

格点量子色动力学中的多强子散射问题

燕浩波

Based on:

1. Haobo Yan et al., *Phys. Rev. Lett* 133 (2024) 211906 on $\pi\pi\pi \rightarrow \omega(782)$ (Editors' Suggestion)
2. Haobo Yan et al., *Phys. Rev. Lett* 136 (2026) 141901 on $\pi\pi\pi \rightarrow \pi(1300)$

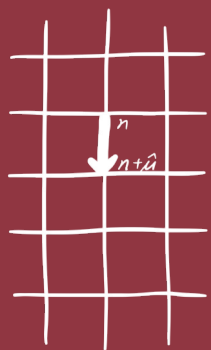


北京大学

PEKING UNIVERSITY

2026年5月13日

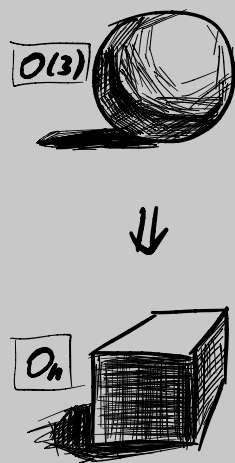
格点谱学 简介



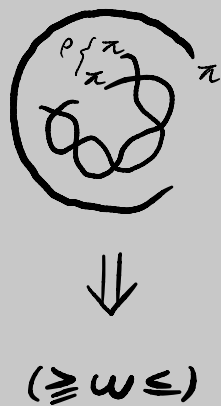
$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U$$

$$e^{-S_F - S_G}$$

格点算符 构造方法



三体散射问题 $\pi\pi\pi \rightarrow \omega(782)$



三体散射问题 $\pi\pi\pi \rightarrow \pi(1300)$



总结

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu}$$

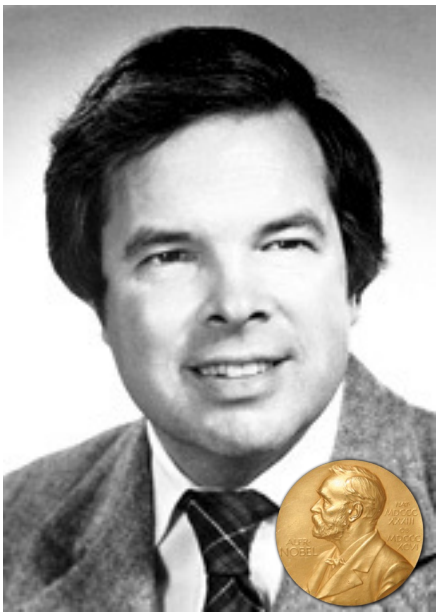
$$+ i \bar{\psi} \not{D} \psi + h.c.$$

$$+ \psi_i \psi_j \psi_k \phi + h.c.$$

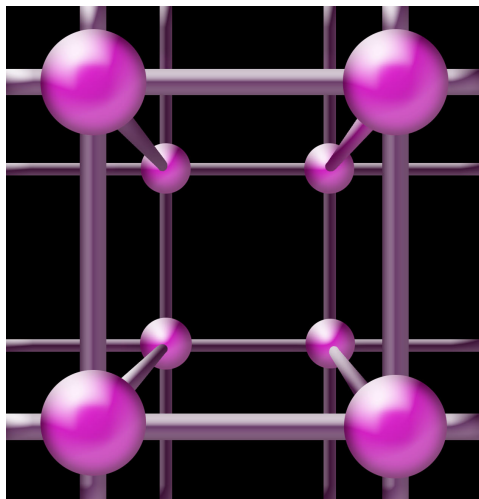
$$+ 1/2 \phi^2 - V(\phi)$$

$$+ ?$$

格点谱学简介



Wilson



- 格点 QCD 是一个从第一性原理出发、能够进行非微扰计算、能够系统改进的方法

格点谱学的里程碑

➤ 格点谱学是格点 QCD 建立以来最早应用的领域，也是它最擅长的领域之一

➤ 两体散射问题

1991: Lüscher 公式

1992-2014: Lüscher 方法各类推广

目前前沿: 重子相关的散射

➤ 三体散射问题

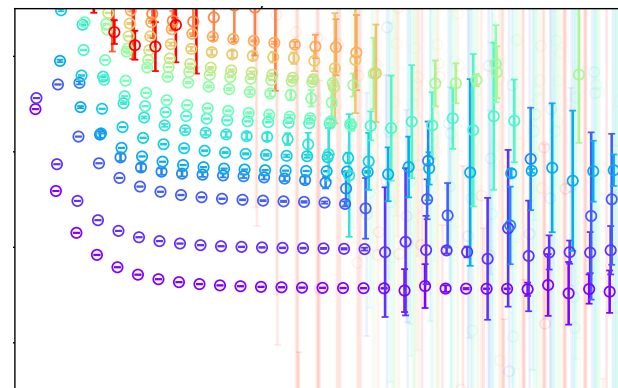
2014-2017: 三体形式理论开始

2021: 纯排斥道 $I = 3 \pi\pi\pi$

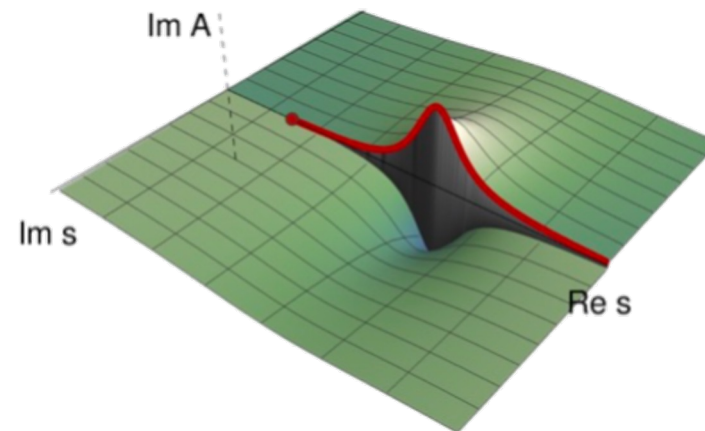
2021: 共振态 $\pi\pi\pi \rightarrow a_1(1260)$

2021-至今: 一些无共振态的三体道, 如 $\pi\pi\pi$ 、 $KK\pi$

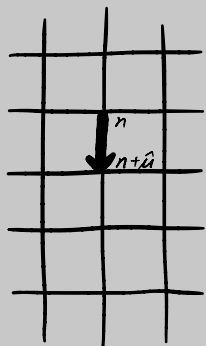
目前前沿: 寻找三体共振态



格点能谱示例



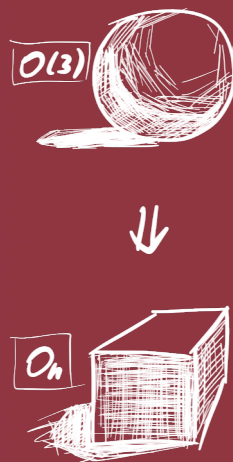
格点谱学 简介



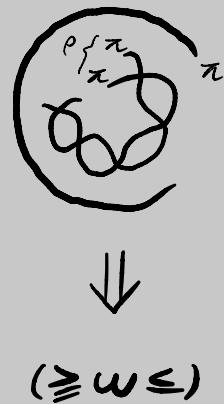
$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U$$

$$e^{-S_F - S_G}$$

格点算符 构造方法



三体散射问题 $\pi\pi\pi \rightarrow \omega(782)$



三体散射问题 $\pi\pi\pi \rightarrow \pi(1300)$



总结

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$+ i \bar{\psi} \not{\partial} \psi + h.c.$$

$$+ \psi_i \psi_j \psi_k \phi + h.c.$$

$$+ 1/2 \phi^2 - V(\phi)$$

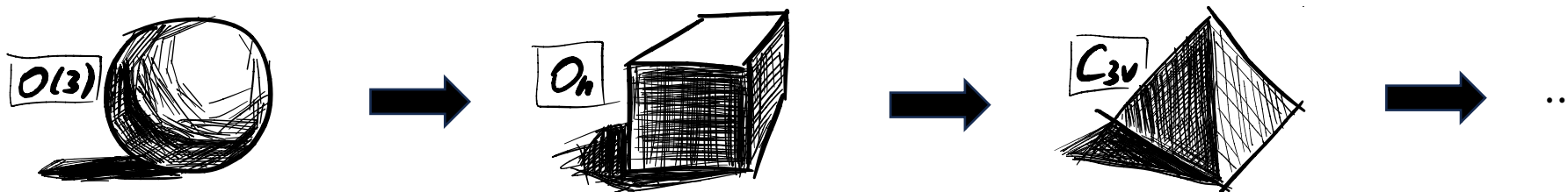
$$+ ?$$

格点算符构造方法 (OpTion)

- 大部分的格点研究需要计算多点函数

$$C(t_1, \dots, t_n) = \langle \Omega | O_1(t_1) O_2(t_2) \cdots O_n(t_n) | \Omega \rangle$$

- 从真空中产生强子态 (例如 $D\pi, \pi\pi\pi, \dots$) 需要特定量子数的算符
- 对称性从连续空间的 $O(3)$ 破缺为立方群 O_h 或更小的小群



- 一般的算符投影写为

$$O_{\Gamma, \mu}(\vec{P}) = \sum_{g \in G[\vec{P}]} T_{\mu, \mu}^{\Gamma}(g) \prod_{i=1}^N (g O_i(\vec{k}_i) g^{\dagger})$$

$\underbrace{\hspace{10em}}_{O(\vec{P}) = \prod_{i=1}^N O_i(\vec{k}_i)}$ 表示一般的 N 体算符

格点算符构造方法 (OpTion)

- Mathematica 软件 OpTion 用以构造任意不可约表示下的任意 N 体算符
- 软件已开源: <https://github.com/wittscien/OpTion>

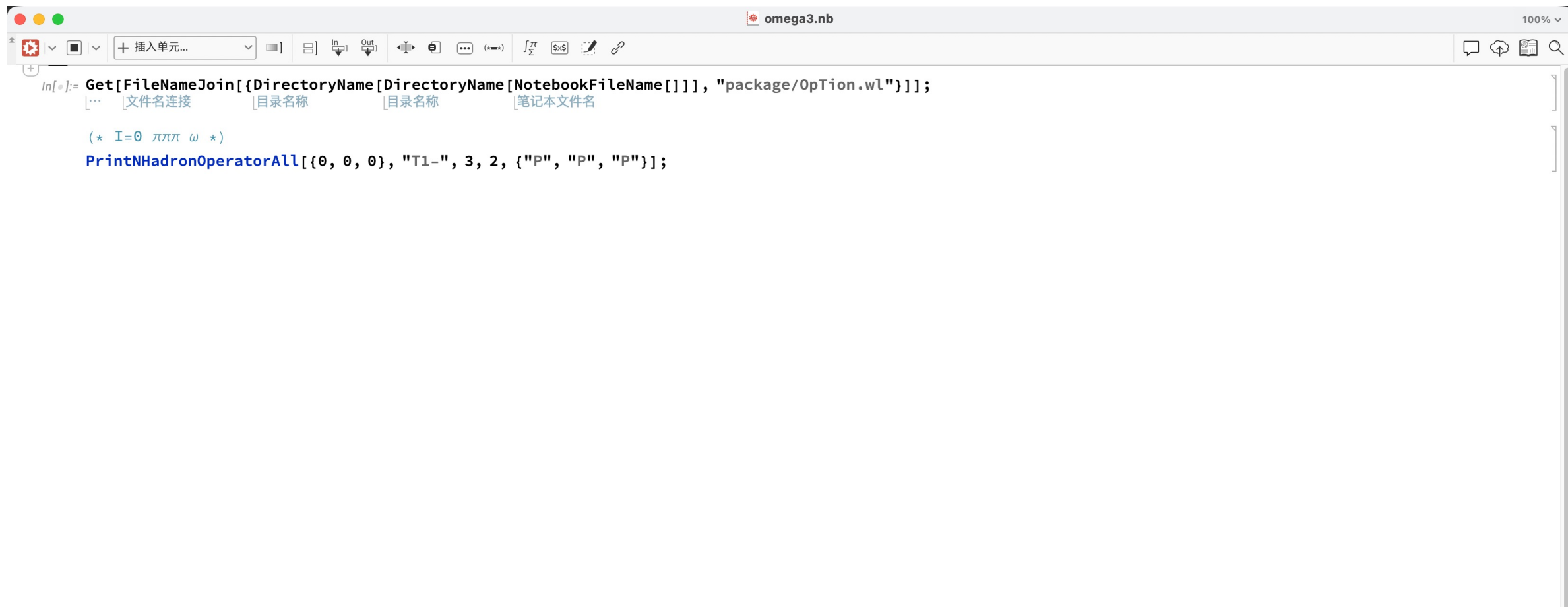
The screenshot displays the GitHub repository for `wittscien/OpTion`. The repository is public and has 14 stars, 0 forks, and 0 notifications. The main content area shows a commit by `wittscien` with 28 commits, titled "Fix Python output when the coeff is complex". Below the commit, there is a file browser view showing the following files and folders:

File/Folder	Description	Last Commit
Manual	Update tutorials	6 months ago
OpTion	Fix Python output when the coeff is complex	5 months ago
Tests	Update tutorials	6 months ago
.gitignore	First commit.	2 years ago
LICENSE.md	First commit.	2 years ago
OpTion.wl	Add option to output Python input	5 months ago
README.md	Update the published paper link	7 months ago

The right sidebar contains an "About" section with the following text: "OpTion (Operator construcTION) is a Mathematica package for building operators in lattice QCD." Below this, there are tags for `physics`, `mathematica`, and `lattice-qcd`. The sidebar also includes links to the Readme, MIT license, Activity, 14 stars, 1 watching, 0 forks, and a "Report repository" button. The "Releases" section shows 2 releases, with the latest being `OpTion 1.1.0` on Jul 24, 2025.

格点算符构造方法 (OpTion)

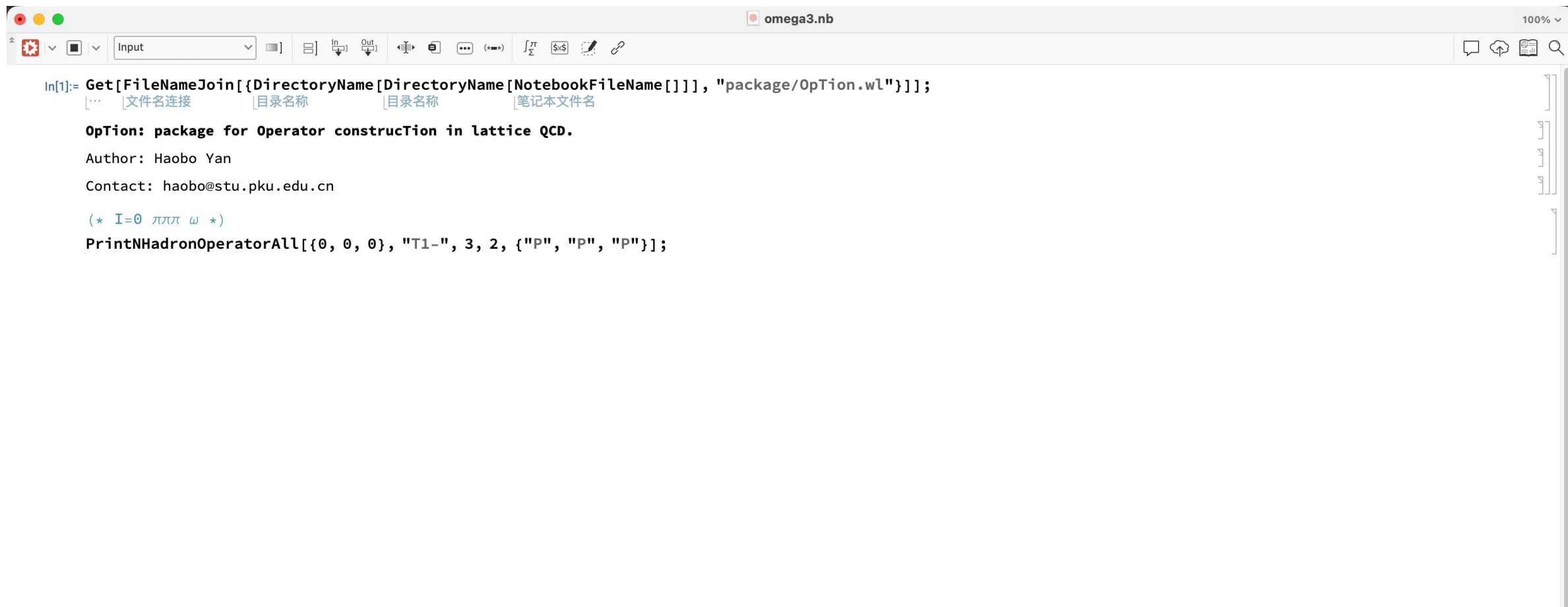
- Mathematica 软件 OpTion 用以构造任意不可约表示下的**任意** N 体算符
- 软件已开源: <https://github.com/wittscien/OpTion>



```
In[*]:= Get[FileNameJoin[{DirectoryName[DirectoryName[NotebookFileName[]]], "package/OpTion.wl"}]];
(* I=0  $\pi\pi\pi$   $\omega$  *)
PrintNHadronOperatorAll[{0, 0, 0}, "T1-", 3, 2, {"P", "P", "P"}];
```

格点算符构造方法 (OpTion)

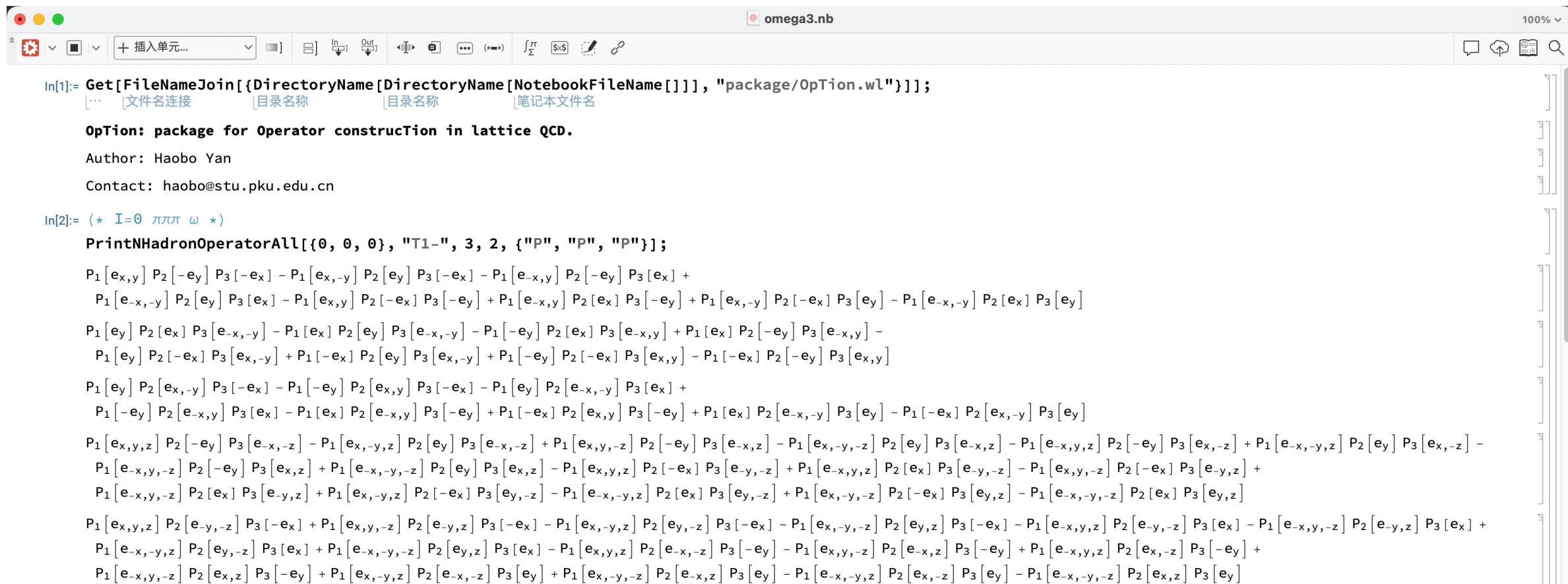
- Mathematica 软件 OpTion 用以构造任意不可约表示下的任意 N 体算符
- 软件已开源: <https://github.com/wittscien/OpTion>



```
In[1]:= Get[FileNameJoin[{DirectoryName[DirectoryName[NotebookFileName[]]], "package/OpTion.wl"}]];
      OpTion: package for Operator construction in lattice QCD.
      Author: Haobo Yan
      Contact: haobo@stu.pku.edu.cn
      (* I=0  $\pi\pi\pi \omega$  *)
      PrintNHadronOperatorAll[{0, 0, 0}, "T1-", 3, 2, {"P", "P", "P"}];
```

格点算符构造方法 (OpTion)

- Mathematica 软件 OpTion 用以构造任意不可约表示下的任意 N 体算符
- 软件已开源: <https://github.com/wittscien/OpTion>



```
In[1]:= Get[FileNameJoin[{DirectoryName[DirectoryName[NotebookFileName[]]], "package/OpTion.wl"}]];
      OpTion: package for Operator construction in lattice QCD.
      Author: Haobo Yan
      Contact: haobo@stu.pku.edu.cn

In[2]:= (* I=0  $\pi\pi\pi$   $\omega$  *)
PrintNHadronOperatorAll[{0, 0, 0}, "T1-", 3, 2, {"P", "P", "P"}];

P1[ex,y] P2[-ey] P3[-ex] - P1[ex,-y] P2[ey] P3[-ex] - P1[-ex,y] P2[-ey] P3[ex] +
  P1[-ex,-y] P2[ey] P3[ex] - P1[ex,y] P2[-ex] P3[-ey] + P1[-ex,y] P2[ex] P3[-ey] + P1[ex,-y] P2[-ex] P3[ey] - P1[-ex,-y] P2[ex] P3[ey]

P1[ey] P2[ex] P3[-ex,-y] - P1[ex] P2[ey] P3[-ex,-y] - P1[-ey] P2[ex] P3[-ex,y] + P1[ex] P2[-ey] P3[-ex,y] -
  P1[ey] P2[-ex] P3[ex,-y] + P1[-ex] P2[ey] P3[ex,-y] + P1[-ey] P2[-ex] P3[ex,y] - P1[-ex] P2[-ey] P3[ex,y]

P1[ey] P2[ex,-y] P3[-ex] - P1[-ey] P2[ex,y] P3[-ex] - P1[ey] P2[-ex,-y] P3[ex] +
  P1[-ey] P2[-ex,y] P3[ex] - P1[ex] P2[-ex,y] P3[-ey] + P1[-ex] P2[ex,y] P3[-ey] + P1[ex] P2[-ex,-y] P3[ey] - P1[-ex] P2[ex,-y] P3[ey]

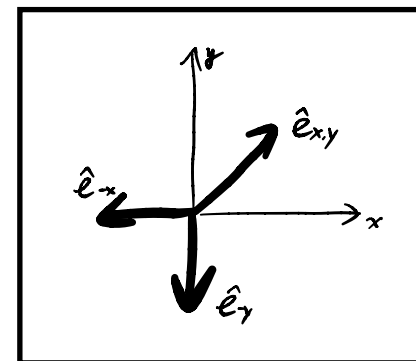
P1[ex,y,z] P2[-ey] P3[-ex,-z] - P1[ex,-y,z] P2[ey] P3[-ex,-z] + P1[ex,y,-z] P2[-ey] P3[-ex,z] - P1[ex,-y,-z] P2[ey] P3[-ex,z] - P1[-ex,y,z] P2[-ey] P3[ex,-z] + P1[-ex,-y,z] P2[ey] P3[ex,-z] -
  P1[-ex,y,-z] P2[-ey] P3[ex,z] + P1[-ex,-y,-z] P2[ey] P3[ex,z] - P1[ex,y,z] P2[-ex] P3[-ey,-z] + P1[-ex,y,z] P2[ex] P3[-ey,-z] - P1[ex,y,-z] P2[-ex] P3[-ey,z] +
  P1[-ex,y,-z] P2[ex] P3[-ey,z] + P1[ex,-y,z] P2[-ex] P3[ey,-z] - P1[-ex,-y,z] P2[ex] P3[ey,-z] + P1[ex,-y,-z] P2[-ex] P3[ey,z] - P1[-ex,-y,-z] P2[ex] P3[ey,z]

P1[ex,y,z] P2[-ey,-z] P3[-ex] + P1[ex,y,-z] P2[-ey,z] P3[-ex] - P1[ex,-y,z] P2[ey,-z] P3[-ex] - P1[ex,-y,-z] P2[ey,z] P3[-ex] - P1[-ex,y,z] P2[-ey,-z] P3[ex] - P1[-ex,-y,-z] P2[-ey,z] P3[ex] +
  P1[-ex,-y,z] P2[ey,-z] P3[ex] + P1[-ex,-y,-z] P2[ey,z] P3[ex] - P1[ex,y,z] P2[-ex,-z] P3[-ey] - P1[ex,y,-z] P2[-ex,z] P3[-ey] + P1[-ex,y,z] P2[ex,-z] P3[-ey] +
  P1[-ex,y,-z] P2[ex,z] P3[-ey] + P1[ex,-y,z] P2[-ex,-z] P3[ey] + P1[ex,-y,-z] P2[-ex,z] P3[ey] - P1[-ex,-y,z] P2[ex,-z] P3[ey] - P1[-ex,-y,-z] P2[ex,z] P3[ey]
```

格点算符构造方法 (OpTion)

- Mathematica 软件 OpTion 用以构造任意不可约表示下的任意 N 体算符
- 软件已开源: <https://github.com/wittscien/OpTion>

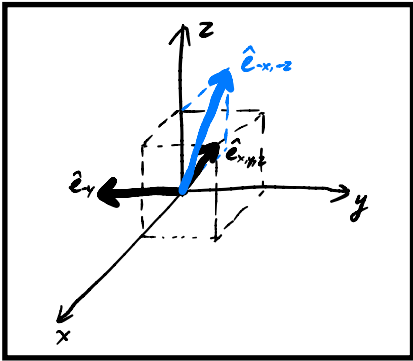
```
omega3.nb 100%
+ 插入单元...
In[1]:= Get[FileNameJoin[{DirectoryName[DirectoryName[NotebookFileName[]]], "package/OpTion.wl"}]];
OpTion: package for Operator construction in lattice QCD.
Author: Haobo Yan
Contact: haobo@stu.pku.edu.cn
In[2]:= (* I=0 πππ ω *)
PrintNHadronOperatorAll[{0, 0, 0}, "T1-", 3, 2, {"P", "P", "P"}];
P1[ex,y] P2[-ey] P3[-ex] - P1[ex,-y] P2[ey] P3[-ex] - P1[-ex,y] P2[-ey] P3[ex] +
P1[-ex,-y] P2[ey] P3[ex] - P1[ex,y] P2[-ex] P3[-ey] + P1[-ex,y] P2[ex] P3[-ey] + P1[ex,-y] P2[-ex] P3[ey] - P1[-ex,-y] P2[ex] P3[ey]
P1[ey] P2[ex] P3[-ex,-y] - P1[ex] P2[ey] P3[-ex,-y] - P1[-ey] P2[ex] P3[-ex,y] + P1[ex] P2[-ey] P3[-ex,y] -
P1[ey] P2[-ex] P3[ex,-y] + P1[-ex] P2[ey] P3[ex,-y] + P1[-ey] P2[-ex] P3[ex,y] - P1[-ex] P2[-ey] P3[ex,y]
P1[ey] P2[ex,-y] P3[-ex] - P1[-ey] P2[ex,y] P3[-ex] - P1[ey] P2[-ex,-y] P3[ex] +
P1[-ey] P2[-ex,y] P3[ex] - P1[ex] P2[-ex,y] P3[-ey] + P1[-ex] P2[ex,y] P3[-ey] + P1[ex] P2[-ex,-y] P3[ey] - P1[-ex] P2[ex,-y] P3[ey]
P1[ex,y,z] P2[-ey] P3[-ex,-z] - P1[ex,-y,z] P2[ey] P3[-ex,-z] + P1[ex,y,-z] P2[-ey] P3[-ex,z] - P1[ex,-y,-z] P2[ey] P3[-ex,z] - P1[-ex,y,z] P2[-ey] P3[ex,-z] + P1[-ex,-y,z] P2[ey] P3[ex,-z] -
P1[-ex,y,-z] P2[-ey] P3[ex,z] + P1[-ex,-y,-z] P2[ey] P3[ex,z] - P1[ex,y,z] P2[-ex] P3[-ey,-z] + P1[-ex,y,z] P2[ex] P3[-ey,-z] - P1[ex,y,-z] P2[-ex] P3[-ey,z] +
P1[-ex,y,-z] P2[ex] P3[-ey,z] + P1[ex,-y,z] P2[-ex] P3[ey,-z] - P1[-ex,-y,z] P2[ex] P3[ey,-z] + P1[ex,-y,-z] P2[-ex] P3[ey,z] - P1[-ex,-y,-z] P2[ex] P3[ey,z]
P1[ex,y,z] P2[-ey,-z] P3[-ex] + P1[ex,y,-z] P2[-ey,z] P3[-ex] - P1[ex,-y,z] P2[ey,-z] P3[-ex] - P1[ex,-y,-z] P2[ey,z] P3[-ex] - P1[-ex,y,z] P2[-ey,-z] P3[ex] - P1[-ex,-y,-z] P2[-ey,z] P3[ex] +
P1[-ex,-y,z] P2[ey,-z] P3[ex] + P1[-ex,-y,-z] P2[ey,z] P3[ex] - P1[ex,y,z] P2[-ex,-z] P3[-ey] - P1[ex,y,-z] P2[-ex,z] P3[-ey] + P1[-ex,y,z] P2[ex,-z] P3[-ey] +
P1[-ex,y,-z] P2[ex,z] P3[-ey] + P1[ex,-y,z] P2[-ex,-z] P3[ey] + P1[ex,-y,-z] P2[-ex,z] P3[ey] - P1[-ex,-y,z] P2[ex,-z] P3[ey] - P1[-ex,-y,-z] P2[ex,z] P3[ey]
```



格点算符构造方法 (OpTion)

- Mathematica 软件 OpTion 用以构造任意不可约表示下的任意 N 体算符
- 软件已开源: <https://github.com/wittscien/OpTion>

```
omega3.nb 100%
+ 插入单元...
In[1]:= Get[FileNameJoin[{DirectoryName[DirectoryName[NotebookFileName[]]], "package/OpTion.wl"}]];
OpTion: package for Operator construction in lattice QCD.
Author: Haobo Yan
Contact: haobo@stu.pku.edu.cn
In[2]:= (* I=0 πππ ω *)
PrintNHadronOperatorAll[{0, 0, 0}, "T1-", 3, 2, {"P", "P", "P"}];
P1[e_x,y] P2[-e_y] P3[-e_x] - P1[e_x,-y] P2[e_y] P3[-e_x] - P1[e_x,y] P2[-e_y] P3[e_x] +
P1[-e_x,-y] P2[e_y] P3[e_x] - P1[e_x,y] P2[-e_x] P3[-e_y] + P1[-e_x,y] P2[e_x] P3[-e_y] + P1[e_x,-y] P2[-e_x] P3[e_y] - P1[-e_x,-y] P2[e_x] P3[e_y]
P1[e_y] P2[e_x] P3[-e_x,-y] - P1[e_x] P2[e_y] P3[-e_x,-y] - P1[-e_y] P2[e_x] P3[-e_x,y] + P1[e_x] P2[-e_y] P3[-e_x,y] -
P1[e_y] P2[-e_x] P3[e_x,-y] + P1[-e_x] P2[e_y] P3[e_x,-y] + P1[-e_y] P2[-e_x] P3[e_x,y] - P1[-e_x] P2[-e_y] P3[e_x,y]
P1[e_y] P2[e_x,-y] P3[-e_x] - P1[-e_y] P2[e_x,y] P3[-e_x] - P1[e_y] P2[e_x,-y] P3[e_x] +
P1[-e_y] P2[-e_x,y] P3[e_x] - P1[e_x] P2[-e_x,y] P3[-e_y] + P1[-e_x] P2[e_x,y] P3[-e_y] + P1[e_x] P2[e_x,-y] P3[e_y] - P1[-e_x] P2[e_x,-y] P3[e_y]
P1[e_x,y,z] P2[-e_y] P3[e_x,-z] - P1[e_x,-y,z] P2[e_y] P3[e_x,-z] + P1[e_x,y,-z] P2[-e_y] P3[e_x,z] - P1[e_x,-y,-z] P2[e_y] P3[e_x,z] - P1[e_x,y,z] P2[-e_y] P3[e_x,-z] + P1[e_x,-y,z] P2[e_y] P3[e_x,-z] -
P1[-e_x,y,-z] P2[-e_y] P3[e_x,z] + P1[-e_x,-y,-z] P2[e_y] P3[e_x,z] - P1[e_x,y,z] P2[-e_x] P3[-e_y,-z] + P1[-e_x,y,z] P2[e_x] P3[-e_y,-z] - P1[e_x,y,-z] P2[-e_x] P3[-e_y,z] +
P1[-e_x,y,-z] P2[e_x] P3[-e_y,z] + P1[e_x,-y,z] P2[-e_x] P3[e_y,-z] - P1[-e_x,-y,z] P2[e_x] P3[e_y,-z] + P1[e_x,-y,-z] P2[-e_x] P3[e_y,z] - P1[-e_x,-y,-z] P2[e_x] P3[e_y,z]
P1[e_x,y,z] P2[-e_y,-z] P3[-e_x] + P1[e_x,y,-z] P2[-e_y,z] P3[-e_x] - P1[e_x,-y,z] P2[e_y,-z] P3[-e_x] - P1[e_x,-y,-z] P2[e_y,z] P3[-e_x] - P1[-e_x,y,z] P2[-e_y,-z] P3[e_x] - P1[-e_x,y,-z] P2[-e_y,z] P3[e_x] +
P1[-e_x,-y,z] P2[e_y,-z] P3[e_x] + P1[-e_x,-y,-z] P2[e_y,z] P3[e_x] - P1[e_x,y,z] P2[-e_x,-z] P3[-e_y] - P1[e_x,y,-z] P2[-e_x,z] P3[-e_y] + P1[-e_x,y,z] P2[e_x,-z] P3[-e_y] +
P1[-e_x,y,-z] P2[e_x,z] P3[-e_y] + P1[e_x,-y,z] P2[-e_x,-z] P3[e_y] + P1[e_x,-y,-z] P2[-e_x,z] P3[e_y] - P1[-e_x,-y,z] P2[e_x,-z] P3[e_y] - P1[-e_x,-y,-z] P2[e_x,z] P3[e_y]
```



规范场组态

组态名	体积	a / fm	β	\tilde{m}_l^b	\tilde{m}_s^b	M_π / MeV	$M_\pi L$
C24P34	$24^3 \times 64$	0.10530(18)	6.200	-0.2770	-0.2310	340	4.38
C24P29	$24^3 \times 72$	0.10530(18)	6.200	-0.2770	-0.2400	292	3.75
C32P29	$32^3 \times 64$	0.10530(18)	6.200	-0.2770	-0.2400	292	5.01
C24P23	$24^3 \times 64$	0.10530(18)	6.200	-0.2790	-0.2400	230	2.93
C32P23	$32^3 \times 64$	0.10530(18)	6.200	-0.2790	-0.2400	228	3.91
C48P23	$48^3 \times 96$	0.10530(18)	6.200	-0.2790	-0.2400	225	5.79
C48P14	$48^3 \times 96$	0.10530(18)	6.200	-0.2825	-0.2310	133	3.56
C48P14	$64^3 \times 128$	0.10530(18)	6.200	-0.2825	-0.2310	133	4.63
E28P35	$28^3 \times 64$	0.08973(20)	6.308	-0.2490	-0.2170	351	4.46
E32P29	$32^3 \times 64$	0.08973(20)	6.308	-0.2490	-0.2170	287	4.19
E32P22	$32^3 \times 96$	0.08973(20)	6.308	-0.2490	-0.2170	215	3.14
F24P30	$24^3 \times 96$	0.07746(18)	6.410	-0.2295	-0.2050	303	2.86
F32P30	$32^3 \times 96$	0.07746(18)	6.410	-0.2295	-0.2050	303	3.81
F48P30	$48^3 \times 96$	0.07746(18)	6.410	-0.2295	-0.2050	305	5.72
F32P21	$32^3 \times 64$	0.07746(18)	6.410	-0.2320	-0.2050	210	2.67
F48P21	$48^3 \times 96$	0.07746(18)	6.410	-0.2320	-0.2050	208	3.91
F64P14	$64^3 \times 128$	0.07746(18)	6.410	-0.2320	-0.2050	136	3.41
G32P35	$32^3 \times 96$	0.06887(12)	6.498	-0.2150	-0.1926	352	3.94
G36P29	$36^3 \times 108$	0.06887(12)	6.498	-0.2150	-0.1926	297	3.73
H48P32	$48^3 \times 144$	0.05187(26)	6.72	-0.1850	-0.1700	321	4.06
I64P31	$64^3 \times 128$	0.03761(08)	7.020	-0.1569	-0.1475	312	3.81
I64P19	$64^3 \times 128$	0.03761(08)	7.020	-0.1585	-0.1475	188	2.29

➤ 中国格点合作组产生的 Wilson-Clover 组态



Hu et al, PRD 09 (2024) 054507

➤ $N_f = 2 + 1$

规范场组态

组态名	体积	a / fm	β	\tilde{m}_l^b	\tilde{m}_s^b	M_π / MeV	$M_\pi L$
C24P34	$24^3 \times 64$	0.10530(18)	6.200	-0.2770	-0.2310	340	4.38
C24P29	$24^3 \times 72$	0.10530(18)	6.200	-0.2770	-0.2400	292	3.75
C32P29	$32^3 \times 64$	0.10530(18)	6.200	-0.2770	-0.2400	292	5.01
C24P23	$24^3 \times 64$	0.10530(18)	6.200	-0.2790	-0.2400	230	2.93
C32P23	$32^3 \times 64$	0.10530(18)	6.200	-0.2790	-0.2400	228	3.91
C48P23	$48^3 \times 96$	0.10530(18)	6.200	-0.2790	-0.2400	225	5.79
C48P14	$48^3 \times 96$	0.10530(18)	6.200	-0.2825	-0.2310	133	3.56
C48P14	$64^3 \times 128$	0.10530(18)	6.200	-0.2825	-0.2310	133	4.63
E28P35	$28^3 \times 64$	0.08973(20)	6.308	-0.2490	-0.2170	351	4.46
E32P29	$32^3 \times 64$	0.08973(20)	6.308	-0.2490	-0.2170	287	4.19
E32P22	$32^3 \times 96$	0.08973(20)	6.308	-0.2490	-0.2170	215	3.14
F24P30	$24^3 \times 96$	0.07746(18)	6.410	-0.2295	-0.2050	303	2.86
F32P30	$32^3 \times 96$	0.07746(18)	6.410	-0.2295	-0.2050	303	3.81
F48P30	$48^3 \times 96$	0.07746(18)	6.410	-0.2295	-0.2050	305	5.72
F32P21	$32^3 \times 64$	0.07746(18)	6.410	-0.2320	-0.2050	210	2.67
F48P21	$48^3 \times 96$	0.07746(18)	6.410	-0.2320	-0.2050	208	3.91
F64P14	$64^3 \times 128$	0.07746(18)	6.410	-0.2320	-0.2050	136	3.41
G32P35	$32^3 \times 96$	0.06887(12)	6.498	-0.2150	-0.1926	352	3.94
G36P29	$36^3 \times 108$	0.06887(12)	6.498	-0.2150	-0.1926	297	3.73
H48P32	$48^3 \times 144$	0.05187(26)	6.72	-0.1850	-0.1700	321	4.06
I64P31	$64^3 \times 128$	0.03761(08)	7.020	-0.1569	-0.1475	312	3.81
I64P19	$64^3 \times 128$	0.03761(08)	7.020	-0.1585	-0.1475	188	2.29

➤ 中国格点合作组产生的 Wilson-Clover 组态

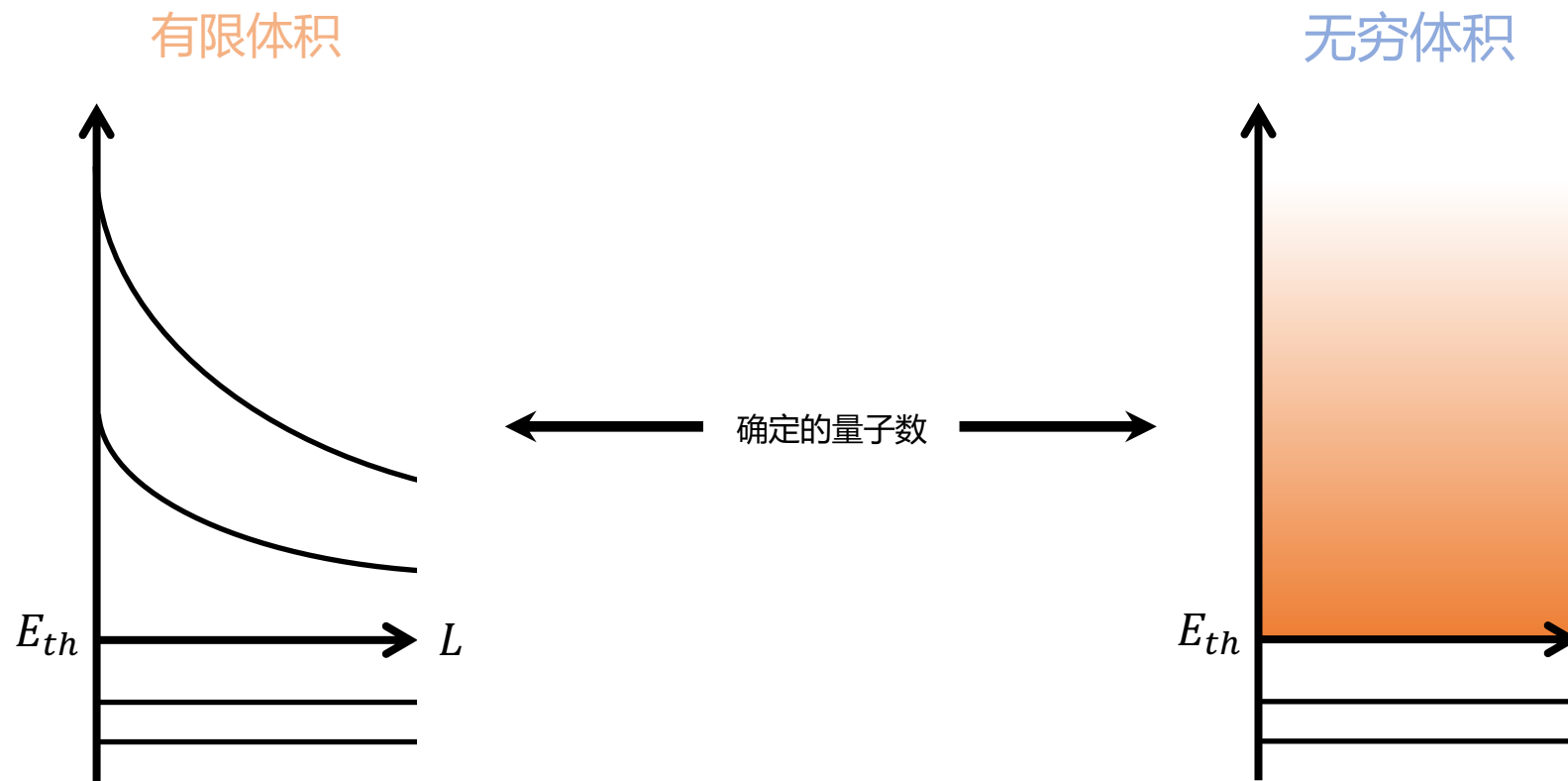


Hu et al, PRD 09 (2024) 054507

- $N_f = 2 + 1$
- 采用 2 个不同的 M_π 来探索手征行为
- 暂不考虑格距误差
- 采用蒸馏算法计算传播子 (perambulator)
- 压低高激发态贡献, 同时大幅减少计算量

Peardon et al PRD 80 (2009) 054506

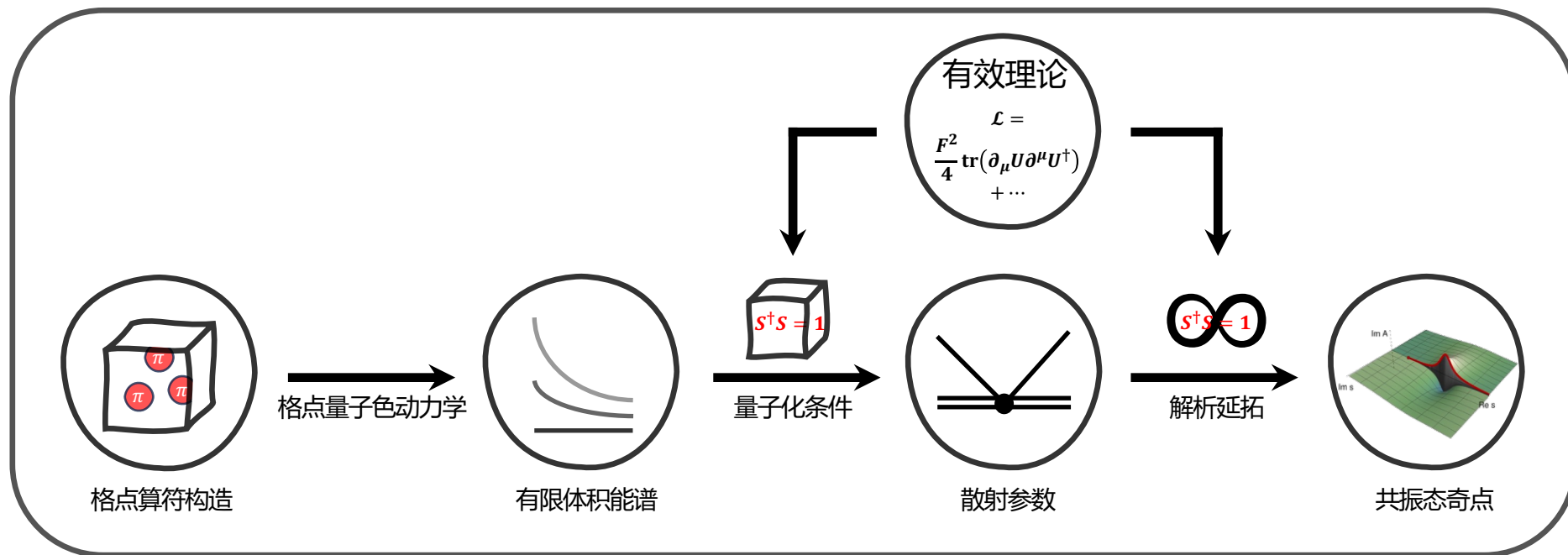
量子化条件



➤ 从有限体积能谱中抽取无穷体积散射振幅

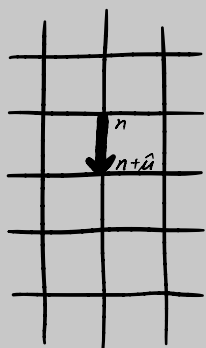
➔ 量子化条件: $f(\mathcal{M}(E_n), \mathcal{F}(E_n, L)) = 0$

➤ 能级 E_n 和体积 L 越多, 对振幅 $\mathcal{M}(E_n)$ 的限制就越强



- 利用第一节开发的软件 OpTion 构造多体算符
- 选择作用量，生成组态，求解夸克传播子
- 维克收缩，计算两点关联函数
- 应用量子化条件，从能谱中抽取散射参数
- 将散射振幅解析延拓到复黎曼面，寻找奇点

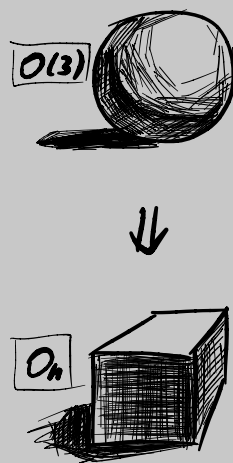
格点谱学 简介



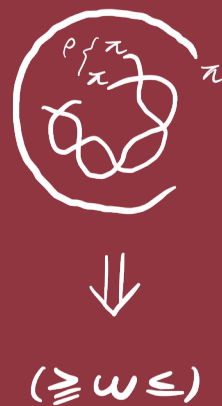
$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U$$

$$e^{-S_F - S_G}$$

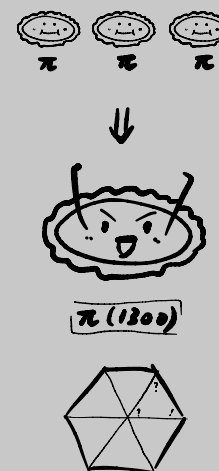
格点算符 构造方法



三体散射问题 $\pi\pi\pi \rightarrow \omega(782)$



三体散射问题 $\pi\pi\pi \rightarrow \pi(1300)$



总结

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$+ i \bar{\psi} \not{\partial} \psi + h.c.$$

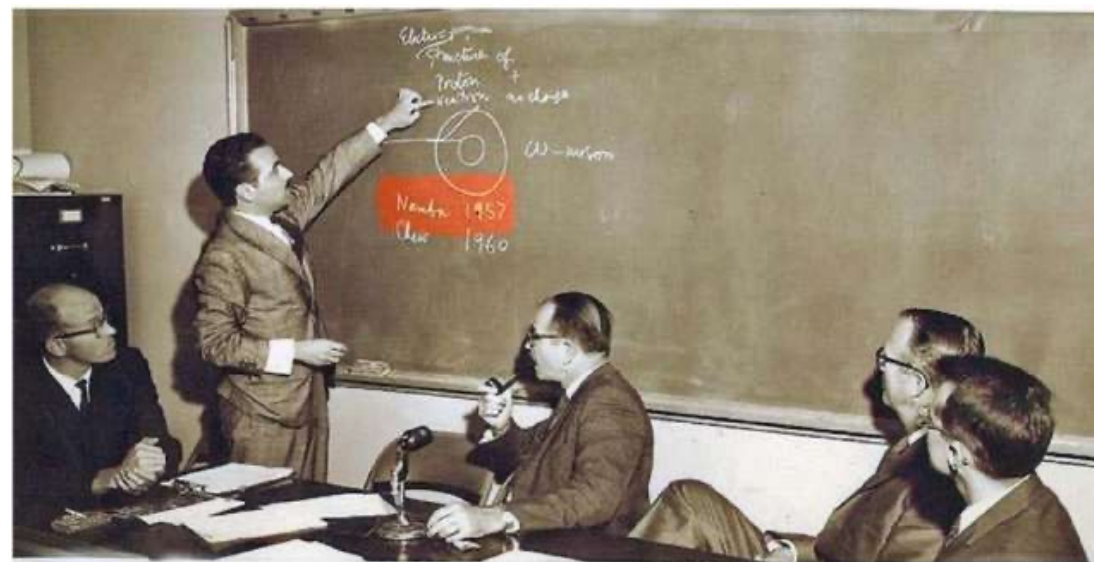
$$+ \psi_i \psi_j \psi_k \phi + h.c.$$

$$+ 1/2 \phi^2 - V(\phi)$$

$$+ ?$$

三体 $\pi\pi\pi \rightarrow \omega(782)$ 散射

- 大多数强子有很大的衰变到三体末态的分宽度
- 诸多**非微扰**强相互作用难题与三体问题相关
 1. Roper共振态: Burkert and Roberts, RMP 2019
 2. T_{cc} 奇特强子态: Hansen, Romero-López, Sharpe, JHEP 2024
($T_{cc}(3875)$ 可在非物理 M_π 下由 DD^* 研究)
- **三体问题**的理论研究十分困难
 1. 动力学更复杂
 2. 数值计算困难得多
 3. 需要单独推导量子化条件
- 国际上三体问题共振态的研究还很初步，仅有一个工作
 $\pi\pi\pi \rightarrow a_1(1260)$: Maxim Mai *et al.*, PRL 2021



Stevenson Maglich MacMillan Alvarez Rosenfeld
PRESS/TV CONFERENCE ON DISCOVERY OF OMEGA MESON
Berkeley, August 31, 1961
Maglic, Alvarez, Rosenfeld & Stevenson, Phys. Rev. Lett. September 1, 1961

Maglic et al (1961)

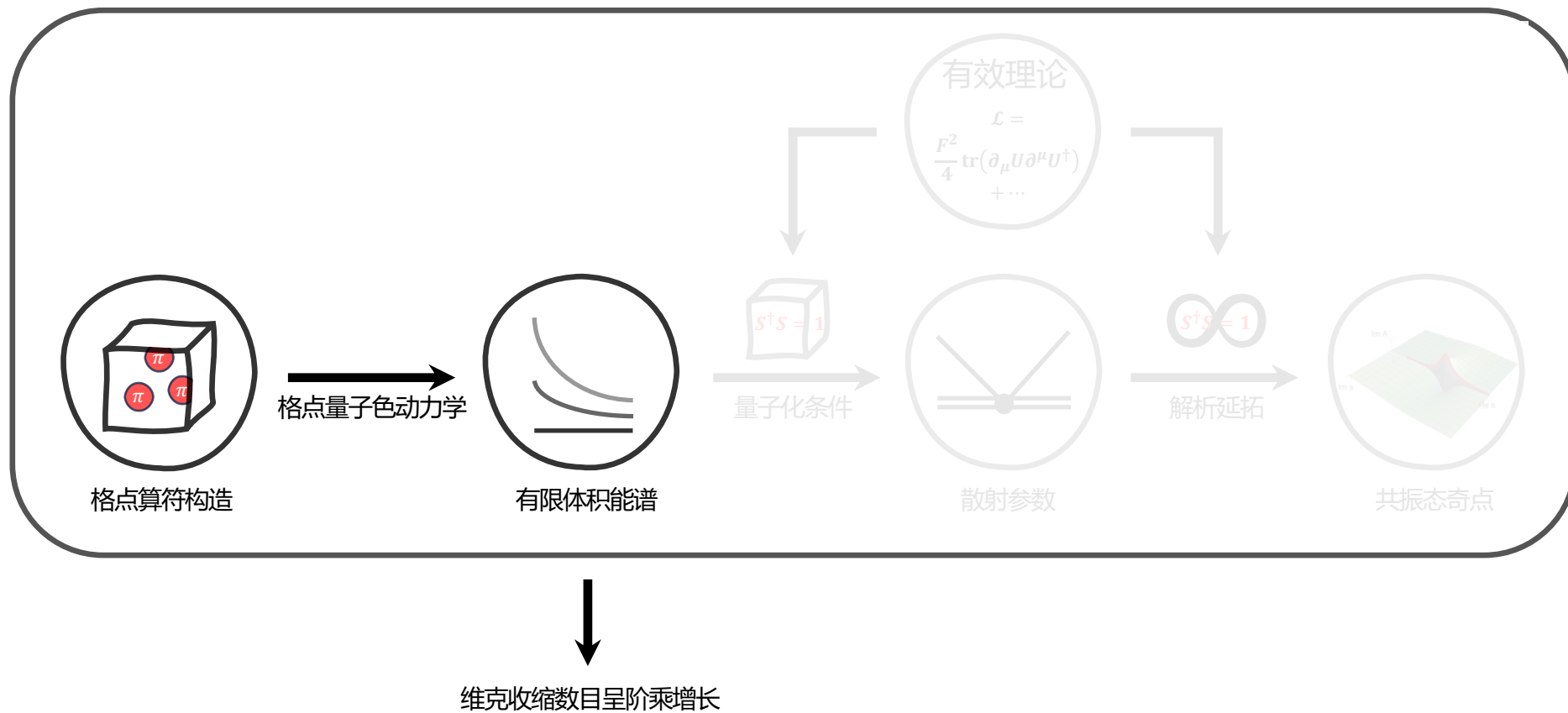
- 考察 $\pi\pi\pi$ 散射，寻找其中的 ω 介子
 - ω 是最轻的会衰变到三体末态的强子
 - ω 在唯象上有重要意义，例如核子间小于 1 fm 时的排斥相互作用、缪子反常磁距...

Sakurai (1960); Erkelenz (1974); Brown and Jackson (1976); Barkov et al., 1985; Connell et al. (1997); Bazavov et al. (2021)

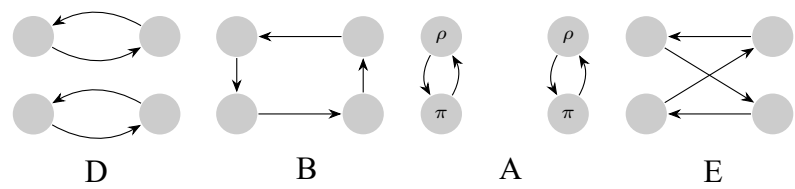
计算流程



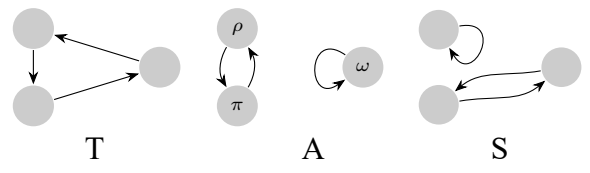
计算流程



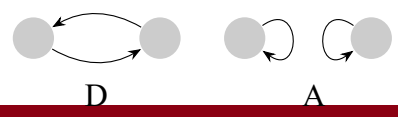
维克收缩



$\sim \mathcal{O}(10)$

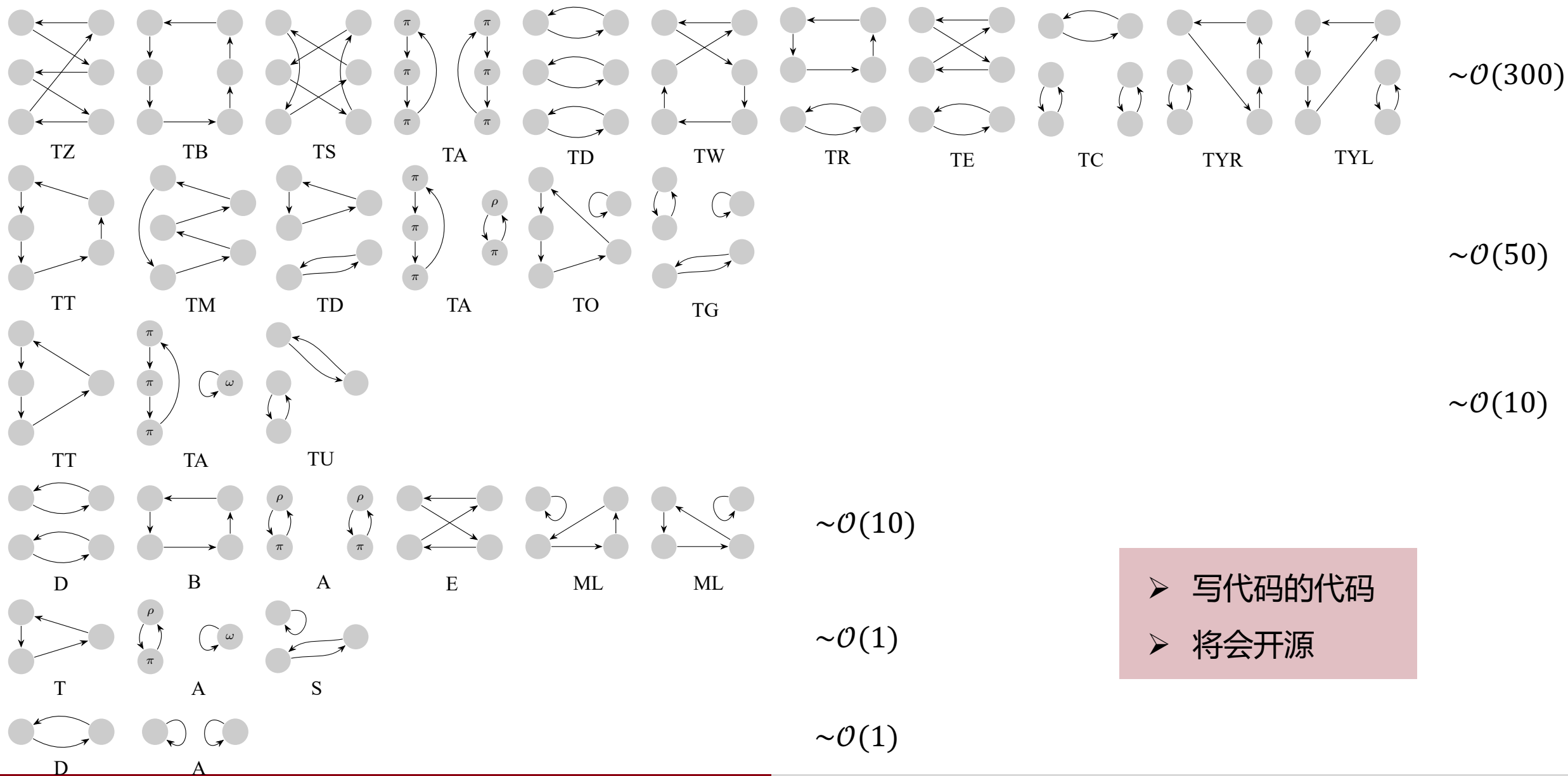


$\sim \mathcal{O}(1)$



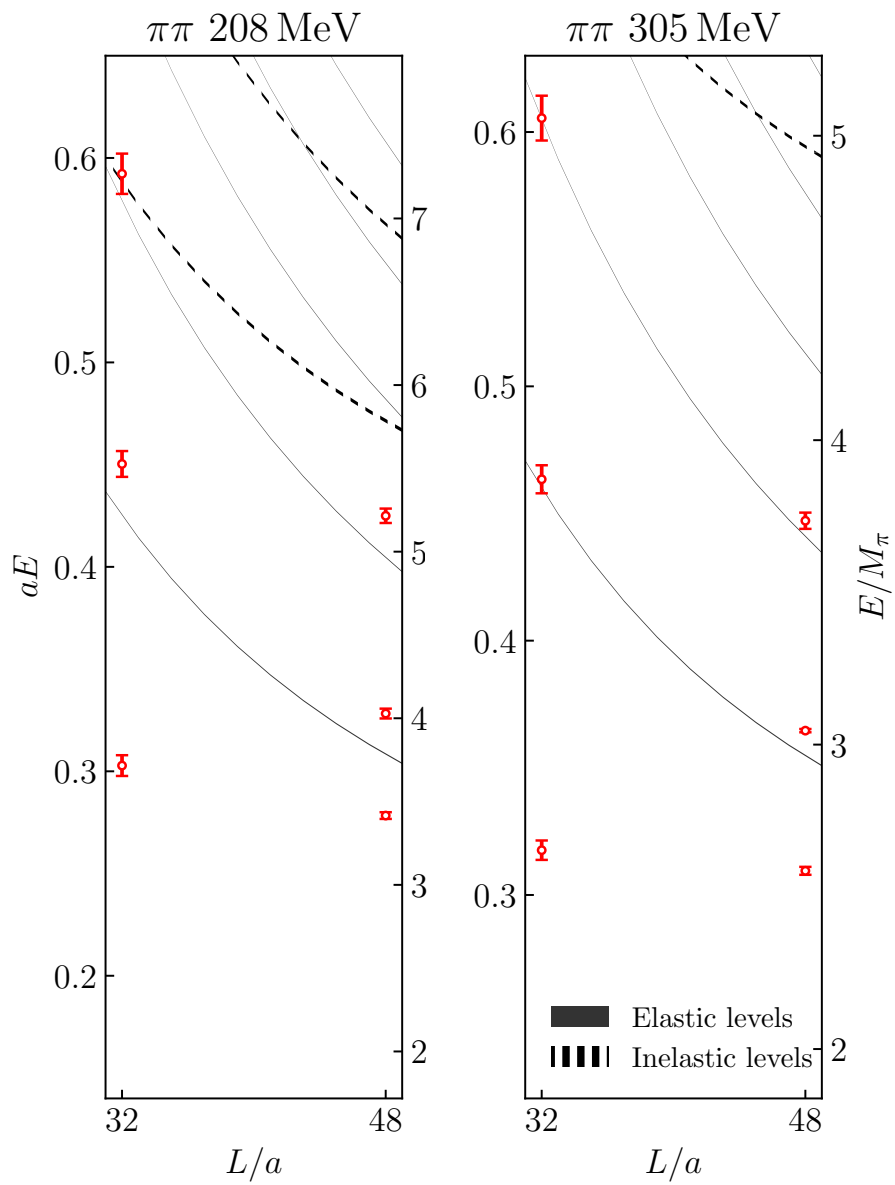
$\sim \mathcal{O}(1)$

维克收缩



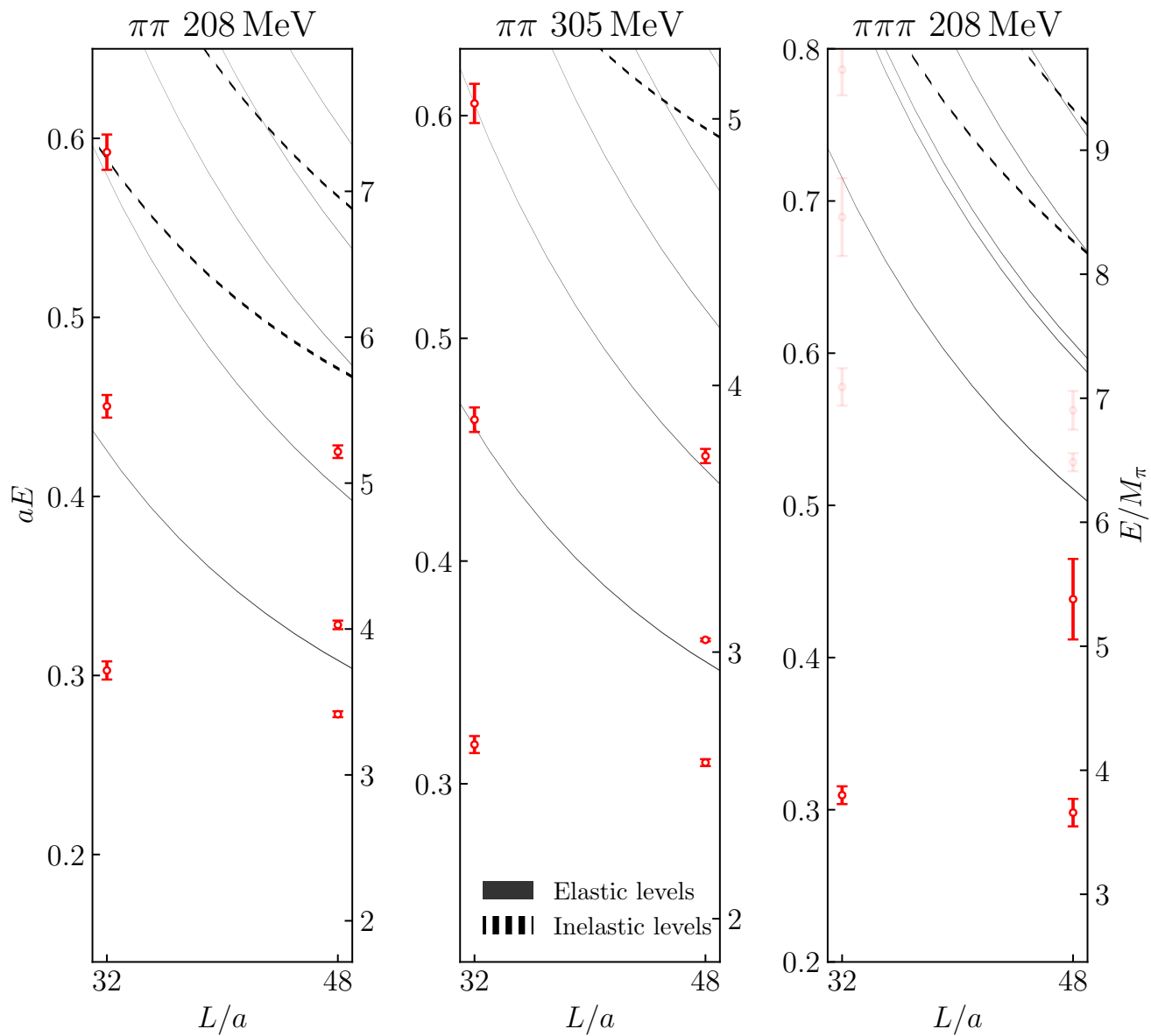
➤ 写代码的代码
➤ 将会开源

有限体积能谱



➤ 两体 $\pi\pi \rightarrow \rho$

有限体积能谱

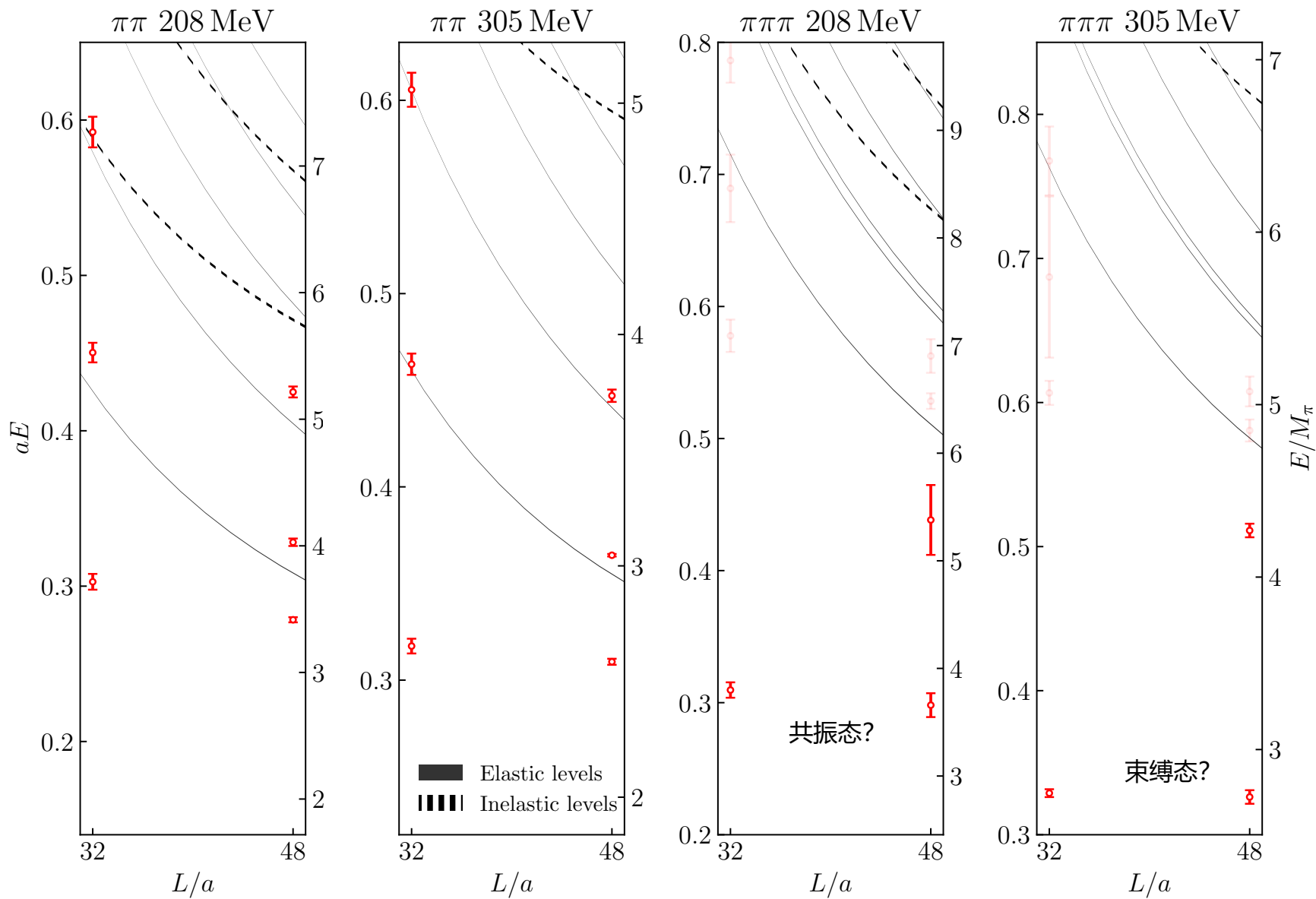


➤ 两体 $\pi\pi \rightarrow \rho$

➤ 三体 $\pi\pi\pi$ 有非平凡的结构

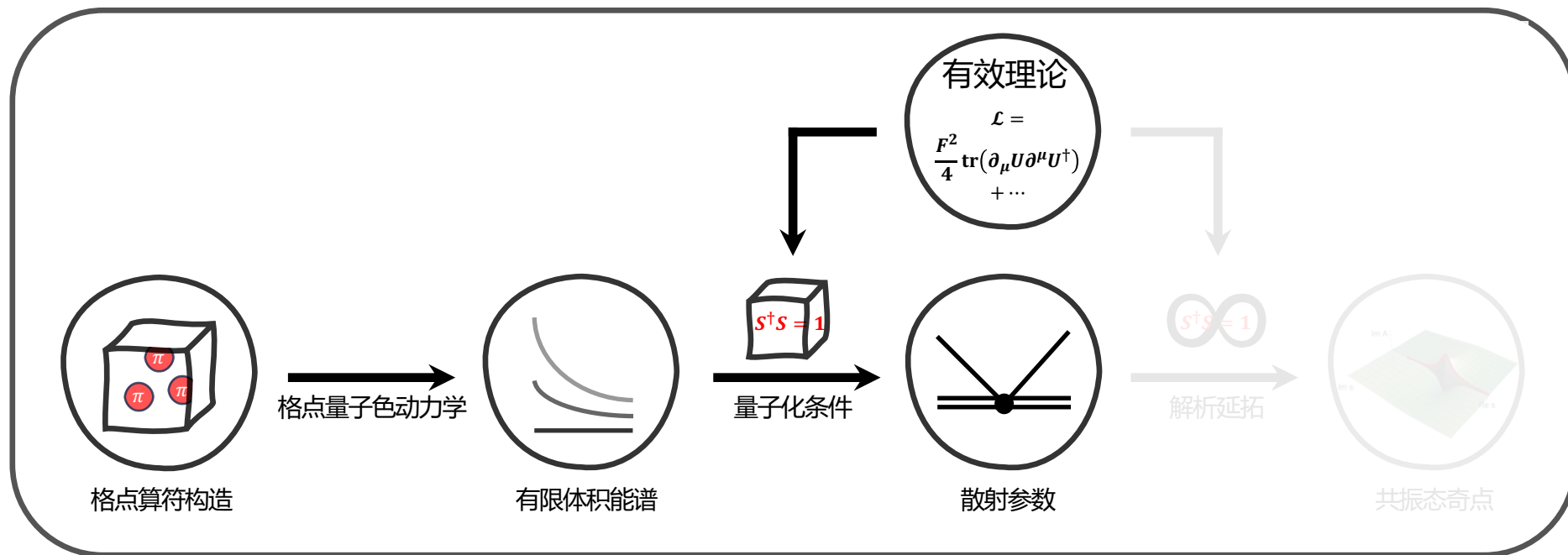
➤ ω ?

有限体积能谱



- 两体 $\pi\pi \rightarrow \rho$
- 三体 $\pi\pi\pi$ 有非平凡的结构
- ω ?

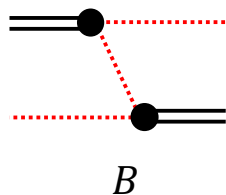
计算流程



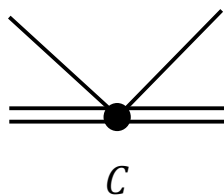
量子化条件

有限体积幺正化理论 (FVU, Finite-Volume Unitarity)

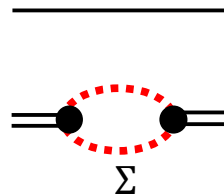
$$\begin{cases} \tilde{K}^{-1}(\sigma) - \Sigma^{FV}(\sigma) = 0 \\ \det[(\tilde{K}^{-1}(s) - \Sigma^{FV}(s))E_L - (\tilde{B}(s) + \tilde{C}(s))] = 0 \end{cases}$$



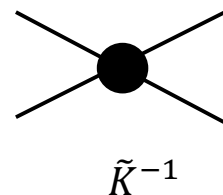
B
幺正性固定



C
三体力



Σ
幺正性固定



\tilde{K}^{-1}
两体力

$$\begin{cases} \text{Diagram C} \sim a_0 + a_1 \sigma + \dots \\ \text{Diagram B} \sim \frac{c_0}{s - M_\omega^2} + c_1 + \dots \end{cases}$$

洛朗展开

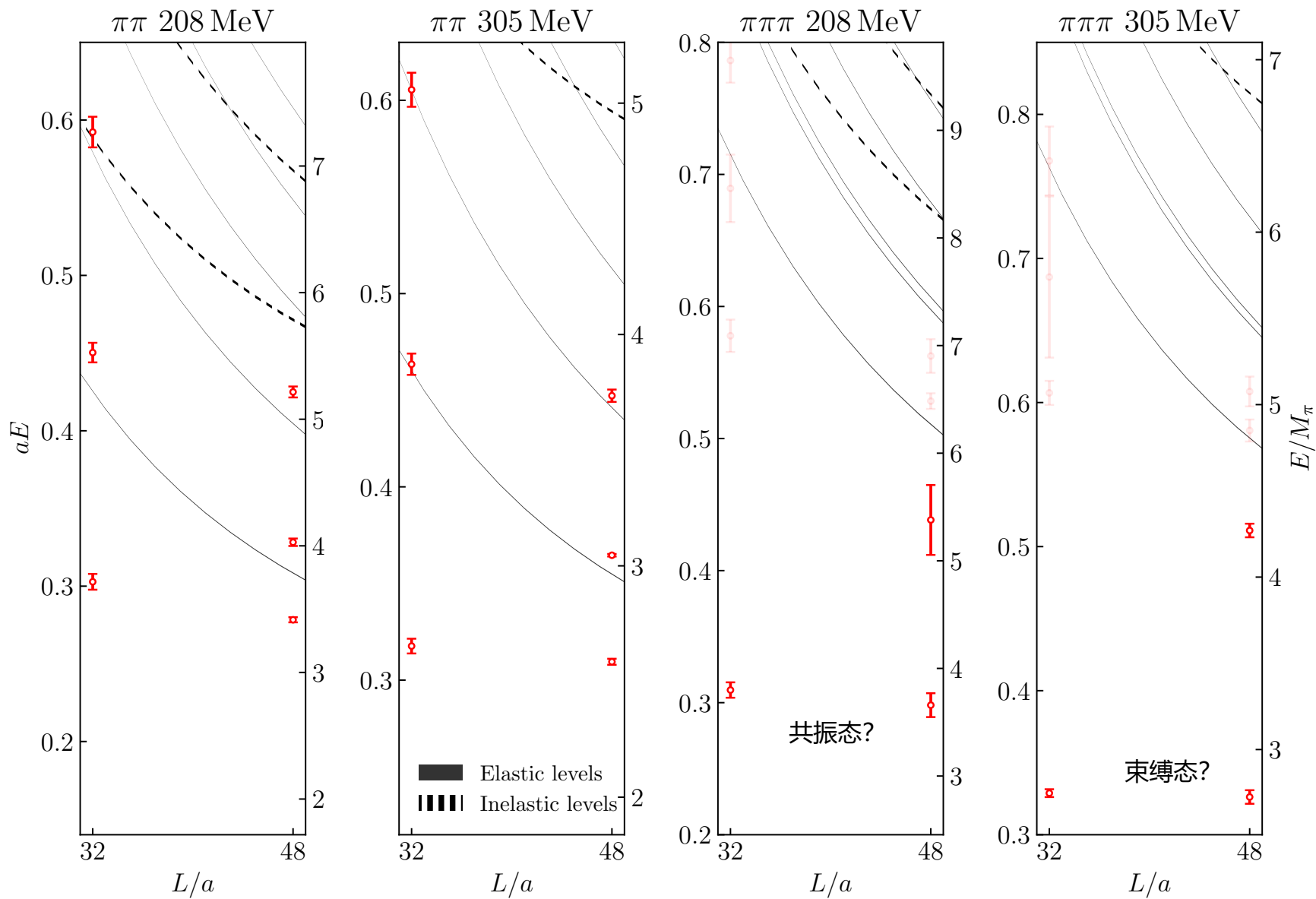
$$\begin{cases} \text{Diagram C} \sim \frac{\sigma - M_\rho^2}{g^2} \\ \text{Diagram B} \sim \frac{s(M_\rho^2 - \sigma_q + 6g^2 f_\pi^2)(M_\rho^2 - \sigma_p + 6g^2 f_\pi^2)}{g^2 f_\pi^6 (s - M_\omega^2)} \end{cases}$$

有效理论形式 (隐对称性方法)

其他形式理论

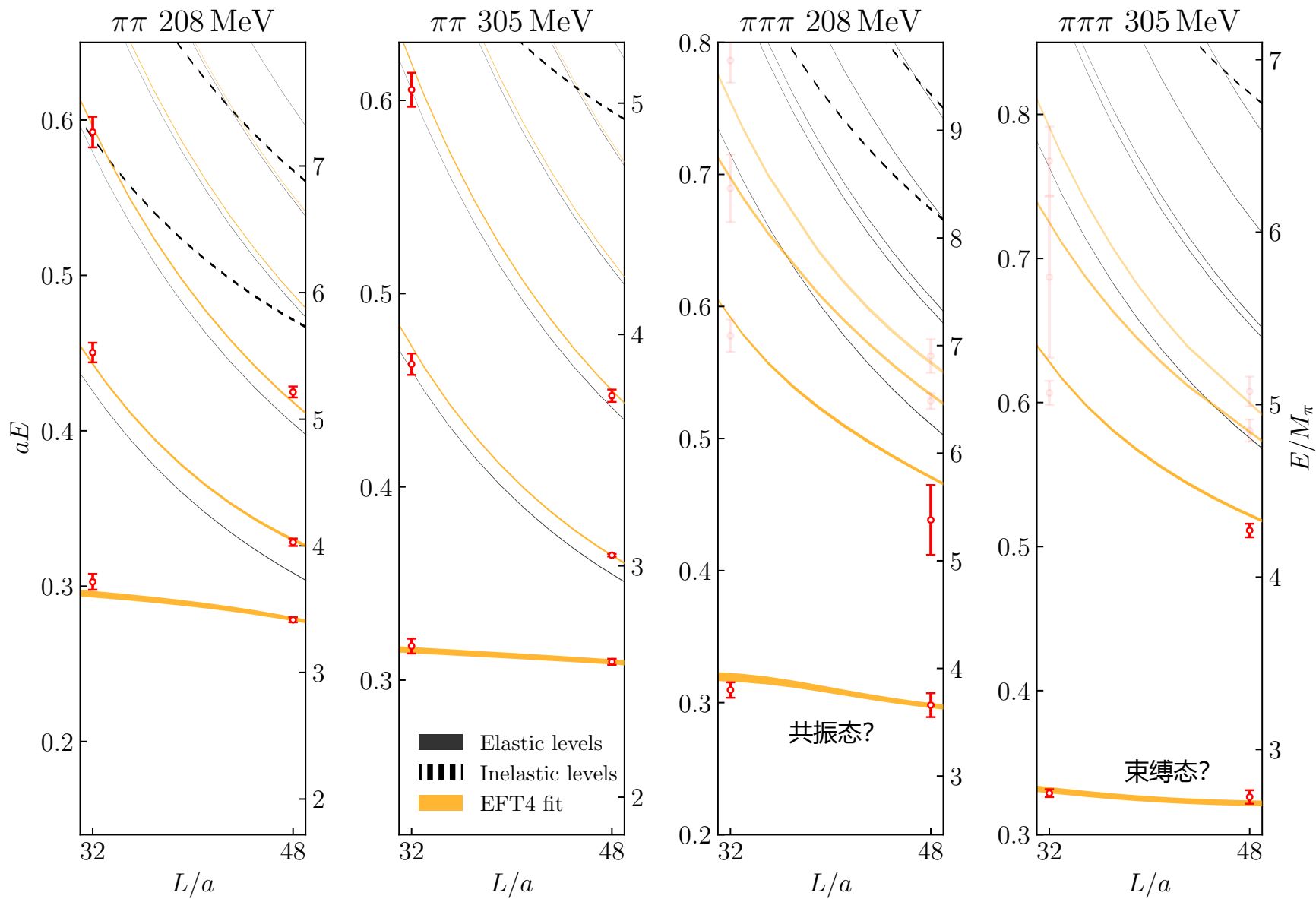
- FVU [Mai and Döring, 2017]
- RFT [Hansen and Sharpe, 2014]
- NREFT [Hammer, Pang, and Rusetsky, 2017]

有限体积能谱



- 两体 $\pi\pi \rightarrow \rho$
- 三体 $\pi\pi\pi$ 有非平凡的结构
- ω ?

有限体积能谱

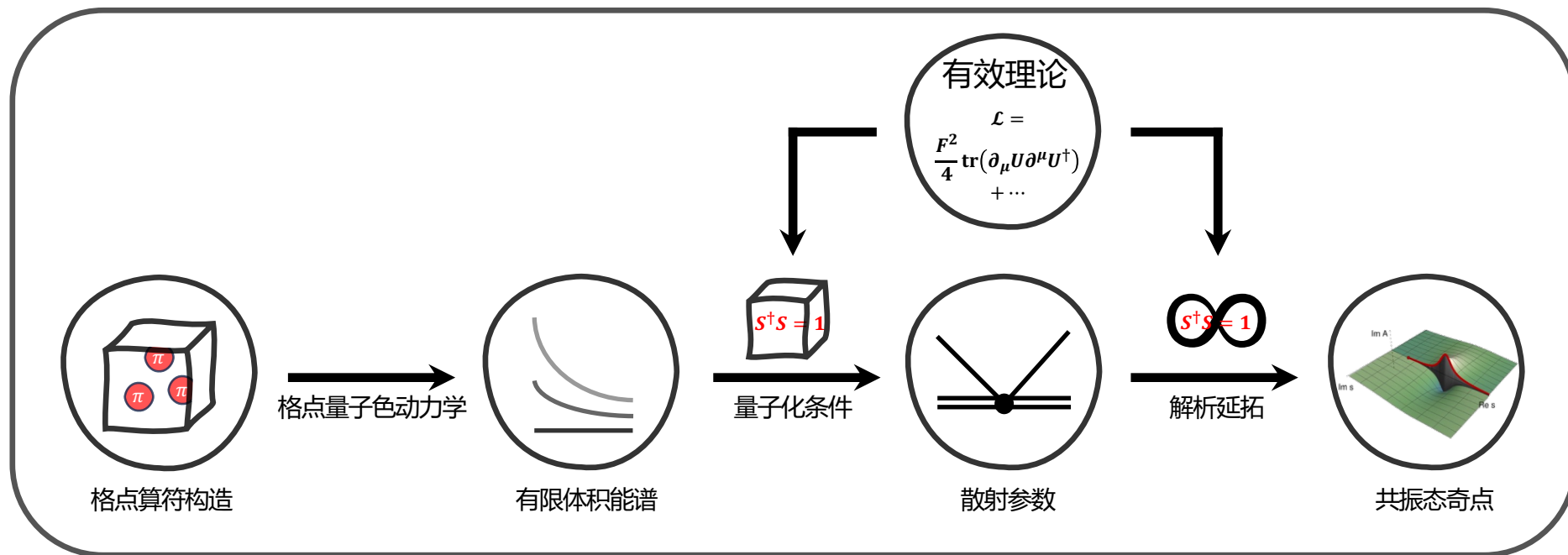


➤ 两体 $\pi\pi \rightarrow \rho$

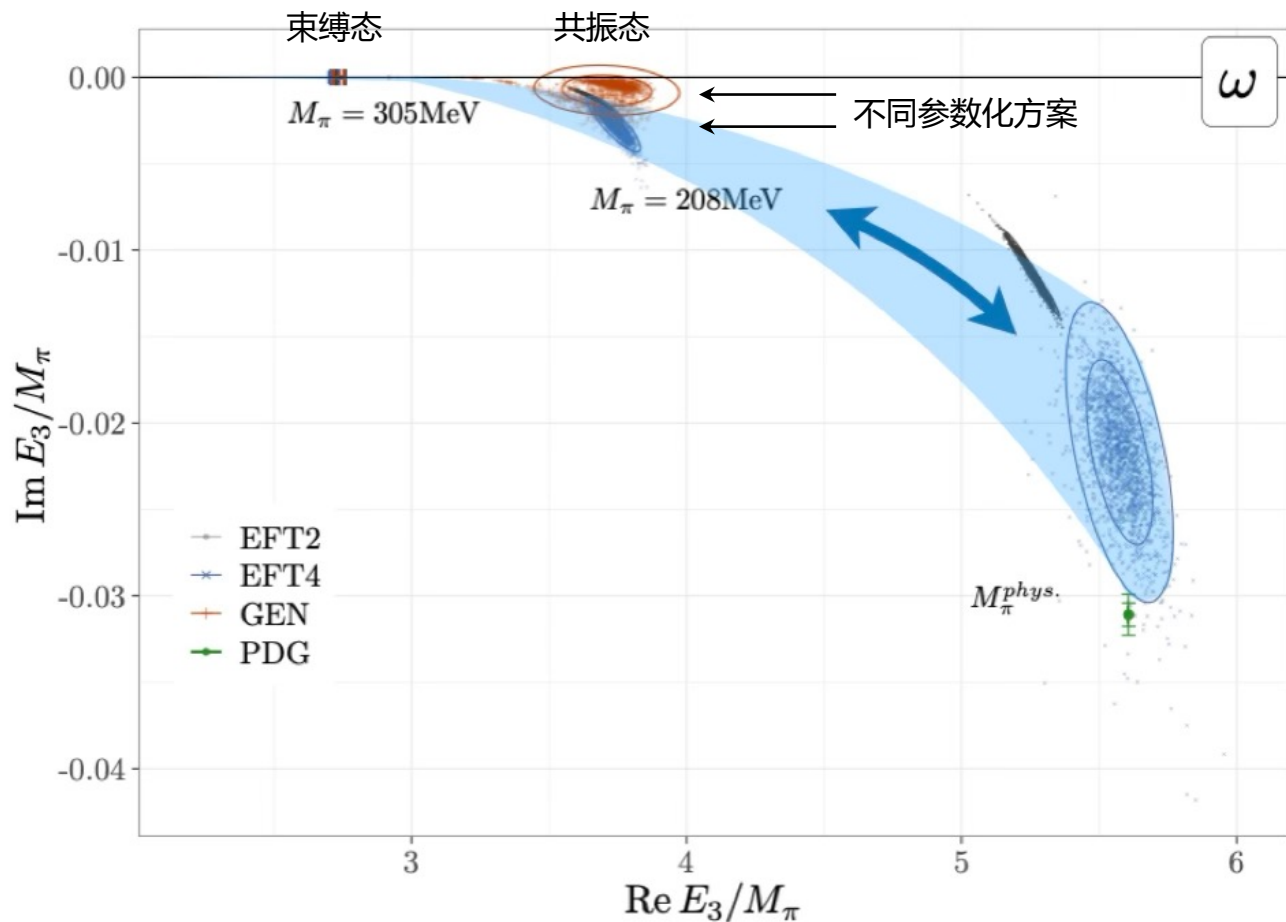
➤ 三体 $\pi\pi\pi$ 有非平凡的结构

➤ ω ?

计算流程



奇点位置



➤ 求解积分方程

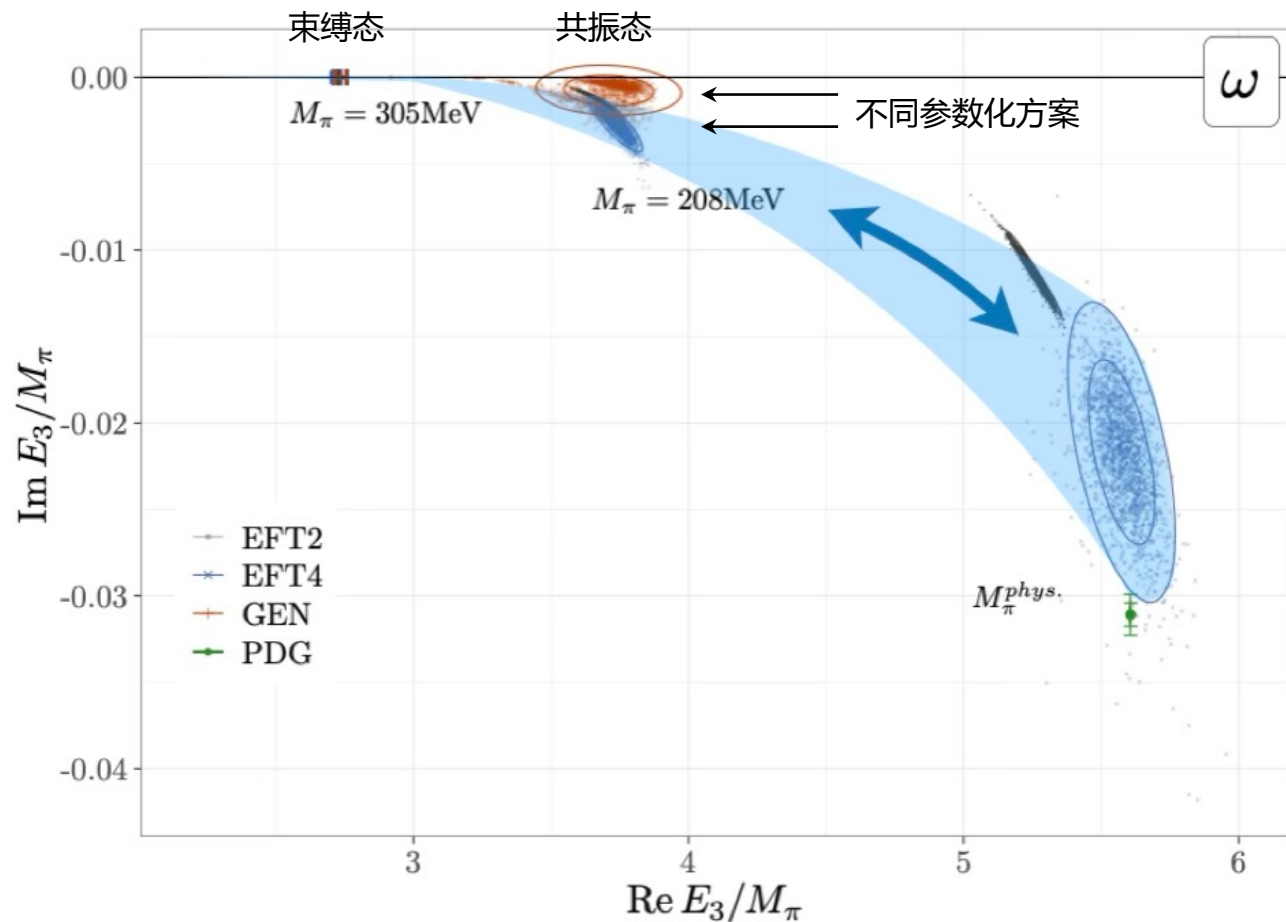
$$T = \tilde{B} + \tilde{C} + \int \frac{d^3l}{(2\pi)^3} \frac{\tilde{B} + \tilde{C}}{2E_l(\tilde{K}^{-1} - \Sigma^{IV})} T$$

➤ 不同参数化方案相符

➤ 外推到物理 M_π

$$M_{\omega(782)} = (778.0 \pm 11.2) - i(3.0 \pm 5) \text{ MeV}$$

奇点位置



➤ 求解积分方程

$$T = \tilde{B} + \tilde{C} + \int \frac{d^3l}{(2\pi)^3} \frac{\tilde{B} + \tilde{C}}{2E_l(\tilde{K}^{-1} - \Sigma^{IV})} T$$

➤ 不同参数化方案相符

➤ 外推到物理 M_π

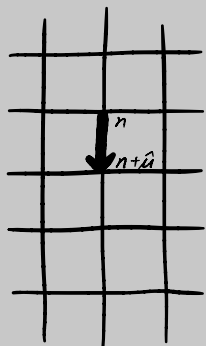
$$M_{\omega(782)} = (778.0 \pm 11.2) - i(3.0 \pm 5) \text{ MeV}$$

➤ 没有估计格距误差

➤ 没有考虑 $K\bar{K}$ 耦合

➤ 正在考察 $\omega - \phi$ 混合问题

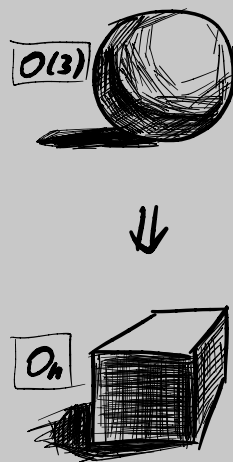
格点谱学 简介



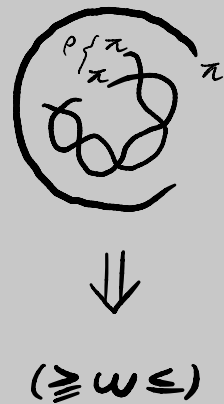
$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U$$

$$e^{-S_F - S_G}$$

格点算符 构造方法



三体散射问题 $\pi\pi\pi \rightarrow \omega(782)$



三体散射问题 $\pi\pi\pi \rightarrow \pi(1300)$



总结

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

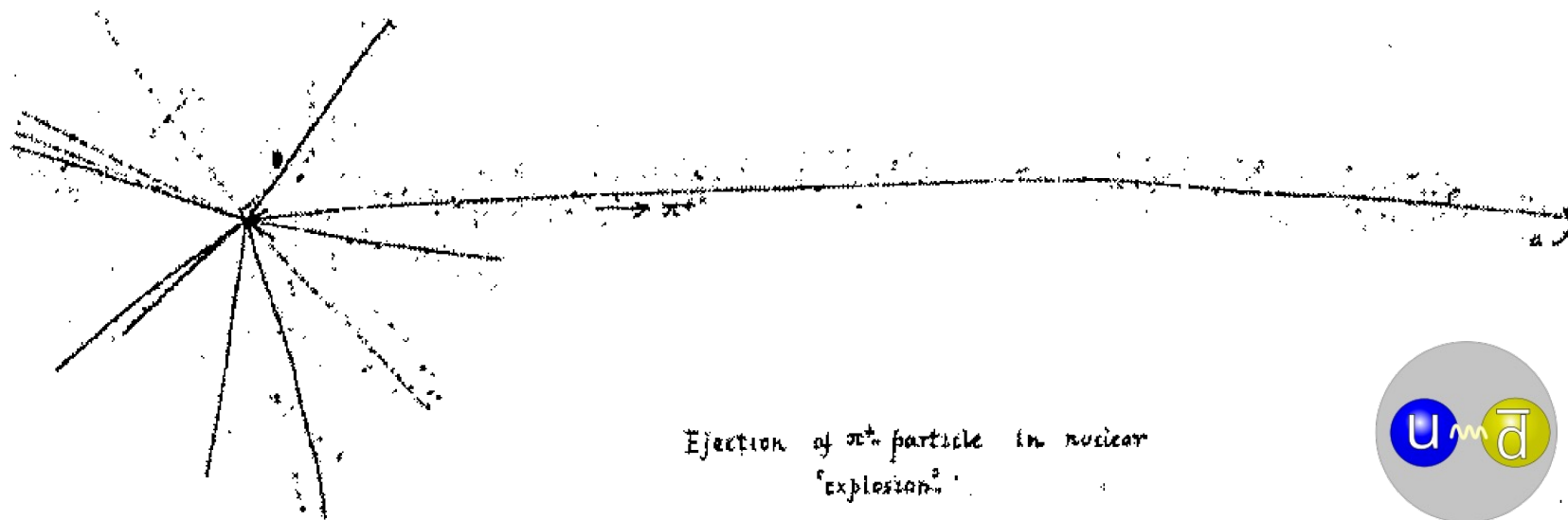
$$+ i \bar{\psi} \not{\partial} \psi + h.c.$$

$$+ \psi_i \psi_j \psi_k \phi + h.c.$$

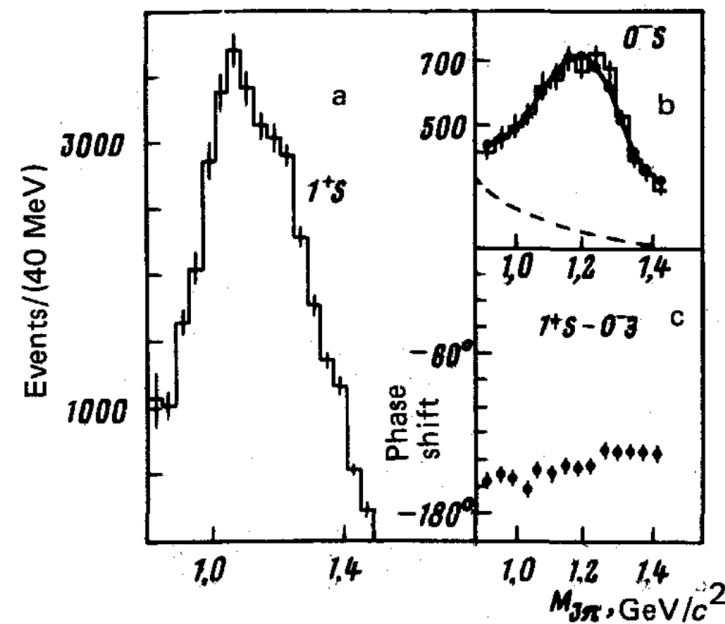
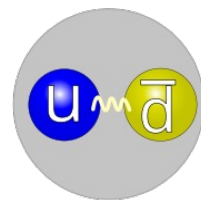
$$+ 1/2 \phi^2 - V(\phi)$$

$$+ ?$$

三体 $\pi\pi\pi \rightarrow \pi(1300)$ 散射



π 介子的发现

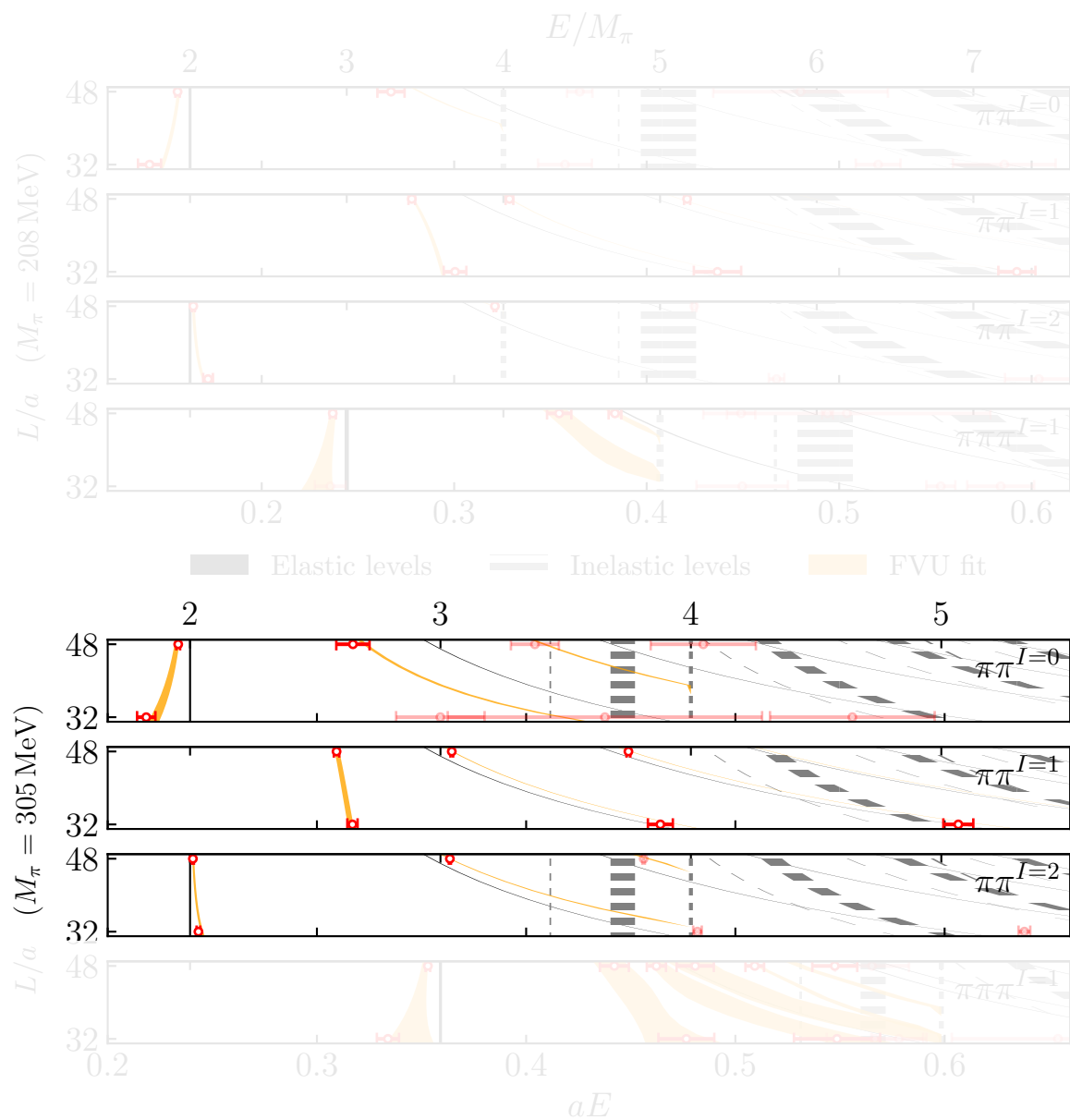


$\pi(1300)$ 的证据

Bellini et al (1961)

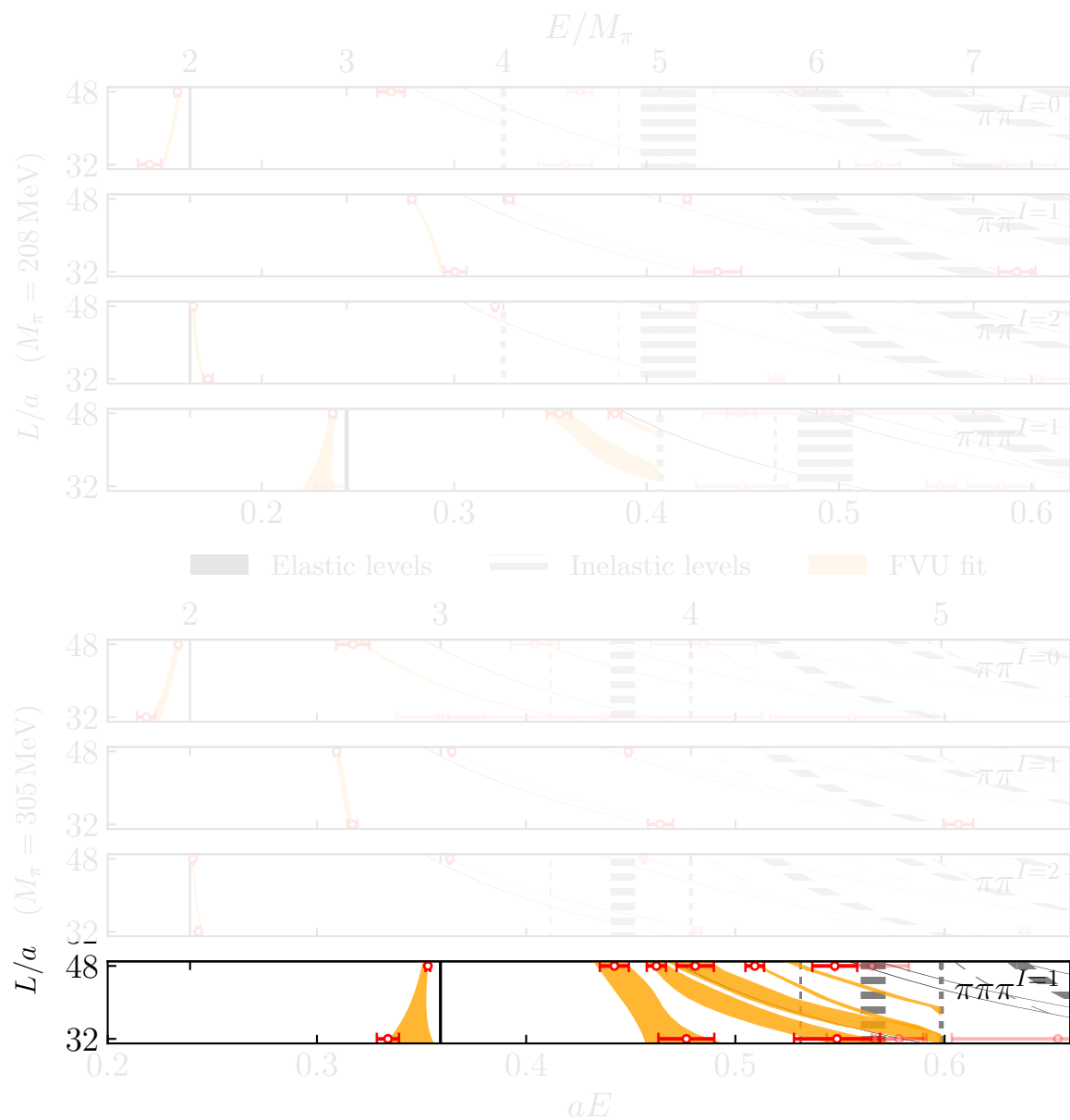
- 下一个挑战: $\pi(1300)$, 比基态 π 重 10 倍
- 其存在性尚存争议 \Rightarrow 超越对已知共振态的事后验证
- 是对软 π 定理的验证; 可能存在 $\bar{q}qg$ 混杂态成分
- 衰变到 $\pi\pi\pi$ 没有离心势, 是研究 Roper 共振的第二步

三体 $\pi\pi\pi \rightarrow \pi(1300)$ 散射



➤ 熟悉的 $I = 0, 1, 2$ $\pi\pi$ 能级

三体 $\pi\pi\pi \rightarrow \pi(1300)$ 散射

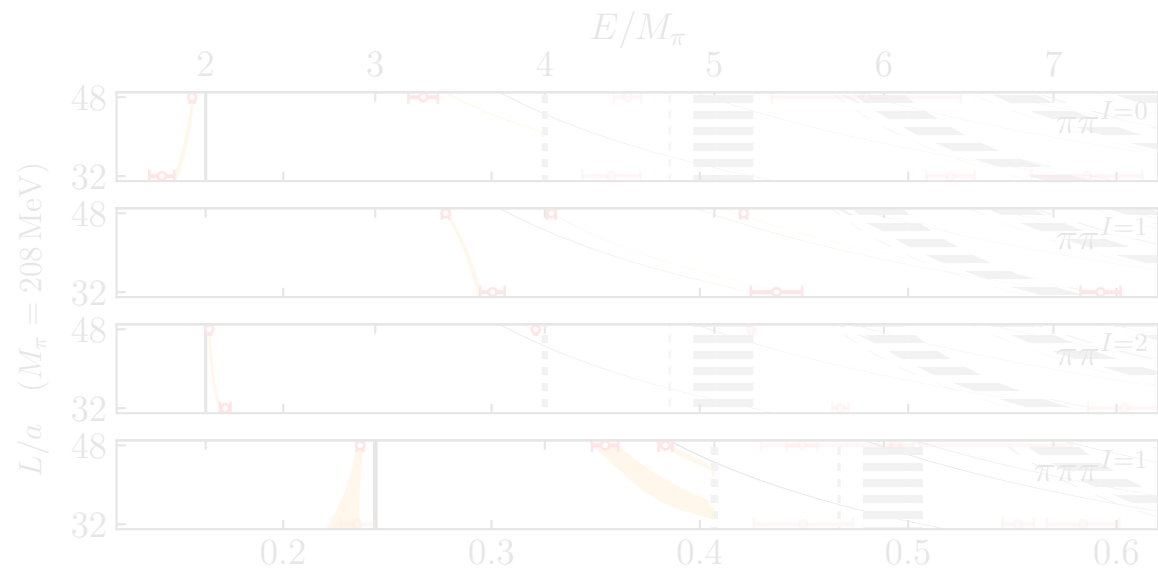


- 熟悉的 $I = 0, 1, 2$ $\pi\pi$ 能级
- 三体 $\pi\pi\pi$ 有大量的新能级聚集



- 暗示着非平庸的结构: 0^- 的共振态

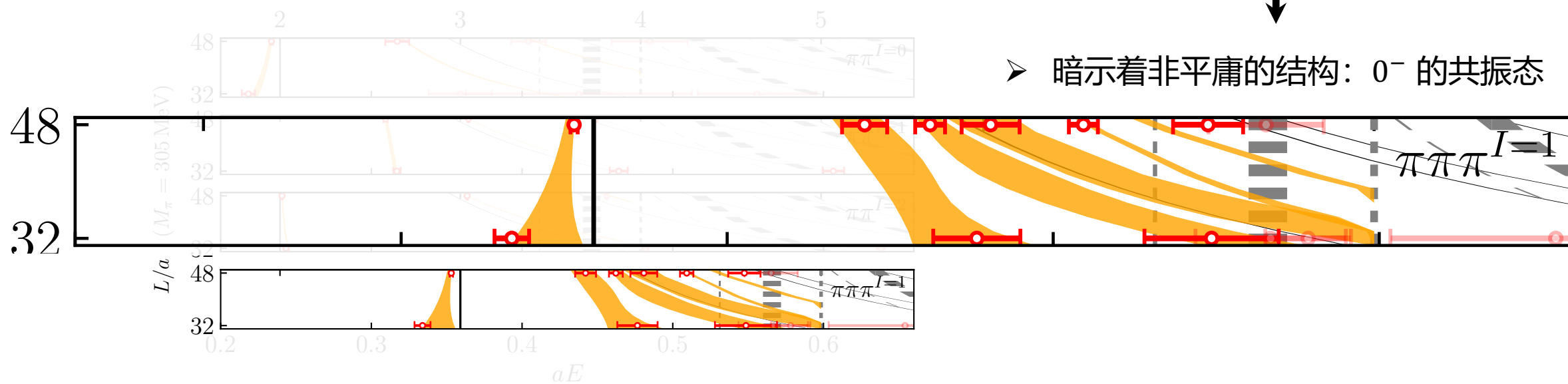
三体 $\pi\pi\pi \rightarrow \pi(1300)$ 散射



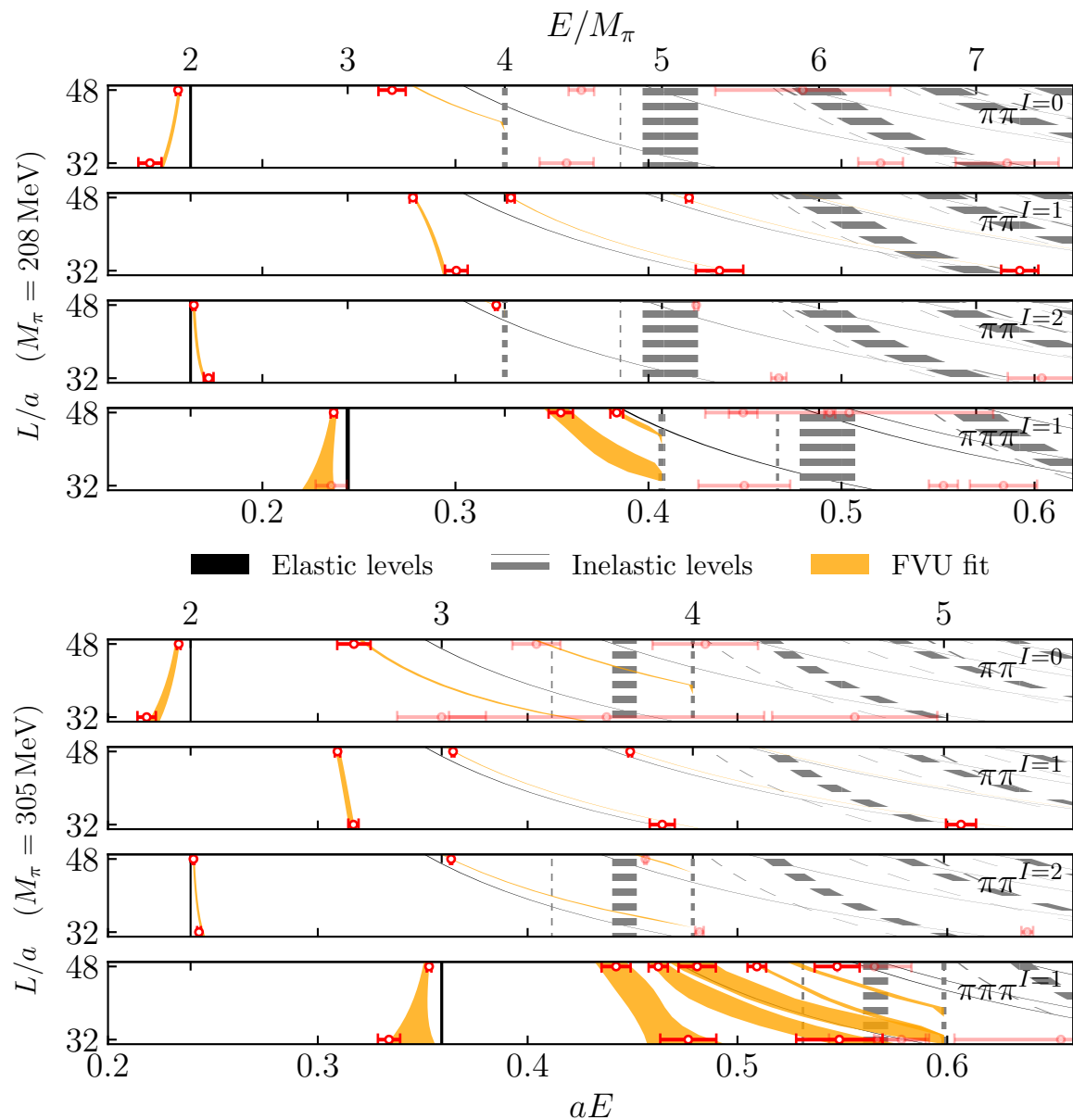
- 熟悉的 $I = 0, 1, 2$ $\pi\pi$ 能级
- 三体 $\pi\pi\pi$ 有大量的新能级聚集



- 暗示着非平庸的结构: 0^- 的共振态



三体 $\pi\pi\pi \rightarrow \pi(1300)$ 散射

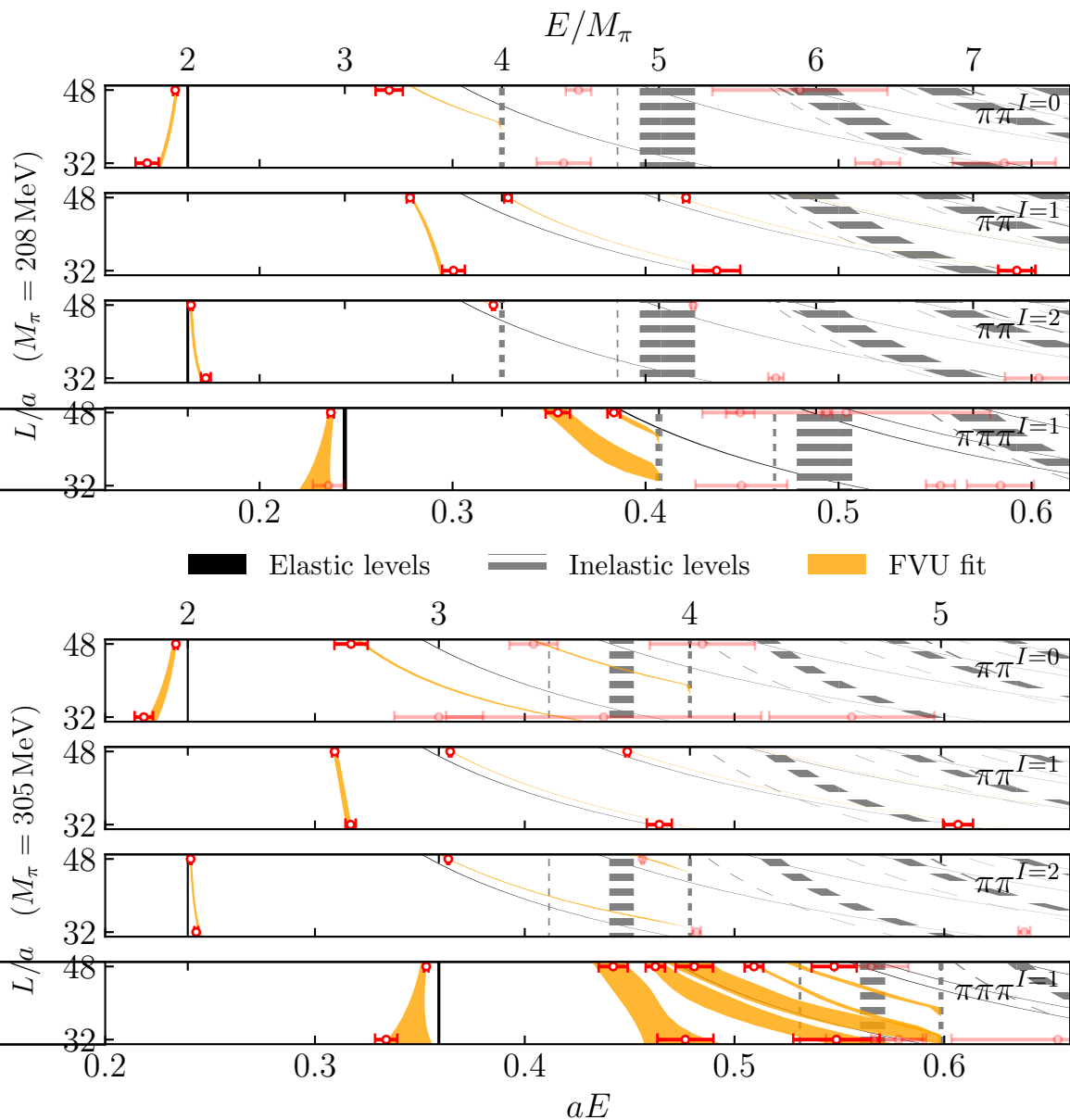


- 熟悉的 $I = 0, 1, 2$ $\pi\pi$ 能级
- 三体 $\pi\pi\pi$ 有大量的新能级聚集



- 暗示着非平庸的结构: 0^- 的共振态

三体 $\pi\pi\pi \rightarrow \pi(1300)$ 散射



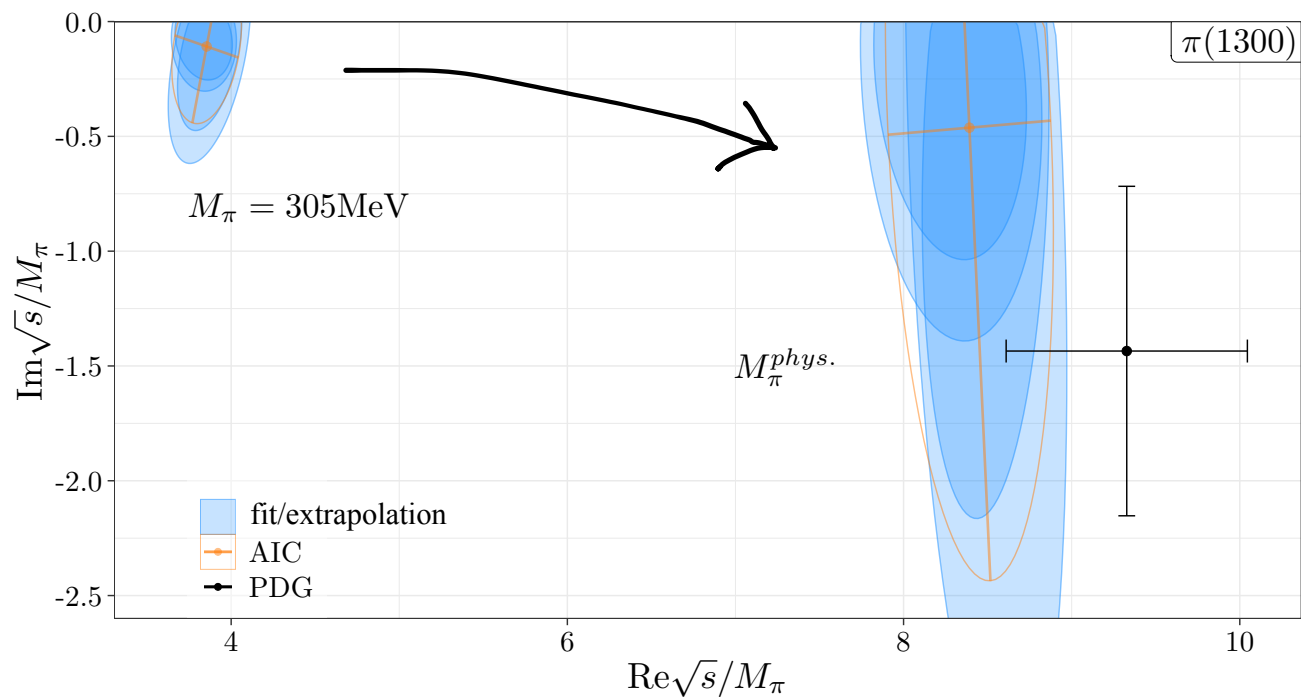
➤ 熟悉的 $I = 0, 1, 2$ $\pi\pi$ 能级

➤ 三体 $\pi\pi\pi$ 有大量的新能级聚集



➤ 暗示着非平庸的结构: 0^- 的共振态

$\pi(1300)$ 奇点位置



- $M_\pi = 305$ MeV 找到共振态
- $M_\pi = 208$ MeV 只有一些样本有共振奇点 ($5M_\pi$ 阈)
- 对两体用 mIAM 方法, 三体力作洛朗展开
- 外推到物理点

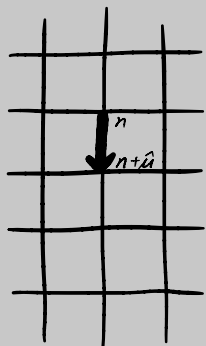
$$M_{\pi(1300)} = (1169 \pm 46) - i(62 \pm 169) \text{ MeV}$$

$$M_{\rho(770)} = (727 \pm 3) - i(72 \pm 1) \text{ MeV}$$

$$M_{f_0(500)} = (433 \pm 7) - i(250 \pm 7) \text{ MeV}$$

- 可以考虑其他径向激发态

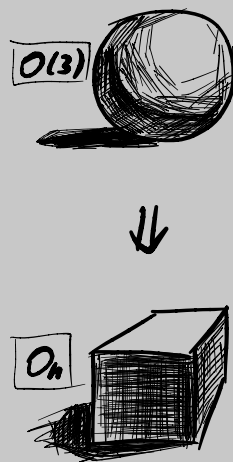
格点谱学 简介



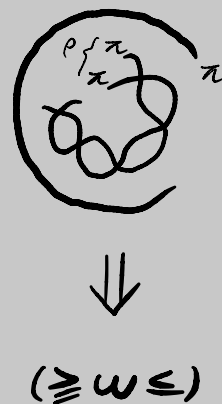
$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U$$

$$e^{-S_F - S_G}$$

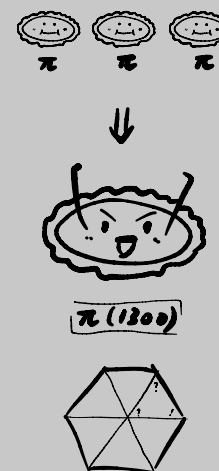
格点算符 构造方法



三体散射问题 $\pi\pi\pi \rightarrow \omega(782)$



三体散射问题 $\pi\pi\pi \rightarrow \pi(1300)$



总结

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu}$$

$$+ i \bar{\psi} \not{\partial} \psi + h.c.$$

$$+ \psi_i \psi_j \psi_k \phi + h.c.$$

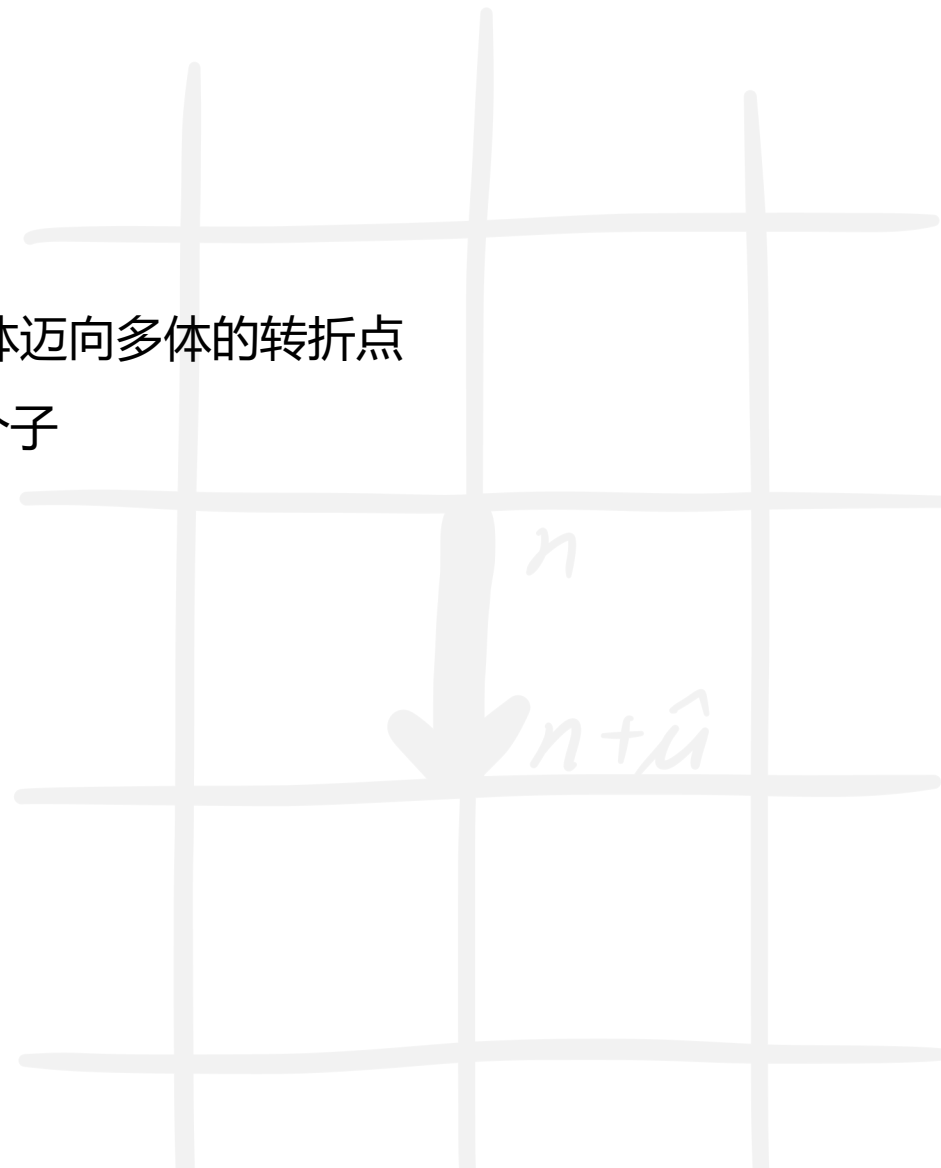
$$+ |D_\mu \phi|^2 - V(\phi)$$

$$+ ?$$

总结

- 格点场论是研究强子谱学强大的方法，我们正处于格点场论由两体迈向多体的转折点
- 第一性原理计算表明 QCD 可以涌现出 $\omega(782)$ 介子和 $\pi(1300)$ 介子
- 相比实验观测，格点知道得还很少

谢谢!



Short introduction to lattice QCD

- **Correlation functions** are calculated by Wick contractions

$$\langle \Omega | O(t) O^\dagger(0) | \Omega \rangle = \frac{1}{Z} \int \mathcal{D}[\psi, \bar{\psi}] \mathcal{D}[U] e^{-S_F[\psi, \bar{\psi}, U] - S_G[U]} (O(t) O^\dagger(0))[\psi, \bar{\psi}, U]$$

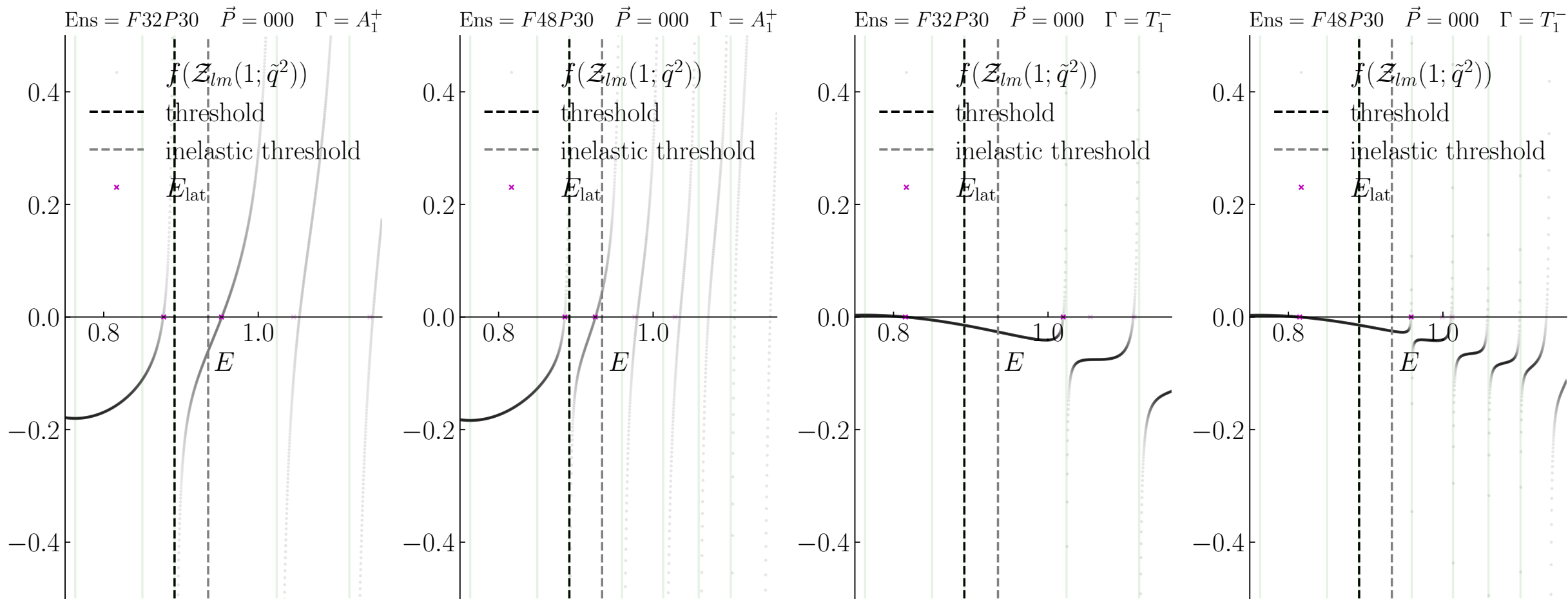
- Impossible to perform the path integral

$$\begin{array}{ccccccc} \text{Dimension} & \sim & 48^3 & \times & 96 & \times & 4 & \times & 3 & \sim & 10^9 \\ & & \text{volume} & & \text{time} & & \text{spin} & & \text{color} & & \end{array}$$

- Generate Monte-Carlo ensembles of U according to the gauge action
- Then measure QCD on them

$$\langle O \rangle \approx \frac{1}{N} \sum_{\substack{U_n \text{ with} \\ \text{probability} \\ \propto e^{-S[U_n]}}} O[U_n]$$

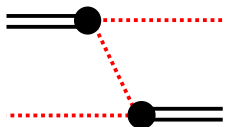
Eg: Finite-volume spectra at $m_\pi \approx 305$ MeV



量子化条件

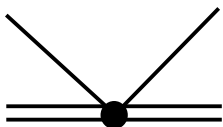
➤ 有限体积幺正化理论 (FVU, Finite-Volume Unitarity)

$$\begin{cases} \tilde{K}^{-1}(\sigma) - \Sigma^{FV}(\sigma) = 0 \\ \det[(\tilde{K}^{-1}(s) - \Sigma^{FV}(s))E_L - (\tilde{B}(s) + \tilde{C}(s))] = 0 \end{cases}$$



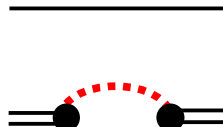
B

幺正性固定



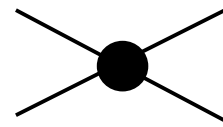
C

三体力



Σ

幺正性固定



\tilde{K}^{-1}

两体力

- 两体是三体的输入
- 需要对旁观粒子动量作截断, 结果并不敏感
- 阈下 $\tilde{K}^{-1} \rightarrow 0$

$$\begin{cases} \text{Diagram } \tilde{K}^{-1} \sim a_0 + a_1\sigma + \dots \\ \text{Diagram } \Sigma \sim \frac{c_0}{s - M_\omega^2} + c_1 + \dots \end{cases}$$

洛朗展开

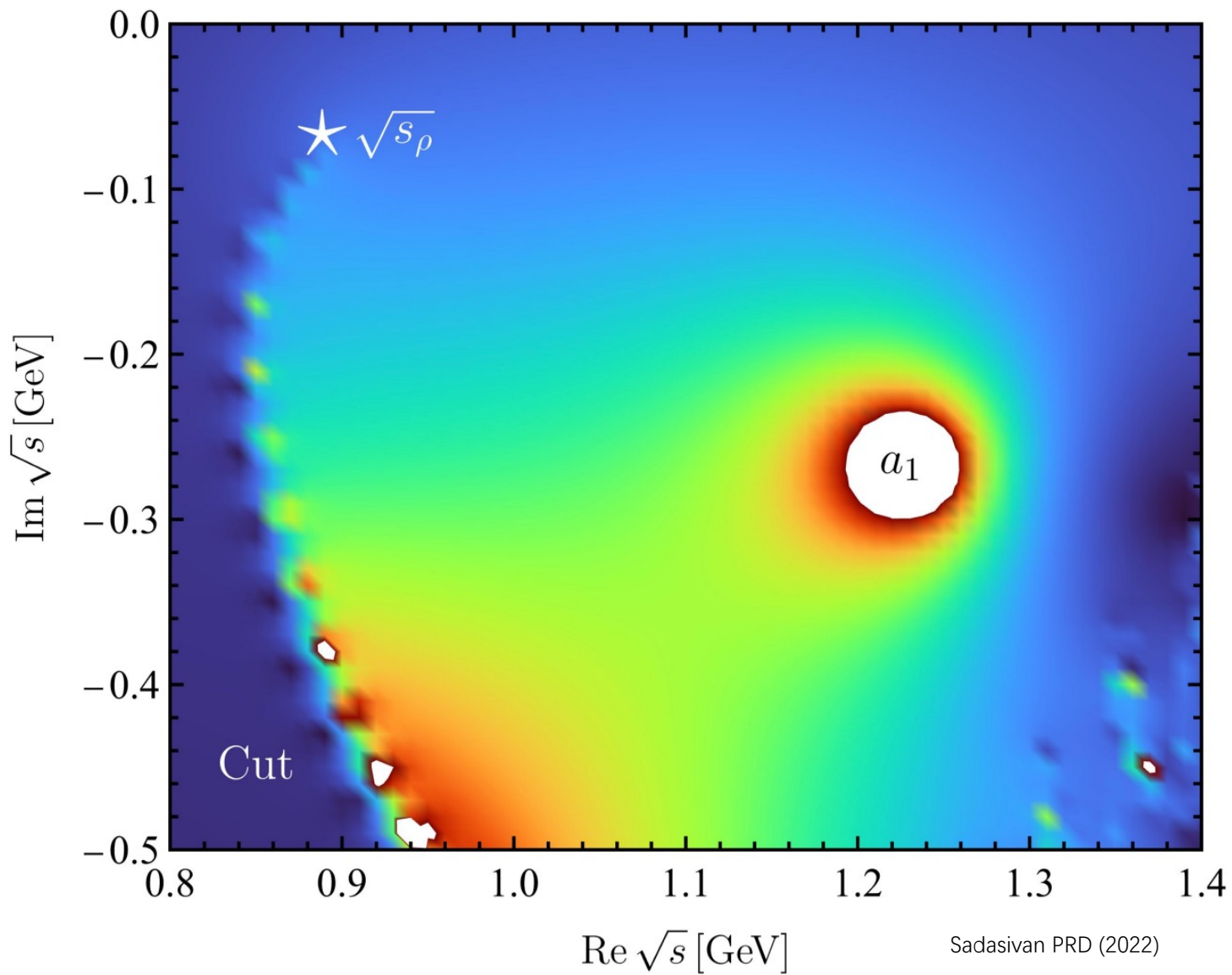
$$\begin{cases} \text{Diagram } \tilde{K}^{-1} \sim \frac{\sigma - M_\rho^2}{g^2} \\ \text{Diagram } \Sigma \sim \frac{s(M_\rho^2 - \sigma_q + 6g^2 f_\pi^2)(M_\rho^2 - \sigma_p + 6g^2 f_\pi^2)}{g^2 f_\pi^6 (s - M_\omega^2)} \end{cases}$$

有效理论形式 (隐对称性方法)

其他形式理论

- FVU [Mai and Döring, 2017]
- RFT [Hansen and Sharpe, 2014]
- NREFT [Hammer, Pang, and Rusetsky, 2017]

Analytic continuation example



- It is possible to do a smooth analytical continuation in our energy of interest
- Spectator momentum cutoff is chosen such that the analytical continuation covers the interested region of the Riemann sheet