NPhiS 2026

Gauge and Goldstone bosons from Flavour Symmetries

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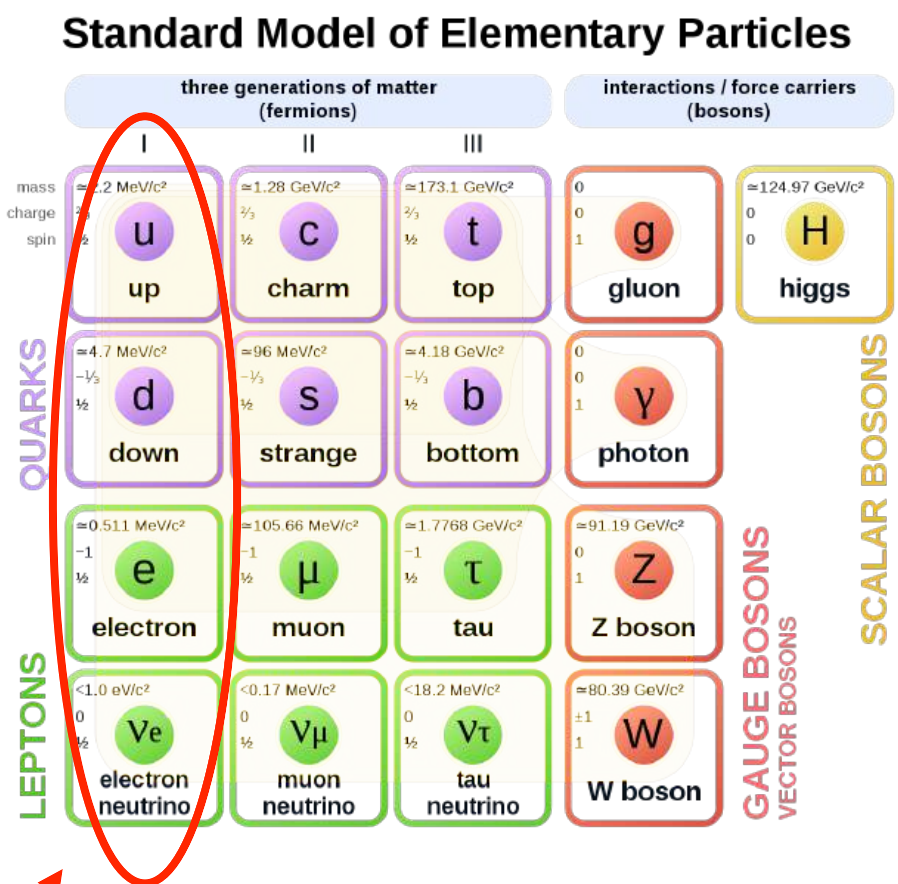
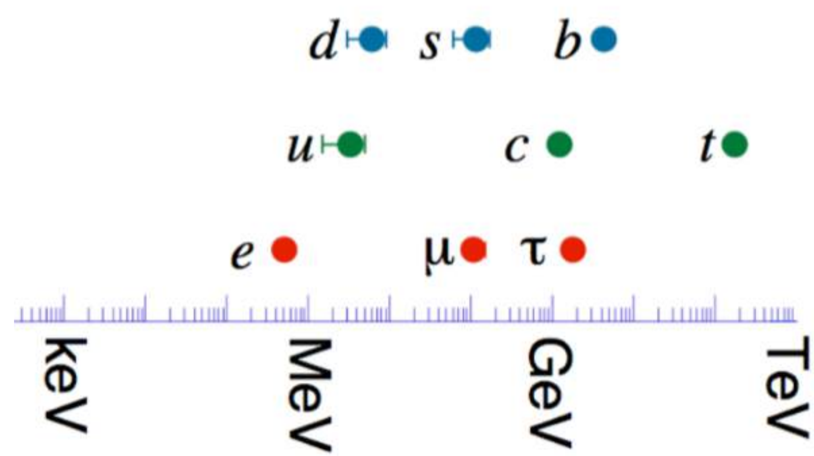
The flavour puzzle

see *e.g.* the reviews
Zupan 1903.05062
Altmannshofer Stangl 2508.03950





3 fermion generations (or families)

Hierarchical fermion masses

You are here (why?)

(why?)

Flavor in the SM

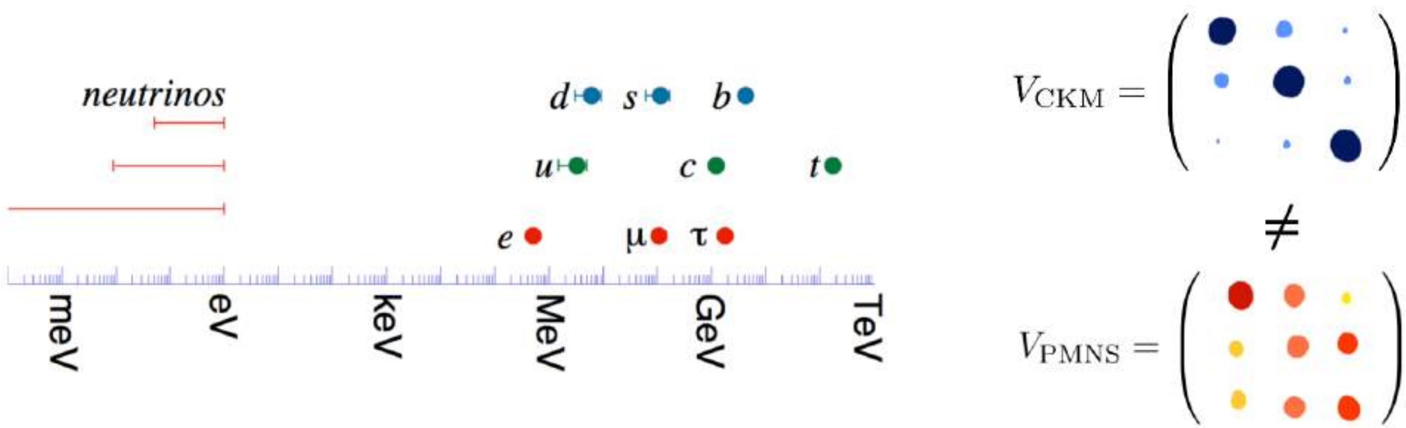
courtesy of O. Sumensari

- The SM **flavor sector** is **loose**: (*even w/o considering neutrinos*)

  $\Rightarrow$ 13 free parameters (masses and quark mixing) — *fixed by data.*

$$\mathcal{L}_{\rm Yuk} = -Y_d^{ij}\,\overline{Q}_i d_{Rj}\, H - Y_u^{ij}\,\overline{Q}_i u_{Rj}\, \widetilde{H} - Y_\ell^{ij}\,\overline{L}_i e_{Rj}\, H + \text{h.c.}$$

  $\Rightarrow$ These (many) parameters exhibit a hierarchical structure which we do not understand.



How to **explain the observed patterns** in terms of **less** and **more fundamental parameters**?

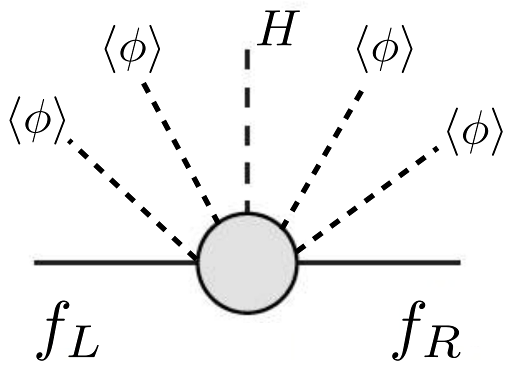
A possible solution: Froggatt-Nielsen flavour models

- SM fermions charged under a new horizontal symmetry G_F
- G_F forbids Yukawa couplings at the renormalisable level
- G_F spontaneously broken by the vev(s) of one or more scalars (the "flavons")
- Yukawas arise as higher dimensional operators

Froggatt Nielsen '79
Leurer Seiberg Nir '92, '93
...

$$-\mathcal{L} = a^u_{ij} \left(\frac{\phi}{\Lambda}\right)^{n^u_{ij}} \overline{Q}_i u_j \tilde{H} + a^d_{ij} \left(\frac{\phi}{\Lambda}\right)^{n^d_{ij}} \overline{Q}_i d_j H$$

flavour-anarchical
O(1) coefficients



$\langle\phi\rangle < \Lambda \Longrightarrow \epsilon \equiv \langle\phi\rangle/\Lambda$ small expansion parameter (Λ=UV scale)

n^f_{ij} dictated by the symmetry

G_F could abelian or non-abelian, continuous or discrete, local or global

→ for a recent review see Altmannshofer Greljo '24

The simplest option: Froggatt-Nielsen U(1)

Quark sector

FN charges

	ϕ	Q_i	u_i	d_i	H
U(1)	1	$\mathcal{Q}_{Q_i}$	$\mathcal{Q}_{u_i}$	$\mathcal{Q}_{d_i}$	0

$\Rightarrow$

$$Y^u_{ij} = a^u_{ij}\,\epsilon^{\mathcal{Q}_{Q_i}-\mathcal{Q}_{u_j}}$$

$$Y^d_{ij} = a^d_{ij}\,\epsilon^{\mathcal{Q}_{Q_i}-\mathcal{Q}_{d_j}}$$

Rotation matrices $Y^f = V^{f\dagger}\hat{Y}^f W^f \Rightarrow V^{u,d}_{ij} \sim \epsilon^{|\mathcal{Q}_{Q_i}-\mathcal{Q}_{Q_j}|} \quad W^{u,d}_{ij} \sim \epsilon^{|\mathcal{Q}_{u_i,d_i}-\mathcal{Q}_{u_j,d_j}|}$

Successful predictions for $V_{\rm CKM} = V^u\, V^{d\,\dagger}$:

$$V_{ud} \approx V_{cs} \approx V_{tb} \approx 1 \qquad V_{ub} \approx V_{td} \approx V_{us} \times V_{cb}$$

(independent of charge assignment)

Example:

$$(\mathcal{Q}_{Q_1},\,\mathcal{Q}_{Q_2},\,\mathcal{Q}_{Q_3}) \;=\; (3,\,2,\,0), \;\; (\mathcal{Q}_{u_1},\,\mathcal{Q}_{u_2},\,\mathcal{Q}_{u_3}) \;=\; (-4,\,-2,\,0), \;\; (\mathcal{Q}_{d_1},\,\mathcal{Q}_{d_2},\,\mathcal{Q}_{d_3}) \;=\; (-4,\,-2,\,-2)$$

$$Y^u \sim \begin{pmatrix} \epsilon^7 & \epsilon^5 & \epsilon^3 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \qquad Y^d \sim \begin{pmatrix} \epsilon^7 & \epsilon^5 & \epsilon^5 \\ \epsilon^6 & \epsilon^4 & \epsilon^4 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \end{pmatrix} \qquad V_{\rm CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

$$\epsilon = \langle\phi\rangle/\Lambda \approx 0.2$$

Global Froggatt-Nielsen U(1)

Spontaneous breaking of a **global** U(1) $\Rightarrow$ Nambu-Goldstone boson

$$\phi = \frac{v_\phi + \varphi}{\sqrt{2}}\, e^{i a / v_\phi}$$

For anomalous charges (QCD anomaly) a behaves like a **QCD axion**:

$\Rightarrow$ "*axiflavon*"

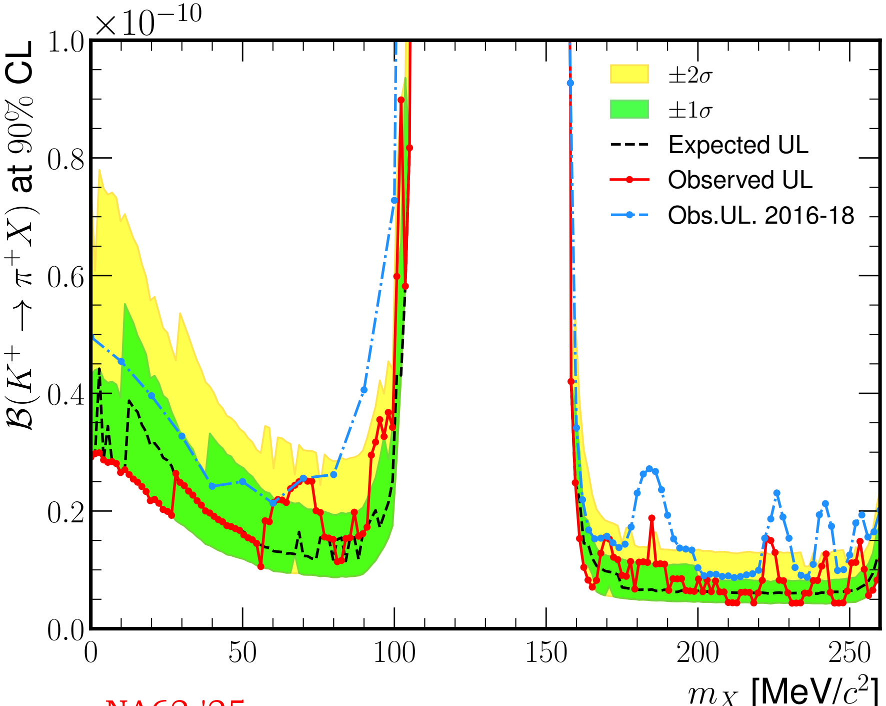
LC Goertz Redigolo Ziegler Zupan '16
Ema Hamaguchi Moroi Nakayama '16

- Acquires mass from the QCD anomaly
- Drives the QCD vacuum to a CP-conserving minimum (solving the strong CP problem)
- Provides Dark Matter through the misalignment mechanism
- Anomaly coefficients (hence, axion coupling to photons) predicted in terms of the fermion mass hierarchies
- By construction, flavour-violating couplings: produced in meson/lepton decays, prominently $K \to \pi\, a$

Flavour-violating decays into light bosons

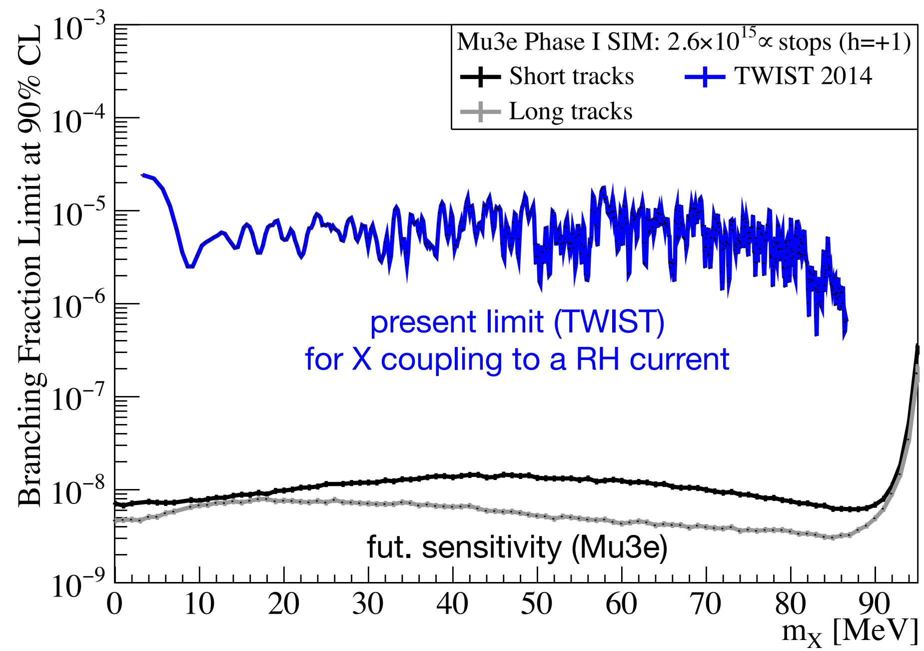
Searches for meson and lepton decays to a light invisible boson X, *e.g.*:

$K^+ \to \pi^+ X$



NA62 '25

$\mu^+ \to e^+ X$



TWIST 2014, Perrevoort (Mu3e) '18

(similar sensitivity at MEGII, Mu2e, COMET)

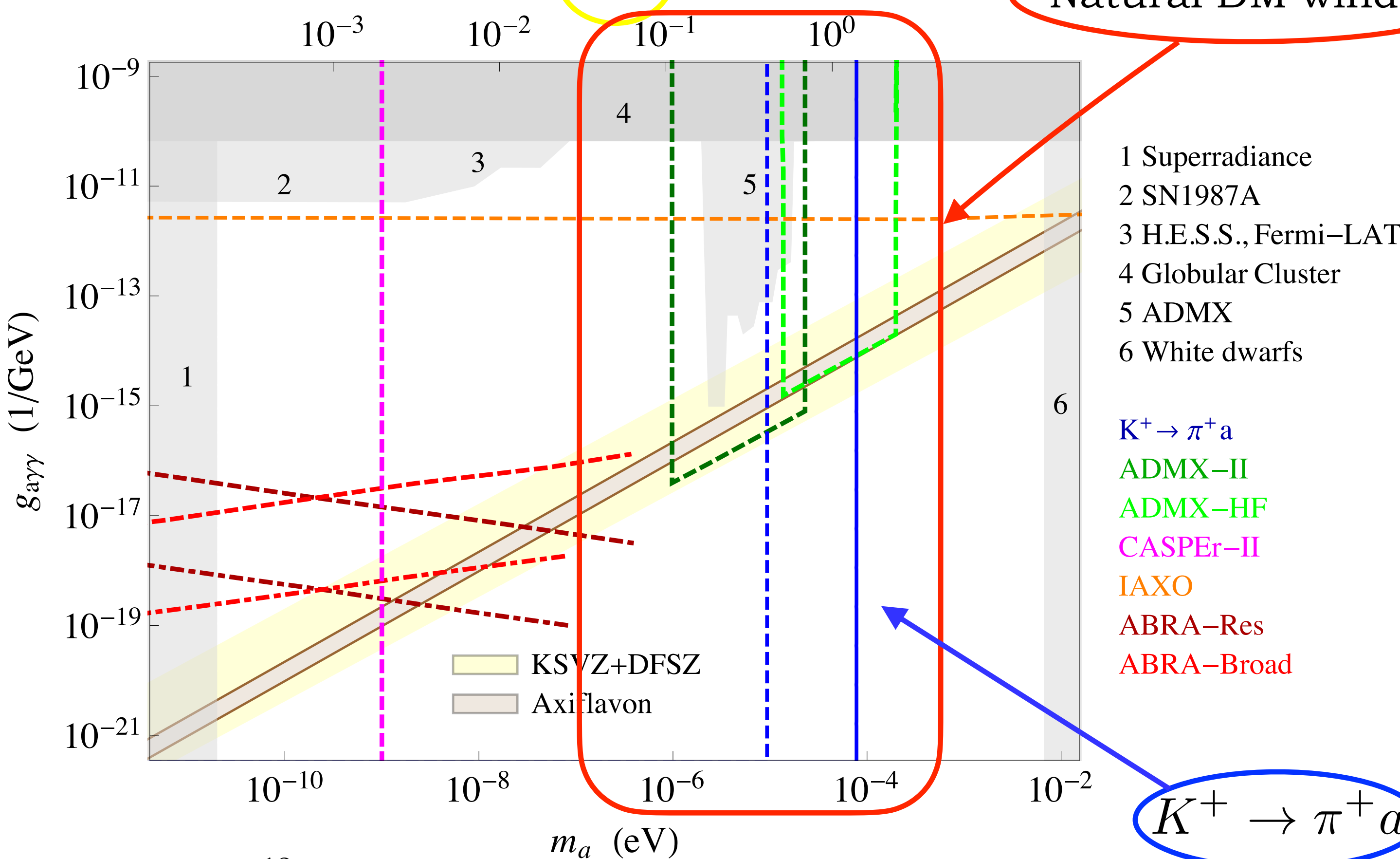
In our case, depending on their lifetime, the light boson X can give signature like these or decay inside the detector *e.g.* as $X \to \mu^+\mu^-, e^+e^-$

Global Froggatt-Nielsen U(1)

The axiflavon can be complementary tested at axion and flavour experiments

Natural DM window

$\frac{1}{4} g_{a\gamma\gamma} a F \tilde{F}$



$K^+ \to \pi^+ a$

$$m_a = 5.7\,\mu\text{eV}\left(\frac{10^{12}\text{GeV}}{f_a}\right)$$

LC Goertz Redigolo Ziegler Zupan '16

U(2) flavour model

Data seems to suggest a tighter flavour structure

"Zeroth-order" pattern:

massive third generation (singlets), *massless* first two families (doublets)

$\Longrightarrow$ **U(2)** natural candidate

Barbieri Dvali Hall '95
Barbieri Hall Romanino '97
...

We focus on the $U(2)_F \simeq SU(2)_F \times U(1)_F$ model proposed in Linster Ziegler '18 providing an excellent fit to fermion masses and mixing (*including neutrinos* $\rightarrow$ Linster et al. '20, Giarnetti et al. '25)

Charge assignment

(as slightly modified in LC Redigolo Ziegler Zupan '20)

	U_a	D_a	Q_a	U_3	D_3	Q_3	E_a	L_a	E_3	L_3	H	ϕ_a	χ
$SU(2)_F$	**2**	**2**	**2**	**1**	**1**	**1**	**2**	**2**	**1**	**1**	**1**	**2**	**1**
$U(1)_F$	1	1	1	0	1	0	1	1	0	-1	0	-1	-1

$$\langle\phi\rangle = \frac{v_\phi}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} \equiv \begin{pmatrix}\varepsilon_\phi\Lambda\\0\end{pmatrix}, \quad \langle\chi\rangle = \frac{v_\chi}{\sqrt{2}} \equiv \varepsilon_\chi\Lambda$$

2 flavons: 1 doublet, 1 singlet

Example: down-quark Yukawas

$$\mathcal{L}_d = \frac{\lambda^d_{11}}{\Lambda^6}\chi^4\left(\phi^*_a Q_a\right)\left(\phi^*_b D_b\right)\widetilde{H} + \frac{\lambda^d_{12}}{\Lambda^2}\chi^2\epsilon_{ab}Q_a D_b\widetilde{H} + \frac{\lambda^d_{13}}{\Lambda^4}\chi^3\left(\phi^*_a Q_a\right)D_3\widetilde{H} +$$
$$\frac{\lambda^d_{22}}{\Lambda^2}\left(\epsilon_{ab}\phi_a Q_b\right)\left(\epsilon_{cd}\phi_c D_d\right)\widetilde{H} + \frac{\lambda^d_{23}}{\Lambda^2}\chi\left(\epsilon_{ab}\phi_a Q_b\right)D_3\widetilde{H} + \frac{\lambda^d_{31}}{\Lambda^3}\chi^2 Q_3\left(\phi^*_a D_a\right)\widetilde{H} +$$
$$\frac{\lambda^d_{32}}{\Lambda}Q_3\left(\epsilon_{ab}\phi_a D_b\right)\widetilde{H} + \frac{\lambda^d_{33}}{\Lambda}\chi Q_3 D_3\widetilde{H}\,,$$

$$\Rightarrow \quad Y^d \approx \begin{pmatrix} \lambda^d_{11}\varepsilon^2_\phi\varepsilon^4_\chi & \lambda^d_{12}\varepsilon^2_\chi & \lambda^d_{13}\varepsilon_\phi\varepsilon^3_\chi \\ -\lambda^d_{12}\varepsilon^2_\chi & \lambda^d_{22}\varepsilon^2_\phi & \lambda^d_{23}\varepsilon_\phi\varepsilon_\chi \\ \lambda^d_{31}\varepsilon_\phi\varepsilon^2_\chi & \lambda^d_{32}\varepsilon_\phi & \lambda^d_{33}\varepsilon_\chi \end{pmatrix} \approx \begin{pmatrix} 0 & \lambda^d_{12}\varepsilon^2_\chi & 0 \\ -\lambda^d_{12}\varepsilon^2_\chi & \lambda^d_{22}\varepsilon^2_\phi & \lambda^d_{23}\varepsilon_\phi\varepsilon_\chi \\ 0 & \lambda^d_{32}\varepsilon_\phi & \lambda^d_{33}\varepsilon_\chi \end{pmatrix}$$

approximate texture zeroes (fewer parameters to fit than in U(1) models)

Rotation matrices

$$V^d_L \sim V^e_R \sim \begin{pmatrix} 1 & \frac{\varepsilon^2_\chi}{\varepsilon^2_\phi} & \frac{\varepsilon^2_\chi}{\varepsilon_\phi} \\ \frac{\varepsilon^2_\chi}{\varepsilon^2_\phi} & 1 & \varepsilon_\phi \\ \frac{\varepsilon^2_\chi}{\varepsilon_\phi} & \varepsilon_\phi & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad V^d_R \sim V^e_L \sim \begin{pmatrix} 1 & \frac{\varepsilon^2_\chi}{\varepsilon^2_\phi} & \varepsilon^2_\chi \\ \frac{\varepsilon^2_\chi}{\varepsilon^2_\phi} & 1 & 1 \\ \frac{\varepsilon^2_\chi}{\varepsilon^2_\phi} & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^5 \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

$\lambda \approx \sin\theta_c \approx 0.2$

good fit with $\varepsilon_\chi = 0.008\,, \quad \varepsilon_\phi = 0.023$

U(2) Nambu-Goldstone bosons

Four Nambu-Goldstone bosons:

$$\phi(x) = \frac{e^{i\,\widetilde{\pi}_i(x)\,\sigma_i/v_\phi}}{\sqrt{2}} \begin{pmatrix} v_\phi + \varphi(x) \\ 0 \end{pmatrix}\,, \quad \chi(x) = \frac{1}{\sqrt{2}}\left(v_\chi + \rho\,(x)\right) e^{i\,\widetilde{a}(x)/v_\chi}$$

Those associated with diagonal generators mix

($U(1)_F \subset U(2)_F$ broken by both $\langle\phi\rangle$ and $\langle\chi\rangle$)

$$\pi'_1 = \widetilde{\pi}_1 \quad \pi'_2 = \widetilde{\pi}_2\,, \quad \pi'_3 = \frac{v_\chi\widetilde{\pi}_3 - v_\phi\widetilde{a}}{\sqrt{v_\chi^2 + v_\phi^2}}\,, \quad a = \frac{v_\chi\widetilde{a} + v_\phi\widetilde{\pi}_3}{\sqrt{v_\chi^2 + v_\phi^2}}$$

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PNGB of the (global) anomalous $U(1)_F \rightarrow$ **axiflavon**!

Again, a behaves like a QCD axion, but $U(2)$ suppresses 1-2 flavour violation:

$$\mathcal{L}_a = \frac{a}{f_a}\frac{\alpha_s}{8\pi} G^a_{\mu\nu}\widetilde{G}^{a\,\mu\nu} + \frac{E}{N}\frac{a}{f_a}\frac{\alpha_{\rm em}}{8\pi} F_{\mu\nu}\widetilde{F}^{\mu\nu} + \frac{\partial_\mu a}{2 f_a}\,\overline{f}_i \gamma^\mu \left(C^V_{f_i f_j} + C^A_{f_i f_j}\gamma_5\right) f_j \qquad \begin{matrix} E = 10 \\ N = 9/2 \end{matrix}$$

$$C^{V,A}_{f_i f_j} = \frac{[f_{R\,a}] \mp [f_{L\,a}]}{2N}\delta_{ij} + \frac{[f_{R\,3}] - [f_{R\,a}]}{2N}\varepsilon^{f_R}_{ij} \mp \frac{[f_{L\,3}] - [f_{L\,a}]}{2N}\varepsilon^{f_L}_{ij}$$

$U(1)_F$ charges

$$\varepsilon^{f_L}_{ij} \equiv (V^f_L)_{3i}(V^f_L)^*_{3j}\,, \quad \varepsilon^{f_R}_{ij} \equiv (V^f_R)_{3i}(V^f_R)^*_{3j}$$

1-2 transitions only from mixing with 3rd generation

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PNGB of the (global) anomalous $U(1)_F \to$ **axiflavon**!

Again, a behaves like a QCD axion, but $U(2)$ suppresses 1-2 flavour violation:

⇨ Strongest limits on the $U(2)_F$ axiflavon come from astrophysics (supernova explosions, white dwarf cooling)

$$f_a \gtrsim 2.0 \times 10^7\,\text{GeV} \quad (K \to \pi a)\,, \quad f_a \gtrsim 5 \times 10^8\,\text{GeV} \quad (\text{SN1987A})\,, \quad f_a \gtrsim 3.5 \times 10^8\,\text{GeV} \quad (\text{WD})$$

$$f_a \equiv \frac{\sqrt{v_\chi^2 + v_\phi^2}}{\sqrt{2}N} = \frac{\sqrt{1 + \varepsilon_\chi^2/\varepsilon_\phi^2}}{\sqrt{2}N}\, v_\phi \simeq 0.17\, v_\phi$$

Linster Ziegler '18

LC Yi '25

U(2) Nambu-Goldstone bosons

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What about the other three bosons?

$SU(2)_F$ is *anomaly free*, so we can consider two cases:

(i) Local $SU(2)_F \Rightarrow$ gauge boson triplet, W'

(ii) Global $SU(2)_F \Rightarrow$ PNGB triplet, π'

LC and Jiangyi Yi 2511.10468, PRD

W' mass and couplings controlled by the $SU(2)_F$ gauge coupling g_F :

$$m_{W'} = \frac{1}{2} g_F v_\phi \qquad \mathcal{L}_{W'ff} = \frac{g_F}{2} W'_{i\,\mu} \left(\overline{f_L}_\alpha C^i_{\alpha\beta} \gamma^\mu f_{L\,\beta} + \overline{f_R}_\alpha C^i_{\alpha\beta} \gamma^\mu f_{R\,\beta} \right)$$

Because of the $SU(2)_F$ structure, **maximal** flavour-violation in the 1-2 sector:

$$C^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad C^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad C^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Rotations to the mass basis induce couplings to 3rd family too, e.g. for LH down quarks:

$$C_L^{d\,1} = V_L^{d\,\dagger} C^1 V_L^d \sim \begin{pmatrix} \lambda & 1 & \lambda^2 \\ 1 & \lambda & \lambda^3 \\ \lambda^2 & \lambda^3 & \lambda^5 \end{pmatrix}, \quad C_L^{d\,2} = V_L^{d\,\dagger} C^2 V_L^d \sim \begin{pmatrix} 0 & 1 & \lambda^2 \\ 1 & 0 & \lambda^3 \\ \lambda^2 & \lambda^3 & 0 \end{pmatrix}, \quad C_L^{d\,3} = V_L^{d\,\dagger} C^3 V_L^d \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^4 \end{pmatrix}$$

Heavy W' best constrained by $K - \bar{K}$ mixing:

$$(\bar{s}_X \gamma^\mu d_X)(\bar{s}_Y \gamma_\mu d_Y) : \quad \sum_i \frac{g_F^2}{4 m_{W'}^2} C^{d\,i}_{X\,sd} C^{d\,i}_{Y\,sd} \xRightarrow{v_\phi = 2 m_{W'}/g_F} \Delta m_K : \quad v_\phi \gtrsim 5.4 \times 10^6\,\text{GeV}$$

Light W' can be produced in meson/lepton decays: $\quad (X = W'_i)$

$$K \to \pi X (\to \ell\ell^{(\prime)}), \quad B \to K(\pi) X (\to \ell\ell^{(\prime)}), \quad \mu \to e X (\to ee), \quad \tau \to \ell X (\to \ell\ell^{(\prime)})$$

Light W' bounds depend on the probability of decaying outside/inside the detector:

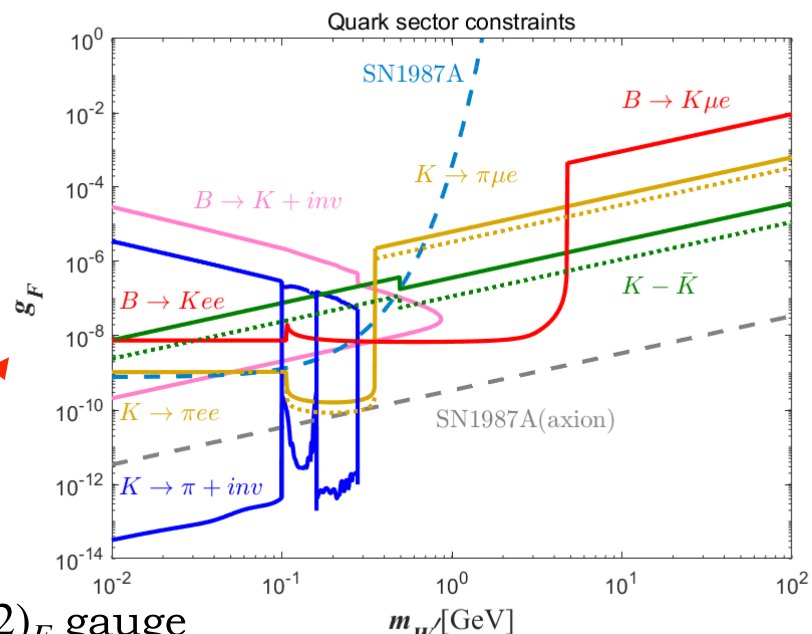
detector size

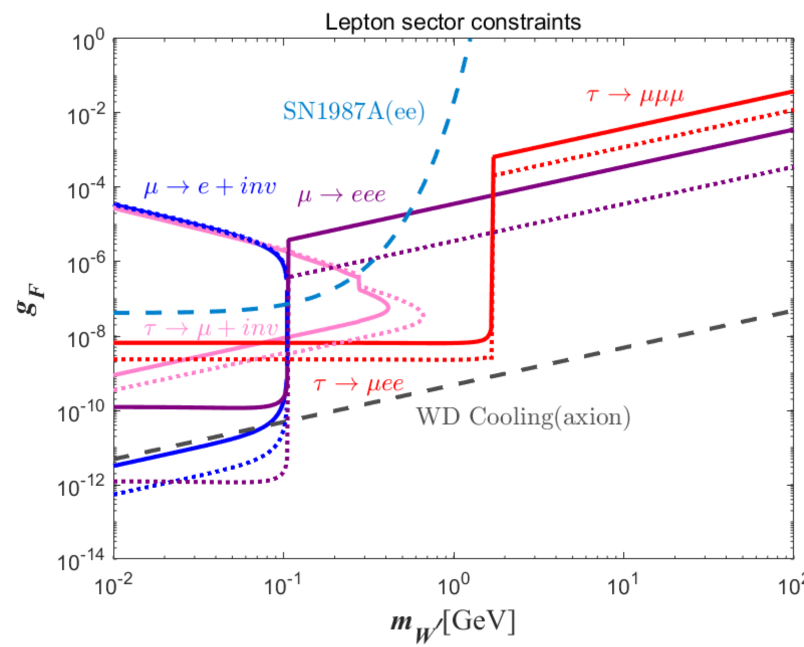
$$\exp\left(-\frac{L}{l_X}\right)\,\mathrm{BR}(P\to P'\,X)\;<\;\mathrm{BR}(P\to P'+\mathrm{inv})^{\mathrm{exp}} \qquad \to \text{(semi) invisible}$$

$$\left[1-\exp\left(-\frac{L}{l_X}\right)\right]\mathrm{BR}(P\to P'\,X)\,\mathrm{BR}(X\to \ell\ell^{(\prime)})\;<\;\mathrm{BR}(P\to P'\,\ell\ell^{(\prime)})^{\mathrm{exp}} \qquad \to \text{visible}$$

$$l_X=\frac{p_X}{m_X\Gamma_X}$$

lab frame decay length





$SU(2)_F$ gauge coupling

W' mass

LC Yi '25

Global SU(2)

PNGB π' mass free parameter from explicit $SU(2)_F$ breaking:

$\delta V = m^2_{\pi'}(\phi^\dagger \sigma_3 \phi)$ (e.g. from Planck suppressed ops $m^2_{\pi'} \sim (v_\phi)^{2n}/M_{\rm Pl}^{2(n-1)}$)

Standard form of the coupling to fermions arises from the kinetic terms after field-dependent transformations, e.g.:

$$Q_a \to e^{i(\widetilde{\pi}_1\sigma_1 + \widetilde{\pi}_2\sigma_2 + \widetilde{\pi}'_3(x)\sigma_3)_{ab}/v_\phi} e^{-i\widetilde{a}/v_\chi} Q_b$$

$$\mathcal{L}_{\pi' f f} = \frac{\partial_\mu \pi'_i}{v_\phi}\left[\overline{f}_\alpha \gamma^\mu \left(\hat{C}^{f\,i}_{V\,\alpha\beta} + \hat{C}^{f\,i}_{A\,\alpha\beta}\gamma_5\right) f_\beta\right] \qquad \hat{C}^{f\,i}_{V,A} = \frac{\hat{C}^{f\,i}_R \pm \hat{C}^{f\,i}_L}{2}$$

Same flavour-violating matrices as for W' !

Because of the derivative, interaction additionally suppressed by ~$m_f/m_{\pi'}$:

$$(\overline{s}_L\gamma^\mu d_L)(\overline{s}_R\gamma_\mu d_R): \quad \sum_i \frac{m_s^2}{4m^2_{\pi'_i}v_\phi^2}\hat{C}^{d\,i}_{L\,sd}\hat{C}^{d\,i}_{R\,sd} \Rightarrow \Delta m_K: \quad v_\phi \gtrsim 5.0\times 10^4 \left(\frac{10\,\text{GeV}}{m_{\pi'}}\right)\,\text{GeV}$$

Heavy PNGBs better constrained by $\mu \to e\gamma: \quad v_\phi \gtrsim 1.4\times 10^5 \left(\frac{10\,\text{GeV}}{m_{\pi'}}\right)\,\text{GeV}$

Global SU(2)

Light PNGBs behave like light gauge bosons (equivalence theorem for $m_X \to 0$)

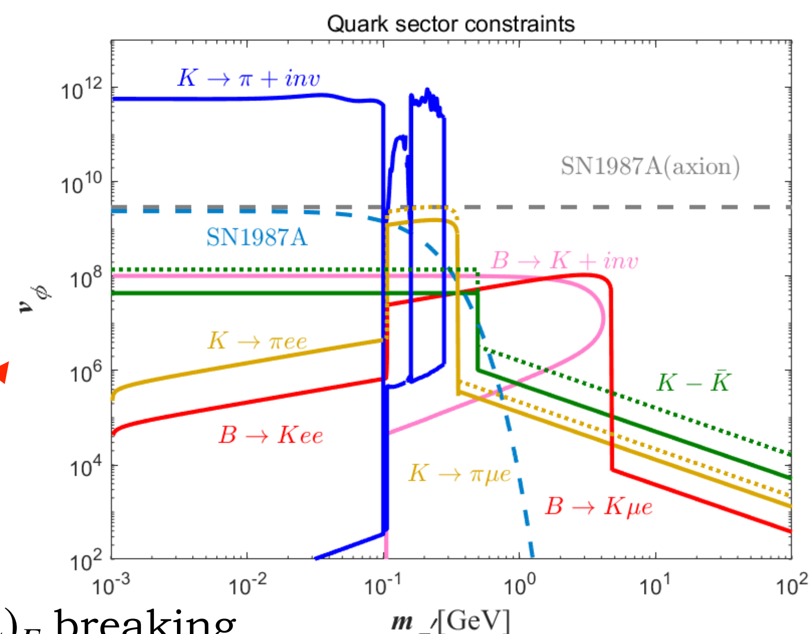
e.g.

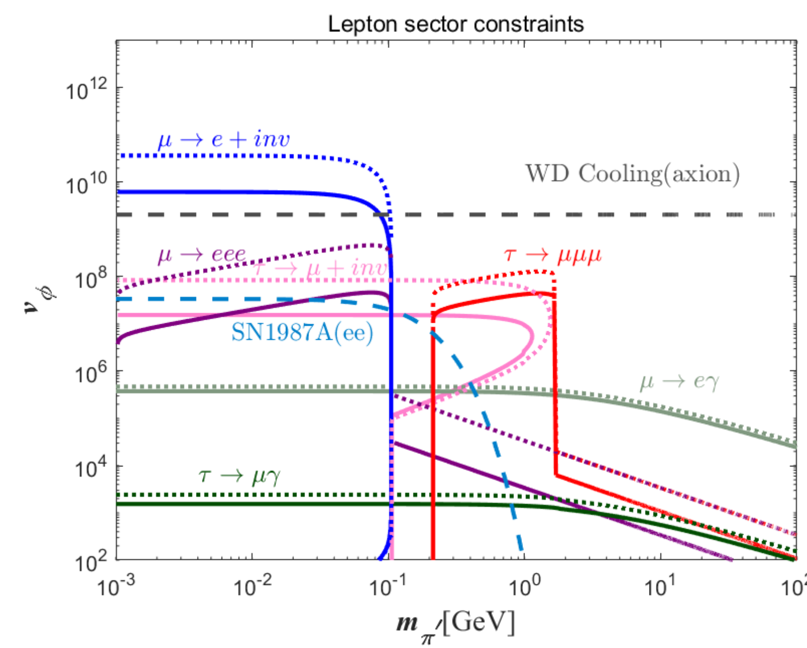
$$\mathrm{BR}(\ell_\alpha \to \ell_\beta\, W'_i) = \frac{g_F^2}{64\pi\, m_{W'}^2}\frac{m_{\ell_\alpha}^3}{\Gamma_{\ell_\alpha}}\left(\left|C^{e\,i}_{V\,\alpha\beta}\right|^2 + \left|C^{e\,i}_{A\,\alpha\beta}\right|^2\right)\left(1+2\frac{m_{W'}^2}{m_{\ell_\alpha}^2}\right)\left(1-\frac{m_{W'}^2}{m_{\ell_\alpha}^2}\right)^2$$

$$v_\phi = 2m_{W'}/g_F$$

$$\mathrm{BR}(\ell_\alpha \to \ell_\beta\, \pi'_i) = \frac{1}{16\pi\, v_\phi^2}\frac{m_{\ell_\alpha}^3}{\Gamma_{\ell_\alpha}}\left(|\hat{C}^{e\,i}_{V\,\alpha\beta}|^2 + |\hat{C}^{e\,i}_{A\,\alpha\beta}|^2\right)\left(1-\frac{m_{\pi'_i}^2}{m_{\ell_\alpha}^2}\right)^2$$

In both cases: $K \to \pi\, X_{\rm inv}\ :\ v_\phi \gtrsim 6\times 10^{11}$ GeV $\quad \mu \to e\, X_{\rm inv}\ :\ v_\phi \gtrsim 6\times 10^{9}$ GeV





$SU(2)_F$ breaking scale

PNGB mass

LC Yi '25

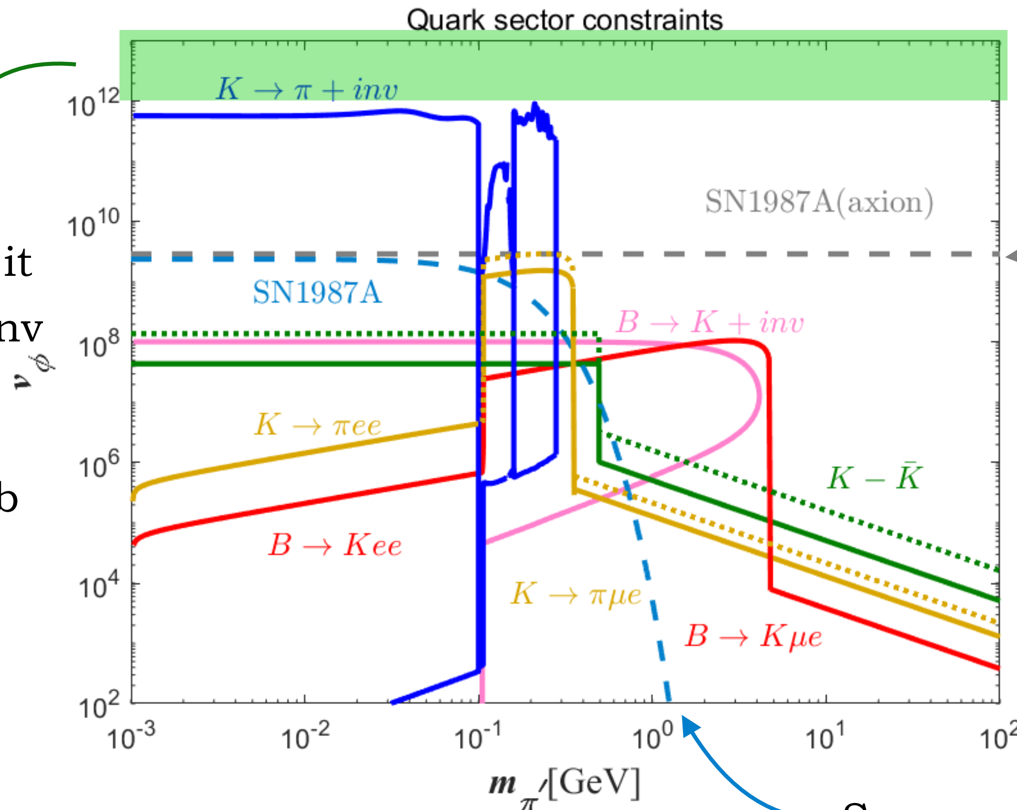
What about cosmology/astrophysics?

DM relic density from axiflavon misalignment: $\Omega_a h^2 \simeq 0.12 \left(\frac{v_\phi}{5.6 \times 10^{12}\ \text{GeV}}\right)^{7/6} \theta^2$

DM region unconstrained!

Best way to test it is via $K \to \pi + X_{\text{inv}}$

It would be a job for HIKE 😢



here a can account for 100% DM

Supernova bound on axiflavon emission

Supernova bound on π' emission (weaker than flavour bounds)

Summary

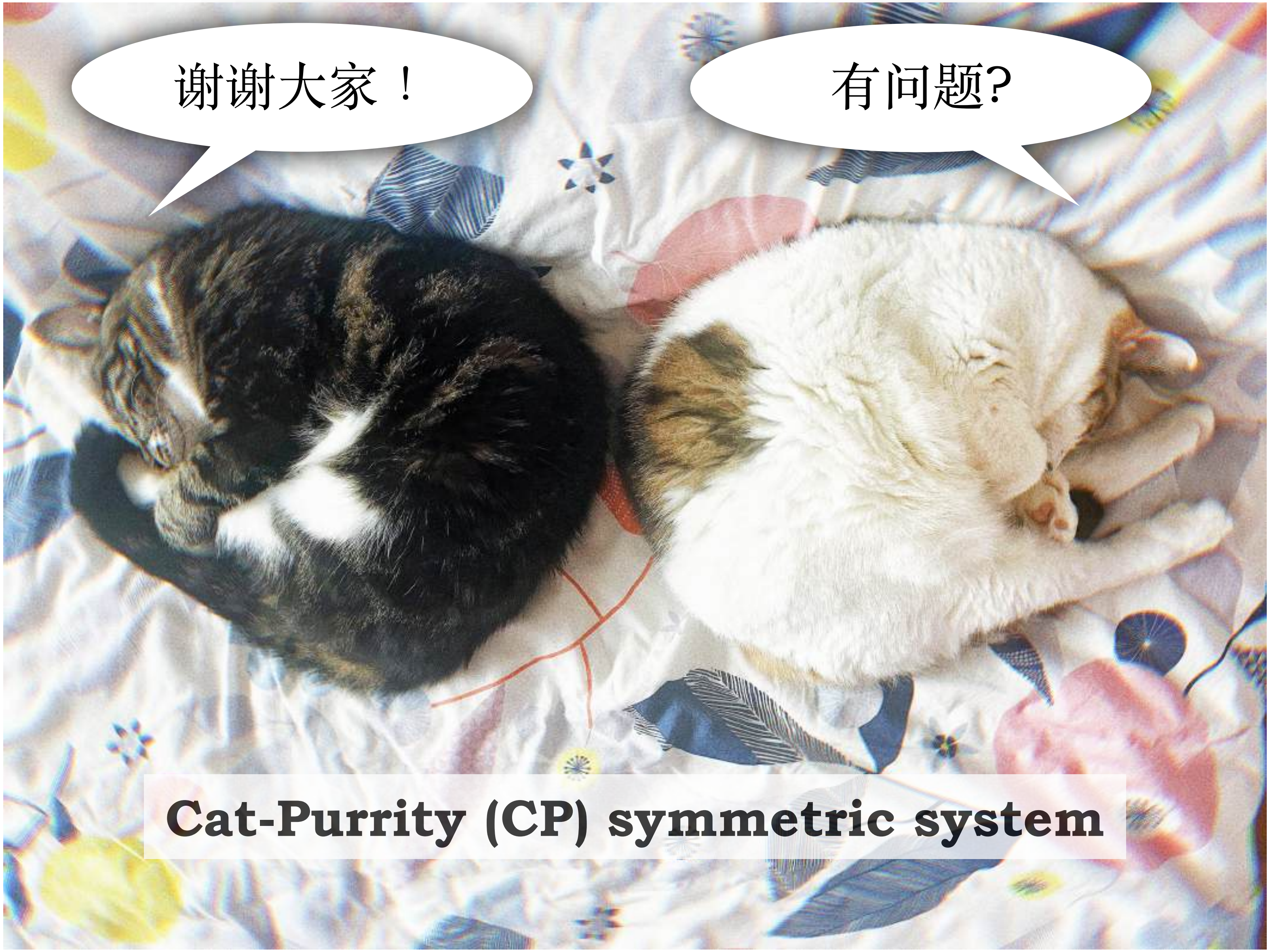
We don't know the origin of the SM flavour sector (dynamical?)

A wide class of flavour models entails light new physics with flavour-violating couplings to SM fermions

Searches for muon/kaon decays into light flavoured particles can test flavour-breaking scales up to $10^{10}/10^{12}$ GeV

Interesting complementary to tau and B decays, astrophysical/cosmological bounds (and gravitational waves)

Huge room for improvement over old limits: next generation flavour experiments may discover light new physics


谢谢大家！
有问题?
Cat-Purrity (CP) symmetric system

Additional slides

Relevant flavour processes

Decay mode	Current limit		Expected limit	
$\mu \to e\gamma$	1.5×10^{-13}	[71]	6×10^{-14}	[78]
$\mu \to ee\bar{e}$	1.0×10^{-12}	[70]	10^{-16}	[79]
$\mu \to e\,X_{\text{inv}}{}^{\dagger}$	2.6×10^{-6}	[80]	7×10^{-8}	[81]
$\tau \to \mu\gamma$	4.2×10^{-8}	[82]	6.9×10^{-9}	[74]
$\tau \to \mu\mu\bar{\mu}$	1.9×10^{-8}	[83]	3.6×10^{-10}	[74]
$\tau \to \mu e\bar{e}$	1.6×10^{-8}	[84]	2.9×10^{-10}	[74]
$\tau \to \mu\,X_{\text{inv}}{}^{\ddagger}$	3.4×10^{-4}	[85]	2×10^{-5}	[45]
$\tau \to e\gamma$	3.3×10^{-8}	[86]	9×10^{-9}	[74]
$\tau \to ee\bar{e}$	2.7×10^{-8}	[87]	4.7×10^{-10}	[74]
$\tau \to e\mu\bar{\mu}$	2.7×10^{-8}	[87]	4.5×10^{-10}	[74]
$\tau \to e\,X_{\text{inv}}{}^{\ddagger}$	2.8×10^{-4}	[85]	8×10^{-6}	[45]
$K^+ \to \pi^+\mu^+e^-$	1.3×10^{-11}	[88]	10^{-12}	[89]
$K^+ \to \pi^+X_{\text{inv}}{}^{\ddagger}$	3×10^{-11}	[26]		
$B^+ \to K^+\mu^+e^-$	6.4×10^{-9}	[90]		
$B^+ \to K^+\tau^\pm\mu^\mp$	4.8×10^{-5}	[91]	3.3×10^{-6}	[73]
$B^+ \to K^+\tau^\pm e^\mp$	3.0×10^{-5}	[91]	2.1×10^{-6}	[73]
$B^+ \to K^+X_{\text{inv}}{}^{\ddagger}$	8×10^{-6}	[64, 92]		

Table 2: Current experimental limits (90% CL) and future prospects for the flavor-violating decay modes relevant for our analysis. † corresponds to a practically massless X with substantial couplings to RH lepton currents; ‡ corresponds to a massless X. See the text for details.

Lepton sector

$$-\mathcal{L} \supset \left[a^{\ell}_{ij} \left(\frac{\langle \phi \rangle}{\Lambda_\ell} \right)^{\mathcal{Q}_{L_i} - \mathcal{Q}_{e_j}} \overline{L}_i e_j H + h.c. \right] + \kappa^{\nu}_{ij} \left(\frac{\langle \phi^* \rangle}{\Lambda_\ell} \right)^{\mathcal{Q}_{L_i} + \mathcal{Q}_{L_j}} \frac{(\overline{L^c_i}\, \tilde{H})(\tilde{H}^T L_j)}{\Lambda_N}$$

$$\Rightarrow \quad Y^\ell = V^\ell \hat{Y}^\ell W^{\ell\dagger}, \qquad m^\nu = V^\nu \hat{m}^\nu V^{\nu T} \qquad V^{\ell,\nu}_{ij} \sim \epsilon_\ell^{\left|\mathcal{Q}_{L_i} - \mathcal{Q}_{L_j}\right|}, \qquad W^\ell_{ij} \sim \epsilon_\ell^{\left|\mathcal{Q}_{e_i} - \mathcal{Q}_{e_j}\right|}$$

LH charges can chosen to give a (quasi-)anarchical $U_{\text{PMNS}} = V^\nu V^{\ell\dagger}$
RH charges then responsible for charged leptons hierarchy

Examples:

Altarelli Feruglio Masina Merlo '12

- Anarchy $\quad (\mathcal{Q}_{L_1}, \mathcal{Q}_{L_2}, \mathcal{Q}_{L_3}) = (\mathcal{Q}_L, \mathcal{Q}_L, \mathcal{Q}_L)$
- Mu-tau anarchy $\quad (\mathcal{Q}_{L_1}, \mathcal{Q}_{L_2}, \mathcal{Q}_{L_3}) = (\mathcal{Q}_L + 1, \mathcal{Q}_L, \mathcal{Q}_L)$
- Hierarchy $\quad (\mathcal{Q}_{L_1}, \mathcal{Q}_{L_2}, \mathcal{Q}_{L_3}) = (\mathcal{Q}_L + 2, \mathcal{Q}_L + 1, \mathcal{Q}_L)$

Charged lepton hierarchy, *e.g.* : $\quad (\mathcal{Q}_{e_1}, \mathcal{Q}_{e_2}, \mathcal{Q}_{e_3}) = (\mathcal{Q}_L - 4, \mathcal{Q}_L - 2, \mathcal{Q}_L - 1)$
(with $\epsilon_\ell \approx \epsilon^2 \approx 0.04$)

FCNC Axion/ALPs

Flavour-violating axions/ALPs couplings to quarks

$$\mathcal{L}_{aff} = \frac{\partial_\mu a}{2f_a}\, \bar{f}_i \gamma^\mu (C^V_{f_i f_j} + C^A_{f_i f_j}\gamma_5) f_j$$

vector coupling

axial coupling

$$P_1 \to P_2 + a \;[\text{e.g. } B \to K + a] \qquad P_1 \to V_2 + a \;[\text{e.g. } B \to K^* + a]$$

(because of parity conservation of strong interactions)

Martin Camalich et al. '20

Decay	sd	cu	bd	bs
$\mathrm{BR}(P_1 \to P_2 + a)$	3×10^{-11} NA62 NEW!	No analysis	4.9×10^{-5} [90]	4.9×10^{-5} [90]
$\mathrm{BR}(P_1 \to P_2 + a)_{\text{recast}}$	No need	8.0×10^{-6} [93]	2.3×10^{-5} [92]	7.1×10^{-6} [91]
				to be updated soon
$\mathrm{BR}(P_1 \to V_2 + a)$	3.8×10^{-5} [98]	No analysis	No analysis	No analysis
$\mathrm{BR}(P_1 \to V_2 + a)_{\text{recast}}$	No need	No data	No data	5.3×10^{-5} [91]

News from $K \to \pi + \text{inv}$ (NA62) and $B \to K + \text{inv}$ (Belle II)

but there are still gaps...

Flavour-violating axions/ALPs couplings to quarks

$$\mathcal{L}_{aff} = \frac{\partial_\mu a}{2 f_a} \bar{f}_i \gamma^\mu (C^V_{f_i f_j} + C^A_{f_i f_j} \gamma_5) f_j$$

vector coupling | axial coupling

$$P_1 \to P_2 + a \; [\text{e.g. } B \to K + a] \qquad P_1 \to V_2 + a \; [\text{e.g. } B \to K^* + a]$$

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$\mathrm{BR}(P_1 \to V_2 + a)$	3.8×10^{-5} [98]	No analysis	No analysis	No analysis
$\mathrm{BR}(P_1 \to V_2 + a)_{\mathrm{recast}}$	No need	No data	No data	5.3×10^{-5} [91]

Example: No dedicated searches for D to axion decays

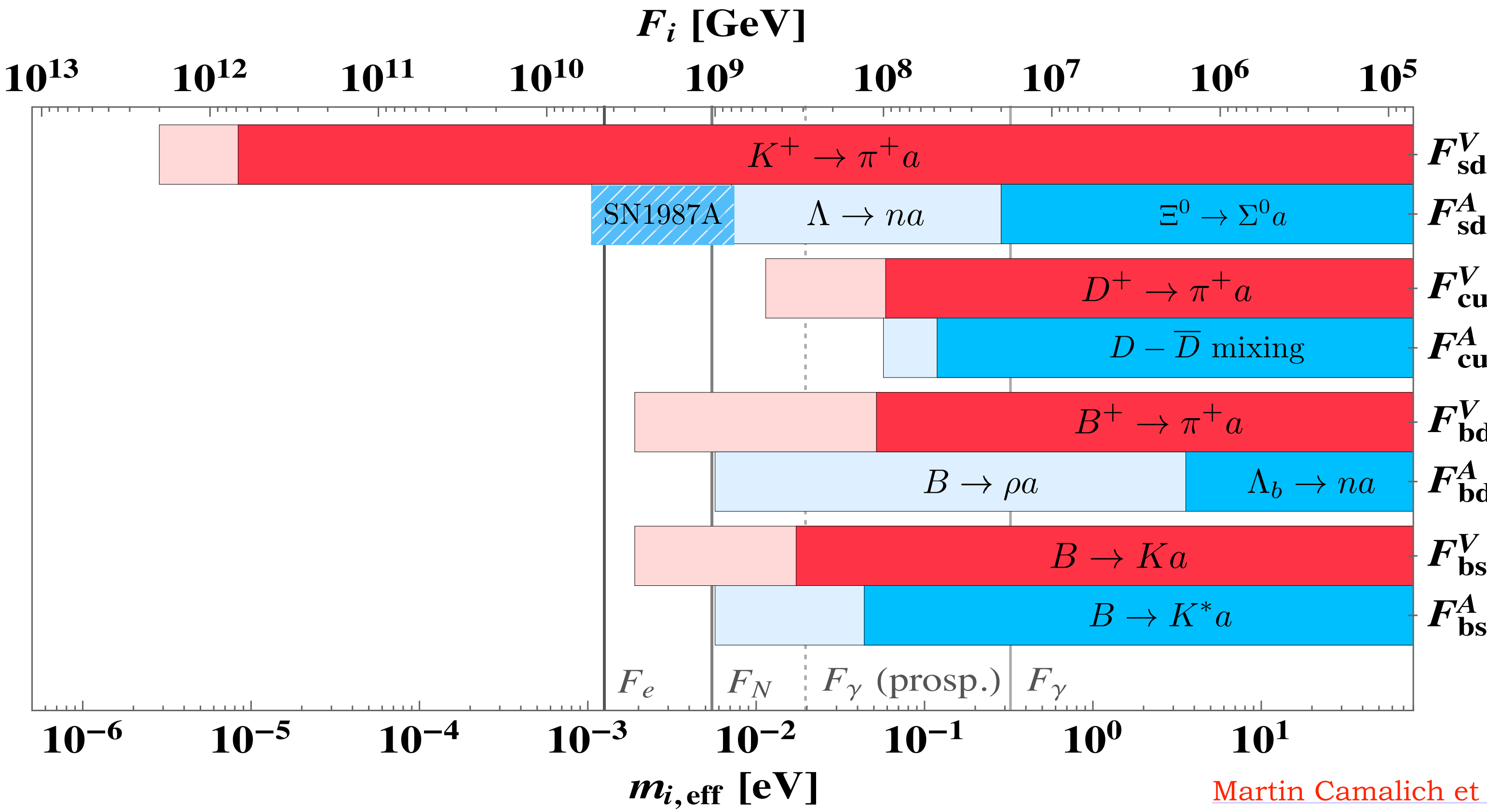
Recasting data from $D^+ \to \tau^+ (\to \pi^+ \nu)\, \nu$ (CLEO 2008):

$$\mathrm{BR}(D^+ \to \pi^+ a) < 8 \times 10^{-6}$$ $\to$ BESIII ? Belle II ?

Flavour-violating axions/ALPs couplings to quarks

$$\mathcal{L}_{aff} = \frac{\partial_\mu a}{2 f_a}\, \bar{f}_i \gamma^\mu (C^V_{f_i f_j} + C^A_{f_i f_j}\gamma_5) f_j$$

$$F^{V,A}_{f_i f_j} \equiv \frac{2 f_a}{C^{V,A}_{f_i f_j}}$$

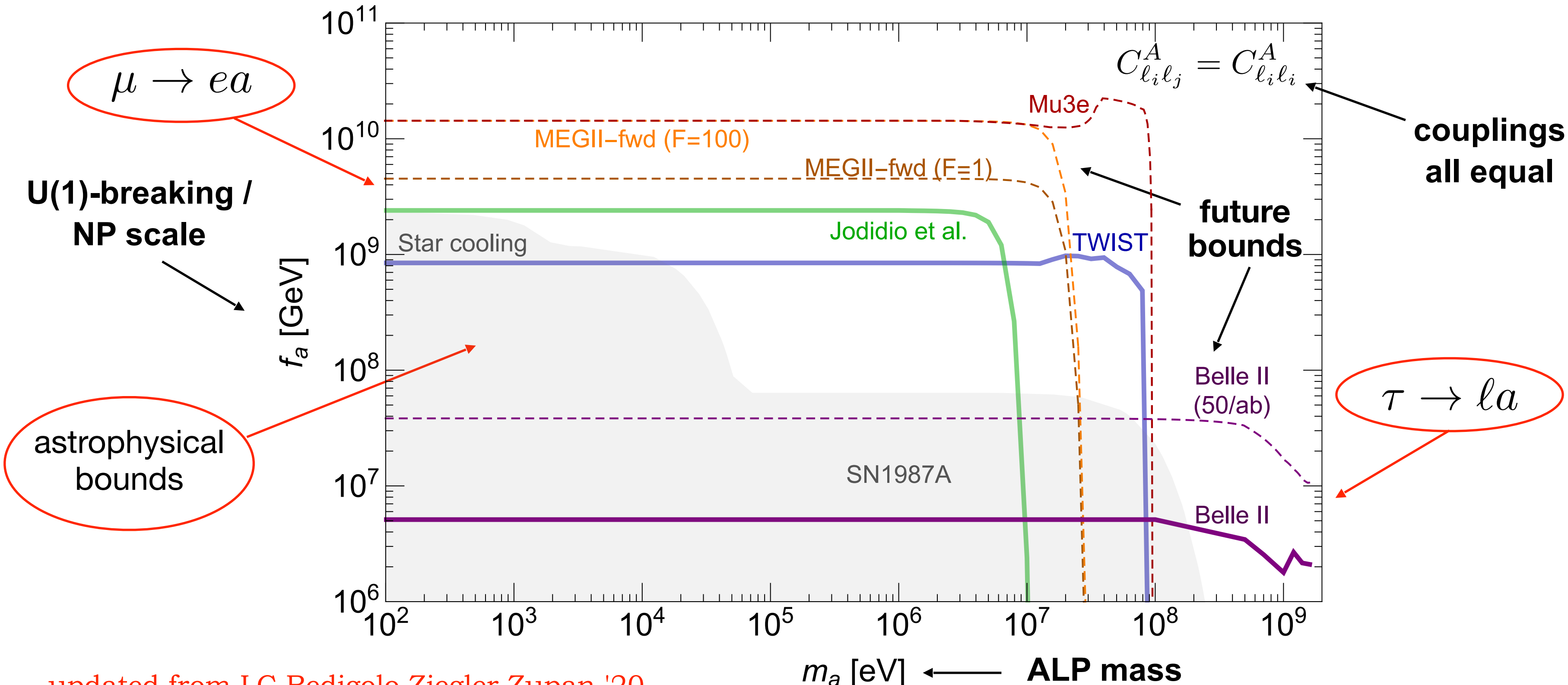


Martin Camalich et al. '20

LFV Axion/ALPs

Summary of searches for light *invisible* LFV ALPs

$$\mathcal{L}_{a\ell\ell} = \frac{\partial^\mu a}{2 f_a} \left(C^V_{ij}\ \bar{\ell}_i \gamma_\mu \ell_j + C^A_{ij}\ \bar{\ell}_i \gamma_\mu \gamma_5 \ell_j \right) \Rightarrow \Gamma(\ell_i \to \ell_j\, a) = \frac{1}{64\pi} \frac{m^3_{\ell_i}}{f_a^2} \left(|C^V_{\ell_i \ell_j}|^2 + |C^A_{\ell_i \ell_j}|^2 \right) \left(1 - \frac{m_a^2}{m_{\ell_i}^2} \right)^2$$



updated from LC Redigolo Ziegler Zupan '20

- Decays mediated by dimension-5 operators: one can reach NP scales even larger than $\mu \to e\gamma$, $\mu \to eee$ etc. (from dim-6 operators)
- Mu/tau/astro interplay: if $m_a > m_\mu$ constraints mainly come from τ decays

Summary of present limits/future prospects

Comparison in the case $m_a \approx 0$

$$\mathcal{L}_{a\ell\ell} = \frac{\partial^\mu a}{2f_a}\left(C^V_{ij}\,\bar{\ell}_i\gamma_\mu\ell_j + C^A_{ij}\,\bar{\ell}_i\gamma_\mu\gamma_5\ell_j\right) \qquad F^{V,A}_{ij} \equiv \frac{2f_a}{C^{V,A}_{ij}} \qquad F_{ij} \equiv \frac{2f_a}{\sqrt{|C^V_{ij}|^2 + |C^A_{ij}|^2}}$$

Present best limits — LC Redigolo Ziegler Zupan '20

Process	BR Limit	Decay constant	Bound (GeV)	Experiment
$\mu \to e\,a$	$2.6 \times 10^{-6*}$	$F_{\mu e}$ (V or A)	4.8×10^9	Jodidio at al. [9]
$\mu \to e\,a$	$2.5 \times 10^{-6*}$	$F_{\mu e}$ ($V + A$)	4.9×10^9	Jodidio et al. [9]
$\mu \to e\,a$	$5.8 \times 10^{-5*}$	$F_{\mu e}$ ($V - A$)	1.0×10^9	TWIST [10]
$\mu \to e\,a\,\gamma$	$1.1 \times 10^{-9*}$	$F_{\mu e}$	$5.1 \times 10^{8\#}$	Crystal Box [47]

Expected future sensitivities

Process	BR Sens.	Decay constant	Sens. (GeV)	Experiment
$\mu \to e\,a$	$7.2 \times 10^{-7*}$	$F_{\mu e}$ (V or A)	9.2×10^9	MEGII-fwd*
$\mu \to e\,a$	$7.2 \times 10^{-8*}$	$F_{\mu e}$ (V or A)	2.9×10^{10}	MEGII-fwd** *magnetic focusing*
$\mu \to e\,a$	$7.3 \times 10^{-8*}$	$F_{\mu e}$ (V or A)	2.9×10^{10}	Mu3e [42]

$\mu \to e\, a$: signal and background

Signal: monochromatic positron with $p_e = \sqrt{\left(\frac{m_\mu^2 - m_a^2 + m_e^2}{2m_\mu}\right)^2 - m_e^2}$

Differential decay rate: $$\frac{\mathrm{d}\Gamma(\ell_i \to \ell_j\, a)}{\mathrm{d}\cos\theta} = \frac{m_{\ell_i}^3}{32\pi F_{\ell_i\ell_j}^2}\left(1 - \frac{m_a^2}{m_{\ell_i}^2}\right)^2 \left[1 + 2P_{\ell_i}\cos\theta \frac{C^V_{\ell_i\ell_j} C^A_{\ell_i\ell_j}}{(C^V_{\ell_i\ell_j})^2 + (C^A_{\ell_i\ell_j})^2}\right]$$

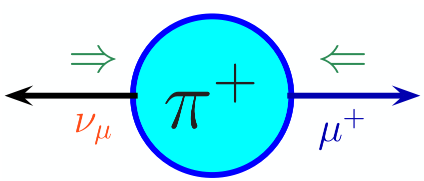
signal anisotropy depends on the chirality of the couplings

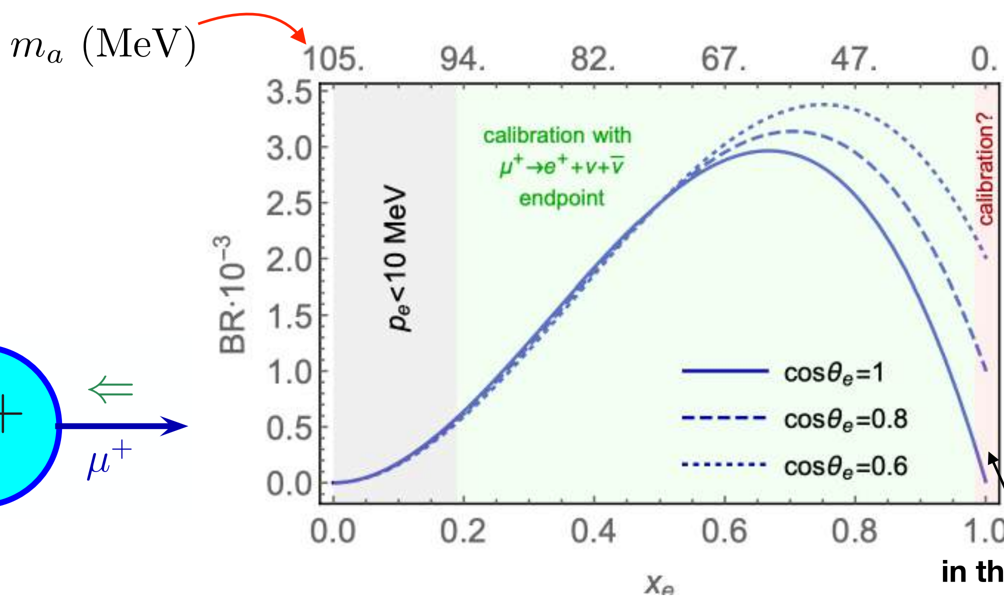
Michel spectrum: $$\frac{\mathrm{d}^2\Gamma(\mu^+ \to e^+\,\nu_e\,\bar\nu_\mu)}{\mathrm{d}x_e\,\mathrm{d}\cos\theta} \simeq \Gamma_\mu\left((3-2x_e) - P_\mu(2x_e - 1)\cos\theta\right)x_e^2$$

$x_e = \frac{2p_e}{m_\mu}$

μ polarisation

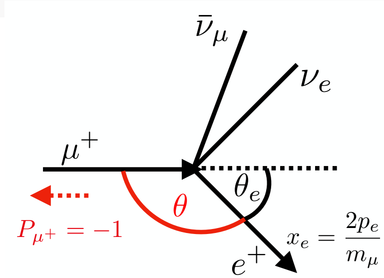
And "surface" muons are highly polarised (produced by pion decays at rest on the surface of the production target) $\to$ the SM background can be suppressed





in the massless case

the bkd goes to zero in the "forward" direction (the direction opposite to the muon polarization)



$\mu \to e\,a$: future prospects

Many ideas and proposals for running and upcoming experiments:

MEG II

- Add a forward calorimeter to perform a Jodidio-like search LC et al. '20
- Run with a dedicated trigger to search for $\mu \to ea\gamma$ Jho Knapen Redigolo '22

Mu3e

- Search performed on e^+ momentum histograms filled with *online* reconstructed short tracks Perrevoort (Mu3e) '18
- Search for $\mu \to 3ea$ from the internal conversion of virtual photon into e^+e^-

Knapen et al. '23

COMET/Mu2e

- search for excess over the Michel spectrum, using data from calibration runs employing *positive* muon beams Hill Plestid Zupan '23
- enlarge the $\mu \to e$ conversion signal window coping with a large (~100kHz?) Michel background rate Xing et al. '22

MACE

- One month dedicated run with μ^+ spin rotation in preparation

Many ideas and proposals for running and upcoming experiments:

MEG

• Ad

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Mu3

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sh ... e) '18

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COM

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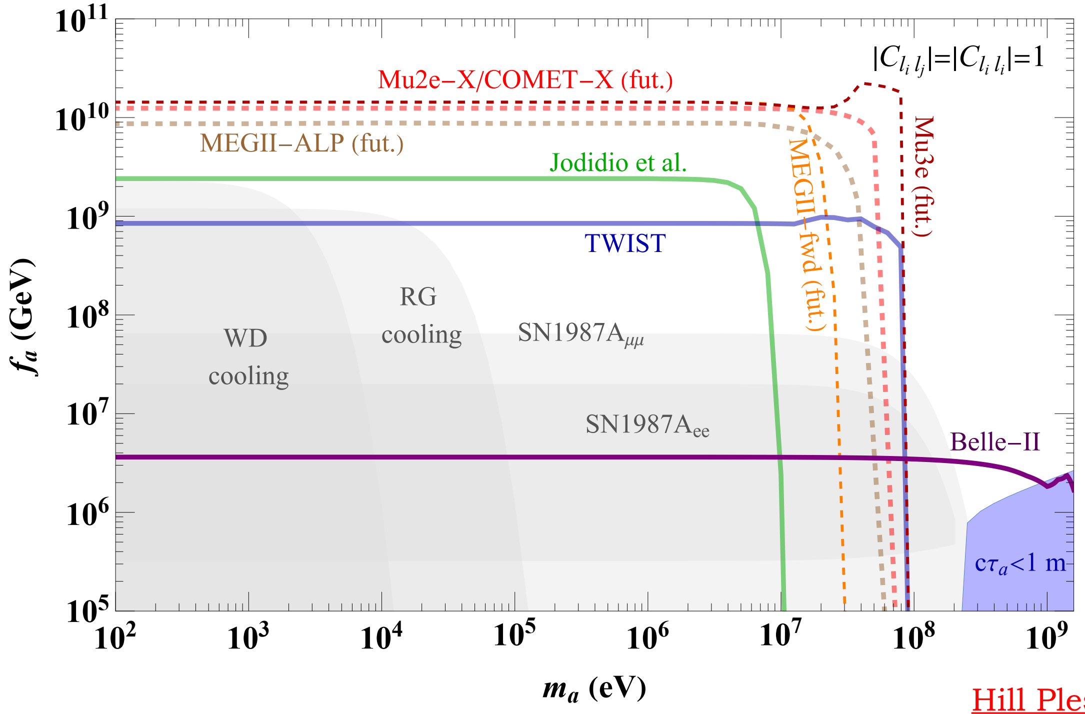
Sensitivities up to BR($\mu \to ea$) $\approx 10^{-8}$,

$f_a \approx 10^{10}$ GeV seem to be feasible!



Hill Plestid Zupan '23

MACE

• One month dedicated run with μ^+ spin rotation

U(1) axiflavon pheno

Axiflavon

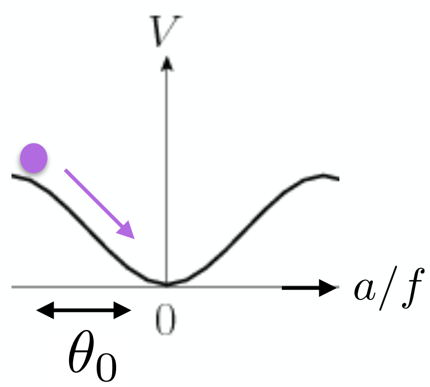
- Another puzzle of the SM is the strong CP problem
- The strong CP problem is elegantly solved by a QCD axion
- The axion field can also provide the correct density of cold DM
- The QCD axion is the PNGB of a colour-anomalous global U(1)
- Can we identify this symmetry with a Froggatt-Nielsen U(1)?

YES!

LC Goertz Redigolo Ziegler Zupan '16
Ema Hamaguchi Moroi Nakayama '16

Minimal realisation of an old idea by Wilczek of using a subgroup of the global $U(3)^5$ flavour symmetry of the SM [as is the PQ U(1)] Wilczek '82 [axion essentially stable for $m_a \lesssim 20\,\mathrm{eV}$]

In early universe axion displaced from minimum



As universe expands axion rolls down and starts oscillating around minimum: energy stored in oscillations contributes to DM relic density

$$\Omega_{\mathrm{DM}} h^2 \approx 0.1 \left(\frac{10^{-5}\mathrm{eV}}{m_a}\right)^{1.18} \theta_0^2$$

Right abundance for $10^{-6}\ \mathrm{eV} \lesssim\ m_a\ \lesssim 10^{-4}\ \mathrm{eV}$

borrowed from R. Ziegler

Axiflavon setup

The axion identified with the Nambu-Goldstone boson of a broken global FN U(1), *i.e.* as the phase of the flavon field → "axiflavon"

$$\Phi = \frac{1}{\sqrt{2}}(V_\Phi + \phi)e^{ia/V_\Phi}$$

Couplings gluons and photons via colour and electromagnetic anomalies:

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\frac{a}{f_a}G\tilde{G} + \frac{E}{N}\frac{\alpha_{\rm em}}{8\pi}\frac{a}{f_a}F\tilde{F} \qquad f_a = V_\Phi/2N$$

Anomaly coefficients given by FN charges:

QCD $$N = \frac{1}{2}\sum_i 2[q]_i + [u]_i + [d]_i\,,$$

E.M. $$E = \sum_i \frac{4}{3}\left([q]_i + [u]_i\right) + \frac{1}{3}\left([q]_i + [d]_i\right) + [l]_i + [e]_i\,,$$

[no contributions from UV completion, if messengers are scalars or vectorlike under U(1)]

Usual axion mass induced by the QCD anomaly:

$$m_a = 5.7\,\mu\text{eV}\left(\frac{10^{12}\text{GeV}}{f_a}\right)$$

Axiflavon setup

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\frac{a}{f_a}G\tilde{G} + \frac{E}{N}\frac{\alpha_{\rm em}}{8\pi}\frac{a}{f_a}F\tilde{F} \qquad f_a = V_\Phi/2N$$

Key observation: FN U(1) to reproduce observed Yukawas is necessarily anomalous and the coefficients are linked to the quark masses:

$$\det m_u \det m_d = \alpha_{ud}\, v^6 \epsilon^{2N}\,,$$

$$\det m_d/\det m_e = \alpha_{de}\,\epsilon^{\frac{8}{3}N-E}\,,$$

$$\alpha_{ud} = \det a_u \det a_d,\;\; \alpha_{de} = \det a_d/\det a_e$$

Ibanez Ross '94, Bineutry Lavignac Ramond '94 '96

$$\Rightarrow \quad \boxed{\frac{E}{N} = \frac{8}{3} - 2\frac{\overbrace{\log\frac{\det m_d}{\det m_e}}^{\approx -0.4} - \overbrace{\log\alpha_{de}}^{\mathcal{O}(1)}}{\underbrace{\log\frac{\det m_u \det m_d}{v^6}}_{\approx -44} - \underbrace{\log\alpha_{ud}}_{\mathcal{O}(1)}}}$$

Sharp prediction for the coupling to photons $\frac{1}{4}g_{a\gamma\gamma}aF\tilde{F}$, independent of U(1) charges and little sensitive to O(1)s:

$$\boxed{\frac{E}{N} \in [2.4, 3.0] \quad \Rightarrow \quad g_{a\gamma\gamma} \in \frac{[1.0, 2.2]}{10^{16}\mathrm{GeV}}\frac{m_a}{\mu\mathrm{eV}}}$$

Compare to DFSZ and KSVZ axions:

$|E/N| \in [0.3, 2.7]$ $\qquad$ $|E/N| \in [0, 6]$

$$m_a = 5.7\,\mu\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)$$

Axiflavon setup

The axion identified with the Nambu-Goldstone boson of a broken global FN U(1), *i.e.* as the phase of the flavon field $\longrightarrow$ "axiflavon"

$$\Phi = \frac{1}{\sqrt{2}}(V_\Phi + \phi)e^{ia/V_\Phi}$$

SM fermions-axiflavon couplings proportional to the Yukawas but not aligned:

$$y^f_{ij} = a^f_{ij}\left(\frac{\langle\Phi\rangle}{M}\right)^{[f_L]_i+[f_R]_j} \Longrightarrow \mathcal{L}_{aff} = \lambda^f_{ij} a \bar{f}_{Li} f_{Rj} + \text{h.c.} \qquad \lambda^f_{ij} = i([f_L]_i + [f_R]_j)\frac{v}{V_\Phi} y^f_{ij}$$

flavour violating!

Or in the usual derivative form:

$$\mathcal{L}_{aff} = \frac{\partial^\mu a}{V_\Phi}\left(C^f_{V\,ij}\,\bar{f}_i\gamma_\mu f_j + C^f_{A\,ij}\,\bar{f}_i\gamma_\mu\gamma_5 f_j\right)$$

$$C^f_{V/A} = V^{f\dagger}_R X^f_R V^f_R \pm V^{f\dagger}_L X^f_L V^f_L$$

non-universal charges $\longrightarrow$ non-vanishing off-diagonal couplings

$$V^{f\dagger}_L Y^f V^f_R = Y^f_{diag} \qquad X^f_{R/L} = \begin{pmatrix} [f_{R/L}]_1 & & \\ & [f_{R/L}]_2 & \\ & & [f_{R/L}]_3 \end{pmatrix}$$

Axiflavon phenomenology

Stellar evolution bounds $f_a > 10^8$ GeV [natural DM window 10^{10} GeV < f_a < 10^{13} GeV]

$\Rightarrow$ flavour processes mediated by the dynamical flavon very suppressed

Despite the tiny couplings low-energy searches for rare processes are sensitive to flavour-violating decays to ultralight axiflavons! *E.g.*:

$$K^+ \to \pi^+ a \qquad B^+ \to K^+ a \qquad \mu^+ \to e^+ a$$

Small rates but strong constraints! Most stringent from Kaons:

$$\Gamma(K^+ \to \pi^+ a) \simeq \frac{m_K}{64\pi} |\lambda^d_{21} + \lambda^{d*}_{12}|^2 B_s^2 \left(1 - \frac{m_\pi^2}{m_K^2}\right)$$

$$|\lambda^d_{21} + \lambda^{d*}_{12}| \approx \frac{\sqrt{m_s m_d}}{f_a} \frac{\kappa_{sd}}{N}$$

$$\boxed{\mathrm{BR}(K^+ \to \pi^+ a) \simeq 1.2 \cdot 10^{-10} \left(\frac{m_a}{0.1\,\mathrm{meV}}\right)^2 \left(\frac{\kappa_{sd}}{N}\right)^2}$$

U(1) Z' pheno

Flavour non-universal **local** U(1) symmetry generating the hierarchies of fermion masses and mixing through the Froggatt-Nielsen mechanism

(anomalies cancelled by suitable UV completions Smolkovič Tammaro Zupan '19 Bonnefoy Dudas Pokorski '19)

Below the cutoff Λ, only **two** new particles:

$$\phi = \frac{v_\phi + \varphi}{\sqrt{2}} e^{i a / v_\phi}$$

longitudinal component of

Physical flavon

$$m_\varphi^2 = \frac{1}{2}\lambda_\phi v_\phi^2$$

$$\mathcal{L} = n_{ij}^f \frac{m_{ij}^f}{v_\phi} \overline{f}_i P_R f_j \varphi$$

U(1) gauge boson, Z'

$$m_{Z'} = \sqrt{2}\, g_F \langle\phi\rangle = g_F\, v_\phi$$

$$\mathcal{L} \supset g_F\, \overline{f} \gamma^\mu (\mathcal{Q}_{f_L} P_L + \mathcal{Q}_{f_R} P_R) f\, Z'_\mu$$

→ both fields decay into SM fermions and are produced in the early universe by thermal interactions (O(1) couplings with the fields at Λ)

→ we have to require their lifetime < 0.1 s in order not to affect **BBN**

Flavour-violating FN Z'

Flavour non-universal **local** U(1) symmetry generating the hierarchies of fermion masses and mixing through the Froggatt-Nielsen mechanism

(anomalies cancelled by suitable UV completions Smolkovič Tammaro Zupan '19 Bonnefoy Dudas Pokorski '19)

Interactions of the new gauge boson Z' **flavour-violating** by construction:

$$\mathcal{L} = g_F\, Z'_\mu \Big[\overline{u}_\alpha\gamma^\mu (C^u_{L\,\alpha\beta}\,P_L + C^u_{R\,\alpha\beta}\,P_R) u_\beta + \overline{d}_\alpha\gamma^\mu (C^d_{L\,\alpha\beta}\,P_L + C^d_{R\,\alpha\beta}\,P_R) d_\beta + \overline{\ell}_\alpha\gamma^\mu (C^\ell_{L\,\alpha\beta}\,P_L + C^\ell_{R\,\alpha\beta}\,P_R) \ell_\beta + \bar{\nu}_\alpha\gamma^\mu\, C^\nu_{L\,\alpha\beta}\,P_L \nu_\beta\Big]\,,$$

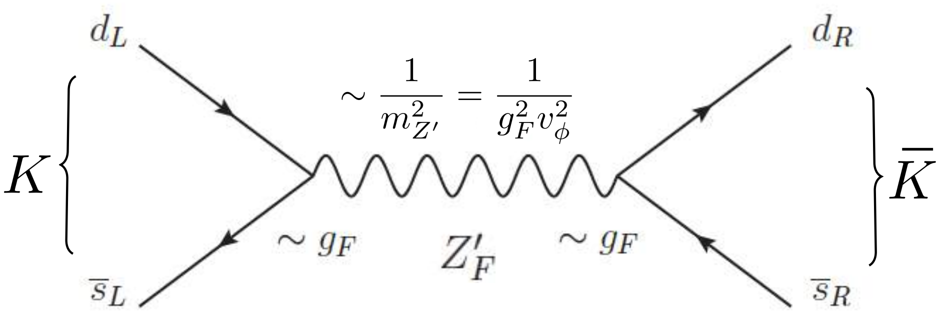
new U(1) gauge coupling

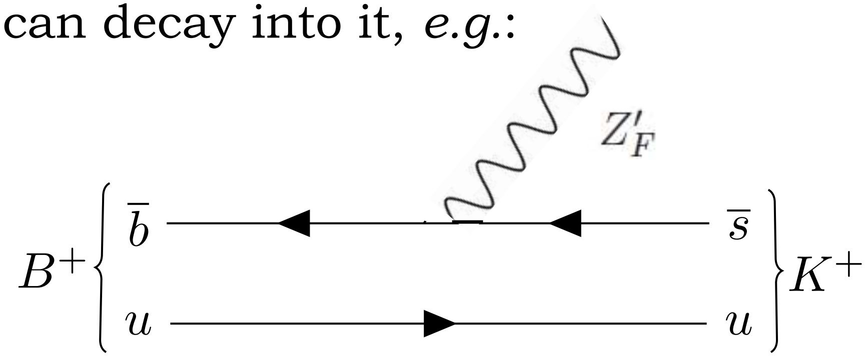
$$C^f_{L\,\alpha\beta} \equiv V^f_{\alpha i}\,\mathcal{Q}_{f_{L_i}}\,V^{f\,*}_{\beta i} \qquad C^f_{R\,\alpha\beta} \equiv W^f_{\alpha i}\,\mathcal{Q}_{f_{R_i}}\,W^{f\,*}_{\beta i} \qquad C^f_{V,A} = \frac{C^f_R \pm C^f_L}{2}$$

unitary rotations to the fermion mass basis

matrices of U(1) charges

⇨ Z' mediates flavour-violating processes and, if light, mesons and leptons can decay into it, *e.g.*:





Flavour non-universal **local** U(1) symmetry generating the hierarchies of fermion masses and mixing through the Froggatt-Nielsen mechanism

(anomalies cancelled by suitable UV completions Smolkovič Tammaro Zupan '19 Bonnefoy Dudas Pokorski '19)

Interactions of the new gauge boson Z' **flavour-violating** by construction:

$$\mathcal{L} = g_F\, Z'_\mu \Big[\overline{u}_\alpha \gamma^\mu (C^u_{L\,\alpha\beta} P_L + C^u_{R\,\alpha\beta} P_R) u_\beta + \overline{d}_\alpha \gamma^\mu (C^d_{L\,\alpha\beta} P_L + C^d_{R\,\alpha\beta} P_R) d_\beta + \overline{\ell}_\alpha \gamma^\mu (C^\ell_{L\,\alpha\beta} P_L + C^\ell_{R\,\alpha\beta} P_R) \ell_\beta + \bar{\nu}_\alpha \gamma^\mu C^\nu_{L\,\alpha\beta} P_L \nu_\beta\Big],$$

new U(1) gauge coupling

$$C^f_{L\,\alpha\beta} \equiv V^f_{\alpha i} \mathcal{Q}_{f_{L\,i}} V^{f\,*}_{\beta i} \qquad C^f_{R\,\alpha\beta} \equiv W^f_{\alpha i} \mathcal{Q}_{f_{R\,i}} W^{f\,*}_{\beta i} \qquad C^f_{V,A} = \frac{C^f_R \pm C^f_L}{2}$$

unitary rotations to the fermion mass basis

matrices of U(1) charges

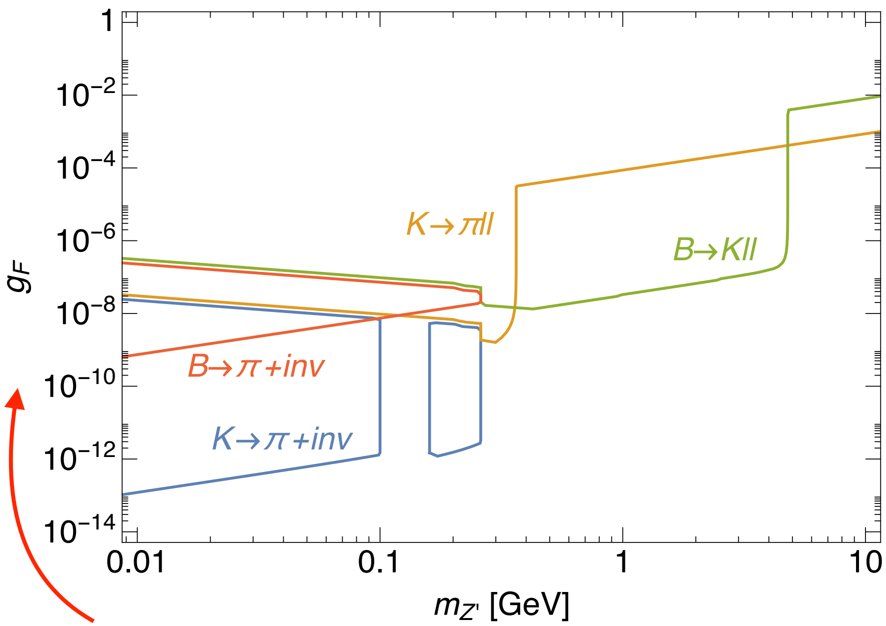
⇨ Z' mediates flavour-violating processes and, if light, mesons and leptons can decay into it, *e.g.*:

$$\mathrm{BR}(K^+ \to \pi^+ Z') = \frac{g_F^2}{16\pi\,\Gamma_K} \frac{m_K^3}{m_{Z'}^2} \left[\lambda\left(1, \frac{m_\pi^2}{m_K^2}, \frac{m_{Z'}^2}{m_K^2}\right)\right]^{\frac{3}{2}} [f_+(m_{Z'}^2)]^2 |C^d_{V\,sd}|^2$$

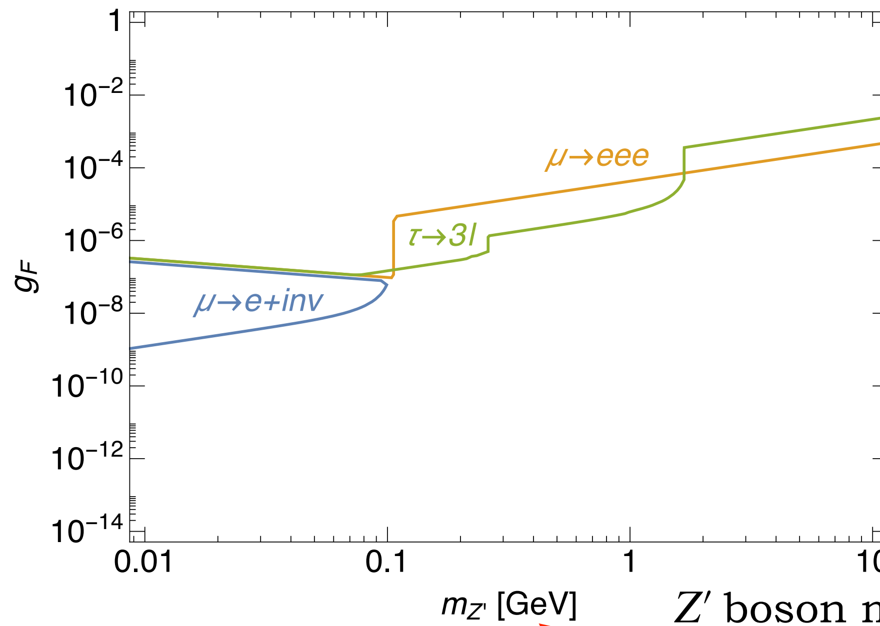
$$\mathrm{BR}(\ell_\alpha \to \ell_\beta Z') = \frac{g_F^2}{16\pi\,\Gamma_{\ell_\alpha}} \frac{m_{\ell_\alpha}^3}{m_{Z'}^2} \left(|C^\ell_{V\,\alpha\beta}|^2 + |C^\ell_{A\,\alpha\beta}|^2\right) \left(1 + 2\frac{m_{Z'}^2}{m_{\ell_\alpha}^2}\right) \left(1 - \frac{m_{Z'}^2}{m_{\ell_\alpha}^2}\right)^2$$

Flavour-violating FN Z' : flavour bounds

Meson decays into Z'



Lepton decays into Z'



U(1) coupling

Z' boson mass

$$m_{Z'} = \sqrt{2}\, g_F \langle\phi\rangle = g_F\, v_\phi$$

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Flavour processes set stringent **lower bounds** on the U(1) breaking scale

$$K^+ \to \pi^+ Z' : \ v_\phi \gtrsim 8.3\times 10^{10}\ \text{GeV}, \qquad B^+ \to K^+ Z' : \ v_\phi \gtrsim 3.0\times 10^{7}\ \text{GeV}$$

light Z'

$$\mu \to e Z' : \ v_\phi \gtrsim 1.3\times 10^{7}\ \text{GeV}, \qquad \tau \to \ell Z' : \ v_\phi \gtrsim 7.6\times 10^{5}\ \text{GeV}$$

$$K - \bar{K}\ \text{mix.} : \ v_\phi \gtrsim 6.5\times 10^{5}\ \text{GeV}$$

heavy Z'

see also Smolkovič Tammaro Zupan '19

Froggatt-Nielsen GWs

What if the U(1) breaking occurs at higher energies?

A new promising direction: **gravitational waves** (GW)

U(1) breaking → cosmic strings → emission of a GW background!

Kibble '76 (for a review: Vilenkin Shellard '00)

Key assumptions:

- After inflation, the universe reheats with $T_{\rm RH} > v_\phi$
  ⇨ FN U(1) unbroken in the early universe
- At $T \sim v_\phi$ the universe undergoes a 2nd order phase transition
  ⇨ gauge strings form

What if the U(1) breaking occurs at higher energies?

A new promising direction: **gravitational waves** (GW)

U(1) breaking → cosmic strings → emission of a GW background!

Kibble '76 (for a review: Vilenkin Shellard '00)

String tension (energy per unit length): $G\mu = \dfrac{\pi v_\phi^2}{8\pi M_p^2} B(\beta)$

it grows quadratically with the U(1) breaking scale

String loops and string network collisions emit GWs

⇨ stochastic GW background with frequency spectrum

$$\Omega_{\rm GW}(f) = \sum_{k=1}^{\infty} \Omega_{\rm GW}^{(k)}(f) = \frac{8\pi}{3H_0^2}(G\mu)^2 f \sum_{k=1}^{\infty} C_k(f) P_k$$

Larger signal for larger tension (higher U(1) breaking scales)

What if the U(1) breaking occurs at higher energies?

A new promising direction: **gravitational waves** (GW)

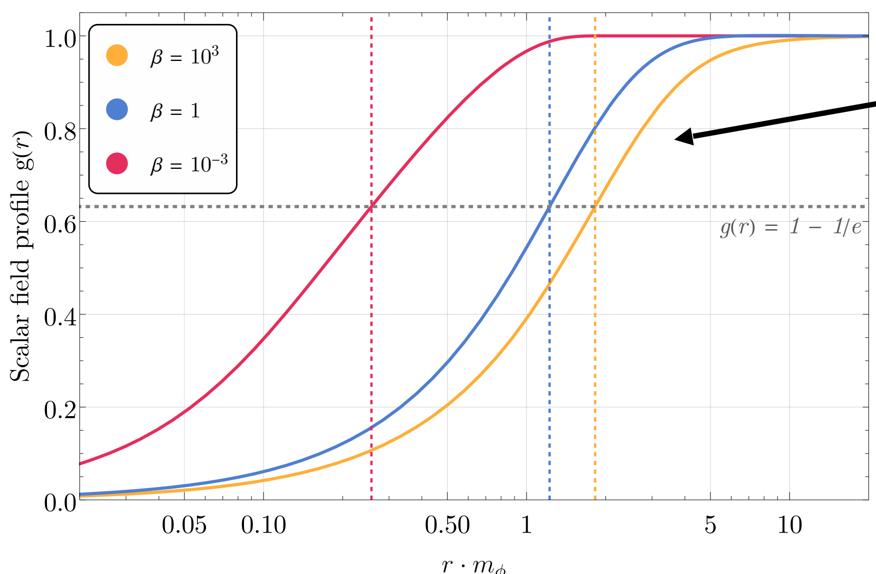
U(1) breaking → cosmic strings → emission of a GW background!

EoM: $$D_\mu D^\mu \phi + \frac{\lambda_\phi}{2}\phi\left(\phi\phi^* - \eta^2\right) = 0\,, \qquad \partial_\mu F'^{\,\mu\nu} = 2g_F\ \mathrm{Im}\left(\phi^* D^\nu \phi\right)$$

static, cylindrically symmetric solutions (strings):

$$\phi_s(\mathbf{r}) = e^{in\theta} g(r)\,, \qquad Z'_{s,\theta}(\mathbf{r}) = -\frac{n}{g_F r}\alpha(r)$$

string profile ($\sim\phi/v_\phi$)



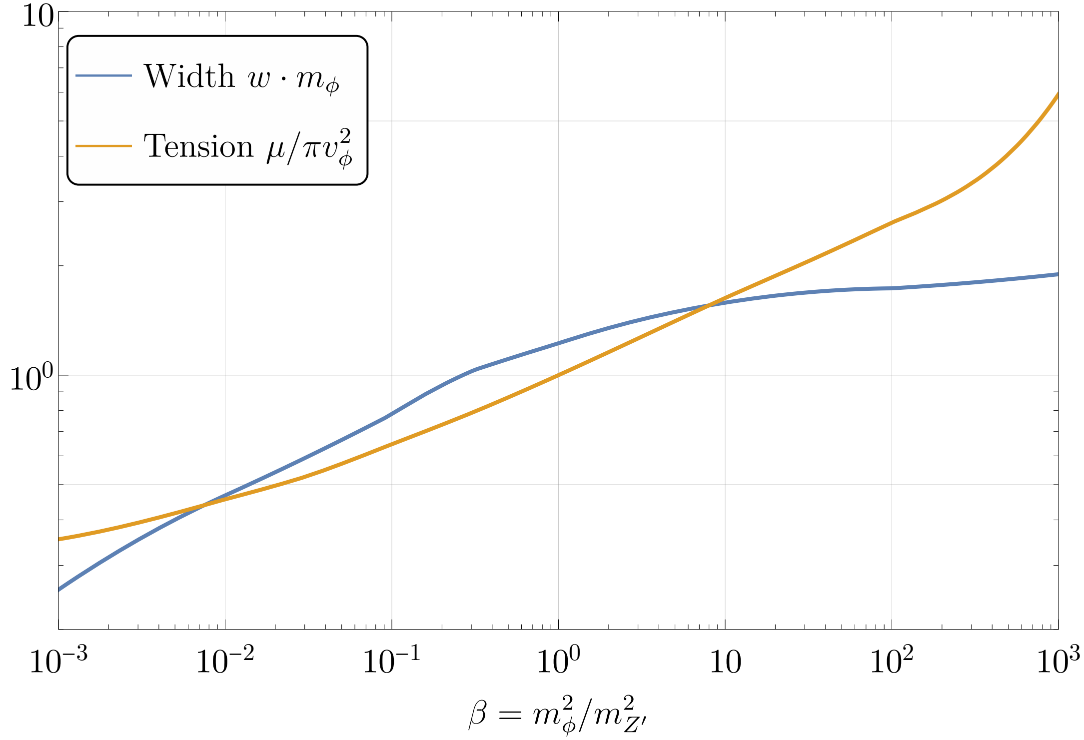
string width depends on

$$\beta \equiv \frac{m_\phi^2}{m_{Z'}^2} = \frac{\lambda_\phi}{2g_F^2}$$

flavon/Z' mass ratio squared

Numerical solutions for the string **width** and **tension**:

$$w = \frac{1}{m_\phi} W(\beta) \qquad\qquad G\mu = \frac{\pi v_\phi^2}{8\pi M_p^2} B(\beta)$$

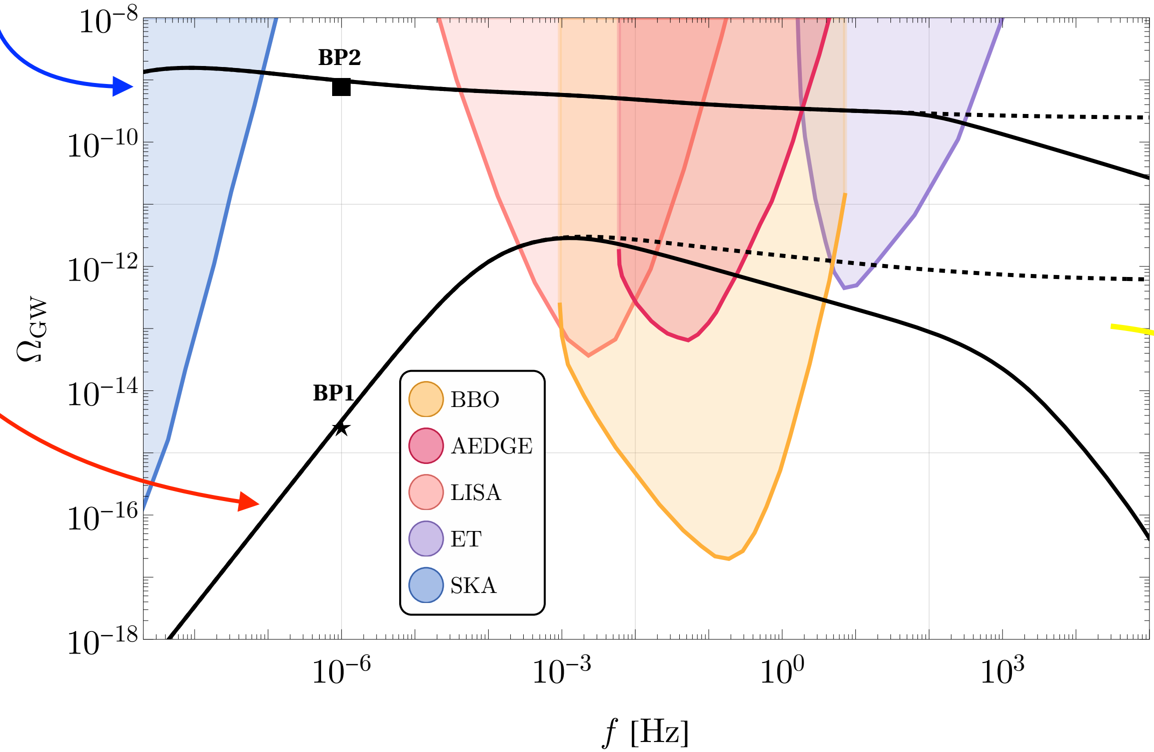


$$\beta \equiv \frac{m_\phi^2}{m_{Z'}^2} = \frac{\lambda_\phi}{2g_F^2}$$

Illustrative GW spectra

BP1 $m_{Z'} = 2 \cdot 10^2$ GeV, $g_F = 10^{-9}$, $\beta = 1$, $v_\phi = \frac{m_{Z'}}{g_F} = 2 \cdot 10^{11}$ GeV

BP2 : $m_{Z'} = 10^7$ GeV, $g_F = 10^{-7}$, $\beta = 1$, $v_\phi = \frac{m_{Z'}}{g_F} = 10^{14}$ GeV



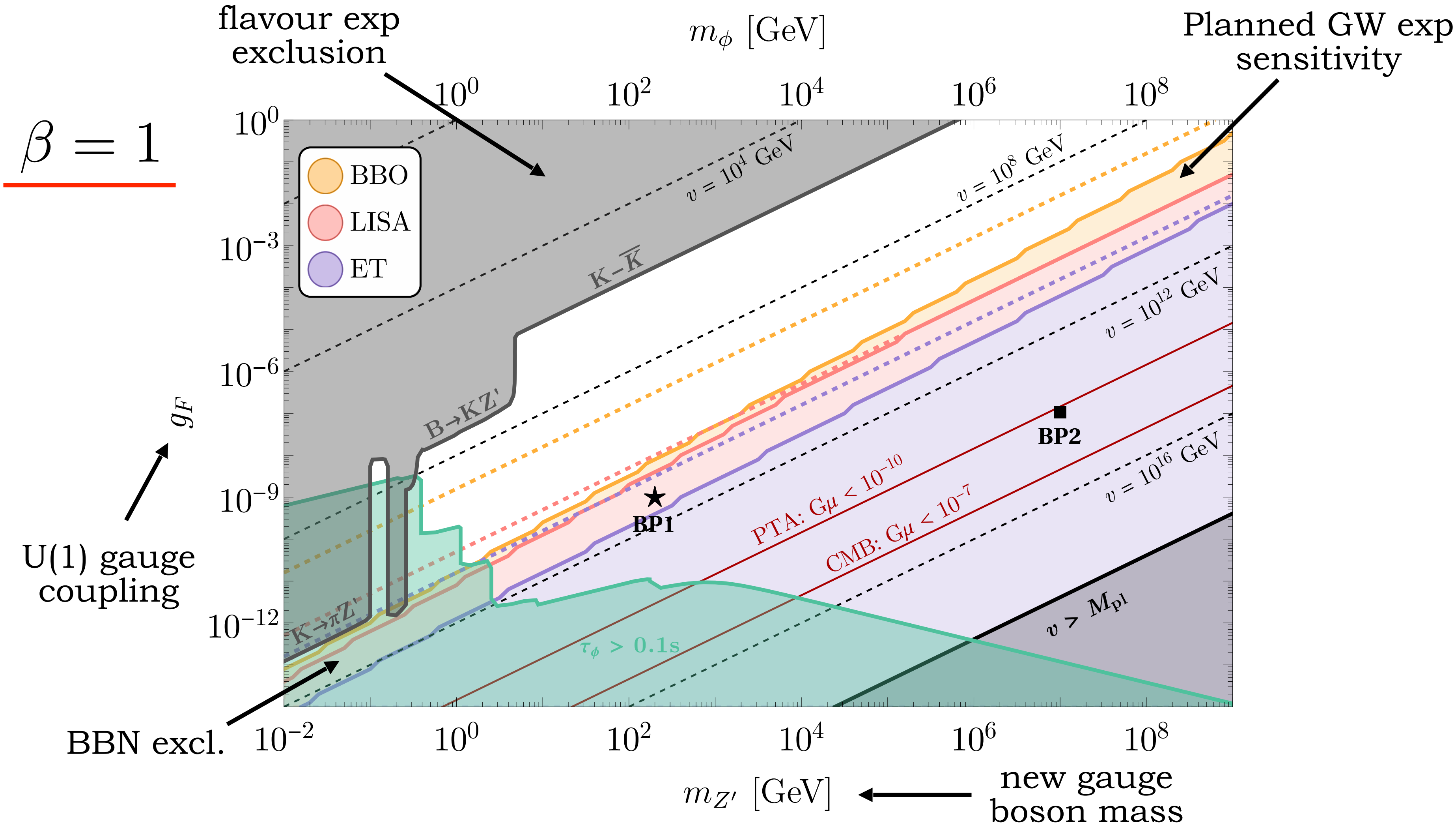
high-frequency cut-off due to large width effects

string loops lose energy mostly through particle (Z’) emission below the critical size: $\ell < \ell_c \sim \frac{w}{(\Gamma G \mu)^2}$

Matsunami et al '19

Flavour limits vs future GW sensitivities

$\beta = 1$



GW and flavour exps. interplay can (almost) close the parameter space!

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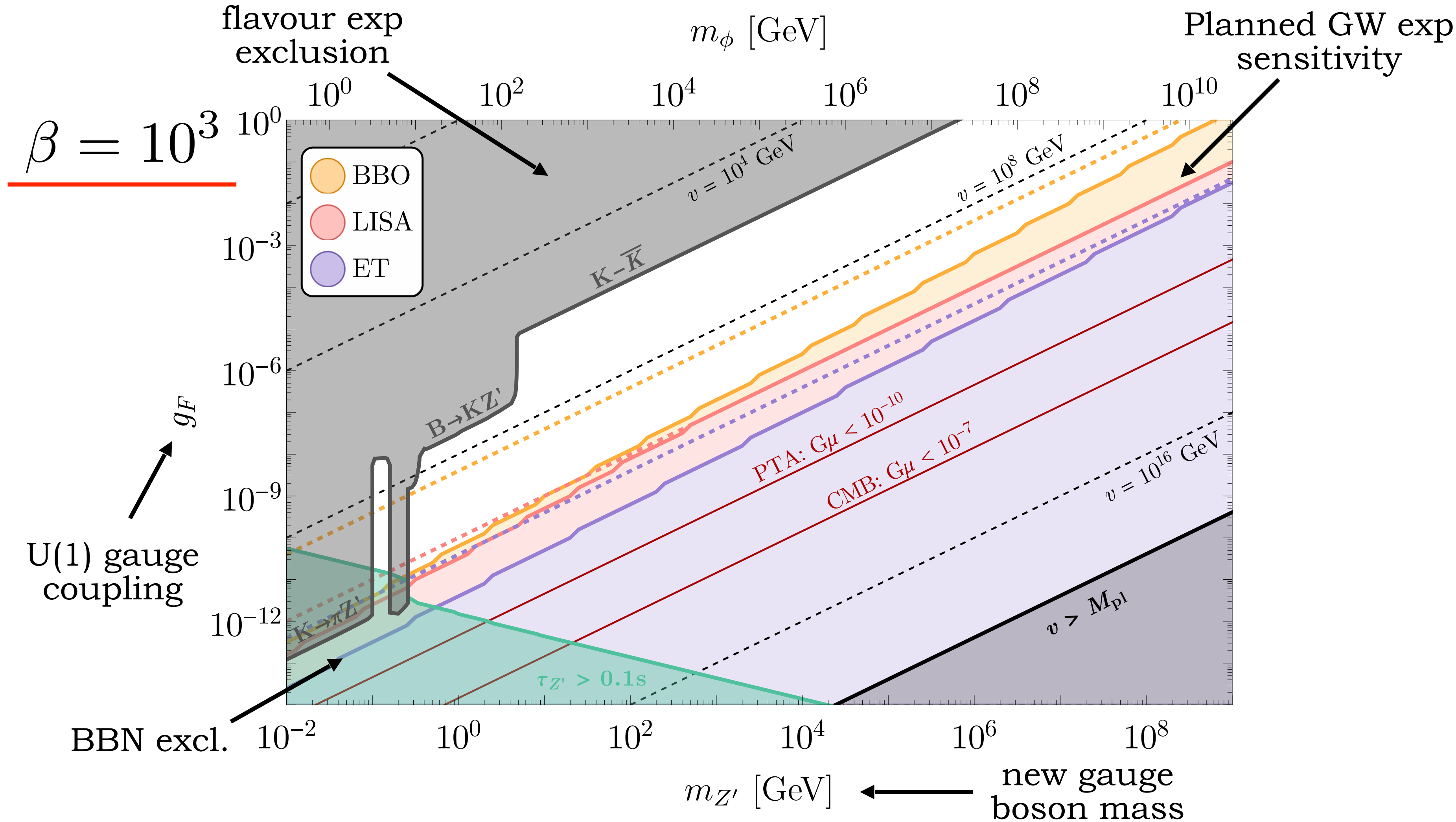
Flavour limits vs future GW sensitivities

$\beta = 10^3$



GW and flavour exps. interplay can (almost) close the parameter space!

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Lifetimes

SU(2) boson decays into SM fermions

$$\Gamma(W_i' \to f_\alpha \overline{f}_\beta) = \frac{N_c^f g_F^2 m_{W'}}{12\pi}\sqrt{\left(1-\frac{(m_{f_\alpha}+m_{f_\beta})^2}{m_{W'}^2}\right)\left(1-\frac{(m_{f_\alpha}-m_{f_\beta})^2}{m_{W'}^2}\right)} \times$$
$$\left[\left(1-\frac{m_{f_\alpha}^2+m_{f_\beta}^2}{2m_{W'}^2}\right)\left(|C_{V\,\alpha\beta}^{f\,i}|^2+|C_{A\,\alpha\beta}^{f\,i}|^2\right)+3\frac{m_{f_\alpha}m_{f_\beta}}{m_{W'}^2}\left(|C_{V\,\alpha\beta}^{f\,i}|^2-|C_{A\,\alpha\beta}^{f\,i}|^2\right)\right]$$

$$\Gamma(\pi_i' \to f_\alpha \bar{f}_\beta) = \frac{N_c^f m_{\pi'}}{8\pi}\left[\left(\frac{m_{f_\alpha}-m_{f_\beta}}{v_\phi}\right)^2|\hat{C}_{V\,\alpha\beta}^{f\,i}|^2 z_+ + \left(\frac{m_{f_\alpha}+m_{f_\beta}}{v_\phi}\right)^2|\hat{C}_{A\,\alpha\beta}^{f\,i}|^2 z_-\right]\sqrt{z_+ z_-}$$
$$z_\pm = 1-(m_{f_\alpha}\pm m_{f_\beta})^2/m_{\pi'}^2$$

$$\Rightarrow \quad \Gamma(\pi' \to f f^{(\prime)}) \sim m_{\pi'}(m_f/v_\phi)^2$$
$$\Gamma(W' \to f f^{(\prime)}) \sim g_F m_{W'} \sim m_{W'}(m_{W'}/v_\phi)^2$$

PNGB relatively more long-lived, searches
for visible final states less important