Charmed $P \to V\ell\nu_\ell$ semileptonic decay with (2+1)-flavor lattice QCD

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CLQCD

Based on arXiv:2510.14478 and arXiv:26xx.xxxxx

2026年轻强子专题研讨会

Outline

- Motivation
- Method: LQCD and scalar function
- $D_s \to \phi \ell \nu_\ell$ channel
- $D \to K^* \ell \nu_\ell$ channel
- Summary

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Motivation

- Precise test for Standard Model and search for new physics: CKM matrix element, lepton flavor universality

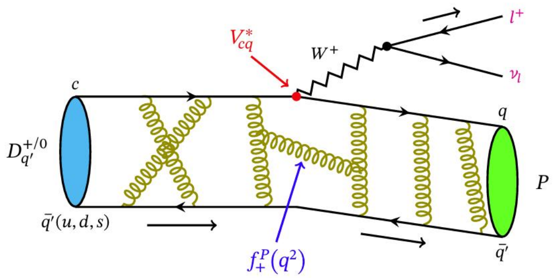
SM parameter

$$\frac{d\Gamma(P\to V\ell\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2|\boldsymbol{p}_\phi|\,q^2}{96\pi^3 M_{D_s}^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2\left[\left(1+\frac{m_\ell^2}{2q^2}\right)\left(|H_+|^2+|H_-|^2+|H_0|^2\right)+\frac{3m_\ell^2}{2q^2}|H_t|^2\right]$$

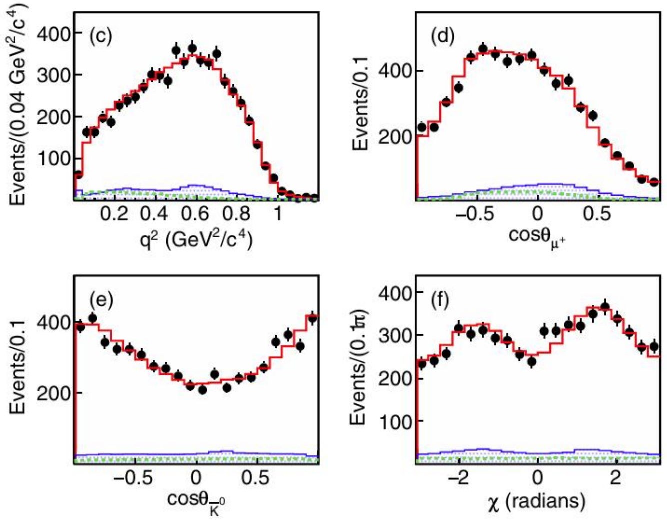
non-perturbative

Experimental data

- High-precison measurements (BESIII, etc) and theoretical calculations
- Vector meson decay brings additional polarization information
- Test the non-perturbative QCD in charm sector



courtesy arXiv:2103.00908



[BESIII, PRL 135, 111803 (2025)]

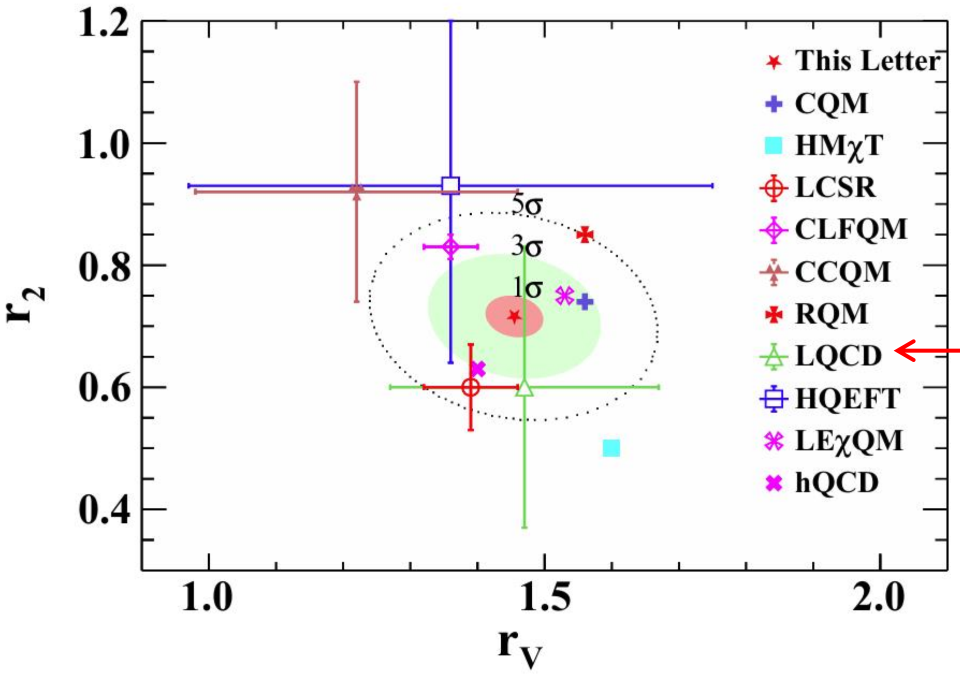
Motivation

- Status of theoretical and experimental studies

No full lattice calculation

Experiments	r_V	r_2
PDG [45]	1.80 ± 0.08	0.84 ± 0.11
This analysis	$1.58 \pm 0.17 \pm 0.02$	$0.71 \pm 0.14 \pm 0.02$
BABAR [26]	$1.807 \pm 0.046 \pm 0.065$	$0.816 \pm 0.036 \pm 0.030$
FOCUS [59]	$1.549 \pm 0.250 \pm 0.148$	$0.713 \pm 0.202 \pm 0.284$
Theory	r_V	r_2
CCQM [5]	1.34 ± 0.27	0.99 ± 0.20
CQM [6]	1.72	0.73
LFQM [7]	1.42	0.86
LQCD [3]	1.72 ± 0.21	0.74 ± 0.12
HMχT [8]	1.80	0.52

2014 → LQCD [3]



2003 → LQCD

$D_s \to \phi$ [BESIII, JHEP 12, 072 (2023)]

$D \to K^*$ [BESIII, PRL 135, 111803 (2025)]

A precise lattice calculation is important!

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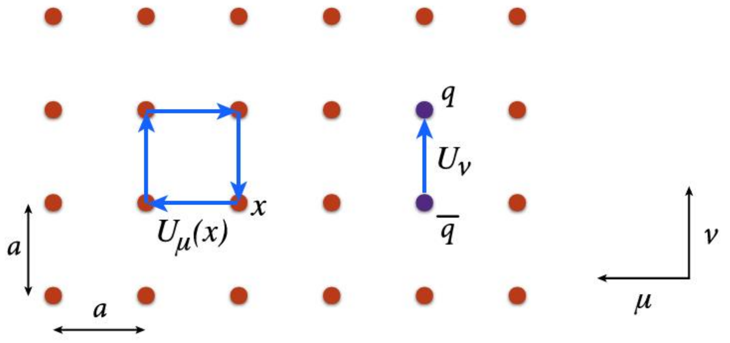
Introduction to lattice QCD

- Path integral in discrete Euclidean space

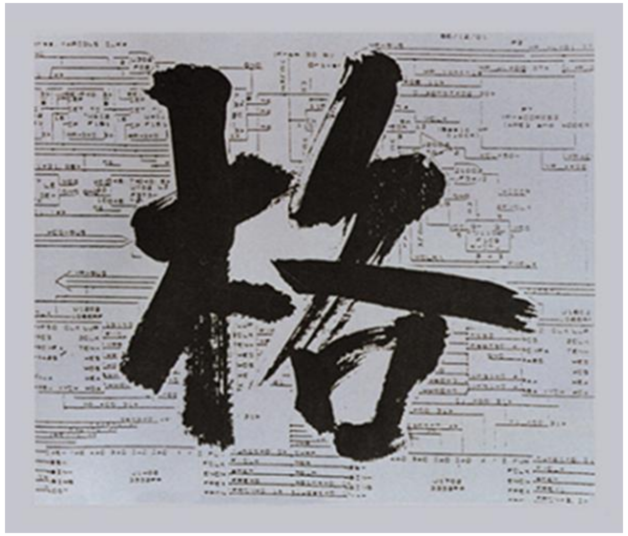
$$Z = \int [dU] \prod_f [dq_f][d\bar{q}_f] e^{-S_g[U] - \sum_f \bar{q}_f (D[U]+m_f) q_f}$$

$$Z = \int [dU] e^{-S_g[U]} \prod_f \det(D[U] + m_f)$$



- Expectation values of gauge-invariant operators, also known as “correlation functions”

$$\langle \mathcal{O}(U,q,\bar{q}) \rangle = (1/Z) \int [dU] \prod_f [dq_f][d\bar{q}_f] \mathcal{O}(U,q,\bar{q}) e^{-S_g[U] - \sum_f \bar{q}_f (D[U]+m_f) q_f}$$

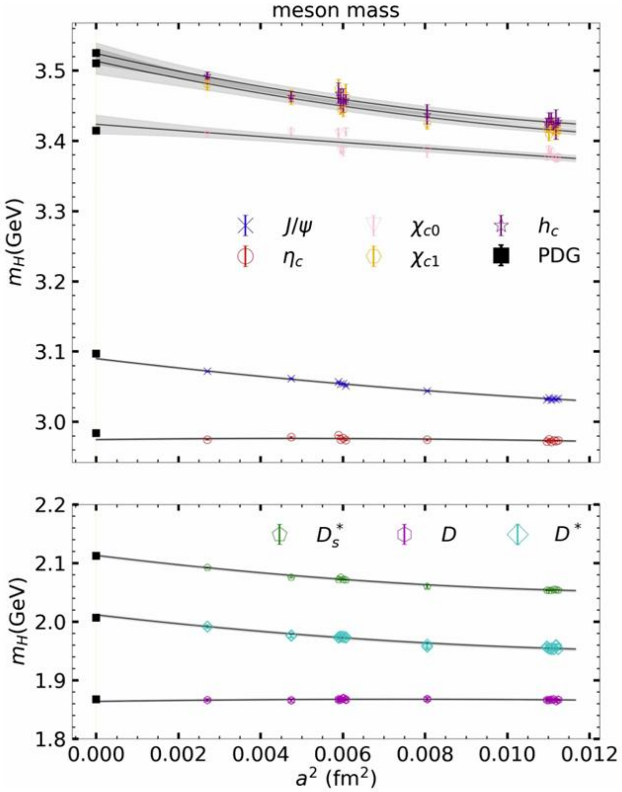


- Monte-Carlo method and data analysis

Lattice set up

- (2+1)-flavor Wilson-clover gauge ensembles [CLQCD, PRD 111, 054504 (2025)]
- Computer resources: “SongShan” supercomputer at Zhengzhou University
- Systematic calculation: four lattice spacing and four pion mass

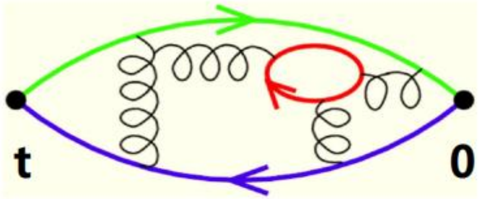
	C24P29	C32P23	C32P29	F32P30	F48P21	G36P29	H48P32
a (fm)		0.10524(05)(62)		0.07753(03)(45)		0.06887(12)(41)	0.05199(08)(31)
am_l	−0.2770	−0.2790	−0.2770	−0.2295	−0.2320	−0.2150	−0.1850
am_s	−0.2400	−0.2400	−0.2400	−0.2050	−0.2050	−0.1926	−0.1700
$am_s^{\rm V}$	−0.2356(1)	−0.2337(1)	−0.2358(1)	−0.2038(1)	−0.2025(1)	−0.1928(1)	−0.1701(1)
$am_c^{\rm V}$	0.4159(07)	0.4190(07)	0.4150(06)	0.1974(05)	0.1997(04)	0.1433(12)	0.0551(07)
aL (fm)	2.53	3.37	3.37	2.48	3.72	2.48	2.50
$L^3 \times T$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 64$	$32^3 \times 96$	$48^3 \times 96$	$36^3 \times 108$	$48^3 \times 144$
$N_{\rm mea}$	$450 \times 72 \times 2$	$333 \times 64 \times 3$	$397 \times 64 \times 2$	$360 \times 96 \times 2$	$241 \times 48 \times 4$	$300 \times 54 \times 2$	$300 \times 72 \times 2$
m_π (MeV)	292.3(1.0)	227.9(1.2)	293.1(0.8)	300.4(1.2)	207.5(1.1)	297.2(0.9)	316.6(1.0)
$m_\pi L$	3.746(13)	3.894(20)	5.008(14)	3.780(15)	3.917(22)	3.731(11)	4.000(12)
t	2 − 17	2 − 20	2 − 20	4 − 22	4 − 26	2 − 32	8 − 30
Z_V^s	0.85184(06)	0.85350(04)	0.85167(04)	0.86900(03)	0.86880(02)	0.87473(05)	0.88780(01)
Z_V^c	1.57353(18)	1.57644(12)	1.57163(14)	1.30566(07)	1.30673(04)	1.23990(13)	1.12882(11)
Z_A/Z_V	1.07244(70)	1.07375(40)	1.07648(63)	1.05549(54)	1.05434(88)	1.04500(22)	1.03802(28)



Correlation function

- 2-point correlation function (2pt), to extract mass and decay constant

$$C^{(2)}(\vec{p}, t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_h(\vec{x}, t)\mathcal{O}_h^\dagger(0)\rangle$$



两点关联函数

- 3-point correlation function (3pt), to extract form factor with 2pt

$$\begin{aligned} C_{\mu\nu}(\vec{x}, t, t_s) &= \langle \mathcal{O}_{V_\nu}(t) J_\mu^W(0) \mathcal{O}_P^\dagger(-t_s)\rangle \\ &= \langle \bar{x}(t)\gamma_\nu s(t)\bar{s}(0)\gamma_\mu(1-\gamma_5)c(0)\bar{c}(-t_s)\gamma_5 x(-t_s)\rangle \\ &= \langle \mathrm{Tr}[\gamma_5\gamma_5 S_x^\dagger(t, -t_s)\gamma_5\gamma_\nu S_s(t, 0)\gamma_\mu(1-\gamma_5)S_c(0, -t_s)]\rangle \end{aligned}$$

Scalar function method

- A_3 is not an independent form factor

$$A_3\left(q^2\right)=\frac{M+m}{2m}A_1\left(q^2\right)-\frac{M-m}{2m}A_2\left(q^2\right)$$

- $A_3(0)=A_0(0)$ is the kinematic constraint

- The parameterization for $P\rightarrow V$ semileptonic matrix element

$$\langle V(\varepsilon,\vec{p})|J_\mu^W(0)|P(p')\rangle=\varepsilon_\nu^*\varepsilon_{\mu\nu\alpha\beta}p'_\alpha p_\beta\frac{2V}{m+M}+(M+m)\varepsilon_\mu^*A_1+\frac{\varepsilon^*\cdot q}{M+m}(p+p')_\mu A_2-2m\frac{\varepsilon^*\cdot q}{q^2}q_\mu(A_0-A_3)$$

$$\langle V_\sigma(\vec{p})|J_\mu^W(0)|P(p')\rangle=\frac{F_0(q^2)}{Mm}\epsilon_{\mu\sigma\alpha\beta}p'_\alpha p_\beta+F_1(q^2)\delta_{\mu\sigma}+\frac{F_2(q^2)}{Mm}p_\mu p'_\sigma+\frac{F_3(q^2)}{M^2}p'_\mu p'_\sigma$$

- Correlation functions ⟶ Scalar functions ⟶ Form factors

$$\langle\phi_\sigma(\vec{p})|J_\mu^W(0)|D_s(p')\rangle\longrightarrow\tilde{\mathcal{I}}_j\longrightarrow V,A_0,A_1,A_2$$

$$\mathcal{I}_0=\frac{1}{M|\vec{p}|^2}\epsilon_{\mu\nu\alpha'\beta'}p'_{\alpha'}p_{\beta'}\tilde{V}_{\mu\nu}=\frac{1}{|\vec{p}|^2}\epsilon_{\mu\nu0\beta}p_\beta\int\mathrm{d}^3\vec{x}\sin(\vec{p}\cdot\vec{x})V_{\mu\nu}(\vec{x},t),$$

$$\mathcal{I}_1=\delta_{\mu\nu}\tilde{A}_{\mu\nu}=\delta_{\mu\nu}\int\mathrm{d}^3\vec{x}e^{-i\vec{p}\cdot\vec{x}}A_{\mu\nu}(\vec{x},t),$$

$$\mathcal{I}_2=\frac{E}{M}\frac{p_\mu p'_\nu}{|\vec{p}|^2}\tilde{A}_{\mu\nu}=-\frac{E^2}{|\vec{p}|^2}\int\mathrm{d}^3\vec{x}\cos(\vec{p}\cdot\vec{x})A_{00}(\vec{x},t)+\frac{E}{|\vec{p}|^2}\int\mathrm{d}^3\vec{x}\sin(\vec{p}\cdot\vec{x})p_iA_{i0}(\vec{x},t),$$

$$\mathcal{I}_3=\frac{p'_\mu p'_\nu}{|\vec{p}|^2}\tilde{A}_{\mu\nu}=-\frac{M^2}{|\vec{p}|^2}\int\mathrm{d}^3\vec{x}\cos(\vec{p}\cdot\vec{x})A_{00}(\vec{x},t).$$

$$\mathcal{I}_0=\frac{2F_0}{m}\times\frac{Z_\phi e^{-Et}}{2E},$$

$$\mathcal{I}_1=\left(3F_1+\frac{E^2-m^2}{m^2}F_3\right)\times\frac{Z_\phi e^{-Et}}{2E},$$

$$\mathcal{I}_2=\left(-\frac{E}{m}F_2-\frac{E^2}{m^2}F_3\right)\times\frac{Z_\phi e^{-Et}}{2E},$$

$$\mathcal{I}_3=\left(\frac{M^2}{m^2}F_1-\frac{EM^2}{m^3}F_2-\frac{M^2}{m^2}F_3\right)\times\frac{Z_\phi e^{-Et}}{2E}.$$

Scalar function method

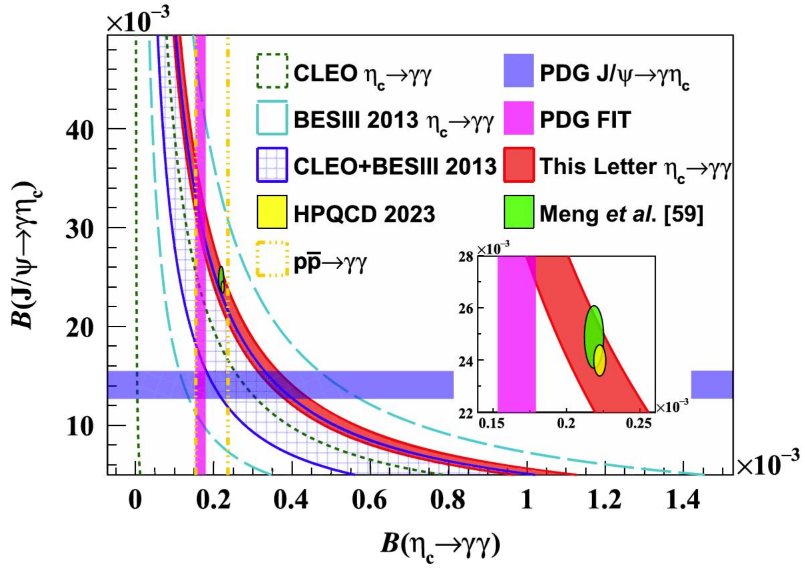
- A similar scalar function scheme has been used for high-precision calculation

  - $\mathrm{Br}(J/\psi \to \gamma \eta_c) = 2.49(11)_{\mathrm{lat}}(5)_{\mathrm{exp}}\%$

  - $\mathrm{Br}(J/\psi \to D e \nu_e) = 1.21(11) \times 10^{-11}$, $\mathrm{Br}(J/\psi \to D \mu \nu_\mu) = 1.18(11) \times 10^{-11}$
    $\mathrm{Br}(J/\psi \to D_s e \nu_e) = 1.90(8) \times 10^{-10}$, $\mathrm{Br}(J/\psi \to D_s \mu \nu_\mu) = 1.84(8) \times 10^{-10}$

  - $\Gamma(D_s^* \to \gamma D_s) = 0.0549(54)\ \mathrm{keV}$

  - $\Gamma(\eta_c \to 2\gamma) = 6.67(16)_{\mathrm{stat}}(6)_{\mathrm{syst}}\ \mathrm{keV}$



[BESIII, PRL 134, 181901 (2025)]

[Y. M et al, Science Bulletin 68, 1880 (2023)]
[Y. M, ..., Z. Liu, et al, PRD 109, 074511 (2024)]
[Y. M et al, PRD 110, 074510 (2024)]
[Y. M et al, PRD 111, 014508 (2025)]

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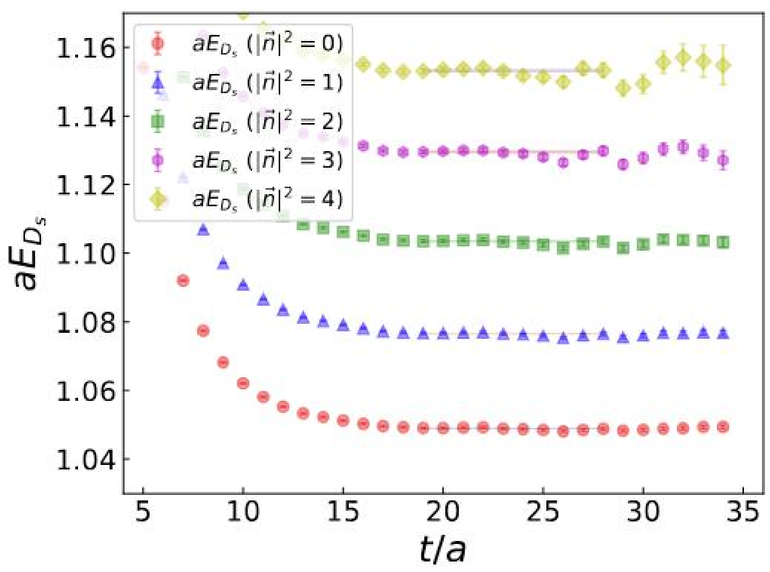
Two-point function of D_s

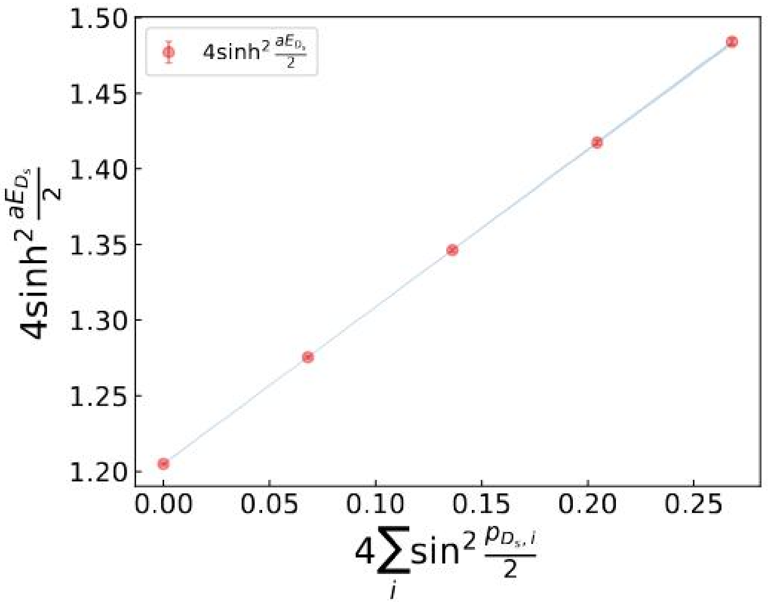
- Least χ^2 fitting considering covariance matrix between configurations and time
- There should be a plateau when meson ground states are dominant
- Check the discrete dispersion relation

$$C^{(2)}(\vec{p}, t) = \frac{Z_h^2}{2E_h}\left(e^{-E_h t} + e^{-E_h(T-t)}\right)$$

$$4\sinh^2\frac{E_h}{2} = 4\sinh^2\frac{m_h}{2} + \mathcal{Z}_{\text{latt}}^h \cdot 4\sum_i \sin^2\frac{p_i}{2}$$



(i) For C24P29 D_s meson.



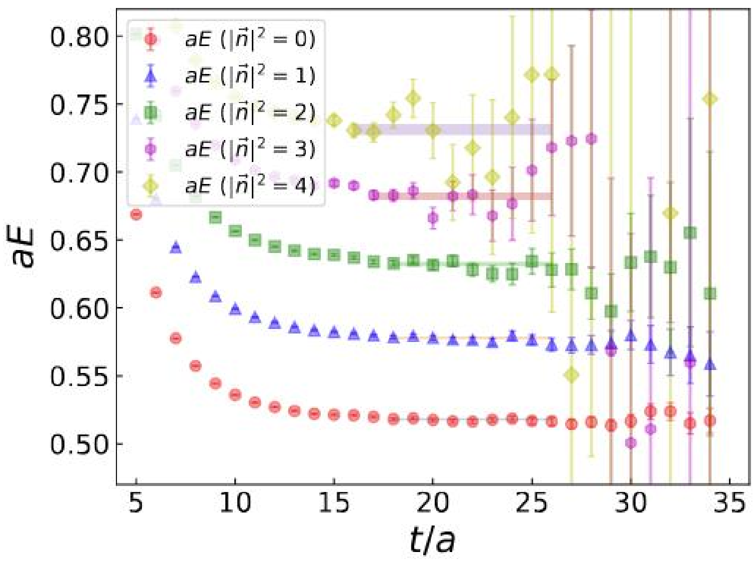
(ii) For C24P29 D_s meson.

Two-point function of ϕ

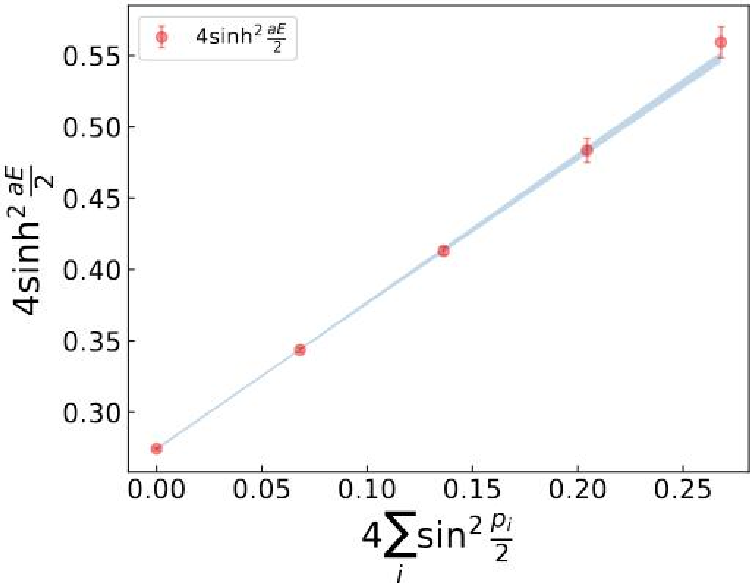
- Least χ^2 fitting considering covariance matrix between configurations and time
- There should be a plateau when meson ground states are dominant
- Check the discrete dispersion relation

$$C^{(2)}(\vec{p},t) = \left(-1 - \frac{|\vec{p}|^2}{3m_h^2}\right)\frac{Z_h^2}{2E_h}\left[e^{-E_h t} + e^{-E_h(T-t)}\right]$$

$$4\sinh^2\frac{E_h}{2} = 4\sinh^2\frac{m_h}{2} + \mathcal{Z}_{\mathrm{latt}}^h \cdot 4\sum_i \sin^2\frac{p_i}{2}$$



(iii) For C24P29 ϕ meson.



(iv) For C24P29 ϕ meson.

Mass and decay constant

- We extrapolate mass and decay constant of ϕ meson to get physical results

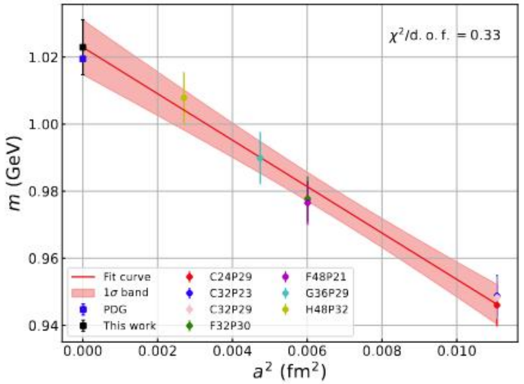
$$f_\phi = \frac{Z_V^S Z_\phi}{m}$$

$$m \text{ or } f_\phi = c + da^2 + f(m_\pi^2 - m_{\pi,\text{phys}}^2)$$

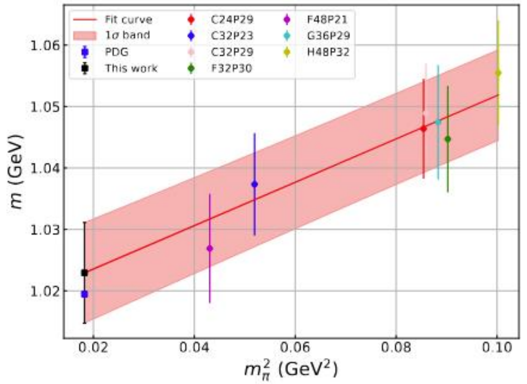
$$m = 1.0229(85)\ \text{GeV}/c^2$$

$$f_\phi = 0.2456(60)\ \text{GeV}$$

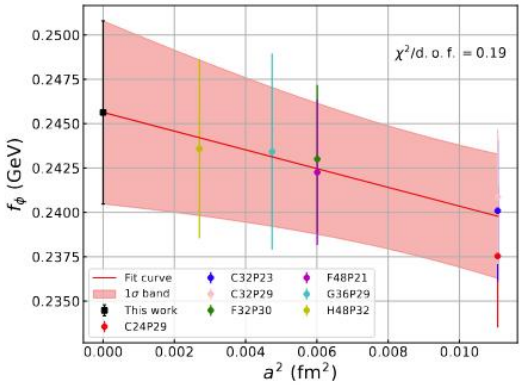
	m (GeV)	f_ϕ (GeV)
Central value	**1.0229**	**0.2456**
Statistical (total)	**0.0077**	**0.0040**
Two-point function	0.0029	0.0033
Lattice spacing	0.0071	0.0023
Systematic (total)	**0.0035**	**0.0045**
Fit range	0.0027	0.0032
Excited state	0.0006	0.0006
Fit ansatz (such as the cross term $a^2 m_\pi^2$ and higher-order term a^4, m_π^4)	0.0019	0.0008
Finite volume	0.0010	0.0029
This work	**1.0229(85)**	**0.2456(60)**



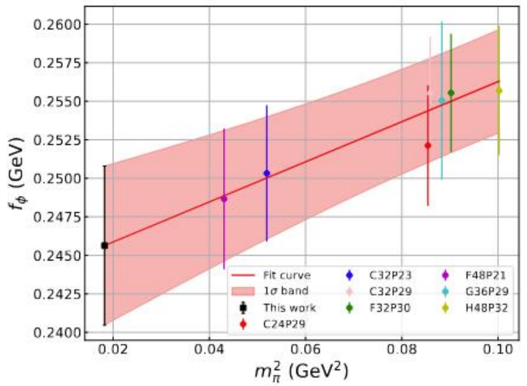
(i) ϕ mass at $m_\pi = 0.135$ GeV.



(ii) ϕ mass at $a = 0.0$ fm.



(iii) ϕ decay constant at $m_\pi = 0.135$ GeV.



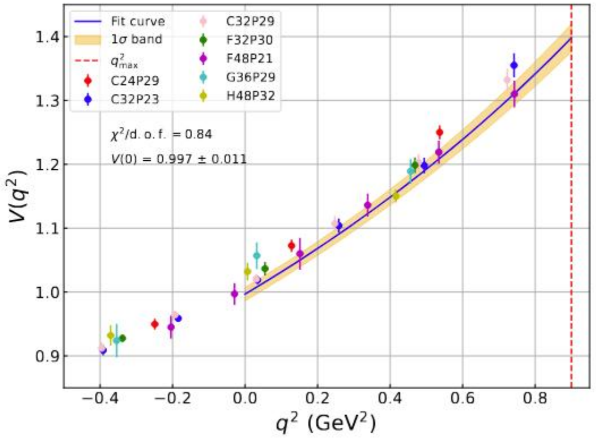
(iv) ϕ decay constant at $a = 0.0$ fm.

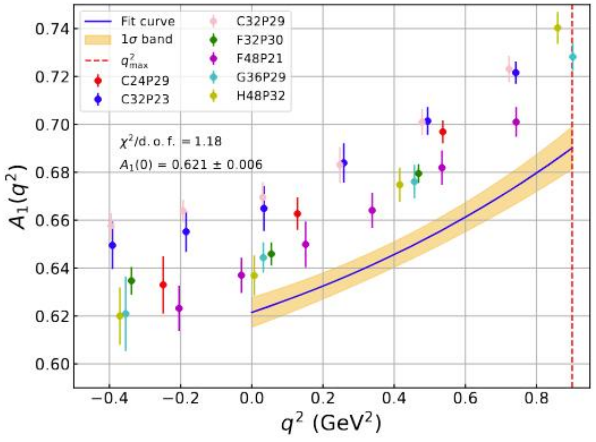
Form factor

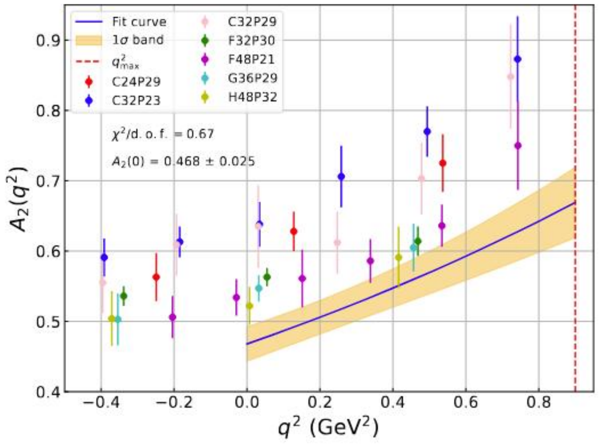
- Extrapolate results to the physical pion mass and continuum limit using z-expansion

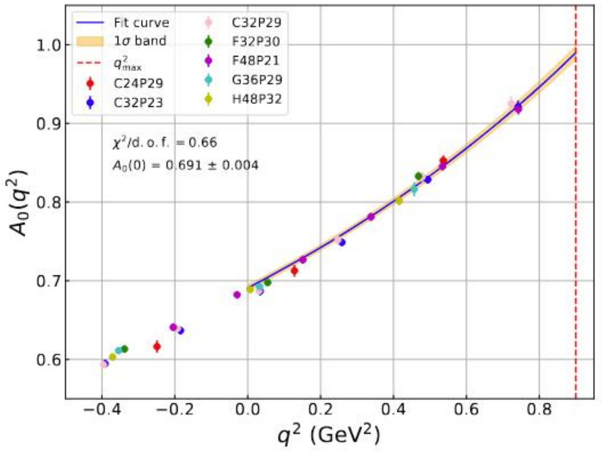
$$F\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m_{\text{pole}}^2} \sum_{i=0}^{2} \left(c_i + d_i a^2\right) \left[1 + f_i \left(m_\pi^2 - m_{\pi,\text{phys}}^2\right)\right] z^i \qquad z\left(q^2, t_0\right) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- $A_3(0) - A_0(0) = 0.002(15)$, consistent with zero



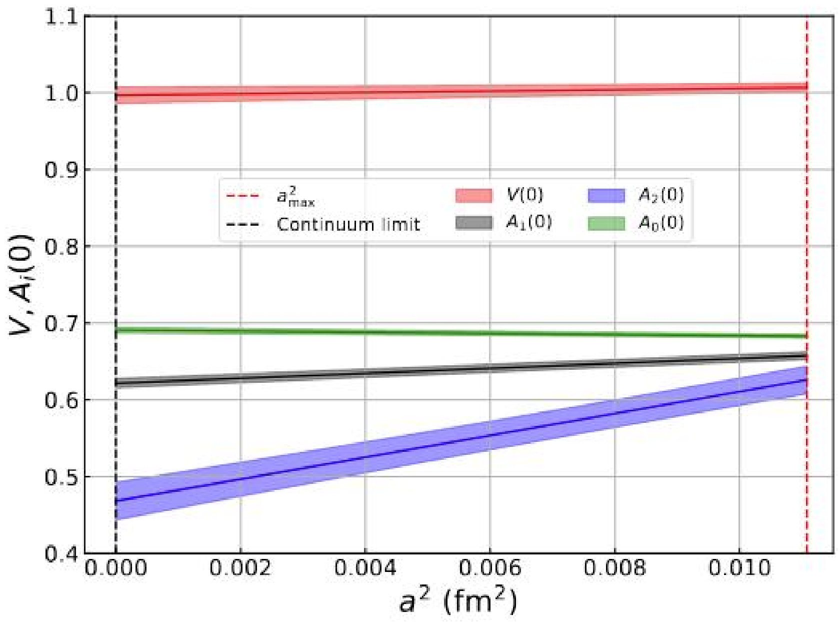




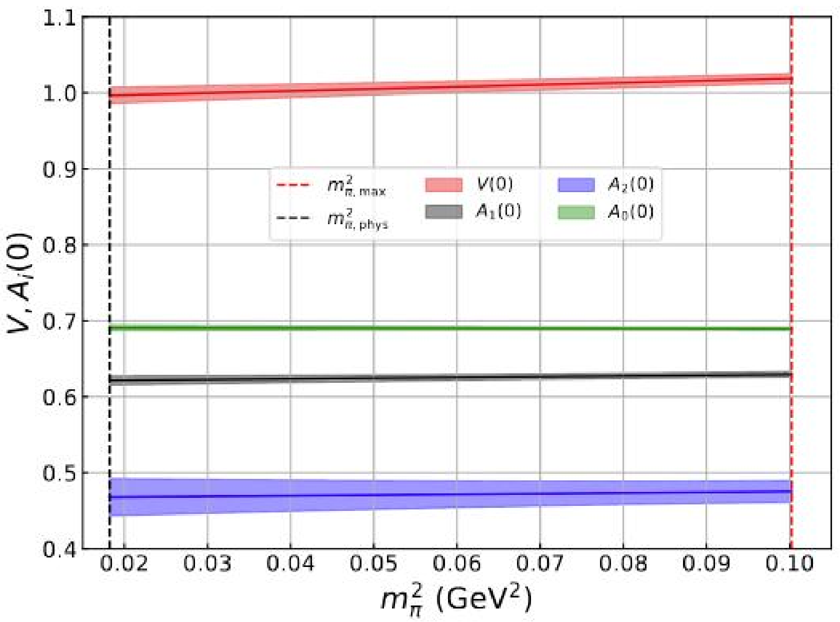


a^2 and m_π^2 dependence

- The form factors can be described well by the a^2-order term and the m_π^2-order term



(i) Form factors at $m_\pi = 0.135$ GeV.



(ii) Form factors at $a = 0.0$ fm.

Systematic uncertainty

- Fit range and excited-state $c + de^{-\delta m_{D_s} t} + fe^{-\delta m_\phi t}$
- Fit ansatz

$$F\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/h^2}\left(c + da^2\right)\left[1 + f\left(m_\pi^2 - m_{\pi,\text{phys}}^2\right)\right]$$

$$F\left(q^2, a, m_\pi\right) = \frac{1}{\left(1 - q^2/m_{\text{pole}}^2\right)\left(1 - hq^2/m_{\text{pole}}^2\right)}\left(c + da^2\right)\left[1 + f\left(m_\pi^2 - m_{\pi,\text{phys}}^2\right)\right]$$

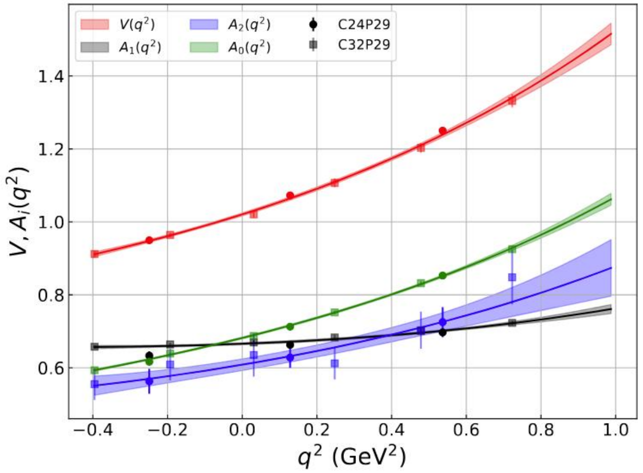
$$F\left(q^2, a, m_\pi\right) = F\left(s_0\right)\left(1 + da^2\right)\left[1 + f\left(m_\pi^2 - m_{\pi,\text{phys}}^2\right)\right]\prod_{n=0}^{\infty}\exp\left(\frac{q^2 - s_0}{s_{\text{th}}}\mathcal{A}_n^F\frac{q^{2n}}{s_{\text{th}}^n}\right)$$

	z-expansion	Single pole	Modified pole	Phase moment
$V(0)$	0.997(13)	0.997(13)	1.000(13)	0.992(13)
$A_1(0)$	0.6215(71)	0.6212(64)	0.6213(64)	0.6180(64)
$A_2(0)$	0.468(27)	0.478(26)	0.479(26)	0.471(26)
$A_0(0)$	0.6908(52)	0.6873(50)	0.6884(50)	0.6877(50)
$A_3(0) - A_0(0)$	0.002(15)	0.001(14)	−0.001(14)	−0.001(14)
r_V	1.604(25)	1.605(23)	1.610(23)	1.606(24)
r_2	0.753(44)	0.769(42)	0.770(42)	0.762(41)
r_0	1.112(14)	1.106(13)	1.108(13)	1.113(13)

- Finite-volume effect

	C24P29	χ^2/d.o.f.	C32P29	χ^2/d.o.f.	Combined	χ^2/d.o.f.
$V(0)$	1.0284(74)	0.10	1.0171(49)	0.25	1.0205(41)	0.51
$A_1(0)$	0.6523(63)	0.10	0.6703(34)	0.10	0.6658(29)	1.10
$A_2(0)$	0.606(21)	0.10	0.614(23)	0.45	0.608(16)	0.31
$A_0(0)$	0.6774(61)	0.10	0.6824(26)	0.26	0.6817(24)	0.31

$$F\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m_{\text{pole}}^2}\sum_{i=0}^{2}\left(c_i + d_i a^2\right)\left[1 + f_i\left(m_\pi^2 - m_{\pi,\text{phys}}^2\right)\left(1 + g_i e^{-m_\pi L}\right)\right]z^i$$

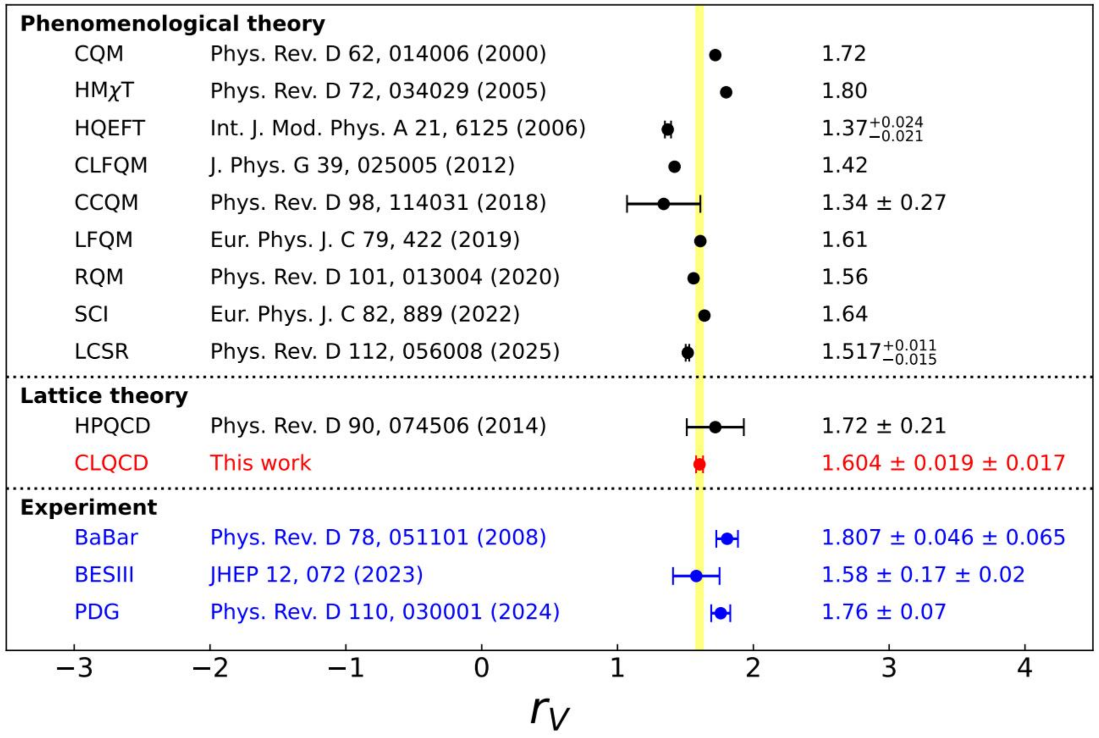


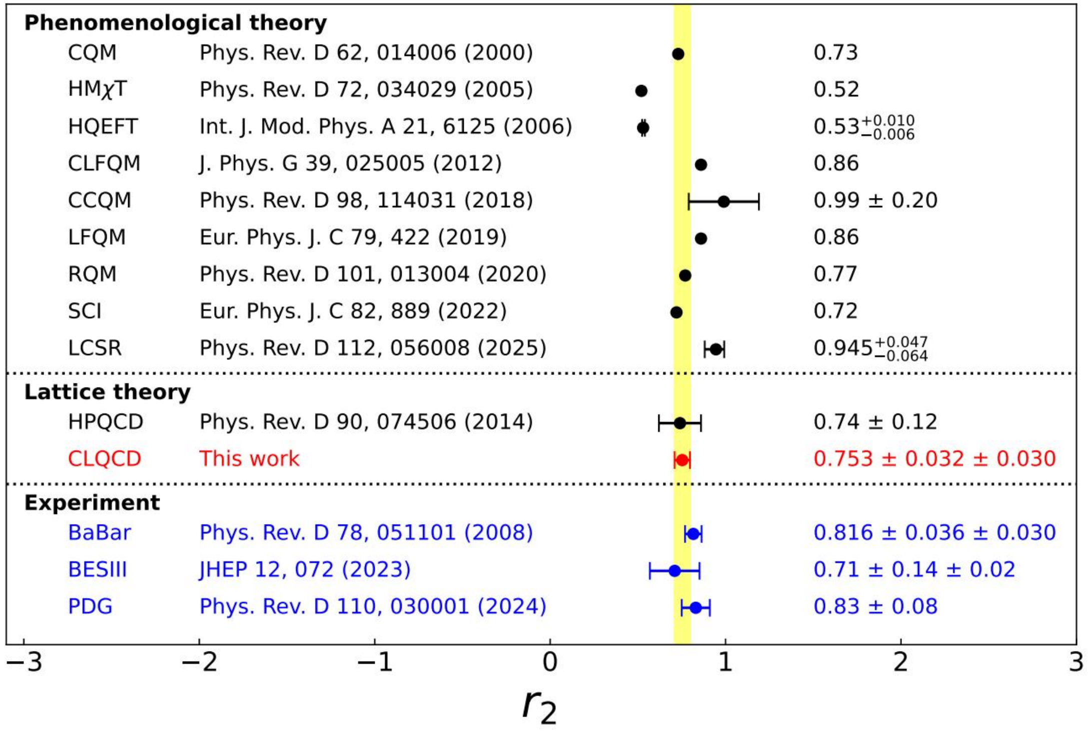
● C24P29, $L = 2.53$ fm

■ C32P23, $L = 3.37$ fm

Comparision with experiments

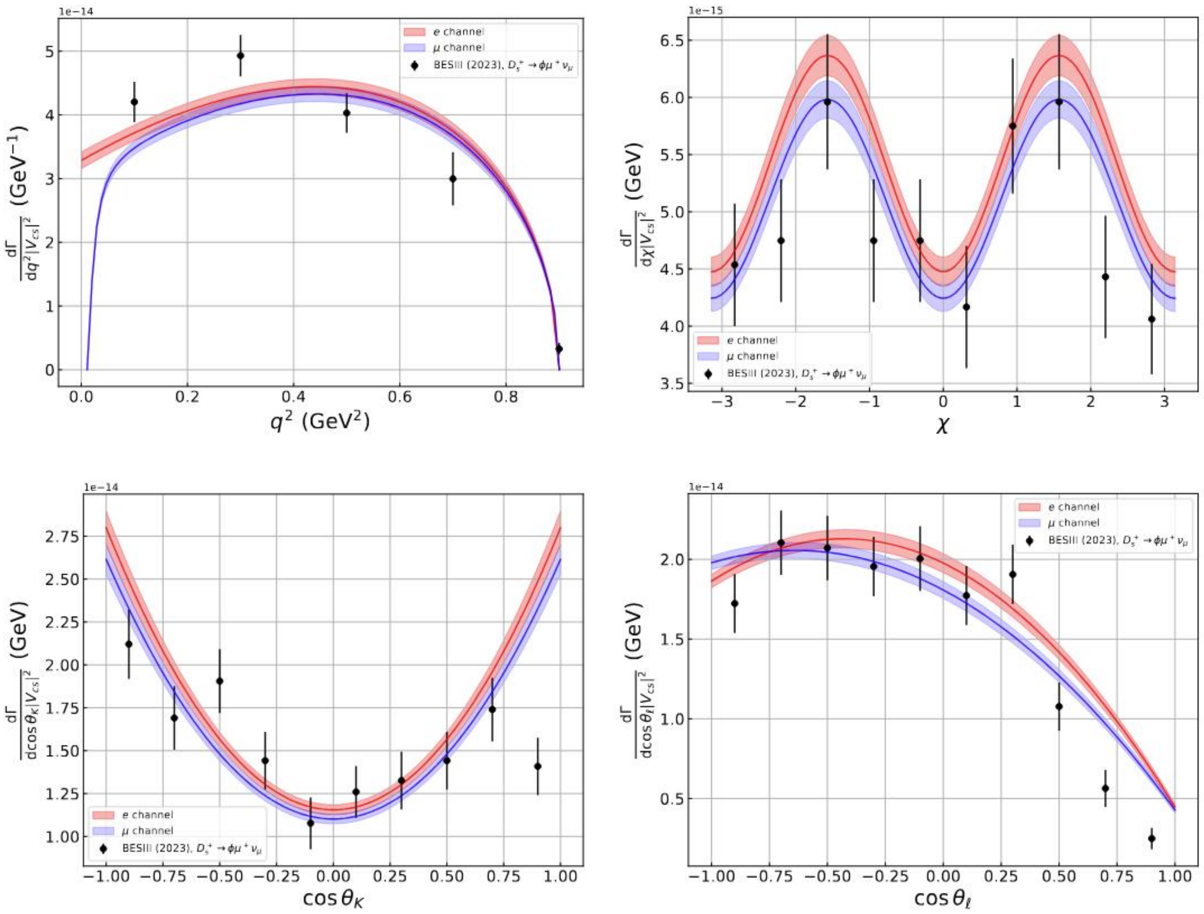
- Key physical quantities: $r_V \equiv \frac{V(0)}{A_1(0)}$ $r_2 \equiv \frac{A_2(0)}{A_1(0)}$

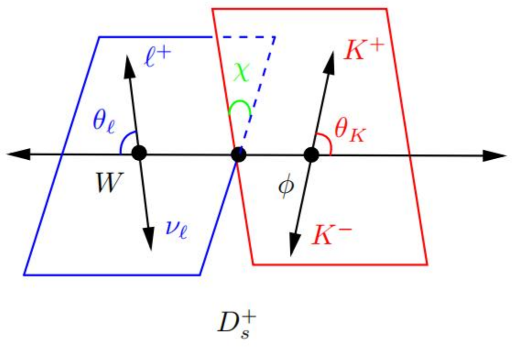




Differential decay width

- Differential decay width [M. Ivanov et al, Front. Phys. (Beijing) 14 (2019) 64401]





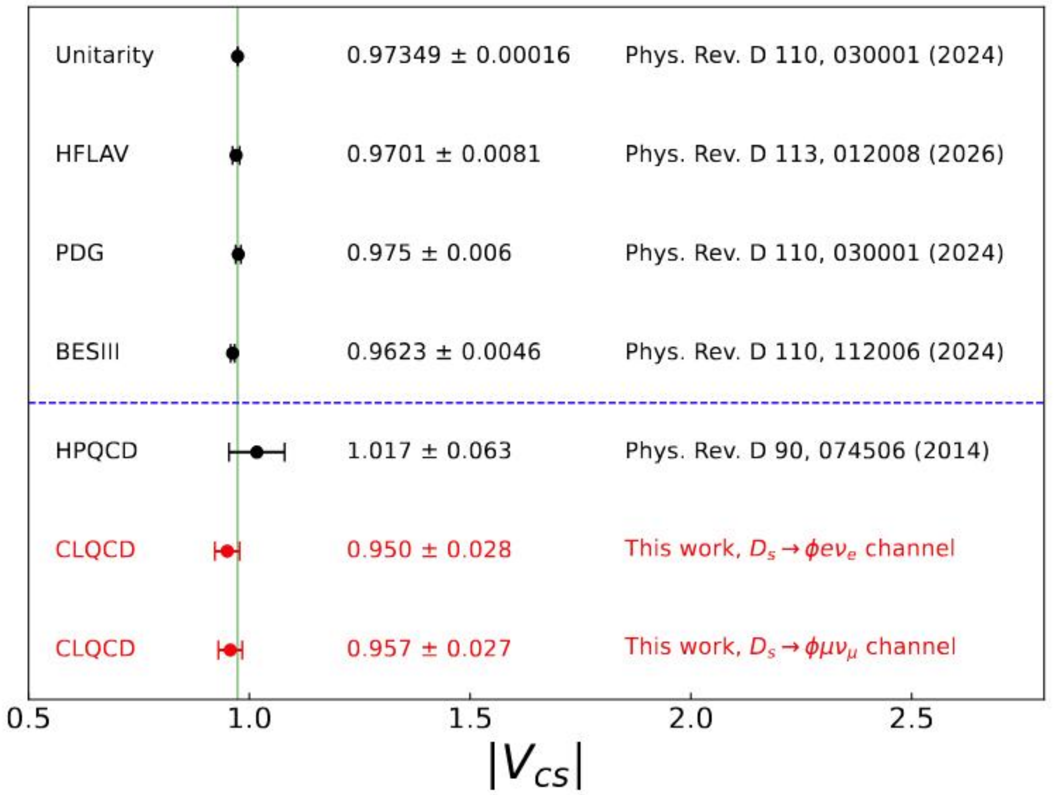
$\mathcal{B}(D_s \to \phi\ell\nu_\ell) \times 10^2$	e channel	μ channel	$\mathcal{R}_{\mu/e}$
This work (z-expansion)	2.466(79)	2.326(74)	0.9432(18)
This work (Single pole)	2.488(69)	2.348(63)	0.9438(18)
This work (Modified pole)	2.489(69)	2.349(63)	0.9438(17)
This work (Phase moment)	2.474(68)	2.335(62)	0.9441(17)
BaBar [9]	2.61(17)	–	–
CLEO [10]	2.14(19)	–	–
BESIII (2018) [11]	2.26(46)	1.94(54)	0.86(29)
BESIII (2023) [12]	–	2.25(11)	–
PDG [42]	2.34(12)	2.24(11)	0.957(68)

CKM matrix element

- Extract $|V_{cs}|$ by comparing with PDG

$$|V_{cs}| = \sqrt{\frac{1}{\Gamma_{\mathrm{latt}}} \times \frac{\mathcal{B}_{\mathrm{PDG}} \times \hbar}{\tau_{D_s}}}$$

	e channel	μ channel
z-expansion	0.950(28)	0.957(27)
Single pole	0.946(27)	0.952(26)
Modified pole	0.945(27)	0.952(26)
Phase moment	0.948(27)	0.955(26)



Outline

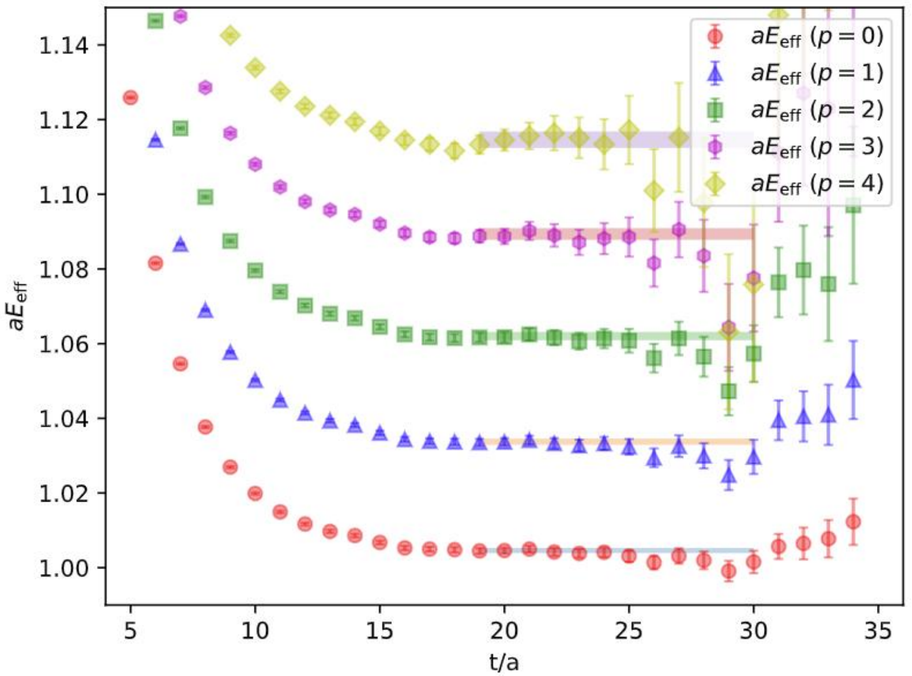
- Motivation
- Method: LQCD and scalar function
- $D_s \to \phi \ell \nu_\ell$ channel
- $D \to K^* \ell \nu_\ell$ channel
- Summary

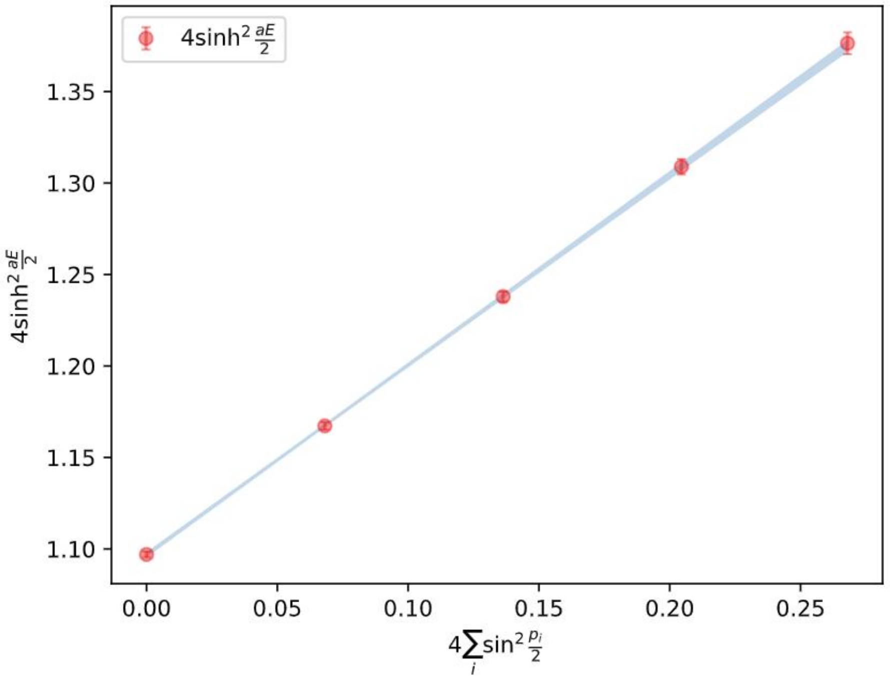
Two-point function of D

- Least χ^2 fitting considering covariance matrix between configurations and time
- There should be a plateau when meson ground states are dominant
- Check the discrete dispersion relation

$$C^{(2)}(\vec{p}, t) = \frac{Z_h^2}{2E_h}\left(e^{-E_h t} + e^{-E_h(T-t)}\right)$$

$$4\sinh^2\frac{E_h}{2} = 4\sinh^2\frac{m_h}{2} + \mathcal{Z}_{\rm latt}^h \cdot 4\sum_i \sin^2\frac{p_i}{2}$$



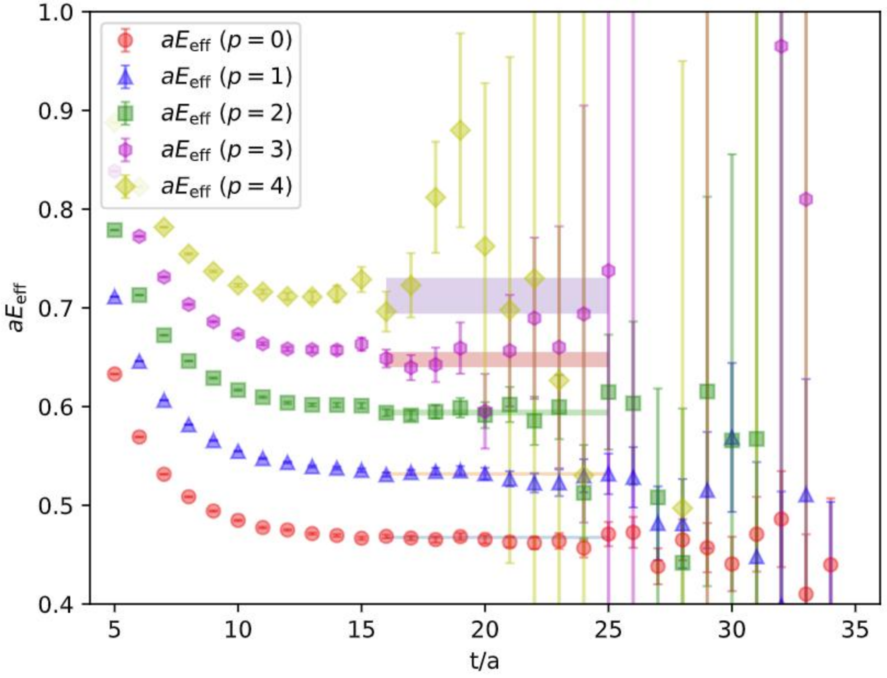


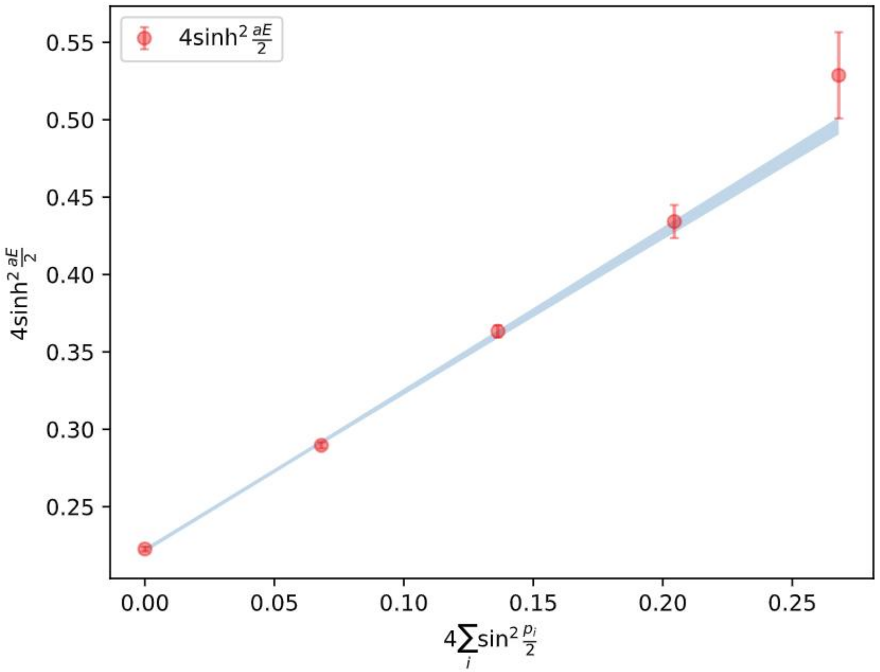
Two-point function of K^*

- Least χ^2 fitting considering covariance matrix between configurations and time
- There should be a plateau when meson ground states are dominant
- Check the discrete dispersion relation

$$C^{(2)}(\vec{p}, t) = \left(-1 - \frac{|\vec{p}|^2}{3m_h^2}\right) \frac{Z_h^2}{2E_h} \left[e^{-E_h t} + e^{-E_h(T-t)}\right]$$

$$4\sinh^2\frac{E_h}{2} = 4\sinh^2\frac{m_h}{2} + \mathcal{Z}^h_{\rm latt}\cdot 4\sum_i \sin^2\frac{p_i}{2}$$



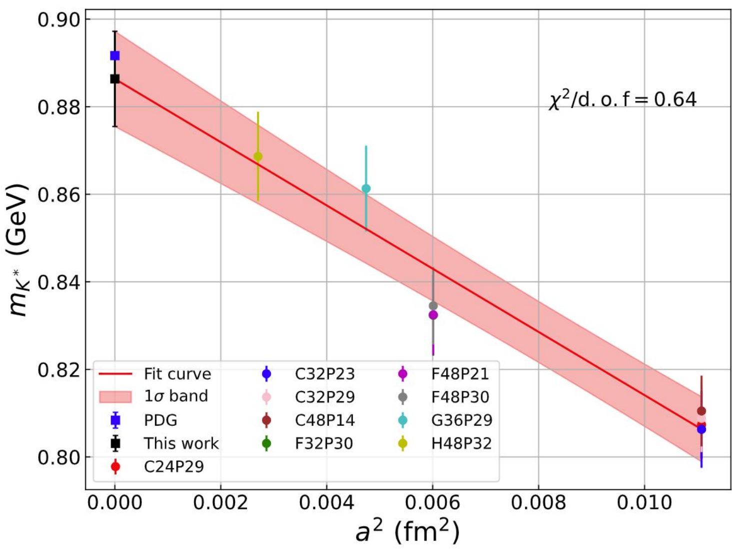


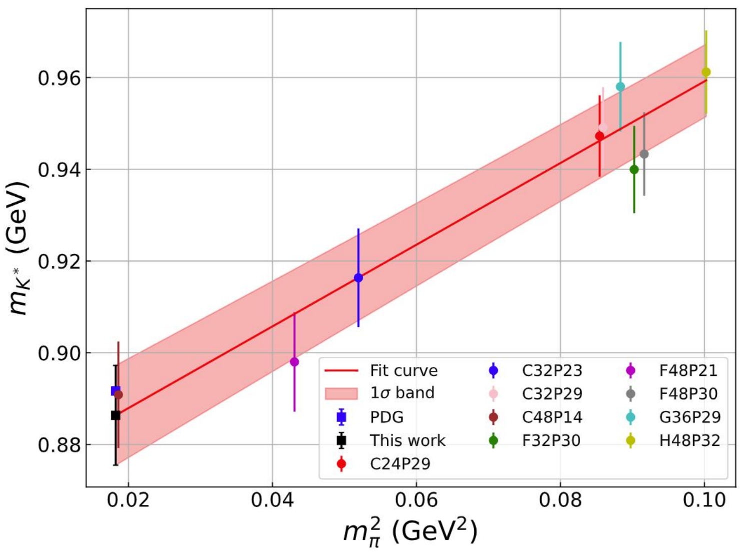
K^* Mass

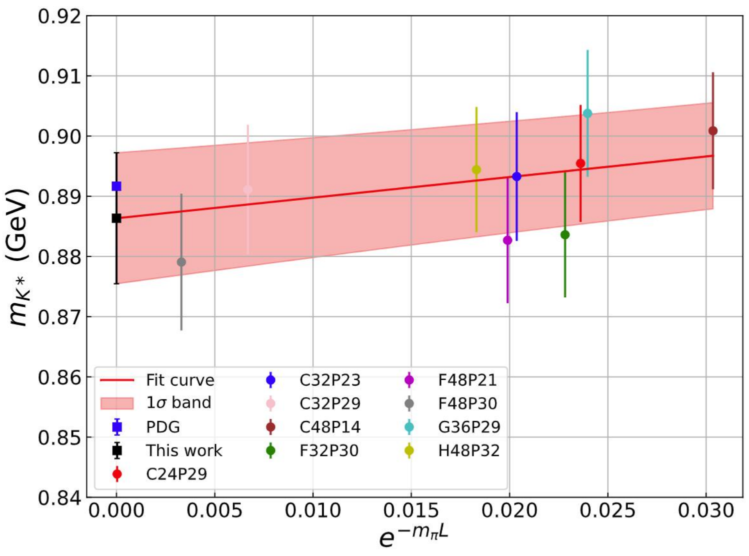
- We extrapolate mass of K^* meson to get physical result

$$m \text{ or } f_{K^*} = c + da^2 + f(m_\pi^2 - m_{\pi,\text{phys}}^2) + ge^{-m_\pi L}$$

$$m = 0.886(11)\ \text{GeV}/c^2$$





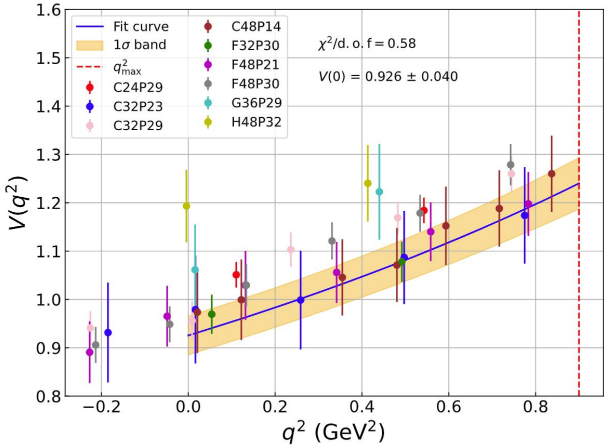


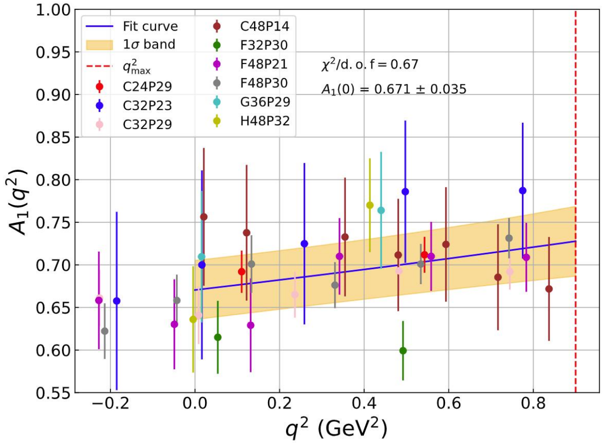
Form factor

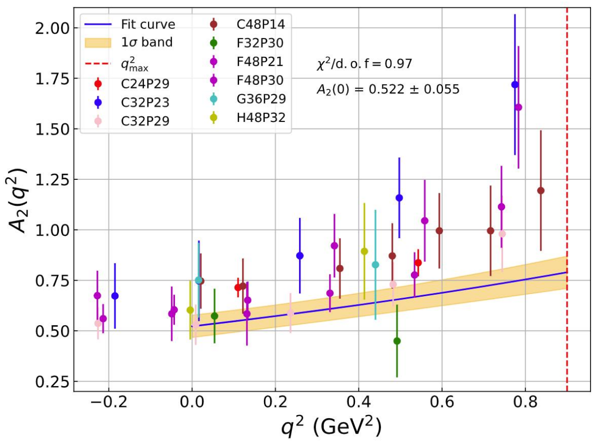
- All form factors are obtanied by $c + de^{-\delta m_D t} + fe^{-\delta m_{K^*} t}$
- Extrapolate results to the physical pion mass, continuum limit and infinite volume using z-expansion

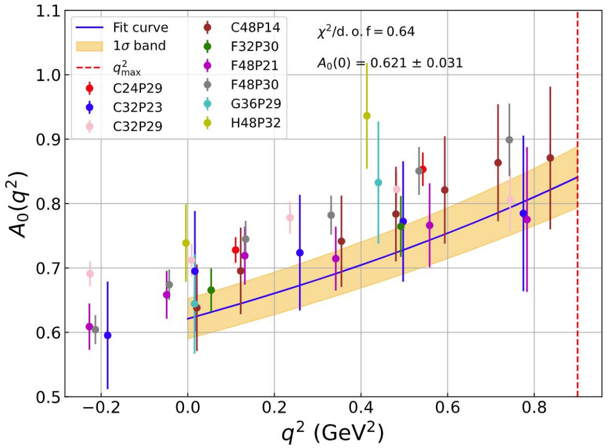
$$F\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m_{\text{pole}}^2} \sum_{i=0}^{2} \left(c_i + d_i a^2\right) \left[1 + f_i \left(m_\pi^2 - m_{\pi,\text{phys}}^2\right) \left(1 + g_i e^{-m_\pi L}\right)\right] z^i \qquad z\left(q^2, t_0\right) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- $A_3(0) - A_0(0) = 0.13(7)$, consistent with zero



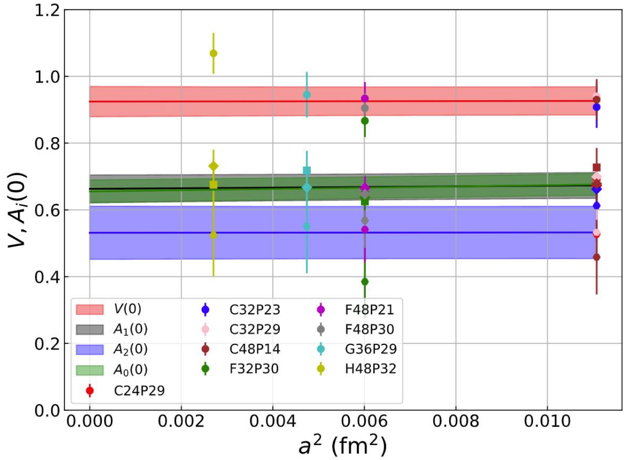


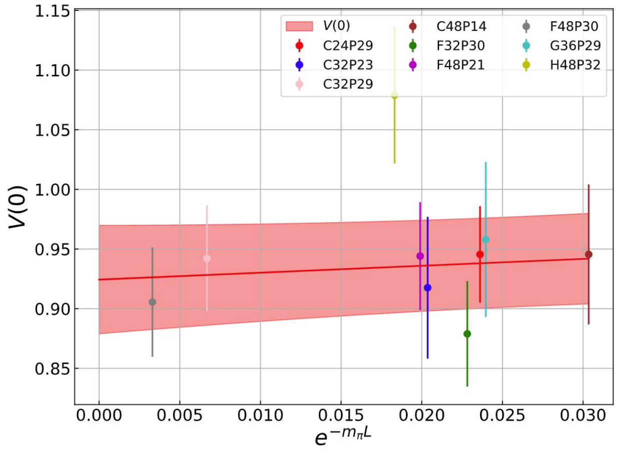


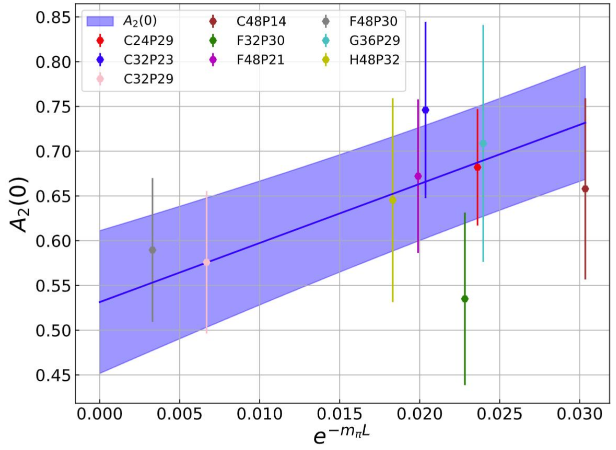


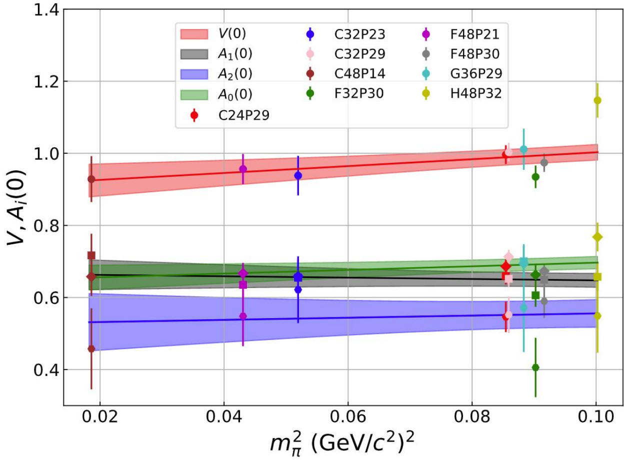
a^2, m_π^2 and $e^{-m_\pi L}$ dependence

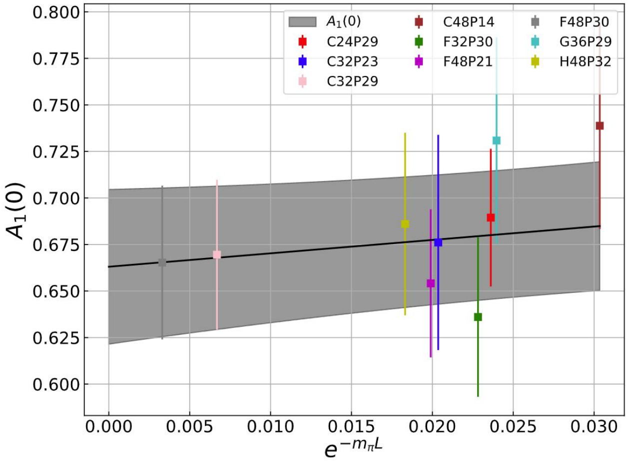
- Check the dependence of the a^2-order term, m_π^2-order term and $e^{-m_\pi L}$-order term

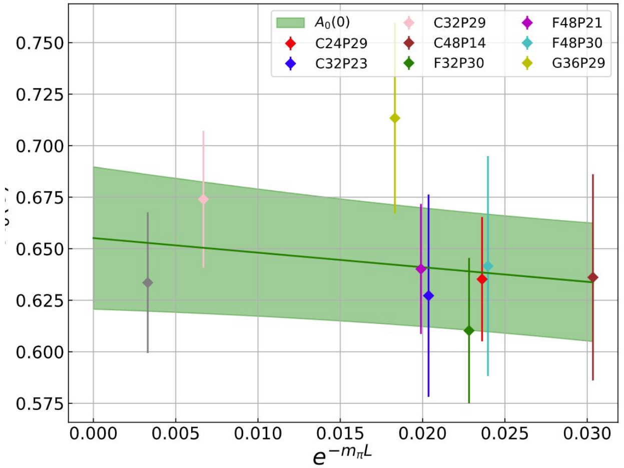








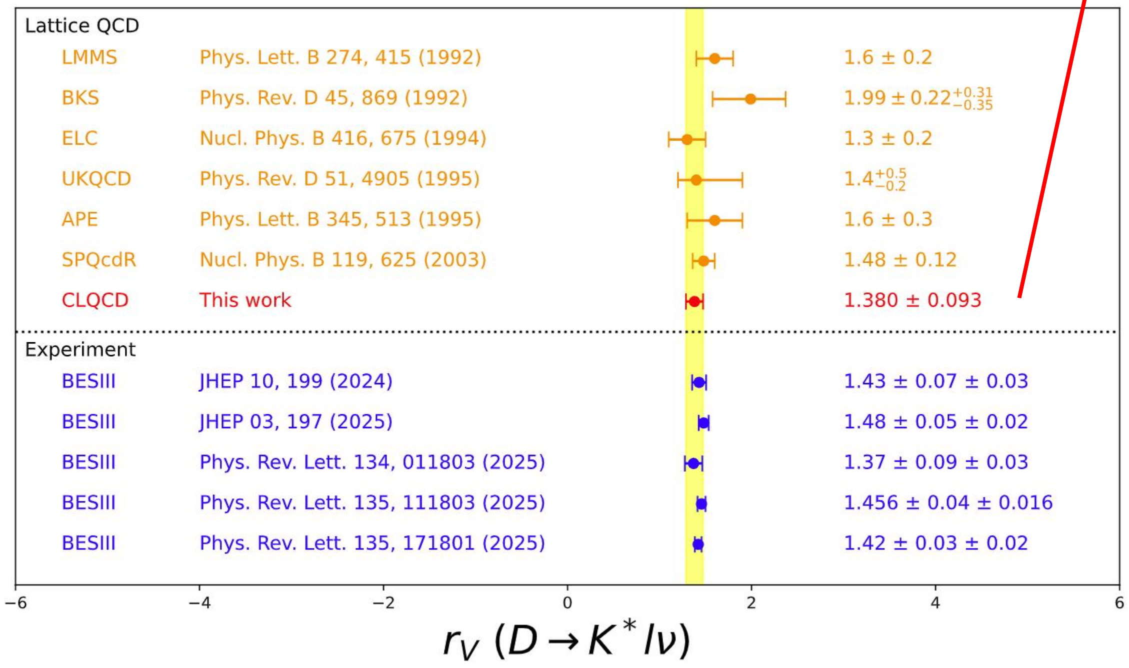


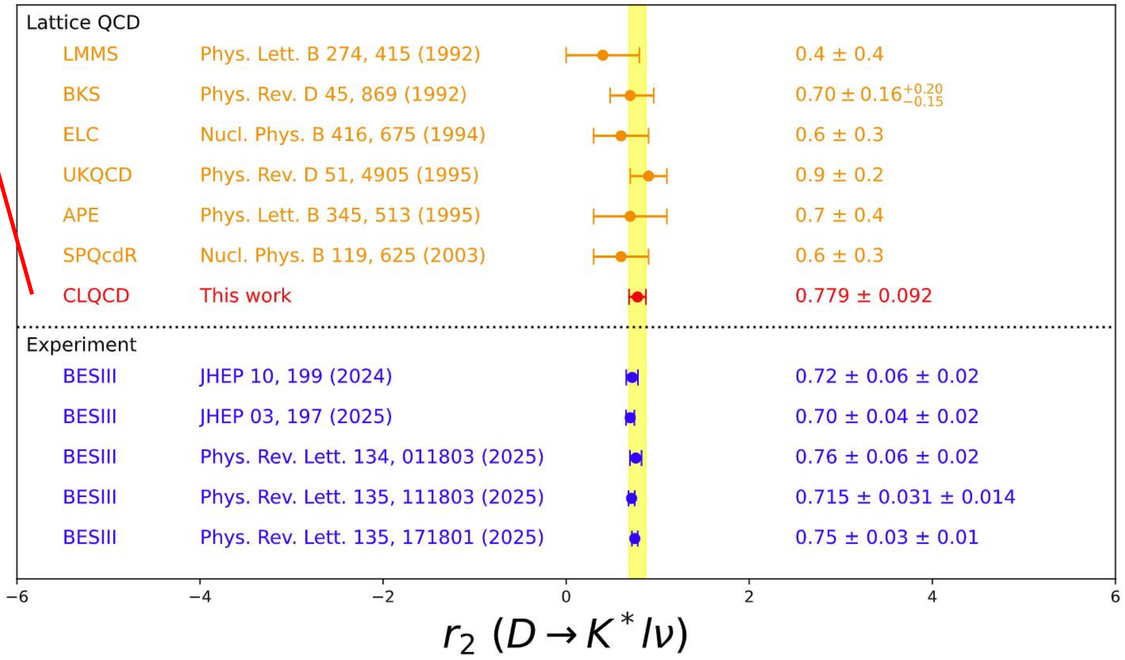


Comparision with experiments

- Key physical quantities: $r_V \equiv \frac{V(0)}{A_1(0)}$ $\quad r_2 \equiv \frac{A_2(0)}{A_1(0)}$

First un-quenched result





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Summary

- Develop scalar function method for $P \to V$ semileptonic decay
- Systematic study on $D_s \to \phi \ell \nu_\ell$ using seven CLQCD ensembles
- Greatly improve the precision (1% - 5%) compared to HPQCD (12% - 16%)
- First full QCD calculation $D \to K^* l \nu$ form factors (Preliminary!)

- Evaluate resonance effect of the vector meson and systematic uncertainties
- Other charmed meson semileptonic decay: $D \to \rho, D_s \to K^* \ldots$

Thank you for your attention!