

Recent theoretical progress on $\phi(2170)/Y(2175)$

报告人：王俊璋

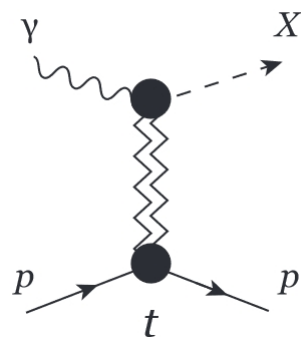
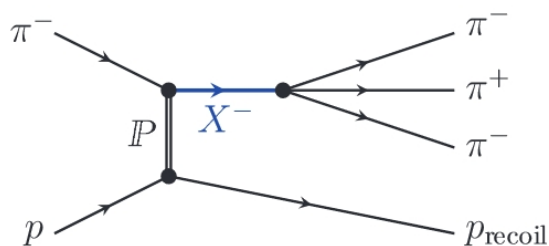
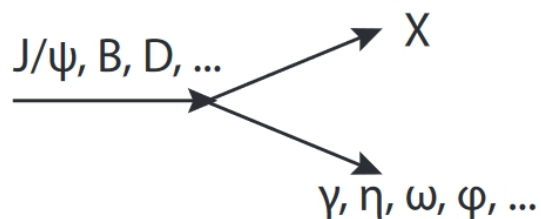
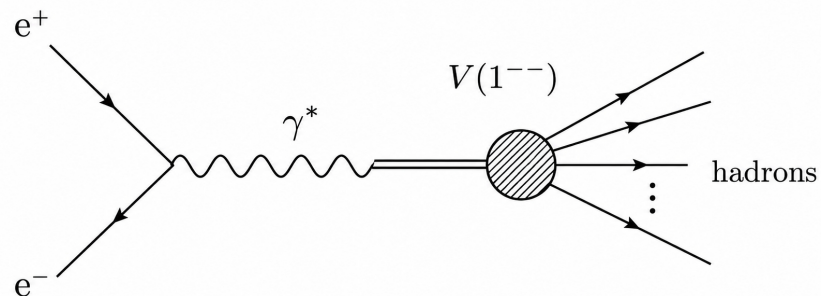
重庆大学



2026.05.15 商丘师范学院@河南商丘

Why vector light mesons?

BESIII, Belle II, COMPASS, GlueX experiment



The vector channel is experimentally special

- Directly accessed in e^+e^- annihilation through γ^* .
- ISR data provide a continuous scan of low-energy hadron spectra.
- Light vector states can also be generated by multiple production modes.

More complicated confinement dynamics

- The confinement regime of the light quark-antiquark interaction is more complicated.
- The relativistic effect is important.

The ϕ -meson family

ρ mesons
 $I^G(J^{PC}) = 1^+(1^{--})$

State	Mass	Width
$\rho(770)^0$	775.26 ± 0.23	147.4 ± 0.8
$\rho(770)^\pm$	775.11 ± 0.34	149.1 ± 0.8
$\rho(1450)^*$	1465 ± 25	400 ± 60
$\rho(1570)^\dagger$	$1570 \pm 36 \pm 62$	$144 \pm 75 \pm 43$
$\rho(1700)^*$	1720 ± 20	250 ± 100
$\rho(1900)^\dagger$	1860 – 1910	10 – 160
$\rho(2150)^\dagger$	2034 – 2255	70 – 460

MeV

ω mesons
 $I^G(J^{PC}) = 0^-(1^{--})$

State	Mass	Width
$\omega(782)$	782.66 ± 0.13	8.68 ± 0.13
$\omega(1420)^*$	1410 ± 60	290 ± 190
$\omega(1650)^*$	1670 ± 30	315 ± 35
$\omega(2220)^\dagger$	2188 ± 21	105 ± 34

MeV

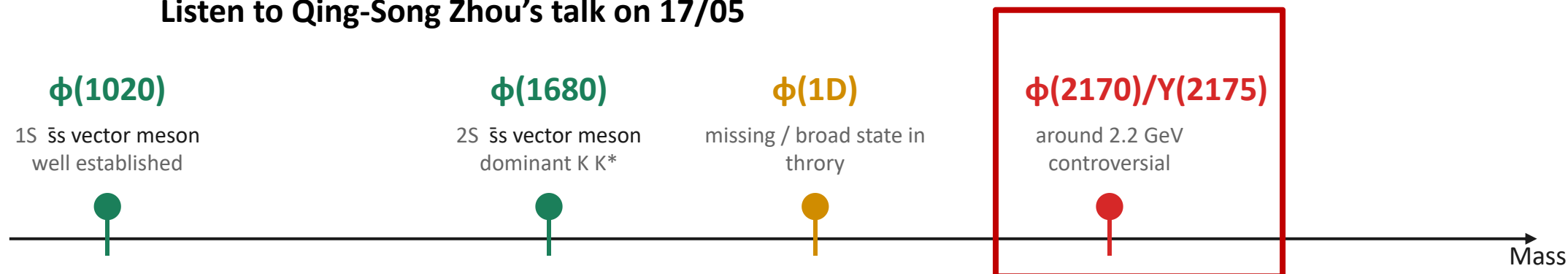
ϕ mesons
 $I^G(J^{PC}) = 0^-(1^{--})$

State	Mass	Width
$\phi(1020)$	1019.460 ± 0.0164	4.249 ± 0.013
$\phi(1680)^*$	1680 ± 20	150 ± 50
$\phi(2170)$	2164 ± 5	88^{+26}_{-21}

MeV

* PDG estimate; \dagger PDG listing omitted from summary table

Listen to Qing-Song Zhou's talk on 17/05



Y(2175) vs Y(4230)

A Structure at 2175-MeV in $e^+e^- \rightarrow \phi f_0(980)$ Observed via Initial-State Radiation #1

Radiation

BaBar Collaboration • Bernard Aubert (Barcelona U., ECM) et al. (Oct, 2006)

Published in: *Phys.Rev.D* 74 (2006) 091103 • e-Print: [hep-ex/0610018](https://arxiv.org/abs/hep-ex/0610018) [hep-ex]

Counted up to 13. 05.2026

pdf links DOI cite claim reference search **234 citations**

Observation of a broad structure in the $\pi^+\pi^- J/\psi$ mass spectrum around 4.26-GeV/c² #16

BaBar Collaboration • Bernard Aubert (Annecy, LAPP) et al. (Jun, 2005)

Published in: *Phys.Rev.Lett.* 95 (2005) 142001 • e-Print: [hep-ex/0506081](https://arxiv.org/abs/hep-ex/0506081) [hep-ex]

Counted up to 13. 05.2026

pdf links DOI cite claim reference search **1,126 citations**

[BaBar], *Phys. Rev. D* 74, 091103 (2006)

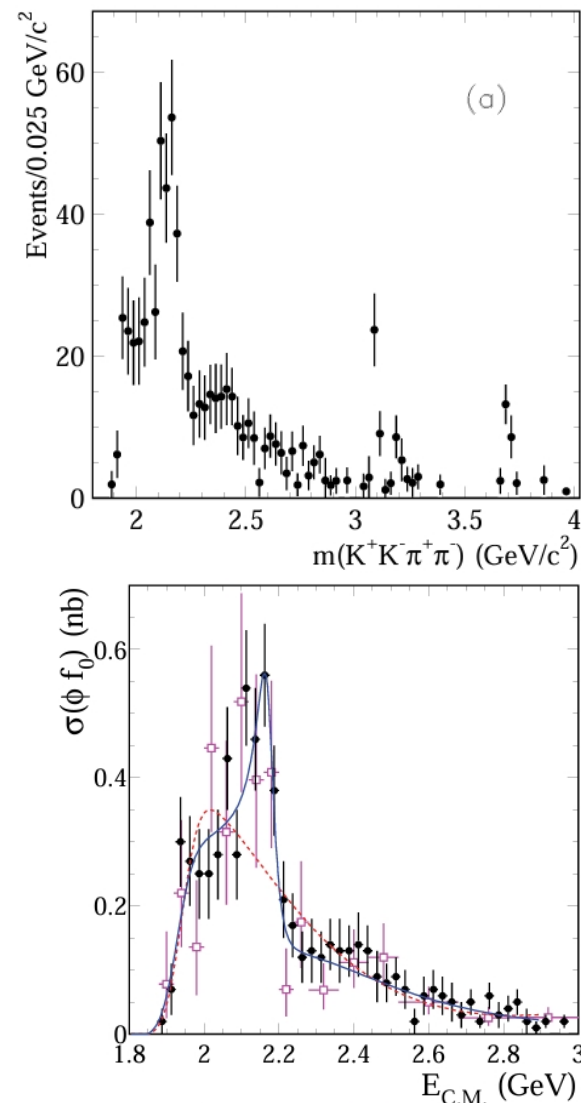
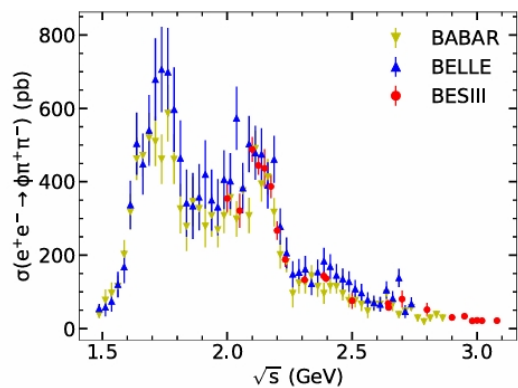
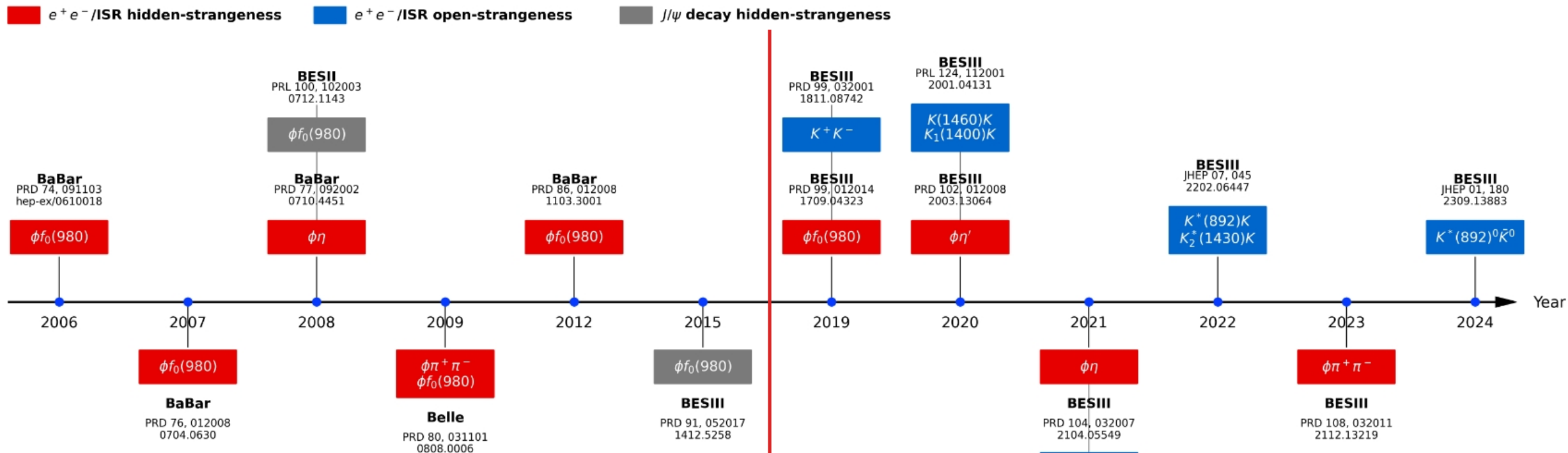
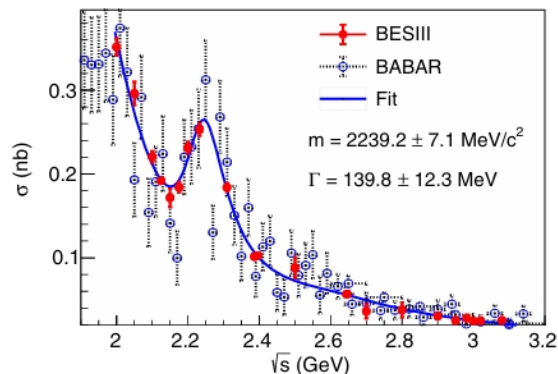


FIG. 6: The $e^+e^- \rightarrow \phi(1020)f_0(980)$ cross section, with about 10% of the $\phi\pi\pi$ contribution, obtained via ISR in the $K^+K^-\pi^+\pi^-$ (circles) and $K^+K^-\pi^0\pi^0$ (squares) final states. The curves represent results of the fits described in the text.

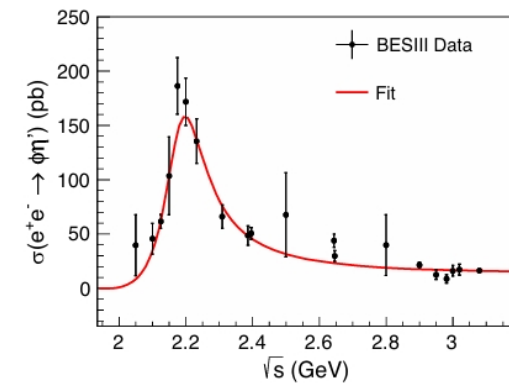
Experimental overview of $Y(2175)$



[BESIII], Phys. Rev. D 108, 032011 (2023)



[BESIII], Phys.Rev.D 99 (2019) 3, 032001



[BESIII], Phys.Rev.D 102 (2020) 1, 012008

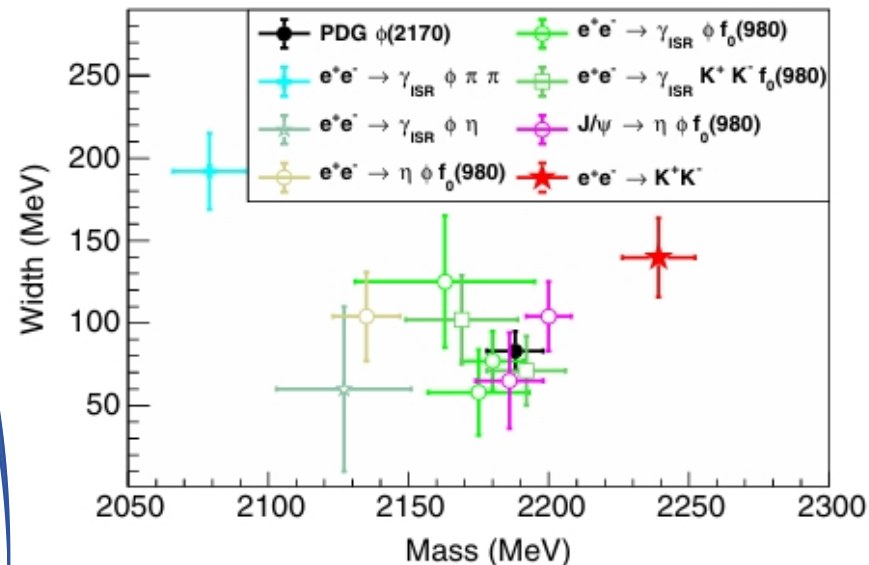
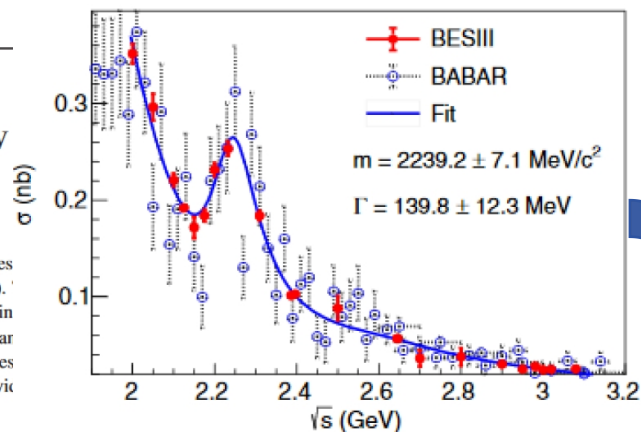
Puzzle I : extracted resonance parameters are channel dependent

PHYSICAL REVIEW D **99**, 032001 (2019)

Measurement of $e^+e^- \rightarrow K^+K^-$ cross section at $\sqrt{s}=2.00-3.08$ GeV

(BESIII Collaboration)

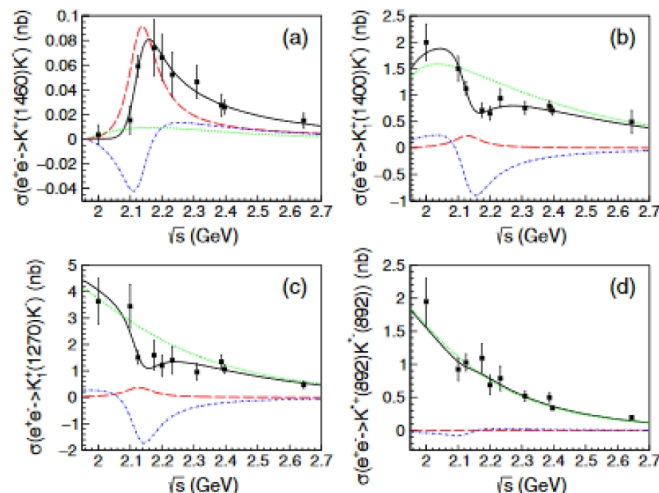
The cross section of the process $e^+e^- \rightarrow K^+K^-$ is measured at a number of center-of-mass energies from 2.00 to 3.08 GeV with the BESIII detector at the Beijing Electron Positron Collider (BEPCII). Results provide the best precision achieved so far. A resonant structure around 2.2 GeV is observed in cross section line shape. A Breit-Wigner fit yields a mass of $M = 2239.2 \pm 7.1 \pm 11.3$ MeV/ c^2 and width of $\Gamma = 139.8 \pm 12.3 \pm 20.6$ MeV, where the first uncertainties are statistical and the second ones systematic. In addition, the timelike electromagnetic form factor of the kaon is determined at the individual center-of-mass energy points.



PHYSICAL REVIEW LETTERS **124**, 112001 (2020)

Observation of a Resonant Structure in $e^+e^- \rightarrow K^+K^-\pi^0\pi^0$

(BESIII Collaboration)



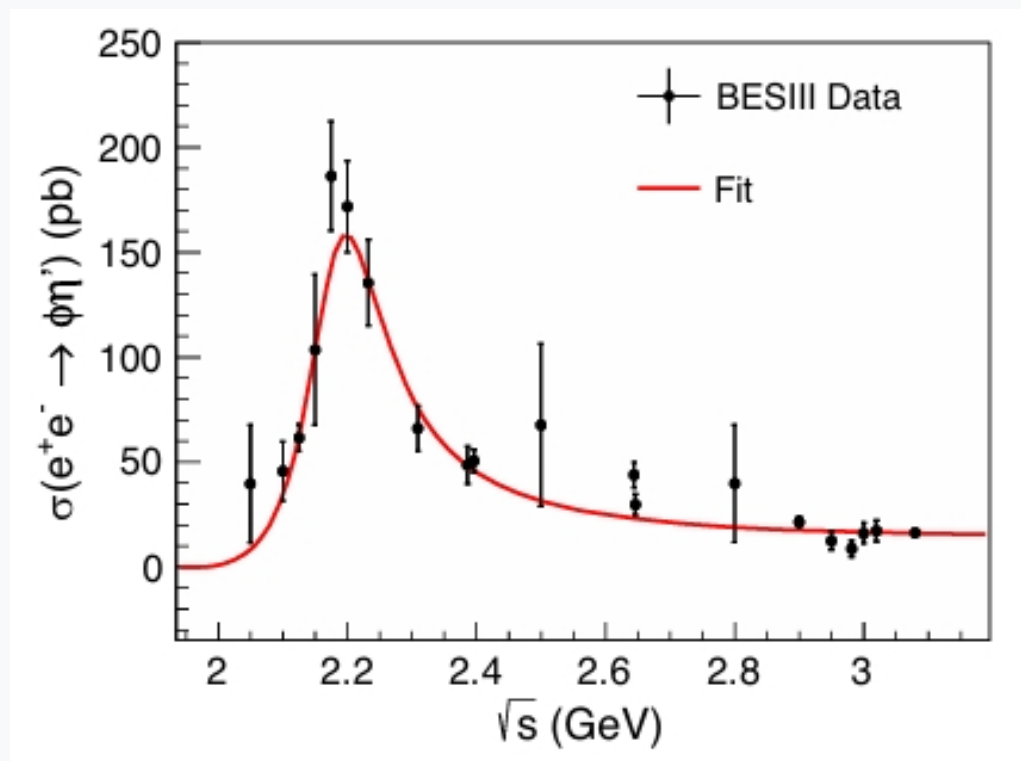
100 MeV mass discrepancy!

**several nearby resonances?
strong channel dynamics?**

$(2126.5 \pm 16.8 \pm 12.4)$ MeV/ c^2 and width $\Gamma = (106.9 \pm 32.1 \pm 28.1)$ MeV

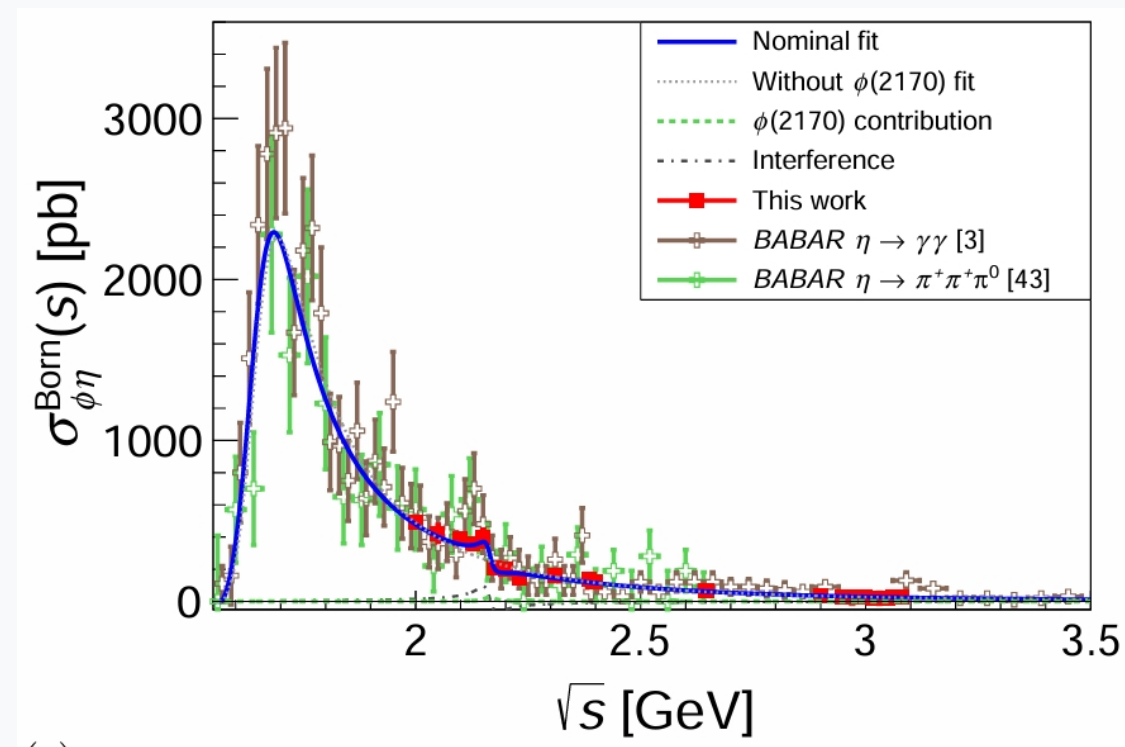
Puzzle II : why does $\phi\eta'$ look so prominent?

BESIII $\phi\eta'$ cross section



[BESIII], Phys.Rev.D 102 (2020) 1, 012008

$\phi\eta$ cross section



[BESIII], Phys.Rev.D 104 (2021) 3, 032007

Puzzle II : why does $\phi\eta'$ look so prominent?

BESIII $\phi\eta'$ cross section

Parameter	Solution I	Solution II
M_R (MeV/ c^2)	$2177.5 \pm 4.8(\text{stat}) \pm 19.5(\text{syst})$	
Γ_{tot}^R (MeV)	$149.0 \pm 15.6(\text{stat}) \pm 8.9(\text{syst})$	
$\mathcal{B}_{\mathcal{R}}\Gamma_{e^+e^-}^R$ (eV)	$7.1 \pm 0.7(\text{stat}) \pm 0.7(\text{syst})$	
φ (rad)	3.13 ± 2.01	-0.01 ± 2.36

this width is consistent with the $Y(2175)$ structure in open-charm channels and ϕf_0

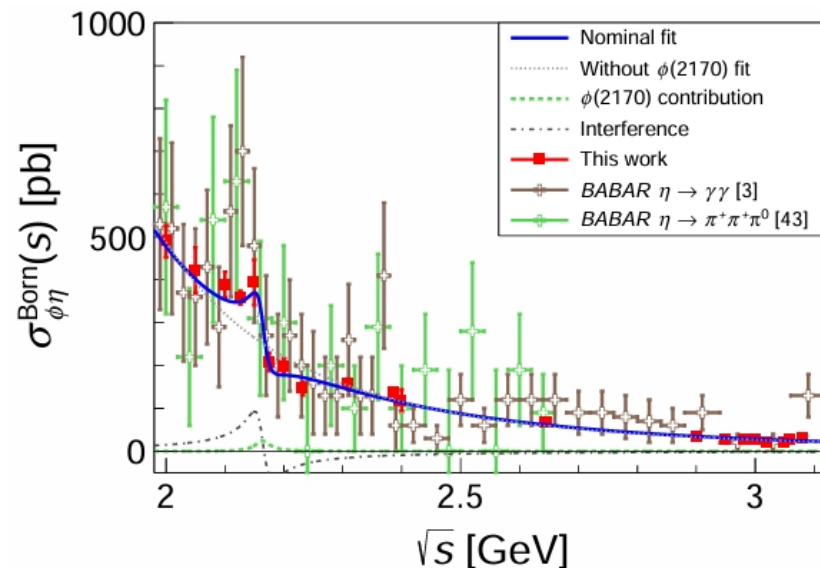
$$R_{\eta/\eta'}^{\text{exp}} = \begin{cases} 0.034_{-0.011}^{+0.018} \text{ solution I,} \\ 1.42_{-0.48}^{+0.58} \text{ solution II,} \end{cases}$$

The broad state is missing for the $\phi\eta$ channel

Ratio of phase space is about 4.3

BESIII $\phi\eta$ cross section

Parameter	Solution I	Solution II
$\chi^2/\text{n.d.f}$	86.8/97	
a_0	$-0.11_{-0.22}^{+0.09}$	-0.24 ± 0.58
a_1	$1.91_{-0.76}^{+0.61}$	$2.54_{-1.55}^{+2.86}$
$\mathcal{B}_{\phi\eta}^{\phi(2170)}\Gamma_{e^+e^-}^{\phi(2170)}$	$0.24_{-0.07}^{+0.12}$ eV	$10.11_{-3.13}^{+3.87}$ eV
$m_{\phi(2170)}$	2163.5 ± 6.2 MeV/ c^2	
$\Gamma_{\phi(2170)}$	$31.1_{-11.6}^{+21.1}$ MeV	
$\Phi_{\phi(2170)}$	$1.82_{-0.31}^{+0.35}$	$-2.92_{-0.06}^{+0.05}$



Theoretical interpretations before the new open-strange constraints

Conventional $\bar{s}s$

$\phi(3S)$ or $\phi(2D)$;
strong-decay pattern
is the central test.

Ding, Yan, PLB 657,
49 (2007)

Wang et al., PRD 85,
074024 (2012)

Coito et al., PRD 80,
094011 (2009)

Pang, PRD 99, 074015
(2019)

Hybrid $\bar{s}sg$

Selection-rule
expectations; open-
strange channels are
decisive.

Ding, Yan, PLB 650,
390 (2007)

Ding, Yan, PLB 657,
49 (2007)

Ho et al., PRD 100,
034012 (2019)

Tetraquark

Mass can be
accommodated

Z. G. Wang, NPA 791,
106 (2007)

Chen et al., PRD 78,
034012 (2008)

Drenska et al., PLB
669, 160 (2008)

Ke, Li, PRD 99,
036014 (2019)

Agaev et al., PRD 101,
074012 (2020)

Two-body molecules

ϕf_0 or $\Lambda\Lambda$
components

Napsuciale et al., PRD
76, 074012 (2007)

Alvarez-Ruso et al.,
PRD 80, 054011
(2009)

Zhao et al., PRD 87,
054034 (2013)

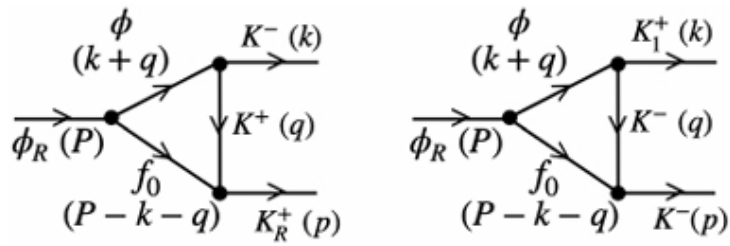
Dong et al., PRD 96,
074027 (2017)

Three-body molecule

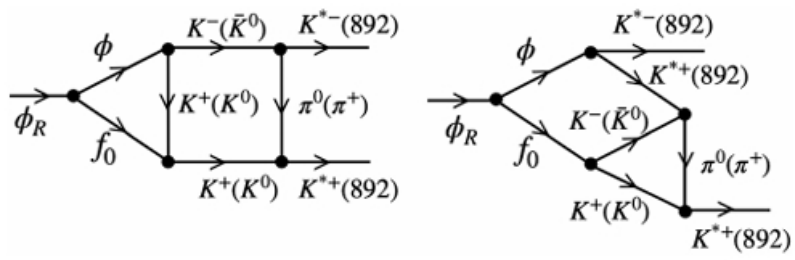
$\phi K \bar{K}$ Faddeev
dynamics predicts
 ϕf_0 -like structure
around 2.15 GeV.

Martínez Torres et al.,
PRD 78, 074031
(2008)

Recent theoretical researches: ϕf_0 or $\phi K \bar{K}$



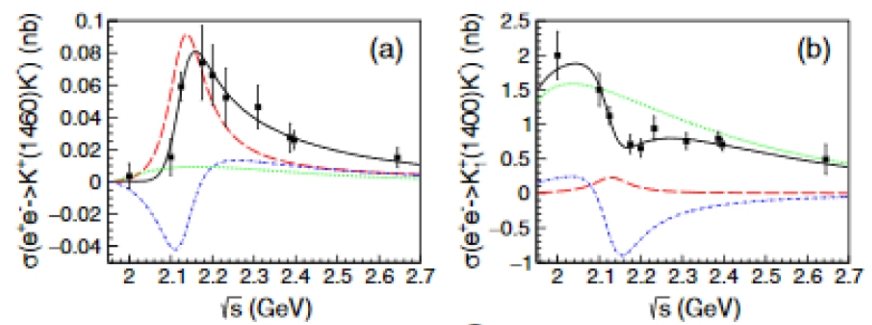
B_1		
Our results	Model B	0.62 ± 0.20
	Model C	0.11 ± 0.04
Experiment	Solution 1	0.64 ± 0.92
	Solution 2	0.03 ± 0.04



$$B_1 \equiv \frac{\Gamma_{\phi_R \rightarrow K^+(1460)K^-}}{\Gamma_{\phi_R \rightarrow K_1^+(1400)K^-}} = \frac{\mathcal{B}r[\phi_R \rightarrow K^+(1460)K^-]}{\mathcal{B}r[\phi_R \rightarrow K_1^+(1400)K^-]}$$

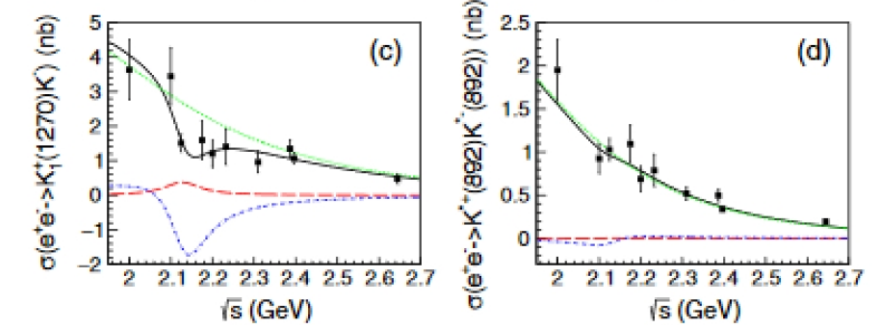
B_2			
Our results	Model A	1.3 ± 0.4	(Poles z_1, z_2)
		3.6 ± 1.2	(Pole z_1)
		8.8 ± 2.8	(Pole z_2)
	Model B	16 ± 6	
		1.2 ± 0.4	(Solution S_1)
	Model C	0.12 ± 0.04	(Solution S_2)
		0.05 ± 0.02	(Solution S_3)
Experiment	Solution 1	0.40 ± 0.54	
	Solution 2	0.02 ± 0.03	

$$B_2 \equiv \frac{\Gamma_{\phi_R \rightarrow K^+(1460)K^-}}{\Gamma_{\phi_R \rightarrow K_1^+(1270)K^-}} = \frac{\mathcal{B}r[\phi_R \rightarrow K^+(1460)K^-]}{\mathcal{B}r[\phi_R \rightarrow K_1^+(1270)K^-]}$$



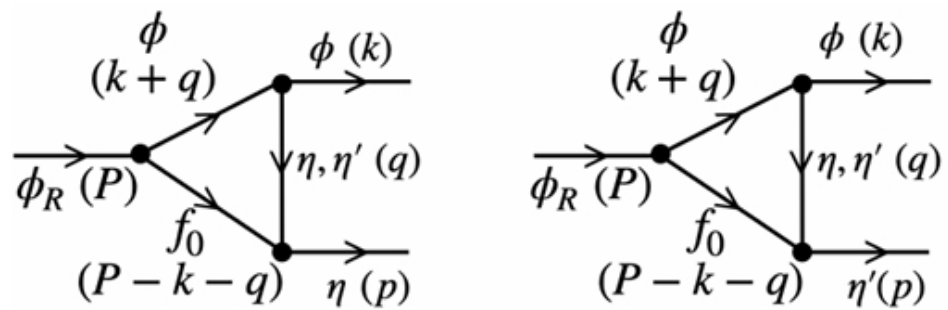
B_3			
Our results	Model B	0.04 ± 0.01	
		0.09 ± 0.02	(Solution S_1)
	Model C	0.96 ± 0.16	(Solution S_2)
		2.40 ± 0.40	(Solution S_3)
Experiment	Solution 1	1.62 ± 1.38	
	Solution 2	1.55 ± 0.19	

$$B_3 \equiv \frac{\Gamma_{\phi_R \rightarrow K_1^+(1270)K^-}}{\Gamma_{\phi_R \rightarrow K_1^+(1400)K^-}} = \frac{\mathcal{B}r[\phi_R \rightarrow K_1^+(1270)K^-]}{\mathcal{B}r[\phi_R \rightarrow K_1^+(1400)K^-]}$$



Malabarba, Ren, Khemchandani and Martinez Torres, Phys. Rev. D 103, 016018 (2021)

Recent theoretical researches: ϕf_0 or $\phi K \bar{K}$ and $\Lambda \bar{\Lambda}$



β (Degree)	-15	-19.47	-22	
$\Gamma_{\phi\eta}^{\phi(2170)}$	LI	4.30 ± 0.93	3.27 ± 0.71	2.76 ± 0.60
	GI	5.14 ± 1.11	3.91 ± 0.85	3.29 ± 0.71
	LII	3.38 ± 0.73	2.59 ± 0.56	2.20 ± 0.48
	GII	4.17 ± 0.91	3.20 ± 0.69	2.71 ± 0.59
$\Gamma_{\phi\eta'}^{\phi(2170)}$	LI	0.84 ± 0.18	0.83 ± 0.18	0.81 ± 0.18
	GI	0.94 ± 0.20	0.93 ± 0.20	0.91 ± 0.20
	LII	0.80 ± 0.18	0.80 ± 0.17	0.79 ± 0.17
	GII	0.95 ± 0.21	0.94 ± 0.20	0.93 ± 0.20

$R_{\eta/\eta'} = 3.9 \pm 1.3.$

$$|\eta\rangle = \cos\beta|\eta_8\rangle - \sin\beta|\eta_1\rangle,$$

$$|\eta'\rangle = \sin\beta|\eta_8\rangle + \cos\beta|\eta_1\rangle,$$

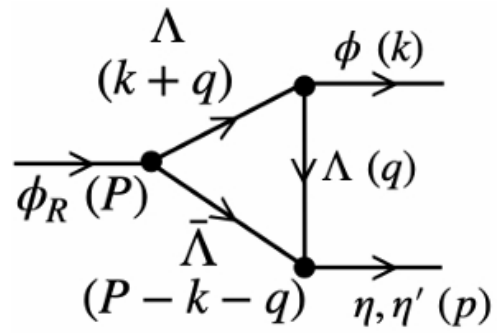
$$\mathbb{P} = \begin{pmatrix} A(\beta)\eta + B(\beta)\eta' + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & A(\beta)\eta + B(\beta)\eta' - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & C(\beta)\eta + D(\beta)\eta' \end{pmatrix},$$

$$A(\beta) = -\frac{\sin\beta}{\sqrt{3}} + \frac{\cos\beta}{\sqrt{6}},$$

$$B(\beta) = \frac{\sin\beta}{\sqrt{6}} + \frac{\cos\beta}{\sqrt{3}},$$

$$C(\beta) = -\frac{\sin\beta}{\sqrt{3}} - \sqrt{\frac{2}{3}}\cos\beta,$$

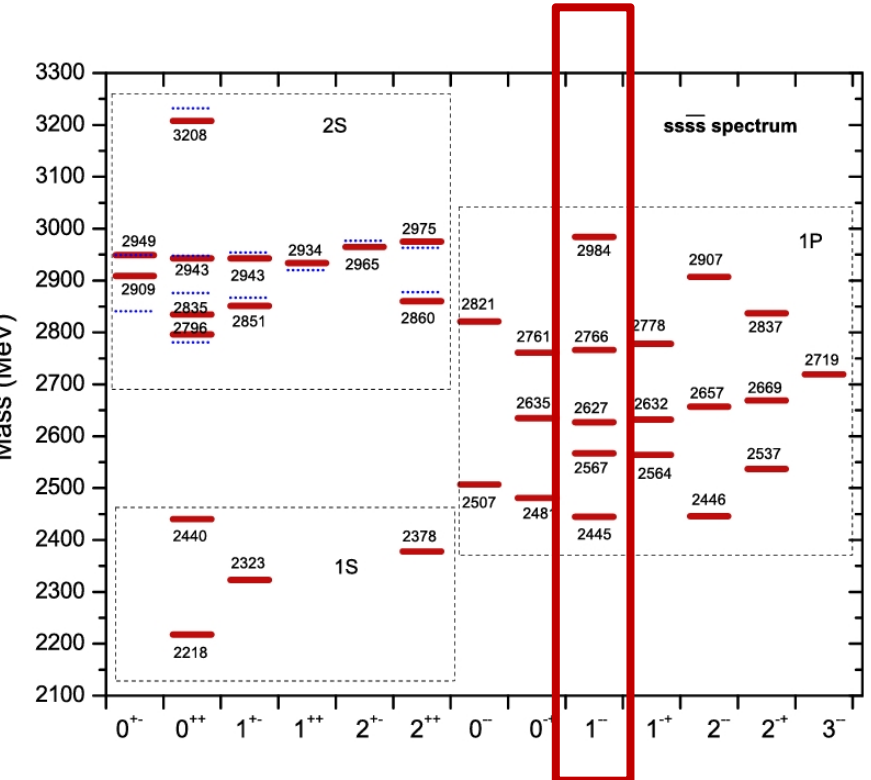
$$D(\beta) = -\sqrt{\frac{2}{3}}\sin\beta + \frac{\cos\beta}{\sqrt{3}},$$



$$R_{\eta/\eta'} \simeq 0.43 - 0.86$$

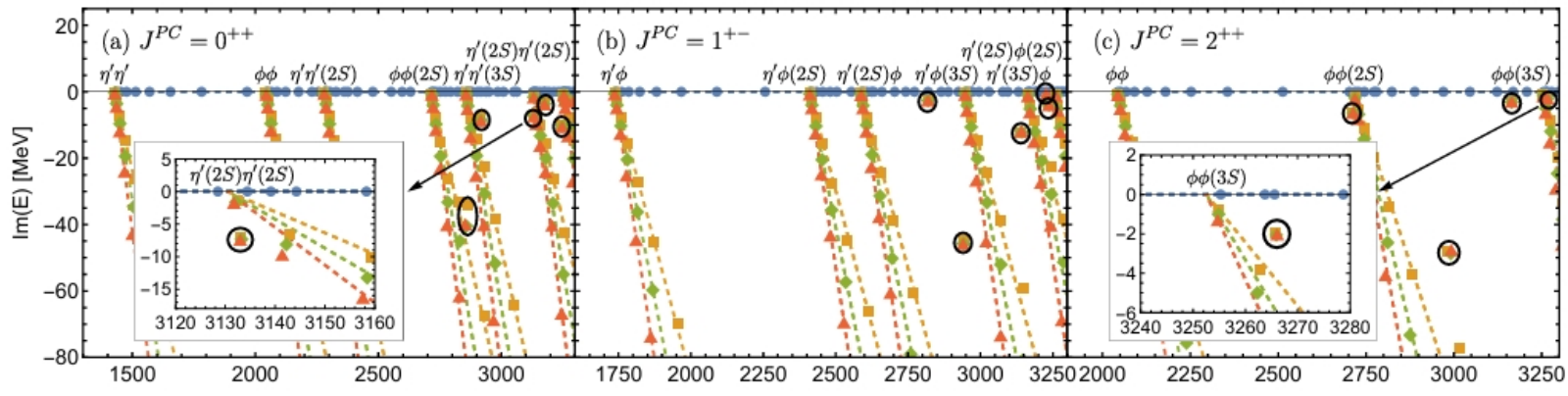
Malabarba, Khemchandani and Torres, Phys. Rev. D 108, 036010 (2023)

Recent theoretical researches: fully-strange tetraquark



Non-relativistic potential model within diquark-antidiquark approximation

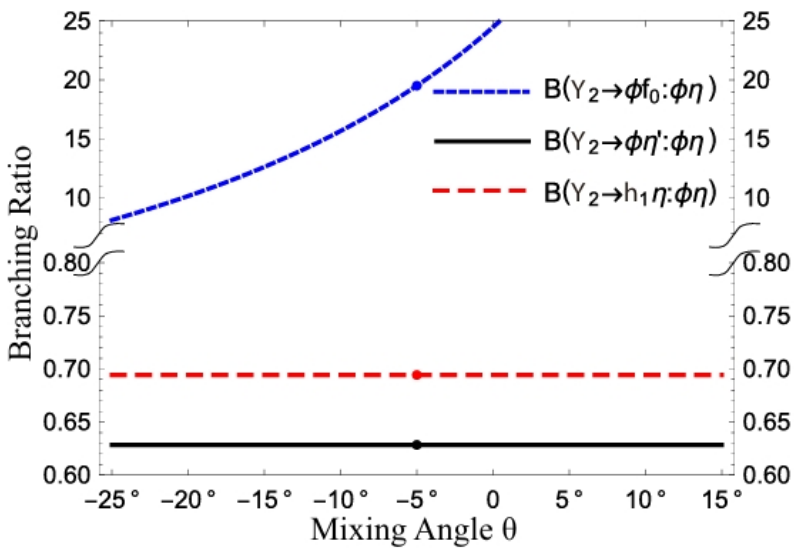
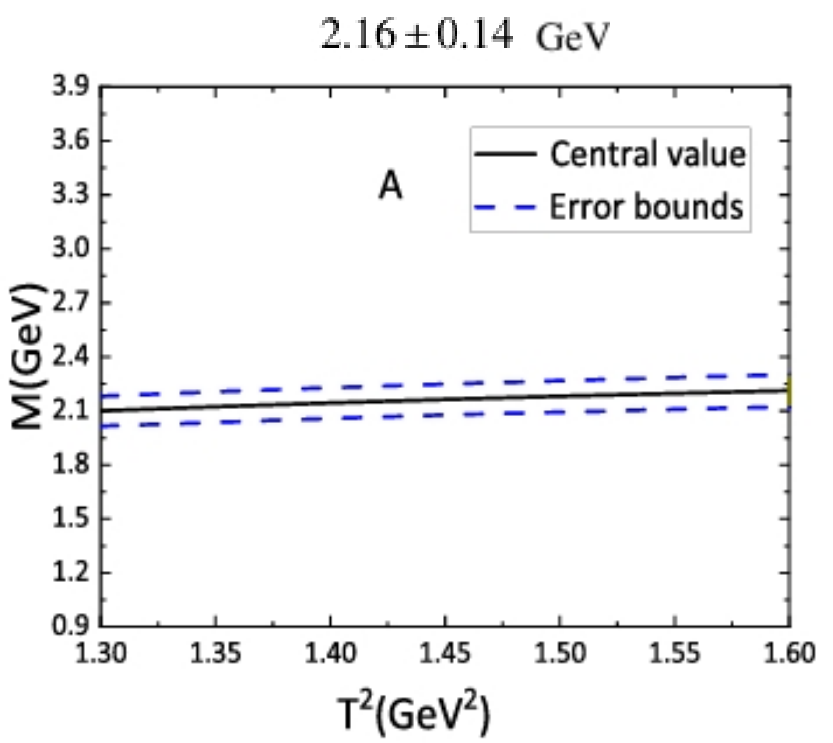
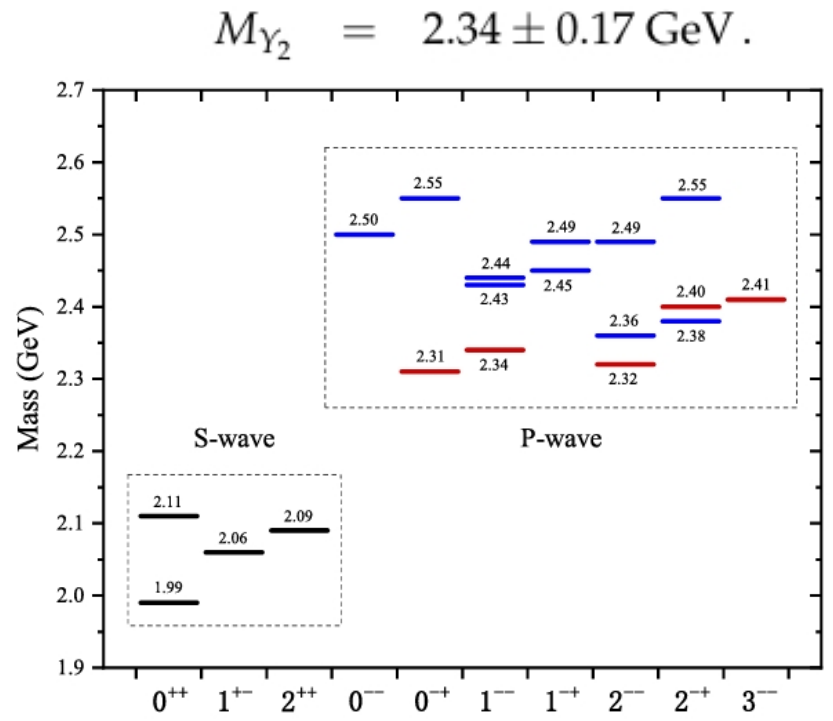
Liu, Liu, Zhong and Zhao, Phys. Rev. D 103, 016016 (2021)



Constituent quark potential model, by solving the full four-body Schrodinger equation and using the complex scaling method to identify resonant states

Ma, Wu, Meng, Chen and Zhu, Phys. Rev. D 110, 074026 (2024)

Recent theoretical researches: fully-strange tetraquark



QCD sum rule method within diquark-antidiquark picture

Su and Chen,
Phys. Rev. D 106, 014023 (2022)

Xin and Wang,
Chin.Phys.C 48 (2024) 3, 033104

$$\begin{aligned}
 B(Y_2 \rightarrow \phi \eta : \phi \eta' : \phi f_0 : h_1(1415)\eta) \\
 = 1 : 0.63^{+0.19}_{-0.27} : 19.52^{+6.05}_{-5.51} : 0.69^{+0.09}_{-0.14}.
 \end{aligned}$$

Jiang, Tan, Chen and Cui,
Symmetry 16, 1021 (2024)

the Fierz rearrangement

Recent theoretical researches: strangeonium hybrid

Table 4. The fitted masses m_n of the 1^{--} states for two different types of operators and different time windows $[t_{\min}, t_{\max}]$ and χ^2 per degree of freedom (χ^2/dof) for the $\beta = 2.4$ and $\beta = 2.8$ lattices. All the masses are converted to values in physical units using the lattice spacing a_s in Table 1.

t_{\min}	χ^2/dof	m_1 /GeV	m_2 /GeV	m_3 /GeV
	$\beta = 2.4$	$t_{\max} = 40$	($s\bar{s}$)	
8	0.84	1.013(1)	1.753(77)	2.16(22)
7	0.81	1.013(1)	1.787(93)	2.08(19)
6	0.83	1.014(1)	1.732(47)	2.13(12)
5	0.92	1.015(1)	1.709(36)	2.11(08)
	$\beta = 2.4$	$t_{\max} = 45$	($s\bar{s}g$)	
16	1.40	1.011(1)	1.72(12)	—
15	1.42	1.011(1)	1.66(10)	—
14	1.42	1.009(1)	1.77(08)	—
13	1.67	1.007(1)	1.83(06)	—
	$\beta = 2.8$	$t_{\max} = 30$	($s\bar{s}$)	
10	1.93	1.001(5)	1.634(114)	2.17(29)
9	1.97	1.003(4)	1.633(86)	2.02(18)
8	2.11	0.999(3)	1.665(81)	2.30(20)
7	2.39	0.998(3)	1.668(52)	2.34(17)
	$\beta = 2.8$	$t_{\max} = 45$	($s\bar{s}g$)	
19	1.17	1.006(4)	1.51(6)	—
18	1.39	1.003(3)	1.55(6)	—
17	1.50	1.005(3)	1.51(5)	—
16	1.45	0.998(2)	1.58(4)	—

$$\frac{\Gamma(\phi(2170) \rightarrow \phi\eta)}{\Gamma(\phi(2170) \rightarrow \phi\eta')} = \tan^2 \theta \left(\frac{k_\eta}{k_{\eta'}} \right)^3,$$

lattice QCD in the quenched approximation

Ma, Chen, Gong and Liu, Chin. Phys. C 45, 013112 (2021)

Recent theoretical researches: strangeonium hybrid

(NLO) QCD sum rule analysis for light hybrid mesons

Parameters	NLO		LO	
	$s\bar{s}g$	$u\bar{u}g$	$s\bar{s}g$	$u\bar{u}g$
$\overline{\chi^2_{\min}}$	1.17×10^{-5}	2.32×10^{-5}	5.99×10^{-5}	4.83×10^{-5}
$s_0(\text{GeV}^2)$	6.15 ± 1.21	5.85 ± 1.21	8.91 ± 1.43	8.45 ± 1.52
$m(\text{GeV})$	2.31 ± 0.23	2.25 ± 0.23	2.77 ± 0.22	2.70 ± 0.24

Table 2. Single-resonance GSR fitting results in different scenarios. The values shown are determined via a Monte Carlo uncertainty analysis (the $\langle \bar{q}q \rangle$ and $\langle \bar{s}s \rangle$ are assumed to have a 10% uncertainties). The $\overline{\chi^2_{\min}}$ denotes the average minimum χ^2 .

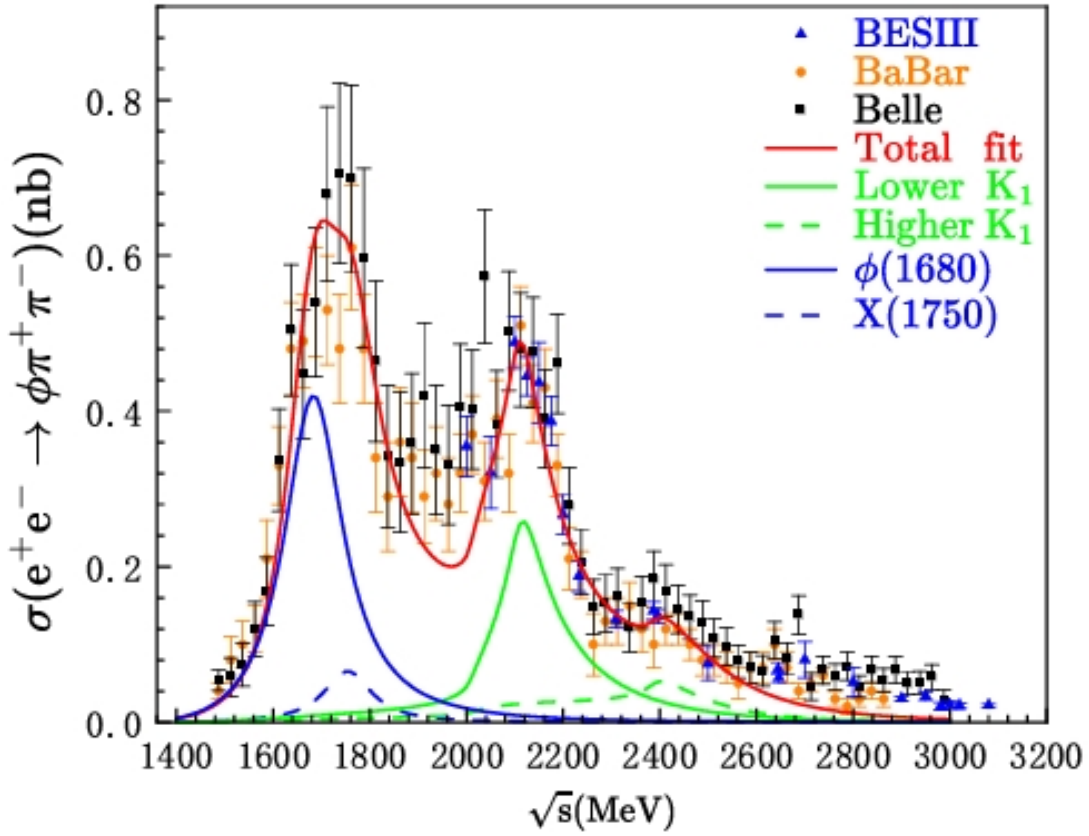
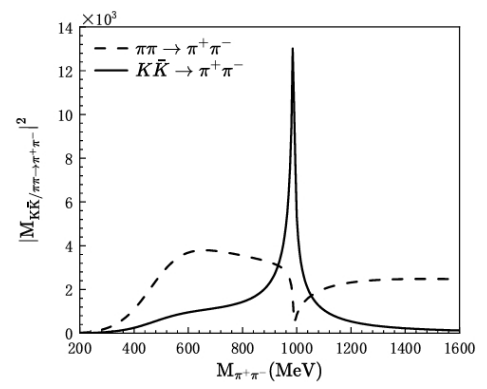
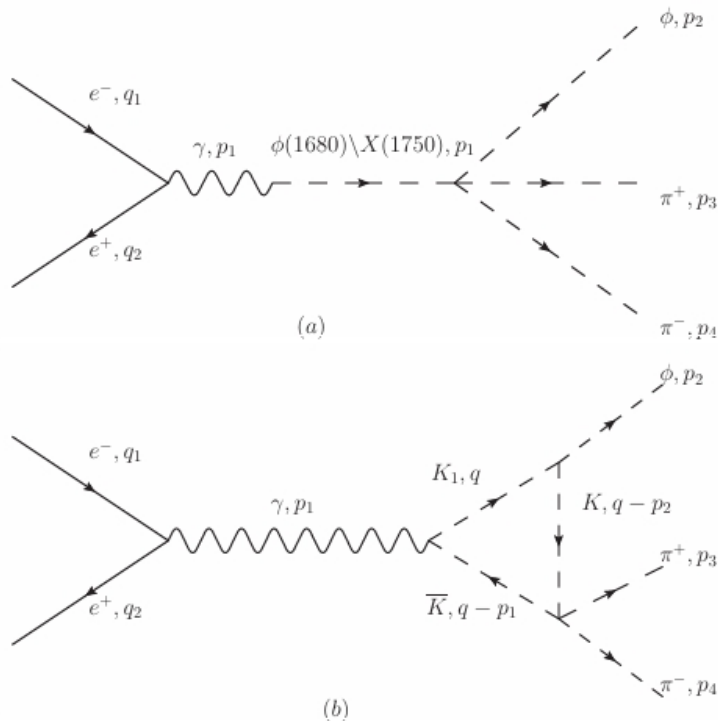
Parameters	$\kappa_6 = 3, \kappa_8 = 2$		$\kappa_6 = 3, \kappa_8 = 5$	
	$s\bar{s}g$	$u\bar{u}g$	$s\bar{s}g$	$u\bar{u}g$
$\overline{\chi^2_{\min}}$	2.85×10^{-5}	7.28×10^{-5}	1.74×10^{-4}	4.30×10^{-4}
$s_0(\text{GeV}^2)$	6.03 ± 1.21	6.02 ± 1.31	6.40 ± 1.60	7.03 ± 2.16
$m(\text{GeV})$	2.29 ± 0.23	2.28 ± 0.24	2.34 ± 0.28	2.43 ± 0.37

Table 3. Single-resonance NLO GSR fitting results for different factorization deviation factors. Obtained in a same way as in table 2.

Listen to Zhuo-Ran Huang's talk on 17/05

Li, Huang, Chen and Jin, JHEP 03, 087 (2026)

Recent theoretical researches: triangular singularity



Wei, Shen, Liu and Xie, Phys. Rev. D 113, 014022 (2026)

Recent theoretical researches: interference effect from strangeonium 3S and 1D

the strangeonium hybrid and excited strangeonium $\phi(3S)$ and $\phi(2D)$ should dominantly decay into open-strange modes, so the newly observed structure around 2.2 GeV in open-strange processes should be good candidate of hybrid and excited ϕ mesons.

Strangeonium hybrid

G. J. Ding and M. L. Yan, $Y(2175)$: Distinguish Hybrid State from Higher Quarkonium, Phys. Lett. B **657**, 49-54 (2007).

The observation of a vector structure around 2.2 GeV in KK and KK(1460) channel does not favor the hybrid explanation, but no signal in K^*K^* is consistent with expectation of hybrid configuration.

$\phi(3S)$ and $\phi(2D)$

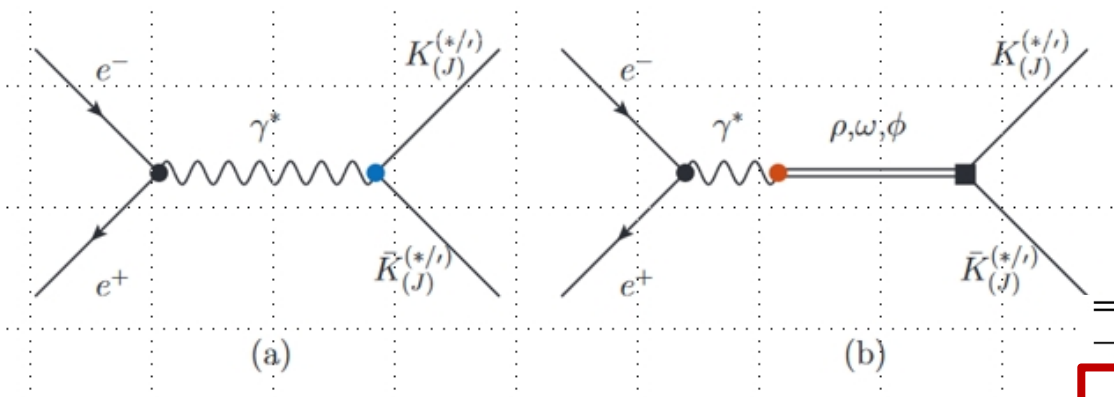
$K1(1270)K$ and K^*K^* were predicted to be dominant decay channels of $\phi(3S)$ and $\phi(2D)$, so the BESIII data does not support these theoretical predictions.

T. Barnes, N. Black and P. R. Page, Strong decays of strange quarkonia, Phys. Rev. D **68**, 054014 (2003).

C. Q. Pang, Excited states of ϕ meson, Phys. Rev. D **99**, no.7, 074015 (2019).

Understanding the puzzle of inconsistent resonance parameter of $Y(2175)$

Recent theoretical researches: interference effect from strangeonium 3S and 1D



There are $\rho(2000)$, $\rho(2150)$ and $\rho(2270)$ (3 ω partner states) and $\phi(3S)$ and $\phi(2D)$ states around 2.0 GeV, so it should be very difficult to analyze the open-strange processes above 2.0 GeV

States	$\rho(2D)$	$\rho(4S)$	$\rho(3D)$	$\rho(5S)$	$\omega(2D)$	$\omega(4S)$	$\omega(3D)$	$\omega(5S)$
Mass (GeV)	2.003	2.180	2.283	2.422	2.003	2.180	2.283	2.422
$\Gamma_{e^+e^-}$ (keV)	0.020	0.063	0.016	0.036	0.0022	0.007	0.0018	0.004
Γ_{Total} (MeV)	179	102	158	80	181	104	94	69
$\mathcal{B}(KK)$	0.006	0.002	0.002	...	0.006	0.002	0.003	...
$\mathcal{B}(KK^*)$	0.001	0.001
$\mathcal{B}(K^*K^*)$...	0.003	...	0.003	...	0.003	0.001	0.003
$\mathcal{B}(KK_1(1270))$...	0.010	...	0.004	...	0.010	...	0.005
$\mathcal{B}(KK^*(1410))$...	0.001	0.001
$\mathcal{B}(K^*K_1(1270))$...	0.004	...	0.004	...	0.004	...	0.005
$\mathcal{B}(KK(1460))$	0.002	0.003	0.003	0.001	0.002	0.002	0.005	0.001

States	$\phi(1D)$	$\phi(3S)$	$\phi(2D)$	$\phi(4S)$	$\phi(3D)$
Mass (GeV)	1.860	2.103	2.236	2.423	2.519
$\Gamma_{e^+e^-}$ (keV)	0.063	0.106	0.017	0.050	0.010
Γ_{Total} (MeV)	515	156	265	140	171
$\mathcal{B}(KK)$	0.076	0.059	0.087	0.047	0.084
$\mathcal{B}(KK^*)$	0.100	0.242	0.067	0.145	0.050
$\mathcal{B}(K^*K^*)$	0.016	0.007	0.105	0.014	0.139
$\mathcal{B}(KK^*(1410))$...	0.180	0.080	0.142	0.052
$\mathcal{B}(KK^*(1680))$	0.004	0.003
$\mathcal{B}(K^*K^*(1410))$	0.045
$\mathcal{B}(KK_2^*(1430))$...	0.115	0.043	0.091	0.043
$\mathcal{B}(K^*K_2^*(1430))$	0.057	0.047
$\mathcal{B}(KK_1(1270))$	0.799	0.185	0.356	0.154	0.293
$\mathcal{B}(K^*K_1(1270))$	0.110	0.018	0.095
$\mathcal{B}(KK_1(1400))$...	0.064	0.026	0.027	0.015
$\mathcal{B}(K^*K_1(1400))$	0.126	0.003
$\mathcal{B}(KK(1460))$...	0.127	0.112	0.073	0.101
$\mathcal{B}(K^*K(1460))$	0.013	0.005
$\mathcal{B}(K^*K_0(1430))$	0.049	...
$\mathcal{B}(KK_3^*(1780))$	0.021	0.014

The scattering amplitude

$$M_{\text{Dir}}^{KK^{(0)}} = [\bar{v}(k_2)ie\gamma^\mu u(k_1)] \frac{-g_{\mu\nu}}{q^2} [-e(p_4^\nu - p_3^\nu)F_{KK^{(0)}}(q^2)],$$

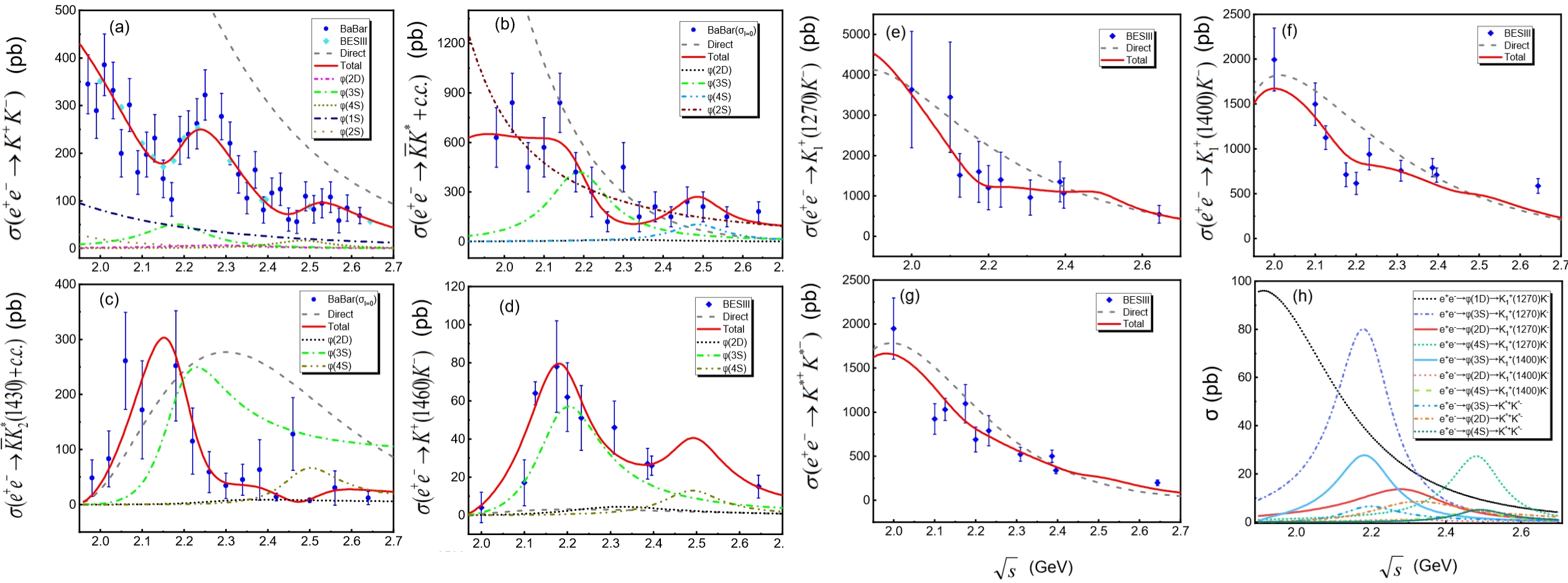
$$M_{\nu}^{KK^{(0)}} = [\bar{v}(k_2)ie\gamma_\mu u(k_1)] \frac{-g^{\mu\rho} - em_\nu^2}{q^2} \frac{-g_{\rho\nu} + q_\rho q_\nu/m_\nu^2}{f_\nu} \times [-g_{\nu KK^{(0)}}(p_4^\nu - p_3^\nu)], \quad (3)$$

$$M_{\text{Dir}}^{KK^*} = [\bar{v}(k_2)ie\gamma^\mu u(k_1)] \frac{-g_{\mu\nu}}{q^2} [e\epsilon^{\alpha\nu\rho\sigma} q_\alpha p_{4\rho} \epsilon_{K^*\sigma}^* F_{KK^*}(q^2)],$$

$$M_{\nu}^{KK^*} = [\bar{v}(k_2)ie\gamma_\mu u(k_1)] \frac{-g^{\mu\rho} - em_\nu^2}{q^2} \frac{-g_{\rho\nu} + q_\rho q_\nu/m_\nu^2}{f_\nu} \times [g_{\nu KK^*} \epsilon^{\alpha\nu\omega\sigma} q_\alpha p_{4\omega} \epsilon_{K^*\sigma}^*], \quad (4)$$

Wang, Wang, Liu and Matsuki,
Phys. Rev. D 104, 054045 (2021)

Recent theoretical researches: interference effect from strangeonium 3S and 1D



Wang, Wang, Liu and Matsuki, Phys. Rev. D 104, 054045 (2021)

Recent theoretical researches: understanding the puzzle phenomenon associated with $\phi\eta$ and $\phi\eta'$

Predicted ratio for $\phi(2170) \rightarrow \phi\eta$ and $\phi\eta'$
 $R_{\eta/\eta'} \equiv \Gamma[\phi(2170) \rightarrow \phi\eta]/\Gamma[\phi(2170) \rightarrow \phi\eta']$

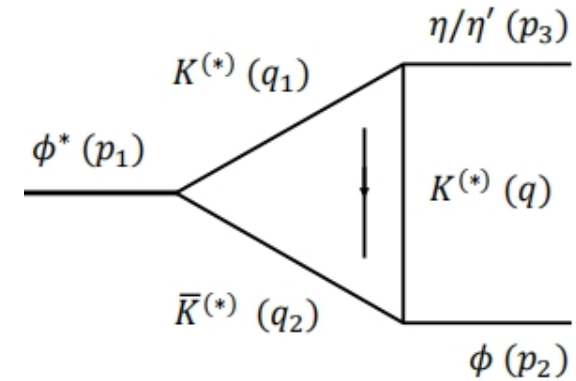
Assignment	Decay model	Reference	$R_{\eta/\eta'}$
$s\bar{s} 3^3S_1$	3P_0 model SHO wave functions	Barnes, Black, Page PRD 68, 054014 (2003)	1.91
$s\bar{s} 3^3S_1$	3P_0 model modified GI wave functions	Pang PRD 99, 074015 (2019)	77
$s\bar{s} 2^3D_1$	3P_0 model modified GI wave functions	Pang PRD 99, 074015 (2019)	9.9
$s\bar{s} 3^3S_1$	nonrelativistic linear-potential quark model	Li, Gui, Liu, Lü, Zhong CPC 45, 023116 (2021)	25
$s\bar{s} 2^3D_1$	nonrelativistic linear-potential quark model	Li, Gui, Liu, Lü, Zhong CPC 45, 023116 (2021)	1.13
$s\bar{s}g$ hybrid	flux-tube / hybrid decay model	Page, Swanson, Szczepaniak PRD 59, 034016 (1999)	9.5-200
$s\bar{s}g$ hybrid	hybrid interpretation of $Y(2175)$	Ding, Yan PLB 650, 390 (2007)	3

Ratio of phase space is about 4.3

Recent theoretical researches: understanding the puzzle phenomenon associated with $\phi\eta$ and $\phi\eta'$

Parameters	$\phi(2S)$	$\phi(2D)$	$\phi(3S)$	$Y(2175)$ (Hybrid)
Mass (MeV)	1680 ± 20	2183 ± 1	2290 ± 3	$2175 \pm 10 \pm 15$
Γ_{Total} (MeV)	150 ± 50	185 ± 4	312 ± 6	148.7
$\Gamma_{e^+e^-}$ (eV)	285	106	17	?
$\mathcal{B}(KK)$	0.108	0.059	0.087	0.237
$\mathcal{B}(K^*K)$	0.851	0.242	0.067	0.025
$\mathcal{B}(K^*K^*)$...	0.007	0.105	
$\mathcal{B}(KK_1(1270))$...	0.185	0.356	0.237
$\mathcal{B}(KK_1(1400))$...	0.064	0.026	0.471
$\mathcal{B}(\phi\eta)$	0.041	0.011	0.003	0.0081
$\mathcal{B}(\phi\eta')$...	0.0003	0.0001	0.0026

The resonance parameters are very reliable from the fit to the cross section of multiple open-charm channels



$$g_{\phi K^* K} = g_{VVP}, \quad g_{\phi K^* K^*} = g_{\phi KK},$$

$$g_{K^* K \eta} = (\alpha + \gamma)g_{\phi KK}, \quad g_{K^* K \eta'} = (\beta + \delta)g_{\phi KK},$$

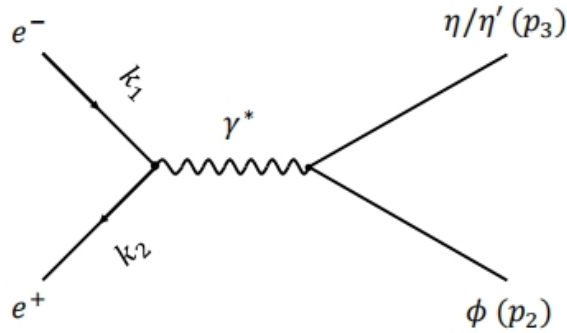
$$g_{K^* K^* \eta} = (\alpha + \gamma)g_{VVP}, \quad g_{K^* K^* \eta'} = (\beta + \delta)g_{VVP},$$

$$\alpha = \frac{\cos \theta - \sqrt{2} \sin \theta}{\sqrt{6}}, \quad \beta = \frac{\sqrt{2} \cos \theta + \sin \theta}{\sqrt{6}},$$

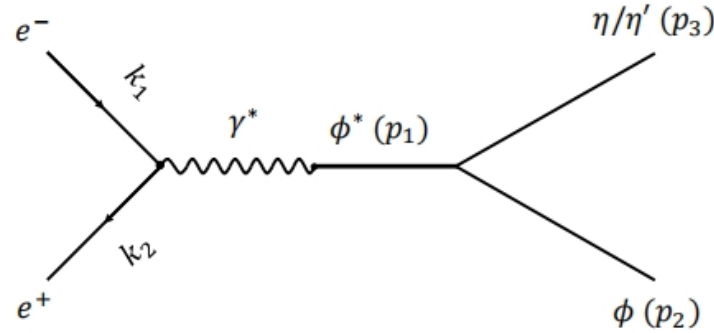
$$\gamma = \frac{-2 \cos \theta - \sqrt{2} \sin \theta}{\sqrt{6}}, \quad \delta = \frac{\sqrt{2} \cos \theta - 2 \sin \theta}{\sqrt{6}}$$

$$\frac{g_{K^{(*)} K^{(*)} \eta}}{g_{K^{(*)} K^{(*)} \eta'}} = \frac{-\cos \theta - 2\sqrt{2} \sin \theta}{2\sqrt{2} \cos \theta - \sin \theta} \approx -0.094$$

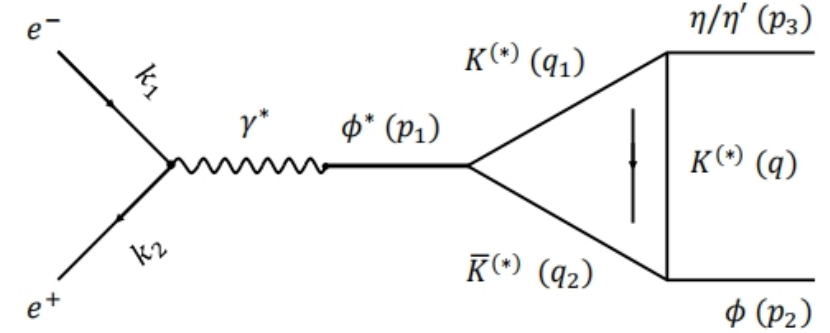
Recent theoretical researches: understanding the puzzle phenomenon associated with $\phi\eta$ and $\phi\eta'$



(a)



(b)



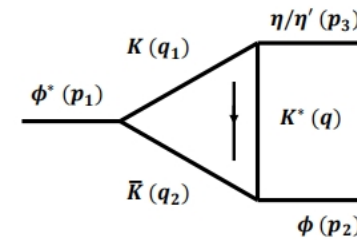
(c)

The reaction amplitude

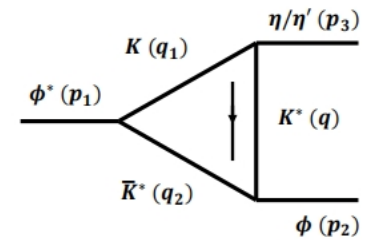
$$\mathcal{M}_{\text{Dir}} = \bar{v}(k_2)(ie\gamma_\rho)u(k_1) \frac{-g^{\rho\nu}}{p_1^2} g_{\gamma\phi\eta^{(\prime)}} \varepsilon_{\mu\nu\alpha\beta} (-ip_1^\mu)(ip_2^\alpha) \varepsilon^{*\beta}(p_2).$$

$$\mathcal{M}_{\text{Tree}}^{\phi_i^*} = \bar{v}(k_2)(ie\gamma_\xi)u(k_1) \frac{-g^{\xi\xi} - em_{\phi^*}^2}{p_1^2} \frac{\tilde{g}_{\xi\mu}(p_1)}{f_{\phi^*} (p_1^2 - m_{\phi^*}^2 + im_{\phi^*}\Gamma_{\phi^*})} \times g_{\phi_i^*\phi\eta^{(\prime)}} \varepsilon_{\mu\nu\alpha\beta} (-ip_1^\mu)(ip_2^\alpha) \varepsilon^{*\beta}(p_2).$$

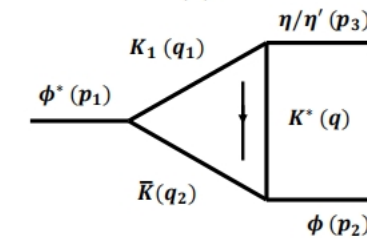
$$\mathcal{M}_{\text{Loop}}^{\phi_i^*} = 4 \times \left(\sum_{j=1}^6 \mathcal{M}^j + g_{\text{phase}} \sum_{j=7}^{10} \mathcal{M}^j \right)$$



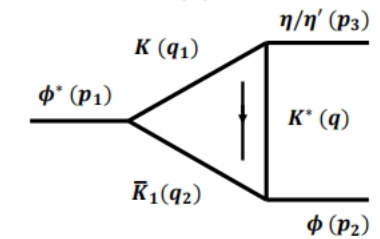
(a)



(b)



(g)

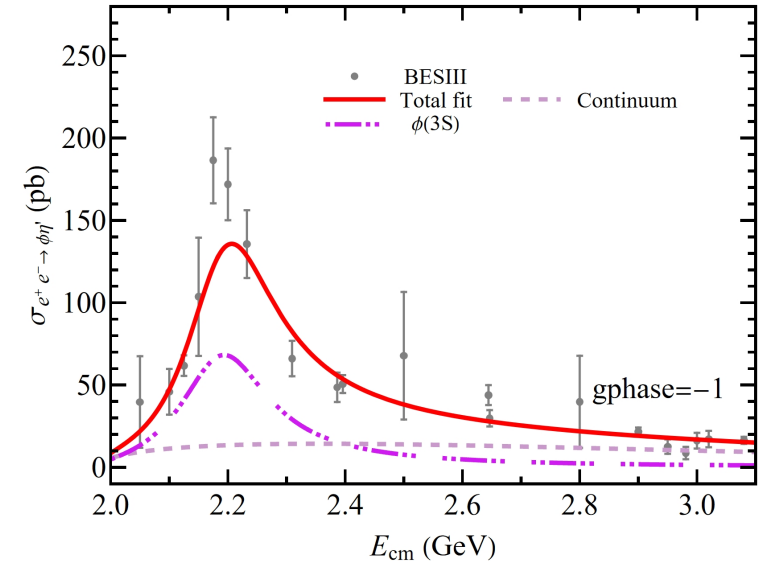
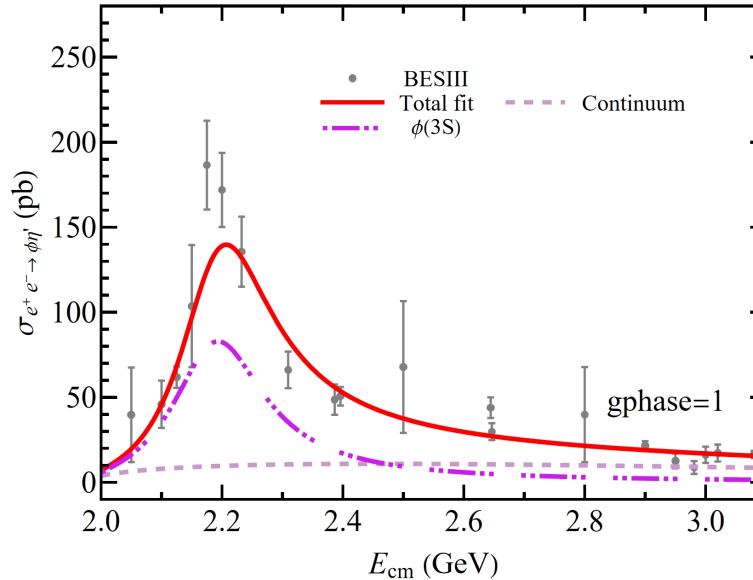
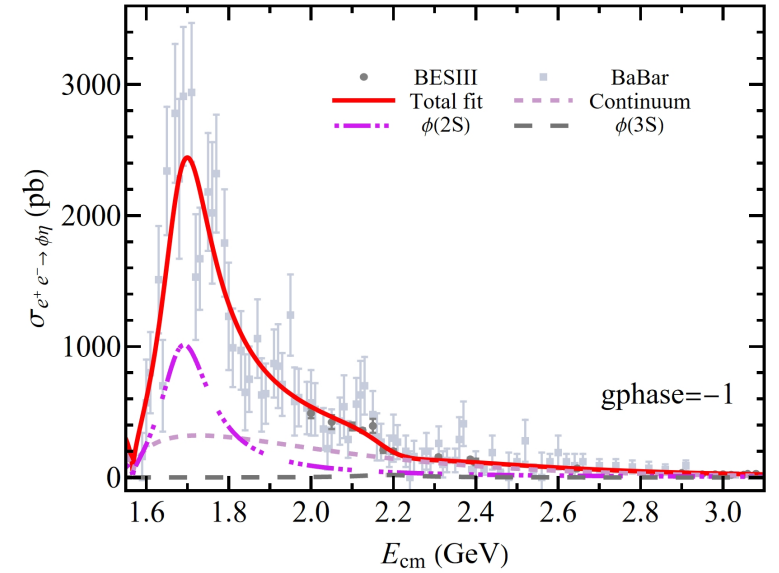
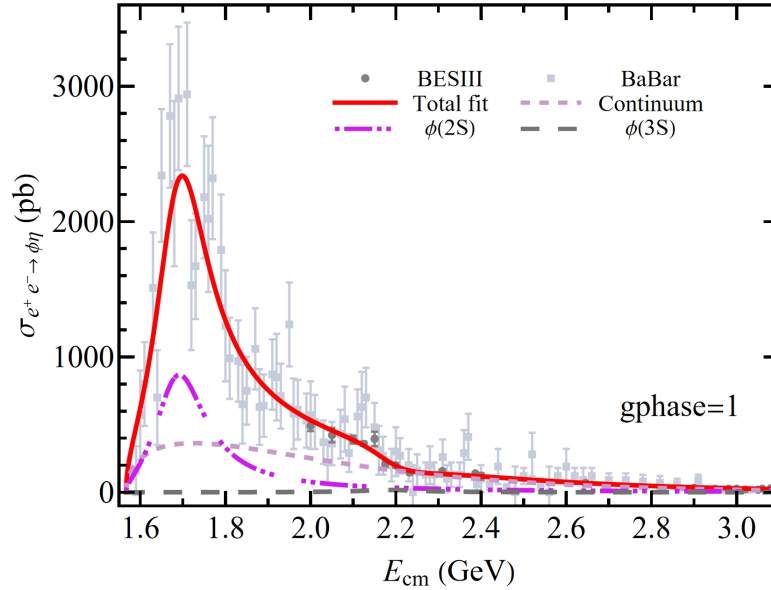


(h)

Recent theoretical researches: understanding the puzzle phenomenon associated with $\phi\eta$ and $\phi\eta'$

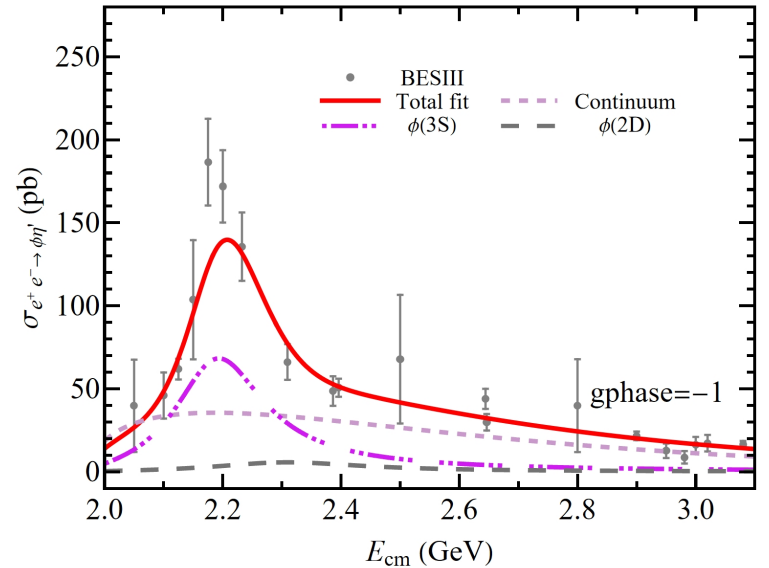
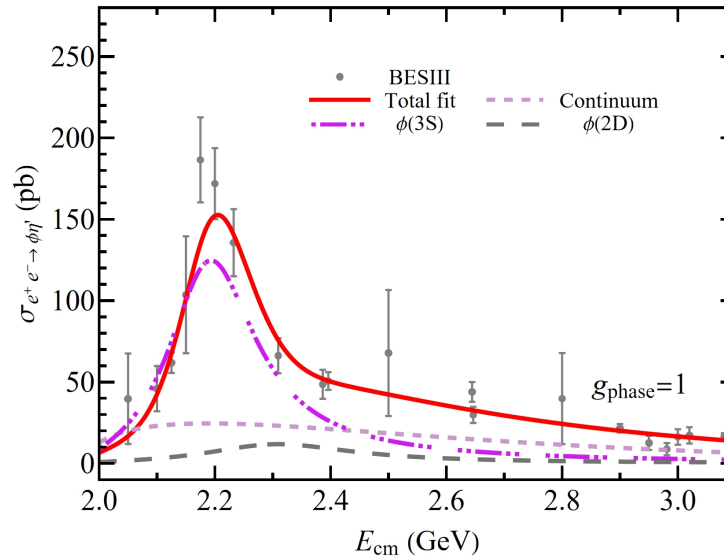
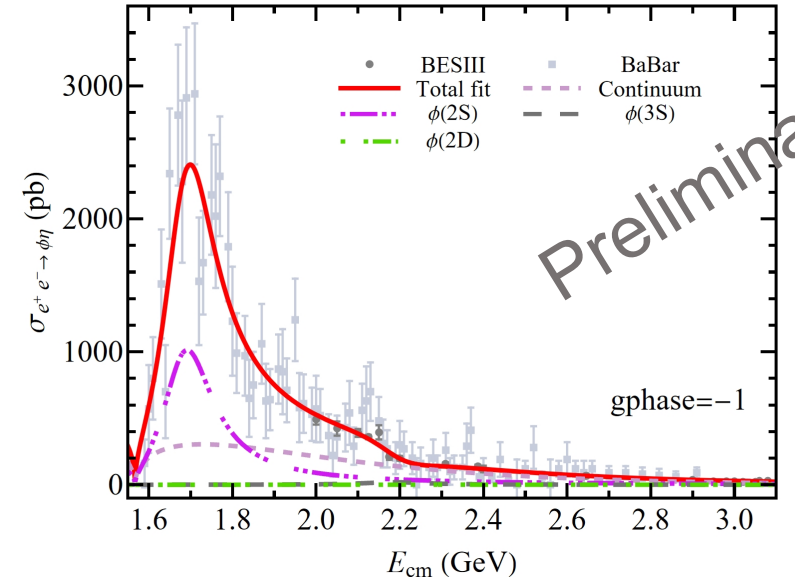
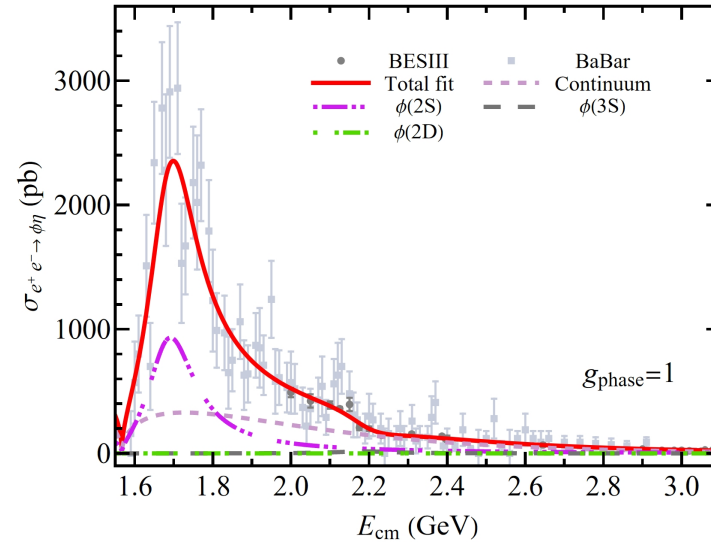
Preliminary

Parameters	$g_{\text{phase}} = 1$	$g_{\text{phase}} = -1$
C_1 ($^{-3}$)	2.14 ± 0.02	2.00 ± 0.02
n_1 (GeV^{-2})	1.68 ± 0.01	1.67 ± 0.01
ϕ_{11} (rad)	1.16 ± 0.03	1.09 ± 0.04
ϕ_{12} (rad)	4.00 ± 0.07	4.00 ± 0.07
C_2 ($^{-3}$)	0.26 ± 0.01	0.31 ± 0.01
n_2 (GeV^{-2})	0.57 ± 0.05	0.71 ± 0.05
ϕ_{22} (rad)	0.80 ± 0.11	0.93 ± 0.10
α	3.62 ± 0.08	5.00 (upper limit)
$\chi^2/\text{n.d.f}$	1.08	1.08
$\Gamma_{\text{Loop}}(\phi(2S) \rightarrow \phi\eta)$ (MeV)	0.79	1.82
$\Gamma_{\text{Total}}(\phi(2S) \rightarrow \phi\eta)$ (MeV)	9.23	10.79
$\Gamma_{\text{Loop}}(\phi(3S) \rightarrow \phi\eta)$ (MeV)	0.23	0.26
$\Gamma_{\text{Total}}(\phi(3S) \rightarrow \phi\eta)$ (MeV)	1.75	1.97
$\Gamma_{\text{Loop}}(\phi(3S) \rightarrow \phi\eta')$ (MeV)	7.97	6.79
$\Gamma_{\text{Total}}(\phi(3S) \rightarrow \phi\eta')$ (MeV)	8.57	7.06



Recent theoretical researches: understanding the puzzle phenomenon associated with $\phi\eta$ and $\phi\eta'$

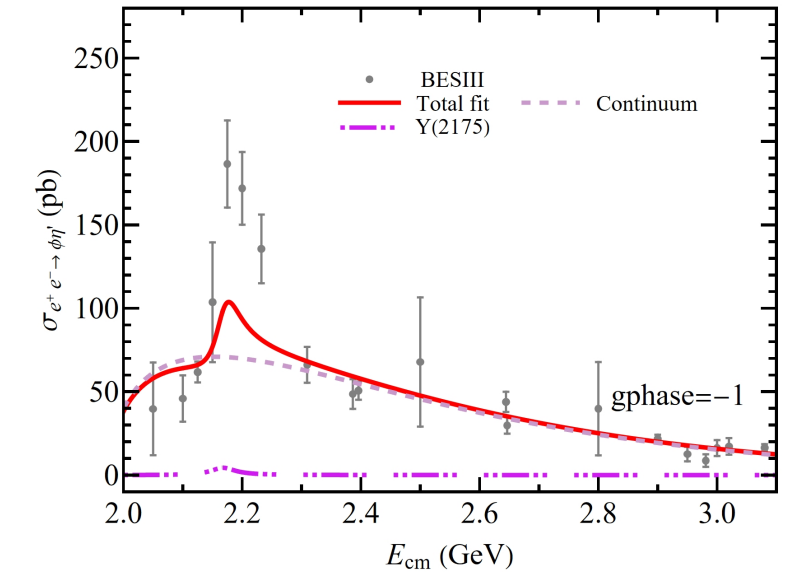
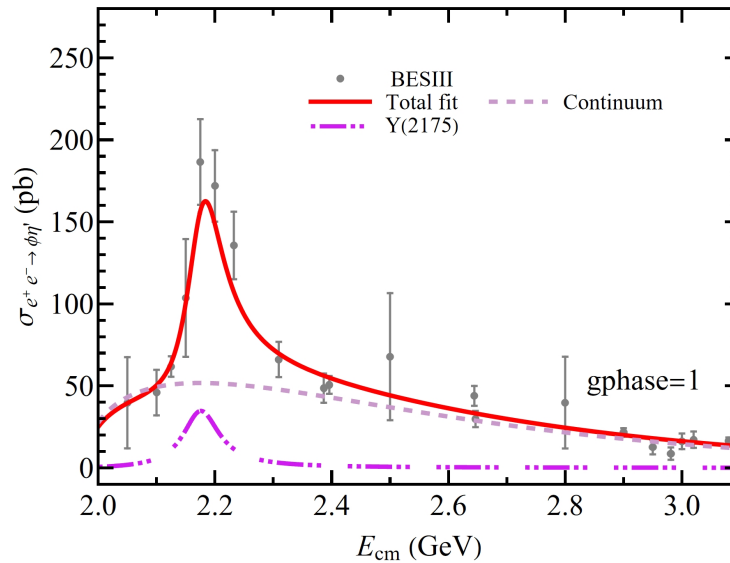
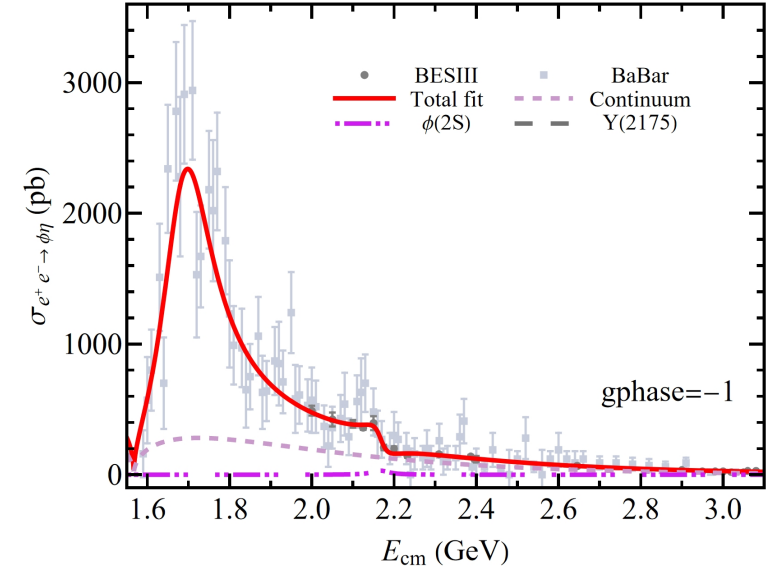
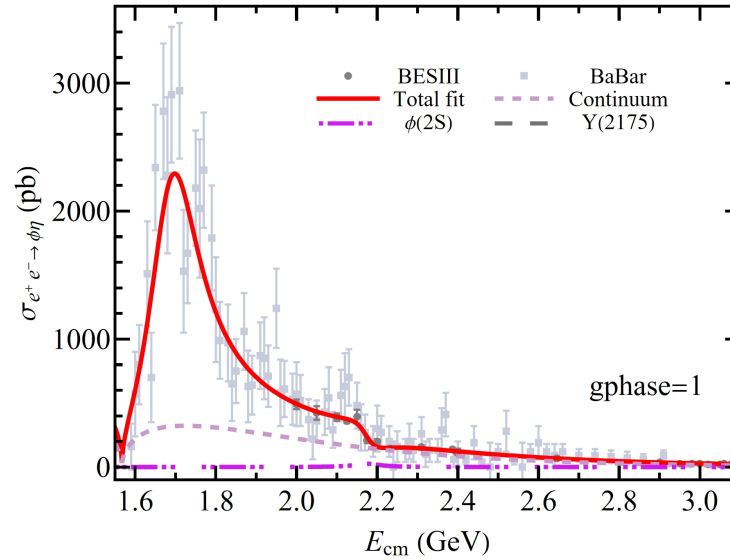
Parameters	$g_{\text{phase}} = 1$	$g_{\text{phase}} = -1$
C_1 ($^{-3}$)	2.03 ± 0.03	1.95 ± 0.03
n_1 (GeV^{-2})	1.67 ± 0.01	1.65 ± 0.02
ϕ_{11} (rad)	1.14 ± 0.04	1.11 ± 0.04
ϕ_{12} (rad)	3.98 ± 0.08	3.97 ± 0.08
ϕ_{13} (rad)	1.87 ± 0.65	1.79 ± 0.47
C_2 ($^{-3}$)	0.48 ± 0.02	0.58 ± 0.02
n_2 (GeV^{-2})	1.24 ± 0.05	1.26 ± 0.04
ϕ_{22} (rad)	0.60 ± 0.07	0.77 ± 0.07
ϕ_{23} (rad)	4.69 ± 0.11	4.54 ± 0.18
α	4.27 ± 0.39	5.00 (upper limit)
$\chi^2/\text{n.d.f}$	0.99	1.02
$\Gamma_{\text{Loop}}(\phi(2S) \rightarrow \phi\eta)$ (MeV)	1.21	1.82
$\Gamma_{\text{Total}}(\phi(2S) \rightarrow \phi\eta)$ (MeV)	9.93	10.79
$\Gamma_{\text{Loop}}(\phi(3S) \rightarrow \phi\eta)$ (MeV)	0.36	0.26
$\Gamma_{\text{Total}}(\phi(3S) \rightarrow \phi\eta)$ (MeV)	1.69	1.97
$\Gamma_{\text{Loop}}(\phi(3S) \rightarrow \phi\eta')$ (MeV)	12.09	6.79
$\Gamma_{\text{Total}}(\phi(3S) \rightarrow \phi\eta')$ (MeV)	12.89	7.06
$\Gamma_{\text{Loop}}(\phi(2D) \rightarrow \phi\eta)$ (MeV)	0.35	0.31
$\Gamma_{\text{Total}}(\phi(2D) \rightarrow \phi\eta)$ (MeV)	0.29	0.52
$\Gamma_{\text{Loop}}(\phi(2D) \rightarrow \phi\eta')$ (MeV)	18.95	9.71
$\Gamma_{\text{Total}}(\phi(2D) \rightarrow \phi\eta')$ (MeV)	23.72	11.39



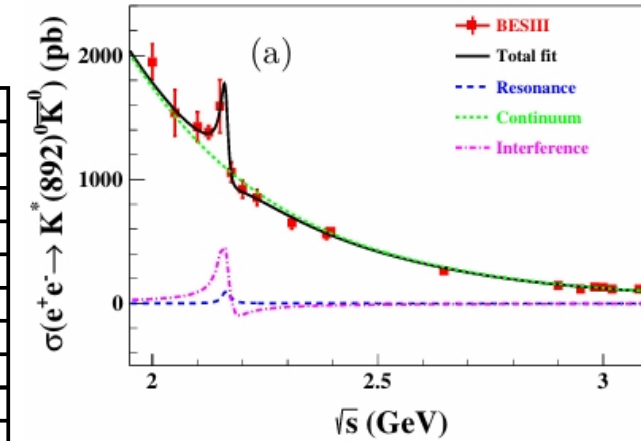
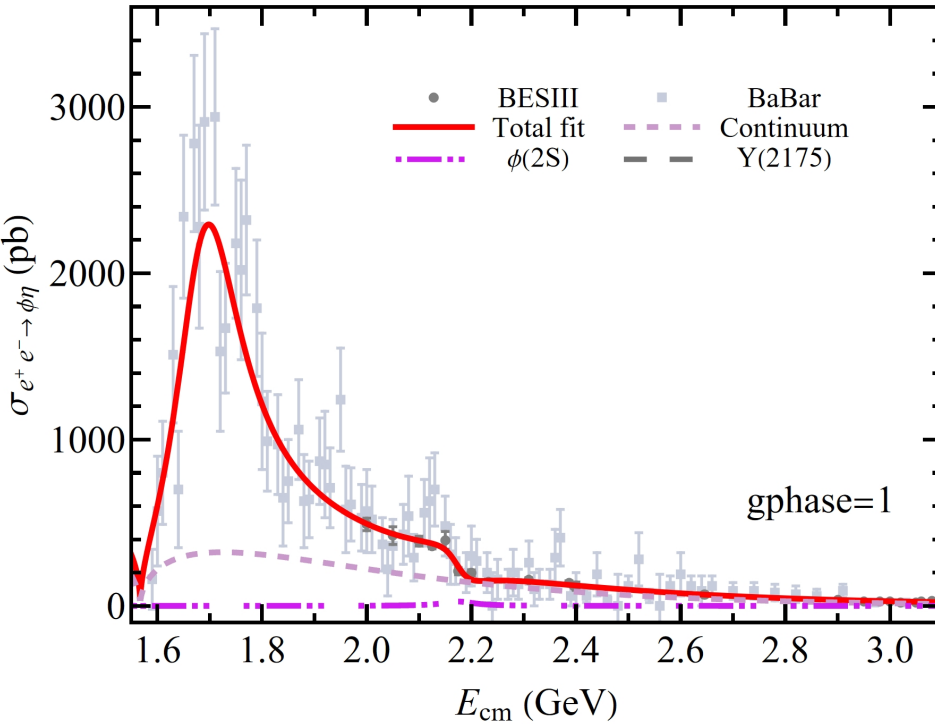
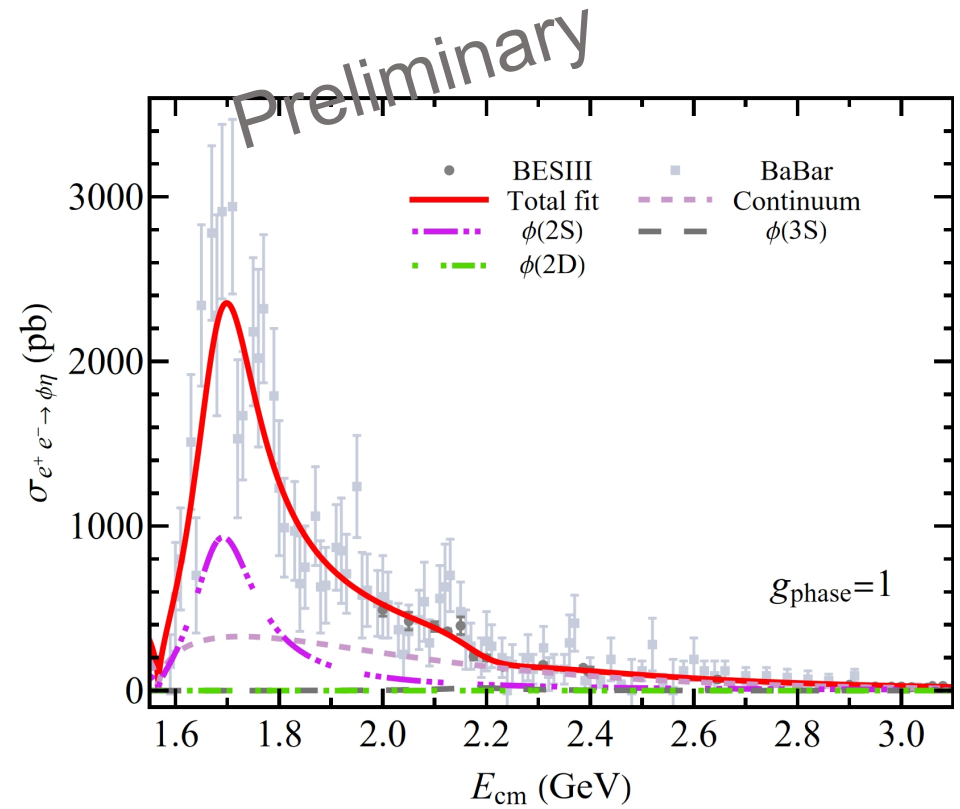
Recent theoretical researches: understanding the puzzle phenomenon associated with $\phi\eta$ and $\phi\eta'$

Preliminary

Parameters	$g_{\text{phase}} = 1$	$g_{\text{phase}} = -1$
C_1 ($^{-3}$)	2.01 ± 0.03	1.89 ± 0.10
n_1 (GeV^{-2})	1.69 ± 0.01	1.69 ± 0.06
ϕ_{11} (rad)	1.17 ± 0.04	1.12 ± 0.15
ϕ_{14} (rad)	3.80 ± 0.09	3.56 ± 0.19
C_2 ($^{-3}$)	0.70 ± 0.02	0.85 ± 0.04
n_2 (GeV^{-2})	1.32 ± 0.04	1.48 ± 0.07
$\Gamma_{Y(2175)}^{e^+e^-}$ (eV)	83.65 ± 11.79	47.57 ± 11.38
$M_{Y(2175)}$ (MeV)	2174.20 ± 3.63	2167.95 ± 4.77
$\Gamma_{Y(2175)}^{\text{Total}}$ (MeV)	75.54 ± 9.92	50 (lower limit)
ϕ_{24} (rad)	0.95 ± 0.10	0.70 ± 0.29
α	3.99 ± 0.11	5.00 (upper limit)
$\chi^2/\text{n.d.f}$	0.98	1.22
$\Gamma_{\text{Loop}}(\phi(2S) \rightarrow \phi\eta)$ (MeV)	1.02	1.82
$\Gamma_{\text{Total}}(\phi(2S) \rightarrow \phi\eta)$ (MeV)	9.62	10.79
$\Gamma_{\text{Loop}}(Y(2175) \rightarrow \phi\eta)$ (MeV)	0.03	0.02
$\Gamma_{\text{Total}}(Y(2175) \rightarrow \phi\eta)$ (MeV)	0.57	0.52
$\Gamma_{\text{Loop}}(Y(2175) \rightarrow \phi\eta')$ (MeV)	1.05	0.18
$\Gamma_{\text{Total}}(Y(2175) \rightarrow \phi\eta')$ (MeV)	1.24	0.12



Recent theoretical researches: understanding the puzzle phenomenon associated with $\phi\eta$ and $\phi\eta'$



Parameter	Solution 1	Solution 2
M_Y (MeV/ c^2)	$2164.7 \pm 9.1 \pm 3.1$	
Γ_Y (MeV)	$32.4 \pm 21.0 \pm 1.8$	
$\Gamma_Y^{e^+e^-} \mathcal{B}$ (eV)	$1.0 \pm 0.2 \pm 0.1$	$73.6 \pm 4.4 \pm 2.0$
ϕ_Y (rad)	$2.5 \pm 0.5 \pm 0.1$	$-1.7 \pm 0.1 \pm 0.1$
Significance	3.2σ	

[BESIII], JHEP01(2024)180

The existence of two Y(2175) around 2170 MeV? (a narrow state and a broad state, and the former may a good candidate of exotic vector light hadron)

Summary

- We reviewed the experimental and theoretical progress on $\phi(2170)$, with emphasis on the new constraints from hidden- and open-strangeness channels after 2019.
- We proposed a unified interpretation of two central puzzles: the channel-dependent resonance parameters and the unexpected enhancement in $\phi\eta'$. Both can be understood from the interference of spectroscopically supported 3S and 2D strangeonium states, with the $\phi\eta'$ enhancement naturally driven by kaon meson-loop effects.
- Although the present $\phi\eta$ and $\phi\eta'$ cross sections can be well described, the data may indicate an additional narrow vector structure. If confirmed, such a state would be a promising exotic candidate in the light vector sector.

Thanks for your attention!