# Accessing baryon-antibaryon generalized distribution amplitudes in $e^+e^- \to B\bar{B}\gamma$ and $e^\pm\gamma \to e^\pm B\bar{B}$


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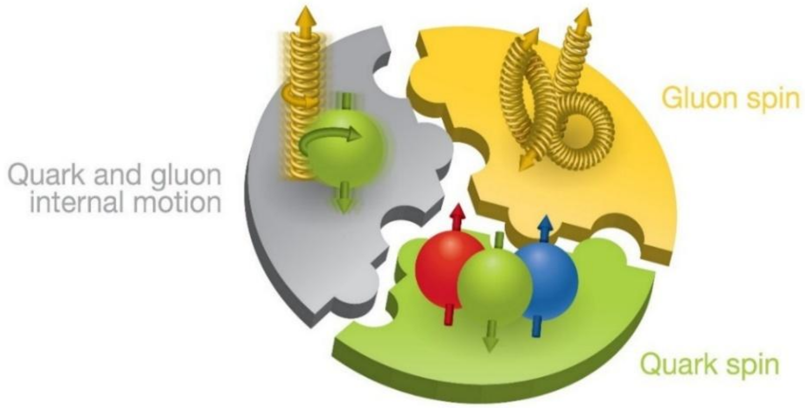
Reference：

J.Han, B.Pire, and Q.-T.Song, PRD 112(2025) , 014048

J.Han, B.Pire, and Q.-T.Song, PRD 113(2026) , 014027




## Proton Spin puzzle

Proton Spin
- $\Delta u^{+}, \Delta d^{+}, \Delta s^{+}$: ~30%
- $L_q, L_g$: rest part

## Generalized Parton Distributions (GPDs)

GPDs provide information on $L_q$ to solve the proton spin puzzle!

Leading-twist proton GPDs:

X.-D. Ji, PRL 78, 610 (1997)

$$\frac{1}{2}\int\frac{dy^-}{2\pi}e^{-ix\bar{P}^+y^-}\langle p_2|\bar{q}\left(y^-\right)\gamma^+q(0)|p_1\rangle$$
$$=\frac{1}{2\bar{P}^+}\left[H^q(x,\xi,t)\bar{u}(p_2)\gamma^+u(p_1)+E^q(x,\xi,t)\bar{u}(p_2)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p_1)\right]$$

# GPDs and Proton spin puzzle

From GPDs to Gravitational Form Factors(**GFFs**):

$$\int_{-1}^{1} dx\, xH^q(x,\xi,t) = A^q(t) + \xi^2 D^q(t)$$

$$\int_{-1}^{1} dx\, xE^q(x,\xi,t) = B^q(t) - \xi^2 D^q(t)$$

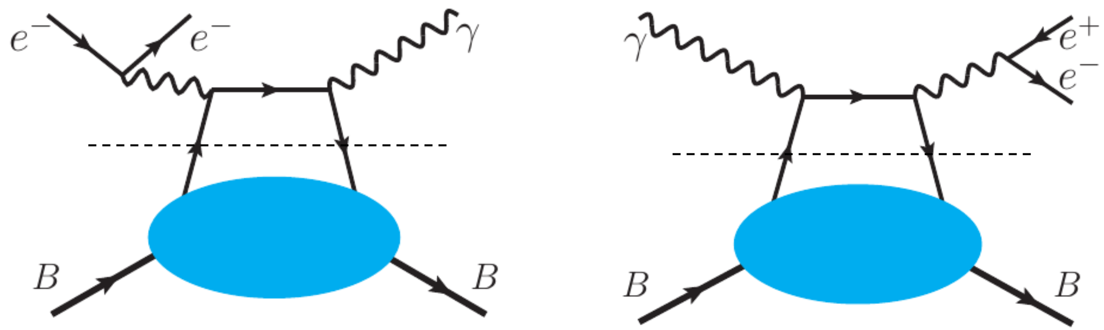
Quark Augular Momentum $J_q$ (Ji's sum rule):

$$J_q = \frac{1}{2}[A(0) + B(0)] = \frac{1}{2}\Delta q + L_q$$

Hadron GPDs are accessed in two-photon processes such as DVCS and TCS under QCD collinear factorization.

The DVCS and TCS amplitudes can be separated into hard and soft parts. The soft part is described by GPDs.

$$\mathcal{M} = \mathrm{GPD} \otimes \mathrm{Hard\ Part}$$



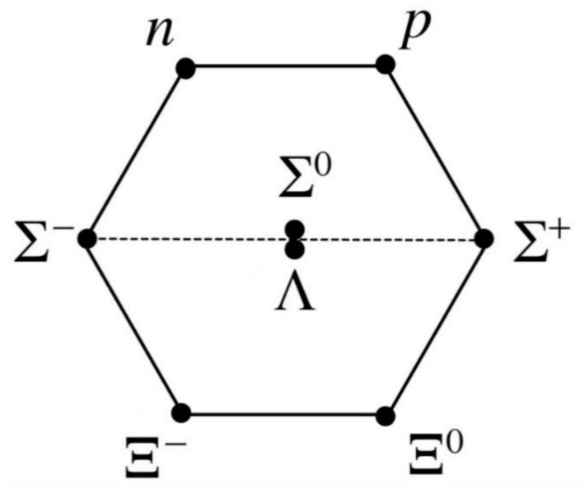

$eB \to e\gamma B$ (DVCS) $\quad$ $\gamma B \to e^+e^- B$ (TCS)

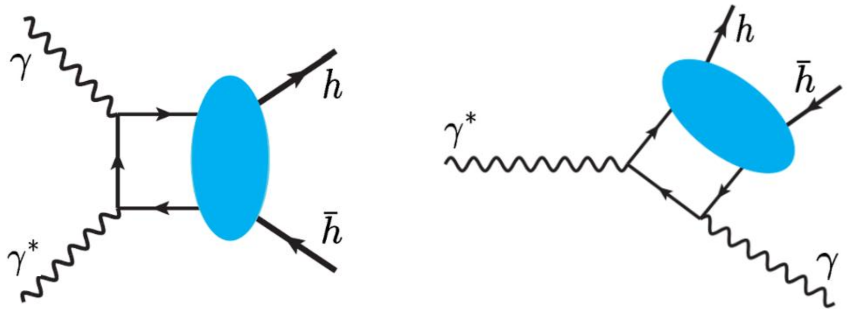
X.-D. Ji, PRL 78, 610 (1997)

E. R. Berger, M. Diehl and B. Pire, EPJC 23 (2002) 675-689

# Hardon GDAs

For unstable hadrons, it is difficult to measure the DVCS and TCS directly. In the baryon octet, only the proton GPDs can be probed.




By studying $\gamma\gamma^* \rightarrow h\bar{h}$ or $\gamma^* \rightarrow \gamma h\bar{h}$ , one can access the Generalized Distribution Amplitudes (**GDAs**) of unstable hadrons.

QCD factorization: $Q^2 \gg s^2, \Lambda^2_{QCD}$.
GDAs describe the amplitude of $q\bar{q} \rightarrow h\bar{h}$.


M. Diehl, T. Gousset, B. Pire, PRD 62 (2000) 073014

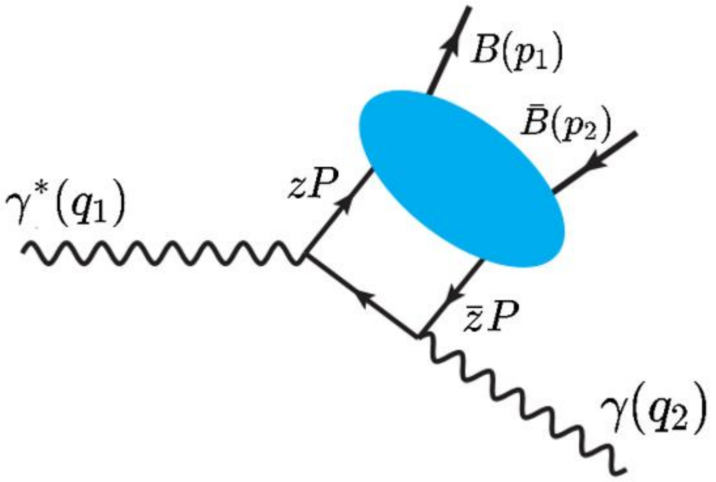
Z. Lu and I. Schmidt, PRD 73 (2006) 094021; 75 (2007) 099902(E)

# Baryon GDAs

The GDAs of Spin 1/2 baryon-antibaryon pair:

$$P^+ \int \frac{dx^-}{2\pi} e^{-izP^+x^-} \langle B(p_1)\bar{B}(p_2) | \bar{q}(\bar{x})\gamma^+ q(0)|0\rangle$$
$$= \Phi_V^q(z,\zeta_0,\hat{s})\bar{u}(p_1)\gamma^+ v(p_2) + \Phi_S^q(z,\zeta_0,\hat{s})\frac{P^+}{2m}\bar{u}(p_1)v(p_2)$$

$$P^+ \int \frac{dx^-}{2\pi} e^{-izP^+x^-} \langle B(p_1)\bar{B}(p_2) | \bar{q}(\bar{x})\gamma^+ \gamma_5 q(0)|0\rangle$$
$$= \Phi_A^q(z,\zeta_0,\hat{s})\bar{u}(p_1)\gamma^+\gamma_5 v(p_2) + \Phi_P^q(z,\zeta_0,\hat{s})\frac{P^+}{2m}\bar{u}(p_1)\gamma_5 v(p_2)$$



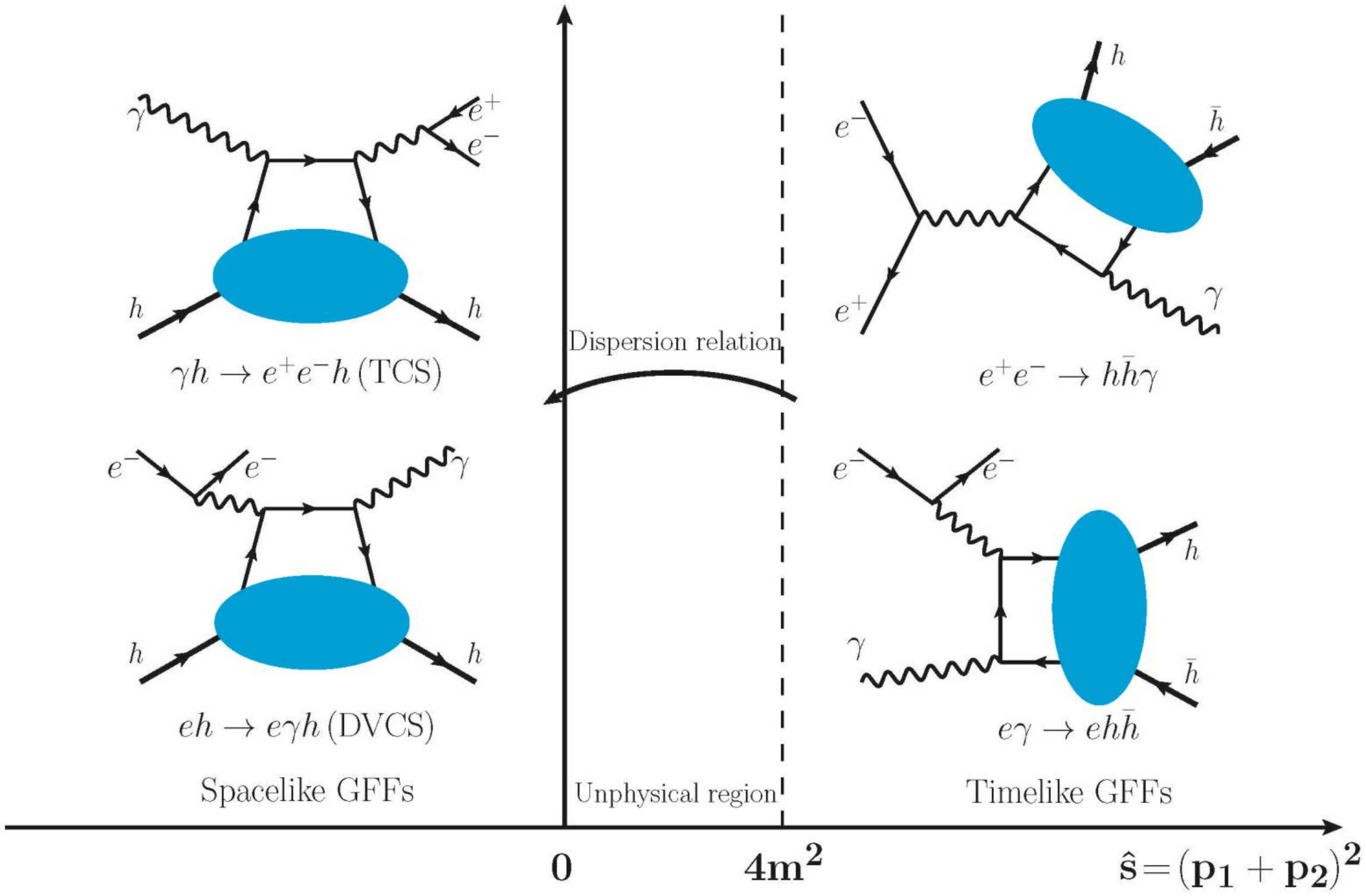

M. Diehl, P. Kroll, and C.Vogt, EPJC 26 (2003) 567

## From GDAs to timelike GFFs:

$$\int_0^1 dz(2z-1)\Phi_V^q(z,\zeta_0,\hat{s}) = -2\zeta_0 J^q(\hat{s}) \qquad \int_0^1 dz(2z-1)\Phi_S^q(z,\zeta_0,\hat{s}) = D^q(\hat{s}) + [A^q(\hat{s}) - 2J^q(\hat{s})](\zeta_0)^2$$

J. Han, B. Pire, and Q.-T. Song, PRD 112(2025) 014048

# GPDs,GDAs and GFFs




Dispersion relation:

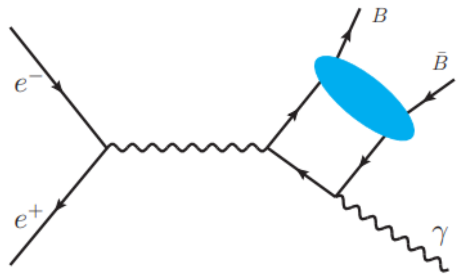
$$\mathcal{F}^B(t) = \int_{4m^2}^{\infty} \frac{d\hat{s}}{\pi} \frac{\mathrm{Im}\mathcal{F}^B(\hat{s})}{\hat{s} - t}$$

G. A. Miller, M. Strikman, and C. Weiss, PRD 83 (2011) 013006

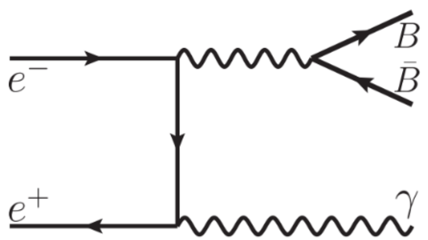
Spacelike and timelike GFFs are probed by different reactions, and the former ones can be obtained from the latter ones using dispersion relation.

# GDAs and $e^-e^+ \to B\bar{B}\gamma$

The process $e^-e^+ \to B\bar{B}\gamma$ includes two subprocesses:




$e^-e^+ \to \gamma^* \to \gamma B\bar{B}$




$e^-e^+ \to \gamma^*\gamma \to \gamma B\bar{B}$

(1) QCD subprocess:$e^-e^+ \to \gamma^* \to B\bar{B}\gamma$, the blob represents the $B\bar{B}$ GDAs.

$$\Phi_i^q(z, \zeta_0, \hat{s}), \quad \text{for } i = V, S, A, P$$

(2) ISR subprocess: $e^-e^+ \to \gamma^*\gamma \to B\bar{B}\gamma$, the $\gamma^* \to B\bar{B}$ vertex is parameterized in terms of the EM form factors (FFs).

$$\langle \bar{B}(p_2)B(p_1)|\, \bar{q}(0)\gamma^\mu q(0)\,|0\rangle = F_V^q(\hat{s})\bar{u}(p_1)\gamma^\mu v(p_2) + F_S^q(\hat{s})\frac{\Delta^\mu}{2m}\bar{u}(p_1)v(p_2),$$

One can extract the baryon GDAs from the process $e^-e^+ \to B\bar{B}\gamma$ with the help the timelike baryon EM FFs.Recently, the EM FFs of the baryon octet family have been extensively studied at BESIII.

M. Ablikim et al. (BESIII Collaboration), PLB 817 (2021)136328.

See the talks of Prof. Ling-Yun Dai and Ju-Jun Xie for the timelike baryon EM FFs!

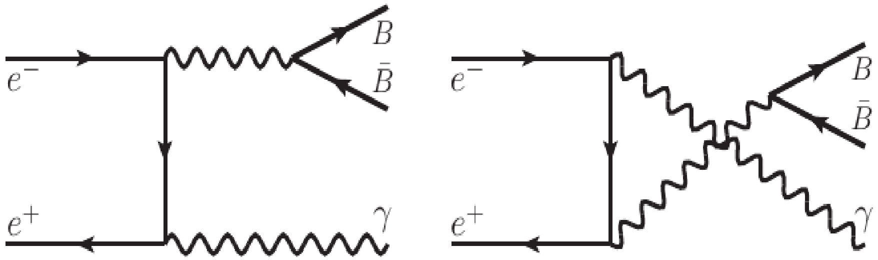
L.-Y. Dai, J. Haidenbauer and U.-G. Meißner, CPL 42 (2025) 030202

B. Yan, C. Chen, X. Li, and J.-J. Xie, PRD 109 (2024) 036033 .

# Cross Sections of $e^-e^+ \to B\bar{B}\gamma$

The contribution of ISR process:

$$e^+e^- \to \gamma^*\gamma \to B\bar{B}\gamma$$




$$\frac{d\sigma_{\rm ISR}}{d\hat{s}\,du\,d(\cos\theta)d\varphi} = \frac{\alpha_{\rm em}^3\beta_0^3}{4\pi s^2}\frac{1}{\epsilon\hat{s}}\left[b_0 + b_1\cos^2\theta + b_2\sin^2\theta + b_3\sin(2\theta)\cos\varphi + b_4\sin^2\theta\cos(2\varphi)\right]$$

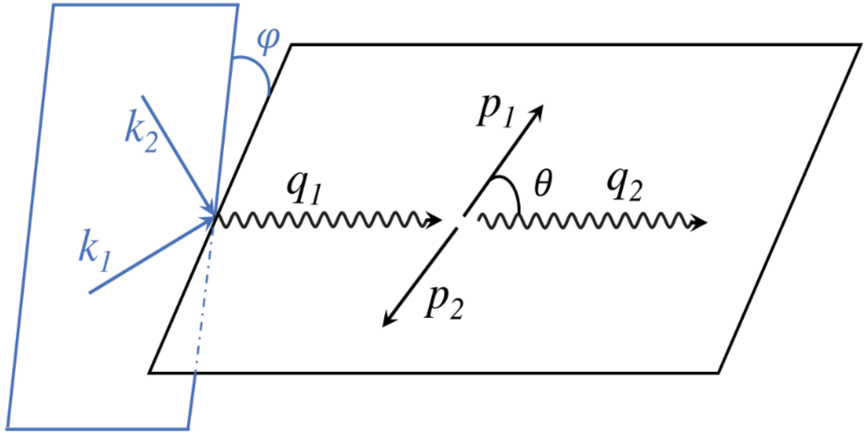
$$b_0 = [1-2x(1-x)(1+\epsilon)](2\lambda-1)|G_M|^2,$$

$$b_1 = [1-2x(1-x)(1-\epsilon)]|G_M|^2 + 4\epsilon x(x-1)(\lambda-1)\left[|G_E|^2 - |G_M|^2\right],$$

$$b_2 = 2\epsilon x(x-1)|G_M|^2 + [1-2x(1-x)](\lambda-1)\left[|G_E|^2 - |G_M|^2\right],$$

$$b_3 = \sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}(2x-1)\mathrm{sgn}(\rho)\left[(\lambda-1)|G_E|^2 - \lambda|G_M|^2\right],$$

$$b_4 = 2\epsilon x(1-x)\left[(\lambda-1)|G_E|^2 - \lambda|G_M|^2\right].$$

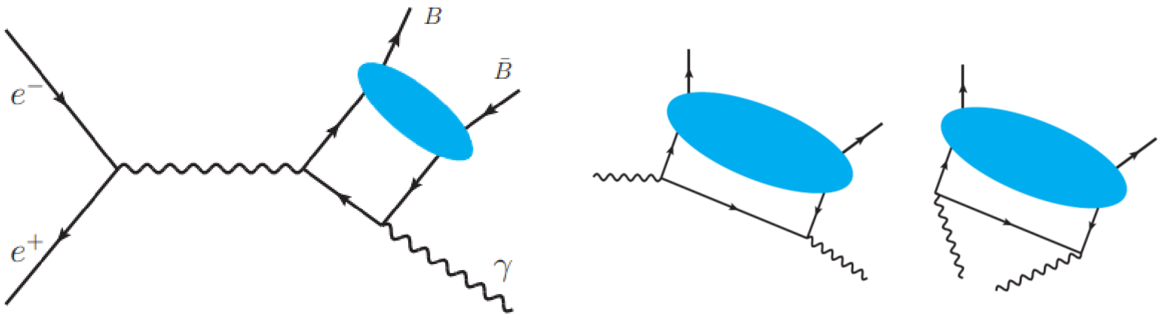



The Center-of-mass frame of $B\bar{B}$

# Cross Sections of $e^-e^+ \to B\bar{B}\gamma$

The QCD subprocess:

$e^-e^+ \to \gamma^* \to B\bar{B}\gamma$




We define the Compton FFs:

$$(\zeta_0)\mathcal{F}_i = \sum_q \frac{e_q^2}{2}\int_0^1 dz \frac{2z-1}{z(1-z)}\Phi_i^q(z,\zeta,\hat{s})(i=V,S), \quad \mathcal{F}_{i'} = \sum_q \frac{e_q^2}{2}\int_0^1 dz \frac{1}{z(1-z)}\Phi_{i'}^q(z,\zeta,\hat{s})(i'=A,P)$$

The hardon tensor (leading twist) is given by

$$T_{\mu\nu} = \frac{-2}{\sqrt{2s}}\left\{g_T^{\mu\nu}\left[\zeta_0\mathcal{F}_V\bar{u}(p_1)\gamma^+v(p_2) + \mathcal{F}_S\frac{P^+}{2m}\bar{u}(p_1)v(p_2)\right] - i\epsilon_T^{\mu\nu}\left[\mathcal{F}_A\bar{u}(p_1)\gamma^+\gamma^5v(p_2) + \mathcal{F}_P\frac{P^+}{2m}\bar{u}(p_1)\gamma^5v(p_2)\right]\right\}$$

Cross section:

$$\frac{d\sigma_G}{d\hat{s}\,du\,d(\cos\theta)d\varphi} = \frac{\alpha_{em}^3\beta_0}{8\pi s^3}\frac{1}{1+\epsilon}\left[|\mathcal{F}_A|^2 - |\mathcal{F}_S|^2 + 2\mathrm{Re}(\mathcal{F}_A\mathcal{F}_P^*) + \frac{\hat{s}\left(|\mathcal{F}_P|^2+|\mathcal{F}_S|^2\right)}{4m^2}\right.$$
$$\left. + (\beta_0)^2\cos^2\theta\left[|\mathcal{F}_V|^2 + 2\mathrm{Re}(\mathcal{F}_S\mathcal{F}_V^*) - |\mathcal{F}_A|^2\right] - (\beta_0)^4\cos^4\theta|\mathcal{F}_V|^2\right]$$

# Cross Sections of $e^-e^+ \to B\bar{B}\gamma$

The interference term of two subprocesses should be also included.

$$|\mathcal{M}_{\mathrm{I}}|^2 = \mathcal{M}_{\mathrm{ISR}}\mathcal{M}_{\mathrm{G}}^* + \mathcal{M}_{\mathrm{ISR}}^*\mathcal{M}_{\mathrm{G}}$$

One can decompose the cross section according to its dependence on angles

$$\frac{d\sigma_{\mathrm{I}}}{d\hat{s}\,du\,d(\cos\theta)d\varphi} = \frac{\alpha_{\mathrm{em}}^3\beta_0}{8\pi s^2}\frac{\sqrt{2}\beta_0}{\sqrt{\hat{s}\,s\epsilon(1+\epsilon)}}\left[c_0\cos\theta + c_1\cos^3\theta + c_2\sin\theta\cos\varphi + c_3\sin(2\theta)\cos\theta\cos\varphi\right]$$

$$\begin{aligned}
c_0 =& 2\mathrm{sgn}(\rho)\sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}[\mathrm{Re}(\mathcal{F}_V G_M^*) + \mathrm{Re}(\mathcal{F}_S G_E^*)],\\
c_1 =& 2(\beta_0)^2\mathrm{sgn}(\rho)\sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}[(\lambda-1)\mathrm{Re}(\mathcal{F}_V G_E^*) - \lambda\mathrm{Re}(\mathcal{F}_V G_M^*)],\\
c_2 =& 2[1-(1-x)(1+\epsilon)]\mathrm{Re}(\mathcal{F}_A G_M^*) + 2[1-(1-x)(1-\epsilon)]\mathrm{Re}(\mathcal{F}_S G_E^*),\\
c_3 =& (\beta_0)^2[1-(1-x)(1-\epsilon)][(\lambda-1)\mathrm{Re}(\mathcal{F}_V G_E^*) - \lambda\mathrm{Re}(\mathcal{F}_V G_M^*)].
\end{aligned}$$

The study of interference terms helps us to extract $B\bar{B}$ GDA.

J. Han, B. Pire, and Q.-T. Song, PRD 112(2025) 014048

# Cross Sections of $e^-e^+ \to B\bar{B}\gamma$

Consider the exchange of $(\theta,\varphi) \to (\pi-\theta,\pi+\varphi)$:

$$d\sigma_{\rm ISR} \longrightarrow d\sigma_{\rm ISR} \qquad d\sigma_{\rm G} \longrightarrow d\sigma_{\rm G} \qquad d\sigma_{\rm I} \longrightarrow -d\sigma_{\rm I}$$

$$d\sigma\big(B,\bar{B}\big) - d\sigma\big(\bar{B},B\big) = 2d\sigma_{\rm I}$$

We can also define the forward-backward asymmetry:

$$A_{\rm FB}(\theta) = \frac{\int_{\pi/2}^{3\pi/2} d\varphi \frac{d\sigma(\theta,\varphi)}{d\cos\theta\, d\varphi} - \int_{3\pi/2}^{2\pi} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi} - \int_{0}^{\pi/2} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi}}{\int_{\pi/2}^{3\pi/2} d\varphi \frac{d\sigma(\theta,\varphi)}{d\cos\theta\, d\varphi} + \int_{3\pi/2}^{2\pi} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi} + \int_{0}^{\pi/2} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi}}$$

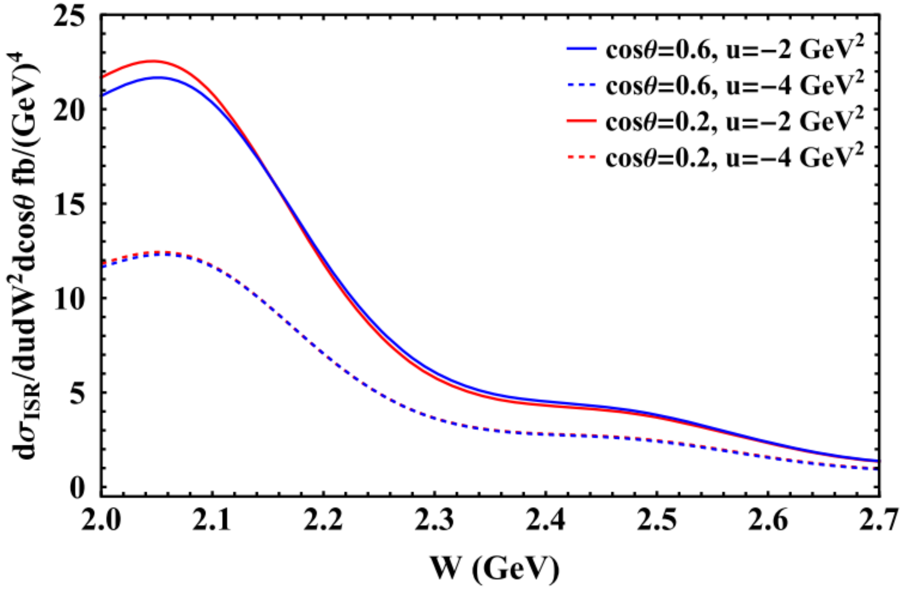
The numerator of $A_{FB}$ is given by

$$\frac{d\sigma_{\rm FB}}{d\hat{s}\,du\,d(\cos\theta)} = \frac{\alpha_{\rm em}^3 \beta_0}{8\pi s^2} \frac{\sqrt{2}\beta_0}{\sqrt{\hat{s}\,s}\,\epsilon(1+\epsilon)} \left[2\pi\big(c_0\cos\theta + c_1\cos^3\theta\big) - 4\big(c_2\sin\theta + c_3\sin(2\theta)\cos\theta\big)\right]$$

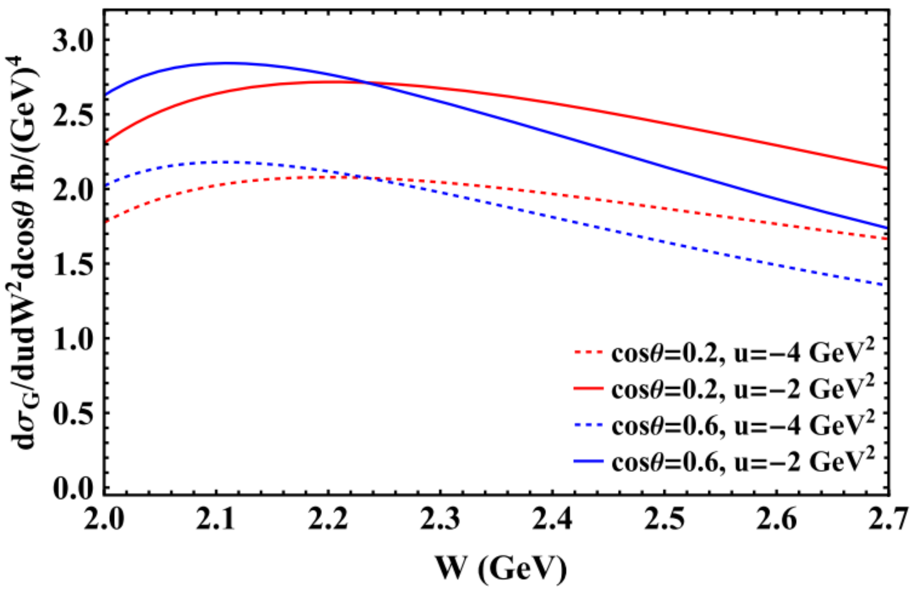
This asymmetry provides another way to obtain interference contribution.

# Numerical estimate of $e^-e^+ \to p\bar{p}\gamma$

We choose $\sqrt{s} = 4$ GeV. This value is typical for BESIII and the proposed STCF. The range of W is set between 2.0 to 2.7 GeV.

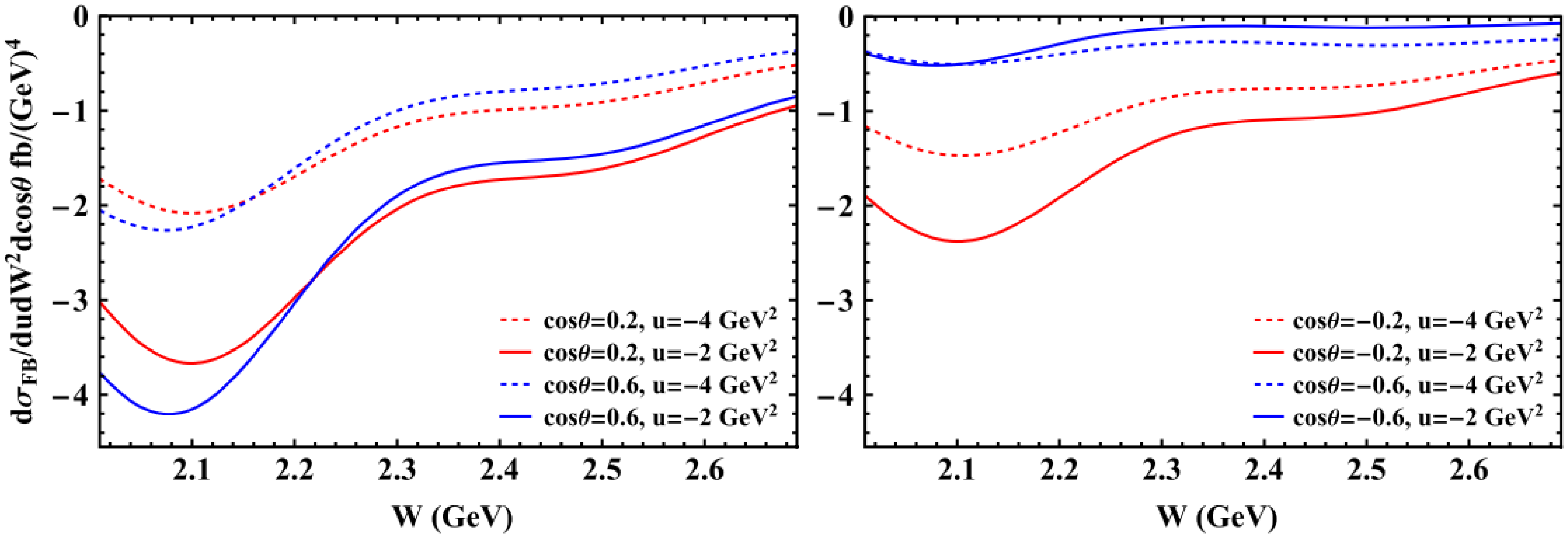



Estimate of ISR contribution




Estimate of $e^+e^- \to \gamma^* \to B\bar{B}\gamma$ contribution

# Numerical estimate of $e^-e^+ \to p\bar{p}\gamma$




Estimate of interference contribution

Our results shows that the interference term is large than pure QCD contribution, it will play an important role in the extraction of baryon GDAs.

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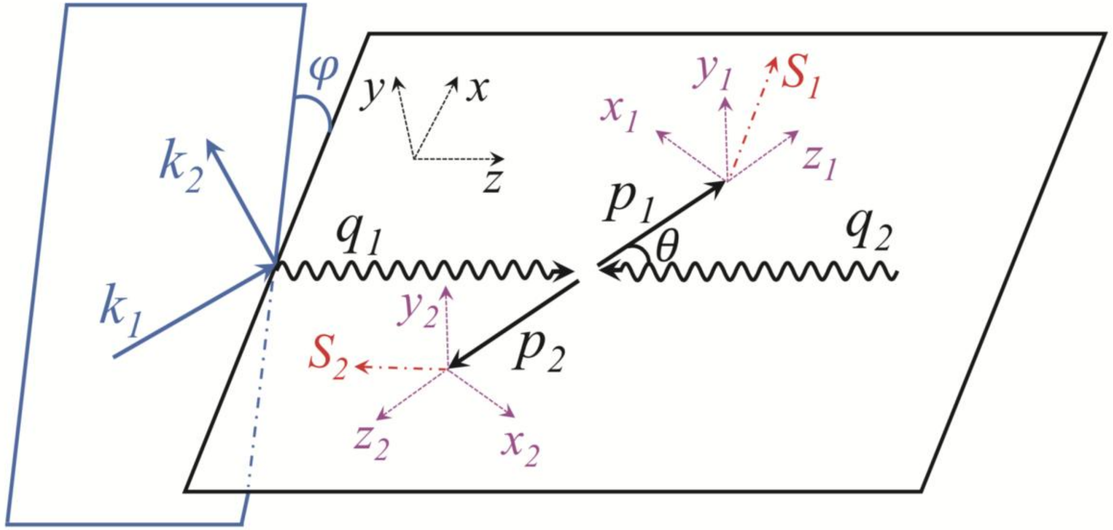
The production of a baryon-antibaryon pair:

$$e^{\pm}(k_1)\gamma(q_2) \to e^{\pm}(k_2)B(p_1, S_1)\bar{B}(p_2, S_2)$$




The Center-of-mass frame of $B\bar{B}$

Here, $S_1$ and $S_2$ are spin vectors that can be determined from the subsequent decays, such as $\Lambda \to N\pi$ and $\Sigma \to N\pi$.

In their own rest frame $x_1y_1z_1$ and $x_2y_2z_2$. These vectors are given by

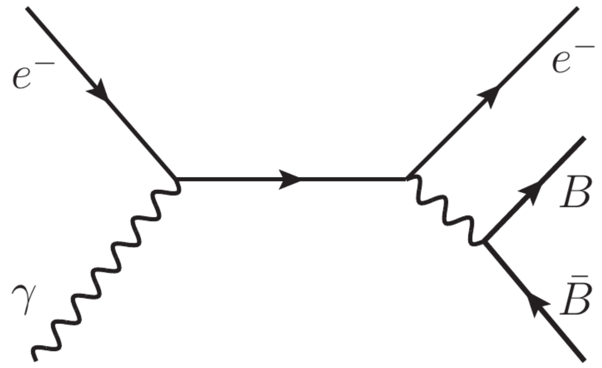
$$S_1^{\mu} = (0, S_1^x, S_1^y, S_1^z),$$
$$S_2^{\mu} = (0, S_2^x, S_2^y, S_2^z).$$

# GDAs and $e^{\pm}\gamma \to e^{\pm}B\bar{B}$

The process includes two subprocesses:

(1) The bremsstrahlung:

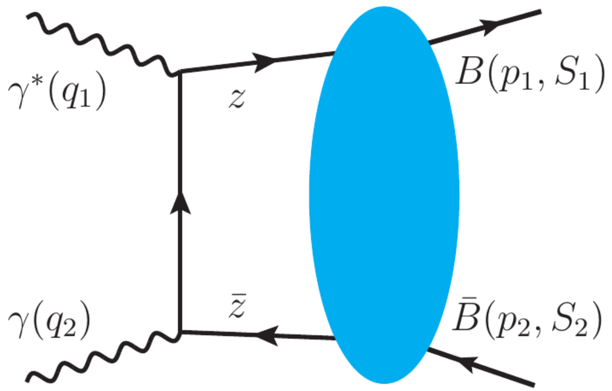
$C$-odd $B\bar{B}$ pairs can be produced in this subprocess.




The bremsstrahlung

(2) The QCD subprocess :

$C$-even $B\bar{B}$ pairs can be produced in this subprocess.




The QCD subprocess $\gamma\gamma^* \to B\bar{B}$

# Unpolarized Cross sections of $e^{\pm}\gamma \rightarrow e^{\pm}B\bar{B}$

Summing over the polarization states of the antibaryon , we get

$$\frac{d\sigma(S_1)}{d\hat{s}dQ^2d(\cos\theta)\,d\varphi} = \frac{1}{2}\frac{d\bar{\sigma}}{d\hat{s}dQ^2d(\cos\theta)\,d\varphi} + \frac{d\hat{\sigma}(S_1)}{d\hat{s}dQ^2d(\cos\theta)\,d\varphi}$$

**Unpolarized cross section** **Single-spin correlation**

Unpolarized cross section:

$$\frac{d\bar{\sigma}_{\rm G}}{d\hat{s}\,dQ^2d(\cos\theta)d\varphi} = \frac{\alpha_{\rm em}^3\beta_0}{8\pi s^2Q^2}\frac{1}{1-\epsilon}\Big\{|\mathcal{F}_A|^2 - |\mathcal{F}_S|^2 + 2{\rm Re}(\mathcal{F}_A\mathcal{F}_P^*) + \frac{\hat{s}}{4m^2}(|\mathcal{F}_P|^2 + |\mathcal{F}_S|^2) + (\beta_0)^2\cos^2\theta\Big[|\mathcal{F}_V|^2 + 2{\rm Re}(\mathcal{F}_S\mathcal{F}_V^*) - |\mathcal{F}_A|^2\Big] - (\beta_0)^4\cos^4\theta|\mathcal{F}_V|^2\Big\},$$

(The QCD process)

$$\frac{d\bar{\sigma}_{\rm B}}{d\hat{s}\,dQ^2d(\cos\theta)d\varphi} = \frac{\alpha_{\rm em}^3\beta_0^3}{4\pi s^2}\frac{1}{\epsilon\hat{s}}\Big\{\omega_1(2\lambda-1)|G_M|^2 + \Big[\omega_2|G_M|^2 + 2\omega_3(\lambda-1)(|G_E|^2 - |G_M|^2)\Big]\cos^2\theta + \Big[\omega_3|G_M|^2 + \omega_4(\lambda-1)(|G_E|^2 - |G_M|^2)\Big]\sin^2\theta + \omega_5\Big[(\lambda-1)|G_E|^2 - \lambda|G_M|^2\Big] \times \sin(2\theta)\cos\varphi - \omega_3\Big[(\lambda-1)|G_E|^2 - \lambda|G_M|^2\Big]\sin^2\theta\cos(2\varphi)\Big\},$$

(The bremsstrahlung)

# Unpolarized Cross sections of $e^{\pm}\gamma \to e^{\pm}B\bar{B}$

The interference term:

$$\frac{d\bar{\sigma}_{\rm I}}{d\hat{s}\,dQ^2 d(\cos\theta)d\varphi} = e_l \frac{\alpha_{\rm em}^3 \beta_0}{8\pi s^2}\frac{\sqrt{2}\beta_0}{Q\sqrt{\hat{s}\epsilon(1-\epsilon)}}\Big\{2\omega_6[{\rm Re}(G_M^*\mathcal{F}_V)+{\rm Re}(G_E^*\mathcal{F}_S)]\cos\theta - 2(\beta_0)^2\omega_6[\lambda{\rm Re}(G_M^*\mathcal{F}_V)$$
$$-(\lambda-1){\rm Re}(G_E^*\mathcal{F}_V)]\cos^3\theta + 2[\omega_7{\rm Re}(G_M^*\mathcal{F}_A)+\omega_8{\rm Re}(G_E^*\mathcal{F}_S)]\sin\theta\cos\varphi$$
$$-(\beta_0)^2\omega_8[\lambda{\rm Re}(G_M^*\mathcal{F}_V)-(\lambda-1){\rm Re}(G_E^*\mathcal{F}_V)]\sin(2\theta)\cos\theta\cos\varphi\Big\},$$

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The forward-backward asymmetry helps us to access baryon GDAs,

$$\frac{d\bar{\sigma}_{\rm FB}}{d\hat{s}\,dQ^2 d(\cos\theta)} = \int_{\Delta\varphi} d\varphi \frac{d\Delta\bar{\sigma}(\theta,\varphi)}{d\cos\theta\, d\varphi}$$
$$= e_l \frac{\alpha_{\rm em}^3 \beta_0}{2\pi s^2}\frac{\sqrt{2}\beta_0}{Q\sqrt{\hat{s}\epsilon(1-\epsilon)}}\Big\{\pi\omega_6[{\rm Re}(G_M^*\mathcal{F}_V)+{\rm Re}(G_E^*\mathcal{F}_S)]\cos\theta - \pi(\beta_0)^2\omega_6[\lambda{\rm Re}(G_M^*\mathcal{F}_V)$$
$$-(\lambda-1){\rm Re}(G_E^*\mathcal{F}_V)]\cos^3\theta - 2[\omega_7{\rm Re}(G_M^*\mathcal{F}_A)+\omega_8{\rm Re}(G_E^*\mathcal{F}_S)]\sin\theta$$
$$+(\beta_0)^2\omega_8[\lambda{\rm Re}(G_M^*\mathcal{F}_V)-(\lambda-1){\rm Re}(G_E^*\mathcal{F}_V)]\sin(2\theta)\cos\theta\Big\}.$$

# The Single-spin correlations of $e^{\pm}\gamma \to e^{\pm}B\bar{B}$

The single-spin correlations of the QCD and bremsstrahlung processes:

$$\frac{d\hat{\sigma}_G(S_1)}{d\hat{s}\,dQ^2 d(\cos\theta) d\varphi} = \frac{\alpha_{\rm em}^3 \beta_0^2}{32\pi s^2 Q^2 (1-\epsilon)} \frac{\sqrt{\hat{s}}}{m} [-\beta_0 \sin(2\theta)\,{\rm Im}(\mathcal{F}_V \mathcal{F}_S^*) + 2\sin\theta\,{\rm Im}(\mathcal{F}_A \mathcal{F}_P^*)] S_1^y.$$

$$\frac{d\hat{\sigma}_B(S_1)}{d\hat{s}\,dQ^2 d(\cos\theta) d\varphi} = \frac{\alpha_{\rm em}^3 \beta_0}{4\pi s^2 \epsilon \hat{s}} \frac{m}{\sqrt{\hat{s}}} {\rm Im}(G_M G_E^*) \Big\{ 2 \big[\omega_3 \sin\theta \sin(2\varphi) - \omega_5 \cos\theta \sin\varphi\big] S_1^x + \big[\omega_2 \sin(2\theta) + 2\omega_5 \cos(2\theta)\cos\varphi - 2\omega_3 \sin(2\theta)\cos^2\varphi\big] S_1^y \Big\},$$

By including baryon polarizations, we gain access to **$\mathrm{Im}(\mathcal{F}_i \mathcal{F}_j^*)$**.

Consider the exchange of : $(\theta,\varphi) \to (\pi-\theta, \pi+\varphi)$, $(S_1^x, S_1^y, S_1^z) \to (-S_2^x, -S_2^y, S_2^z)$.We can obtain the single-spin correlation that is dependent on $S_2$.

# The Single-spin correlations of $e^{\pm}\gamma \to e^{\pm}B\bar{B}$

The interference contribution of two subprocesses:

$$
\begin{aligned}
\frac{d\hat{\sigma}_{\rm I}(S_1)}{d\hat{s}\,dQ^2d(\cos\theta)d\varphi} = \frac{e_l\alpha_{\rm em}^3\beta_0}{8\pi s^2Q\sqrt{2\hat{s}\epsilon(1-\epsilon)}}\Bigg\{\Bigg[\beta_0\Big(\omega_8{\rm Im}(G_M^*\mathcal{F}_S)\sin\varphi + \omega_8\frac{4m^2}{\hat{s}}{\rm Im}(G_M^*\mathcal{F}_V)\cos^2\theta\sin\varphi \\
+\omega_7\frac{4m^2}{\hat{s}}{\rm Im}(G_E^*\mathcal{F}_A)\sin^2\theta\sin\varphi\Big) + \omega_7{\rm Im}\Big(G_M^*(\mathcal{F}_P + \frac{4m^2}{\hat{s}}\mathcal{F}_A)\Big)\cos\theta\sin\varphi\Bigg]\frac{\sqrt{\hat{s}}}{m}S_1^x \\
+\Bigg[\beta_0\Big(\omega_6{\rm Im}(G_M^*\mathcal{F}_S)\sin\theta - \omega_8{\rm Im}\Big(G_M^*(\mathcal{F}_S + \frac{4m^2}{\hat{s}}\mathcal{F}_V)\Big)\cos\theta\cos\varphi + \omega_6\frac{2m^2}{\hat{s}} \\
\times{\rm Im}\Big((G_M^* - G_E^*)\mathcal{F}_V\Big)\sin(2\theta)\cos\theta + \omega_8\frac{2m^2}{\hat{s}}{\rm Im}\Big((G_M^* - G_E^*)\mathcal{F}_V\Big)\sin(2\theta)\sin\theta\cos\varphi\Big) \\
-\omega_7{\rm Im}\Big(G_M^*(\mathcal{F}_P + \frac{4m^2}{\hat{s}}\mathcal{F}_A)\Big)\cos\varphi\Bigg]\frac{\sqrt{\hat{s}}}{m}S_1^y + \Bigg[2\omega_7{\rm Im}\Big(G_E^*(\mathcal{F}_P + \frac{4m^2}{\hat{s}}\mathcal{F}_A)\Big)\sin\theta\sin\varphi \\
-\beta_0\big(\omega_7{\rm Im}(G_M^*\mathcal{F}_A) - \omega_8{\rm Im}(G_M^*\mathcal{F}_V)\big)\sin(2\theta)\sin\varphi\Bigg]S_1^z\Bigg\}.
\end{aligned}
$$

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# Numerical estimate of $e^{\pm}\gamma \to e^{\pm}p\bar{p}$

We take the y-axis to be the polarization direction in the baryon's rest frame. The spin vectors of the spin-up and spin-down baryon can be expressed as:

$$S^{\mu}_{1\uparrow} = (0,0,1,0) \qquad S^{\mu}_{1\downarrow} = (0,0,-1,0)$$

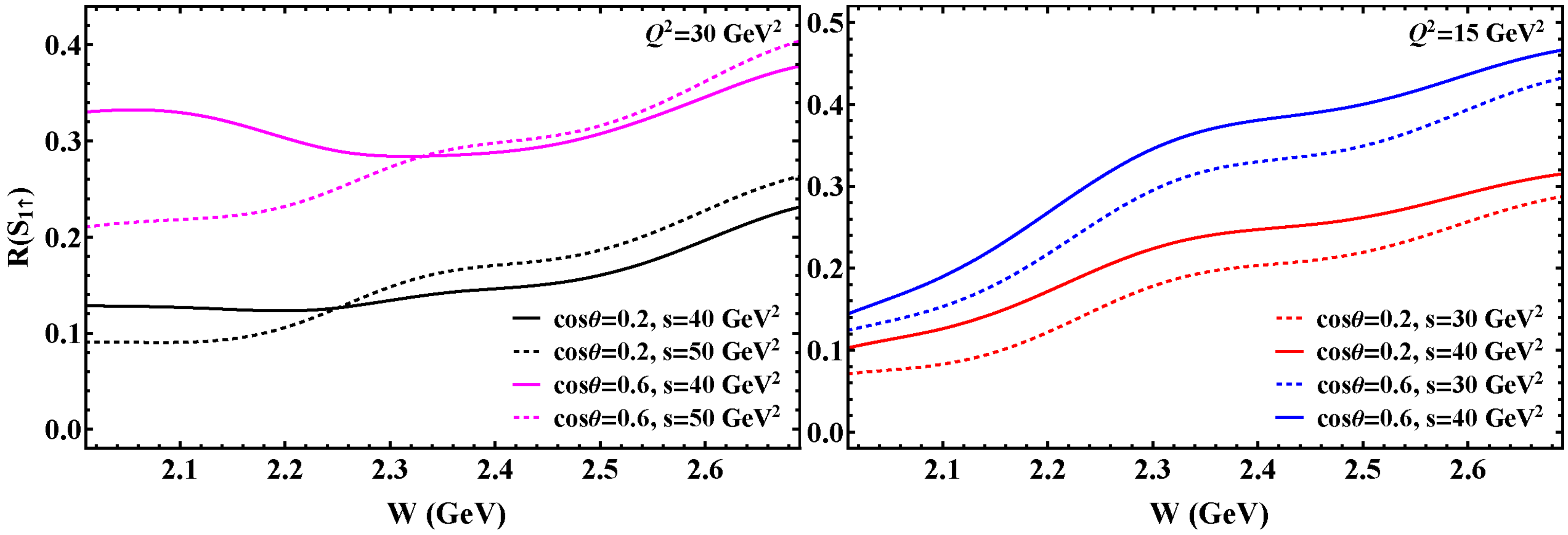
The cross sections for the production of spin-up and spin-down baryons are given by

$$d\sigma(S_{1\uparrow}) = \frac{1}{2}d\bar{\sigma} + d\hat{\sigma}(S_{1\uparrow}),$$
$$d\sigma(S_{1\downarrow}) = \frac{1}{2}d\bar{\sigma} - d\hat{\sigma}(S_{1\uparrow}).$$

We can define the following ratio:

$$R(S_{1\uparrow}) = \left[\frac{d\hat{\sigma}(S_{1\uparrow})}{d\hat{s}\,dQ^2 d(\cos\theta)}\right] \Big/ \left[\frac{1}{2}\,\frac{d\bar{\sigma}}{d\hat{s}\,dQ^2 d(\cos\theta)}\right].$$

# Numerical estimate of $e^{\pm}\gamma \rightarrow e^{\pm}p\bar{p}$




Estimate of ratio $R(S_{1\uparrow})$

The results show that the polarization effect is sizable.

# Summary

⇨ Baryon GDAs can help us access the GFFs of unstable baryons

⇨ We calculated the cross sections of $e^+e^- \to B\bar{B}\gamma$ and $e^\pm\gamma \to e^\pm B\bar{B}$, which can be expressed as Compton FFs and EM FFs

⇨ We provided numerical estimate of the $e^+e^- \to p\bar{p}\gamma$ and $e^\pm\gamma \to e^\pm p\bar{p}$, which will be helpful for future measurements at BESIII, Belle II , and STCF.

# Thank you very much