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Insights on the structures of N^* and Δ resonances

2026 年轻强子专题研讨会

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Part I: Introduction

Hadron structures

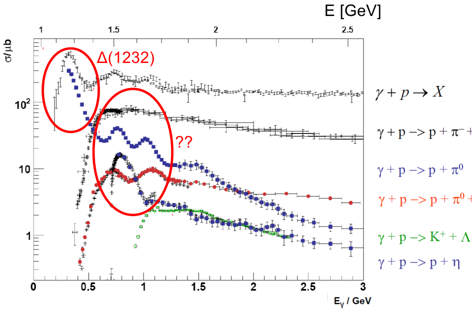
Part I: Introduction

- Structures of matters $\rightarrow$ fundamental task of physics
- Structures of hadrons $\rightarrow$ extremely difficult
  - Not direct observables
  - Non-perturbative nature of QCD at low & middle energies
  - Uncertainties of the hadron spectroscopy
- A bridge between the data and the structures?
  World data $\rightarrow$??? $\rightarrow$ Spectra $\rightarrow$ Indications of the structures
- One possible option: comprehensive data-driven models
  $\rightarrow$ the Jülich-Bonn/Jülich-Bonn-Washington Model for N^* and Δ
- Two studies:
  - Compositeness criteria
    [Y.F. Wang et. al., Phys. Rev. C 109, 015202 (2024)]
  - Baryon transition form factors from the proton
    [Y.F. Wang et. al. (Jülich-Bonn-Washington Collaboration), Phys. Rev. Lett. 133, 101901 (2024)]

Partial Wave Analyses

Part I: Introduction

- Medium energy $\longrightarrow$ involved dynamics & line-shapes
- Extraction of resonances
  $\longrightarrow$ partial wave analyses (PWA)
- PW decomposition $T \sim T^{JLS} \longrightarrow$ fit to the data
  $\longrightarrow$ meaningful outputs
- Various methods
  - Unitary isobar models
    $\longrightarrow$ unitary amplitudes + BW
    [MAID, Yerevan/JLab, KSU]
  - K-matrix Unitarization
    $\longrightarrow$ on-shell intermediate states
    [GWU/SAID, BnGa, Gießen]
  - Dynamical coupled-channel (DCC) approaches
    $\longrightarrow$ interaction potentials + scattering equations (off-shell intermediate states)
    [ANL-Osaka (EBAC), Dubna-Mainz-Taipeh]
    & Jülich-Bonn Model
  - ...



[spectra: PDG 2000. quark model calculations: Löring *et. al.*, EPJA 10, 395 (2001)]

The Jülich-Bonn Model

Part I: Introduction

A comprehensive coupled-channel model, fitting to a worldwide collection of data

Hadronic part ($\pi N \to \cdots$)

- Early origins $\to$ studies of $K^- N$ and $\pi\pi$
  [Müller-Groeling *et. al.*, NPA 513, 557 (1990)] [Lohse *et. al.*, NPA 516, 513 (1990)][Pearce *et. al.*, NPA 541, 663 (1992)]
- The πN elastic scatterings [Schütz *et. al.*, PRC 51, 1374 (1995)] [Schütz *et. al.*, PRC 49, 2671 (1994)]
- Extended to $\pi\pi N$ and ηN [Schütz *et. al.*, PRC 57, 1464 (1998)] [Krehl *et. al.*, PRC 62, 025207 (2000)] [Gasparyan *et. al.*, PRC 68, 045207 (2003)]
- Extended to $K\Lambda$ and $K\Sigma$ [Döring *et. al.*, NPA 851, 58 (2011)] [Rönchen *et. al.*, EPJA 49, 44 (2013)]
- Extended to ωN [Wang *et. al.*, PRD 106, 094031 (2022)]
- Analytical continuation for searching poles [Döring *et. al.*, NPA 829, 170 (2009)]

Photo- & Electroproduction

- Photoproduction
  [Rönchen *et. al.*, EPJA 50, 101 (2014)] [Rönchen *et. al.*, EPJA 51, 70 (2015)][Rönchen *et. al.*, EPJA 54, 110 (2018)] [Rönchen *et. al.*, EPJA 558, 229 (2022)]
- Electroproduction (Jülich-Bonn-Washington Model)
  [Mai *et. al.*, PRC 103, 065204 (2021)] [Mai *et. al.*, PRC 106, 015201 (2022)] [Mai *et. al.*, EPJA 59, 286 (2023)]

Theoretical framework

Part I: Introduction

Hadron dynamics

Lippmann-Schwinger-like equation

$$T_{\mu\nu}(p'', p', z) = V_{\mu\nu}(p'', p', z) + \sum_\kappa \int_0^\infty p^2 dp V_{\mu\kappa}(p'', p, z) G_\kappa(p, z) T_{\kappa\nu}(p, p', z)$$

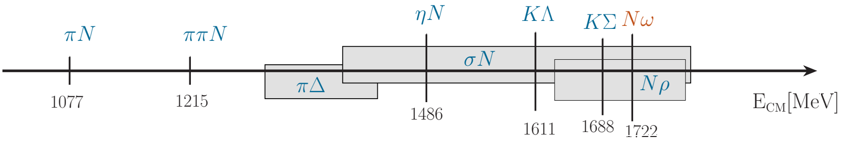
- One-dimensional: time-ordered perturbation theory $+$ JLS basis [Jacob & Wick, Annals Phys. 7, 404 (1959)]
- Effective Lagrangians $\rightarrow$ SU(3), chiral, CP...
- Effective three-body channels: $\rho N, \sigma N, \pi\Delta$

Photo- & electroproduction

Construction from final state interactions

$$M_{\mu\gamma^*}(Q^2) = V_{\mu\gamma^*}(Q^2) + \sum_\kappa \int p^2 dp T_{\mu\kappa} G_\kappa V_{\kappa\gamma^*}(Q^2)$$

- γ^*: the $\gamma^* N$ channel for electroproduction
- Q^2: photon virtuality
- $V_{\kappa\gamma^*} \rightarrow$ phenomenologically parameterized
- Constraints: Siegert's theorem, kinematics, etc.
- Photoproduction $\rightarrow Q^2 = 0$



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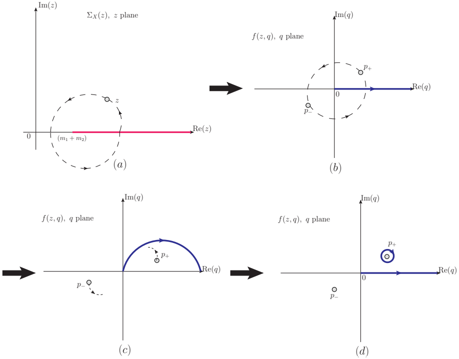
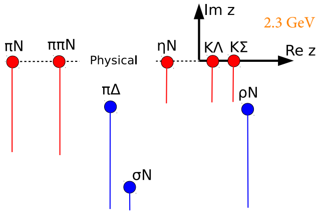
Pole searching

Part I: Introduction

- Resonances $\to$ poles on the second Riemann sheet
- Analytical continuation $\to$ contour deformation
- Pole position $z_r = M_r - i\Gamma_r/2$. Coupling strengths $\to$ normalized residues [PDG, RPP]

$$\tau^{II}_{\mu\nu} \sim \frac{R_\mu R_\nu}{z_r - z} + \cdots, NR_\mu \equiv \frac{2R_{\pi N}}{\Gamma_r} \times R_\mu$$





$p_\pm$: singularities in $G = (z - E_1 - E_2)^{-1}$

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Part II: Compositeness Criteria

Weinberg's Criterion and Beyond

Part II: Compositeness Criteria

Weinberg's criterion

- Hadrons $\to$
  quarks & gluons v.s. hadron exchanges?
- Weinberg's criterion (deuteron)
  [Weinberg, PR 137, B672 (1965)]

$$a = -\frac{2(1-Z)}{2-Z}R + \mathcal{O}(L)$$
$$r = -\frac{Z}{1-Z}R + \mathcal{O}(L)$$

Z: "elementariness". $R \sim 4.3$ fm: deuteron radius.
$L \sim m_\pi^{-1}$ interaction range.
a: scattering length. r: effective range.

- Experimental results $\to Z \simeq 0$
- Conditions: S-wave, near-threshold, stable
- Model-independent [van Kolck, Symmetry 14, 1884 (2022)]

Modern generalization

- Modern hadron resonances $\to$ unstable, coupled-channel
- Interpretations $\to$ genuine states v.s. hadronic molecules [Guo *et al.*, RMP 90, 015004 (2018)], etc.
- Criteria $\to$ qualitative or model-dependent
  - spectral density functions (SDFs)
    [Baru *et al.*, PLB 586, 53 (2004)]
  - complex compositeness (CC)
    [Hyodo, PRL 111, 132002 (2013)]
    [Guo and Oller, PRD 93, 096001 (2016)]
    [Sekihara, PRC 95, 025206 (2017)]
  - ...
- Jülich-Bonn model $\to$ data-driven SDFs & CCs!

Criteria

Part II: Compositeness Criteria

Spectral Density Functions

- Weinberg's Z for bound states: $Z = |\langle\psi_0|\Psi_B\rangle|^2$
- Lower decay channels: $Z \to w(E) = -\mathrm{Im}D(E)/\pi$ $\to$ distribution on energy E
- Källén-Lehmann spectral of the propagator D
- Ideal quasi-bound state $E = E_R - i\Gamma_R/2$: $w(E \simeq E_R) = \frac{Z_B}{\pi}\frac{\Gamma_R/2}{(E-E_R)^2+(\Gamma_R/2)^2}$
- $\lim_{\Gamma_R\to 0} w(E) = Z_B\delta(E-E_R)$: compatible with Weinberg's criterion
- Finite width estimation:

$$Z \simeq \frac{\int_{E_R-\Gamma_R}^{E_R+\Gamma_R} w(E)dE}{\int_{E_R-\Gamma_R}^{E_R+\Gamma_R} BW(E)dE}, BW(E) \equiv \frac{1}{\pi}\frac{\Gamma_R/2}{(E-E_R)^2+(\Gamma_R/2)^2}$$

Complex Compositeness

- Resonances
  - Unphysical Gamow states
    [Gamow, Zeitschrift für Physik 51, 204 (1928)]
    [Civitarese and Gadella, Physics Reports 396, 41(2004)]
  - Complex eigenstates of the Hamiltonian $\hat{H}|\Psi_R) = E_R|\Psi_R)$
  - Non-normalizable
- Complex elementariness: $Z_R = (\Psi_R^*|\psi_0\rangle\langle\psi_0|\Psi_R)$, complex-valued
- Complex compositeness $\to$ off-shell residues $r(k)$

$$T(E, p_i, p_f) \sim \frac{r(p_i)r(p_f)}{E-E_R}$$

$$1 - Z_R = \int_C \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{r^2(k)}{[\mathcal{E}_R-k^2/(2\mu)]^2}$$

[Sekihara, PRC 95, 025206 (2017)]

Applications on the Jülich-Bonn Model

Part II: Compositeness Criteria

Study of the structures

- Jülich-Bonn model
  - coupled-channel
  - more s-channel bare states
  - hadron-exchange potentials
- Spectral density functions → readily extracted from propagators $w_i(z) = -\frac{1}{\pi}\mathrm{Im}D_i(z)$
- "Total elementariness" → $Z = 1 - \prod_i (1 - Z_i)$
- Partial compositeness (Gamow states) → off-shell residues $X_\kappa = \int_C p^2 dp\, r_\kappa^2(p) G_\kappa^2(p, z_{\text{pole}})$
- Model solution: JüBo2022 ($K\Sigma$ photoproduction, 72,000 data points)

[Rönchen *et. al.*, EPJA 58, 229 (2022)]

Study of the structures

- Selection of the resonances
  - For N^* $J \leq 5/2$, for Δ $J \leq 3/2$
  - Width $\Gamma_R < 300$ MeV
- Uncertainties → comparison of three results
  - SDFs directly from the model
  - CCs of the Gamow state → naive measure: $\tilde{X}_\kappa \equiv \frac{|X_\kappa|}{\sum_\alpha |X_\alpha| + |Z|}$, $\tilde{Z} \equiv \frac{|Z|}{\sum_\alpha |X_\alpha| + |Z|}$
  - Locally constructed SDFs only from pole parameters (L_κ: loop functions)

$$w^{\text{lc}}(z) = -\frac{1}{\pi}\mathrm{Im}\left[z - M_0 - \sum_\kappa g_\kappa^2 L_\kappa(z)\right]^{-1}$$

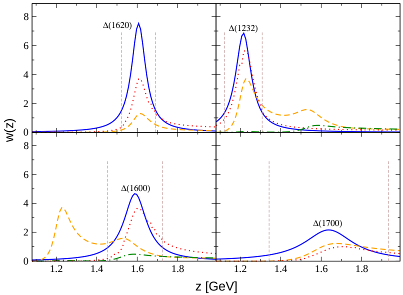
- Failure of the local construction → big uncertainty

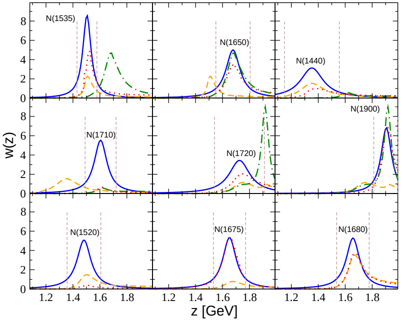
Results

Part II: Compositeness Criteria

- Blue solid line: the Breit-Wigner denominator.
- Orange dashed (green dash-dotted) line: the 1st (2nd) spectral density function (model).
- Red dotted line: the locally constructed function.

- Vertical lines: integral regions.





Results

Part II: Compositeness Criteria

State	Pole position (MeV)	$\mathcal{Z}_{\text{tot}}$	$\mathcal{Z}^{\text{lc}}$	$\tilde{Z}$
$N(1535)\,\frac{1}{2}^-$	$1504-37i$	29.0%	50.8%	39.4%
$N(1650)\,\frac{1}{2}^-$	$1678-64i$	92.8%	70.5%	8.5%
$N(1440)\,\frac{1}{2}^+$	$1353-102i$	49.5%	31.5%	36.9%
$N(1710)\,\frac{1}{2}^+$	$1605-58i$	20.6%	10.2%	40.3%
$N(1720)\,\frac{3}{2}^+$	$1726-93i$	79.3%	62.5%	41.4%
$N(1900)\,\frac{3}{2}^+$	$1905-47i$	100%	99.9%	38.5%
$N(1520)\,\frac{3}{2}^-$	$1482-63i$	29.4%	7.2%	40.4%
$N(1675)\,\frac{5}{2}^-$	$1652-60i$	16.6%	(F)	61.8%
$N(1680)\,\frac{5}{2}^+$	$1657-60i$	67.9%	69.9%	55.0%
$\Delta(1620)\,\frac{1}{2}^-$	$1607-42i$	18.9%	50.0%	69.4%
$\Delta(1232)\,\frac{3}{2}^+$	$1215-46i$	53.8%	(F)	30.5%
$\Delta(1600)\,\frac{3}{2}^+$	$1590-68i$	47.8%	77.5%	69.7%
$\Delta(1700)\,\frac{3}{2}^-$	$1637-148i$	59.7%	44.9%	47.8%

Discussions

Part II: Compositeness Criteria

Results

- At least two results suggest **high compositeness**: $N(1535)\frac{1}{2}^-$, $N(1440)\frac{1}{2}^+$, $N(1710)\frac{1}{2}^+$, $N(1520)\frac{3}{2}^-$
- At least two results suggest **high elementariness**: $N(1650)\frac{1}{2}^-$, $N(1900)\frac{3}{2}^+$, $N(1680)\frac{5}{2}^+$, $\Delta(1600)\frac{3}{2}^+$
- Hints of compositions (Gamow states)
  - $N^*(1535)$: $\tilde{X}_{\eta N} = 35.8\%$
  - $N^*(1440)$: $\tilde{X}_{\pi N} = 59.0\%$
  - $N^*(1710)$: $\tilde{X}_{\eta N} = 44.9\%$
  - $N^*(1520)$: $\tilde{X}_{\pi\pi N} = 43.7\%$
- The compositions may be model-dependent:

  in this model σN bare couplings are switched off

On the states

- $N(1535)$
  - overlap with $N^*(1650)$
  - might be dynamically generated

    [Kaiser *et. al.*, PLB 362, 23 (1995)]
  - ωN might be important

    [Wang *et. al.*, PRD 106, 094031 (2022)]
- $N(1440)$
  - always dynamically generated here

    [Krehl *et. al.*, PRC 62, 025207 (2000)]
  - pure "radial excitation core" not favoured

    [Meißner and Durso, NPA 430, 670 (1984)]
  - σN component [Sekihara, PRC 104, 035202 (2021)]
- Three $\Delta(1232)$'s (s-channel, initial/final, u-channel)

  $\rightarrow$ technical difficulties in this study

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Part III: Baryon Transition Form Factors

Baryon Transition Form Factors

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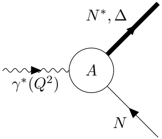
Electromagnetic Probes

- EM interactions
  $\longrightarrow$ clean probes of structures
- Electroproduction
  $\longrightarrow$ additional energy scale Q^2 (photon virtuality)
- Transition form factors (TFFs) $\longrightarrow$ "pictures of hadrons"

  [Ramalho, Peña, Prog. Part. Nucl. Phys. 136, 104097 (2024)]

  — Lower Q^2: meson clouds etc.
  — Higher Q^2: the quark core
  — Related to transverse charge densities

  [Tiator & Vanderhaeghen, PLB 672, 344 (2009)]



Towards the TFFs

- Predictions at quark level
  — Quark models & Schwinger-Dyson equations

  [Segovia *et. al.*, Few Body Syst. 55, 1185 (2014)]

  [Burkert, Roberts RMP 91, 011003 (2019)]

  [Eichmann *et. al.*, Prog. Part. Nucl. Phys. 91, 1 (2016)]

  — Lattice QCD [Agadjanov *et. al.*, NPB 886, 1199 (2014)]
- Extraction from data
  — Experimental facilities @ JLAB
  — Data accumulation

  [Mokeev *et. al.*., PRC 93, 025206 (2016)]

  — Unitary isobar models $\longrightarrow$ real-valued, depending on BW parameters

  [Drechsel, Kamalov, Tiator, EPJA 34, 69 (2007)]

  [Tiator *et. al.*, EPJST 198, 141 (2011)]

  — DCC $\longrightarrow$ complex TFFs at the poles

  [Kamano, Few Body Syst. 59, 24 (2018)]

TFFs in Jülich-Bonn-Washington Model

Part III: Baryon Transition Form Factors

The latest JBW results

- $\gamma^* p$ initial state
- Coupled-channel study of πN, ηN, and $K\Lambda$
  [Mai *et. al.*, EPJA 59, 286 (2023)]
- Based on the JüBo2017 solution
  [Rönchen *et. al.*, EPJA 54, 110 (2018)]
- C.M. energy range $z \in [1.13, 1.8]$ GeV
- Virtuality $Q^2 \in [0, 8]$ GeV^2
- Orbital angular momentum $L \leq 3$
- Database
  - 10^5 data points for the electroproduction
  - 5×10^4 data points from the photoproduction/hadronic part
- Four fit solutions → estimation of uncertainties

Highlights of this study

- TFFs defined at the poles (A, S)

$$H_h = C_I \sqrt{\frac{p_{\pi N}}{\omega_\gamma} \frac{2\pi(2J+1)z_p}{m_N \widetilde{R}_{\pi N}}} \widetilde{\mathcal{R}}_h$$

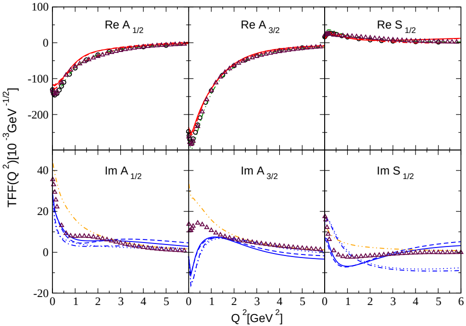
- First simultaneous multi-channel study of TFFs → channel-independent results
- Outputs → TFFs of twelve states at the poles
- Results for some higher states are given for the first time
- Exploring the parameter space
  → realistic uncertainties

A comprehensive data-driven investigation!

Results of $\Delta(1232)$

Part III: Baryon Transition Form Factors

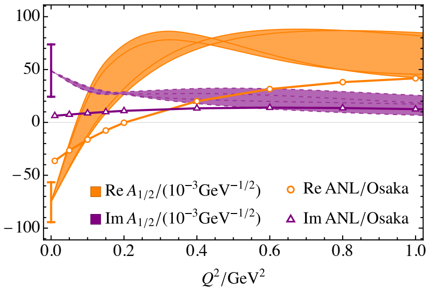
- Solid, dashed, dotted, dash-dotted curves: four fit solutions
- Dash-double-dotted curves: "L+P" extraction from MAID analyses [Workman, Tiator, Sarantsev, PRC 87, 068201 (2013)]
- Triangles: ANL-Osaka[Kamano, Few Body Syst. 59, 24 (2018)]

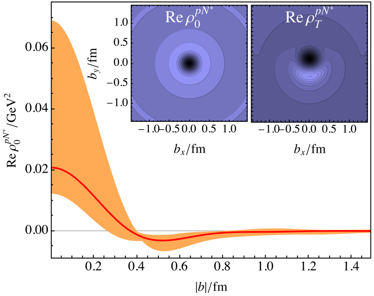


Results of $N^*(1440)$

Part III: Baryon Transition Form Factors

- $\mathrm{Re}A_{1/2}$ TFF $\to$ a zero crossing at small Q^2
- ρ_0, ρ_T: unpolarized, polarized transverse charge densities (estimated by taking the real part)
  [Tiator & Vanderhaeghen, PLB 672, 344 (2009)]
- Red solid line: result of MAID2007[Drechsel, Kamalov, Tiator, EPJA 34, 69 (2007)]

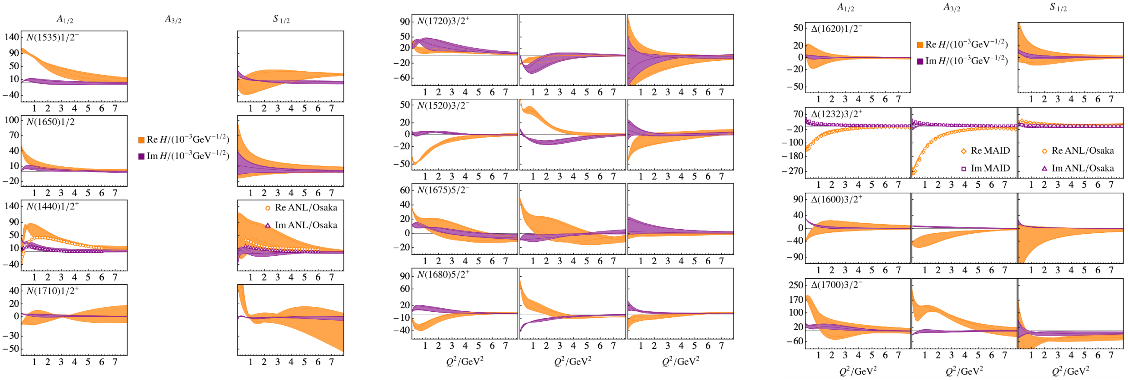




Summary of the results

Part III: Baryon Transition Form Factors

Data files on `https://jbw.phys.gwu.edu`



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Part IV: Conclusion & Outlook

Conclusion & Outlook

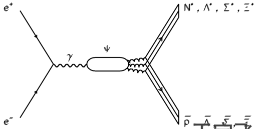
Part IV: Conclusion & Outlook

Conclusions

- The Jülich-Bonn Model
  - Comprehensive dynamical coupled-channel approaches
  - Data driven PWA $\rightarrow$ resonance spectra
  - Connecting experimental observations to hadron structures!
- Study 1: compositeness criterion
  - Generalization of Weinberg's criterion
  - Four states $\rightarrow$ larger hadronic components
  - Four states $\rightarrow$ larger quark/gluon components
- Study 2: transition form factors
  - Determined by multi-channel data
  - Defined at the poles
  - Realistic uncertainties
  - Outputs for twelve states

Outlook

- Compositeness of P_c states $\rightarrow$ already published
  [Y.F. Wang et. al., Phys. Rev. D 112, 074010 (2025)]
- The hyperon spectroscopy in the near future
- JBW model $\rightarrow$ energy extension to 1.95 GeV
- ωN photoproduction underway $\rightarrow$ more modern data!
- Broader applications
  - BES: $J/\psi(\psi') \rightarrow \bar{N}N^*$ [BES, PRL 97, 062001 (2006)] [BES, PRL 110, 022001 (2013)] [BESIII, CPC 44, 040001 (2020)]
  - HHaS @ HIAF: π beams for πN interaction
    [Chen *et al.*, arXiv: 2511.22864 (2025)]



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Thank you

Backups

Details of the scattering equation

Backups

The Lippmann-Schwinger-like equation

$$T_{\mu\nu}(p'',p',z) = V_{\mu\nu}(p'',p',z) + \sum_\kappa \int_0^\infty p^2 dp V_{\mu\kappa}(p'',p,z) G_\kappa(p,z) T_{\kappa\nu}(p,p',z)$$

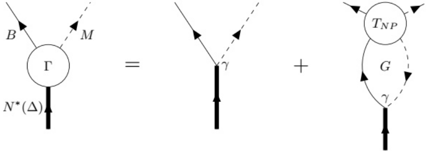
- Reaction channels $\nu \to \mu$ (after PW and isospin projection, JLS basis [Jacob & Wick, Annals Phys. 7, 404 (1959)],$J \leq 9/2$)
- Intermediate channel: κ
- CM initial (final) momentum: $p'(p'')$. CM energy: z
- Potential (kernel): V. Amplitude: T ($\to$ observables)
- Propagator: G ($\pi\pi N$ channel: effective channels $\rho N, \sigma N, \pi\Delta$. E/ω - energy of the baryon/meson.)

$$G_\kappa(z,p) = \begin{cases} (z - E_\kappa - \omega_\kappa + i0^+)^{-1} & \text{(if } \kappa \text{ is a two-body channel)}\ , \\ \left[z - E_\kappa - \omega_\kappa - \Sigma_\kappa(z,p) + i0^+\right]^{-1} & \text{(if } \kappa \text{ is an effective channel)}\ . \end{cases}$$

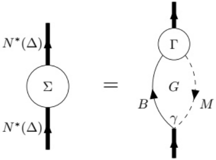
Details of the scattering equation

Backups

- Separating the amplitude $\rightarrow$ with/without s-channel poles $T = T^P + T^{NP}$
- Reconstruction of the amplitude $\rightarrow$ $T^{NP} = V^{NP} + \sum \int p^2 dp V^{NP} G T^{NP}$,
  $T^P_{\mu\nu}(p'', p', z) = \sum_{i,j} \Gamma^a_{\mu,i}(p'') D_{ij}(z) \Gamma^c_{\nu,j}(p')$, $(D^{-1})_{ij} = \delta_{ij}(z - m_i^b) - \Sigma_{ij}(z)$
  - $\Gamma(\gamma)$: the dressed (bare) vertices (a - annihilation, c - creation)
  - Σ: coupled-channel self-energy functions of the s-channel states



(a) The vertex.



(b) The self energy.

Details of the scattering equation

Backups

Potentials $\rightarrow$ field-theoretical construction

Parameters $\rightarrow$ determined by fits

The NP part

- Tree-level potentials
  - t-channel + u-channel + contact
  - Stemming from effective Lagrangians $\rightarrow$ SU(3) flavour symmetry, CP conservation, chiral symmetry
  - Established by time-ordered perturbation theory (TOPT) $\rightarrow$ stationary perturbation in Schrödinger picture
  - TOPT+partial wave $\rightarrow$ one-dimensional integral $\int p^2 dp$
  - Regulators for every vertex $\rightarrow$ to make the integral converge: $F(q) \sim \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + q^2}\right)^n$

    m: the mass of the exchanged particle. Λ: cut-off (fit parameter)
- Beyond tree-level $\rightarrow$ correlated two-pion exchanges [Schütz *et. al.*, PRC 49, 2671 (1994)] [Schütz *et. al.*, PRC 51, 1374 (1995)]

The P part

- Stemming from effective Lagrangians with CP conservation (tree-level bare vertices)
- Phenomenological contact terms $\rightarrow D \sim (1 - \Sigma)^{-1}$ [Rönchen *et. al.*, EPJA 51, 70 (2015)]
- Renormalization of the nucleon mass

Local construction of SDFs

Backups

- Local simulation of the amplitude:
  $T^{\mathrm{lc}}_{\alpha\beta}(z) = \frac{c g_\alpha g_\beta f^a_\alpha(q_{\alpha z}) f^c_\beta(q_{\beta z})}{z - M_0 - \sum_\kappa g^2_\kappa L_\kappa(z)} + \cdots$
- Loop functions (f: vertex function in this model):
  $L_\kappa(z) \equiv \int_0^\infty p^2 dp\, G_\kappa(p,z) f^a_\alpha(q_{\kappa z}) f^c_\alpha(q_{\kappa z})$
- Parameters

$$h_\kappa \equiv \frac{g^2_\kappa}{g^2_1} = \left|\frac{r_\kappa f^a_1}{r_1 f^a_\kappa}\right|^2, g^2_1 = -\frac{\Gamma_R}{2\sum_\kappa h_\kappa \mathrm{Im}(L^{II}_\kappa)}$$

$$M_0 = M_R - g^2_1 \sum_\kappa h_\kappa \mathrm{Re}(L^{II}_\kappa), c = \frac{r^2_1}{g^2_1 f^a_1 f^c_1}\left(1 - g^2_1 \sum_\kappa h_\kappa \frac{d}{dz} L^{II}_\kappa \Big|_{z=M_R - i\Gamma_R/2}\right)$$

- Estimation:
  $w^{\mathrm{lc}}(z) = -\frac{1}{\pi}\mathrm{Im}\left[z - M_0 - \sum_\kappa g^2_\kappa L_\kappa(z)\right]^{-1}$
- The failure of the local construction $\rightarrow g^2_1 < 0$
- Plan B: taking g_κ's as the absolute values of normalized residues

  $\rightarrow$ constant width in M_0

Formulae & Extraction

Backups

Origianl definition

[Ramalho, Peña, Prog. Part. Nucl. Phys. 136, 104097 (2024)]

$$A_h = \sqrt{\frac{2\pi\alpha}{K}} \left\langle R, h \middle| \epsilon_+ \cdot J \middle| N, h-1 \right\rangle$$

$$S_{\frac{1}{2}} = \frac{|\mathbf{q}|}{Q} \sqrt{\frac{2\pi\alpha}{K}} \left\langle R, \tfrac{1}{2} \middle| \epsilon_0 \cdot J \middle| N, \tfrac{1}{2} \right\rangle$$

- A, S: helicity transition amplitudes
- $h = 1/2, 3/2$: the helicity
- α: fine structure constant
- $\epsilon(J)$: virtual photon polarization vector (current)
- $\mathbf{q}$: 3-momentum of the virtual photon
- $M_R(m_N)$: mass of the excitation state R (nucleon)
- $K = (M_R^2 - m_N^2)/(2M_R)$

At the pole

[Workman, Tiator, Sarantsev, PRC 87, 068201 (2013)]

$$H_h = C_I \sqrt{\frac{p_{\pi N}}{\omega_0} \frac{2\pi(2J+1)z_p}{m_N \widetilde{R}}} \widetilde{\mathcal{H}}_h$$

- H is either A or S
- C_I: isospin factor, $C_{1/2} = -\sqrt{3}$ and $C_{3/2} = \sqrt{2/3}$
- $p_{\pi N}$: πN c.m. momentum
- ω_0: photon energy at $Q^2 = 0$
- $z_p = M_R - i\Gamma_R/2$ the pole position
- $\widetilde{R}$, $\widetilde{\mathcal{H}}$: the residues of πN, $\gamma^* N$ channels
- Understanding: the $|R\rangle \rightarrow |R)$ defined from the Gamow state
- Complex-valued

Transverse charge distributions: definition

Backups

[Tiator & Vanderhaeghen, PLB 672, 344 (2009)]

- The light front frame:
  - Large momentum along $P = (p_{N^*} + p_N)/2$ (as z-axis)
  - Light front component $v^{\pm} \equiv v^0 \pm v^3$
  - Symmetric frame $q^+_{\gamma^*} = 0$, the transverse component on xOy plane $\mathbf{q}_\perp^2 = Q^2$
- The transverse charge density for the transition:

$$\rho(\mathbf{b}) \equiv \int \frac{d^2\mathbf{b}}{(2\pi)^2} \frac{1}{2P^+} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}} \left\langle P^+, \frac{\mathbf{q}_\perp}{2}, \lambda_{N^*} \middle| J^+(0) \middle| P^+, -\frac{\mathbf{q}_\perp}{2}, \lambda_N \right\rangle$$

  - λ: helicity
  - J^+: quark charge current, "$+$" component
  - $\mathbf{b}$: 2D position on xOy plane
- The quark charge distribution that is responsible for the $N \to N^*$ transition
- Two independent densities
  - ρ_0: unpolarized $\to$ only depends on $|\mathbf{b}|$
  - ρ_T: polarized along x-axis, $|\lambda\rangle = \frac{1}{\sqrt{2}} \left(|+\tfrac{1}{2}\rangle + |-\tfrac{1}{2}\rangle\right)$

Transverse charge distributions: calculation

Backups

[Tiatior & Vanderhaeghen, PLB 672, 344 (2009)]

- Helicity TFFs in terms of Pauli-Dirac TFFs

$$A_{1/2} = \frac{eQ_-}{\sqrt{4Km_N M_R}}(F_1 + F_2)$$

$$S_{1/2} = \frac{eQ_-}{\sqrt{8Km_N M_R}} \frac{Q_+ Q_-}{2M_R} \frac{M_R + m_N}{Q^2} \left[F_1 - \frac{Q^2}{(m_N + M_R)^2} F_2\right]$$

  with $Q_\pm = \sqrt{(M_R \pm m_N)^2 + Q^2}$
- Unpolarized (J_n: cylindrical Bessel function)

$$\rho_0(\mathbf{b}) = \int_0^{+\infty} \frac{dQ}{2\pi} Q\, J_0(|\mathbf{b}|Q)\, F_1(Q^2)$$

- Polarized ($\sin\phi = b_y/|\mathbf{b}|$)

$$\rho_T(\mathbf{b}) = \rho_0(\mathbf{b}) + \sin\phi \int_0^{+\infty} \frac{dQ}{2\pi} \frac{Q^2}{m_N + M_R} J_1(|\mathbf{b}|Q)\, F_2(Q^2)$$