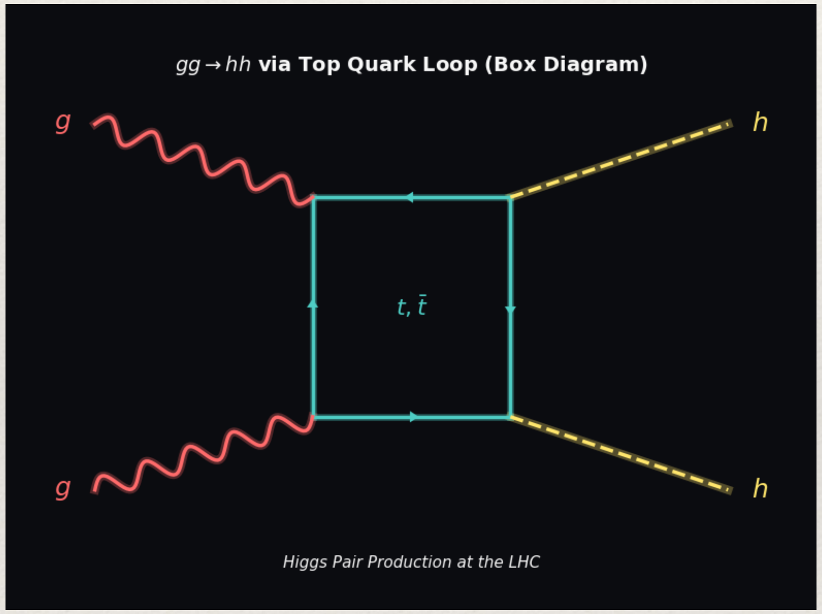
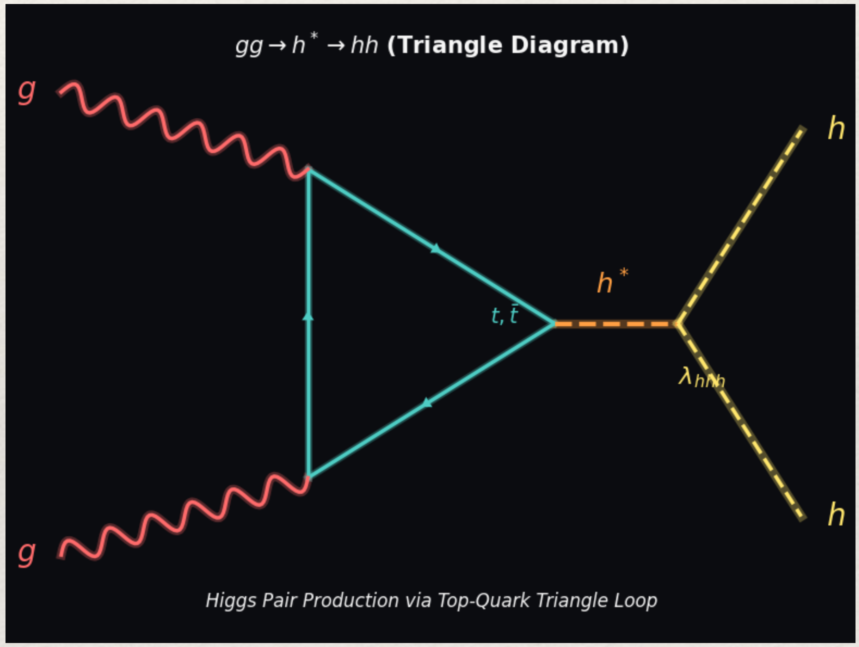
Higgs pair production at the LHC



Jian Wang (王健)
Shandong University



NPhiS 2026
Nanjing, 05.17

Higgs sector in SM

In the Standard Model (SM), the Higgs sector is described by

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 (\phi^\dagger \phi) - \lambda (\phi^\dagger \phi)^2 \,,$$

After symmetry breaking, the Higgs field can be written as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

In the broken phase the Lagrangian takes the form

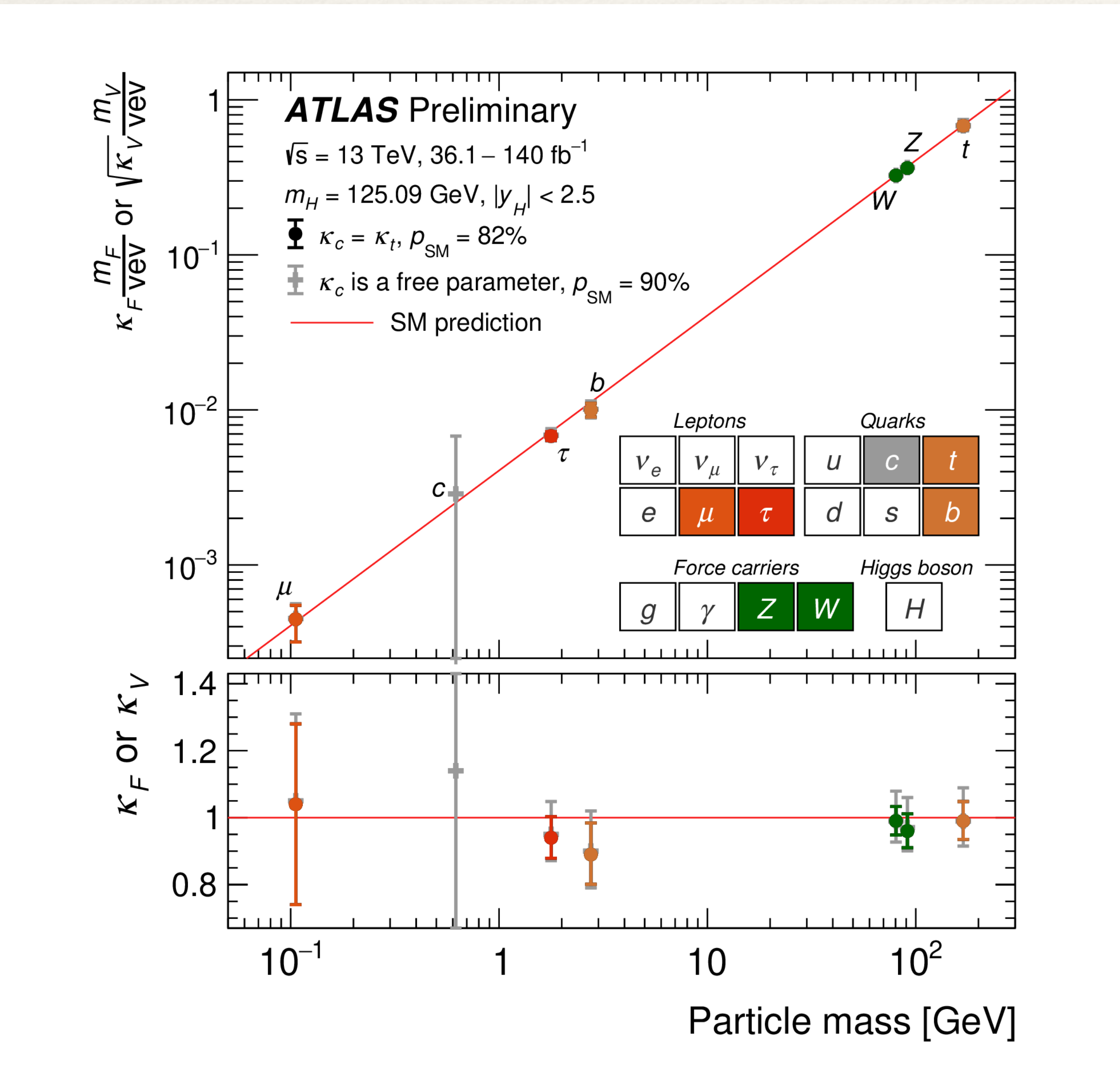
$$\mathcal{L} \supset m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu - \frac{1}{2} m_H^2 H^2 - v\lambda H^3 - \frac{\lambda}{4} H^4$$

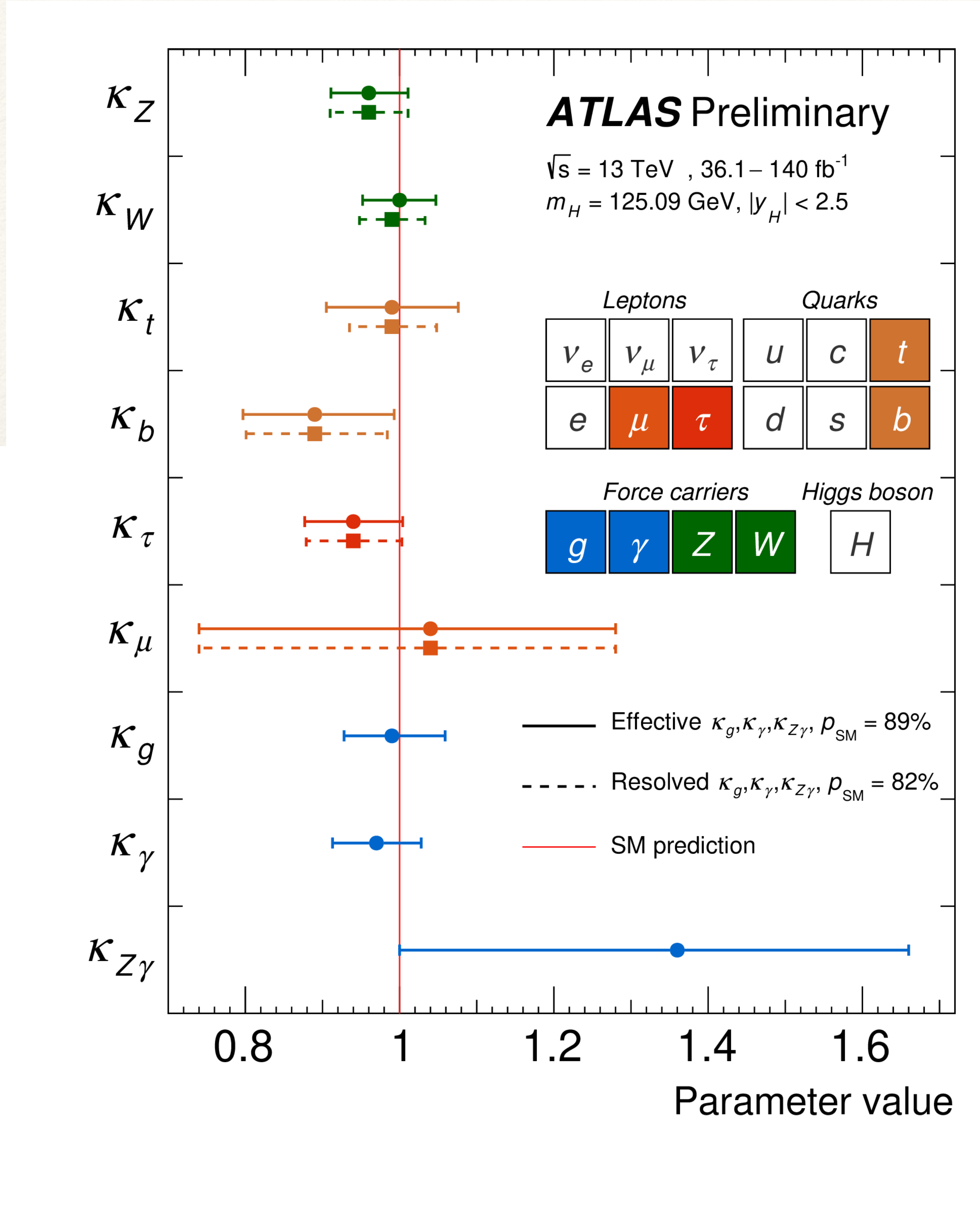
The EW gauge boson and Higgs masses are expressed as

$$m_W = \frac{g_2 v}{2} \,, m_Z = \frac{\sqrt{g_1^2 + g_2^2}\, v}{2} \,, m_H = \sqrt{2\lambda} v$$

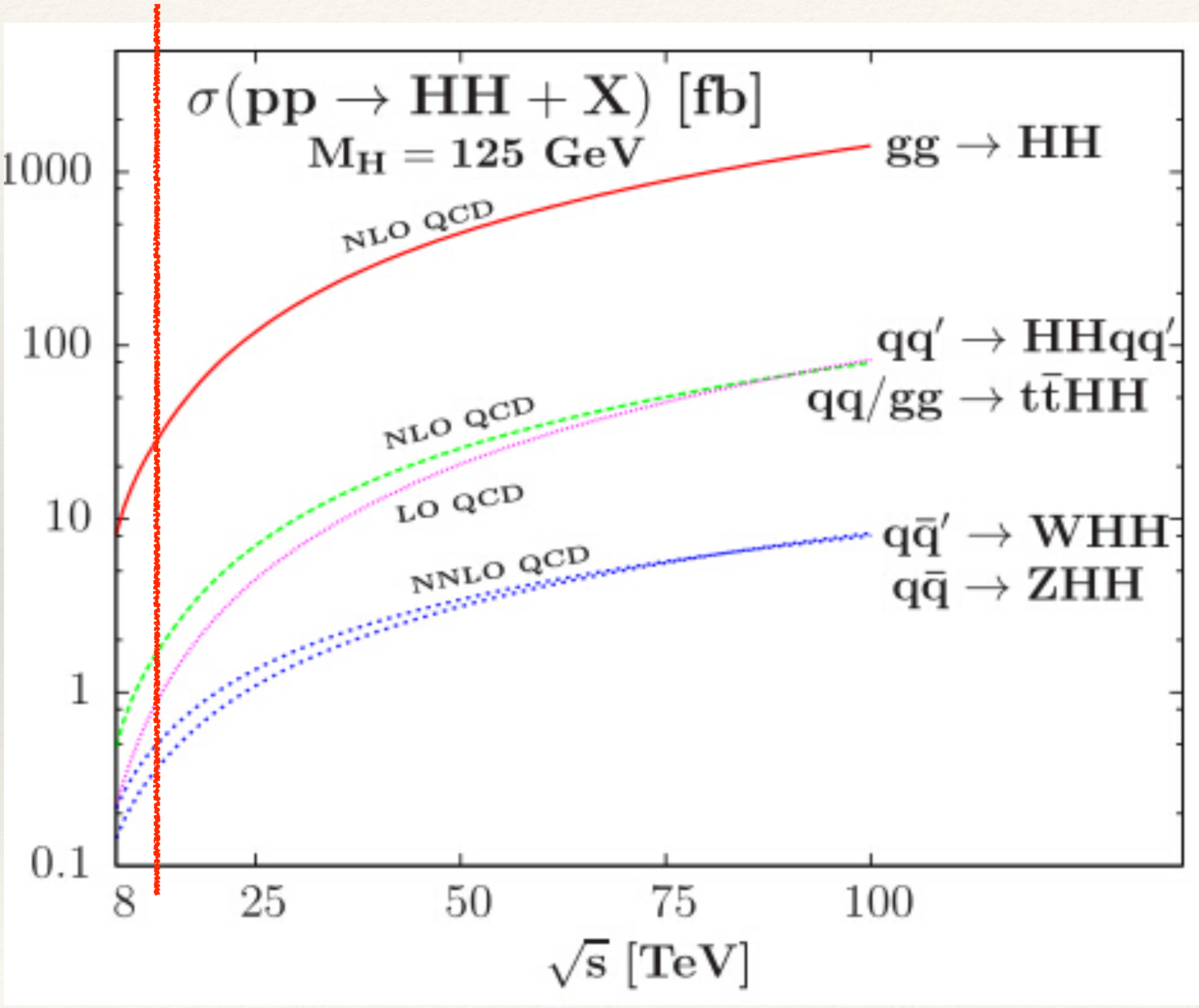
So far, the H(125) properties are consistent with the SM expectation!

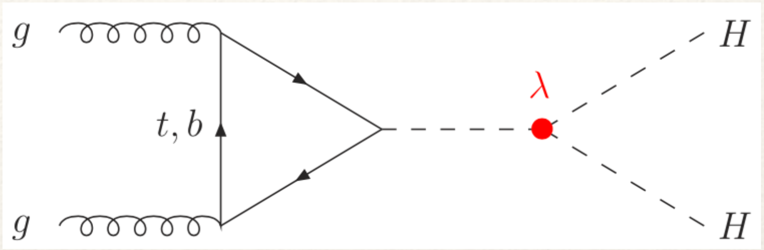
Its couplings with SM particles are proportional to their masses; Higgs mechanism

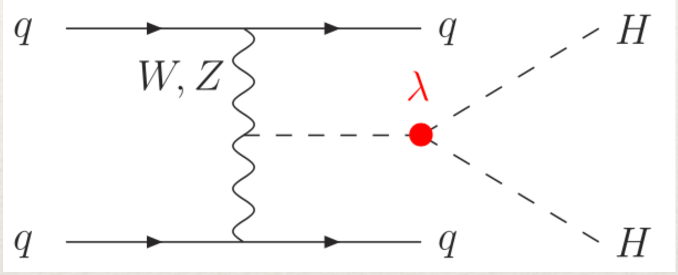


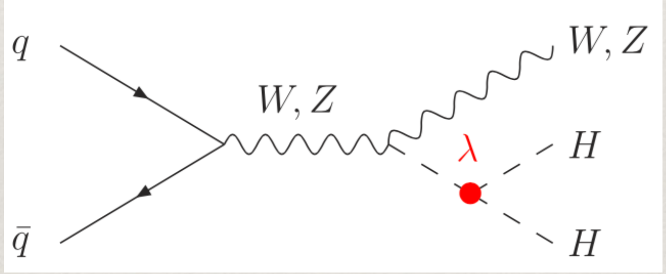


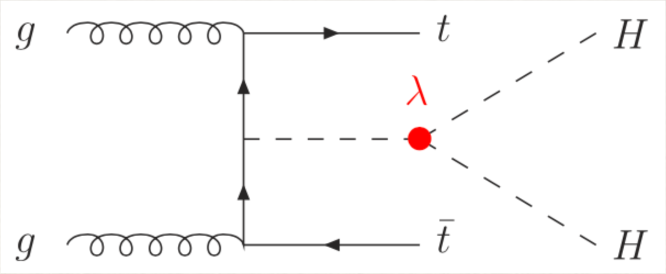
Higgs pair production at the LHC



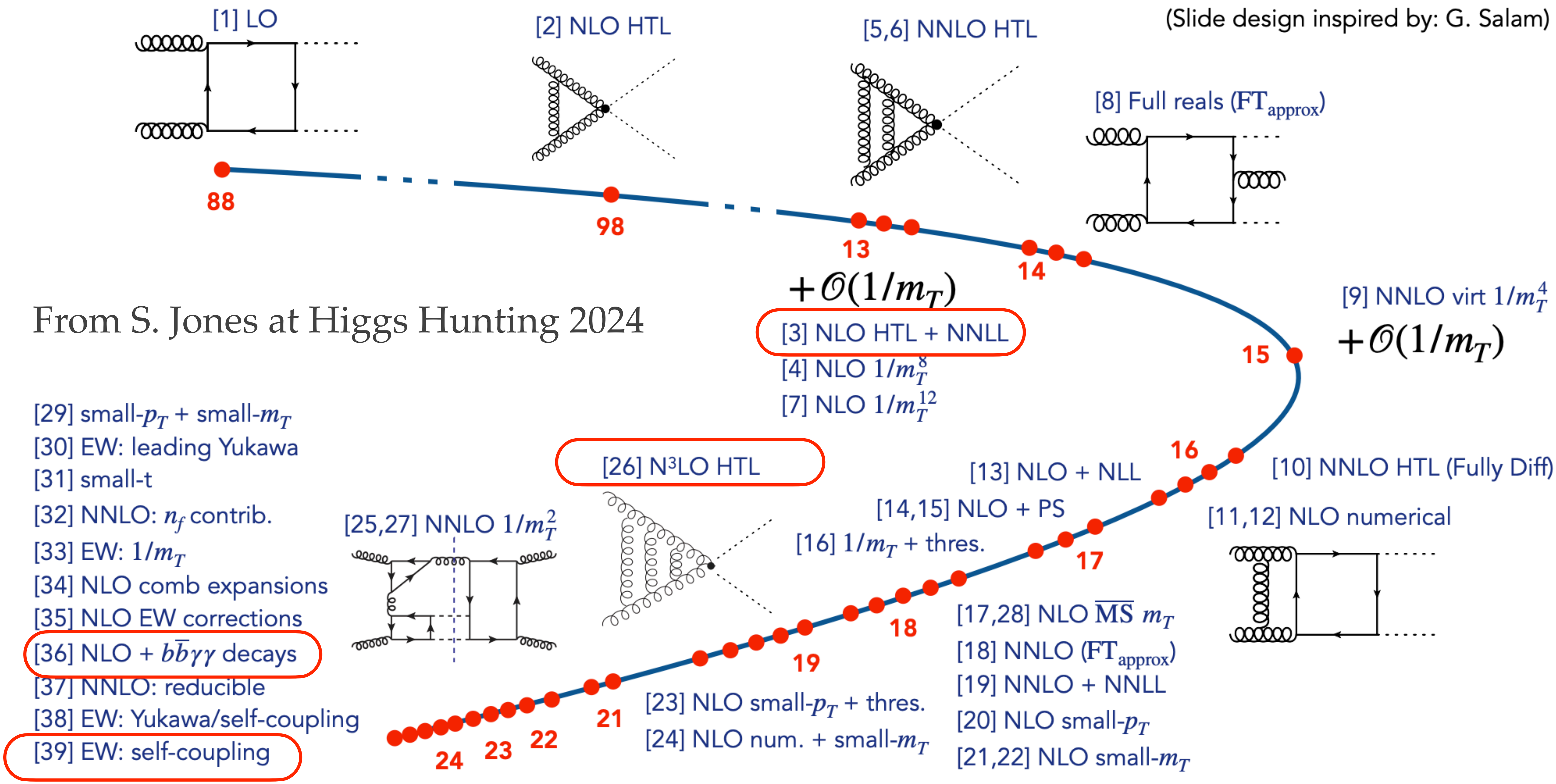






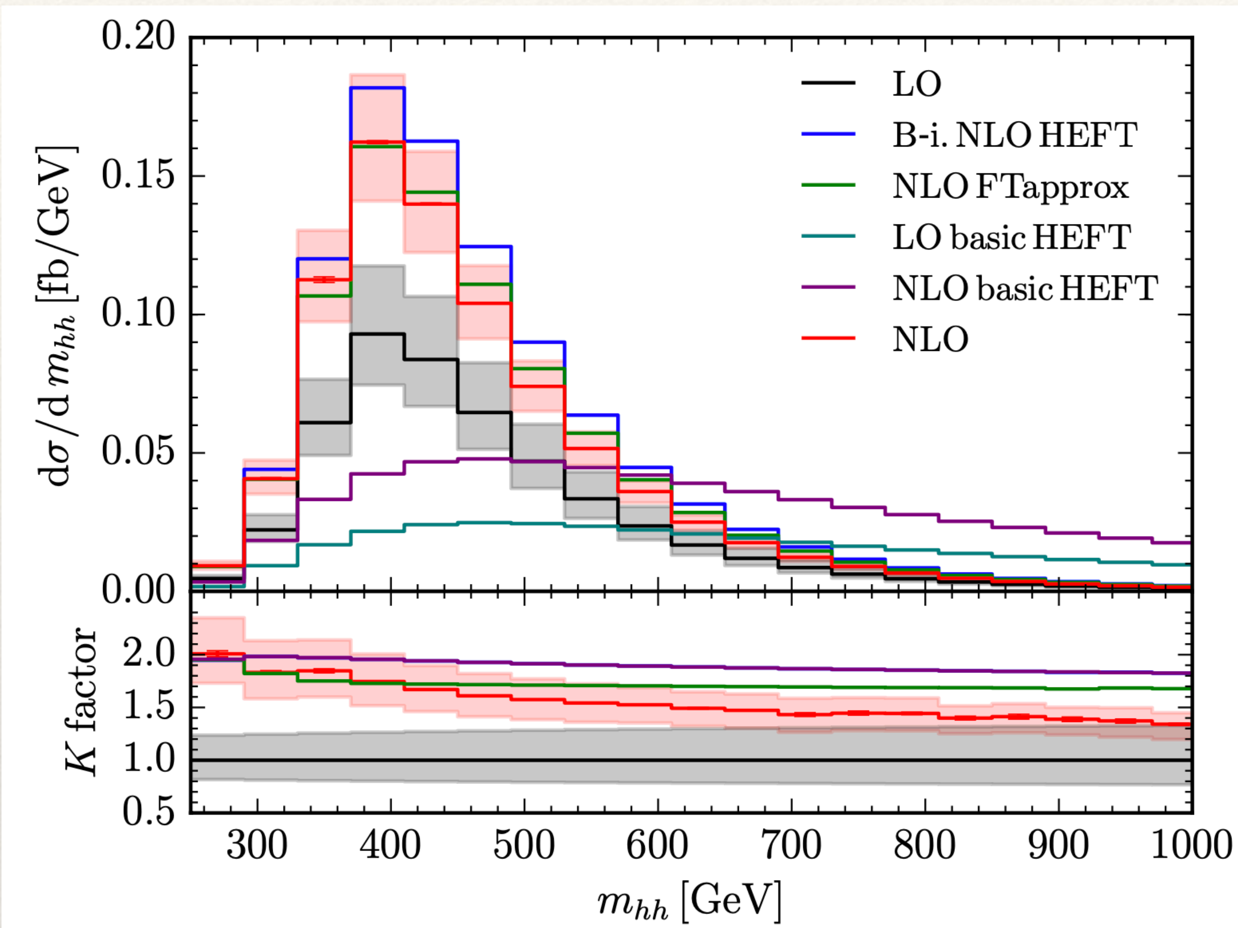


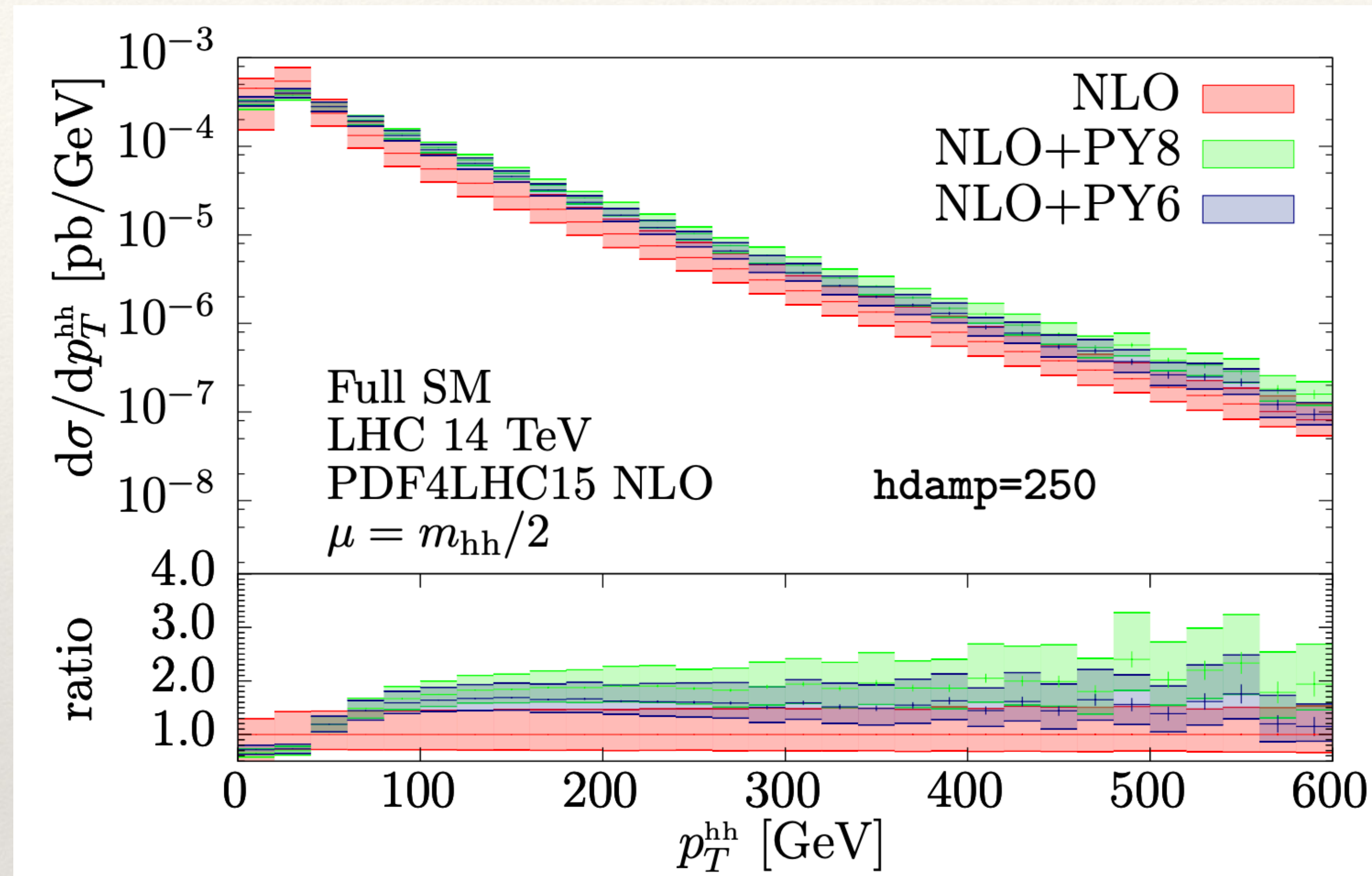
Long history in high precision calculations



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagnaschi, Degrassi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23 [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, SPJ, Kerner, Stone, Vestner [39] Li, Si, Wang, Zhang, Zhao 24

NLO QCD Corrections





- The NLO K-factor is around 1.5-2.0 for the invariant mass distribution.
- The high energy region is not well described by the heavy-top-limit.
- The parton shower effect is large and different in different schemes.

Borowka et al., 1608.04798

Heinrich et al., 1703.09252

Top mass scheme uncertainty

LO

$$\left.\frac{d\sigma(gg\to hh)}{dQ}\right|_{Q=300\ \mathrm{GeV}} = 0.01656^{+62\%}_{-2.4\%}\ \mathrm{fb/GeV},$$
$$\left.\frac{d\sigma(gg\to hh)}{dQ}\right|_{Q=400\ \mathrm{GeV}} = 0.09391^{+0\%}_{-20\%}\ \mathrm{fb/GeV},$$
$$\left.\frac{d\sigma(gg\to hh)}{dQ}\right|_{Q=600\ \mathrm{GeV}} = 0.02132^{+0\%}_{-48\%}\ \mathrm{fb/GeV},$$
$$\left.\frac{d\sigma(gg\to hh)}{dQ}\right|_{Q=1200\ \mathrm{GeV}} = 0.0003223^{+0\%}_{-56\%}\ \mathrm{fb/GeV}$$

NLO

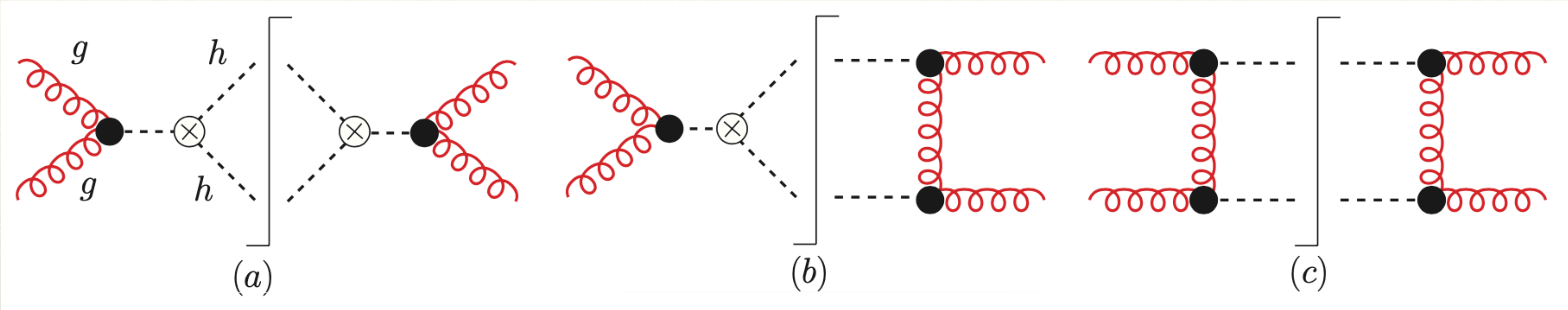
$$\left.\frac{d\sigma(gg\to hh)}{dQ}\right|_{Q=300\ \mathrm{GeV}} = 0.02978(7)^{+6\%}_{-34\%}\ \mathrm{fb/GeV},$$
$$\left.\frac{d\sigma(gg\to hh)}{dQ}\right|_{Q=400\ \mathrm{GeV}} = 0.1609(7)^{+0\%}_{-13\%}\ \mathrm{fb/GeV},$$
$$\left.\frac{d\sigma(gg\to hh)}{dQ}\right|_{Q=600\ \mathrm{GeV}} = 0.03204(9)^{+0\%}_{-30\%}\ \mathrm{fb/GeV},$$
$$\left.\frac{d\sigma(gg\to hh)}{dQ}\right|_{Q=1200\ \mathrm{GeV}} = 0.000435(6)^{+0\%}_{-35\%}\ \mathrm{fb/GeV}$$

- The $\overline{\mathrm{MS}}$ mass $\bar{m}_t(\mu_t)$ with $\mu_t = \bar{m}_t$ or [Q/4,Q].
- The uncertainties drop by roughly a factor of two from LO to NLO.
- They constitute an important contribution to the total theoretical uncertainties.
- Resummation of $\ln^n m_t^2/s$ at leading power leading logarithmic level can reduce the scheme dependence at high energy.

Baglio et al., 1811.05692, 2003.03227, 2008.11626

Jaskiewicz et al., 2501.00587

NNNLO QCD Corrections



$$d\sigma_{hh} = d\sigma_{hh}^{a} + d\sigma_{hh}^{b} + d\sigma_{hh}^{c}$$

	$\sigma_{\rm NLO}$	$\sigma_{\rm NNLO}$	$\sigma_{\rm N^3LO}$
Inclusive	$31.89^{+18\%}_{-15\%}$ fb	$37.55^{+5.2\%}_{-7.6\%}$ fb	$38.65^{+0.5\%}_{-2.7\%}$ fb
Fiducial	$27.87^{+18\%}_{-15\%}$ fb	$32.74^{+5.2\%}_{-7.3\%}$ fb	$34.00(4)^{+1.4\%}_{-2.9\%}$ fb

$p_{T,h_1} > 30$ GeV, $\quad p_{T,h_2} > 20$ GeV, $\quad |y_h| < 2.4$

+17%

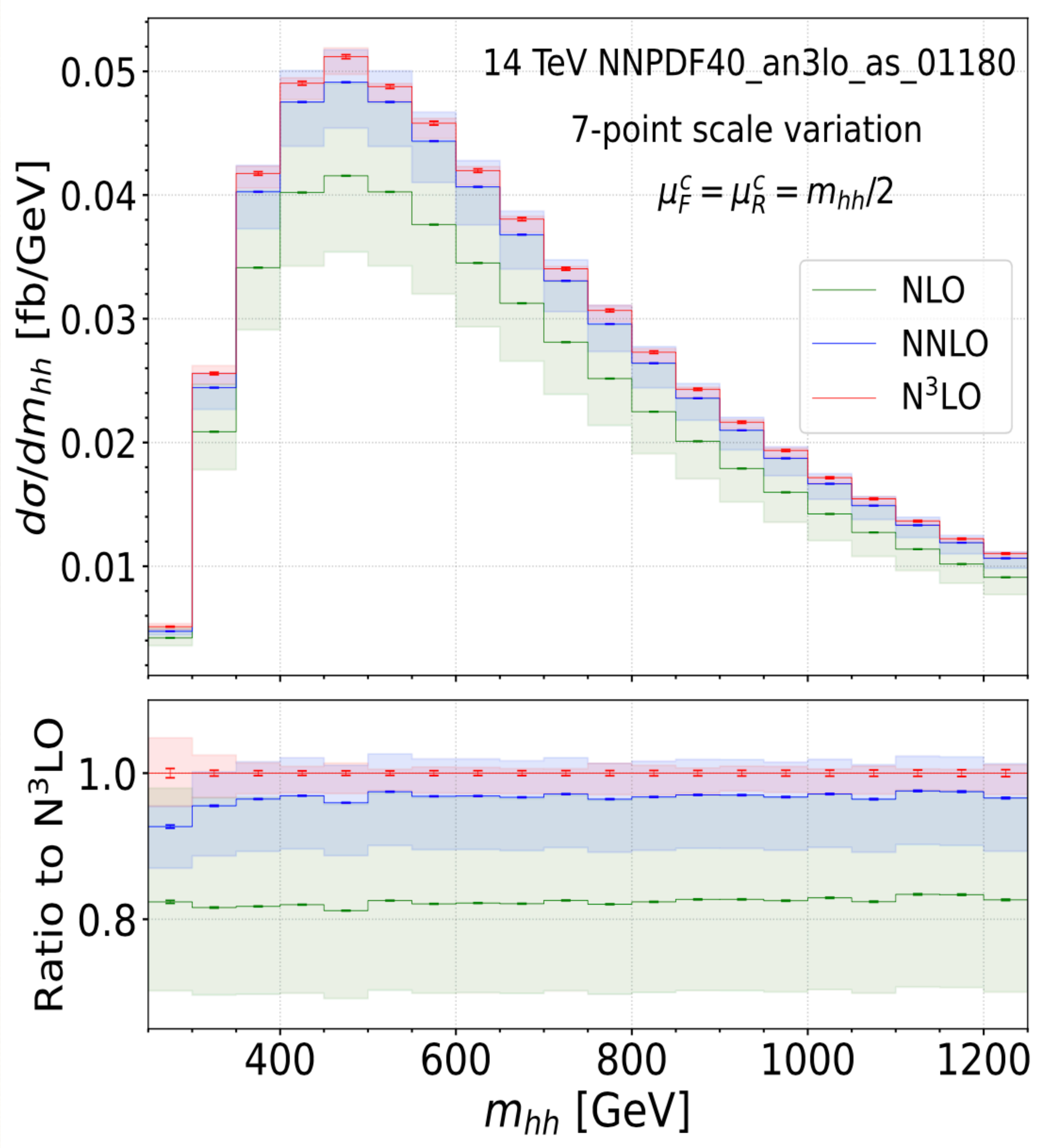
+4%

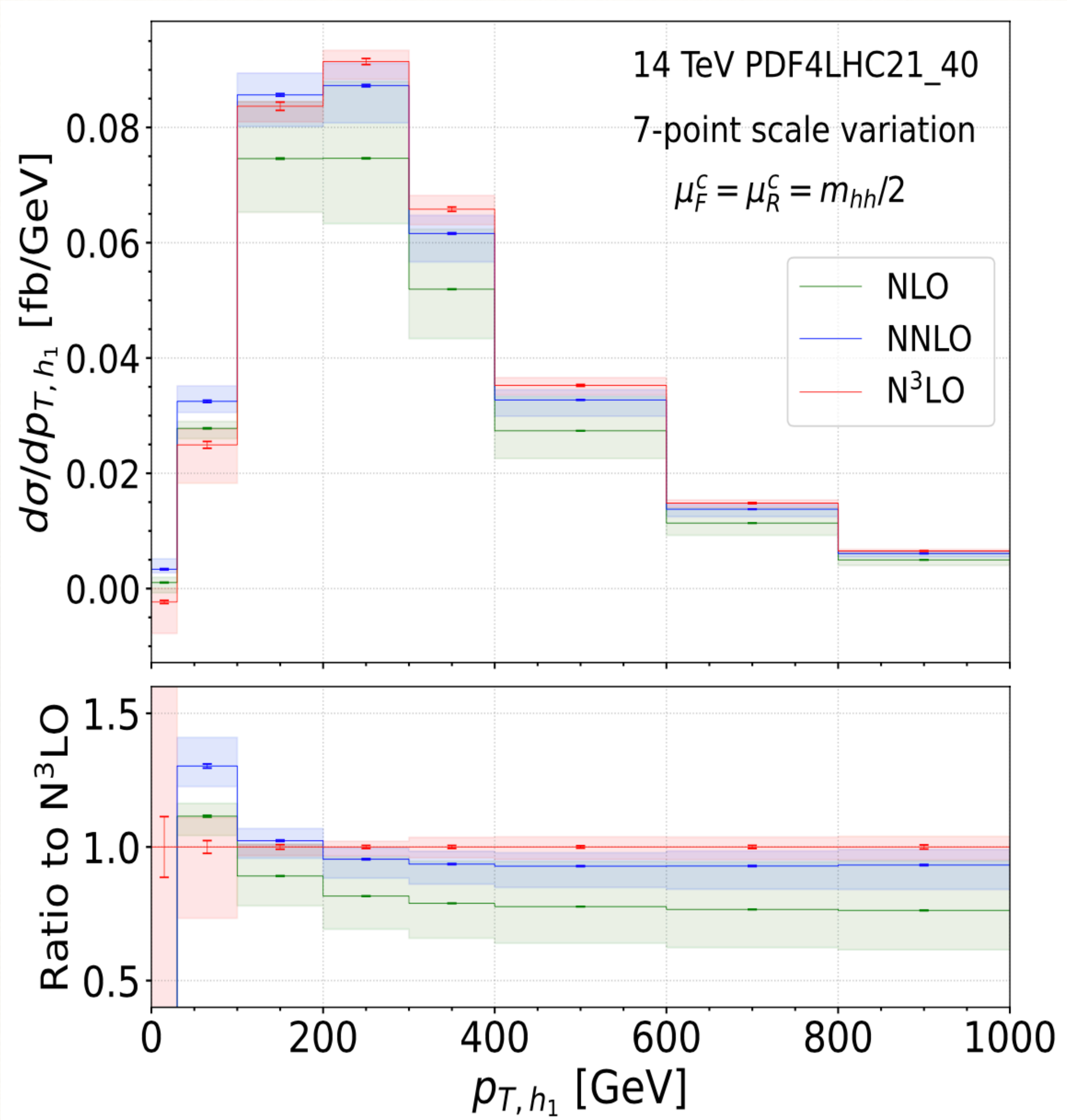
-3×

Chen, Li, Shao, JW, 1909.06808, 1912.13001

Chen, Dai, Li, Li, Shao, JW, 2601.19990

NNNLO QCD Corrections





Chen, Li, Shao, jW, 1909.06808, 1912.13001

Chen, Dai, Li, Li, Shao, JW, 2601.19990

NNNLO QCD Corrections

Contributions	Inclusive cross section [fb]	Fiducial cross section [fb]
NLO_{m_t}	$32.64^{+13.5\%}_{-12.5\%}$	$28.44^{+14\%}_{-12\%}$
$\mathrm{NNLO} \otimes \mathrm{NLO}_{m_t}$	$38.42^{+5.2\%}_{-7.6\%}$	$33.40^{+5.2\%}_{-7.3\%}$
$(\mathrm{NNLO} + \mathrm{NNLL}) \otimes \mathrm{NLO}_{m_t}$	$39.42^{+3.0\%}_{-3.4\%}$	-
$\mathrm{N^3LO} \otimes \mathrm{NLO}_{m_t}$	$39.56^{+0.50\%}_{-2.7\%}$	$34.68(4)^{+1.4\%}_{-2.9\%}$
$(\mathrm{N^3LO} + \mathrm{N^3LL}) \otimes \mathrm{NLO}_{m_t}$	$39.60^{+0.85\%}_{-0.87\%}$	-

+21%

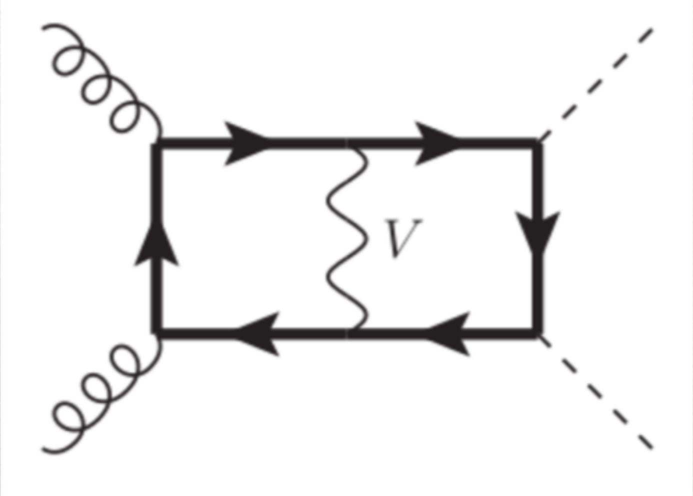
$$d\sigma_{\mathrm{N}^k\mathrm{LO}\otimes\mathrm{NLO}_{m_t}} \equiv d\sigma_{\mathrm{NLO}_{m_t}} \frac{d\sigma_{\mathrm{N}^k\mathrm{LO}}}{d\sigma_{\mathrm{NLO}}}$$

- Scale uncertainty ~3%
- Soft gluon effect is marginal

Chen, Li, Shao, JW, 1909.06808, 1912.13001

Chen, Dai, Li, Li, Shao, JW, 2601.19990

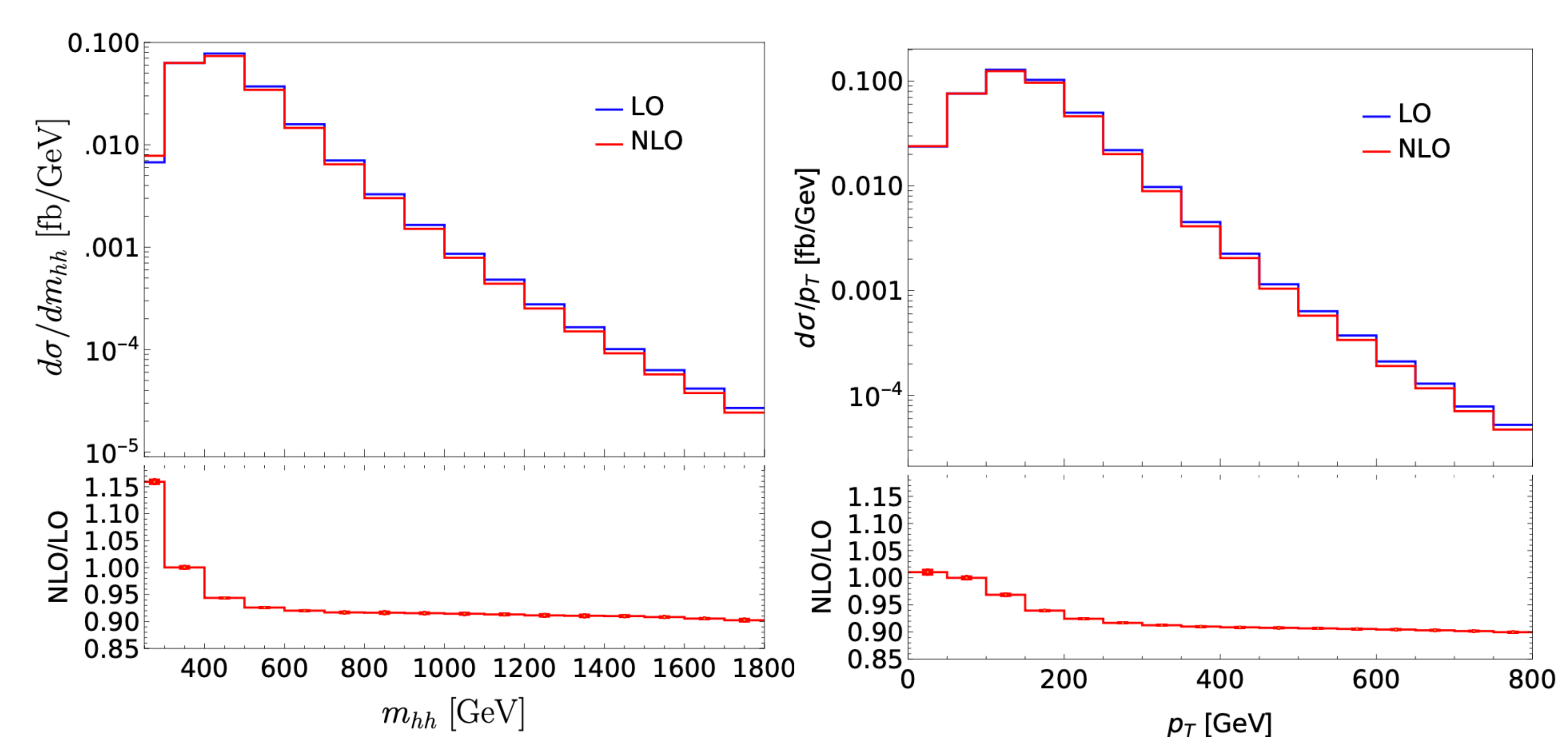
NLO EW Corrections



schemes	G_μ	α_0	α_{m_Z}
LO	19.96	18.84	21.28
NLO	19.12	19.19	19.03
$\mathcal{K}$-factor	0.958	1.019	0.894

1%

13%

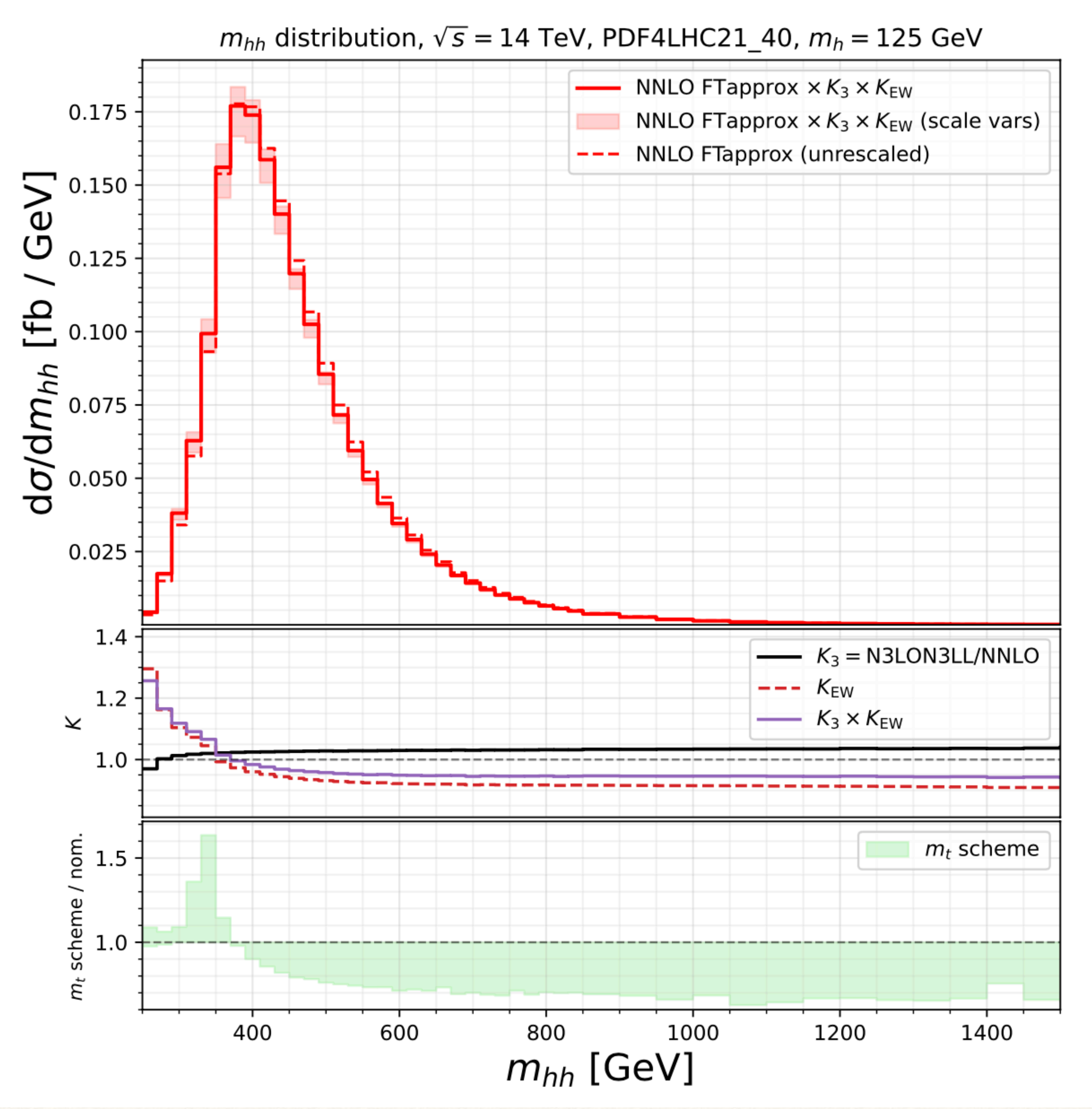


At threshold, +15% effect

At high region, -10% effect

Bi, Huang, Huang, Ma, Yu, 2311.16963

LHCHWG Recommendation for future analyses



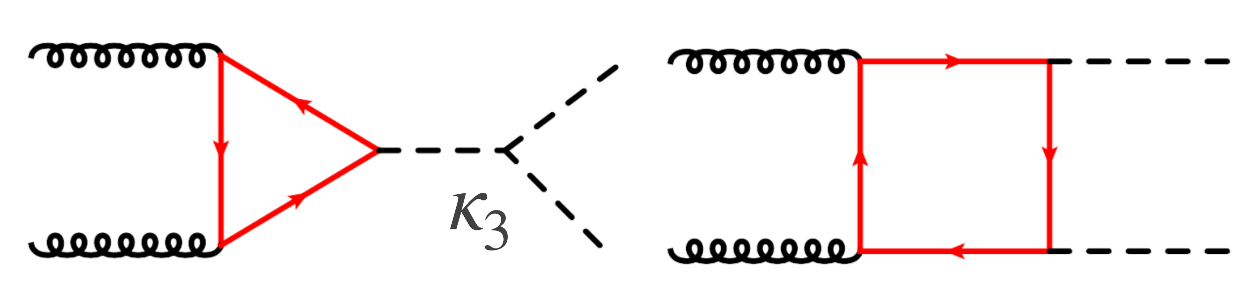
Ajjath A H et al, to appear

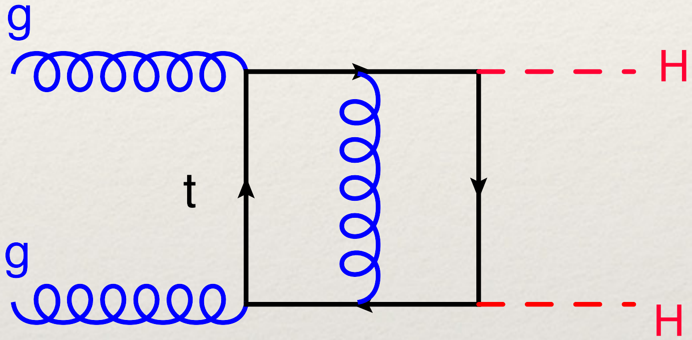
Higgs sector in BSM

In the κ framework

"Handbook of LHC Higgs Cross Sections" 1307.1347

$$\mathcal{L} \supset m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu - \frac{1}{2} m_H^2 H^2 - \kappa_3 v \lambda H^3 - \kappa_4 \frac{\lambda}{4} H^4$$



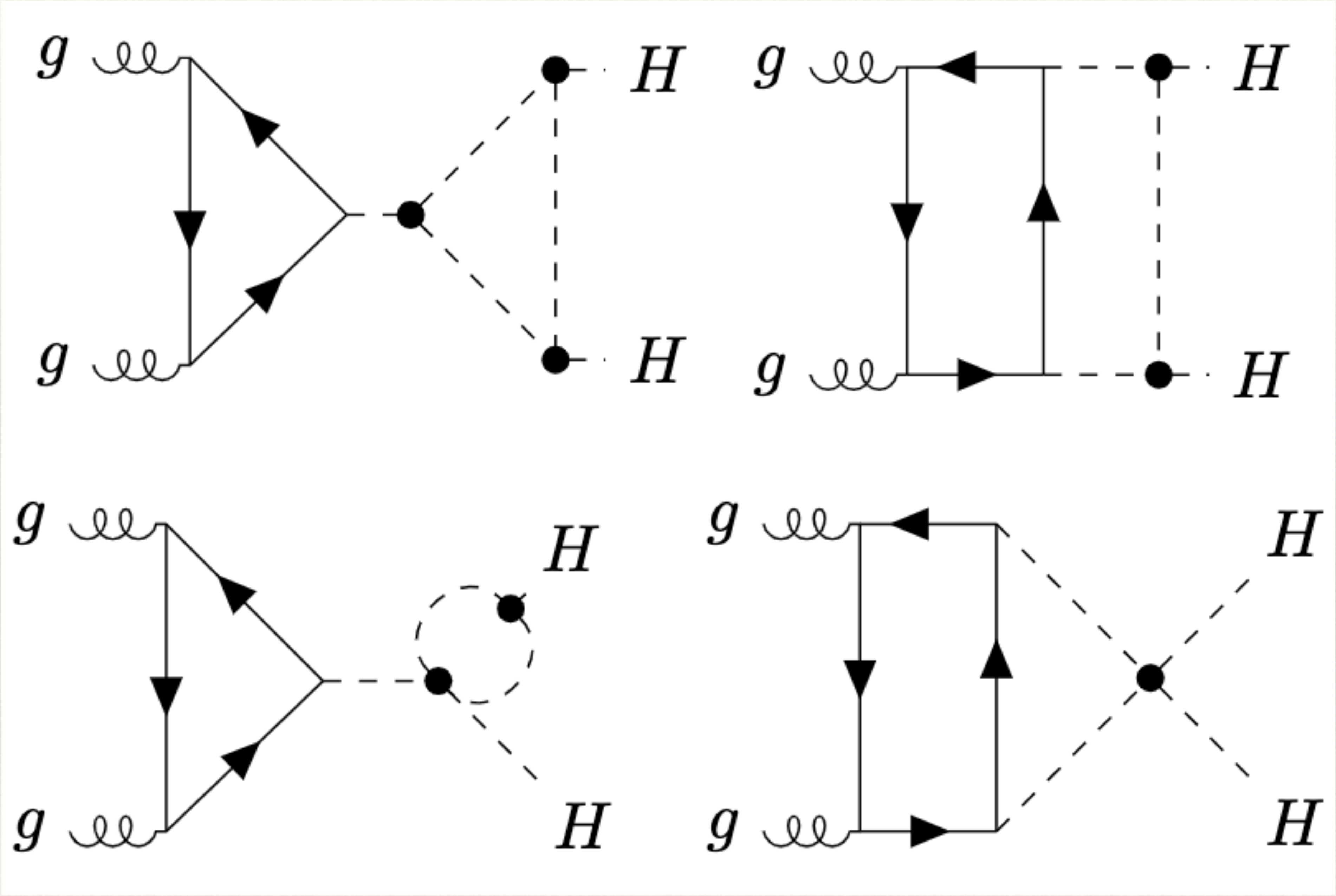


$$\sigma_{HH} = A + B\kappa_3 + C\kappa_3^2$$

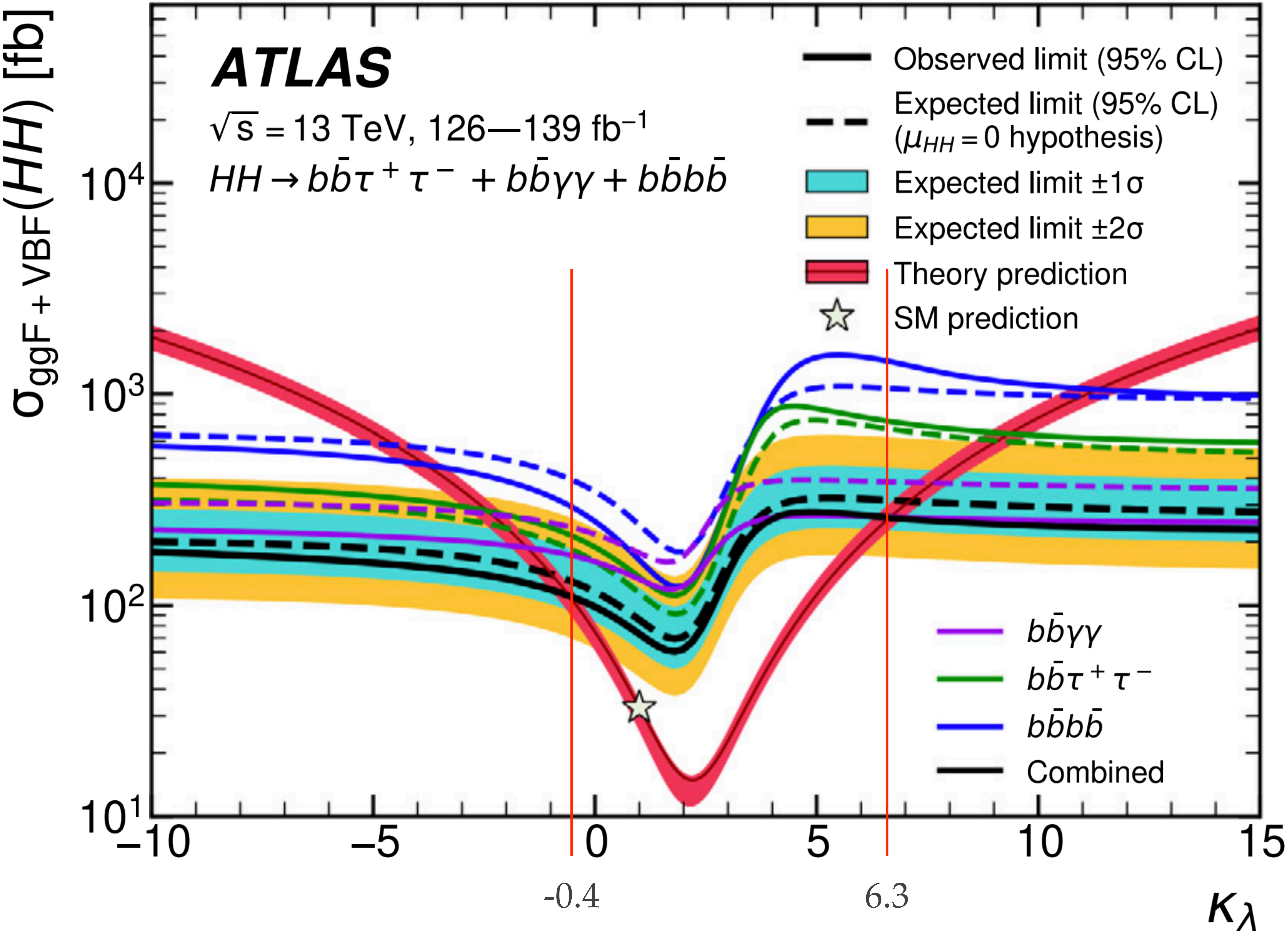
computation	A [fb]	A/A(LO)	B [fb]	B/B(LO)	C [fb]	C/C(LO)
LO m_t fin	35.0		-23.0		4.73	
NLO m_t fin	62.6	1.79	-44.4	1.93	9.64	2.04
NLO m_t fin × NNLO SM FTApprox	70.0	2.00	-49.6	2.16	10.8	2.28
NNLO + NNLL $m_t \to \infty$ × NNLO+NLL SM (partial m_t fin)	71.3	2.04	-47.7	2.08	9.93	2.10

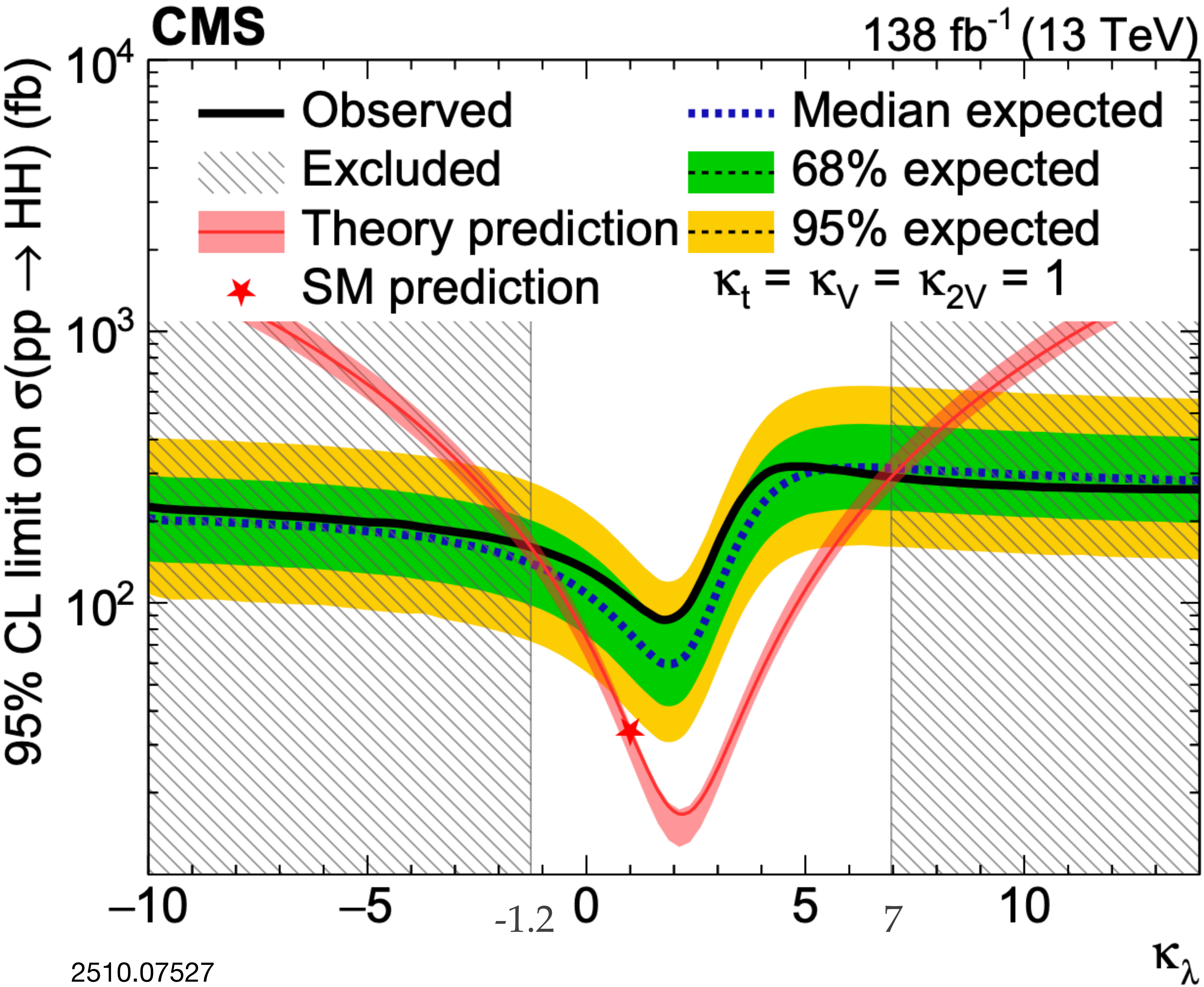
1910.00012

A more realistic function form



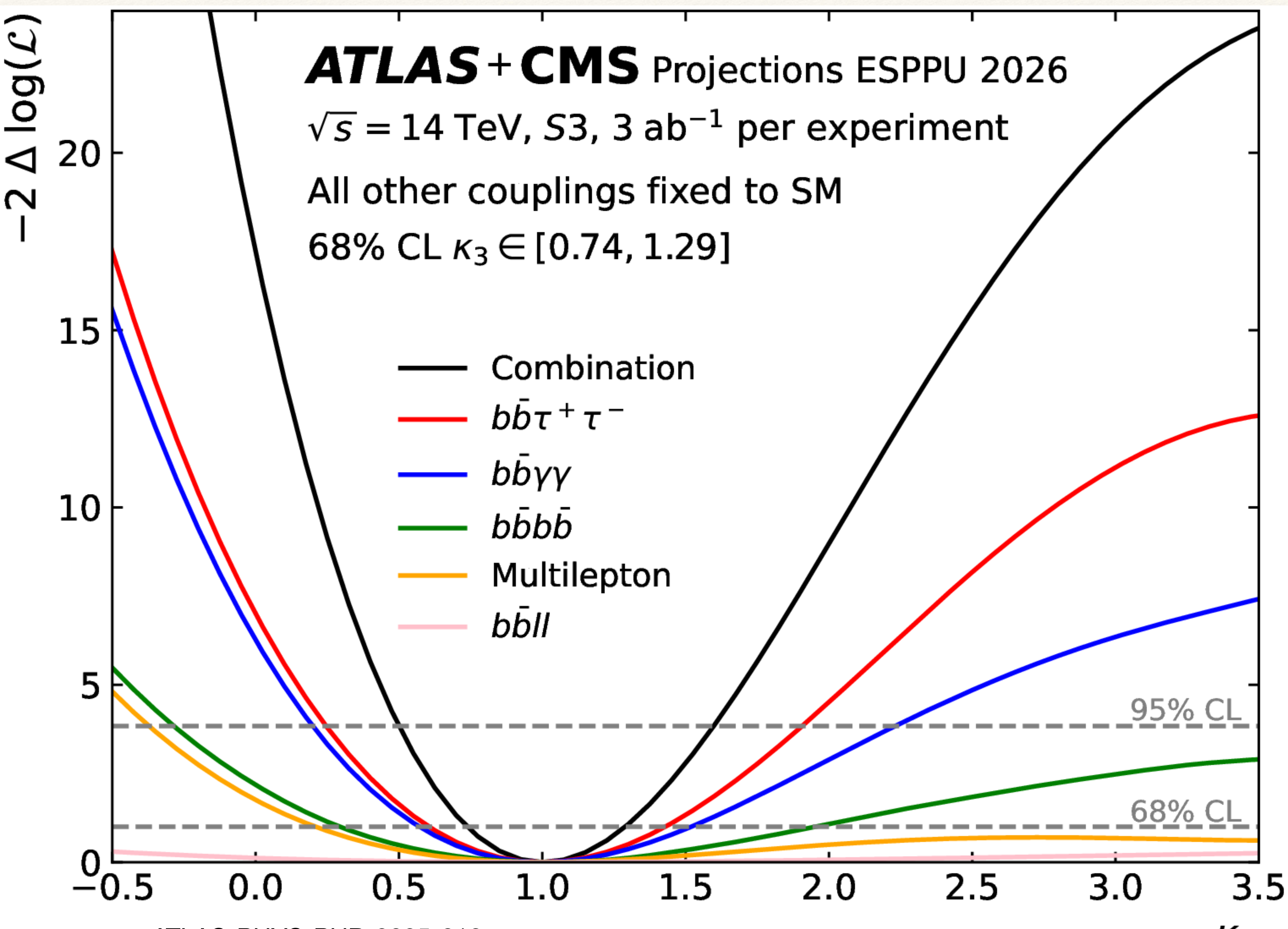
$$\sigma_{HH} = A + B\kappa_3 + C\kappa_3^2 + D\kappa_3^3 + E\kappa_3^4 + F\kappa_3^2\kappa_4 + G\kappa_3\kappa_4 + H\kappa_4 + \cdots$$


ATLAS
$\sqrt{s} = 13$ TeV, 126—139 fb^{-1}
$HH \to b\bar{b}\tau^+\tau^- + b\bar{b}\gamma\gamma + b\bar{b}b\bar{b}$
Observed limit (95% CL)
Expected limit (95% CL)
($\mu_{HH} = 0$ hypothesis)
Expected limit ±1σ
Expected limit ±2σ
Theory prediction
SM prediction
$b\bar{b}\gamma\gamma$
$b\bar{b}\tau^+\tau^-$
$b\bar{b}b\bar{b}$
Combined
$\sigma_{\mathrm{ggF+VBF}}(HH)$ [fb]
10^4
10^3
10^2
10^1
−10
−5
0
5
10
15
-0.4
6.3
κ_λ



CMS
138 fb^{-1} (13 TeV)
95% CL limit on σ(pp → HH) (fb)
Observed
Median expected
Excluded
68% expected
Theory prediction
95% expected
SM prediction
$\kappa_t = \kappa_V = \kappa_{2V} = 1$
10^4
10^3
10^2
–10
–5
-1.2
0
5
7
10
κ_λ

2510.07527



ATLAS-PHYS-PUB-2025-018

HEFT

- To describe the new physics beyond the SM, we often take a consistent effective field theory framework.
- HEFT: Higgs field as a singlet, non-linear, no explicit constraints on couplings

$$\mathcal{L}_{\mathrm{HEFT}} = \mathcal{L}_2 + \mathcal{L}_4$$

$$\begin{aligned}\mathcal{L}_2 = \frac{v^2}{4}\left(1 + 2a\frac{H}{v} + b\left(\frac{H}{v}\right)^2 + \cdots\right)\mathrm{Tr}[D_\mu U^\dagger D^\mu U] \\ + \frac{1}{2}\partial_\mu H \partial^\mu H - V(H) - \frac{1}{2g^2}\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] \\ - \frac{1}{2g'^2}\mathrm{Tr}[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}] + \mathcal{L}_{GF} + \mathcal{L}_{FP}.\end{aligned}$$

$$U(\pi^a) = e^{i\pi^a\tau^a/v}$$

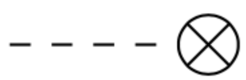
$$\begin{aligned}V(H) = (-\mu^2 + \lambda v^2)vH + \frac{1}{2}(-\mu^2 + 3\lambda v^2)H^2 \\ + \kappa_3 \lambda v H^3 + \kappa_4 \frac{\lambda}{4}H^4.\end{aligned}$$

Buchalla et al, 1307.5017

Renormalization

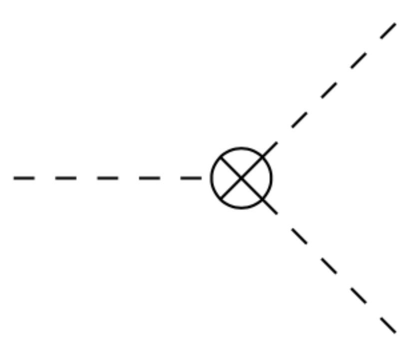
The renormalized Lagrangian after EW gauge symmetry breaking:

$$\mathscr{L}_{\mathrm{H}}^{\kappa} = \frac{1}{2} Z_\phi (\partial_\mu H)^2 - \left(-\frac{1}{2} Z_{\mu^2} Z_\phi Z_v^2 \mu^2 v^2 + \frac{1}{4} Z_\lambda Z_\phi^2 Z_v^4 \lambda v^4 \right) - (Z_\lambda Z_\phi^2 Z_v^3 \lambda v^3 - Z_{\mu^2} Z_\phi Z_v \mu^2 v) H$$
$$- \left(\frac{3}{2} Z_\lambda Z_\phi^2 Z_v^2 \lambda v^2 - \frac{1}{2} Z_{\mu^2} Z_\phi \mu^2 \right) H^2 - Z_{\kappa_{3\mathrm{H}}} Z_\lambda Z_\phi^2 Z_v \lambda_{3\mathrm{H}} v H^3 - \frac{1}{4} Z_{\kappa_{4\mathrm{H}}} Z_\lambda Z_\phi^2 \lambda_{4\mathrm{H}} H^4 + \cdots$$



$$= -2i\lambda v^3 (\delta Z_{m^2} - 2\delta Z_v - \delta Z_\lambda)\,,$$

$$= i\left[p^2 \delta Z_H - (\delta Z_{m^2} + \delta Z_H) m_H^2\right],$$



$$= -6i\kappa_3 \lambda v \left(\frac{3}{2} \delta Z_H + \delta Z_\lambda + \delta Z_v + \delta Z_{\kappa_3} \right).$$

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Renormalization

We choose the renormalization scheme in which there is no tadpole contributions.

$m_h^2 = 2\lambda v^2$ and $(\delta Z_{m^2} - \delta Z_\lambda - 2\delta Z_v)\mu^2 v + \delta T = 0$ with δT the one-loop diagrams.

$$\delta T = \frac{3\lambda_{3\mathrm{H}} v}{16\pi^2} m_H^2 \left(\frac{1}{\epsilon} + \ln \frac{\mu_R^2}{m_H^2} + 1 \right)$$

We choose the on-shell renormalization scheme for the Higgs mass.

$$\delta Z_{m^2} = \frac{3\lambda_{4\mathrm{H}}}{16\pi^2} \left(\frac{1}{\epsilon} + \ln \frac{\mu_R^2}{m_H^2} + 1 \right) + \frac{9\lambda_{3\mathrm{H}}^2 v^2}{m_H^2} \frac{1}{8\pi^2} \left(\frac{1}{\epsilon} + \ln \frac{\mu_R^2}{m_H^2} + 2 - \frac{\pi}{\sqrt{3}} \right)$$

$$\delta Z_\phi = \frac{9\lambda_{3\mathrm{H}}^2 v^2}{8\pi^2} \frac{\sqrt{3} - 2\pi/3}{\sqrt{3} m_H^2}$$

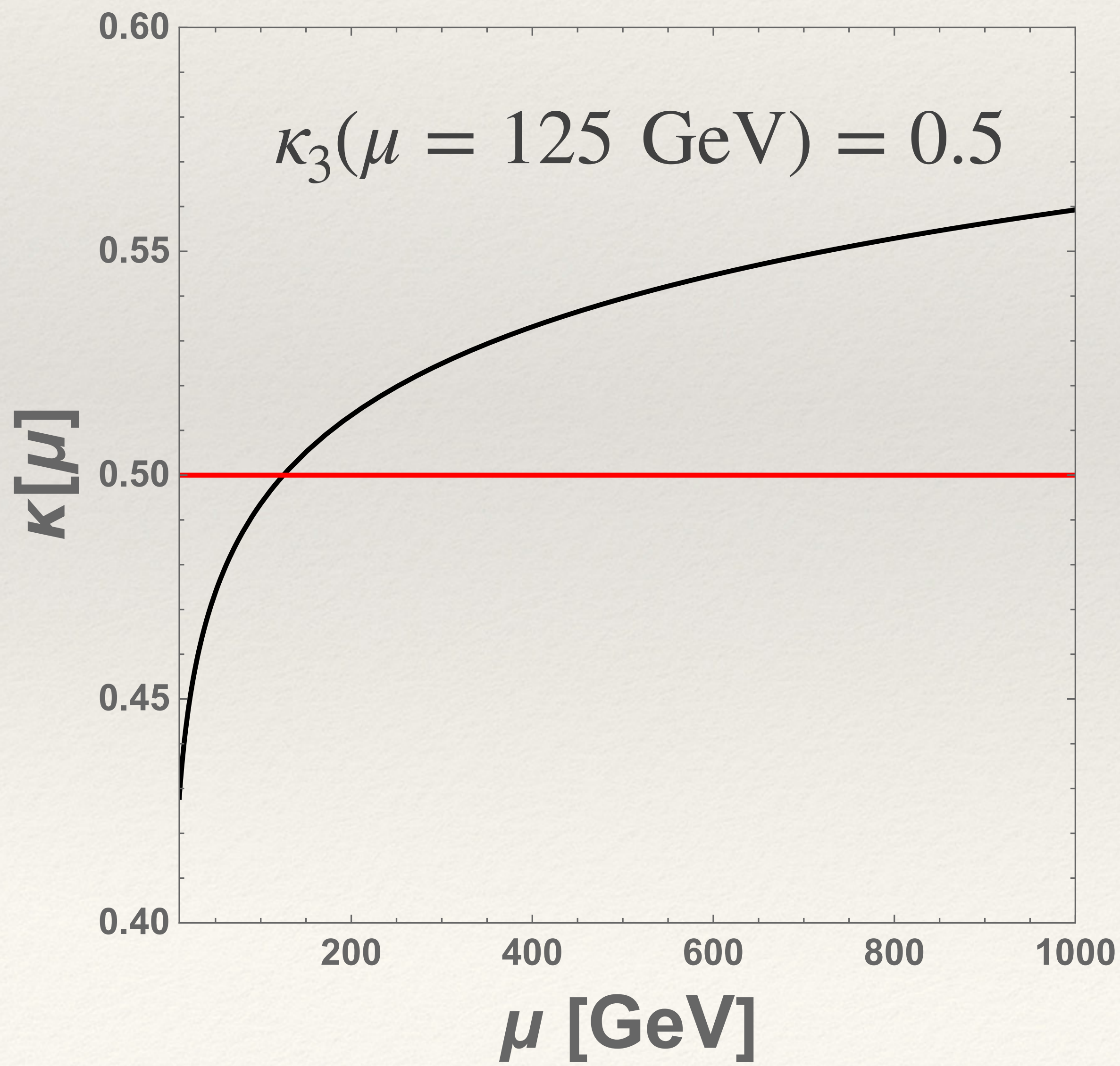
Since W/Z mass corrections are not affected by the Higgs self-couplings at one-loop, we can simply take $\delta Z_v = 0$.

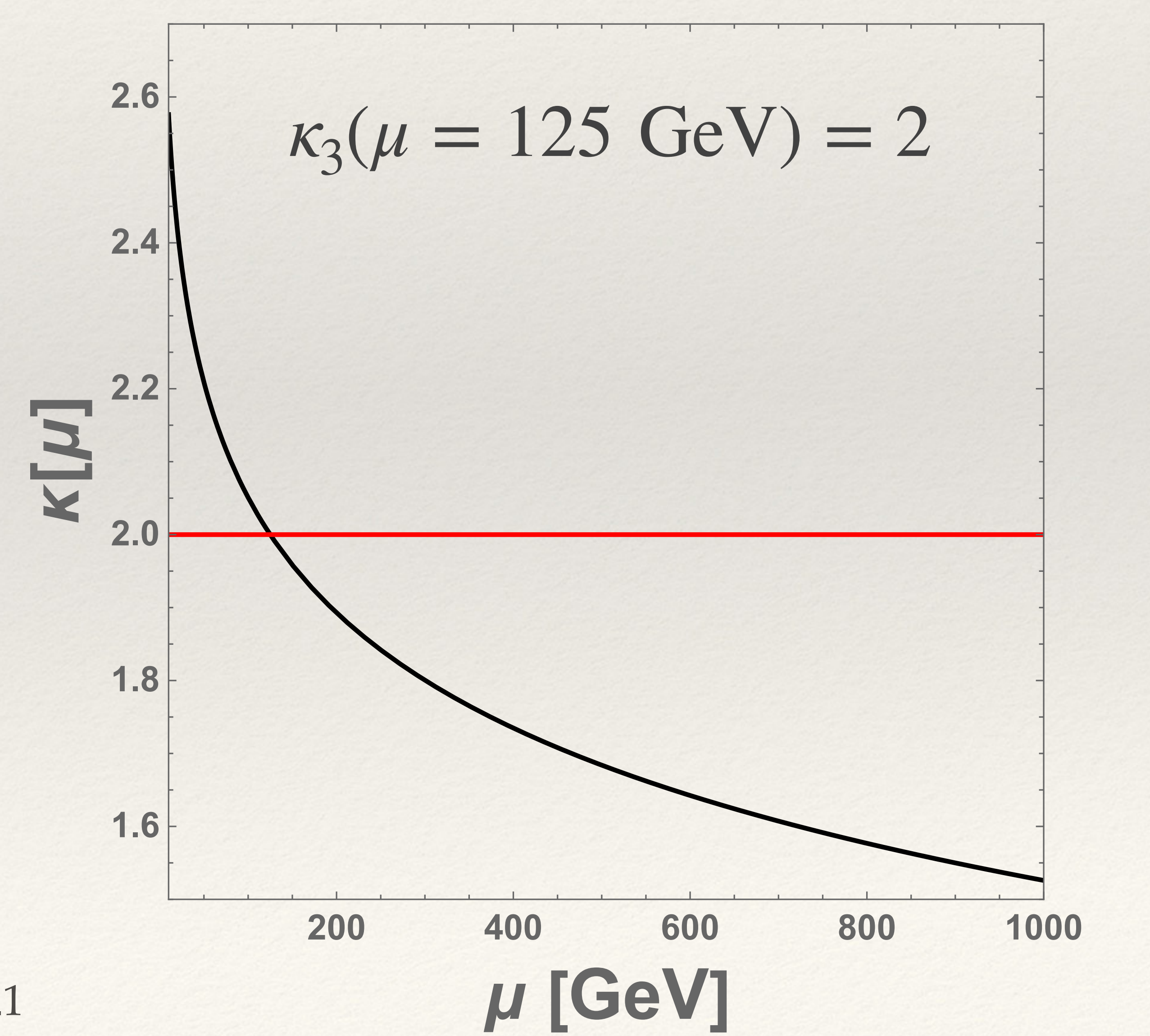
H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Renormalization

The coupling modifier κ_3 is renormalized in the $\overline{\text{MS}}$ scheme.

$$\delta Z_{\kappa_3} = \frac{3\lambda}{16\pi^2}\frac{1}{\epsilon}\left(\kappa_3 + 2\kappa_4 - 3\kappa_3^2\right)$$





Updated function forms

The λ dependent correction is

$$\delta\sigma^{\kappa_\lambda}_{\text{ggF,EW}} = (0.075\kappa^4_{\lambda_{3\text{H}}} - 0.158\kappa^3_{\lambda_{3\text{H}}} - 0.006\kappa^2_{\lambda_{3\text{H}}}\kappa_{\lambda_{4\text{H}}} - 0.058\kappa^2_{\lambda_{3\text{H}}} + 0.070\kappa_{\lambda_{3\text{H}}}\kappa_{\lambda_{4\text{H}}} - 0.149\kappa_{\lambda_{4\text{H}}}) \ \ \text{fb}$$

$$\delta\sigma^{\kappa_\lambda}_{\text{VBF,EW}} = (0.0215\kappa^4_{\lambda_{3\text{H}}} - 0.0324\kappa^3_{\lambda_{3\text{H}}} - 0.0019\kappa^2_{\lambda_{3\text{H}}}\kappa_{\lambda_{4\text{H}}} - 0.0043\kappa^2_{\lambda_{3\text{H}}} + 0.0151\kappa_{\lambda_{3\text{H}}}\kappa_{\lambda_{4\text{H}}} - 0.0211\kappa_{\lambda_{4\text{H}}}) \ \ \text{fb}$$

$\kappa_{\lambda_{3\text{H}}}$	$\kappa_{\lambda_{4\text{H}}}$	ggF			VBF		
		$\sigma^{\kappa_\lambda}_{\text{LO}}$	$\sigma^{\kappa_\lambda}_{\text{NNLO-FT}}$	$\delta\sigma^{\kappa_\lambda}_{\text{EW}}$	$\sigma^{\kappa_\lambda}_{\text{LO}}$	$\sigma^{\kappa_\lambda}_{\text{NNNLO}}$	$\delta\sigma^{\kappa_\lambda}_{\text{EW}}$
1	1	16.7	31.2	-0.225	1.71	1.69	-2.30×10^{-2}
3	1	8.59	18.4	1.28	3.59	3.53	8.35×10^{-1}
6	1	67.3	161	60.6	25.1	24.6	20.7
1	3	16.7	31.2	-0.393	1.71	1.69	-3.89×10^{-2}
1	6	16.7	31.2	-0.646	1.71	1.69	-6.27×10^{-2}
3	3	8.59	18.4	1.30	3.59	3.53	8.50×10^{-1}
6	6	67.3	161	61.0	25.1	24.6	20.7

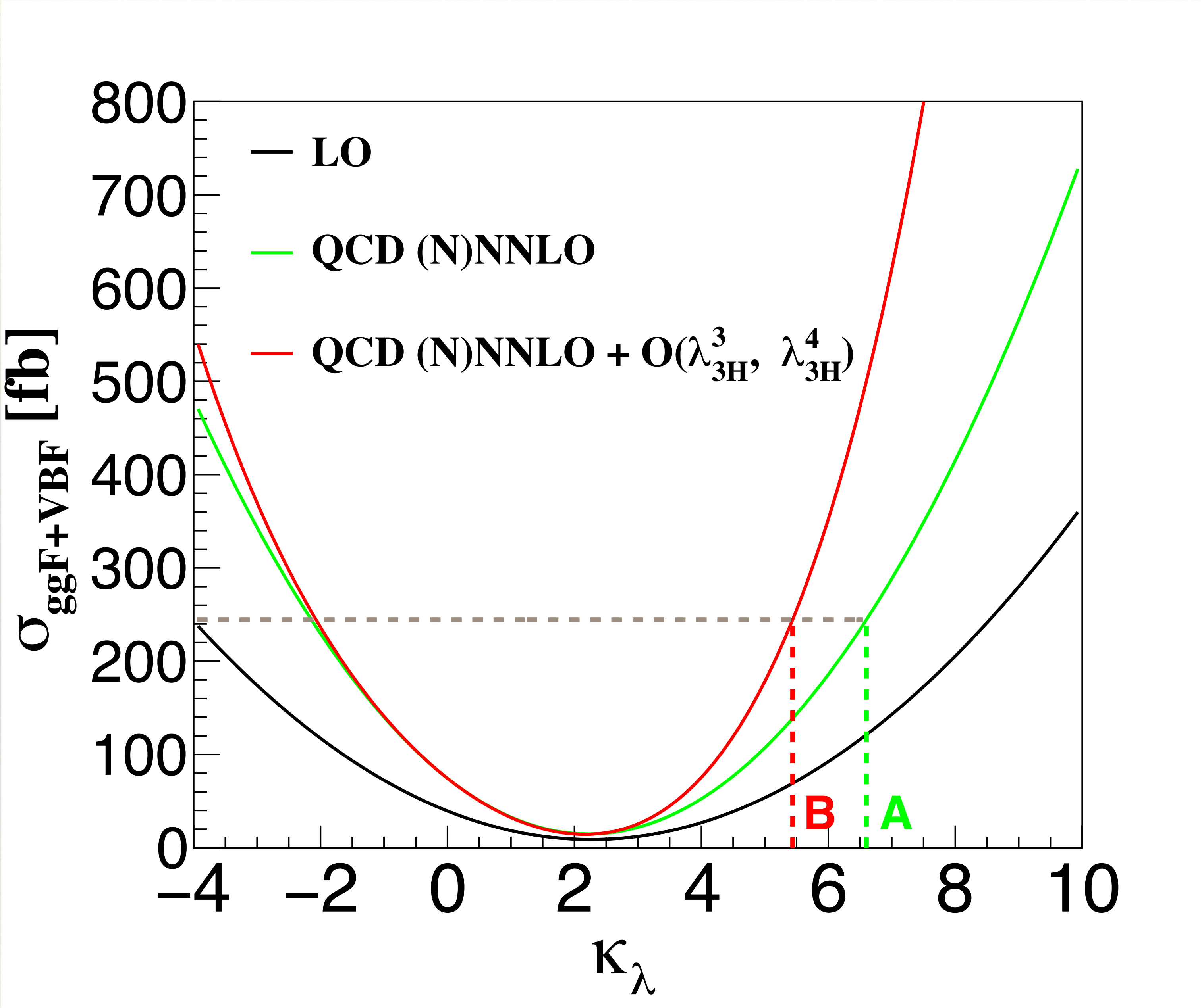
The QCD corrections are significant in ggF, but not sensitive to κ_3.

The EW corrections are 91% (82%) in ggF (VBF) for $\kappa_3 = 6$.

The dependence on κ_4 is weak.

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

More stringent constraint



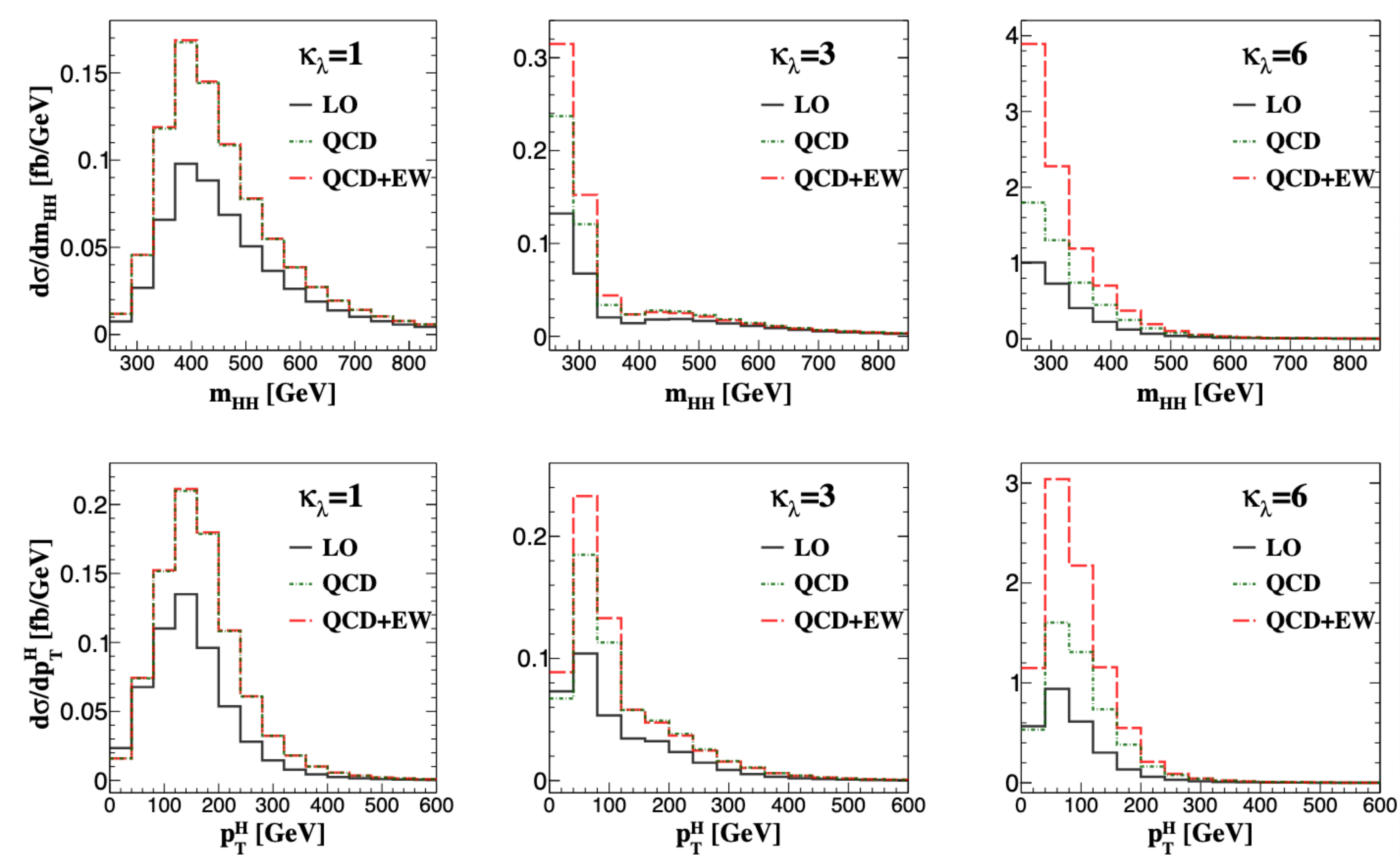
ATLAS (CMS) limit

6.6 (6.49)

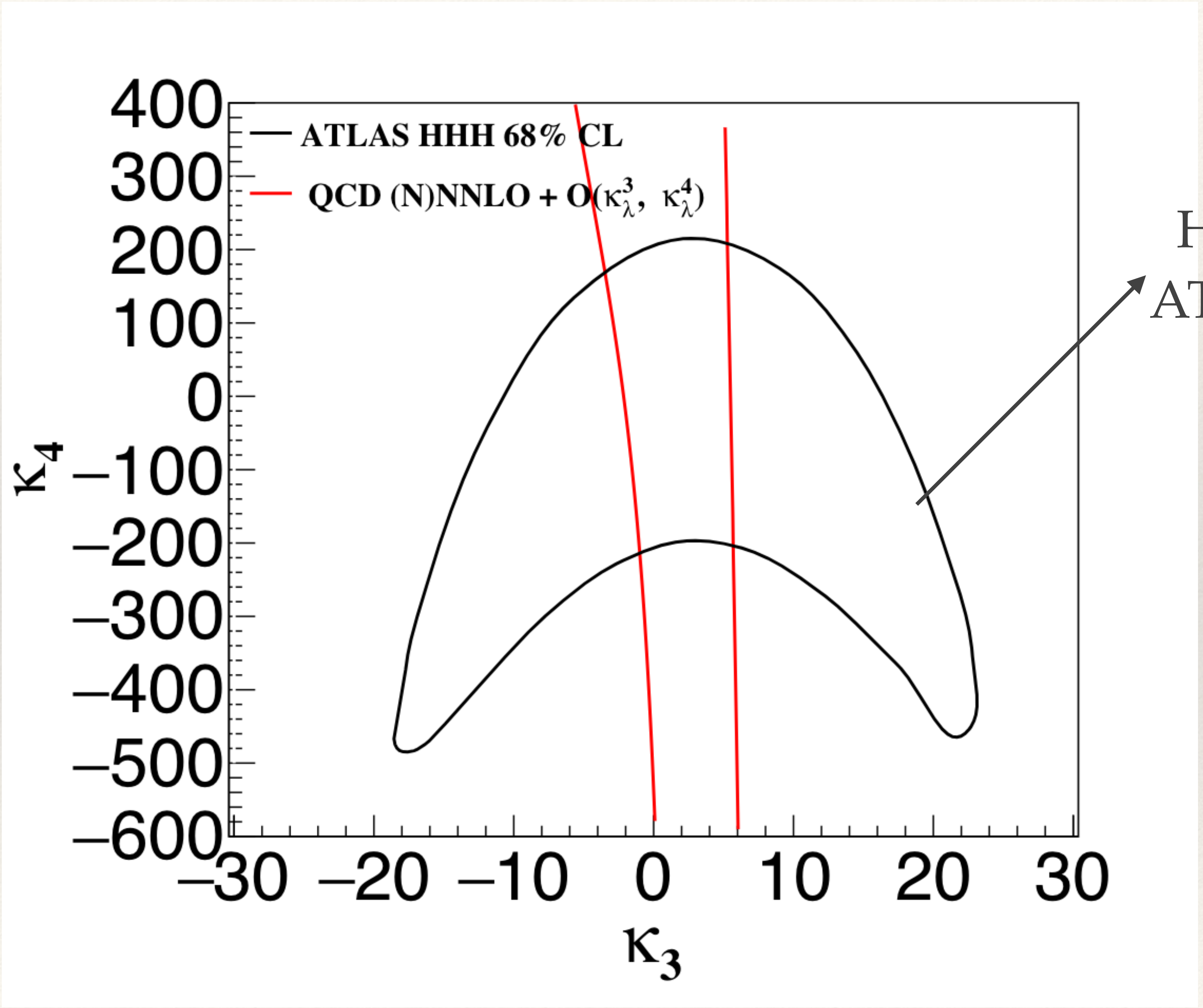
5.4 (5.37)

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Differential distributions



Sensitivity to κ_4



HHH constraint
ATLAS 2411.02040

Summary

Compared with the Higgs couplings to EW gauge bosons or third-generation fermions, the Higgs self-couplings remain only weakly constrained at LHC Run 2.

Probing the Higgs self-couplings at the HL-LHC and future colliders via multi-Higgs production explores largely uncharted model and parameter space and provides genuine discovery potential, even if single-Higgs observables show no anomalies.

Fully exploiting the discovery potential of multi-Higgs production channels requires including higher-order perturbative corrections induced by both QCD and EW interactions in the corresponding theoretical predictions.

Thanks a lot for your attention!

Back-up slides

HEFT

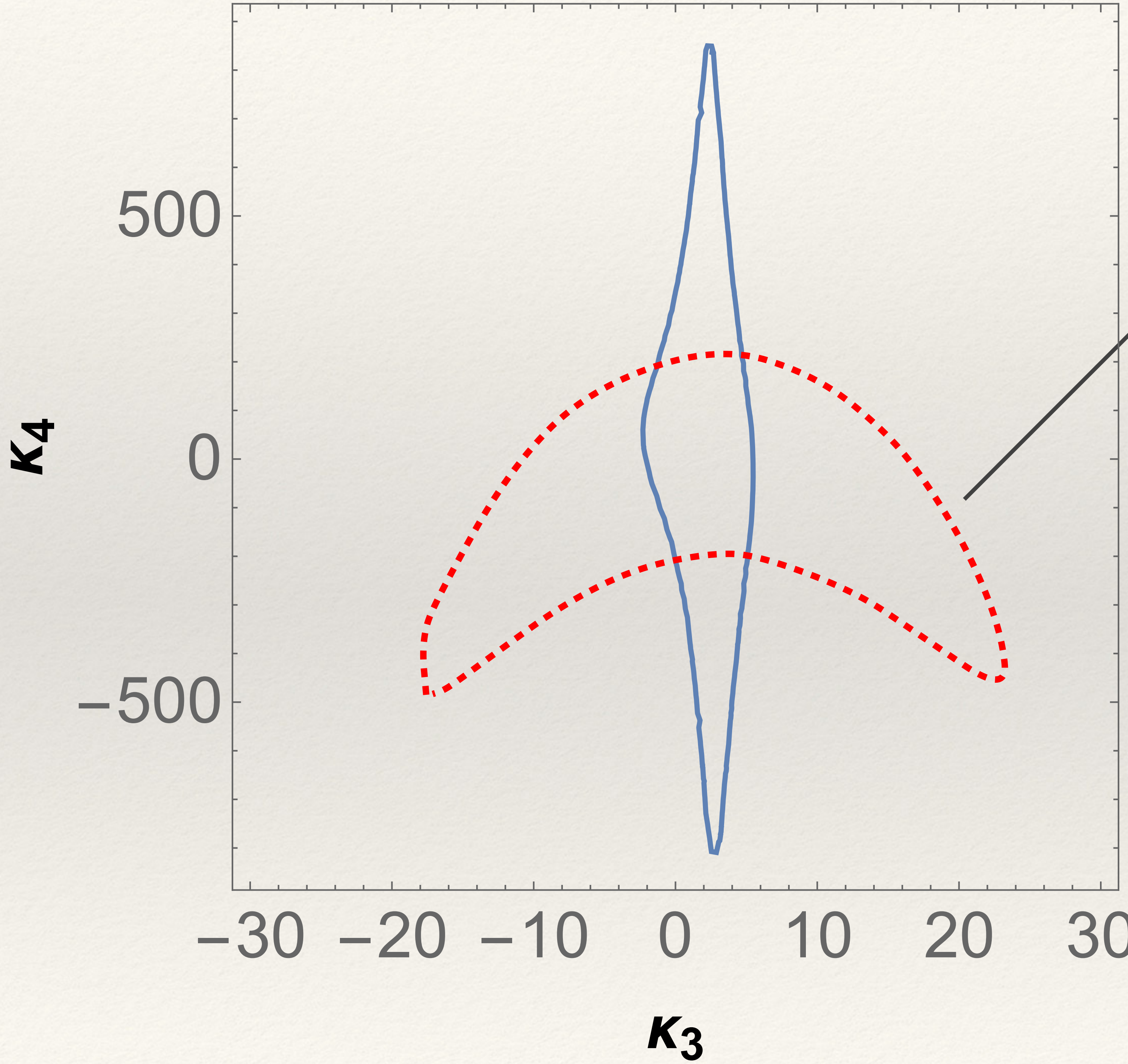
$$\begin{aligned}\mathcal{L}_4 =& -a_{dd\mathcal{V}\mathcal{V}1}\frac{\partial^\mu H\partial^\nu H}{v^2}\mathrm{Tr}[\mathcal{V}_\mu\mathcal{V}_\nu] - a_{dd\mathcal{V}\mathcal{V}2}\frac{\partial^\mu H\partial_\mu H}{v^2}\mathrm{Tr}[\mathcal{V}^\nu\mathcal{V}_\nu]\\
&+\left(a_{11}+a_{H11}\frac{H}{v}+a_{HH11}\frac{H^2}{v^2}\right)\mathrm{Tr}[\mathcal{D}_\mu\mathcal{V}^\mu\mathcal{D}_\nu\mathcal{V}^\nu] - \frac{m_{\mathrm{H}}^2}{4}\left(2a_{H\mathcal{V}\mathcal{V}}\frac{H}{v}+a_{HH\mathcal{V}\mathcal{V}}\frac{H^2}{v^2}\right)\mathrm{Tr}[\mathcal{V}^\mu\mathcal{V}_\mu]\\
&-\left(a_{HWW}\frac{H}{v}+a_{HHWW}\frac{H^2}{v^2}\right)\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] + i\left(a_{d2}+a_{Hd2}\frac{H}{v}\right)\frac{\partial^\nu H}{v}\mathrm{Tr}[\hat{W}_{\mu\nu}\mathcal{V}^\mu]\\
&+\left(a_{\Box\mathcal{V}\mathcal{V}}+a_{H\Box\mathcal{V}\mathcal{V}}\frac{H}{v}\right)\frac{\Box H}{v}\mathrm{Tr}[\mathcal{V}_\mu\mathcal{V}^\mu] + \left(a_{d3}+a_{Hd3}\frac{H}{v}\right)\frac{\partial^\nu H}{v}\mathrm{Tr}[\mathcal{V}_\nu\mathcal{D}_\mu\mathcal{V}^\mu]\\
&+\left(a_{\Box\Box}+a_{H\Box\Box}\frac{H}{v}\right)\frac{\Box H\Box H}{v^2}+a_{dd\Box}\frac{\partial^\mu H\partial_\mu H\Box H}{v^3}\\
&+\left(a_{Hdd}\frac{m_{\mathrm{H}}^2}{v^2}+a_{ddW}\frac{m_{\mathrm{W}}^2}{v^2}+a_{ddZ}\frac{m_{\mathrm{Z}}^2}{v^2}\right)\frac{H}{v}\partial^\mu H\partial_\mu H.\end{aligned}$$

Brivio et al, JHEP, 1403, 024
Gavela, Kanshin, Machado, Saa, JHEP, 1503, 043

Renormalization:

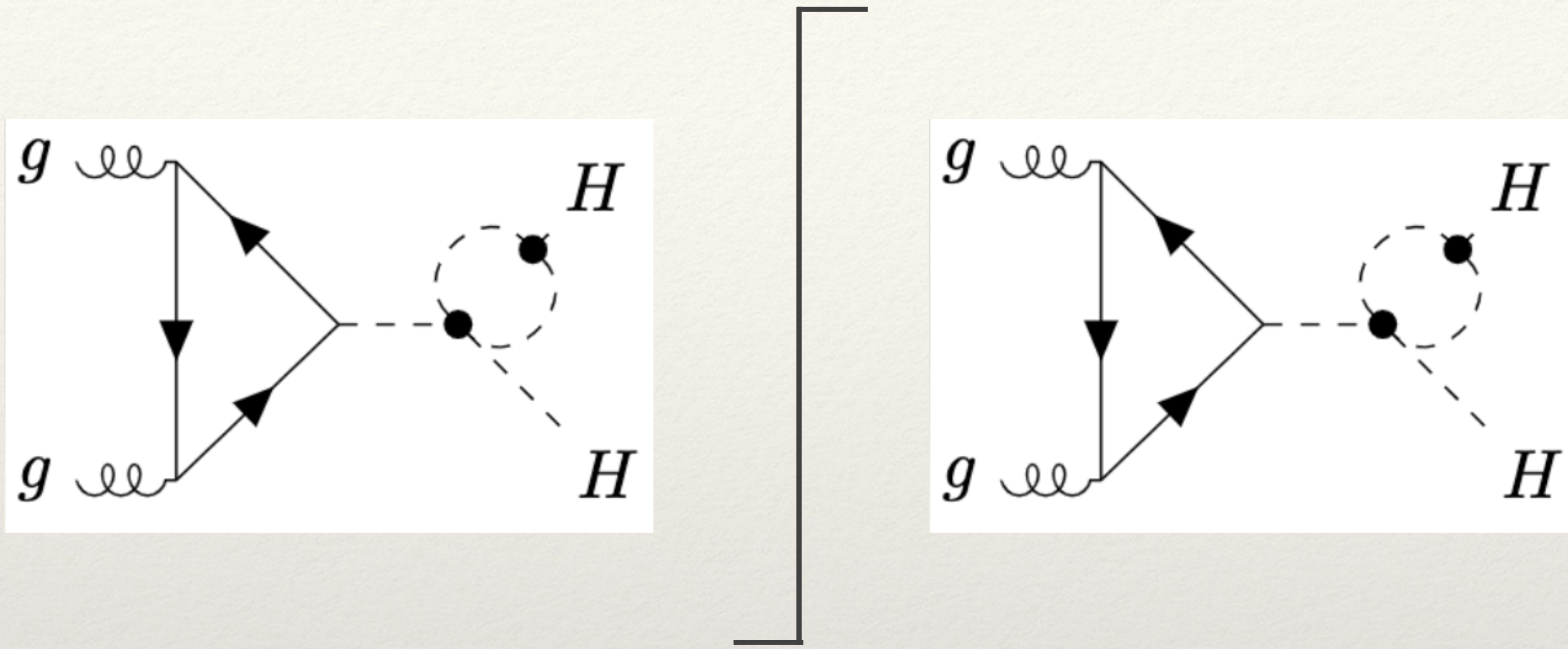
- Background field method: Guo, Ruiz-Femenia, Sanz-Cillero, PRD92,074005(2015); Buchalla, et al, NPB, 928,93 (2018), PRD104,076005(2021)
- Scattering amplitude: Herrero, Morales, PRD106,073008 (2022)

Sensitivity to κ_4



HHH constraint
ATLAS 2411.02040

Sensitivity to κ_4



$$\delta\sigma^{\kappa_\lambda}_{\mathrm{ggF,EW}} = (0.075\kappa^4_{\lambda_{3H}} - 0.158\kappa^3_{\lambda_{3H}} - 0.006\kappa^2_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.058\kappa^2_{\lambda_{3H}} + 0.070\kappa_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.149\kappa_{\lambda_{4H}})\quad \mathrm{fb}$$

$$+[0.887\kappa^6_{\lambda_{3H}} - 2.439\kappa^5_{\lambda_{3H}} + 3.200\kappa^3_{\lambda_{3H}} + \kappa_{\lambda_{4H}}(-0.579\kappa^4_{\lambda_{3H}} + 2.821\kappa^3_{\lambda_{3H}} - 2.870\kappa^2_{\lambda_{3H}})$$

$$+\kappa^2_{\lambda_{4H}}(0.135\kappa^2_{\lambda_{3H}} - 0.742\kappa_{\lambda_{3H}} - 1.179)] \times 10^{-3}\quad \mathrm{fb}$$

This is only partial NNLO correction!

Comparison with SMEFT

- SMEFT: Higgs field as a doublet, Wilson coefficients suppressed by Λ

$$V(\phi_b) = -\mu_b^2(\phi^\dagger\phi)_b + \lambda_b(\phi^\dagger\phi)_b^2 + \frac{d_6^b}{\Lambda^2}\left[(\phi^\dagger\phi)_b - \frac{v^2}{2}\right]^3 + \frac{d_8^b}{\Lambda^4}\left[(\phi^\dagger\phi)_b - \frac{v^2}{2}\right]^4 + \frac{d_{10}^b}{\Lambda^6}\left[(\phi^\dagger\phi)_b - \frac{v^2}{2}\right]^5.$$

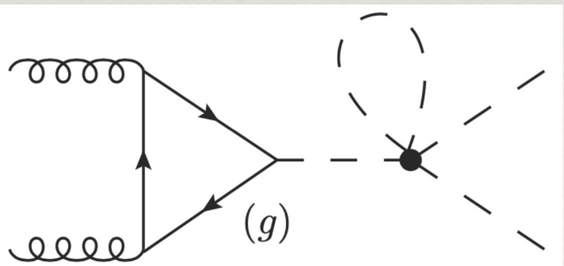
Borowka et al, 1811.12366

$$\mathcal{L}_{3H} = -\kappa_3 \lambda v H^3 - (\delta Z_\lambda + \frac{3}{2}\kappa_3 \delta Z_\phi + \bar{d}_6 \delta Z_{d_6})\lambda v H^3,$$

$$\kappa_3 \equiv 1 + \bar{d}_6$$

$$d_6^b = Z_{d_6}\frac{\lambda\Lambda^2}{v^2}\bar{d}_6$$

$$\mathcal{L}_{4H} + \mathcal{L}_{5H} = -\frac{1}{4}\kappa_4 \lambda H^4 - \kappa_5 \frac{\lambda}{v} H^5 + \cdots$$



$$\kappa_4 \equiv 1 + 6\bar{d}_6 + \bar{d}_8 \qquad \kappa_5 \equiv \tfrac{1}{4}(3\bar{d}_6 + 2\bar{d}_8 + \bar{d}_{10})$$

If $\bar{d}_{10} = 0$, $\kappa_5 = (7 - 9\kappa_3 + 2\kappa_4)/4$.

Comparison with SMEFT

If the new operators are renormalized in the $\overline{\text{MS}}$ scheme. We find that the amplitude of $gg \to H^* \to HH$ is different from that in HEFT.

$$\frac{i\mathcal{M}_{\text{SMEFT2}} - i\mathcal{M}_{\kappa_5}}{i\mathcal{M}_{\text{LO}}} = \frac{3m_H^2(\kappa_3 - 1)\left[-(3-\sqrt{3}\pi)\kappa_3^2 + (\kappa_3 - 3\kappa_3^2 - \kappa_4)(1 + \ln\frac{\mu_R^2}{m_H^2})\right]}{32\pi^2 v^2 \kappa_3}$$

If we require that the amplitude remains the same, we have to change the renormalization condition. The renormalization of the dimension-six operator is not in $\overline{\text{MS}}$ any more.

$$(\delta Z_{d_6})_{\text{finite}} = (\delta Z_\lambda)_{\text{finite}}$$

$\overline{\text{MS}}$ in HEFT is not $\overline{\text{MS}}$ in SMEFT!