \sim \left(\frac{4\times 10^5\mathrm{GeV}}{\Lambda_{\mu e}}\right)^{10} / (10^{34}\,\mathrm{yr})$$

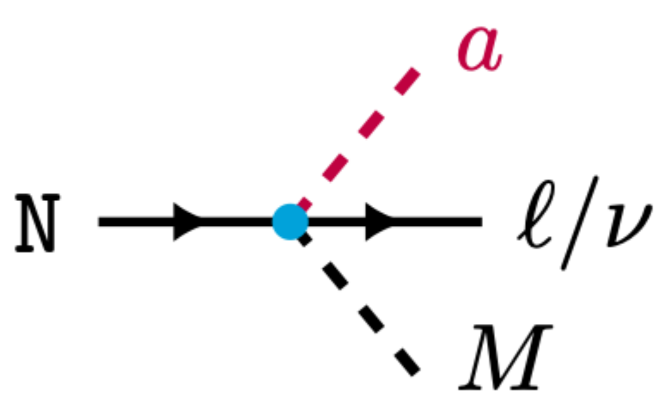
Phenomenological applications to ALPs

- Shift symmetry is assumed
- The LO operators appear at dim-8, $lqqq\partial a$
- Operators involved $\mathbf{6}_{\rm L}\otimes\mathbf{3}_{\rm R}$ ($\mathbf{3}_{\rm L}\otimes\mathbf{6}_{\rm R}$) are (partially) neglected in the literature, 14 out of 20

T. Li, M. A. Schmidt, and C.-Y. Yao, JHEP 08 (2024)



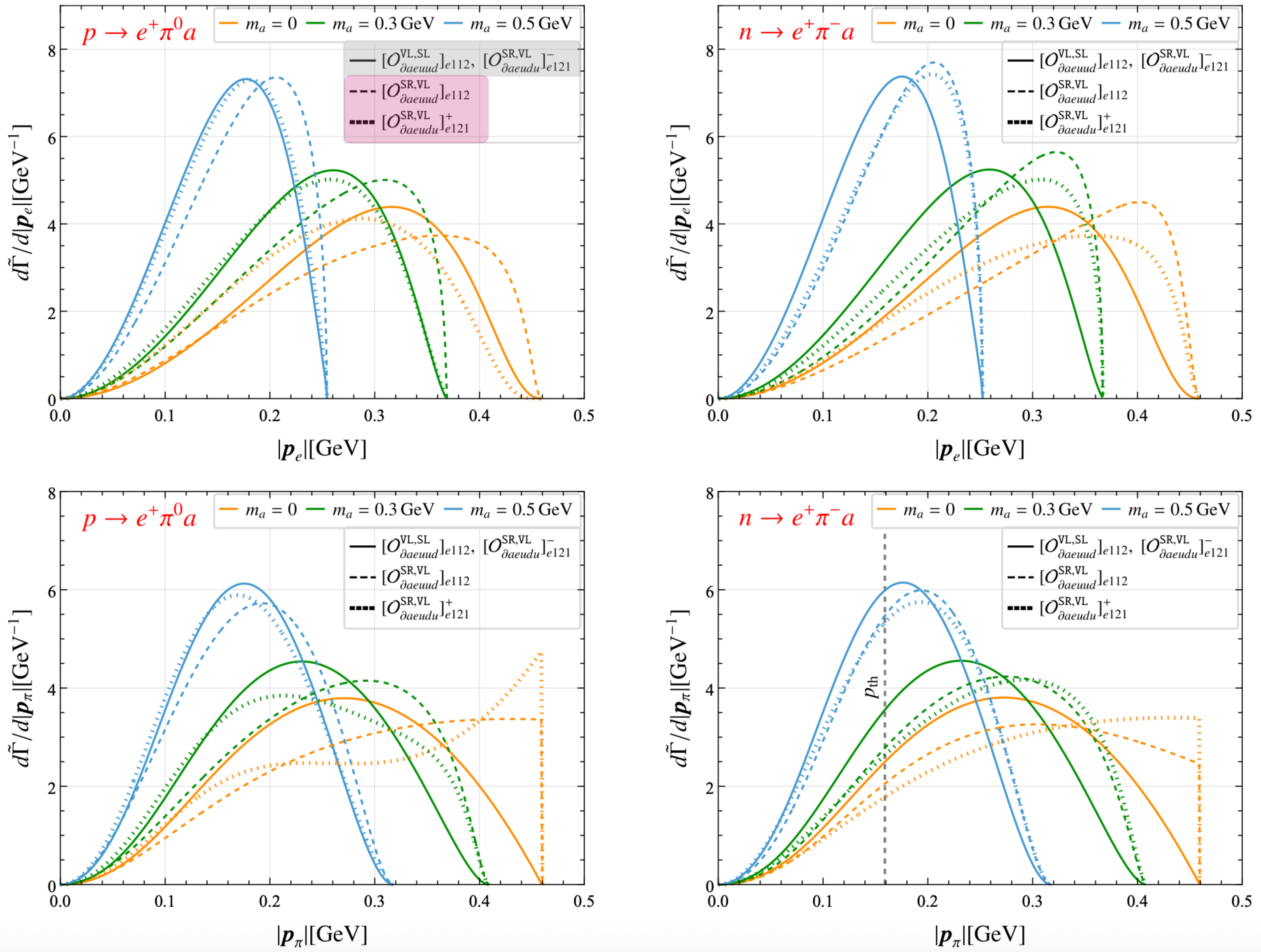




*W.-Q. Fan, Y. Liao, X.-D. Ma, and **HLW,** CPC 50 (2026)*

	Notation	Operator	Chiral Irrep.	# of operators	Comparison with [18]
$\Delta(B-L)=0$	$\mathcal{O}^{\rm VL,SL}_{\partial aeuud}$	$\partial_\mu a(\overline{e^{\rm C}_{\rm R}}\gamma^\mu u^\alpha_{\rm L})(\overline{u^{\beta\rm C}_{\rm L}}d^\gamma_{\rm L})\epsilon_{\alpha\beta\gamma}$	$\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}$	$n_e n_u^2 n_d\,[2n_e]$	$-\,\mathcal{O}^{\rm VL,SL}_{\underline{\partial aeudu}}$
	$\mathcal{O}^{\rm SL,VR}_{\partial aeuud}$	$\partial_\mu a(\overline{e^{\rm C}_{\rm L}}u^\alpha_{\rm L})(\overline{u^{\beta\rm C}_{\rm L}}\gamma^\mu d^\gamma_{\rm R})\epsilon_{\alpha\beta\gamma}$	$\mathbf{6}_{\rm L}\otimes\mathbf{3}_{\rm R}$	$n_e n_u^2 n_d\,[2n_e]$	$-[\mathcal{O}^{\rm VL,SR}_{\partial audue}]^\dagger$
	$\mathcal{O}^{\rm SL,VR}_{\partial aeudu}$	$\partial_\mu a(\overline{e^{\rm C}_{\rm L}}u^\alpha_{\rm L})(\overline{d^{\beta\rm C}_{\rm L}}\gamma^\mu u^\gamma_{\rm R})\epsilon_{\alpha\beta\gamma}$	$\mathbf{6}_{\rm L}\otimes\mathbf{3}_{\rm R}\oplus\bar{\mathbf{3}}_{\rm L}\otimes\mathbf{3}_{\rm R}$	$n_e n_u^2 n_d\,[2n_e]$	$-[\mathcal{O}^{\rm VL,SR}_{\partial aduue}]^\dagger$
	$\mathcal{O}^{\rm SL,VR}_{\partial aeduu}$	$\partial_\mu a(\overline{e^{\rm C}_{\rm L}}d^\alpha_{\rm L})(\overline{u^{\beta\rm C}_{\rm L}}\gamma^\mu u^\gamma_{\rm R})\epsilon_{\alpha\beta\gamma}$	$\mathbf{6}_{\rm L}\otimes\mathbf{3}_{\rm R}\oplus\bar{\mathbf{3}}_{\rm L}\otimes\mathbf{3}_{\rm R}$	$n_e n_u^2 n_d\,[2n_e]$	$-[\mathcal{O}^{\rm VL,SR}_{\partial auude}]^\dagger$
	$\mathcal{O}^{\rm VR,SR}_{\partial aeuud}$	$\partial_\mu a(\overline{e^{\rm C}_{\rm L}}\gamma^\mu u^\alpha_{\rm R})(\overline{u^{\beta\rm C}_{\rm R}}d^\gamma_{\rm R})\epsilon_{\alpha\beta\gamma}$	$\mathbf{1}_{\rm L}\otimes\mathbf{8}_{\rm R}$	$n_e n_u^2 n_d\,[2n_e]$	$\mathcal{O}^{\rm VR,SR}_{\underline{\partial aeuud}}$
	$\mathcal{O}^{\rm SR,VL}_{\partial aeuud}$	$\partial_\mu a(\overline{e^{\rm C}_{\rm R}}u^\alpha_{\rm R})(\overline{u^{\beta\rm C}_{\rm R}}\gamma^\mu d^\gamma_{\rm L})\epsilon_{\alpha\beta\gamma}$	$\mathbf{3}_{\rm L}\otimes\mathbf{6}_{\rm R}$	$n_e n_u^2 n_d\,[2n_e]$	$\mathcal{O}^{\rm VR,SR}_{\partial adue}$
	$\mathcal{O}^{\rm SR,VL}_{\partial aeudu}$	$\partial_\mu a(\overline{e^{\rm C}_{\rm R}}u^\alpha_{\rm R})(\overline{d^{\beta\rm C}_{\rm R}}\gamma^\mu u^\gamma_{\rm L})\epsilon_{\alpha\beta\gamma}$	$\mathbf{3}_{\rm L}\otimes\mathbf{6}_{\rm R}\oplus\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}$	$n_e n_u^2 n_d\,[2n_e]$	$\mathcal{O}^{\rm VL,SR}_{\partial adueu}$
	$\mathcal{O}^{\rm SR,VL}_{\partial aeduu}$	$\partial_\mu a(\overline{e^{\rm C}_{\rm R}}d^\alpha_{\rm R})(\overline{u^{\beta\rm C}_{\rm R}}\gamma^\mu u^\gamma_{\rm L})\epsilon_{\alpha\beta\gamma}$	$\mathbf{3}_{\rm L}\otimes\mathbf{6}_{\rm R}\oplus\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}$	$n_e n_u^2 n_d\,[2n_e]$	$\mathcal{O}^{\rm VL,SR}_{\partial adueu}-\mathcal{O}^{\rm VL,SR}_{\underline{\partial aeudu}}$
	$\mathcal{O}^{\rm VR,SR}_{\partial a\nu ddu}$	$\partial_\mu a(\overline{\nu^{\rm C}_{\rm L}}\gamma^\mu d^\alpha_{\rm R})(\overline{d^{\beta\rm C}_{\rm R}}u^\gamma_{\rm R})\epsilon_{\alpha\beta\gamma}$	$\mathbf{1}_{\rm L}\otimes\mathbf{8}_{\rm R}$	$n_\nu n_u n_d^2\,[4n_\nu]$	$-\,\mathcal{O}^{\rm VL,SR}_{\underline{\partial ad\nu du}}$
	$\mathcal{O}^{\rm SL,VR}_{\partial a\nu ddu}$	$\partial_\mu a(\overline{\nu^{\rm C}_{\rm L}}d^\alpha_{\rm L})(\overline{d^{\beta\rm C}_{\rm L}}\gamma^\mu u^\gamma_{\rm R})\epsilon_{\alpha\beta\gamma}$	$\mathbf{6}_{\rm L}\otimes\mathbf{3}_{\rm R}\oplus\bar{\mathbf{3}}_{\rm L}\otimes\mathbf{3}_{\rm R}$	$n_\nu n_u n_d^2\,[4n_\nu]$	$-[\mathcal{O}^{\rm VL,SR}_{\partial adud\nu}]^\dagger$
	$\mathcal{O}^{\rm SL,VR}_{\partial a\nu dud}$	$\partial_\mu a(\overline{\nu^{\rm C}_{\rm L}}d^\alpha_{\rm L})(\overline{u^{\beta\rm C}_{\rm L}}\gamma^\mu d^\gamma_{\rm R})\epsilon_{\alpha\beta\gamma}$	$\mathbf{6}_{\rm L}\otimes\mathbf{3}_{\rm R}\oplus\bar{\mathbf{3}}_{\rm L}\otimes\mathbf{3}_{\rm R}$	$n_\nu n_u n_d^2\,[4n_\nu]$	$-\left(\mathcal{O}^{\rm VL,SL}_{\underline{\partial ad\nu ud}}+[\mathcal{O}^{\rm VL,SR}_{\partial add\nu u}]^\dagger\right)$
	$\mathcal{O}^{\rm SL,VR}_{\partial a\nu udd}$	$\partial_\mu a(\overline{\nu^{\rm C}_{\rm L}}u^\alpha_{\rm L})(\overline{d^{\beta\rm C}_{\rm L}}\gamma^\mu d^\gamma_{\rm R})\epsilon_{\alpha\beta\gamma}$	$\mathbf{6}_{\rm L}\otimes\mathbf{3}_{\rm R}\oplus\bar{\mathbf{3}}_{\rm L}\otimes\mathbf{3}_{\rm R}$	$n_\nu n_u n_d^2\,[4n_\nu]$	$-[\mathcal{O}^{\rm VL,SR}_{\partial add\nu u}]^\dagger$
$\Delta(B+L)=0$	$\mathcal{O}^{\rm VL,SL}_{\partial a\nu ddu}$	$\partial_\mu a(\overline{\nu_{\rm L}}\gamma^\mu d^\alpha_{\rm L})(\overline{d^{\beta\rm C}_{\rm L}}u^\gamma_{\rm L})\epsilon_{\alpha\beta\gamma}$	$\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}$	$n_\nu n_u n_d^2\,[4n_\nu]$	$\mathcal{O}^{\rm VL,SL}_{\underline{\partial a\nu ddu}}$
	$\mathcal{O}^{\rm SR,VL}_{\partial a\nu ddu}$	$\partial_\mu a(\overline{\nu_{\rm L}}d^\alpha_{\rm R})(\overline{d^{\beta\rm C}_{\rm R}}\gamma^\mu u^\gamma_{\rm L})\epsilon_{\alpha\beta\gamma}$	$\mathbf{3}_{\rm L}\otimes\mathbf{6}_{\rm R}\oplus\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}$	$n_\nu n_u n_d^2\,[4n_\nu]$	$\mathcal{O}^{\rm VR,SR}_{\partial aud\nu d}$
	$\mathcal{O}^{\rm SR,VL}_{\partial a\nu dud}$	$\partial_\mu a(\overline{\nu_{\rm L}}d^\alpha_{\rm R})(\overline{u^{\beta\rm C}_{\rm R}}\gamma^\mu d^\gamma_{\rm L})\epsilon_{\alpha\beta\gamma}$	$\mathbf{3}_{\rm L}\otimes\mathbf{6}_{\rm R}\oplus\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}$	$n_\nu n_u n_d^2\,[4n_\nu]$	$\mathcal{O}^{\rm VR,SR}_{\partial adu\nu d}$
	$\mathcal{O}^{\rm SR,VL}_{\partial a\nu udd}$	$\partial_\mu a(\overline{\nu_{\rm L}}u^\alpha_{\rm R})(\overline{d^{\beta\rm C}_{\rm R}}\gamma^\mu d^\gamma_{\rm L})\epsilon_{\alpha\beta\gamma}$	$\mathbf{3}_{\rm L}\otimes\mathbf{6}_{\rm R}\oplus\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}$	$n_\nu n_u n_d^2\,[4n_\nu]$	$\mathcal{O}^{\rm VR,SR}_{\partial add\nu u}$
	$\mathcal{O}^{\rm VR,SR}_{\partial aeddd}$	$\partial_\mu a(\overline{e_{\rm R}}\gamma^\mu d^\alpha_{\rm R})(\overline{d^{\beta\rm C}_{\rm R}}d^\gamma_{\rm R})\epsilon_{\alpha\beta\gamma}$	$\mathbf{1}_{\rm L}\otimes\mathbf{8}_{\rm R}$	$\frac{1}{3}n_e n_d(n_d^2-1)\,[2n_e]$	$\mathcal{O}^{\rm VR,SR}_{\underline{\partial aeddd}}$
	$\mathcal{O}^{\rm SR,VL}_{\partial aeddd}$	$\partial_\mu a(\overline{e_{\rm L}}d^\alpha_{\rm R})(\overline{d^{\beta\rm C}_{\rm R}}\gamma^\mu d^\gamma_{\rm L})\epsilon_{\alpha\beta\gamma}$	$\mathbf{3}_{\rm L}\otimes\mathbf{6}_{\rm R}\oplus\mathbf{3}_{\rm L}\otimes\bar{\mathbf{3}}_{\rm R}$	$n_e n_d^3\,[8n_e]$	$\tilde{\mathcal{O}}^{\rm VR,SR}_{\partial aeddd}$
	$\mathcal{O}^{\rm VL,SL}_{\partial aeddd}$	$\partial_\mu a(\overline{e_{\rm L}}\gamma^\mu d^\alpha_{\rm L})(\overline{d^{\beta\rm C}_{\rm L}}d^\gamma_{\rm L})\epsilon_{\alpha\beta\gamma}$	$\mathbf{8}_{\rm L}\otimes\mathbf{1}_{\rm R}$	$\frac{1}{3}n_e n_d(n_d^2-1)\,[2n_e]$	$\mathcal{O}^{\rm VL,SL}_{\underline{\partial aeddd}}$
	$\mathcal{O}^{\rm SL,VR}_{\partial aeddd}$	$\partial_\mu a(\overline{e_{\rm R}}d^\alpha_{\rm L})(\overline{d^{\beta\rm C}_{\rm L}}\gamma^\mu d^\gamma_{\rm R})\epsilon_{\alpha\beta\gamma}$	$\mathbf{6}_{\rm L}\otimes\mathbf{3}_{\rm R}\oplus\bar{\mathbf{3}}_{\rm L}\otimes\mathbf{3}_{\rm R}$	$n_e n_d^3\,[8n_e]$	$-[\mathcal{O}^{\rm VL,SR}_{\partial addde}]^\dagger$

Phenomenological applications to ALPs



- Consider the normalized distributions

$$\frac{d\tilde\Gamma}{d|\mathbf{p}_\ell|} = \frac{1}{\Gamma}\frac{d\Gamma}{d|\mathbf{p}_\ell|}, \quad \frac{d\tilde\Gamma}{d|\mathbf{p}_M|} = \frac{1}{\Gamma}\frac{d\Gamma}{d|\mathbf{p}_M|}$$

- $[\mathscr{O}^{\rm VL,SL}_{\partial aeuud}]_{x112} \in \mathbf{8}_{\rm L} \otimes \mathbf{1}_{\rm R}$, $[\mathscr{O}^{\rm SR,VL}_{\partial aeudu}]^-_{x121} \in \mathbf{3}_{\rm L} \otimes \bar{\mathbf{3}}_{\rm R}$
- $[\mathscr{O}^{\rm SR,VL}_{\partial aeuud}]_{x112}$, $[\mathscr{O}^{\rm SR,VL}_{\partial aeudu}]^+_{x121} \in \mathbf{3}_{\rm L} \otimes \mathbf{6}_{\rm R}$
- Newly identified chiral irreps $\mathbf{6}_{\rm L} \otimes \mathbf{3}_{\rm R}$ ($\mathbf{3}_{\rm L} \otimes \mathbf{6}_{\rm R}$) exhibit markedly different behavior:
  - ✳ Tend to be concentrated at higher $|\mathbf{p}_\pi|$
  - ✳ Different isospin components

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Phenomenological applications to ALPs

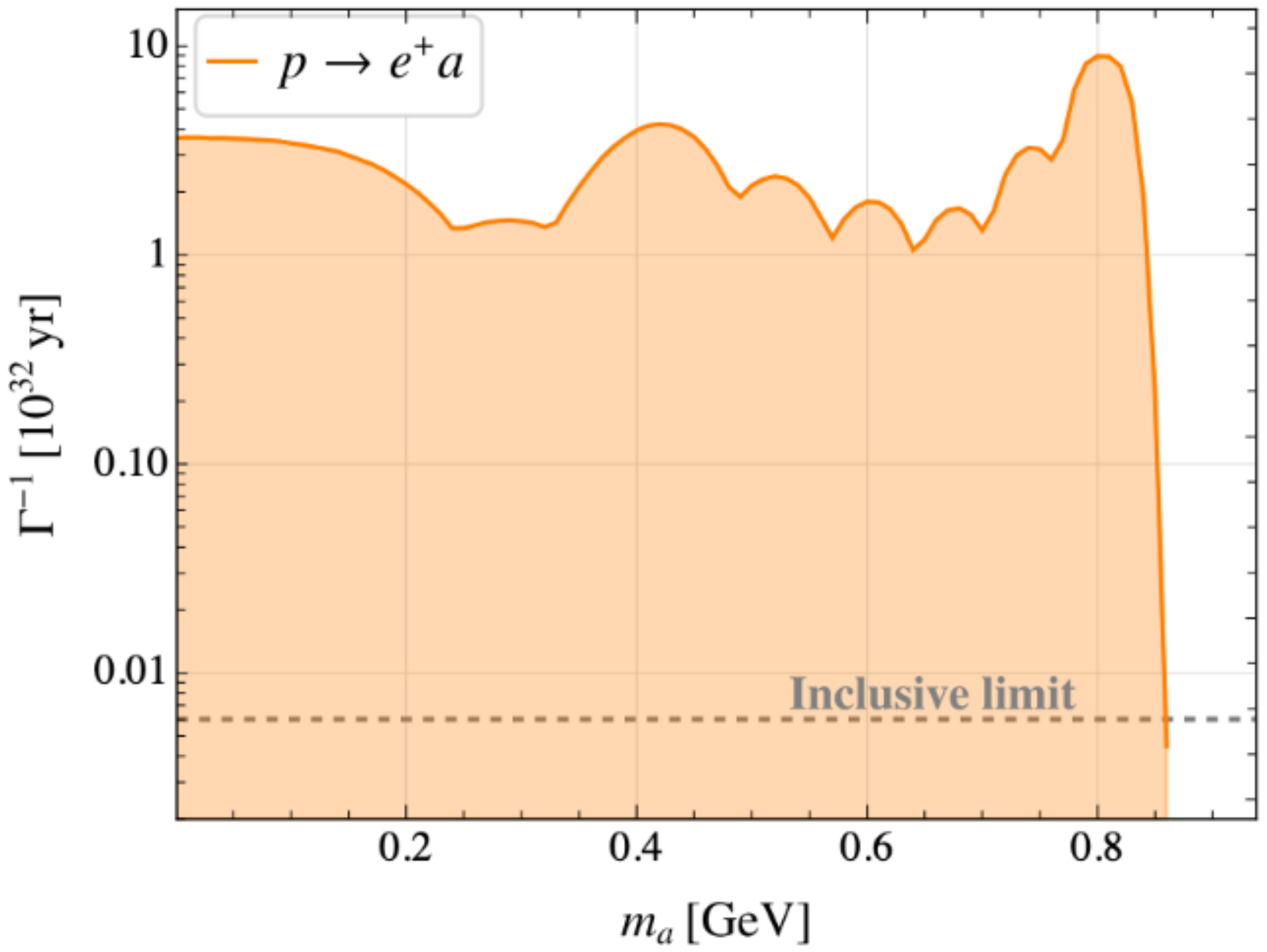
- Existing constraints on nucleon decays involving new light invisible particles

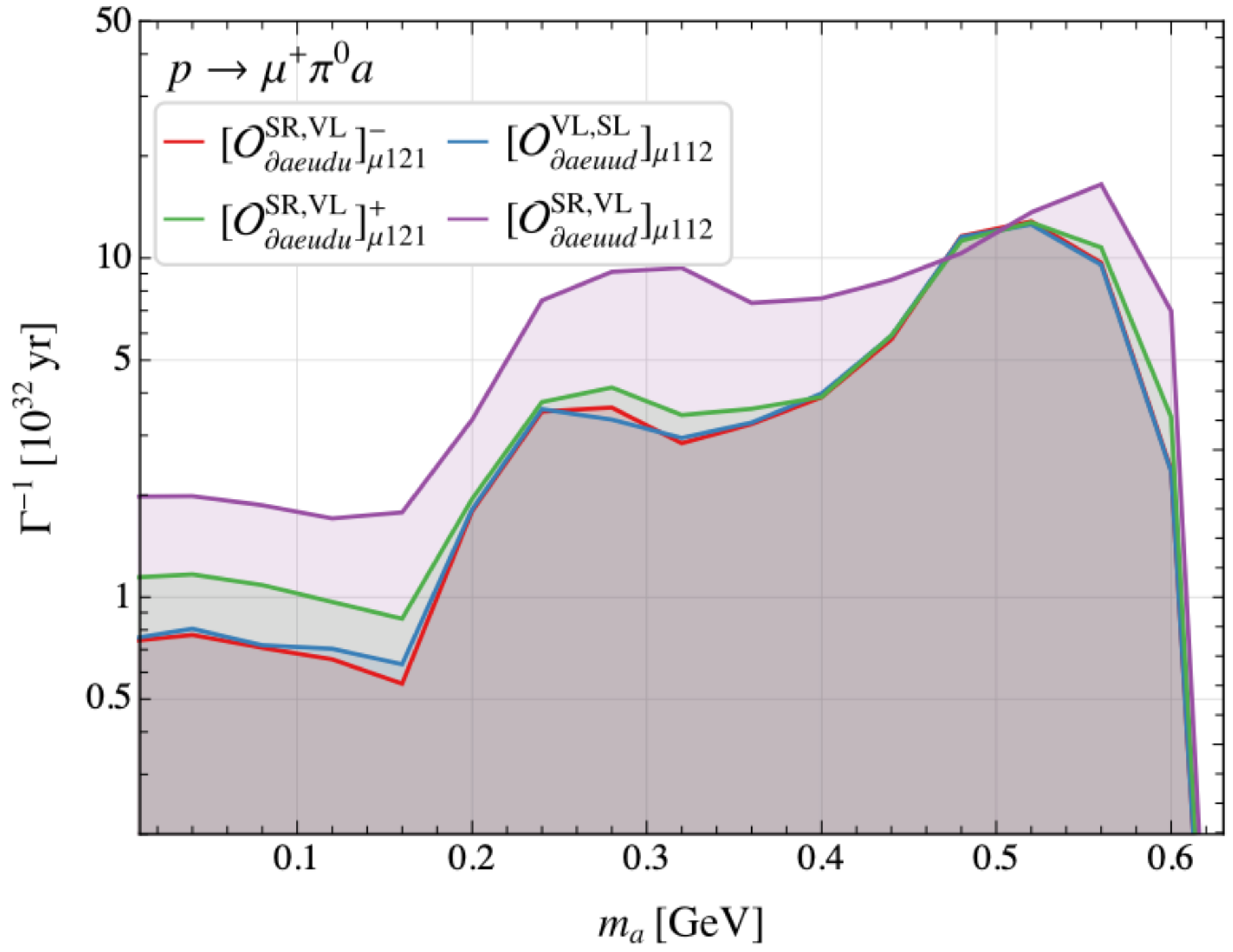
Super-Kamiokande (2015) $\Gamma^{-1}(p \to e^+ + X) > 7.9 \times 10^{32}\,\mathrm{yr}, \Gamma^{-1}(p \to \mu^+ + X) > 4.1 \times 10^{32}\,\mathrm{yr}$ $\boxed{m_X = 0}$

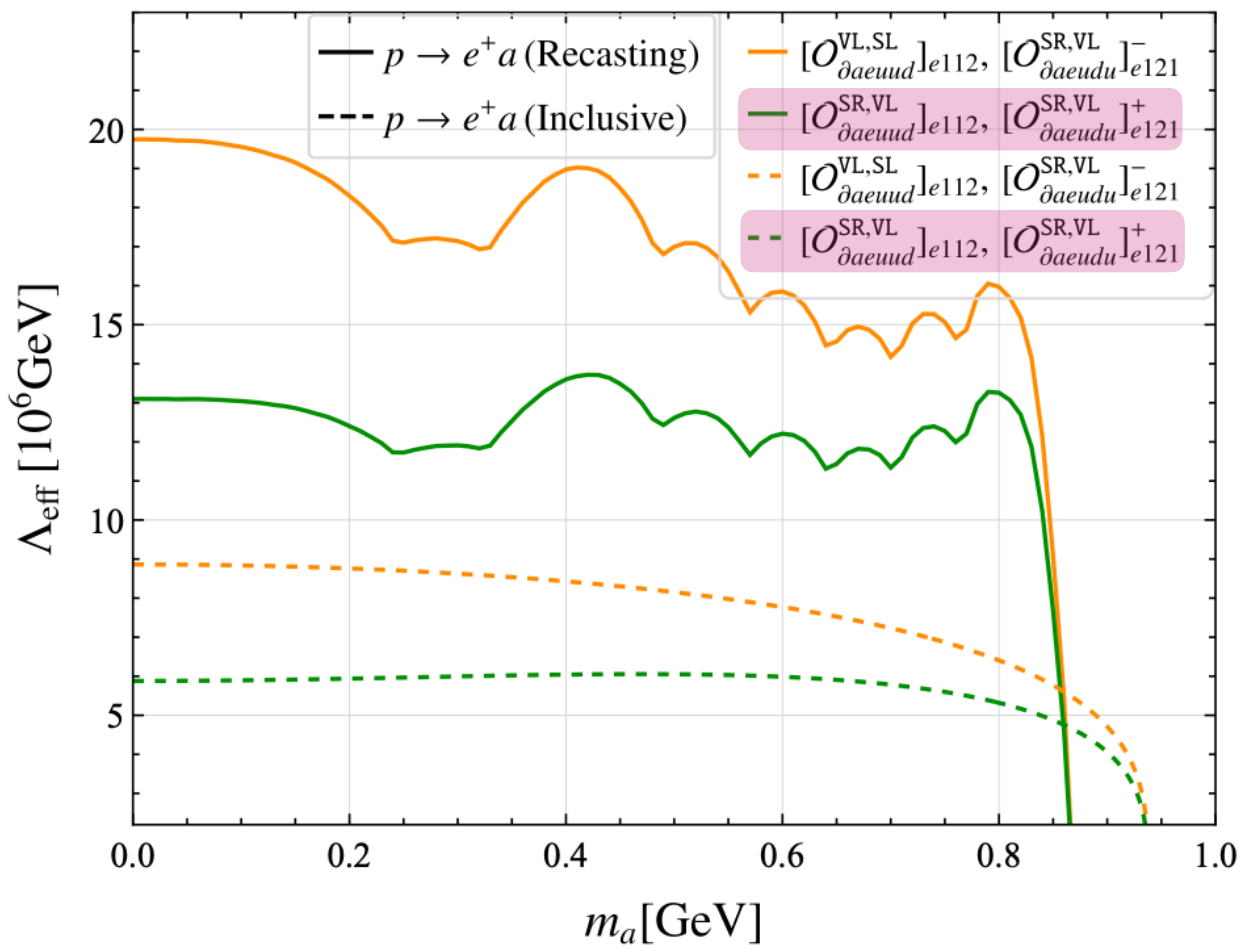
- Constraints on inclusive decay modes *SNO+ (2022)*

$$\Gamma^{-1}(n \to \text{invisible}) > 9.0 \times 10^{29}\,\mathrm{yr}, \Gamma^{-1}(N \to e^+ + \text{anything}) > 0.6 \times 10^{30}\,\mathrm{yr}, \Gamma^{-1}(N \to \mu^+ + \text{anything}) > 12 \times 10^{30}\mathrm{yr}$$

- *Existing experimental data from conventional channels (SM final states) can be reinterpreted to constrain processes involved new light invisible particles*







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Summary

- We classified general triple-quark interactions and carried out their chiral matching.
- We identified new chiral structures that induce exotic nucleon decays beyond the conventional ones.
- We estimated all of four LECs by NDA, which show consistency with lattice results on two known LECs. We encourage the lattice community to compute the new LECs.
- We discuss the phenomenological applications, especially for the ALP.

Thank You!

Backup: The chiral lagrangian involving decuplet baryons

$$\mathcal{L}_T^{\not{B},0} = \tilde{c}_3\left[\mathcal{P}^{\mathrm{LR},\mu}_{yzw}\Gamma^{\mathrm{L}}_{\mu\nu}T^{\nu}_{ijk}\xi_{yi}\xi_{zj}\xi^{*}_{kw} - \mathcal{P}^{\mathrm{RL},\mu}_{yzw}\Gamma^{\mathrm{R}}_{\mu\nu}T^{\nu}_{ijk}\xi^{*}_{iy}\xi^{*}_{jz}\xi_{wk}\right]$$
$$+\frac{\tilde{c}_4}{\Lambda_\chi}\left[\mathcal{P}^{\mathrm{LL},\mu\nu}_{yzw}\hat{\Gamma}^{\mathrm{L}}_{\mu\nu\alpha\beta}iD^{\alpha}T^{\beta}_{ijk}\xi_{yi}\xi_{zj}\xi_{wk} - \mathcal{P}^{\mathrm{RR},\mu\nu}_{yzw}\hat{\Gamma}^{\mathrm{R}}_{\mu\nu\alpha\beta}iD^{\alpha}T^{\beta}_{ijk}\xi^{*}_{iy}\xi^{*}_{jz}\xi^{*}_{kw}\right] + \mathrm{h.c.}$$

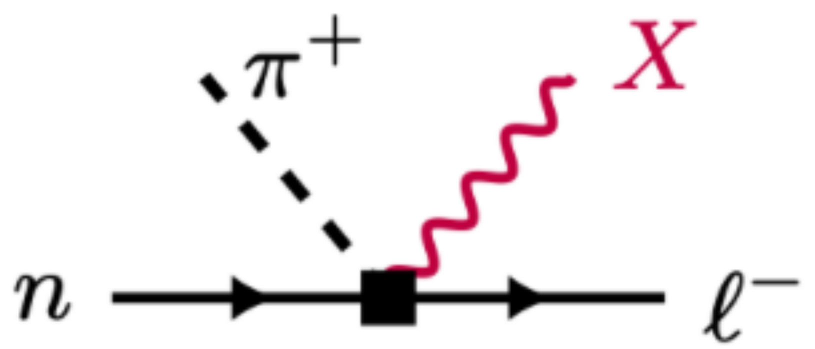
NDA: $\tilde{c}_{3,4} \sim \Lambda_\chi^3/(4\pi)^2 \approx 0.011\,\mathrm{GeV}^3$

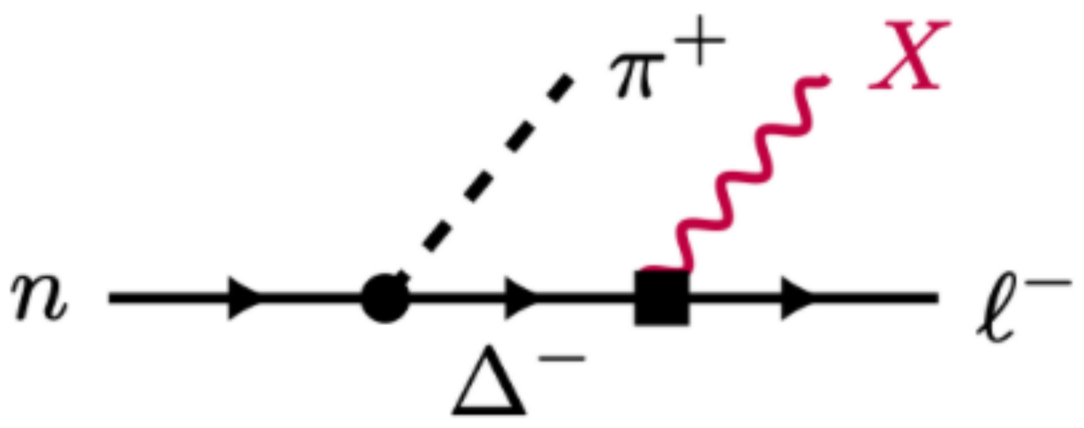
$$T_{111} = \Delta^{++},\ T_{112} = \frac{\Delta^+}{\sqrt{3}},\ T_{122} = \frac{\Delta^0}{\sqrt{3}},\ T_{222} = \Delta^-,$$
$$T_{113} = \frac{\Sigma^{*+}}{\sqrt{3}},\ T_{123} = \frac{\Sigma^{*0}}{\sqrt{6}},\ T_{223} = \frac{\Sigma^{*-}}{\sqrt{3}},\ T_{133} = \frac{\Xi^{*0}}{\sqrt{3}},\ T_{233} = \frac{\Xi^{*-}}{\sqrt{3}},\ T_{333} = \Omega^-.$$

Backup: $n \to \ell^- \pi^+ X^\mu$ as an example

- $\Delta I = 3/2$ process with $\ell = e, \mu$

$$\boxed{\mathcal{O}_{X\ell} = X_{\mu\nu}(\overline{\ell_{\mathrm{R}}} d_{\mathrm{L}}^{\alpha})(\overline{d_{\mathrm{L}}^{\beta\mathrm{C}}} \sigma^{\mu\nu} d_{\mathrm{L}}^{\gamma})\epsilon_{\alpha\beta\sigma} \quad \in \mathbf{10}_{\mathrm{L}} \otimes \mathbf{1}_{\mathrm{R}}} \quad \rightarrow \quad \mathscr{P}_{ddd}^{\mathrm{LL},\mu\nu} = \Lambda_{X\ell}^{-4} X^{\mu\nu} \overline{\ell_{\mathrm{R}}}$$





$$\Gamma_{n\to e^-\pi^+ X} = \left(0.88\,|\kappa_4|^2 + 0.27\,|\tilde{\kappa}_4|^2 - 0.82\,\Re(\kappa_4 \tilde{\kappa}_4^*)\right)\left(\frac{3\times 10^6\,\mathrm{GeV}}{\Lambda_{X\ell}}\right)^8 \frac{1}{10^{30}\,\mathrm{yr}}$$

$\kappa_4 \equiv c_4/c_1 \qquad \tilde{\kappa}_4 \equiv \tilde{c}_4/c_1$