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KN and $\bar{K}N$ interactions in a renormalizable Chiral EFT

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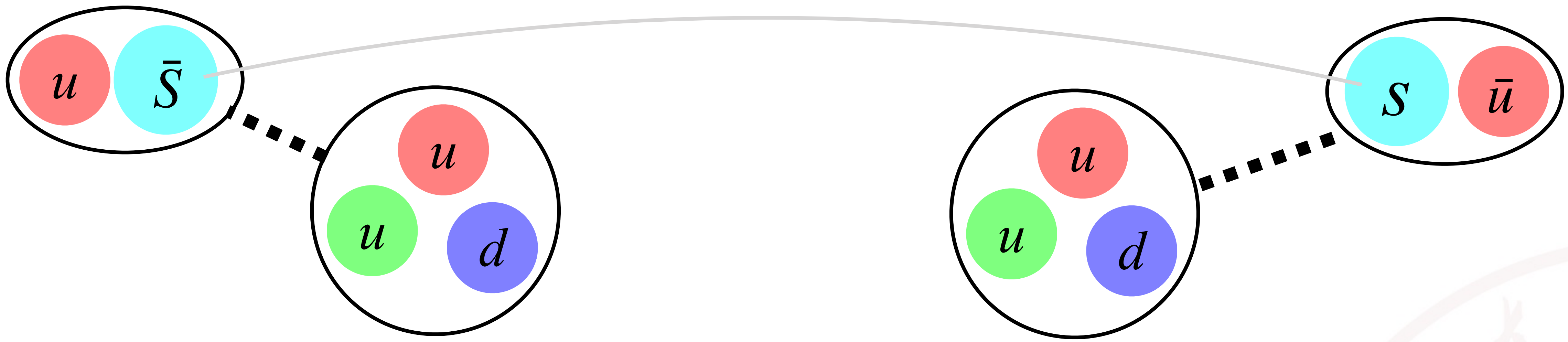
OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary

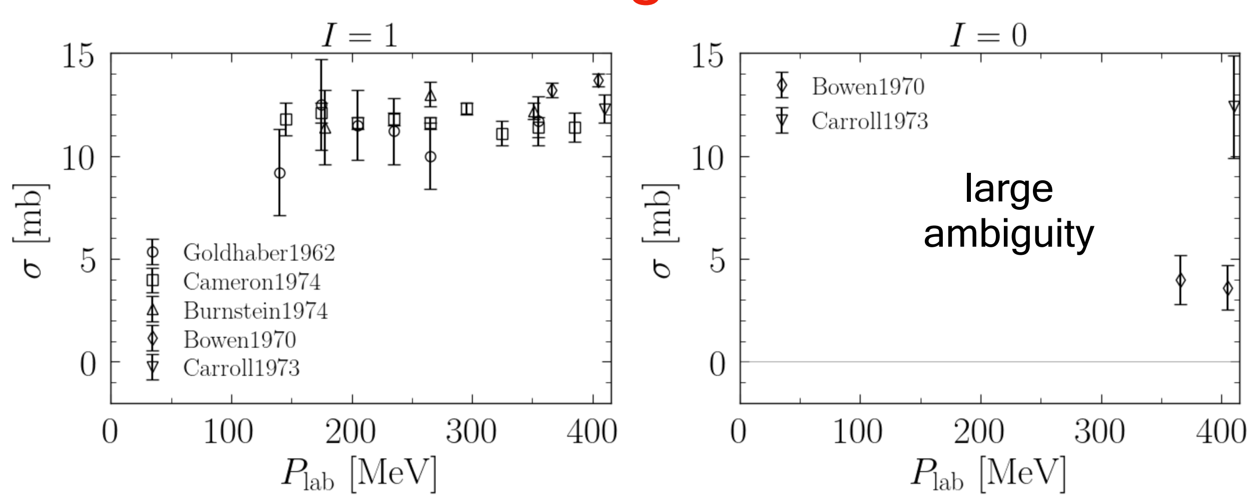
KN and $\bar{K}N$ interactions



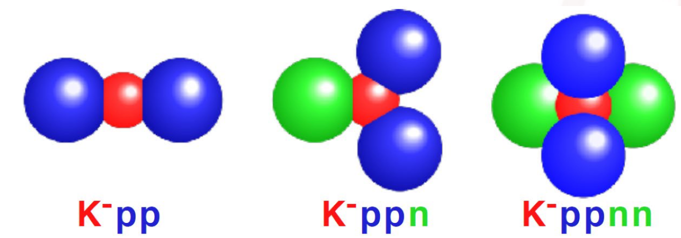
- KN interaction is weak
  - **Kaon penetrates deep into the nuclear interior**

    ✓ sensitive probe of nuclear structure
  - **Clean/single channel**

    ✓ free from nearby resonances
  - Lack of scattering data near threshold

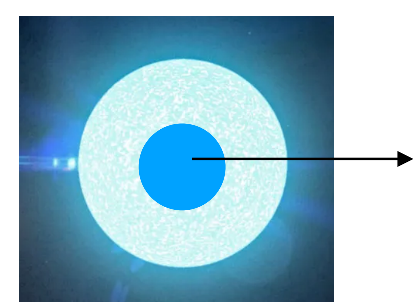


- $\bar{K}N$ interaction is strongly attractive
  - **New form** of nuclei/atoms:



J-PARC, DAΦNE, GSI...

  - **Kaon-condensate** in neutron star



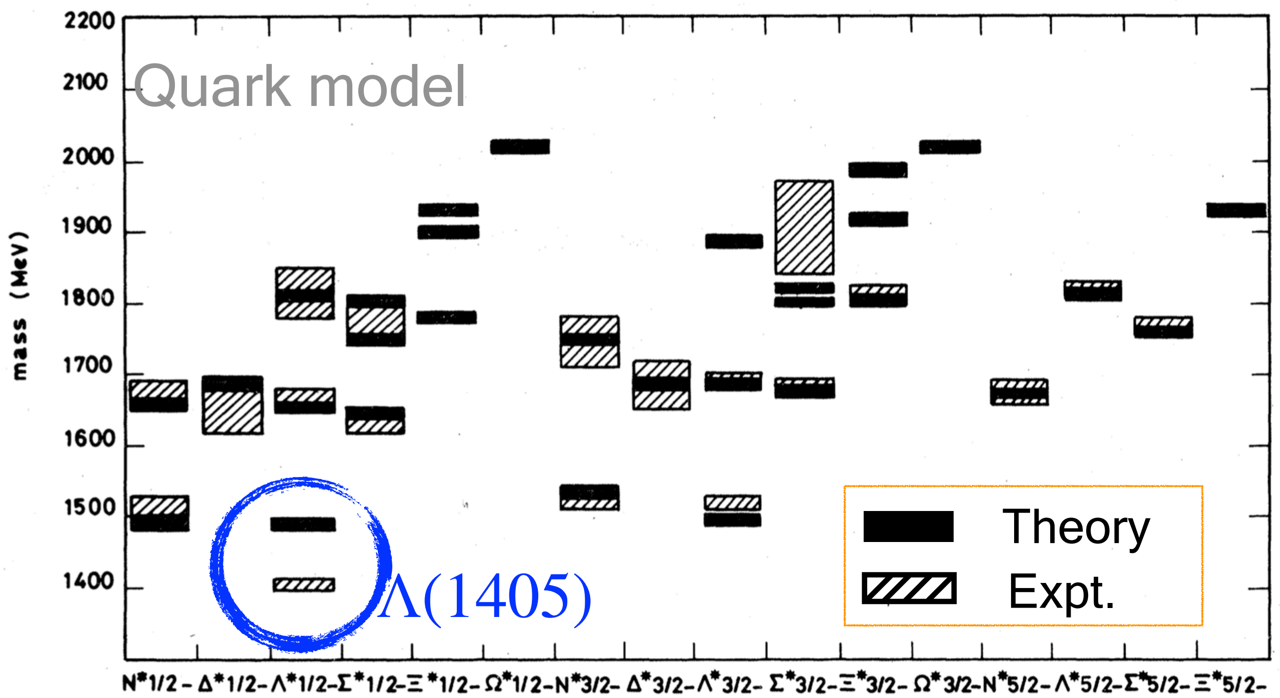
Inner core
- hyperonic matter
- Kaon condensate
- Quark matter

  - **Exotic** $\Lambda(1405)$ resonance

    → important to determine the $\bar{K}N$ amplitude in free space

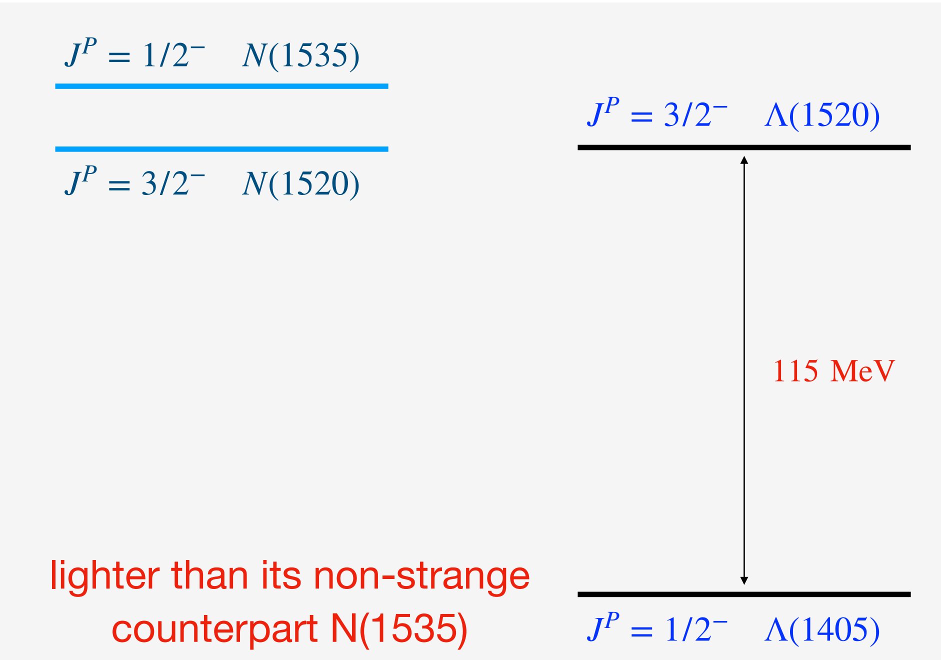
Λ(1405) resonance

- Λ(1405) state is an exotic candidate



N. Isgur and G. Karl, Phys.Rev.D 18 (1978) 4187

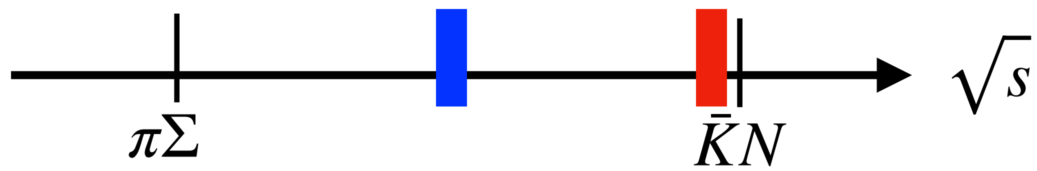
- **Lightest excited baryon with** $J^P = \frac{1}{2}^-$



- Variety of theoretical studies
  - ✓ Chiral SU(3) quark model *F. Huang, PRC2007…*
  - ✓ QCD sum rules *L.S. Kisslinger, EPJA2011…*
  - ✓ Phenomenological potential model *A. Cieplý, NPA2015…*
  - ✓ Skyrme model *T. Ezoe, PRD2020…*
  - ✓ Hamiltonian effective field theory *Z.-W. Liu, PRD2017…*
  - ✓ Chiral unitary approach *N.Kaiser,NPA1995; E.Oset,NPA1998; J.A.Oller&U.-G.Meißner,PLB2001…*

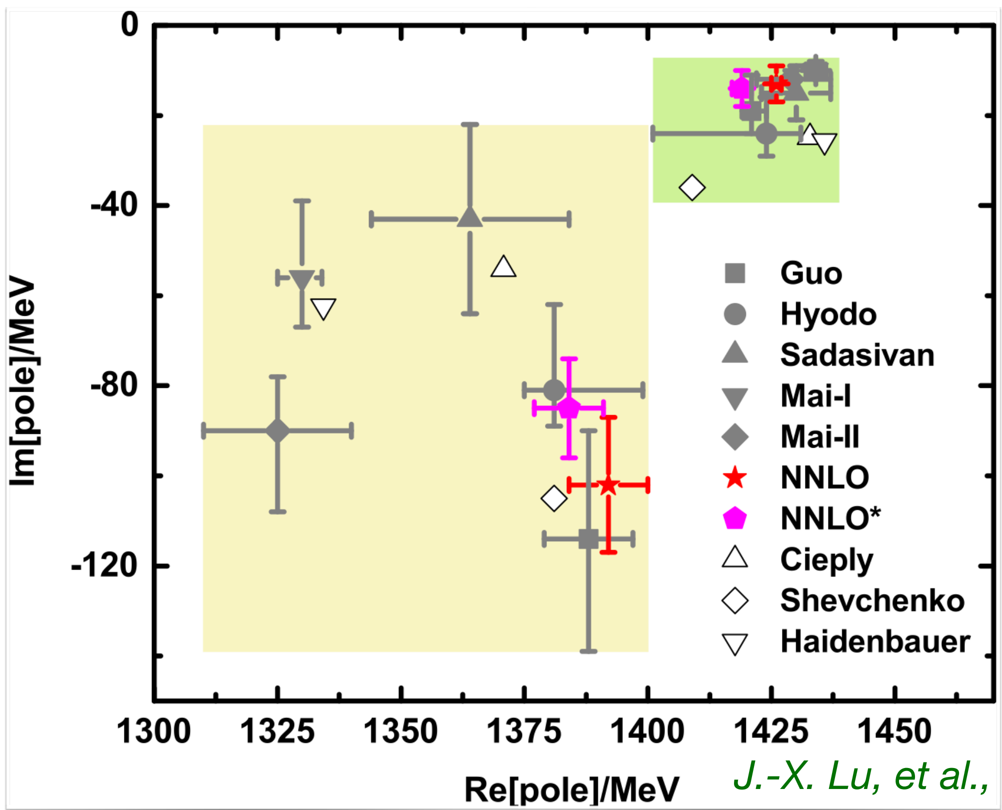
Structure of $\Lambda(1405)$ resonance

- Double-pole predicted by **chiral unitary approach**



$\Lambda(1405)\ 1/2^-$ $\quad I(J^P) = 0(\frac{1}{2}^-)$ Status: ∗∗∗∗

$\Lambda(1380)\ 1/2^-$ $\quad J^P = \frac{1}{2}^-$ Status: ∗∗

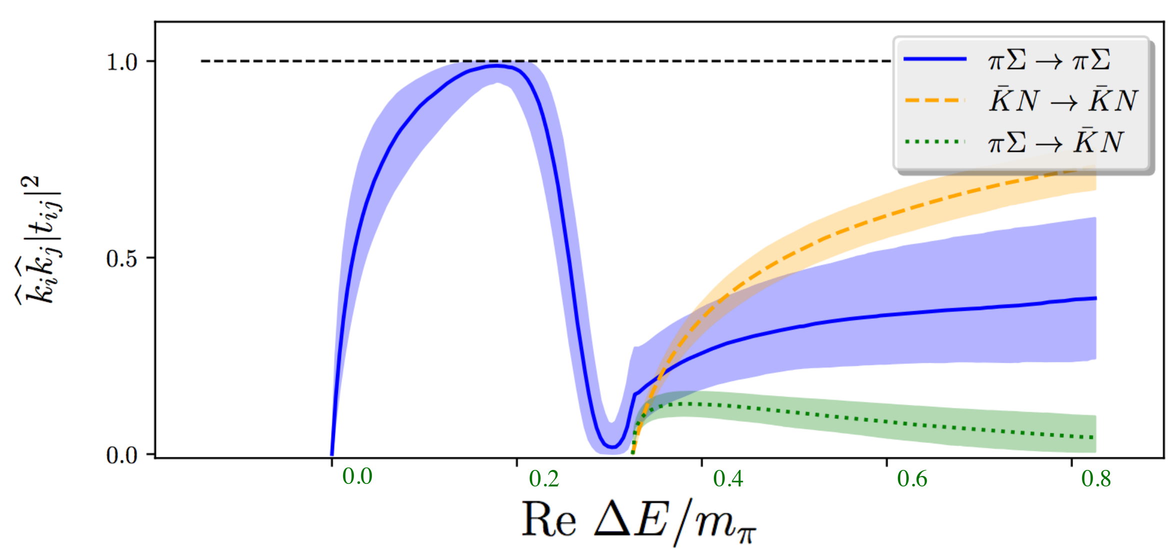


J.-X. Lu, et al., PRL130(2023)071902

✓ Pole 1: Λ(1405) is around 1420 MeV

✓ Pole 2: Λ(1380) needs further studies to fix its position

- Double-pole structure verified by LQCD



$m_\pi \approx 200$ MeV, $m_K \approx 487$ MeV

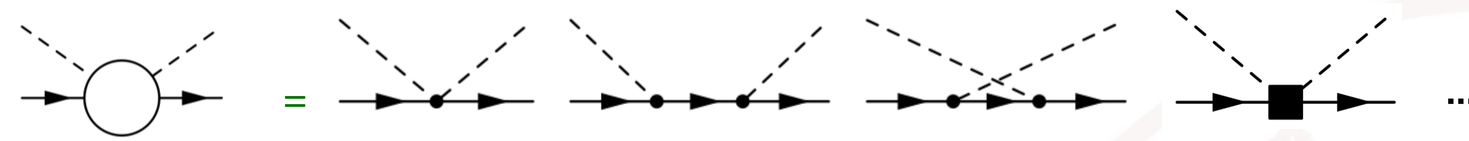
$$\text{Lower Pole}: \quad E_1 = 1392(9)_{\rm stat}(2)_{\rm model}(16)_{\rm a}\ \text{MeV}$$

$$\text{Higher Pole}: \quad E_2 = 1455(13)_{\rm stat}(2)_{\rm model}(17)_{\rm a} - i11.5(4.4)_{\rm stat}(4.0)_{\rm model}(0.1)_{\rm a}\ \text{MeV}$$

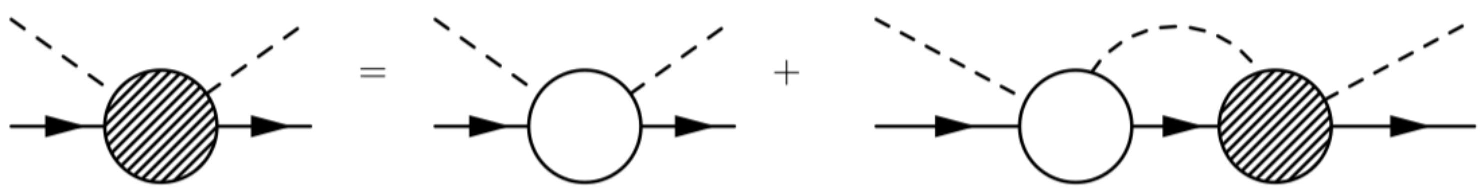
Baryon Scattering Coll., PRL132(2024)051901

Chiral Unitary approach

- **Chiral symmetry** of low-energy QCD + **Unitary Relation**
  *J.A.Oller et al.,PPNP45(2000)157-242; T.Hyodo et al.,PPNP*120 (2021)103868 …
- Interaction kernel V: calculate in ChPT order by order
  - Leading, next-to-leading order, …



- Scattering T-matrix: solve scattering equations



  - Lippmann-Schwinge equation or Bethe-Salpeter equation

$$T(p',p) = V(p',p) + i\int \frac{d^{3(4)}k}{(2\pi)^{3(4)}} V(p',k)\, G(k)\, T(k,p)$$

  - On-shell factorization $\rightarrow V(p',p) + V(p',p)\left(i\int \frac{d^4k}{(2\pi)^4} G(k)\right) T(p',p)$

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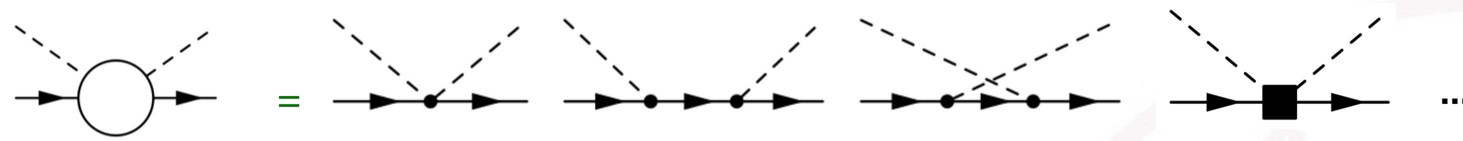

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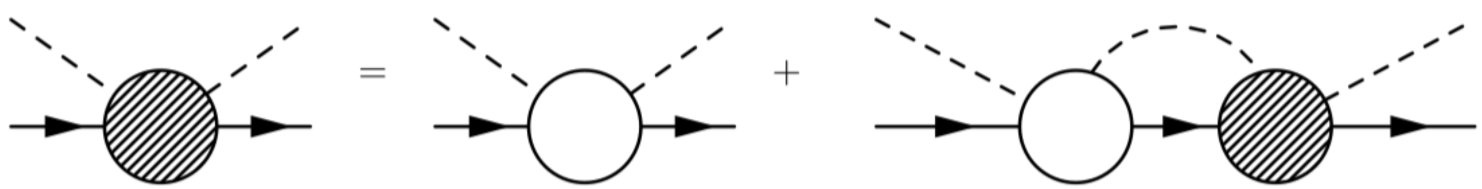

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- Form factor with Finite cutoff / subtraction constant to solve scattering equations $\longrightarrow$ **Cutoff / Model dependence**

In this work

- Facing the rapid progress of precision experiments, **a model-independent formalism would be needed** ALICE, AMADEUS, J-PARC, STAR…

- We tentatively propose **a renormalizable framework** of Chiral EFT for meson-baryon scattering by using **time-ordered perturbation theory** with covariant chiral Lagrangians
  - Leading order studies
    - ✓ pion-nucleon scattering
      XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406
    - ✓ $\bar{K}N$ scattering at LO and $\Lambda(1405)$ state
      XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582
    - ✓ Light-quark mass dependence of $\Lambda(1405)$ XLR, Phys. Lett. B 855 (2024) 138802
  - Next-to-leading order studies
    - ✓ KN scattering XLR, Phys. Rev. D (2026) in press, arXiv: 2512.24721
    - ✓ pion-nucleon scattering XLR, et al., in progress

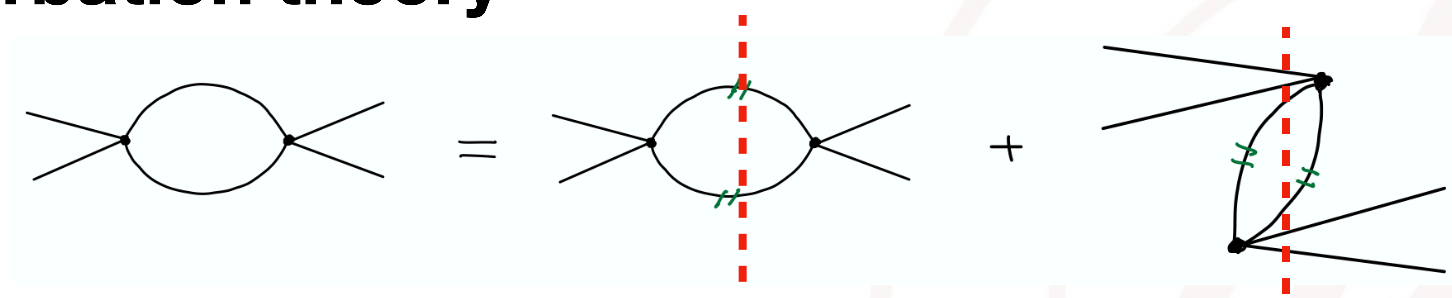
Theoretical framework

Time-ordered perturbation theory

Definition

S. Weinberg, Phys.Rev.150(1966)1313
G.F. Sterman, "An introduction to quantum field theory", Cambridge (1993)

- Re-express the Feynman integral in a form that makes the connection with on-mass-shell (off-energy shell) state explicit.
  - ✓ Instead the propagators for internal lines as the energy denominators for intermediate states
- **TOPT or old-fashioned perturbation theory**



Advantages

- Explicitly show the unitarity
- Easily to tell the contributions of a particular diagram

Obtain the rules for time-ordered diagrams

- Perform Feynman integrations over the zeroth components of the loop momenta
- Decompose Feynman diagram into sums of time-ordered diagrams
- Match to the rules of time-ordered diagrams

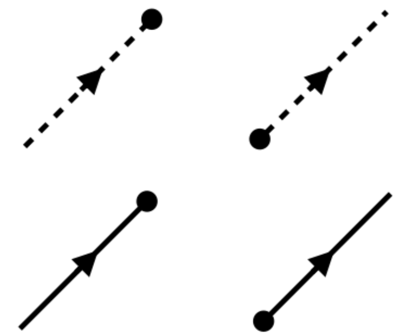
Diagrammatic rules in TOPT

XLR, PoS(CD2021)007

- External lines

Spin 0 boson (in, out)

Spin 1/2 fermion (in, out)



Spin 0 boson: **1**

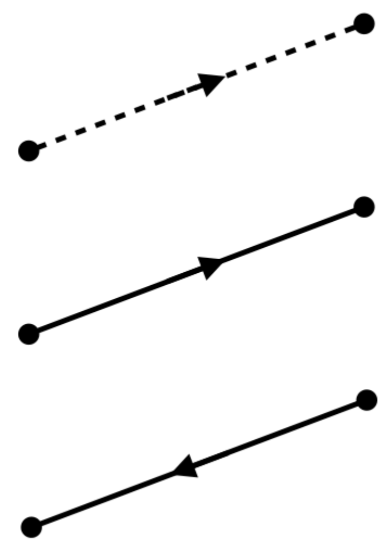
Spin 1/2 fermion: $u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$

- Internal lines

Spin 0 (anti-)boson

Spin 1/2 fermion

anti-fermion



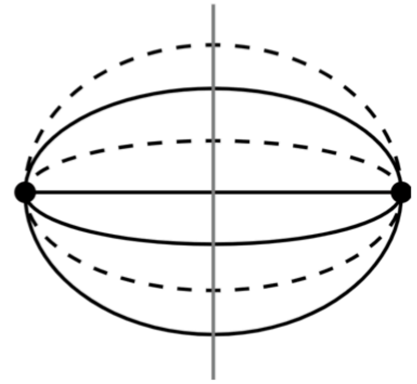
Spin 0 (anti-)boson: $\frac{1}{2\,\epsilon_q}$ $\qquad \epsilon_q \equiv \sqrt{\mathbf{q}^2 + M^2}$

Spin 1/2 fermion: $\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p})$ $\qquad \omega_p \equiv \sqrt{\mathbf{p}^2 + m^2}$

anti-fermion: $\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$

- Intermediate state

A set of lines between two vertices



$$\frac{1}{E - \sum_i \omega_{p_i} - \sum_j \epsilon_{q_j} + i\epsilon}$$

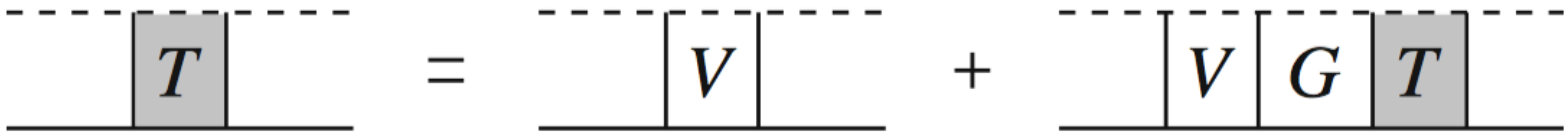
- Interaction vertices: **the standard Feynman rules**
  - Take care of zeroth components of integration momenta

✓ particle $p^0 \to \omega(p, m)$

✓ antiparticle $p^0 \to -\omega(p, m)$

Meson–baryon scatterings in TOPT

- Interaction kernel / potential V
  - **Define:** sum up the one-meson and one-baryon **irreducible diagrams**
  - **Power counting:** Q/Λ_χ systematic ordering of all graphs
- Scattering equation



  - Coupled-channel integral equation for T-matrix

$$T_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) = V_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) + \sum_{MB}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}V_{M_jB_j,MB}(\boldsymbol{p}',\boldsymbol{k};E)\,G_{MB}(E)\,T_{MB,M_iB_i}(\boldsymbol{k},\boldsymbol{p};E)$$

  - **Meson-baryon Green function in TOPT**

$$G_{MB}(E) = \frac{m}{2\omega(k,M)\,\omega(k,m)}\frac{1}{E-\omega(k,M)-\omega(k,m)+i\epsilon}$$

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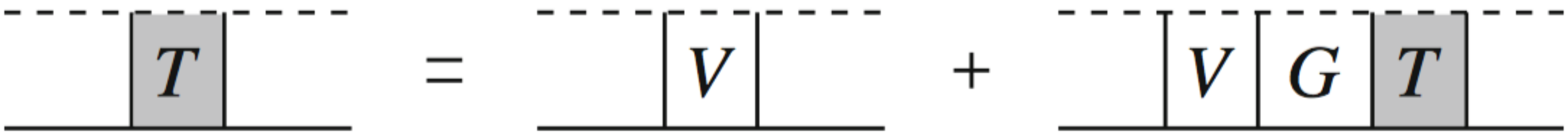

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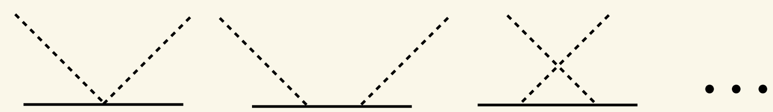
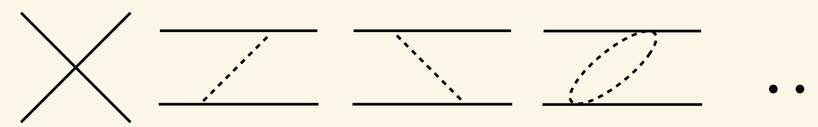
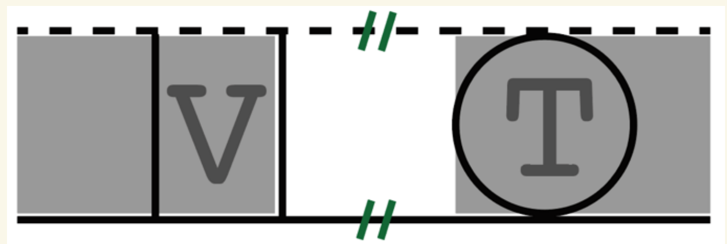
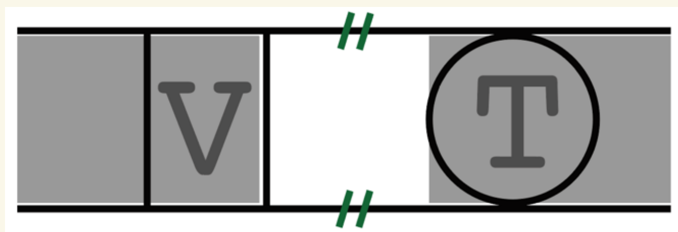
- **Meson-baryon Green function in TOPT**

$$G_{MB}(E) = \frac{m}{2\omega(k,M)\,\omega(k,m)}\,\frac{1}{E-\omega(k,M)-\omega(k,m)+i\epsilon}$$

Potential and scattering equation are obtained on an equal footing!

Baryon–baryon scatterings in TOPT

	Meson-baryon scattering	Baryon-baryon scattering
Potential TOPT diagrams		
Green function	 $G^{MB}(E) = \frac{m}{2\omega_M\,\omega_m}\frac{1}{E-\omega_M-\omega_m+i\epsilon}$	 $G^{BB}_{ij}(E) = \frac{m_i\,m_j}{\omega_{m_i}\,\omega_{m_j}}\frac{1}{E-\omega_{m_i}-\omega_{m_j}+i\epsilon}$

- Extend to the baryon-baryon scatterings
  - Formulate NN and YN interactions at leading order

    V. Baru, E. Epelbaum, J. Gegelia, XLR*, Phys. Lett. B 798,134987 (2019)

    XLR, E.Epelbaum, J.Gegelia, Phys. Rev. C 101,034001 (2020)
  - Extend NN interaction to **next-to-next-to-leading order**

    XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)

    XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 113, 034002(2026)

Leading order studies

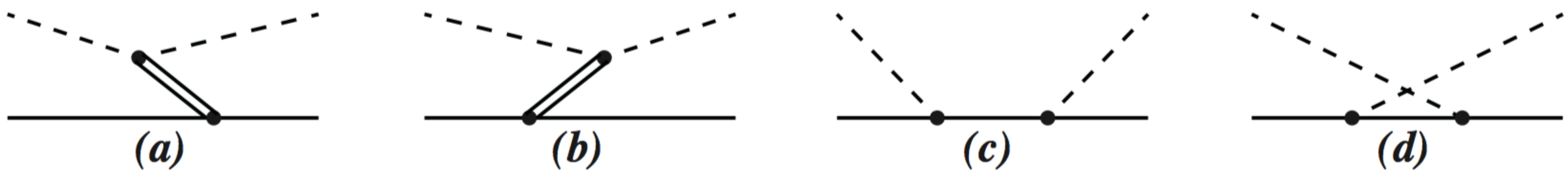
- XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406;
- XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582;
- XLR, Phys. Lett. B 855 (2024) 138802

Leading order potential

- Chiral effective Lagrangian

$$\mathcal{L}_{\rm LO} = \frac{F_0^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + \langle \bar{B}\left(i\gamma_\mu\partial^\mu - m\right)B\rangle + \frac{D/F}{2}\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]_\pm\rangle$$
$$-\frac{1}{4}\left\langle V_{\mu\nu}V^{\mu\nu} - 2\mathring{M}_V^2\left(V_\mu - \frac{i}{g}\Gamma_\mu\right)\left(V^\mu - \frac{i}{g}\Gamma^\mu\right)\right\rangle + g\,\langle \bar{B}\gamma_\mu[V^\mu, B]\rangle$$

- Time ordered diagrams



- **Vector mesons included as explicit degrees of freedom**
  - ✓ One-vector meson exchange potential instead of the Weinberg-Tomozawa term
  - ✓ Improve the ultraviolet behaviour without changing the low-energy physics

- LO potential in TOPT

- Dirac spinor is decomposed as $u_B(p,s) = u_0 + [u(p) - u_0] \equiv (1,0)^\dagger \chi_s$+high order

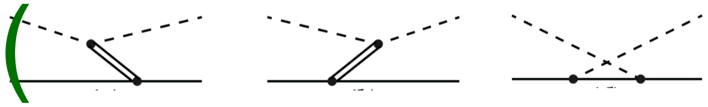
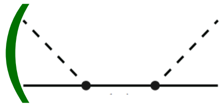
$$V^{(a+b)}_{M_jB_j,M_iB_i} = -\frac{1}{32F_0^2}\sum_{V=K^*,\rho,\omega,\phi} C^V_{M_jB_j,M_iB_i}\frac{\mathring{M}_V^2}{\omega_V(q_1-q_2)}\left(\omega_{M_i}(q_1)+\omega_{M_j}(q_2)\right)$$
$$\times\left[\frac{1}{E-\omega_{B_i}(p_1)-\omega_V(q_1-q_2)-\omega_{M_j}(q_2)}+\frac{1}{E-\omega_{B_j}(p_2)-\omega_V(q_1-q_2)-\omega_{M_i}(q_1)}\right]$$

$$V^{(c)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} C^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(P)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)}{E-\omega_B(P)}.$$

$$V^{(d)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} \tilde{C}^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(K)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)}{E-\omega_{M_i}(q_1)-\omega_{M_j}(q_2)-\omega_B(K)}.$$

Subtractive renormalization

- LO potential: **one-baryon irreducible and reducible parts**

$$V_{\text{LO}} = V_I \Big($$  $$\Big) + V_R \Big($$  $$\Big)$$

- LO T-matrix

$$T_{\text{LO}} = V_{\text{LO}} + V_{\text{LO}}\, G\, T_{\text{LO}} \quad \Longrightarrow \quad \begin{cases} T_{\text{LO}} = T_I + (1+T_I\, G)\, T_R\, (1 + G\, T_I) \\ T_I = V_I + V_I\, G\, T_I \\ T_R = V_R + V_R\, G\, (1 + T_I\, G)\, T_R \end{cases}$$

- Irreducible part: $T_I \xrightarrow{\Lambda \sim \infty}$ Finite
- Reducible part: $T_R \xrightarrow{\Lambda \sim \infty}$ Divergent
  - ✓ **Potential can be rewritten as separable form**

$$V_R(p', p; E) = \xi^T(p')\, C(E)\, \xi(p)$$

C(E): constant $\quad \xi^T(q) := (1, q)$

  - ✓ T_R can be rewritten as $\quad T_R(p', p; E) = \xi^T(p')\, \chi(E)\, \xi(p) \quad \chi(E) = \left[C^{-1} - \xi\, G\, \xi^T - \xi\, G\, T_I^S\, G\, \xi^T\right]^{-1}$

D.B.Kaplan, et al.,NPB478,629(1996); E. Epelbaum, et al.,EPJA51,71(2015)

  - ✓ Using **subtractive renormalization,** replacing Green function $\quad G^{Rn} = G(E) - G(m_B)$

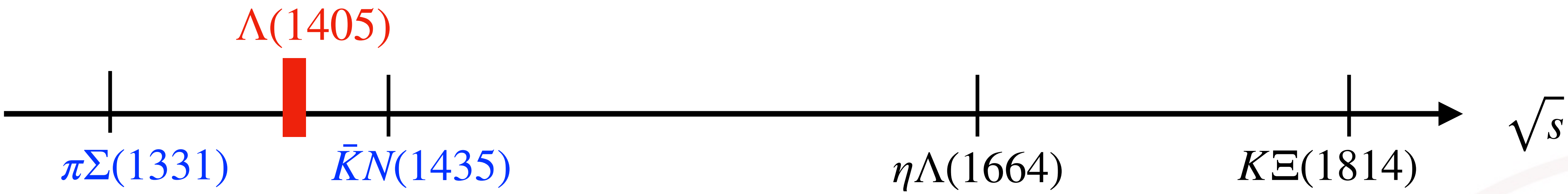
E. Epelbaum, et al., EPJA56(2020)152

Renormalized LO T-matrix

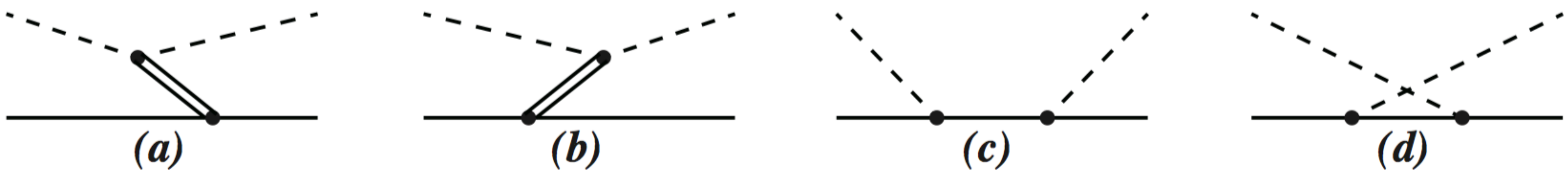
$$T_{\text{LO}}^{Rn} = T_I + \left(\xi^T + T_I\, G^{Rn}\, \xi^T\right)\, \chi^{Rn}(E)\, \left(\xi + \xi\, G^{Rn}\, T_I\right)$$

S=−1 meson-baryon scattering

- Four coupled channels $\bar{K}N, \pi\Sigma, \eta\Lambda, K\Xi$ in isospin limit



- Focus on the S-wave potential



  - Born term (p-wave) does not contribute
  - Crossed-Born term ∼5% of VME contribution
  - VME potential couplings

C^V	$\pi\Sigma$	$\bar{K}N$	$\eta\Lambda$	$K\Xi$
$\pi\Sigma$	$C^{\rho} = -16$	$C^{K^*} = 2\sqrt{6}$	0	$C^{K^*} = -2\sqrt{6}$
$\bar{K}N$	attractive	$C^{\{\rho,\omega,\phi\}} = \{-6, -2, -4\}$	$C^{K^*} = -6\sqrt{2}$	0
$\eta\Lambda$		attractive	0	$C^{K^*} = 6\sqrt{2}$
$K\Xi$				$C^{\{\rho,\omega,\phi\}} = \{-6, -2, -4\}$

S=-1 meson-baryon scattering

◻ P. W. scattering equation

$$T^{LJ}_{M_jB_j,M_iB_i}(p',p) = V^{LJ}_{M_jB_j,M_iB_i}(p',p) + \sum_{MB}\int \frac{dk\,k^2}{(2\pi)^3} V^{LJ}_{M_jB_j,MB}(p',k)\frac{1}{2\,\omega_M\omega_B}\frac{m_B}{E-\omega_M-\omega_B+i\epsilon} T^{LJ}_{MB,M_iB_i}(k,p)$$

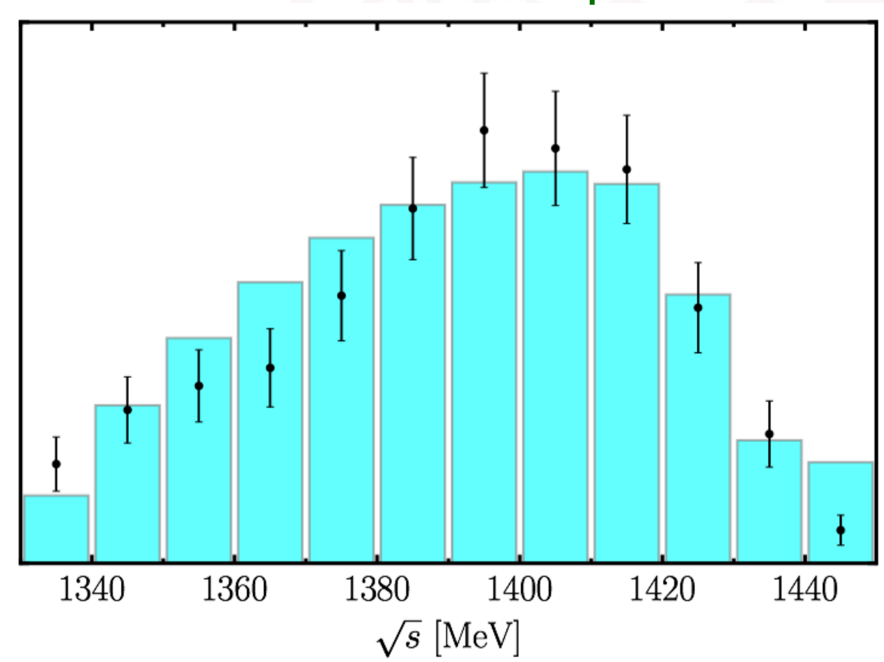
- Take into account **the off-shell effects of potential**
- Use **subtractive reormalization** to obtain the renormalized T-matrix
  - ➡ Cutoff-independent: $\Lambda \to \infty$

No free parameters needed to be fitted!

◻ Two pole positions of $\Lambda(1405)$

		lower pole	higher pole
This work (LO)	$F_0 = F_\pi$	$1337.7 - i\,79.1$	$1430.9 - i\,8.0$
	$F_0 = 103.4$	$1348.2 - i\,120.2$	$1436.3 - i\,0.7$
NLO	*Y. Ikeda,NPA(2012)*	$1381^{+18}_{-6} - i81^{+19}_{-8}$	$1424^{+7}_{-23} - i26^{+3}_{-14}$
	Z.-H.Guo,PRC(2013)-Fit II	$1388^{+9}_{-9} - i114^{+24}_{-25}$	$1421^{+3}_{-2} - i19^{+8}_{-5}$
	M.Mai,EPJA(2015)-sol-2	$1330^{+4}_{-5} - i56^{+17}_{-11}$	$1434^{+2}_{-2} - i10^{+2}_{-1}$
	M.Mai,EPJA(2015)-sol-4	$1325^{+15}_{-15} - i90^{+12}_{-18}$	$1429^{+8}_{-7} - i12^{+2}_{-3}$

πΣ invariant mass spectrum



XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582

Apply our framework to unphysical world

- BaSc results provide an ideal playground

PRL 132, 051901 (2024); PRD109,014511(2024)

- First lattice study of $\Lambda(1405)$ pole positions

Virtual bound state

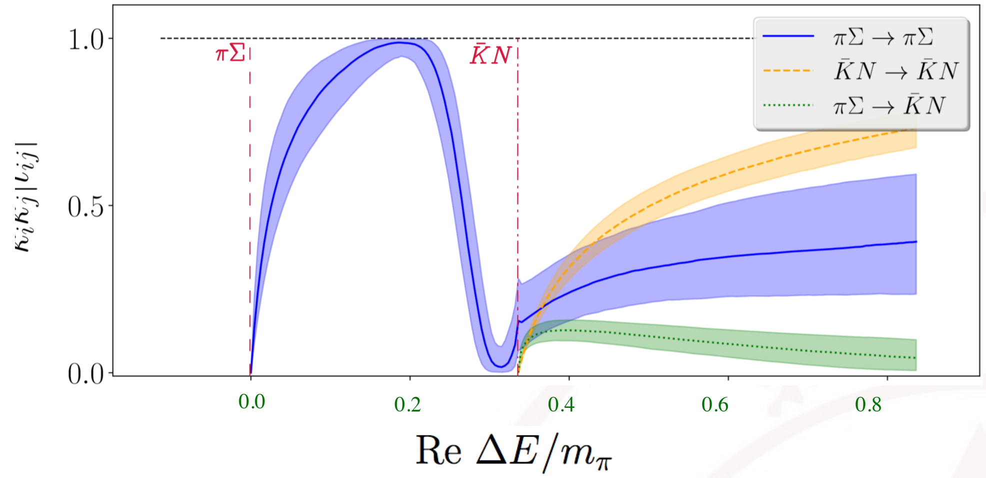
$$E_1 = 1392(9)_{\rm stat}(2)_{\rm model}(16)_{\rm a}\text{MeV}$$

$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 1.9(4)_{\rm stat}(6)_{\rm model}$$

Resonance

$$E_2 = \left[1455(13)_{\rm stat}(2)_{\rm model}(17)_{\rm a} - i11.5(4.4)_{\rm stat}(4.0)_{\rm model}(0.1)_{\rm a}\right]\text{MeV}$$

$$\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2)}_{\bar{K}N}}\right| = 0.53(9)_{\rm stat}(10)_{\rm model}$$



- **Check/verify the predictive power of existing chiral unitary approaches**

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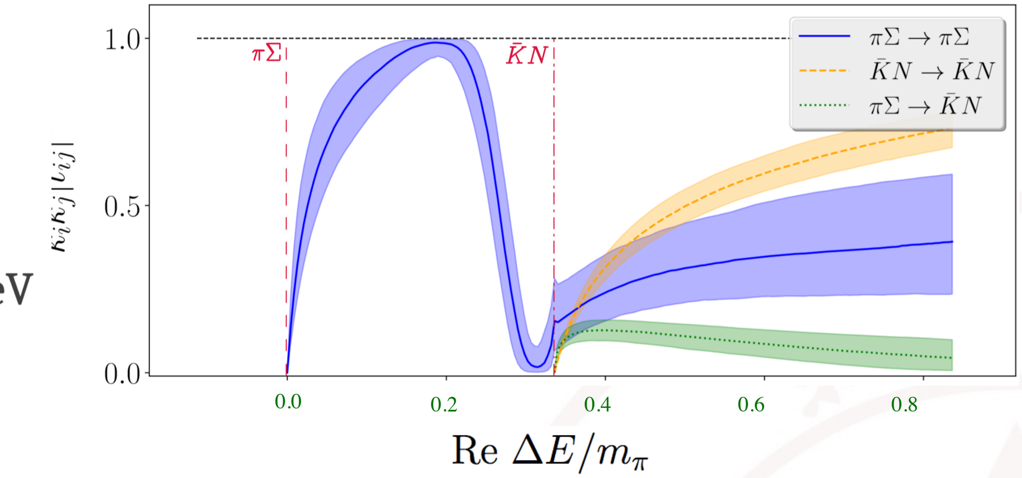
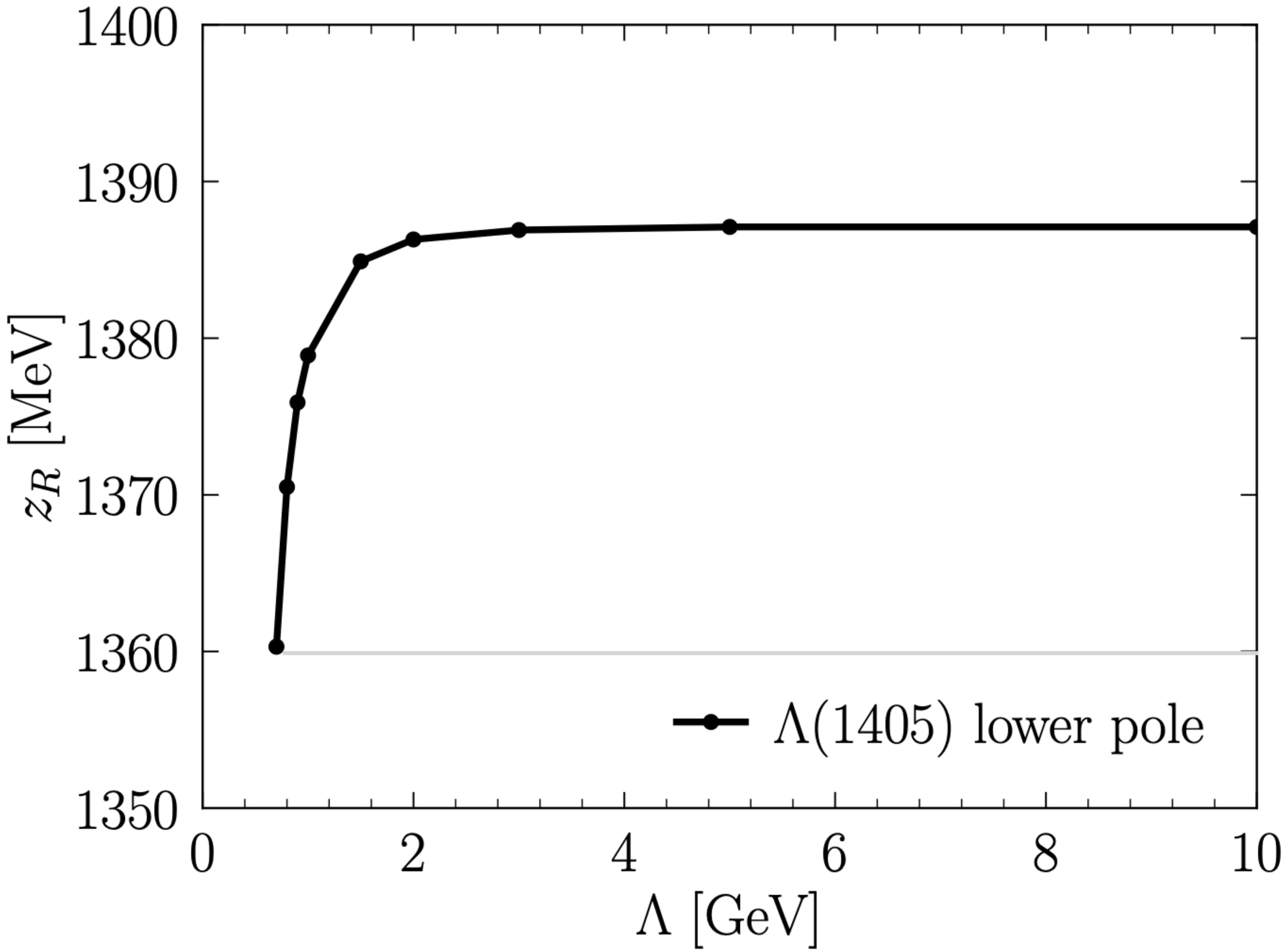


- **Check/verify the predictive power of existing chiral unitary approaches**

- Extend our calculation to the unphysical quark mass region
  - Use the **same** meson and baryon masses as the BaSc study
  - Focus on the $\pi\Sigma - \bar{K}N$ coupled channels
  - **Consistent with the BaSc results** $\Lambda \to \infty$, **no free parameters**

	BaSc [PRL2024]	This work			
$\Lambda(1405)$	z_R [MeV]	z_R [MeV]	$g_{\pi\Sigma}$	$g_{\bar{K}N}$	$\lvert g_{\pi\Sigma}\rvert/\lvert g_{\bar{K}N}\rvert$
Lower pole	**1392(18)**	1387.14	$0.021 + i1.87$	$0.017 + i1.55$	1.21
Higher pole	**1455(21) − i11.5(6.0)**	1469.86 − i4.71	$0.038 + i0.98$	$1.51 - i1.22$	0.50

Cutoff independence of pole position



Finite-cutoff artifact
~ 30 MeV

XLR, Phys. Lett. B 855 (2024) 138802

Next-to-leading order studies

- XLR, Phys. Rev. D (2026) in press, arXiv: 2512.24721;
- XLR, et al., In progress

Beyond leading order

- **Maintain the scattering T-matrix renormalizable**
  - Take LO potential non-perturbatively
  - Include Higher order corrections perturbatively

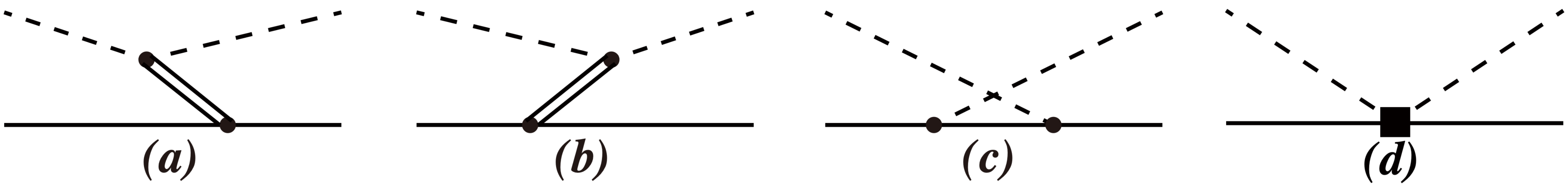
- Up to next-to-leading order
  - Potential: $V = V_{\rm LO} + V_{\rm NLO}$
  - T-matrix: $T = T_{\rm LO} + T_{\rm NLO}$

$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO} G T_{\rm LO}$$ (non-perturbative)

$$T_{\rm NLO} = V_{\rm NLO} + T_{\rm LO} G V_{\rm NLO} + V_{\rm NLO} G T_{\rm LO} + T_{\rm LO} G V_{\rm NLO} G T_{\rm LO}$$

  - Use **the subtractive renormalization** to remove divergent terms and power-counting breaking terms

KN scattering up to NLO



- Chiral effective Lagrangians at NLO
  - For S-wave scattering

$$\mathscr{L}_{\rm NLO} = b_0\langle\chi_+\rangle\langle\bar{B}B\rangle + b_D\langle\bar{B}\{\chi_+,B\}\rangle + b_F\langle\bar{B}[\chi_+,B]\rangle + d_1\langle\bar{B}\{u_\mu,[u^\mu,B]\}\rangle + d_2\langle\bar{B}[u_\mu,[u^\mu,B]]\rangle + d_3\langle\bar{B}u_\mu\rangle\langle u^\mu B\rangle + d_4\langle\bar{B}B\rangle\langle u^\mu u_\mu\rangle,$$

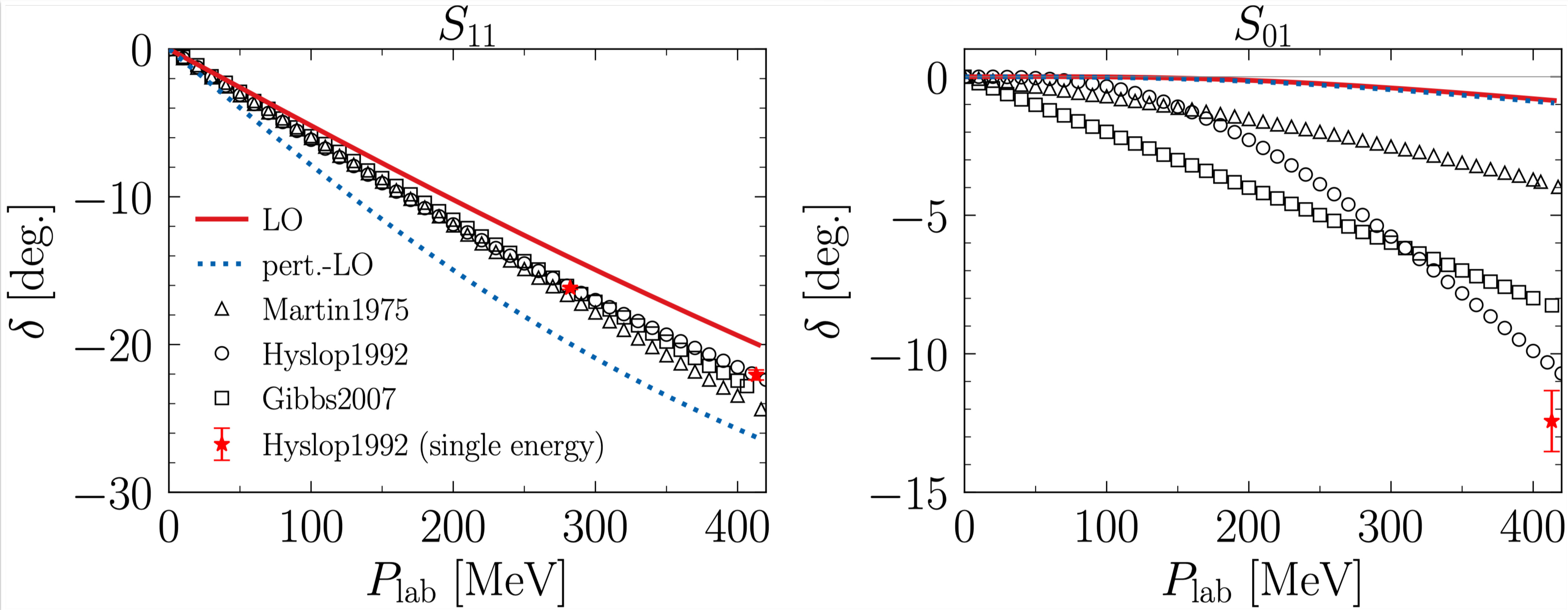
- Effective potential at NLO

$$V = V_{\rm LO} + V_{\rm NLO} = V^{(a+b+c)}\big|_{u=u_0\sim(1,0)^\dagger} + V^{(a+b+c)}\big|_{u=u_1\sim\mathcal{O}(p)} + V^{(d)}\big|_{u=u_0\sim(1,0)^\dagger}$$

$$u_N(p,s) = u_0 + u_1 + \cdots = \left[\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{2m_N}\begin{pmatrix}0\\ \boldsymbol{\sigma}\cdot\boldsymbol{p}\end{pmatrix} + \ldots\right]\chi_s$$

KN: Leading order results

- Prediction of s-wave phase shifts
  - $T_{\mathrm{LO}} = V_{\mathrm{LO}} + V_{\mathrm{LO}} G T_{\mathrm{LO}}$ vs. perturbative ($T_{\mathrm{LO}} = V_{\mathrm{LO}}$)

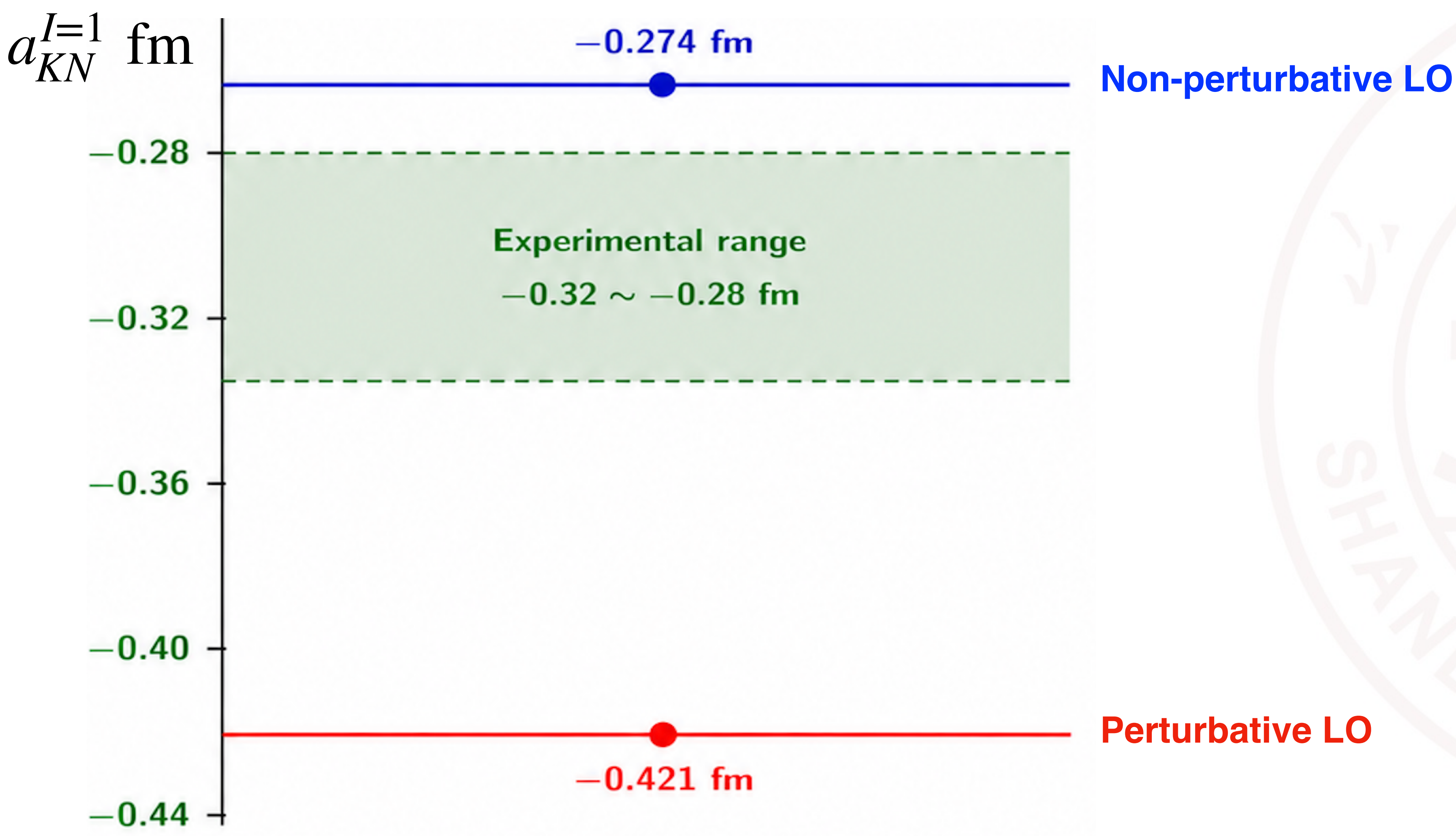


- **S11 channel**: non-pert. result better than the perturbative ones
- **S01 channel**: LO potential is very weak & PWAs are not consistent

$\bar{K}N$: Leading order results

- Effective range expansion

$$p_{\rm cm}\cot\delta(p_{\rm cm}) = \frac{1}{a} + \frac{1}{2} r(p_{\rm cm})\, p_{\rm cm}^2 + \mathcal{O}(p_{\rm cm}^4)$$



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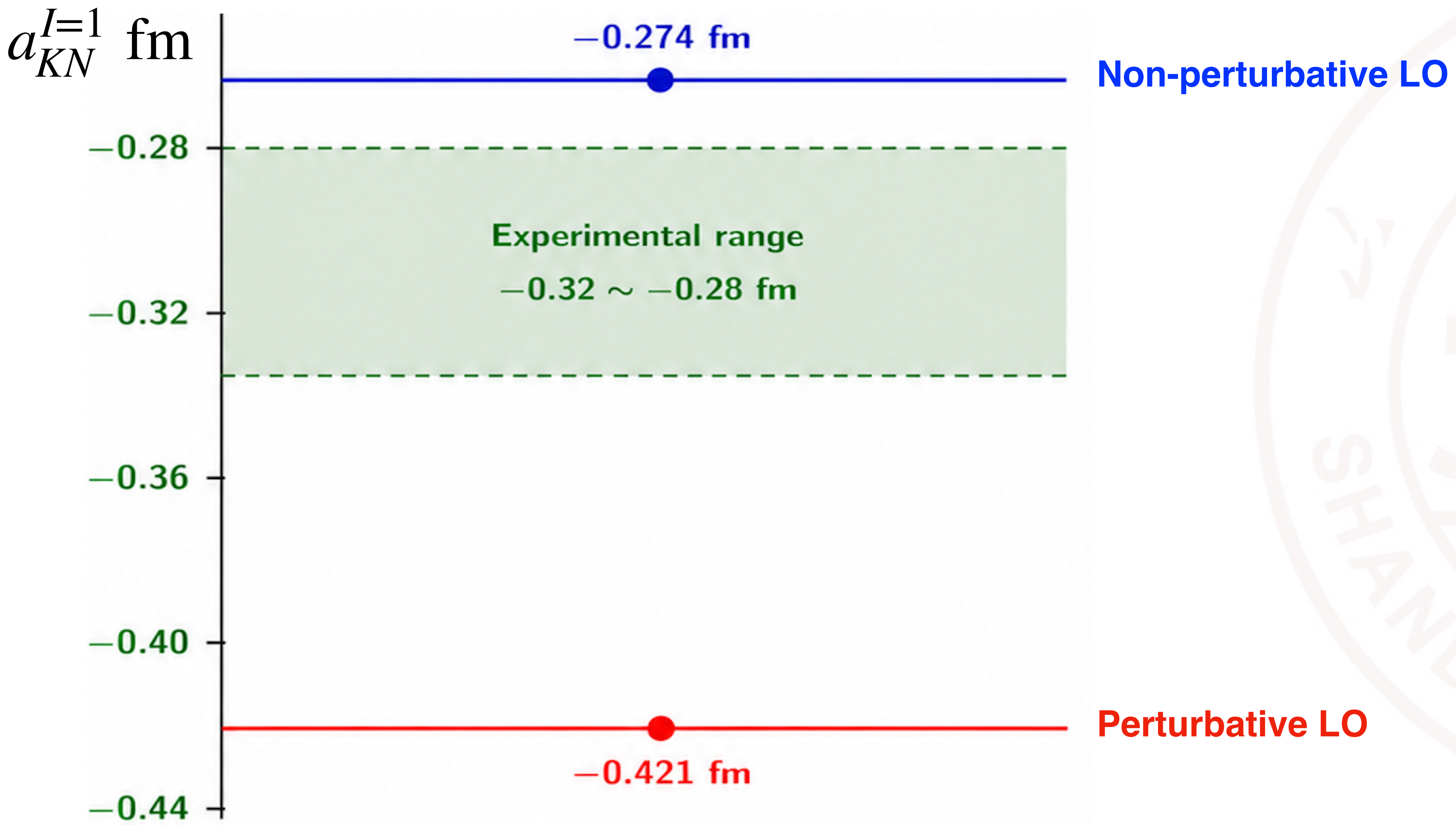


Non-perturbative treatment is essential, at least at lowest order, in the SU(3) sector of KN scattering

KN: Next-to-leading order results

- Four parameters at NLO

$$b^{I=0} = 4(b_0 - b_F), \quad d^{I=0} = 2(2d_1 + d_3 - 2d_4) \rightarrow 0.635(759)\ \text{GeV}^{-1}$$

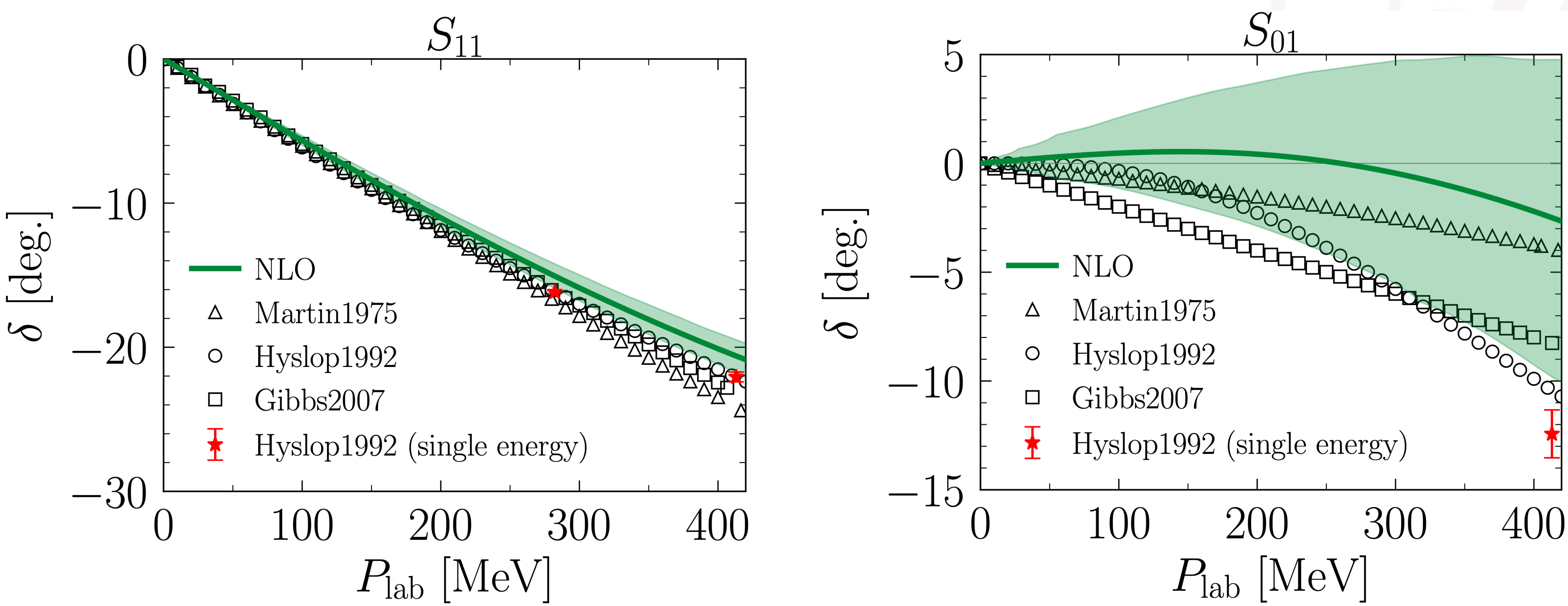
$$b^{I=1} = 4(b_0 + b_D), \quad d^{I=1} = 2(2d_2 + d_3 + 2d_4) \rightarrow -1.486(360)\ \text{GeV}^{-1}$$

Fixed by RQCD's $b_{0,D,F}$

Determined by S11 and S_{01} scattering lengths

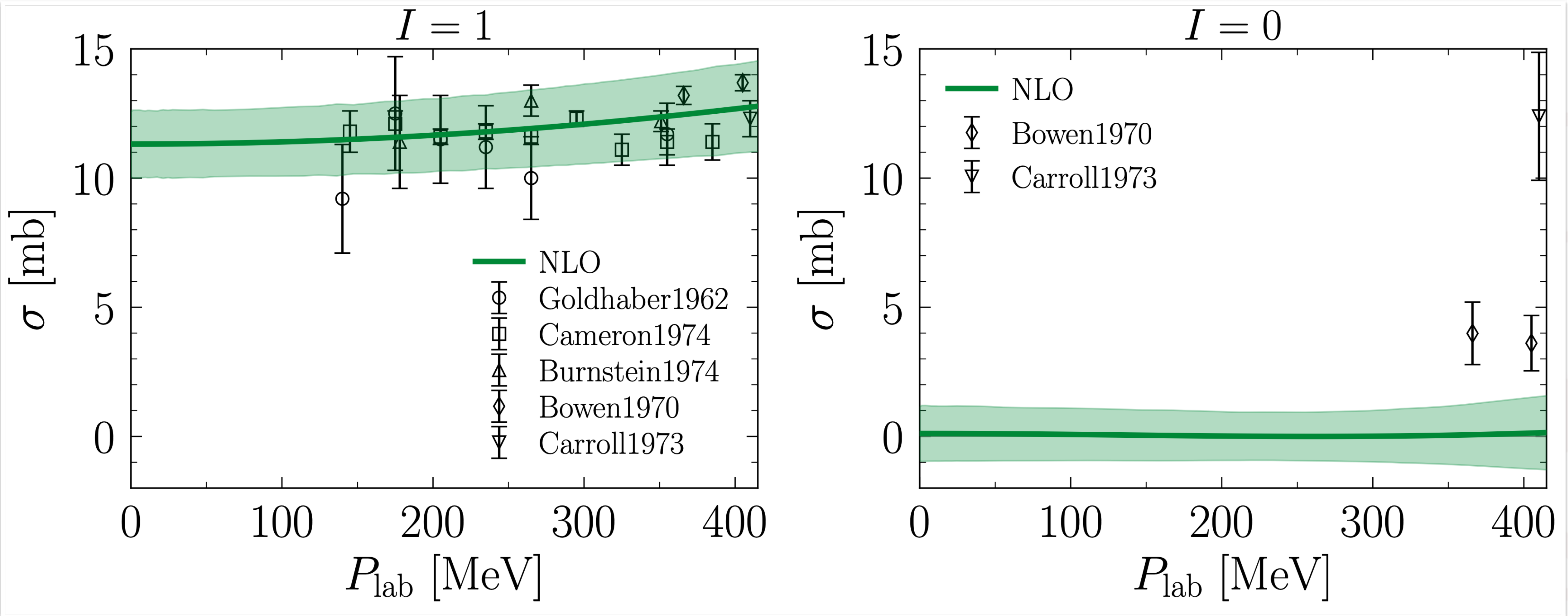
$a_{KN}^{I=0,\,\text{exp}} = 0.03 \pm 0.15\text{fm}$ and $a_{KN}^{I=1,\,\text{exp}} = -0.30 \pm 0.03\text{fm}$

- Prediction of s-wave phase shifts



$\bar{K}N$: Next-to-leading order results

- S-wave contributions to the total cross sections



- **<u>I=1 channel</u>**: NLO s-wave result **agrees well** with experimental data and slowly increases with Plab
- **<u>I=0 channel</u>**: NLO s-wave contribution is too small to reproduce the observed total cross section

  ✓ P-wave contribution dominant !

Summary

- **A renormalized framework** for MB scattering is proposed
  - Time-ordered perturbation theory + Covariant chiral Lagrangians
  - Take into account the off-shell effects of potential
  - Employ the subtractive renormalization
    - ✓ **Achieve T-matrix cutoff-independent**
- Leading order studies
  - πN scattering; $\bar{K}N$ scattering with coupled channels
  - Obtain the **two-pole structure** of $\Lambda(1405)$
  - **Predictive power:** consistent with LQCD results
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  - Higher order corrections are perturbatively included
  - $KN, \pi N$ scatterings: improve the description of phase shifts

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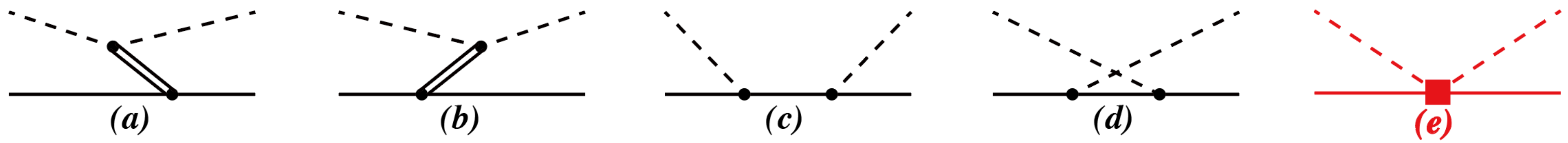
  - Working on $\bar{K}N$ scattering/$\Lambda(1405)$ at NLO

Thank you for your attention!

Back up

πN scattering at NLO



- Chiral effective Lagrangian

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi_+ \rangle - \frac{c_2}{4m^2} \langle u^\mu u^\nu \rangle \left(D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[u_\mu, u_\nu \right] \right\} \Psi_N$$

  - Fix $c_1 = -0.74,\ c_2 = 1.81,\ c_3 = -3.61,\ c_4 = 2.17\ \mathrm{GeV}^{-1}$

D. Siemens, et al., PLB770 (2017) 27-34

- NLO potential

$$V = V_{\rm LO} + V_{\rm NLO}$$
$$= V^{(a+b+c+d)}\big|_{u=u_0\sim(1,0)^\dagger} + V^{(a+b+c+d)}\big|_{u=u_1\sim\mathcal{O}(p)} + V^{(e)}\big|_{u=u_0\sim(1,0)^\dagger}$$

- Prediction for the πN phase shifts