Quantum Tomography of $e^+e^- \to f\bar{f}$: Effects from Beam Polarizations

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NPhiS 2026 May 15-19, 2026

Y G, Tao Han, Matthew Low, Youle Su, arXiv: 2602.02719

Quantum Information meets High-Energy Physics

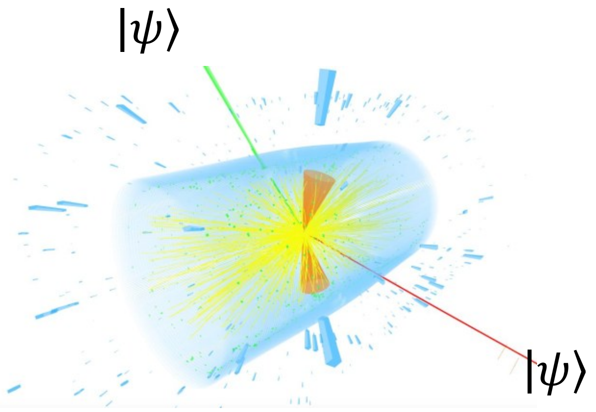
The quantum nature of the SM is so pervasive in the daily life of the particle physicist, that we often forget about it. Naively:

$$Z = \int D\phi \; e^{\frac{i}{\hbar} S[\phi]}$$

- **tree level** → order $\hbar^0$
- **1-loop** → order $\hbar^1$
- **2-loop** → order $\hbar^2$
- etc.

We have extensively validated QFT at different scales, and always found it a consistent way of making predictions in HEP.

- What is the information-theoretic structure of QFT?
- Can quantum-information observables reveal new aspects of fundamental interactions?
- Can quantum information provide new probes of physics beyond the SM?
- How does quantum information flow in time?



Quantum Information meets High-Energy Physics

Letter | Published: 06 May 2019

Polarization and entanglement in baryon–antibaryon pair production in electron–positron annihilation

The BESIII Collaboration

Nature Physics **15**, 631–634 (2019) | Cite this article

8157 Accesses | **190** Citations | **63** Altmetric | Metrics

Article | Open access | Published: 01 June 2022

Probing CP symmetry and weak phases with entangled double-strange baryons

The BESIII Collaboration

Nature **606**, 64–69 (2022) | Cite this article

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Entangled double-strange baryons $\Lambda\bar{\Lambda}$ system

CP violation

Quantum Information meets High-Energy Physics

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PAPER • OPEN ACCESS

Observation of quantum entanglement in top quark pair production in proton–proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration

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Reports on Progress in Physics, Volume 87, Number 11

Citation The CMS Collaboration 2024 *Rep. Prog. Phys.* **87** 117801

DOI 10.1088/1361-6633/ad7e4d

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Article | Open access | Published: 18 September 2024

Observation of quantum entanglement with top quarks at the ATLAS detector

The ATLAS Collaboration

Nature **633**, 542–547 (2024) | Cite this article

87k Accesses | **19** Citations | **476** Altmetric | Metrics

Entangled $t\bar{t}$ system

- Many papers with new ideas have appeared.
- New and exciting ideas for measurements at colliders.
- A rapidly growing interplay between HEP and QIS.

Quantum Information meets High-Energy Physics

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Observation of quantum entanglement in top quark pair production in proton–proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration

PAPER • OPEN ACCESS

Observation of a cross-section enhancement near the $t\bar{t}$ production threshold in $\sqrt{s} = 13$ TeV pp collisions with the ATLAS detector

The ATLAS Collaboration

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Reports on Progress in Physics, Volume 89, Number 5

Citation The ATLAS Collaboration 2026 *Rep. Prog. Phys.* **89** 057801

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Nature **633**, 542–547 (2024) | Cite this article

"quasi-bound state"
toponium

For details, see Haifeng Li's talk

Quantum Information meets High-Energy Physics

- Entanglement and Bell nonlocality at colliders — Zhang, et al. 2504.01496; Ruzi, Wu, Ding, Li, 2506.16077; Wu, et al. 2410.17025; Bi, et al. 2307.14895; Ma and Li. 2309.08103; Gao, et al. 2411.12518 …
- QI in Particle Physics — Pei, Li, Wu, Hao, Wang, 2510.0803; Cheng, Han, Low, 2410.08303; Qian, Li, Khan, Qiao, 2002.04283 …
- QCD Frontier — Cao, et al.. 2509.18276; Cheng, Yan. 2501.03321; Hong, et al. 2512.22837; Feng, et al. 2504.15798; Li, et al. 2309.09487; Wu, et al. 2406.16298; Shi, Yang, 1909.10626…
- Decohering — Gu, et al. 2510.13951; Aoude, et al. 2504.07030 …
- Flavor — Cheng, Han, Low, Wu. 2507.12513; Chen, Xing, Zhu, 2407.19242 …
- CP — Du, et al., 2409.15418; Fabbrichesi, et al. 2512.07939;
- Symmetry and Computation — Liu, et al. 2511.17321; Hu, et al. 2506.08960; Carena, et al. 2505.00873; Carena, et al. 2307.08112 …
- Magic — Liu, Low, Yin, 2509.18251; Gargalionis, et al. 2508.14967; Liu, Low, ZY, 2502.17550; White, White. 2406.07321…
- New Physics & SMEFT — Rufo, et al. 2503.19072; Fabbrichesi, et al. 2405.09201; Fabbrichesi, et al. 2208.11723; …
- Phase transitions … — Liu, et al. 2505.06001…

Apologies to many recent papers

Quantum Tomography

- For a **single qubit** $|\psi\rangle$ describes two-level state $|\uparrow\rangle, |\downarrow\rangle$ or $|1\rangle, |0\rangle$ in terms of **Bloch vector** $\mathbf{B}_i$:

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle \xrightarrow{\text{Pauli decomposition}} \rho = \frac{1}{2}(\mathbb{I}_2 + \sum_i B_i\sigma_i)$$

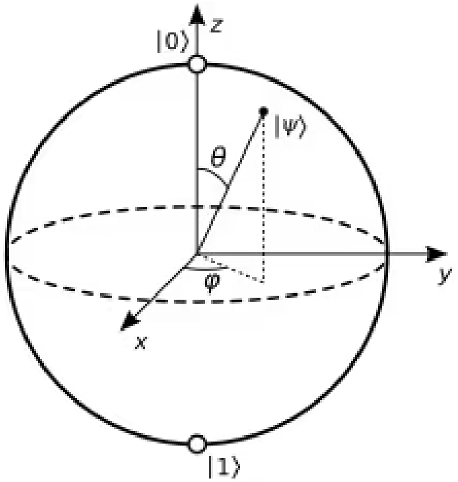
- For **two qubits**, use **Fano-Bloch decomposition** a bipartite system described by

$$\rho = \frac{1}{4}\left(\mathbb{I}_4 + \sum_i \underline{B_i^+\sigma_i \otimes \mathbb{I}_2} + \sum_j \underline{B_j^-\mathbb{I}_2 \otimes \sigma_j} + \sum_{ij} \underline{C_{ij}\sigma_i \otimes \sigma_j}\right)$$

B_i^+ **Qubit 1 polarization** (3 parameters)

B_j^- **Qubit 2 polarization** (3 parameters)

C_{ij} **Spin correlations** (9 parameters)



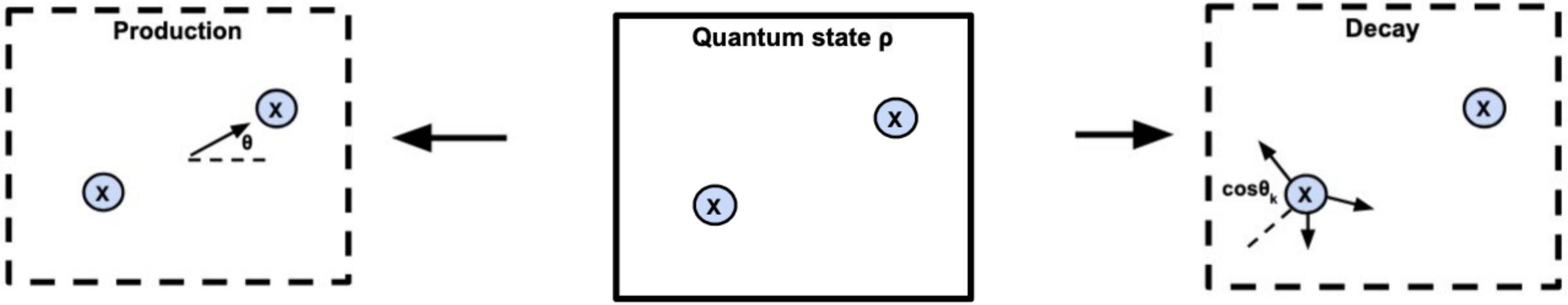
The **15** coefficients for the bipartite → **Quantum Tomography**

which encodes the full quantum information.

Quantum Tomography

Two paths to proceed to quantum tomography

$$\rho = \frac{1}{4}\left(\mathbb{I}_2 \otimes \mathbb{I}_2 + \sum_i B_i^+ \sigma_i \otimes \mathbb{I}_2 + \sum_j B_j^- \mathbb{I}_2 \otimes \sigma_i + \sum_{ij} C_{ij} \sigma_i \otimes \sigma_j\right)$$



Decay Approach

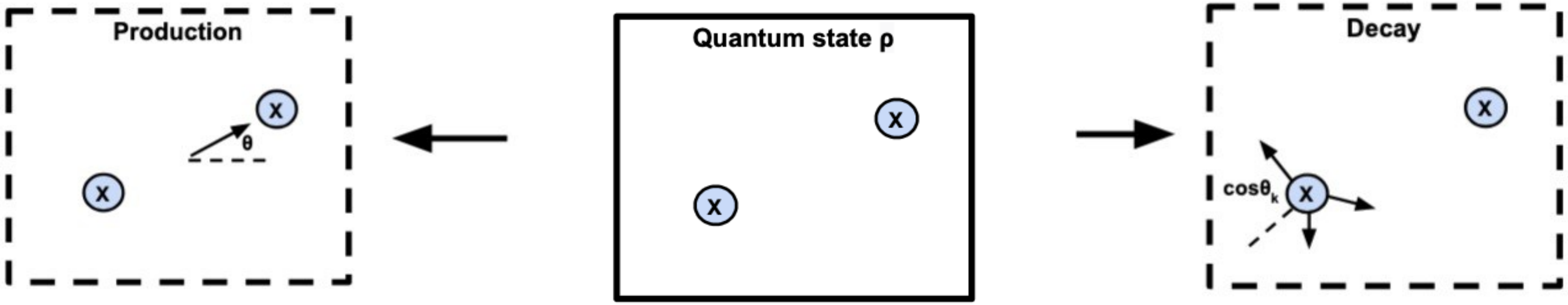
Afik, de Nova. 2003.02280

- Theory input on the decay
- Spin is related with decay distribution
- Angles in rest frames of decay products
- Useful for particles with tracked decay products

Quantum Tomography

Two paths to proceed to quantum tomography

$$\rho = \frac{1}{4}\left(\mathbb{I}_2 \otimes \mathbb{I}_2 + \sum_i B_i^+ \sigma_i \otimes \mathbb{I}_2 + \sum_j B_j^- \mathbb{I}_2 \otimes \sigma_i + \sum_{ij} C_{ij} \sigma_i \otimes \sigma_j\right)$$



Kinematic Approach

Cheng, Han, Low 2410.08383

- Theory input on the production
- Spin is related with scattering kinematic
- Initial states and scattering angle **θ**, speed of particle **β**
- Useful for stable particles

Decay Approach

Afik, de Nova. 2003.02280

- Theory input on the decay
- Spin is related with decay distribution
- Angles in rest frames of decay products
- Useful for particles with tracked decay products

Historically, the determination of the quark spin in $e^-e^+ \to q\bar{q}$ and the gluon spin in 3-jet events $e^-e^+ \to q\bar{q}g$ are inferred from **kinematic** distributions.

Quantum Information @ Colliders

Concurrence

- Quantity: **Entanglement**

Separable vs. entangled states

Einstein, Podolsky, Rosen 1935

$$|\psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \qquad |\psi_1\rangle = |\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

  - First qubit cannot be described independently from second qubit

- Measured at a collider using the **concurrence C**

Afik, de Nova 2003.02280

$$\mathcal{C}(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

λ_i is eigenvalues of $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}, \quad \tilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2)$

$$0 \leq \mathcal{C}(\rho) \leq 1$$

separable ↓ (0) Maximumly entangled ↓ (1)

Quantum Information @ Colliders

Bell Nonlocality

$$|E_{11} - E_{12} + E_{21} + E_{22}| \leq 2$$

- Each term calculable from **measured density matrix**

$$E_{11} = \vec{a}_1 \cdot C \cdot \vec{b}_1$$

Reference axis for qubit A   Reference axis for qubit B

- For a bipartite qubit system, Bell's inequality is the CHSH inequality $\vec{a}_i, \vec{b}_i$

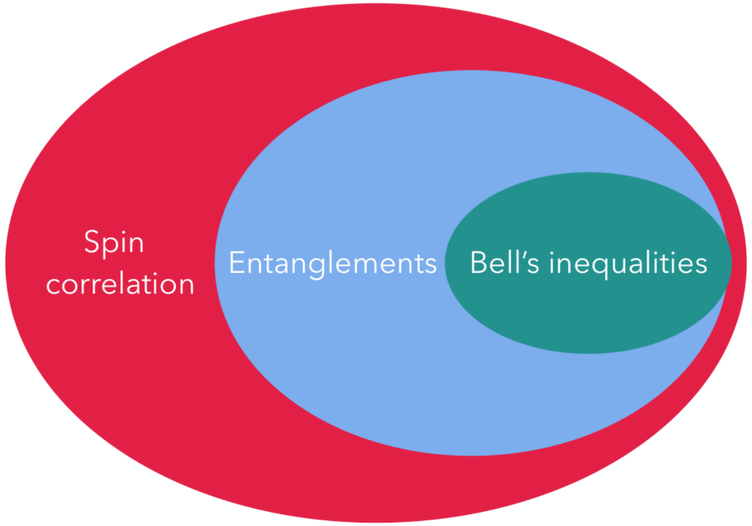
$$|\langle \vec{a}_1 \cdot \vec{\sigma} \otimes \vec{b}_1 \cdot \vec{\sigma} \rangle - \langle \vec{a}_1 \cdot \vec{\sigma} \otimes \vec{b}_2 \cdot \vec{\sigma} \rangle + \langle \vec{a}_2 \cdot \vec{\sigma} \otimes \vec{b}_1 \cdot \vec{\sigma} \rangle + \langle \vec{a}_2 \cdot \vec{\sigma} \otimes \vec{b}_2 \cdot \vec{\sigma} \rangle| \leq 2.$$

The violation of Bell's inequality corresponds to

$$\mathcal{B} = \left| \vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2) \right| > 2$$

- **Approximately optimal** of choice of axes yields   Severi et al 2110.10112

$$\sqrt{2}|C_{ii} \pm C_{jj}| \leq 2$$



$\mathcal{C} > 0$ and $\mathcal{B} \geq 2$

- mathematically useful
- conceptually different

Quantum Magic

- **Stabilizer states** that can be efficiently simulated by a classical computer.
  - Entanglement is not enough to guarantee the advantage of quantum computing
  - Bell states have $C = 1$ but $M_2 = 0$!
- **Magic** States: A second layer of "**quantumness**" where quantum computing has advantages

$$\mathcal{M}_2 = -\log_2 \frac{1 + \sum_i (B_i^+)^4 + \sum_j (B_j^-)^4 + \sum_{ij} (C_{ij})^4}{1 + \sum_i (B_i^+)^2 + \sum_j (B_j^-)^2 + \sum_{ij} (C_{ij})^2}.$$

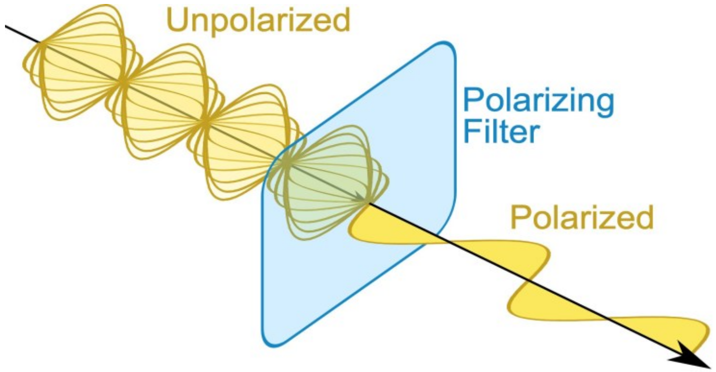
Stabilizer state $\quad \mathcal{M}_2 = 0$

Magic state $\quad 0 < \mathcal{M}_2 \leq \log_2 \frac{16}{7} \approx 1.192$

Liu, Low, and Yin, 2502.17550

Motivation

Y G, Tao Han, Matthew Low, Youle Su, arXiv: 2602.02719



- Simplest processes to explore / play
- High colliding energy & luminosity
- Control beam polarizations (60%-80%)
- Highly successful programs:

  LEP1, LEP2, SLC, SuperKEKB, BEPC II, DAϕNE …
- Next HEP collider: the Higgs factory

  STCF, FCC-ee, CEPC, ILC, LCF …

Quantum Tomography of $e^+e^- \to f\bar{f}$

- For the fully-polarized LR ($e_L^- e_R^+$) or RL($e_R^- e_L^+$) the final state $\left|f\bar{f}\right\rangle$ is a pure two-qubit state.

$$|\psi\rangle = a|\uparrow\uparrow\rangle + b|\downarrow\downarrow\rangle + c|\uparrow\downarrow\rangle + d|\downarrow\uparrow\rangle, \quad \text{with} \; |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$$

- Depending on the coefficients,

Separable state: $ab = cd$; **Entangled state**: $ab \neq cd$

- Full tomography

$$B_i^+ = \begin{pmatrix} da^* + cb^* + bc^* + ad^* \\ -i\,da^* + i\,cb^* - i\,bc^* + i\,ad^* \\ aa^* - bb^* + cc^* - dd^* \end{pmatrix}, \quad B_j^- = \begin{pmatrix} ca^* + db^* + ac^* + bd^* \\ -i\,ca^* + i\,db^* + i\,ac^* - i\,bd^* \\ aa^* - bb^* - cc^* + dd^* \end{pmatrix},$$

$$C_{ij} = \begin{pmatrix} ba^* + ab^* + dc^* + cd^* & -i\,ba^* + i\,ab^* + i\,dc^* - i\,cd^* & da^* - cb^* - bc^* + ad^* \\ -i\,ba^* + i\,ab^* - i\,dc^* + i\,cd^* & -ba^* - ab^* + dc^* + cd^* & -i\,da^* - i\,cb^* + i\,bc^* + i\,ad^* \\ ca^* - db^* + ac^* - bd^* & -i\,ca^* - i\,db^* + i\,ac^* + i\,bd^* & aa^* + bb^* - cc^* - dd^* \end{pmatrix}$$

Dynamics for $e^+e^- \to \gamma^*, Z^* \to f\bar{f}$

- Effective chiral couplings:

  (**BSM** will modify them!)

$$f_V^L = Q_e Q_f + \frac{g_L^e(g_L^f + g_R^f)}{2c_W^2 s_W^2}\frac{s}{s - m_Z^2 + i\Gamma_Z m_Z},$$
$$f_A^L = -\frac{g_L^e(g_L^f - g_R^f)}{2c_W^2 s_W^2}\frac{s}{s - m_Z^2 + i\Gamma_Z m_Z},$$
$$f_V^R = Q_e Q_f + \frac{g_R^e(g_L^f + g_R^f)}{2c_W^2 s_W^2}\frac{s}{s - m_Z^2 + i\Gamma_Z m_Z},$$
$$f_A^R = -\frac{g_R^e(g_L^f - g_R^f)}{2c_W^2 s_W^2}\frac{s}{s - m_Z^2 + i\Gamma_Z m_Z},$$

- ff polarizations & Spin correlation:

$$B_k^\pm = \frac{2(f_V \pm f_A c_\theta)(f_A \pm f_V c_\theta)}{(f_V^2 + f_A^2)(1 + c_\theta^2) \pm 4 f_V f_A c_\theta},$$

$$C_{ij} = \frac{1}{\mathcal{D}}\begin{pmatrix} s_\theta^2\left(f_V^2(2-\beta^2) - f_A^2\beta^2\right) & 0 & -2(f_V^2 c_\theta \pm f_V f_A \beta) s_\theta \sqrt{1-\beta^2} \\ 0 & (f_A^2 - f_V^2)\beta^2 s_\theta^2 & 0 \\ -2(f_V^2 c_\theta \pm f_V f_A \beta) s_\theta \sqrt{1-\beta^2} & 0 & f_V^2(2c_\theta^2 + \beta^2 s_\theta^2) + f_A^2\beta^2(1 + c_\theta^2) \pm 4 f_V f_A \beta \end{pmatrix}$$

with normalization factor $\mathcal{D} = f_V^2\left(2 - \beta^2 s_\theta^2\right) \pm 4\beta f_V f_A c_\theta + \beta^2 f_A^2\left(1 + c_\theta^2\right)$

We will not demand to reconstruct the particle decay!

Dynamics for $e^+e^- \to \gamma^*, Z^* \to f\bar{f}$

- Effective chiral couplings:

  (**BSM** will modify them!)

$$f_V^L = Q_e Q_f + \frac{g_L^e(g_L^f + g_R^f)}{2c_W^2 s_W^2}\frac{s}{s - m_Z^2 + i\Gamma_Z m_Z},$$
$$f_A^L = -\frac{g_L^e(g_L^f - g_R^f)}{2c_W^2 s_W^2}\frac{s}{s - m_Z^2 + i\Gamma_Z m_Z},$$
$$f_V^R = Q_e Q_f + \frac{g_R^e(g_L^f + g_R^f)}{2c_W^2 s_W^2}\frac{s}{s - m_Z^2 + i\Gamma_Z m_Z},$$
$$f_A^R = -\frac{g_R^e(g_L^f - g_R^f)}{2c_W^2 s_W^2}\frac{s}{s - m_Z^2 + i\Gamma_Z m_Z},$$

- ff polarizations & Spin correlation:

$$B_k^{\pm} = \frac{2(f_V \pm f_A c_\theta)(f_A \pm f_V c_\theta)}{(f_V^2 + f_A^2)(1 + c_\theta^2) \pm 4 f_V f_A c_\theta},$$

$$C_{ij} = \frac{1}{\mathcal{D}}\begin{pmatrix} s_\theta^2\,(f_V^2(2-\beta^2) - f_A^2\beta^2) & 0 & -2(f_V^2 c_\theta \pm f_V f_A \beta) s_\theta \sqrt{1-\beta^2} \\ 0 & (f_A^2 - f_V^2)\beta^2 s_\theta^2 & 0 \\ -2(f_V^2 c_\theta \pm f_V f_A \beta) s_\theta \sqrt{1-\beta^2} & 0 & f_V^2(2c_\theta^2 + \beta^2 s_\theta^2) + f_A^2\beta^2(1+c_\theta^2) \pm 4 f_V f_A \beta \end{pmatrix}$$

with normalization factor $\mathcal{D} = f_V^2(2 - \beta^2 s_\theta^2) \pm 4\beta f_V f_A c_\theta + \beta^2 f_A^2 (1 + c_\theta^2)$

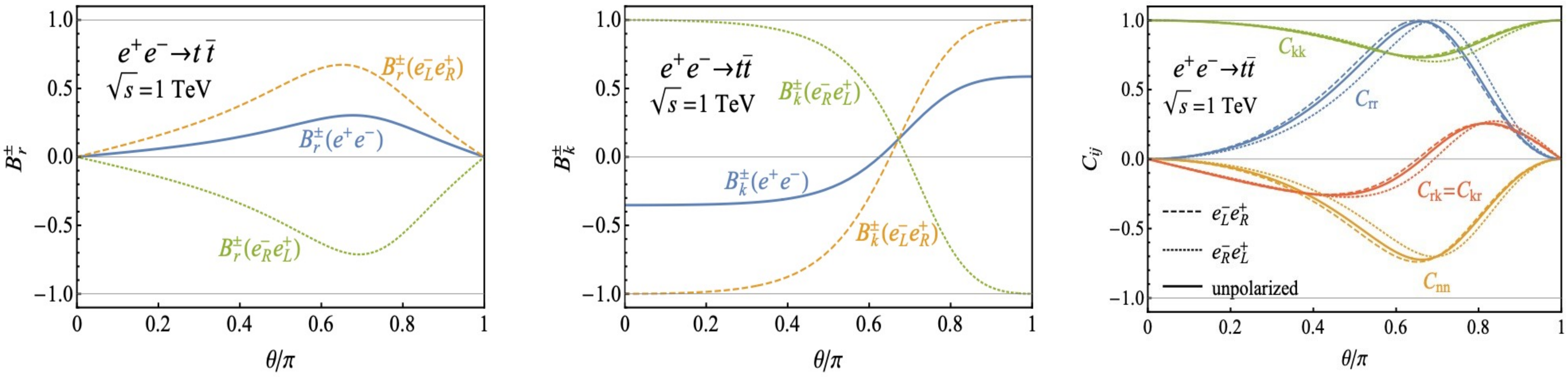
- Concurrence: $\mathcal{C} = \frac{\beta^2 s_\theta^2 (f_V^2 - f_A^2)}{\mathcal{D}}$ & Bell: $\mathcal{B} = 2\sqrt{1 + \mathcal{C}^2}.$

reaching the maximum $\mathcal{C} \to 1$, $\mathcal{B} \to 2\sqrt{2}$ at $\cos\theta = \mp f_A/f_V$.

Applicable to $f\bar{f}$ with / without decay!

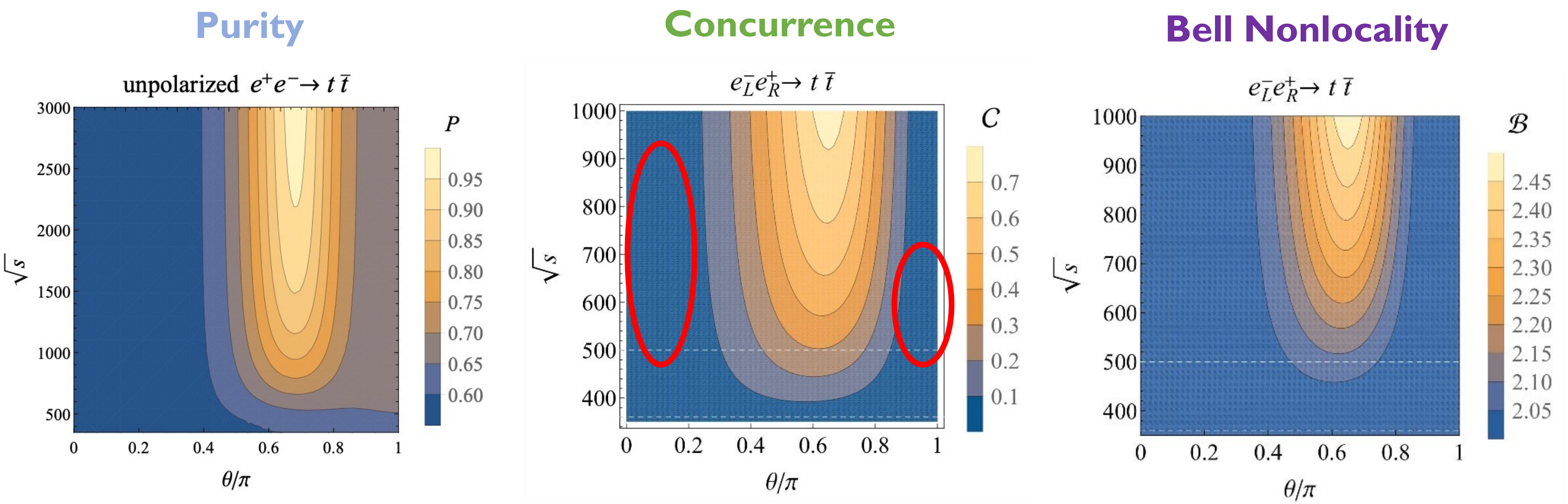
(1) $e^+e^- \to t\bar{t}$ at the threshold & above

- Fano coefficients of $e^-e^+ \to t\bar{t}$



RL $\Longleftrightarrow$ LR flip single-particle polarization vectors $\mathrm{B_i}$; spin-correlation matrix $\mathrm{C_{ij}}$ insensitive to polarization

(1) $e^+e^- \to t\bar{t}$ at the threshold & above



Radiative Correction:

Aoude, et al., 2604.16268

- Strongly correlated with the state purity $P = \mathrm{Tr}(\rho^2)$.
- TeV scale lepton collider can generate stronger entangled $t\bar{t}$ than that observed at the LHC
- No entanglement near threshold, nor in backward/forward scattering.
- Maximum at an angle $\cos\theta = \mp\beta\, f_A/f_V$ (with $f_A^R/f_V^R \approx 0.6$ and $f_A^L/f_V^L \approx -0.5$)

(1) $e^+e^- \to t\bar{t}$ the threshold & above

- At the high-energy (β → 1) **Helicity basis**

$$|\psi\rangle \propto g_L\, d^1_{1,1}(\theta)\ |\uparrow\uparrow\rangle + g_R\, d^1_{-1,1}(\theta)\ |\downarrow\downarrow\rangle\,, \qquad g_{L,R} = (f_V \pm f_A)/2.$$

$|\psi\rangle = a|\uparrow\uparrow\rangle + b|\downarrow\downarrow\rangle + c|\uparrow\downarrow\rangle + d|\downarrow\uparrow\rangle$, with $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$

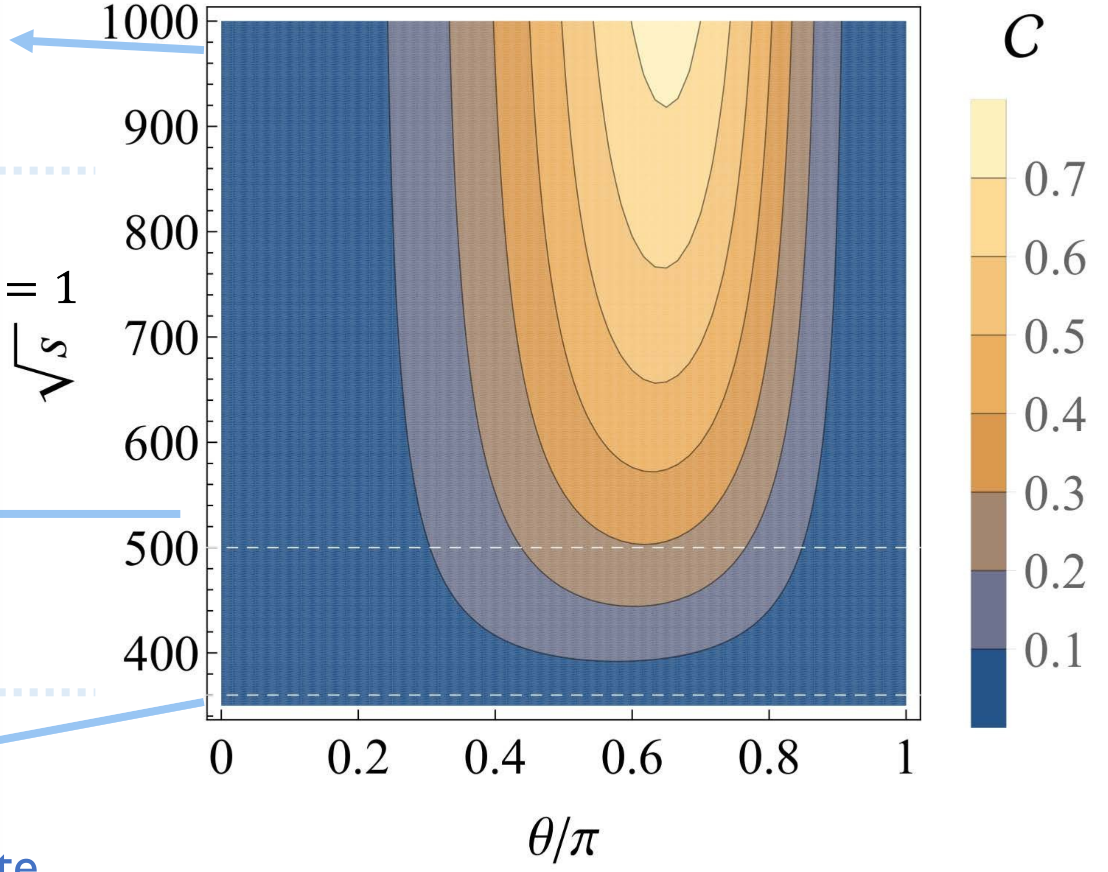
$$a = e^2\left(f_V^{R/L} + f_A^{R/L}\beta\right)(\cos\theta \pm 1),$$
$$b = e^2\left(f_V^{R/L} - f_A^{R/L}\beta\right)(\cos\theta \mp 1),$$
$$c = d = \mp e^2 f_V^{R/L}\sqrt{1-\beta^2}\,\sin\theta, \qquad c \propto m_t^2$$

- Near threshold (β → 0) **Fixed beam basis**

$t\bar{t}$ state $\quad |\psi_{\rm out}\rangle = |\downarrow\downarrow\rangle$ or $|\uparrow\uparrow\rangle$ is separable state

e^-e^+ beam $\quad |\psi_{\rm in}\rangle = |LR\rangle$ or $|RL\rangle$

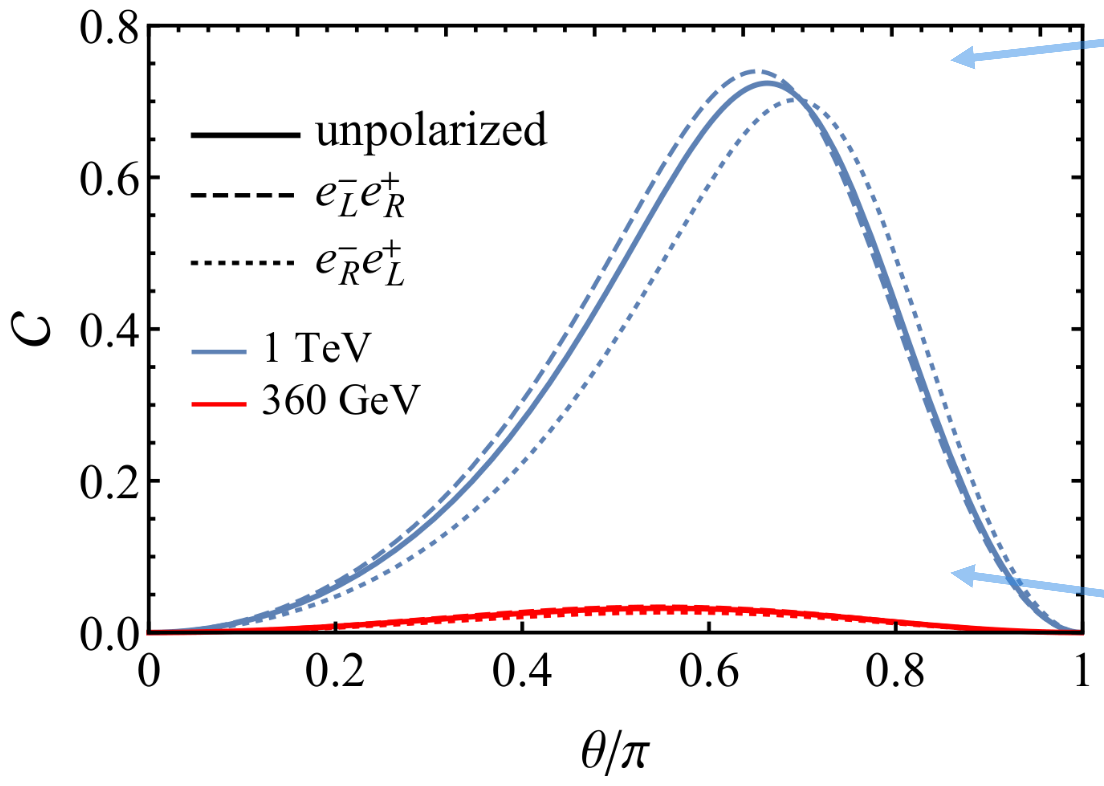
Concurrence

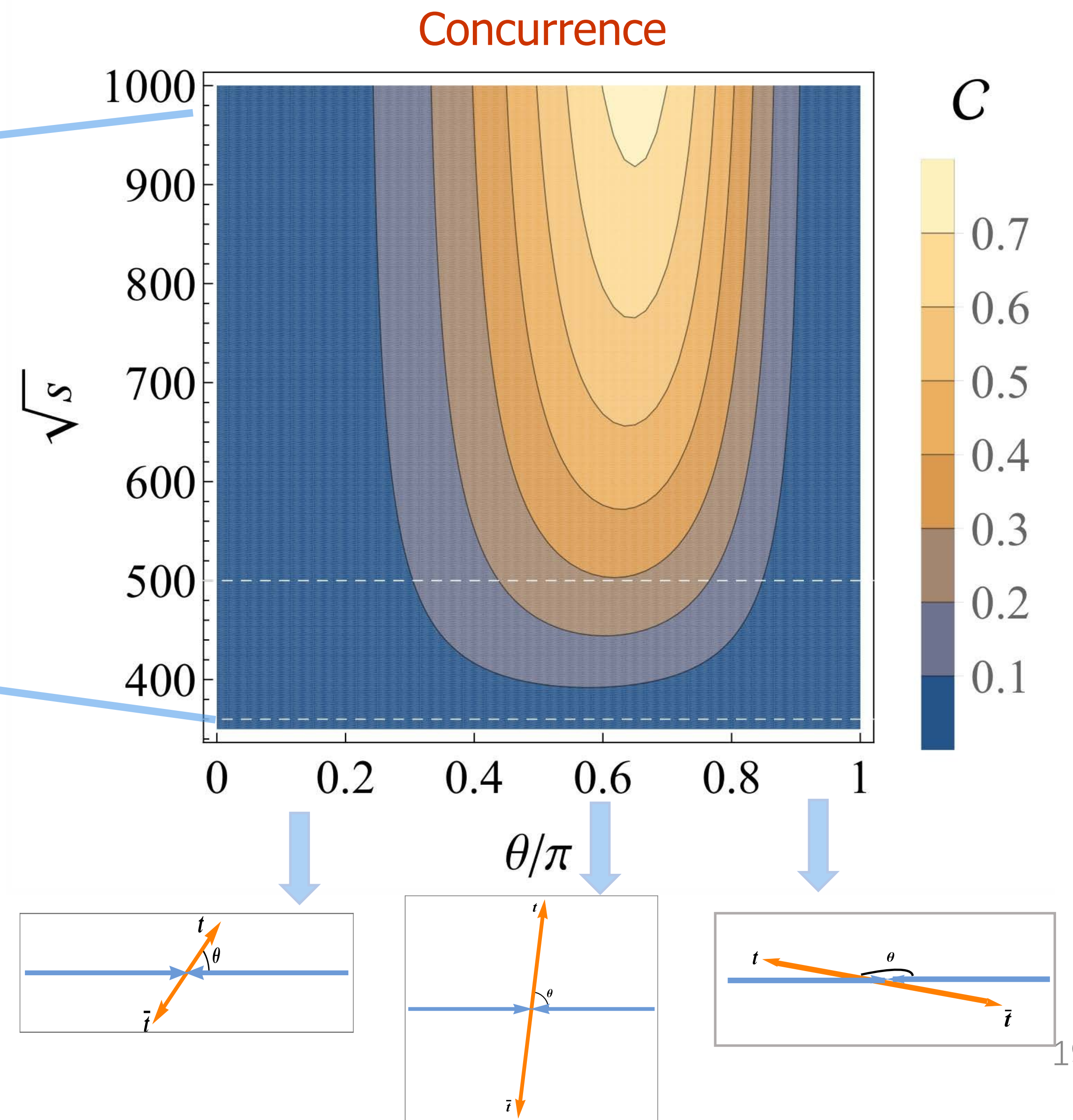


No entanglement at threshold,
nor in backward/forward scattering!

(1) $e^+e^- \to t\bar{t}$ at the threshold & above

$$|\psi\rangle \propto \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) + \epsilon\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \quad \epsilon \propto \frac{\sqrt{1-\beta^2}}{\sin\theta_{\max}}.$$



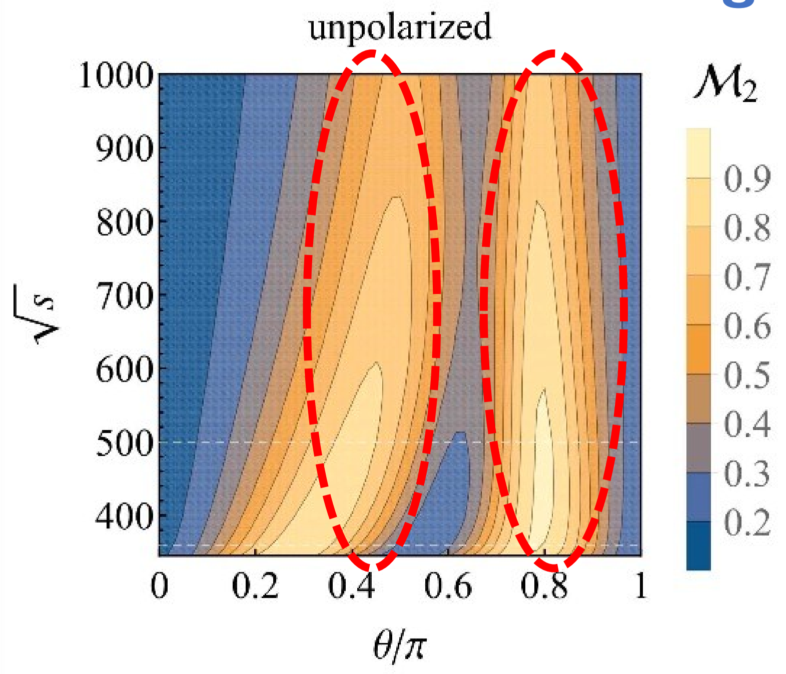


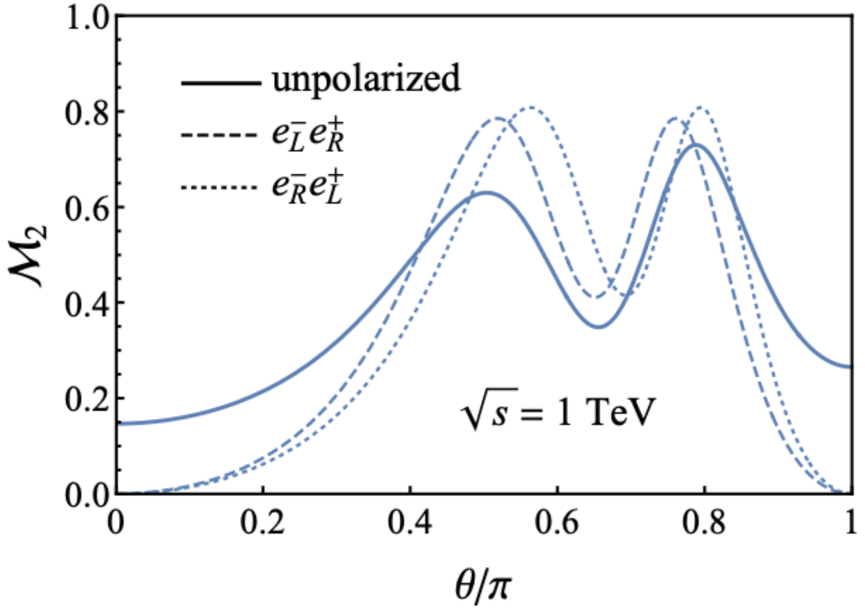
$\epsilon \to 0$ ➡ **Bell triplet state** ➡ $\mathcal{C} \to 1$

$\theta_{\max}^{\mathrm{LR}} < \theta_{\max}^{\mathrm{RL}}$ ➡ $|\epsilon_{LR}| < |\epsilon_{RL}|$ ➡ $\mathcal{C}_{\max}^{\mathrm{LR}} > \mathcal{C}_{\max}^{\mathrm{RL}}$

(1) $e^+e^- \to t\bar{t}$ at the threshold & above

Magic: the quantum resource

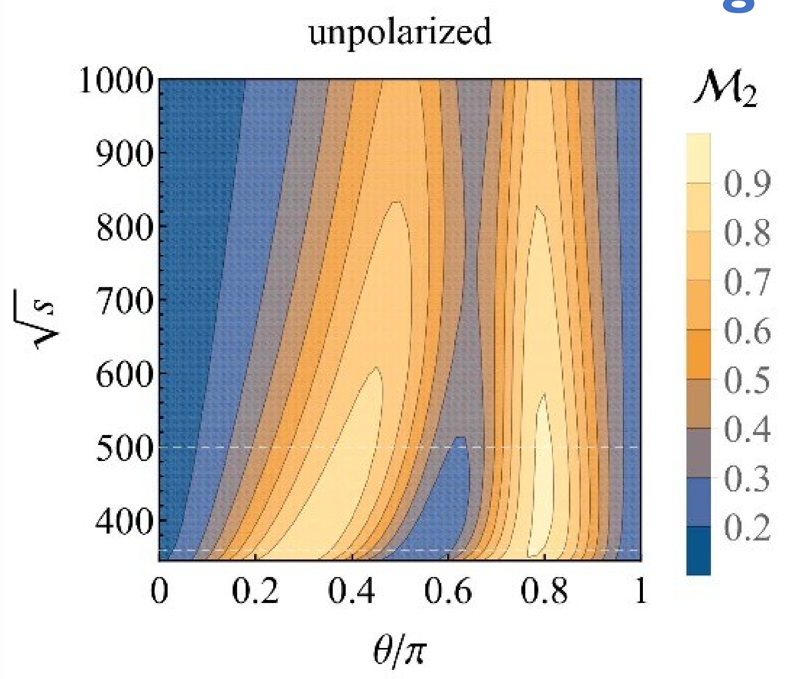


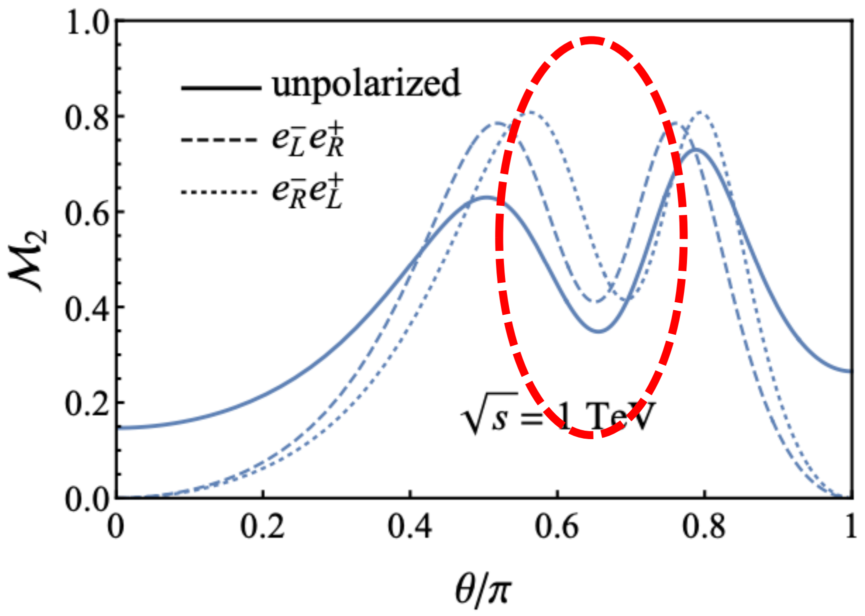


- Twin-peak of Magic
- Near Bell-state region $\Rightarrow$ Magic close to min!
- Zero entanglement may have non-zero magic!
- Polarization helps enhance magic!

(1) $e^+e^- \to t\bar{t}$ at the threshold & above

Magic: the quantum resource





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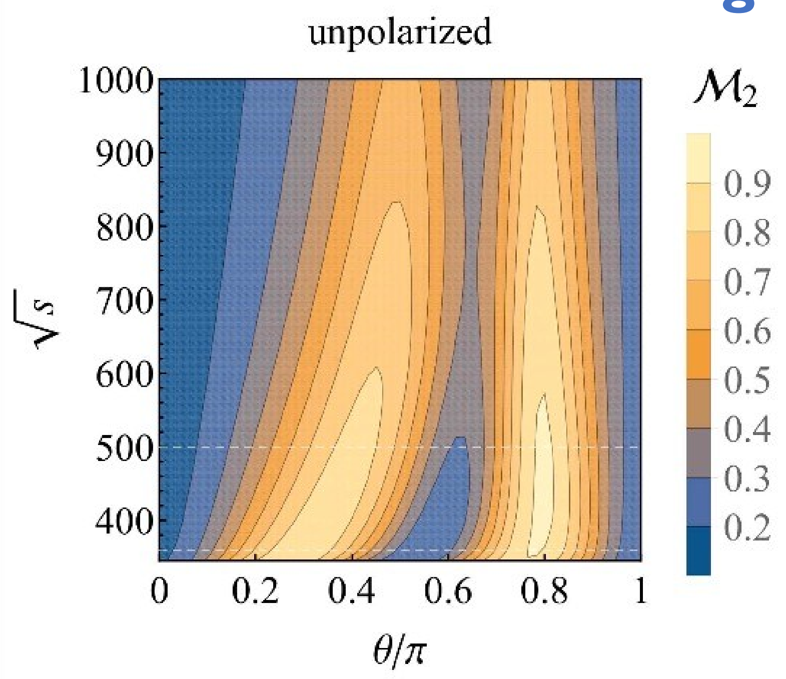
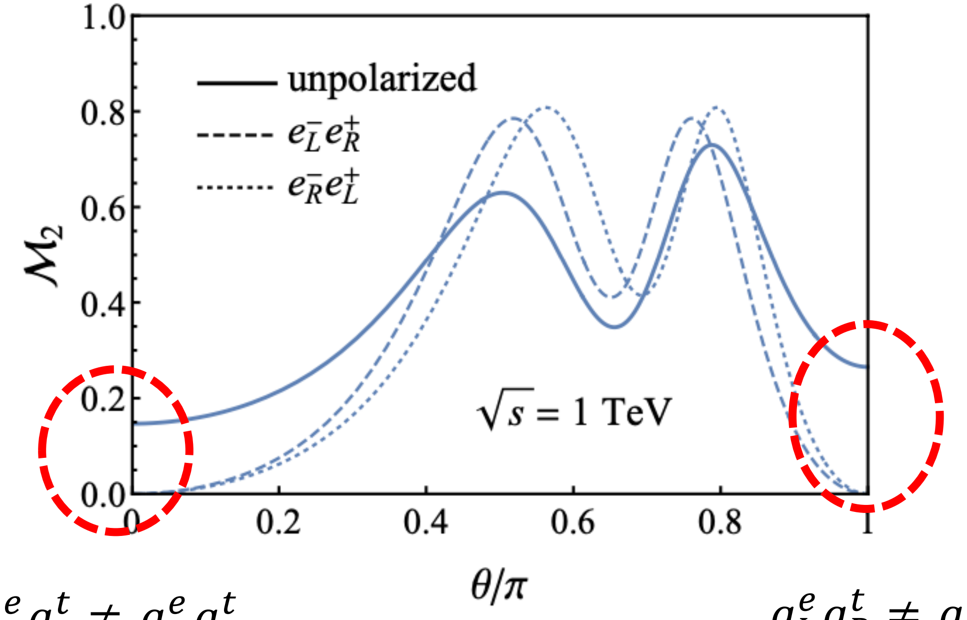
Magic: the quantum resource



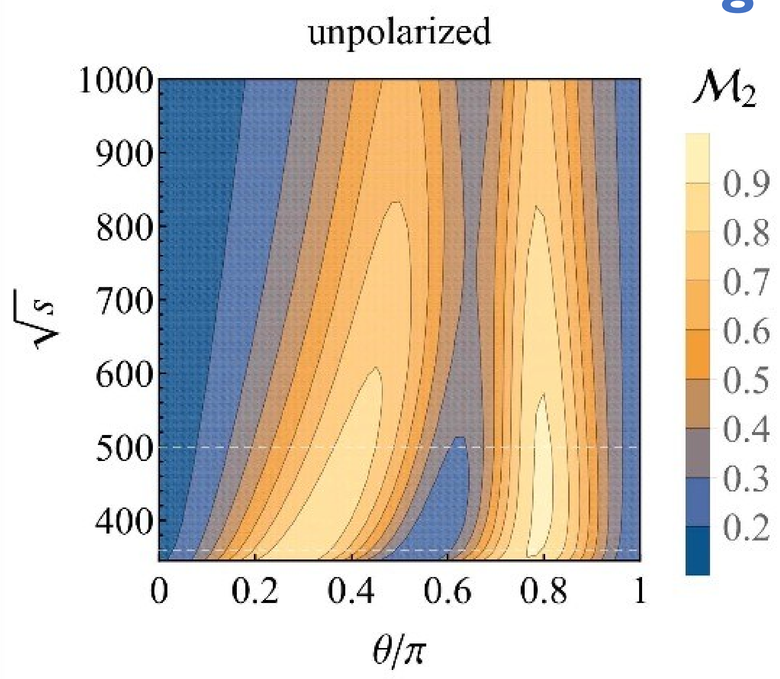


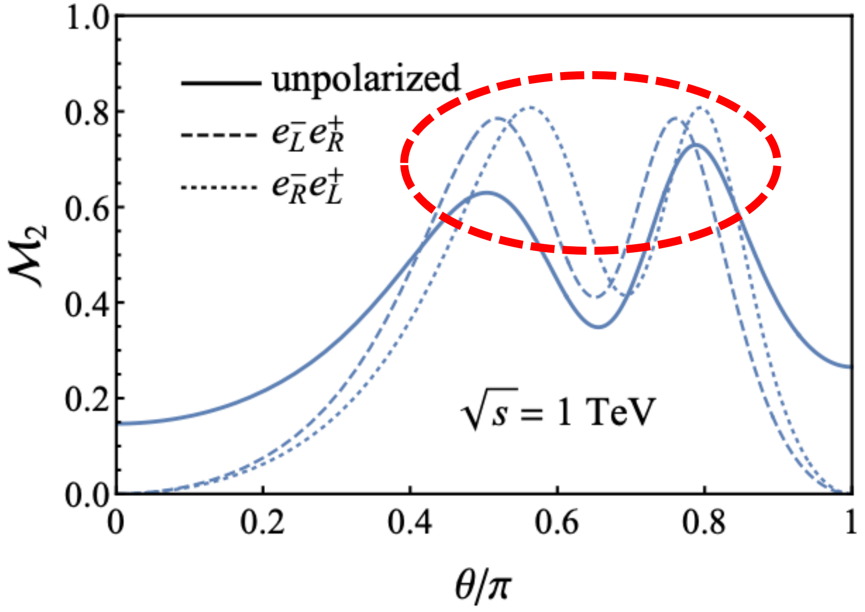
$g_L^e g_L^t \neq g_R^e g_R^t$ $\rightarrow$ $B_i^{\pm} \neq 0{,}1$ $\leftarrow$ $g_L^e g_R^t \neq g_R^e g_L^t$

- Twin-peak of Magic
- Near Bell-state region $\Rightarrow$ Magic close to min!
- Zero entanglement may have non-zero magic! C = 0 & $M_2 > 0$
- Polarization helps enhance magic!

(1) $e^+e^- \to t\bar{t}$ at the threshold & above

Magic: the quantum resource





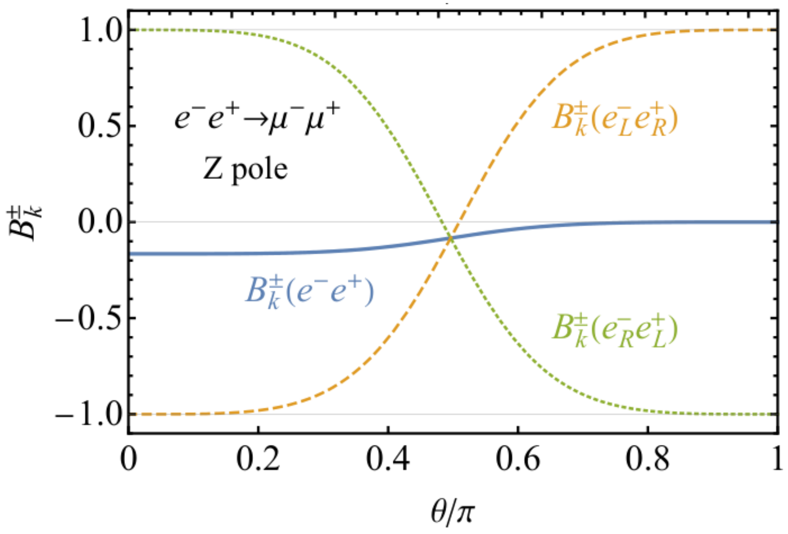
- Twin-peak of Magic
- Near Bell-state region $\Rightarrow$ Magic close to min!
- Zero entanglement may have non-zero magic!
- Polarization helps enhance magic!

(2) $e^+e^- \to \mu^+\mu^-/\tau^+\tau^-$ at the Z pole & beyond

- ## Quamtum states for $e^-e^+ \to \mu^-\mu^+$

$|\psi\rangle = a|\uparrow\uparrow\rangle + b|\downarrow\downarrow\rangle$, with a and b for polarization RL (LR)

$$a \propto \left(1 + \frac{f_A^{R/L}}{f_V^{R/L}}\right)(1 \pm \cos\theta), \quad b \propto \left(1 - \frac{f_A^{R/L}}{f_V^{R/L}}\right)(1 \mp \cos\theta),$$

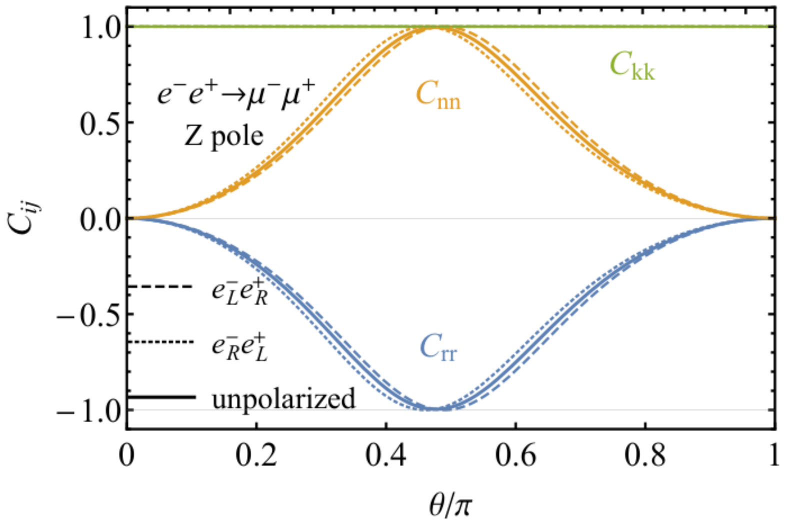


RL $\Longleftrightarrow$ LR flip f-polarization $\mathrm{B_i}$;

- ## Density matrix

The unpolarized process is a mixture of polarized processes.

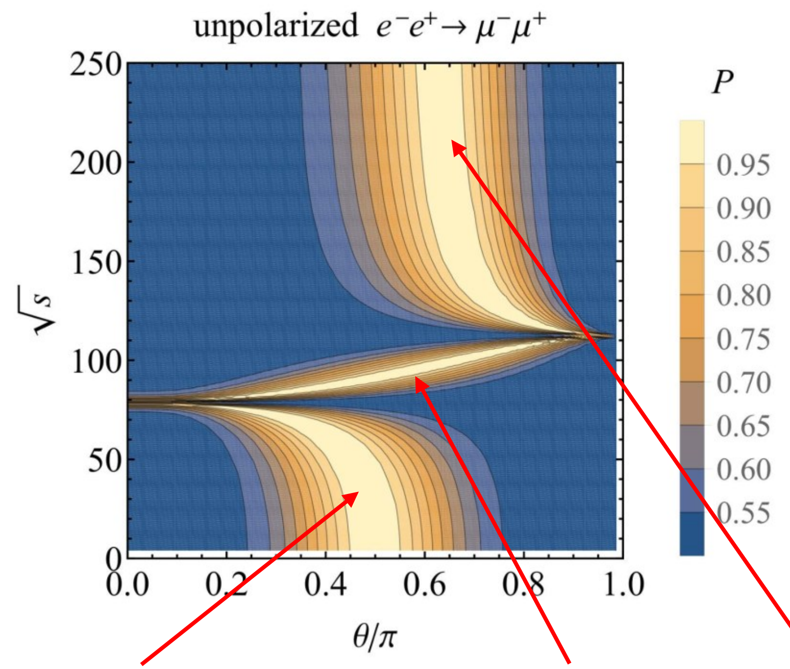
$$\rho_{\text{mixed}} = \begin{pmatrix} \rho_{\uparrow\uparrow}^{\uparrow\uparrow} & 0 & 0 & \rho_{\downarrow\downarrow}^{\uparrow\uparrow} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{\uparrow\uparrow}^{\downarrow\downarrow} & 0 & 0 & \rho_{\downarrow\downarrow}^{\downarrow\downarrow} \end{pmatrix}.$$



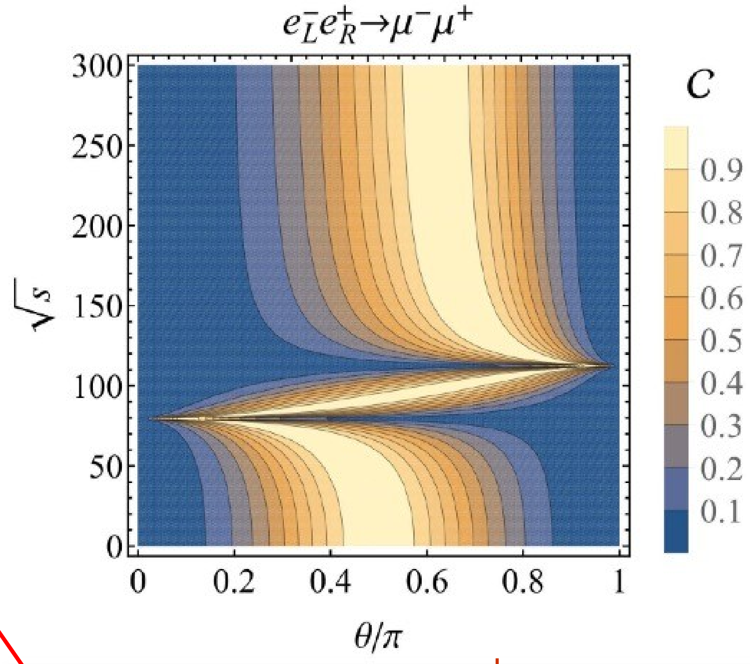
$\mathrm{C_{ij}}$ insensitive to polarization

(2) $e^+e^- \to \mu^+\mu^-/\tau^+\tau^-$ at the Z pole & beyond

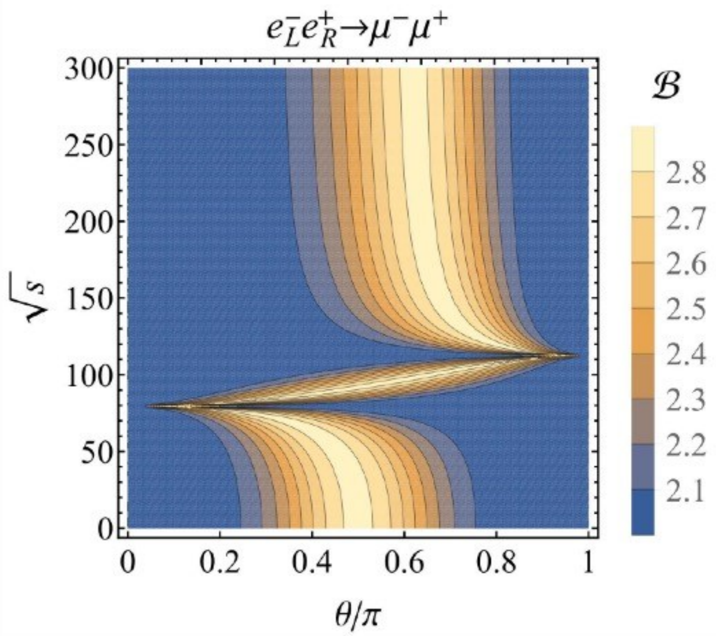
Purity



Concurrence



Bell Nonlocality



QED symmetric

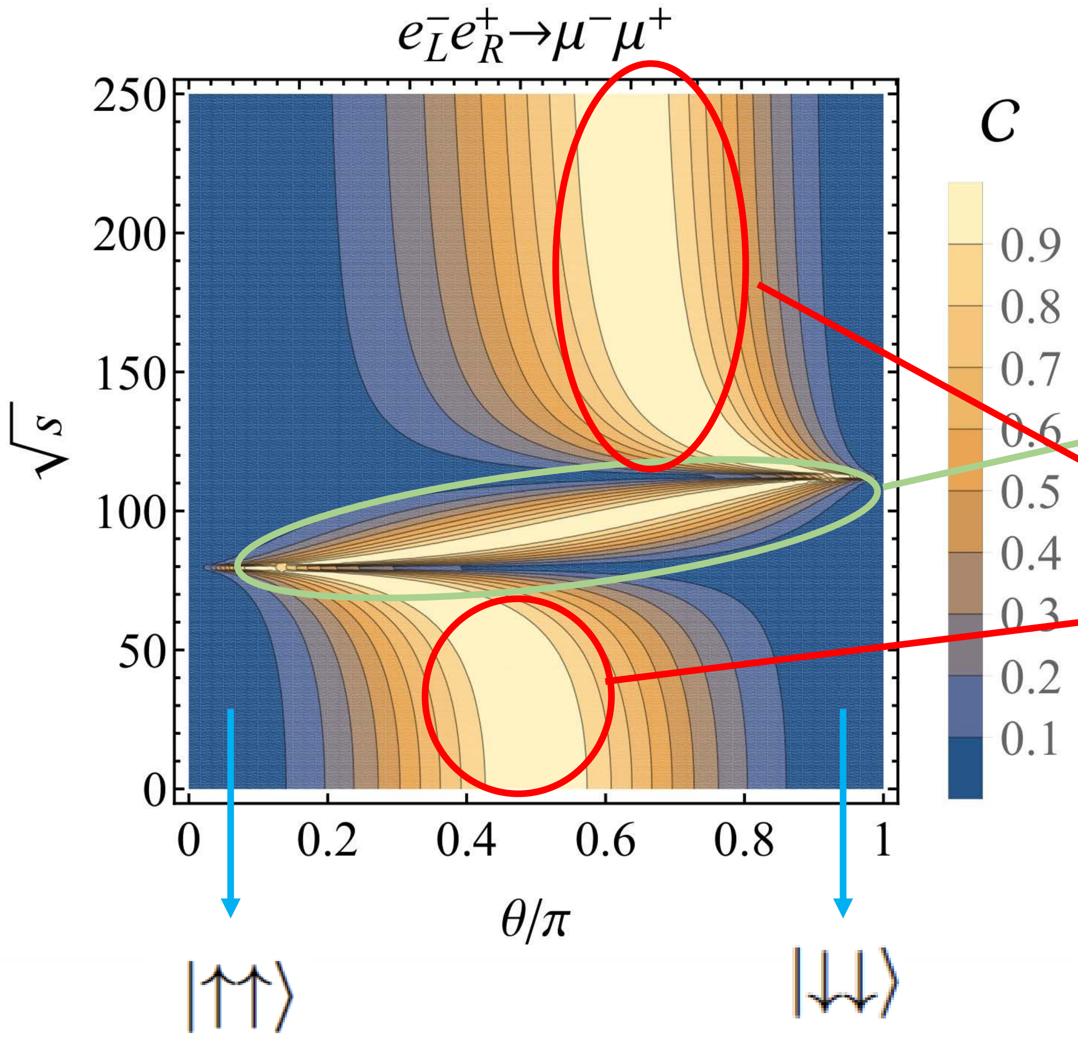
Weak chiral

$\mu^+\mu^-$ pol's dictates entanglement

Interference of QED and EW.

parity-violating shift θ to backward region (cos θ < 0).

(2) $e^+e^- \to \mu^+\mu^-/\tau^+\tau^-$ at the Z pole & beyond



Different Bell state at different energy scales:

governed by the vector/axial vector couplings

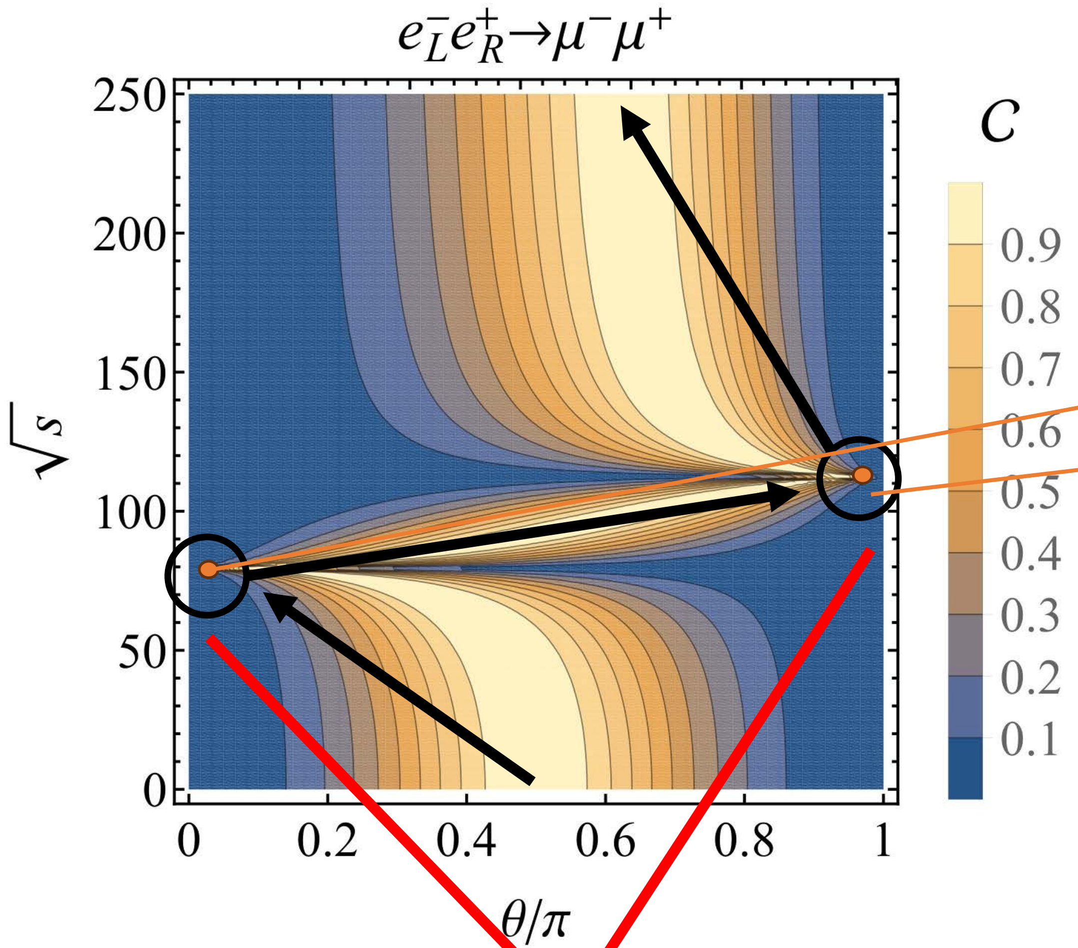
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)$$

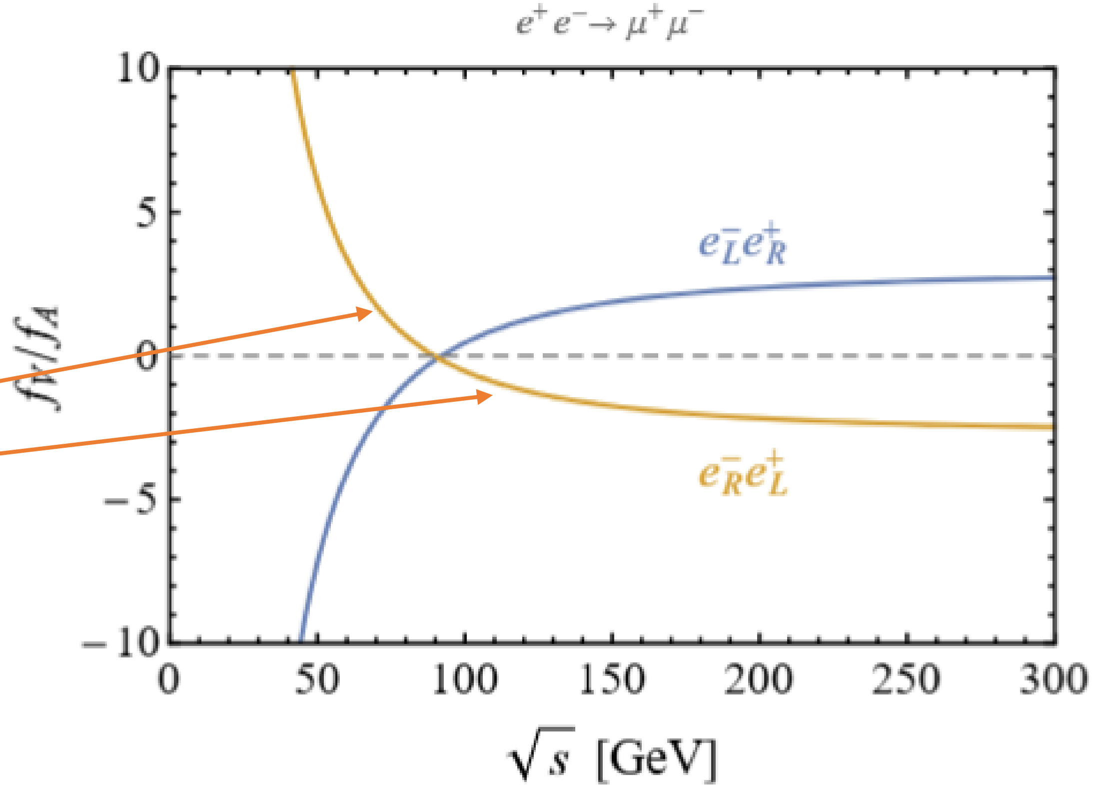
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

Unpolarized Beam:

- Classical mixing of fully polarized case
- Max Entanglement close to pure state

(2) $e^+e^- \to \mu^+\mu^-/\tau^+\tau^-$ at the Z pole & beyond





Ratio between vector and axial coupling strength controls whether the state becomes separable or entangled

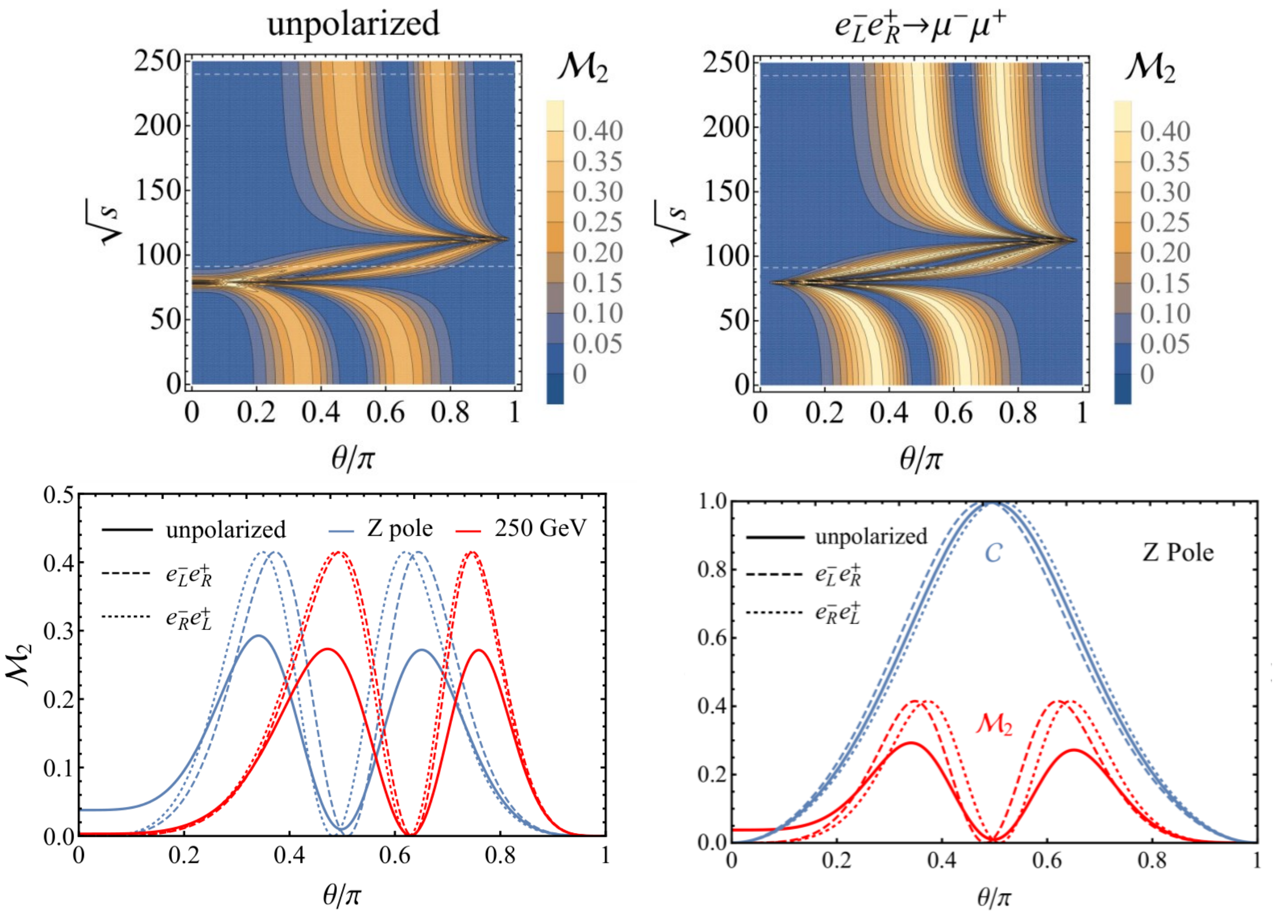
$f_A/f_V = \pm 1$ Max Parity Violation

separable state ↑↑ and ↓↓ , C = 0 and B = 2

$$\frac{f_V^{RL}}{f_A^{RL}} \sim -\frac{f_V^{LR}}{f_A^{LR}} \Rightarrow$$ LR similar to RL

(2) $e^+e^- \to \mu^+\mu^-/\tau^+\tau^-$ at the Z pole & beyond

Magic



- Two Peaks of Magic & Max entanglement → low magic
- Magic is more sensitive to polarization

(3) Bhabha scattering & t-s interference

- unpolarized e^+e^- system

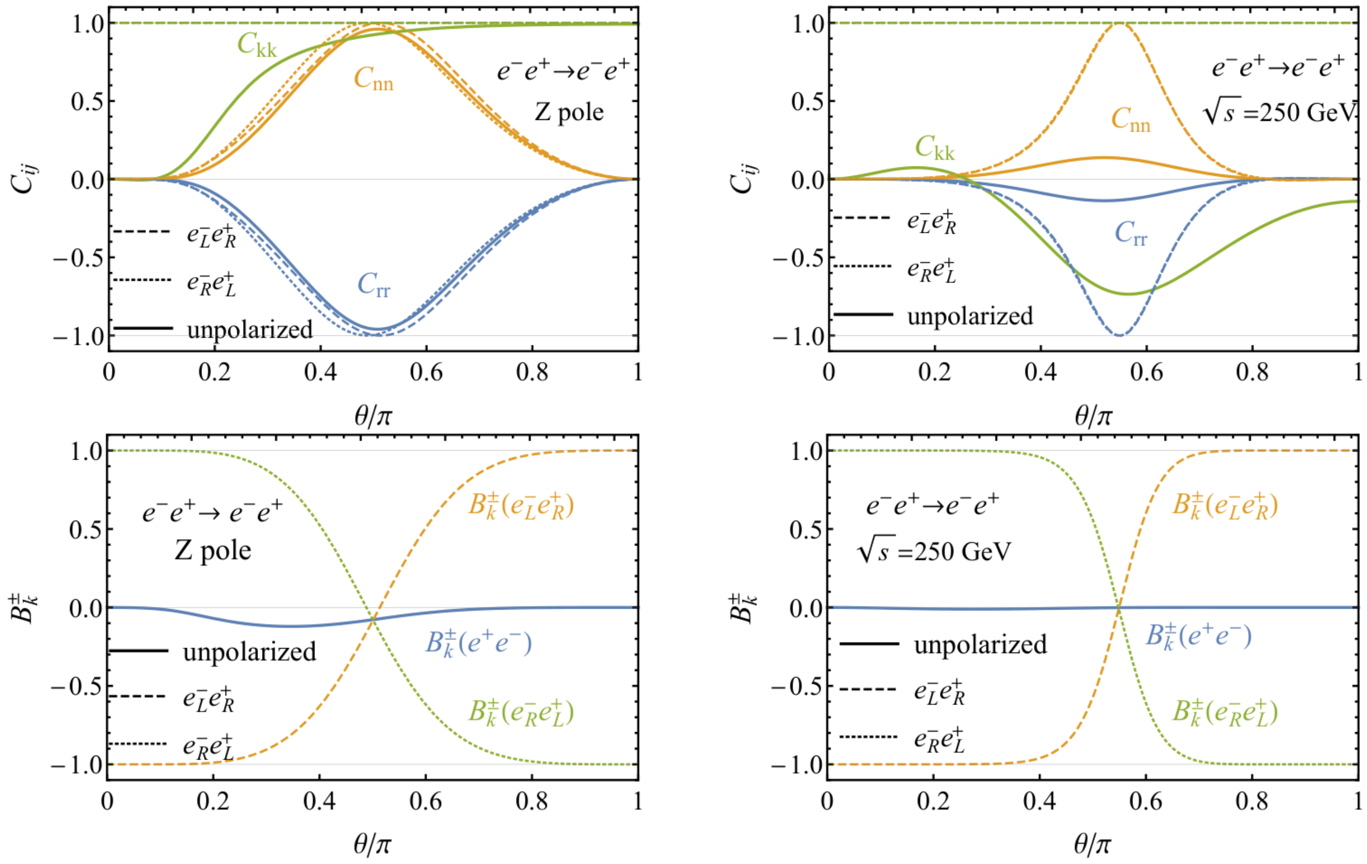
$$\rho_{\text{mixed}} = \begin{pmatrix} \rho_{\uparrow\uparrow,\uparrow\uparrow} & 0 & 0 & \rho_{\uparrow\uparrow,\downarrow\downarrow} \\ 0 & \rho_{\uparrow\downarrow,\uparrow\downarrow} & 0 & 0 \\ 0 & 0 & \rho_{\downarrow\uparrow,\downarrow\uparrow} & 0 \\ \rho_{\downarrow\downarrow,\uparrow\uparrow} & 0 & 0 & \rho_{\downarrow\downarrow,\downarrow\downarrow} \end{pmatrix}$$

- Fano coefficients of $e^-e^+ \to e^-e^+$

$$C_{ij} = \begin{pmatrix} C_{rr} & 0 & 0 \\ 0 & C_{nn} & 0 \\ 0 & 0 & C_{kk} \end{pmatrix}, \qquad B_i^{\pm} = \begin{pmatrix} 0 \\ 0 \\ B_k \end{pmatrix}$$

- Concurrence: $\mathcal{C} = \max\left\{0, C_{rr} + \frac{1}{2}C_{kk} - \frac{1}{2}\right\}$

- Bell: $\mathcal{B} = \begin{cases} 2\sqrt{C_{rr}^2 + C_{kk}^2}, & \text{if } |C_{kk}| > |C_{rr}|, \\ 2\sqrt{2}|C_{rr}|, & \text{if } |C_{kk}| < |C_{rr}|. \end{cases}$



Key difference from s-channel-only processes:

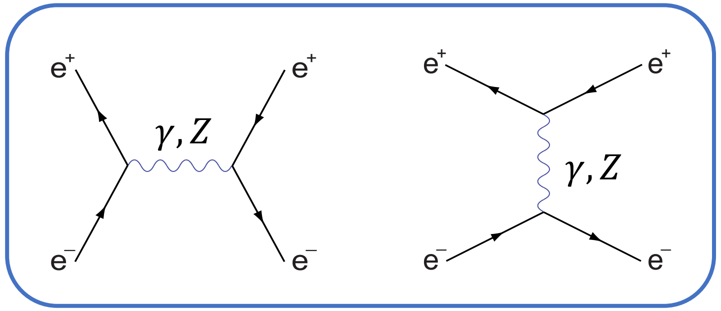
C_{ij} itself changes with polarization in Bhabha, polarization reweights s vs t channel contributions.

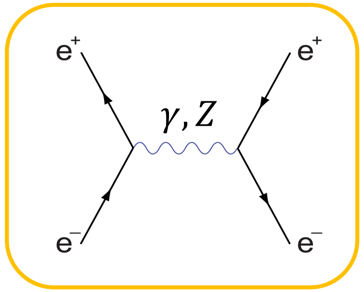
(3) Bhabha scattering & t-s interference

Bhabha scattering has both s and t channels

Helicity amplitudes separate into 3 categories:

- s+t interference: M(LR, ↓↓) = M(RL, ↑↑)
- s-channel only: M(LR, ↑↑) and M(RL, ↓↓)
- t-channel only: M(LL, ↓↑) = M(RR, ↑↓)





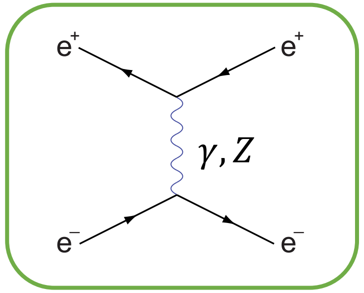
Entangled State

$|\psi\rangle = \alpha|\uparrow\uparrow\rangle + \beta|\downarrow\downarrow\rangle$

☐ How t-channel influences QI?

✓ Owing to the helicity conservation in massless scattering, t-channel results in separable states

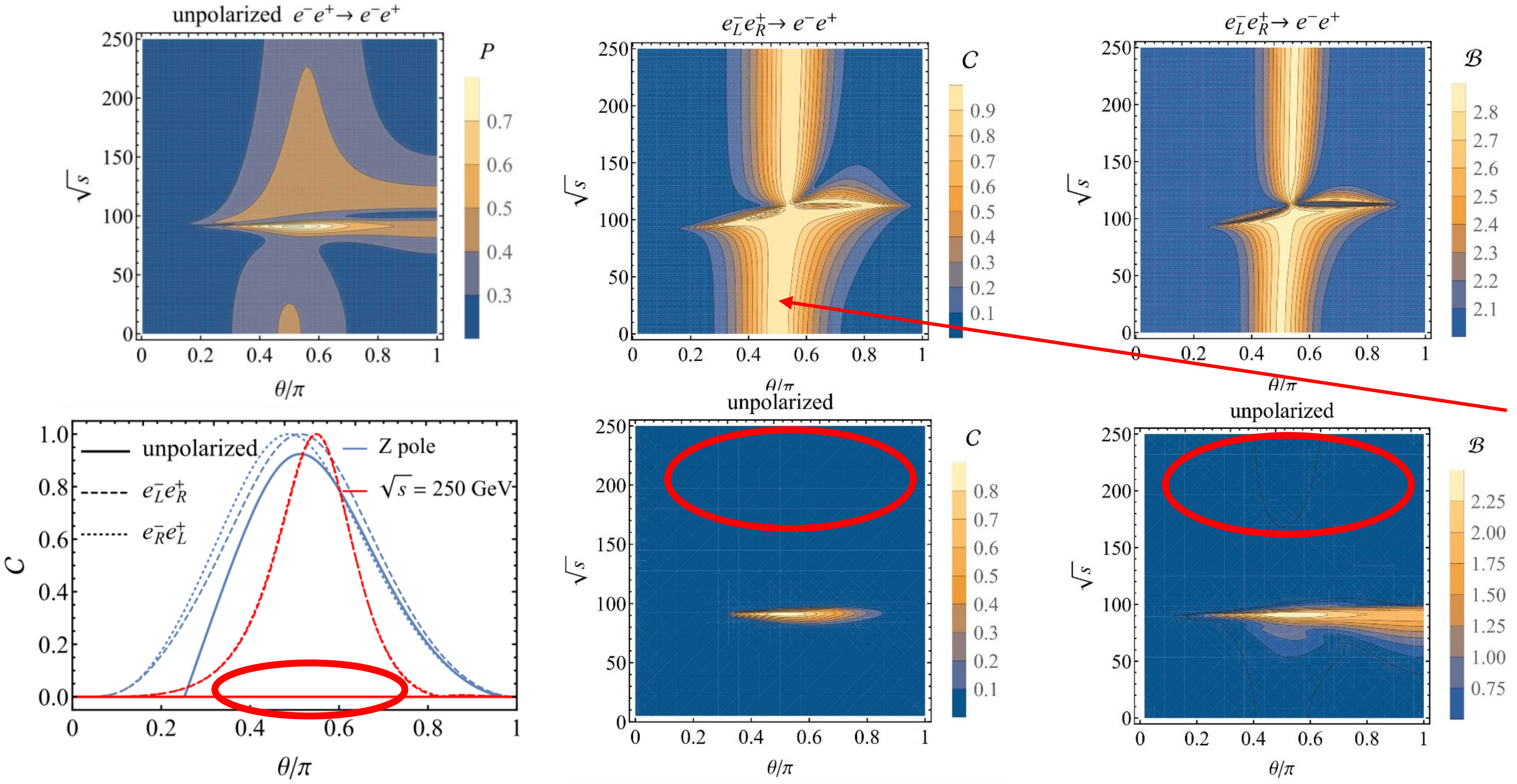
$$|\psi_{RR}\rangle = c_{RR}\,|\uparrow\downarrow\rangle\,, \quad |\psi_{LL}\rangle = d_{LL}\,|\downarrow\uparrow\rangle$$



Separable State

$|\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle$

(3) Bhabha scattering & t-s interference

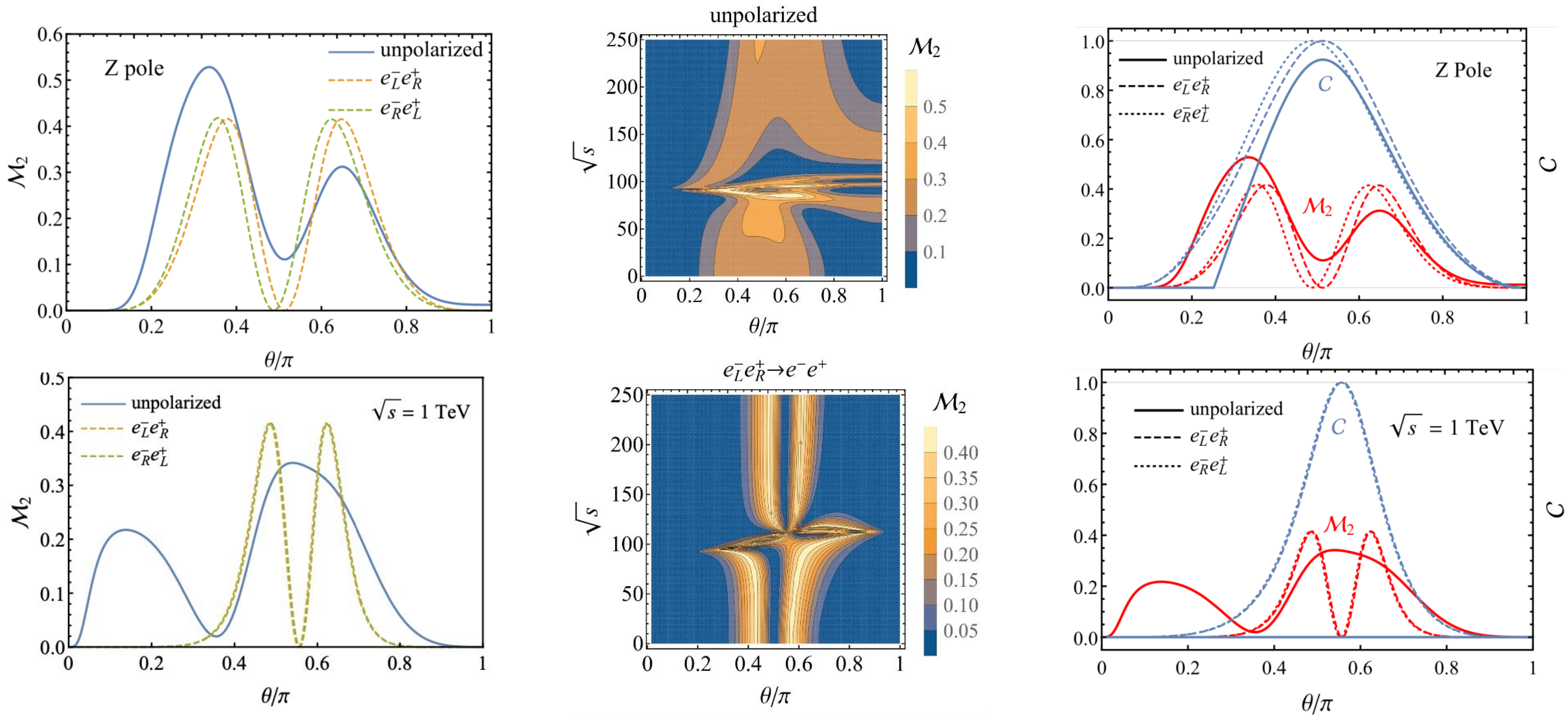


High Entanglement

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)$$

- Z-pole and RL, LR beams similar to $e^-e^+ \to \mu^-\mu^+$
- Unpolarized largely washed out by t-channel RR, LL
- Complete polarization will significantly enhance entanglement.

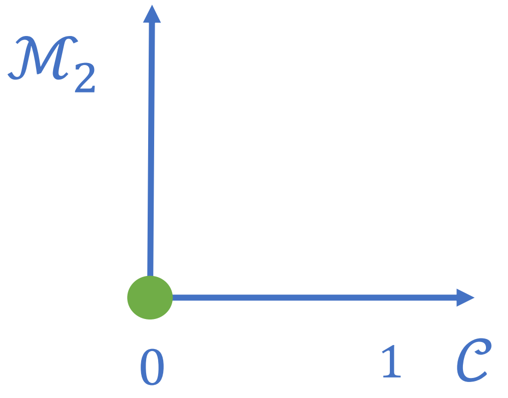
(3) Bhabha scattering & t-s interference



- Magic exhibits twin-peak structure
- Some interesting area: C = 0 but M_2>0;

Entanglement vs. Magic in $f\bar{f}$ systems

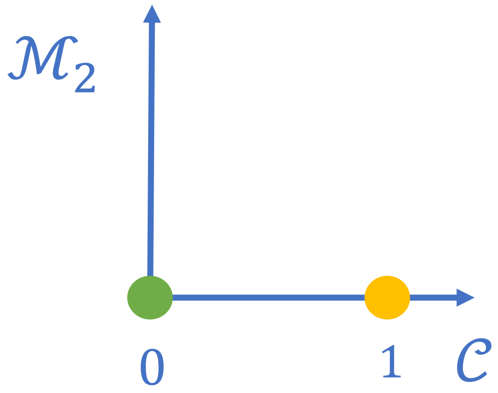
- Magic is more polarization-tunable
- Parameter space can be classified into 4 regimes:



C →0 M_2 →0	C → 1 M_2 →0	0 < C < 1 M_2 > 0	C = 0 M_2 > 0
Separable States + Stabilizer State	Maximally Entangled + Stabilizer State	Entangled + Non-stabilizer State	Separable States + Non-stabilizer State
polarized $\theta \to 0, \pi$	$\cos\theta \approx \mp\beta\, f_A/f_V$	twin-peak	1. unpolarized $\theta \to 0, \pi$ 2. s/t Interference dominate

- In the forward and backward scattering limits, Wigner D-matrix lead state to $|\downarrow\downarrow\rangle$ or $|\uparrow\uparrow\rangle$
- Quantum states aligned with the stabilizer axes, efficiently simulatable classically.

Entanglement vs. Magic in $f\bar{f}$ systems



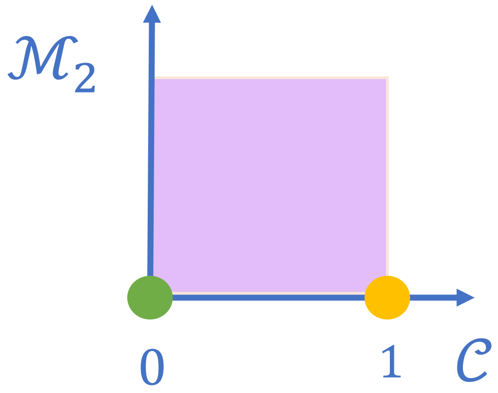
- Magic is more polarization-tunable
- Parameter space can be classified into 4 regimes:

$C \to 0$ $M_2 \to 0$	$C \to 1$ $M_2 \to 0$	$0 < C < 1$ $M_2 > 0$	$C = 0$ $M_2 > 0$
Separable States + Stabilizer State	Maximally Entangled + Stabilizer State	Entangled + Non-stabilizer State	Separable States + Non-stabilizer State
polarized $\theta \to 0, \pi$	$\cos\theta \approx \mp\beta\ f_A/f_V$	twin-peak	1. unpolarized $\theta \to 0, \pi$ 2. s/t Interference dominate

- Bell State $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)$, or as joint eigenstates of Pauli operators
- This limiting case could only be reached with massless fermions $\beta \to 1$

Entanglement vs. Magic in $f\bar{f}$ systems

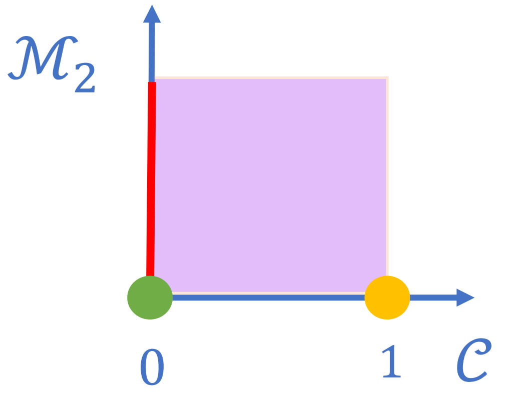
- Magic is more polarization-tunable
- Parameter space can be classified into 4 regimes:



$C \to 0$ $M_2 \to 0$	$C \to 1$ $M_2 \to 0$	$0 < C < 1$ $M_2 > 0$	$C = 0$ $M_2 > 0$
Separable States + Stabilizer State	Maximally Entangled + Stabilizer State	Entangled + Non-stabilizer State	Separable States + Non-stabilizer State
polarized $\theta \to 0, \pi$	$\cos\theta \approx \mp\beta\, f_A/f_V$	twin-peak	1. unpolarized $\theta \to 0, \pi$ 2. s/t Interference dominate

- The general and common case in the transition zones between the product-state limit and the Bell-state limit.

Entanglement vs. Magic in $f\bar{f}$ systems



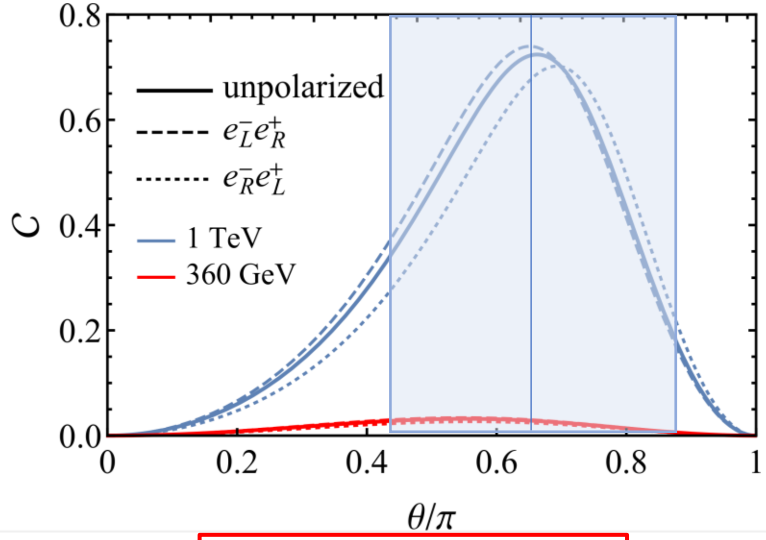
- Magic is more polarization-tunable
- Parameter space can be classified into 4 regimes:

$C \to 0$ $M_2 \to 0$	$C \to 1$ $M_2 \to 0$	$0 < C < 1$ $M_2 > 0$	$C = 0$ $M_2 > 0$
Separable States + Stabilizer State	Maximally Entangled + Stabilizer State	Entangled + Non-stabilizer State	Separable States + Non-stabilizer State
polarized $\theta \to 0, \pi$	$\cos\theta \approx \mp\beta\ f_A/f_V$	twin-peak	1. unpolarized $\theta \to 0, \pi$ 2. s/t Interference dominate

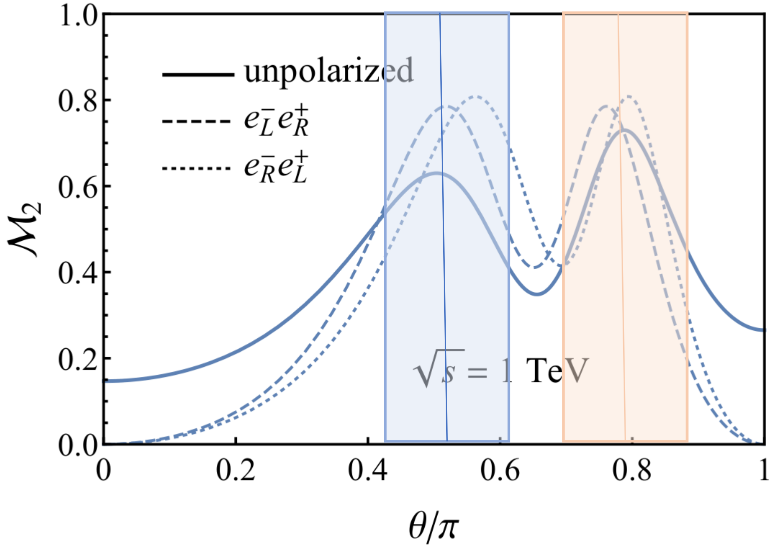
- Unpolarized beams lead to an incoherent mixture in the forward/backward limits
- Coherent superpositions with phase differences and weights that cannot be reduced to stabilizer structures

QI Measurement at Colliders

$e^+e^- \to t\bar{t}$



$$\theta_c - \frac{\Delta\theta}{2} < \theta_t < \theta_c + \frac{\Delta\theta}{2}.$$



Decay Approach for comparison

$\sqrt{s} = 1$ TeV		Entanglement Measure ($\mathcal{C}$)					Bell Nonlocality ($\mathcal{B}-2$)				
(P_{e^-}, P_{e^+})	Δ_{sys}	$\mathcal{C}$	$\Delta_{tot}(\mathcal{C})$	$\mathcal{S},\ \mathcal{S}^{-1}$	θ_c	$\Delta\theta$	$\mathcal{B}-2$	$\Delta_{tot}(\mathcal{B})$	$\mathcal{S},\ \mathcal{S}^{-1}$	θ_c	$\Delta\theta$
$(0,0)$	1%	0.49	0.022	$> 5\sigma$, 4.5%	119°	92°	0.37	0.090	4.2σ	119°	37°
	2%	0.55	0.035	$> 5\sigma$, 6.4%	119°	75°	0.41	0.133	3.1σ	119°	29°
$(+0.8, -0.6)$	1%	0.41	0.025	$> 5\sigma$, 6.1%	124°	110°	0.34	0.10	3.3σ	124°	38°
	1%	0.41	0.032	$> 5\sigma$, 7.9%	124°	110°	0.37	0.14	2.6σ	124°	31°
$(-0.8, +0.6)$	1%	0.49	0.020	$> 5\sigma$, 4.2%	116°	101°	0.41	0.074	$> 5\sigma$, 18%	116°	34°
	2%	0.49	0.032	$> 5\sigma$, 6.5%	116°	101°	0.45	0.12	3.7σ	116°	25°

$\sqrt{s} = 1$ TeV		Central Region					Backward Region				
(P_{e^-}, P_{e^+})	Δ_{sys}	$\mathcal{M}_2$	$\Delta_{tot}(\mathcal{M}_2)$	$\mathcal{S},\ \mathcal{S}^{-1}$	θ_c	$\Delta\theta$	$\mathcal{M}_2$	$\Delta_{tot}(\mathcal{M}_2)$	$\mathcal{S},\ \mathcal{S}^{-1}$	θ_c	$\Delta\theta$
$(0,0)$	1%	0.58	0.040	$> 5\sigma, 2.4\%$	91°	48°	0.66	0.058	$> 5\sigma, 3.0\%$	141°	48°
	2%	0.60	0.061	$> 5\sigma, 3.5\%$	91°	35°	0.66	0.070	$> 5\sigma, 3.7\%$	141°	48°
$(+0.8, -0.6)$	1%	0.81	0.035	$> 5\sigma, 1.5\%$	99°	48°	0.78	0.067	$> 5\sigma, 3.0\%$	145°	40°
	2%	0.82	0.052	$> 5\sigma, 2.2\%$	99°	36°	0.78	0.077	$> 5\sigma, 3.4\%$	145°	40°
$(-0.8, +0.6)$	1%	0.79	0.029	$> 5\sigma, 1.3\%$	92°	45°	0.77	0.039	$> 5\sigma, 1.7\%$	137°	40°
	2%	0.79	0.049	$> 5\sigma, 2.1\%$	92°	34°	0.78	0.055	$> 5\sigma, 2.4\%$	137°	37°

Summary: All Processes Exceed 5σ

Outlook: QI as a probe of BSM interactions

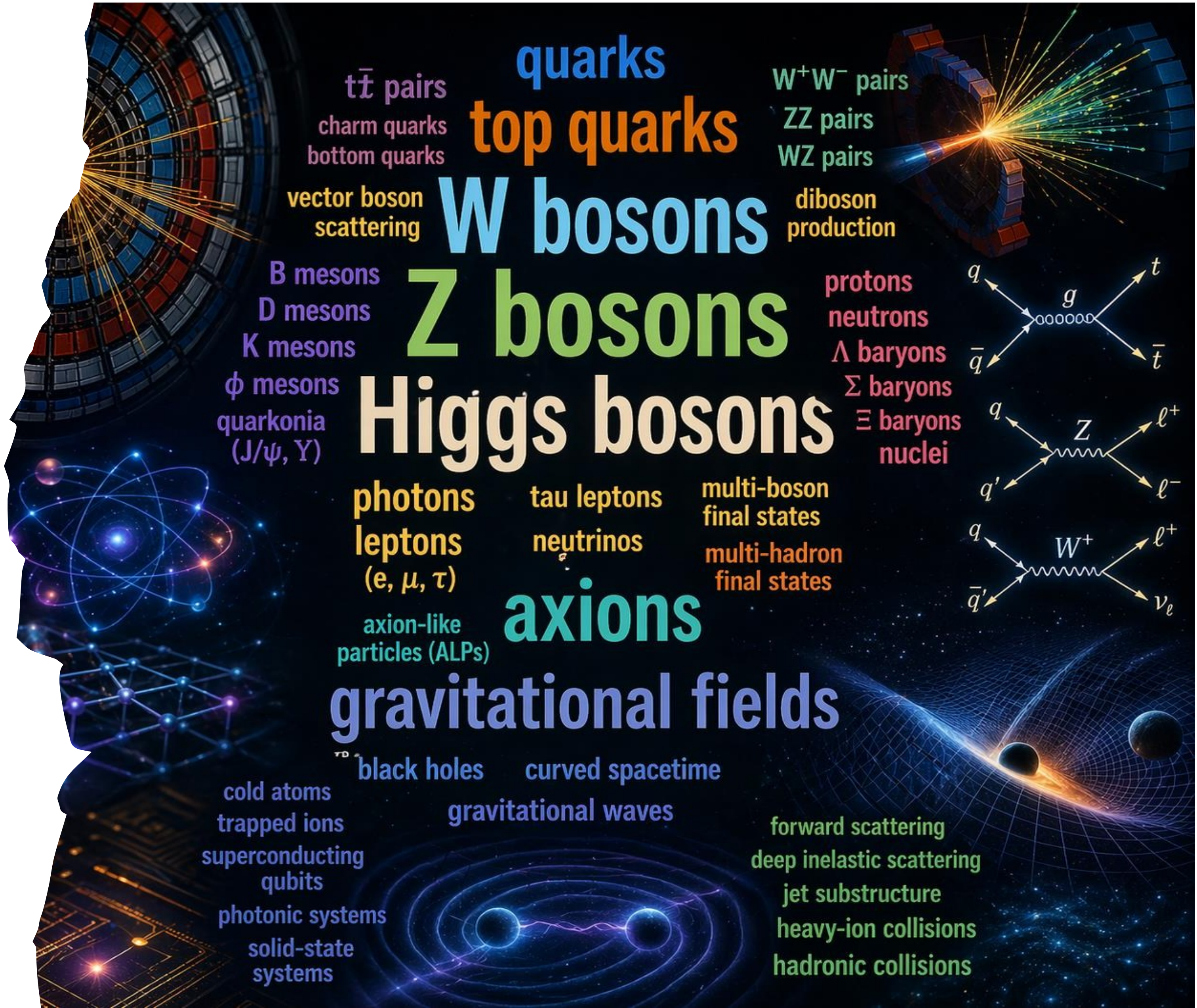
BSM Lorentz

$\Downarrow$

Contributes to B_i, C_{ij}

$\Downarrow$

Distinct pattern in the spin density matrix



Outlook: Quantum is everywhere

- **Flavor**

Meson pair system — Cheng, Han, Low, Wu. 2507.12513

$$\rho = \frac{a}{4}\left(\mathbb{1}_4 + b_i^{\mathcal{A}}\,\sigma_i \otimes \mathbb{1}_2 + b_i^{\mathcal{B}}\,\mathbb{1}_2 \otimes \sigma_i + C_{ij}\,\sigma_i \otimes \sigma_j\right)$$

Spin-Flavor structure — Du, Geng, He, Liu, Liu. 2605.09682

$$\rho_{\vec{k}} = \frac{1}{4}\left[\mathbb{1}_F \otimes \mathbb{1}_s + \mathbb{1}_F \otimes \left(\vec{B}_s \cdot \vec{\sigma}_s\right) + (\boldsymbol{B}_D \cdot \boldsymbol{\sigma}) \otimes \mathbb{1}_s + \sum_{i,j} C_{ij}\,\boldsymbol{\sigma}_i \otimes \sigma_s^j\right]$$

- **Understanding sources**

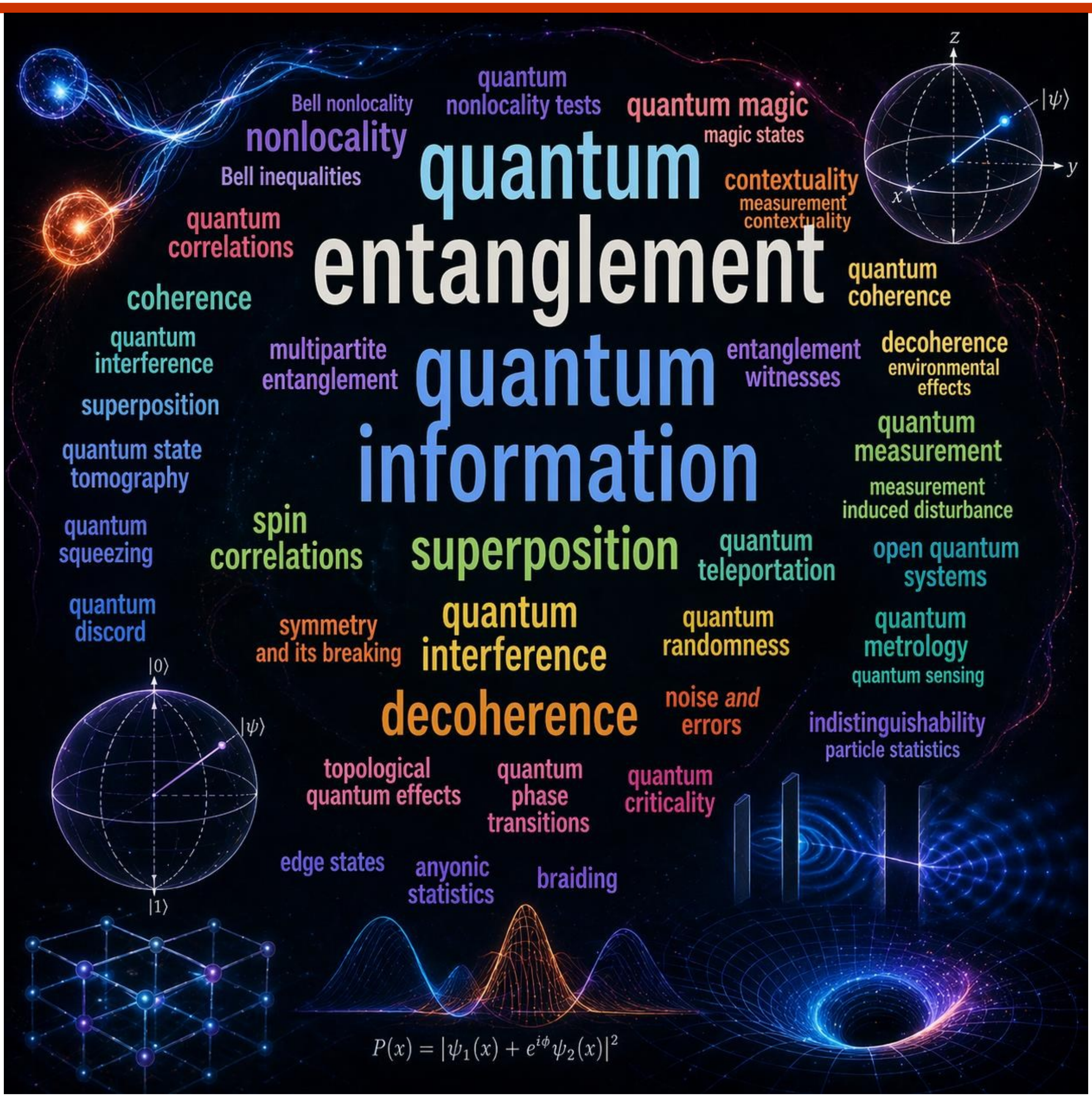
Does Entanglement Suppression Lead to Enhanced Symmetries?

- **Decoherence** — See Dingyu Shao's talk

Radiation, leaking information from the "system", decoherence in spin correction

Spin in hadrons, CP violation, Magic, Quantum swapping,

Quantum teleportation…



Summary

- Collider experiments produce a large number of quantum states with stronger correlations between particles than what is classically allowed.

- ee collisions: Polarization Effect on Tomography ⇒ good control of quantum states.

  Longitudinal/Transverse beam polarization

  Recently: M. Altakach et al., 2601.09558;
  Zhang, Cao, Feng. 2602.10389;
  Fang et al. 2604.11887;
  Liu, He, Xiao. 2604.17756

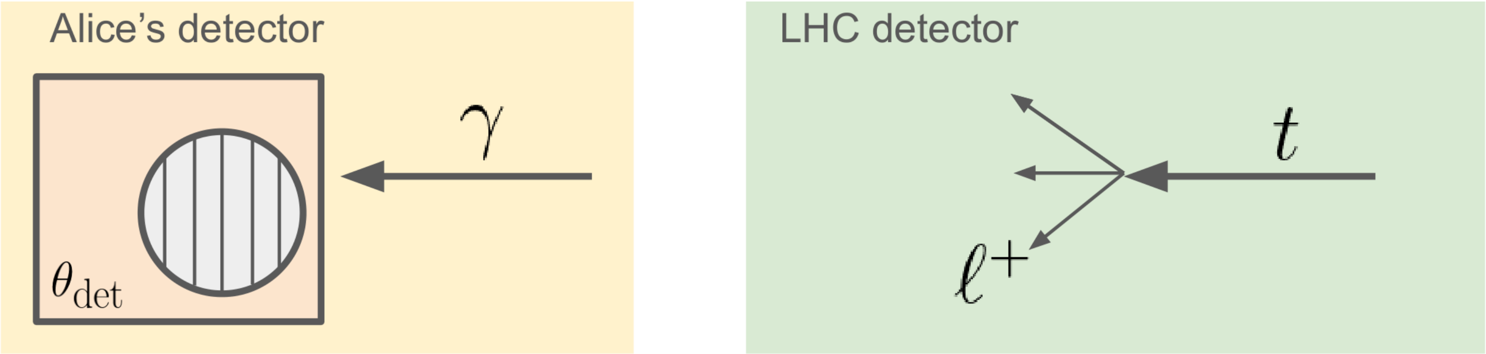
- Theory of Quantum Resources:
  - ✓ Entanglement ⇒ communication resource
  - ✓ Magic ⇒ computational resource

- Understand quantum & seek for BSM effects.

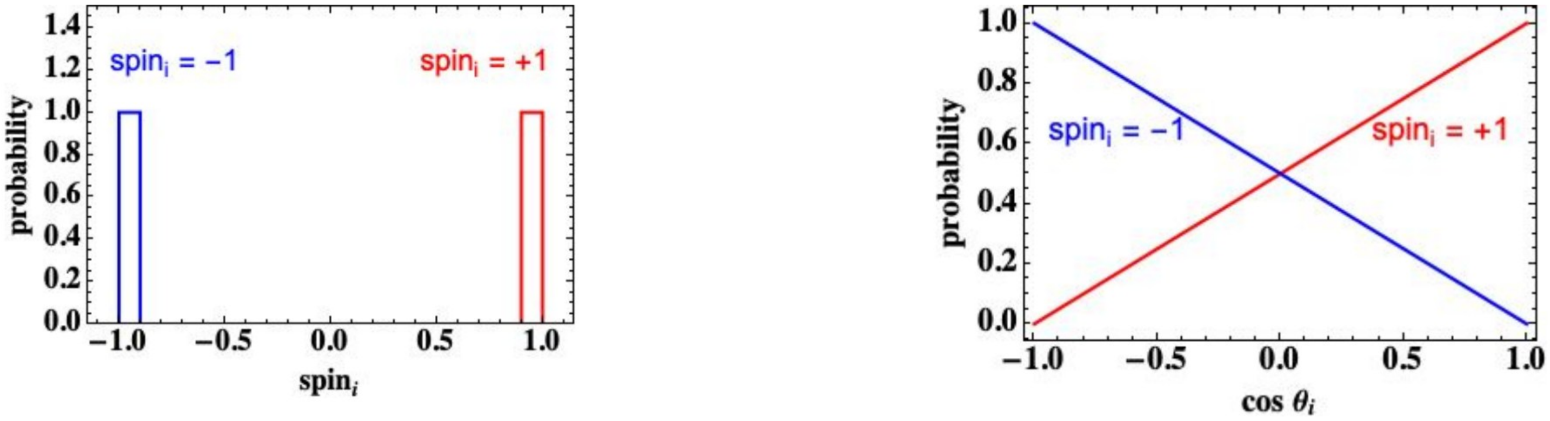
We're just at the beginning of the journey!

Back up $|2026\rangle$

Quantum Tomography @ Colliders



- Can perform **quantum tomography** through **angular distribution** measurements



- Random events in space-time.
- Thus statistically: " **fictitious states**" !

Afik, de Nova, 2003.02280;
Cheng, Han, Low, 2407.01672.

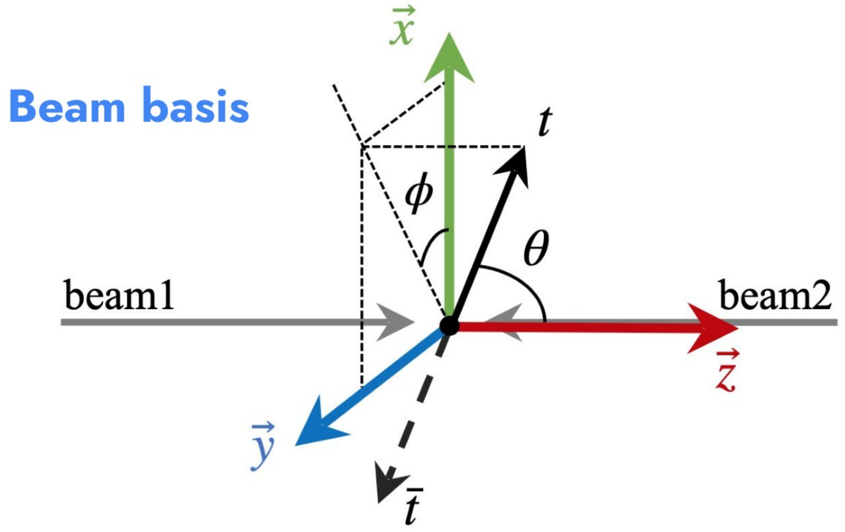
Quantum Tomography @ Colliders

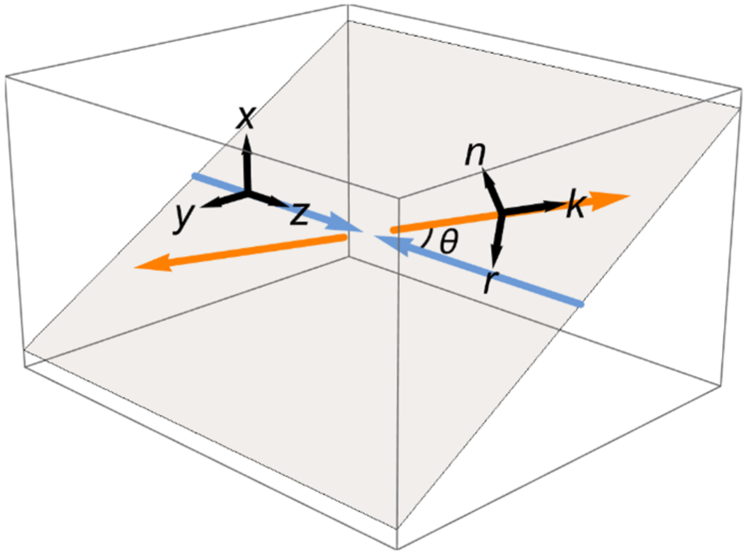
Choice of **Basis**

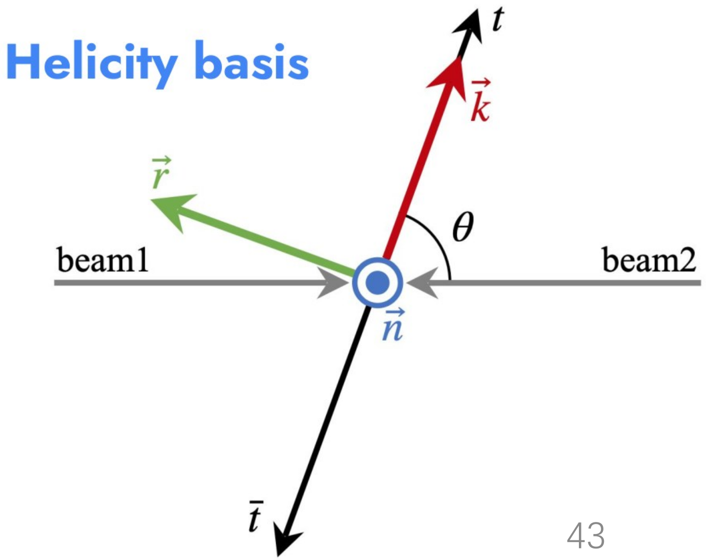
Cheng, Han, Low 2311.09166
Cheng, Fang, Han, Low 2502.14065

- Equivalent to **axes** for spin measurement
- Beam basis {x,y,z}: cancellation leads to null result
- Helicity basis {r,k,n} : entanglement with no cancellations after averaging over phase space

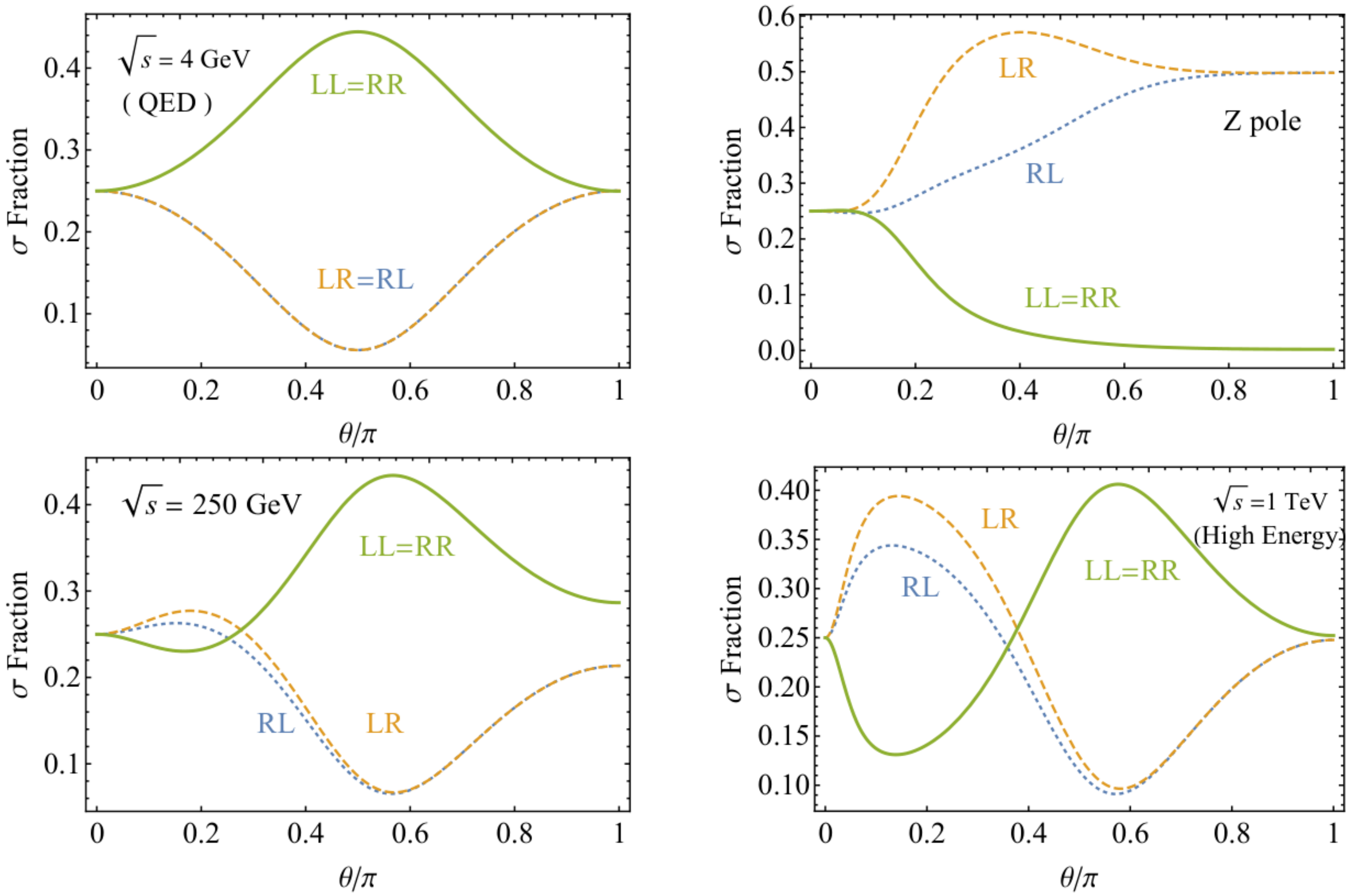
Chirality ≈ helicity at high energy, simplifies physical interpretation







(3) Bhabha scattering & t-s interference



- QED and 250 GeV: t-channel dominates forward region.
- Z-pole: s-channel dominates similar with only s-channel process

Outlook: QI as a probe of BSM interactions

For $e^+e^- \to f\bar{f}$ in the high-energy limit $m_f^2 \ll s$, the final-state spins form a two-qubit state:

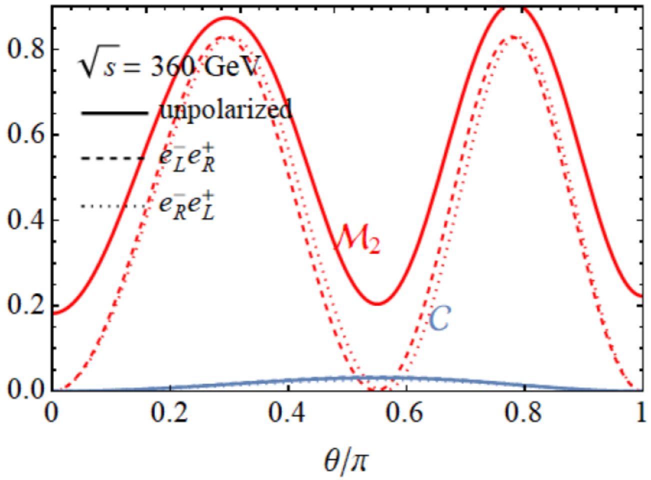
$$|\psi\rangle = a|\uparrow\uparrow\rangle + b|\downarrow\downarrow\rangle + c|\uparrow\downarrow\rangle + d|\downarrow\uparrow\rangle$$

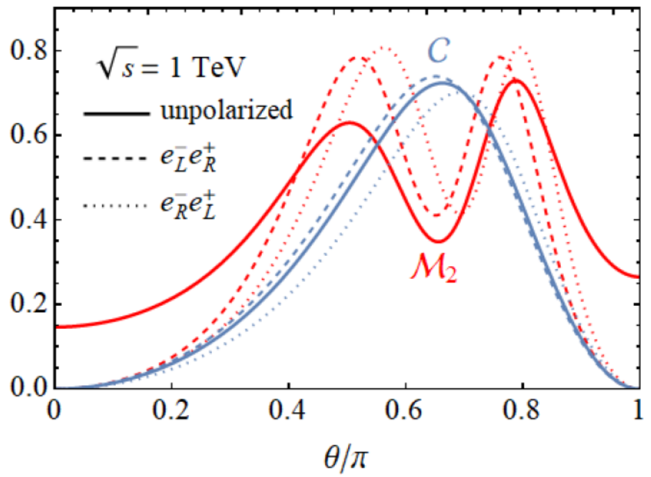
Interaction structure $\Longrightarrow$ Bell-state sector

Lorentz structure	Dominant spin sector	Entanglement pattern	Physics message
Vector / axial-vector ($\gamma/Z/Z'$)	$c, d \simeq 0$	$\|\Phi^{\pm}\rangle = (\|\uparrow\uparrow\rangle \pm \|\downarrow\downarrow\rangle)/\sqrt{2}$	Gauge-like helicity structure
CP-even scalar (H)	$a, b \simeq 0$	$\|\Psi^{-}\rangle = (\|\uparrow\downarrow\rangle - \|\downarrow\uparrow\rangle)/\sqrt{2}$	Chirality-flipping scalar coupling
CP-odd pseudoscalar (A^0)	$a, b \simeq 0$	$\|\Psi^{+}\rangle = (\|\uparrow\downarrow\rangle + \|\downarrow\uparrow\rangle)/\sqrt{2}$	Relative phase/sign carries CP information

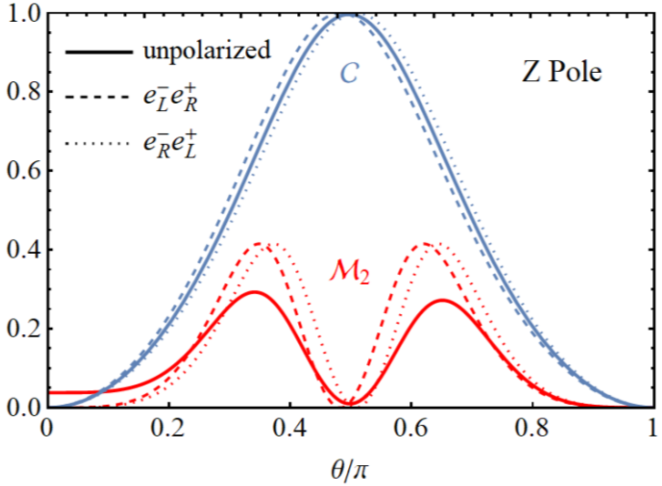
Entanglement vs. Magic

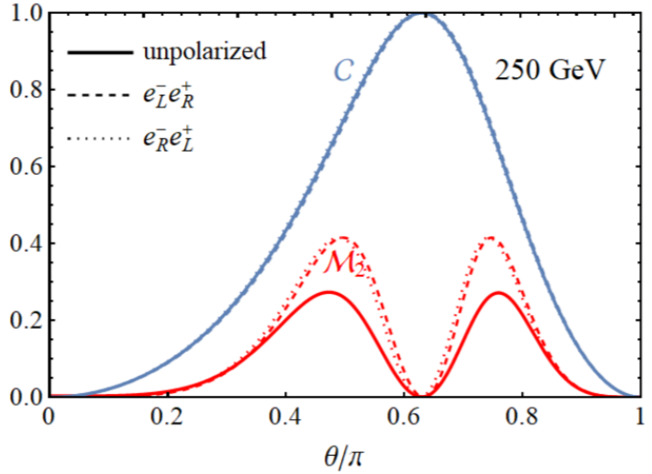
$e^+e^- \to t\bar{t}$





$e^+e^- \to \mu^+\mu^-$





$e^+e^- \to e^+e^-$