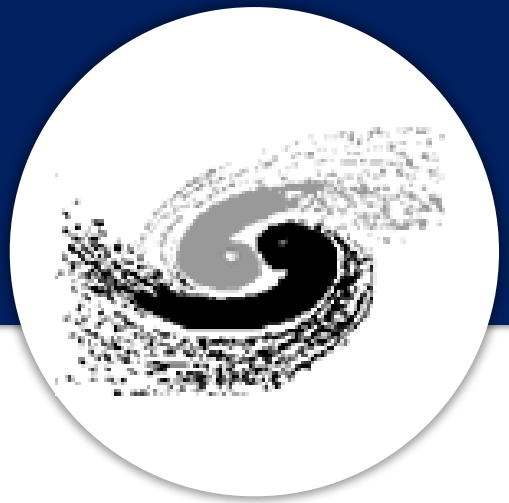


双π介子阈值行为的实验研究

向本后

中国科学院高能物理研究所

2026.05.16

轻强子专题研讨会

CONTENTS

$\pi\pi$ threshold physics

- Quantum Chromodynamics (QCD) becomes non-perturbative at low energies and cannot be treated with standard perturbation theory.
- Chiral perturbation theory is an important effective theory of QCD at low energies.
- Soft gluon produced pion pairs serve as a powerful probe of final-state interactions and the Adler zero.

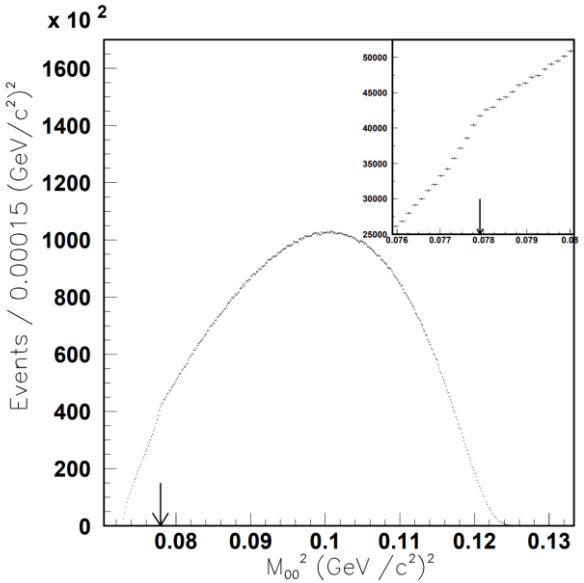
Cusp effect in $\pi\pi$ threshold

- The cusp effect, a sharp structure in invariant mass spectra at a kinematic threshold, provides a direct signature of final-state interactions.
- NA48/2 observe cusp effect in $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$.

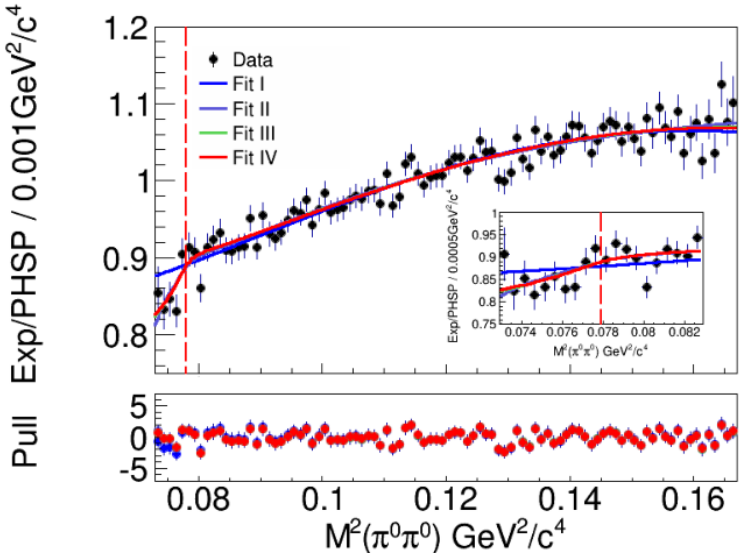
$$a_0 - a_2 = 0.264 \pm 0.004_{\text{sta.}} \pm 0.013_{\text{sys.}}$$

- BESIII use $\eta' \to \pi^0\pi^0\eta$ study cusp effect with 3.5σ.

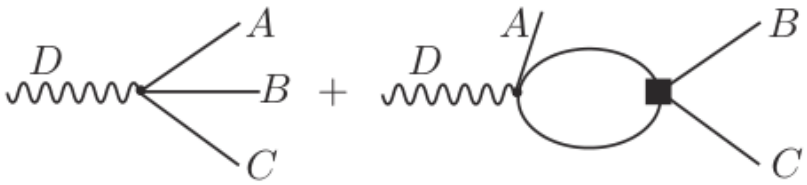
$$a_0 - a_2 = 0.226 \pm 0.060_{\text{sta.}} \pm 0.013_{\text{sys.}}$$



Phys.Lett.B 633 (2006) 173-182

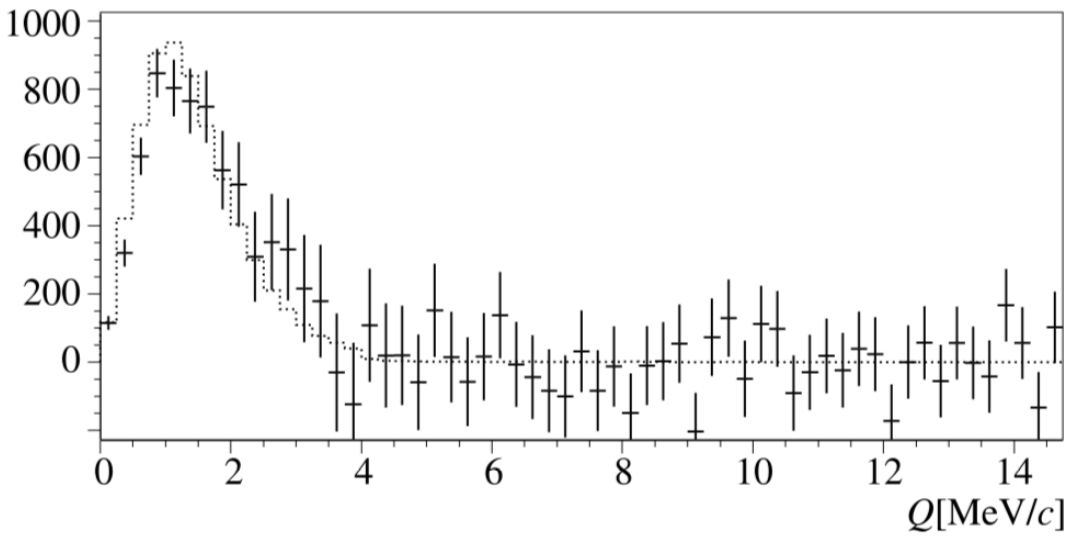


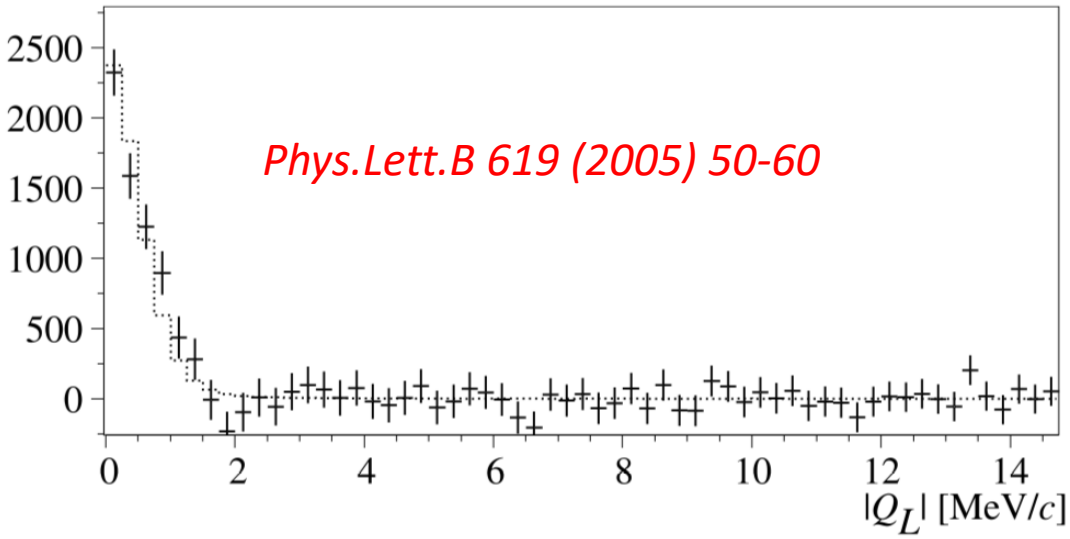
PRL130,081901(2023)



$\pi\pi$ atom in DIRAC experiment

- DIRAC produces $\pi^+\pi^-$ pairs by firing high-energy (24 GeV) protons at a target.
- Pairs with extremely low relative momentum ($Q < 1\ \mathrm{MeV}/c$) are bound by the Coulomb force into hydrogen-like atoms (Pionium).
- Measurement the lifetime: $\tau = 0.45^{+1.08}_{-0.30} \times 10^{-11}$ s.

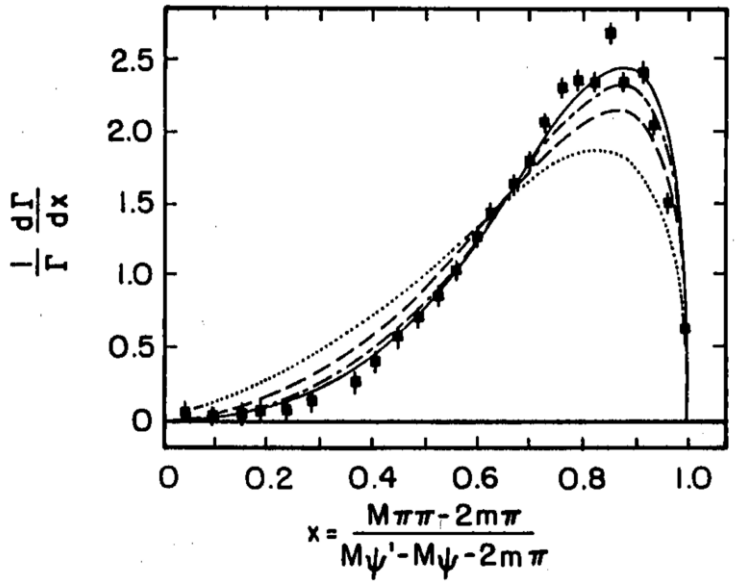




Phys.Lett.B 619 (2005) 50-60

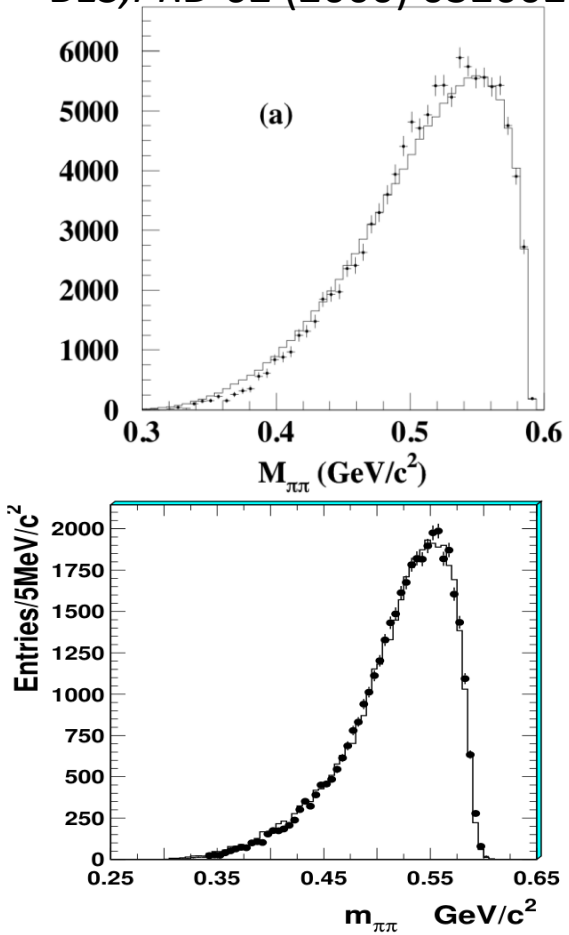
Status of $\psi(3686) \to \pi^+\pi^- J/\psi$

- Experimentally focusing on BF and cross section measurements
- Theoretically focusing on the shape of $\pi\pi$ high invariant mass



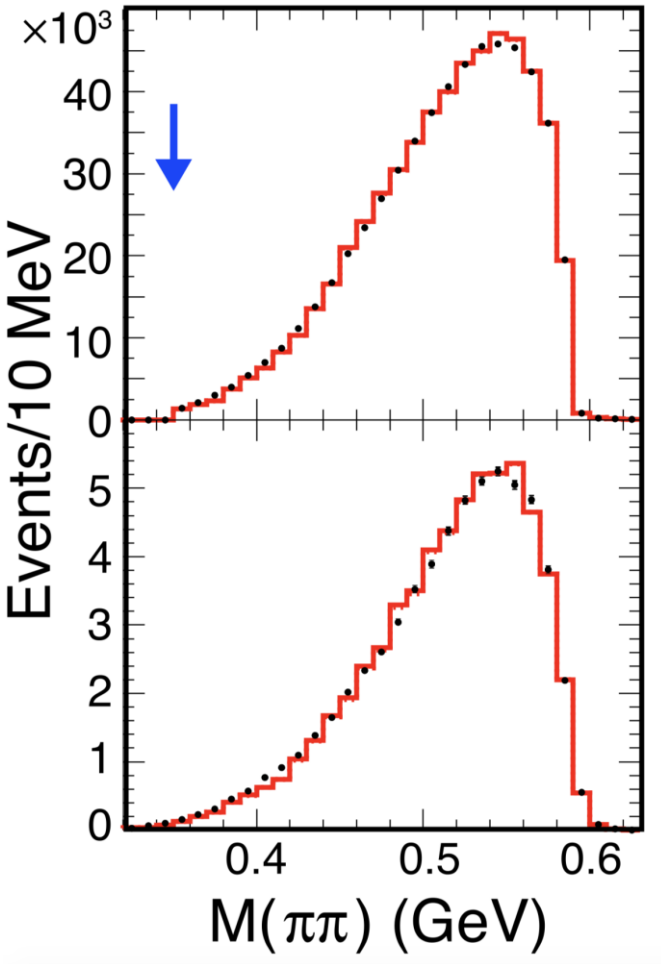
*Y.P. Kuang, PRD*44 (1991) 756-769

BES,PRD 62 (2000) 032002



BES, PLB645 (2007) 19-25

CLEO, PRD 78 (2008) 011102



Motivation of $\psi(3686) \to \pi^+\pi^- J/\psi$

- Mixture of Charmonium:
  - ✓ Because of decay width of $\frac{\Gamma_{\psi(3770)\to e^+e^-}}{\Gamma_{J/\psi\to e^+e^-}} \gg \frac{\Gamma_{1D\to e^+e^-}}{\Gamma_{1S\to e^+e^-}}$. The observed $\psi(3686)$ and $\psi(3770)$ are mixtures of $|2^3S_1>$ and $|1^3D_1>$:

$$\psi(3686) = |2^3S_1> \cos\theta - |1^3D_1> \sin\theta$$
$$\psi(3770) = |2^3S_1> \sin\theta + |1^3D_1> \cos\theta$$

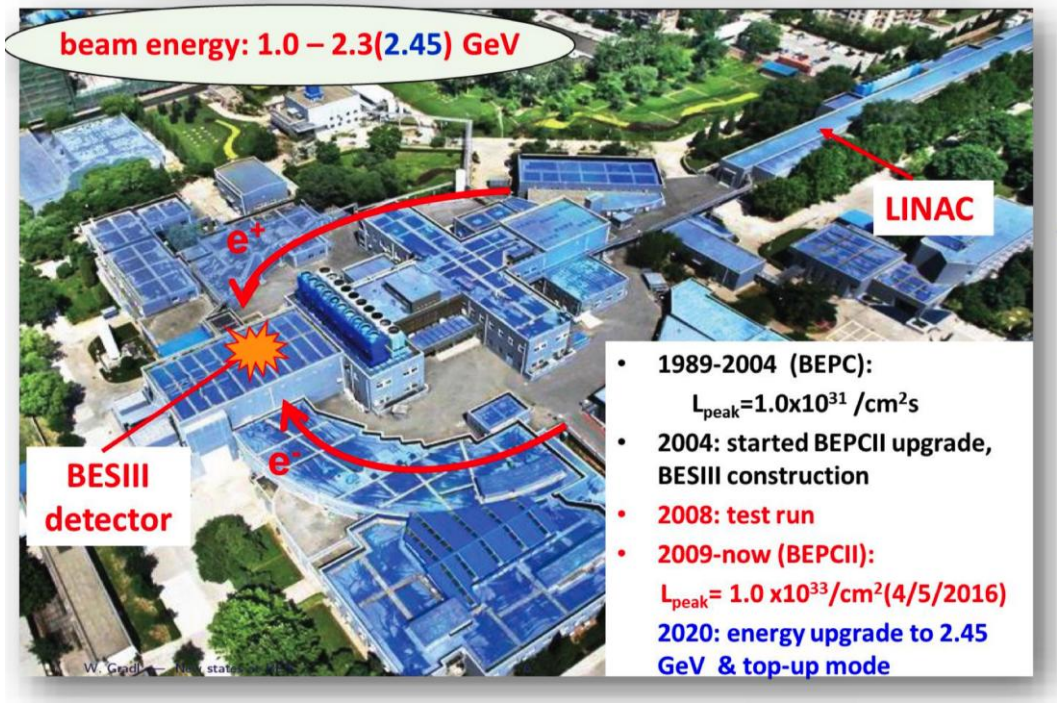
  where θ is mixing angle.

- $\pi\pi$ transition:
  - ✓ A good place to study Chiral Symmetry and Final state interaction.
  - ✓ High statistics of $\pi\pi$ production provide chance to search for pionium.

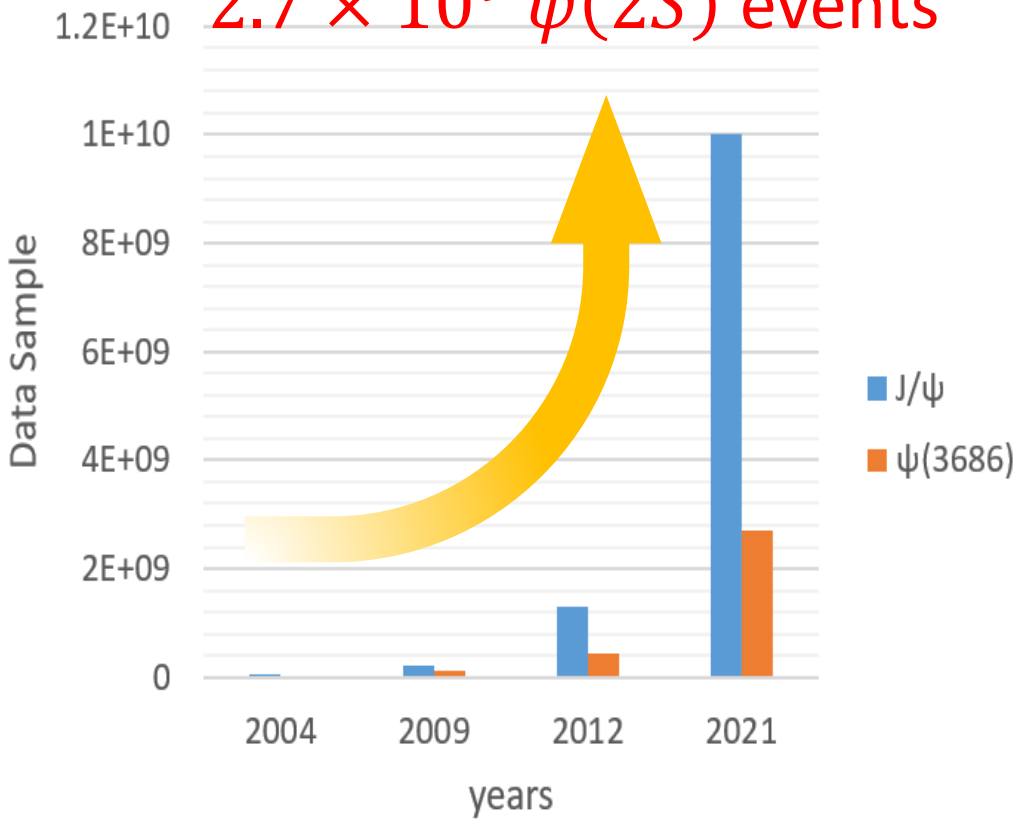
- Test CPV and CEDM?

BESIII status

Beijing Electron Positron Collider (BEPCII)



2.7×10^9 $\psi(2S)$ events

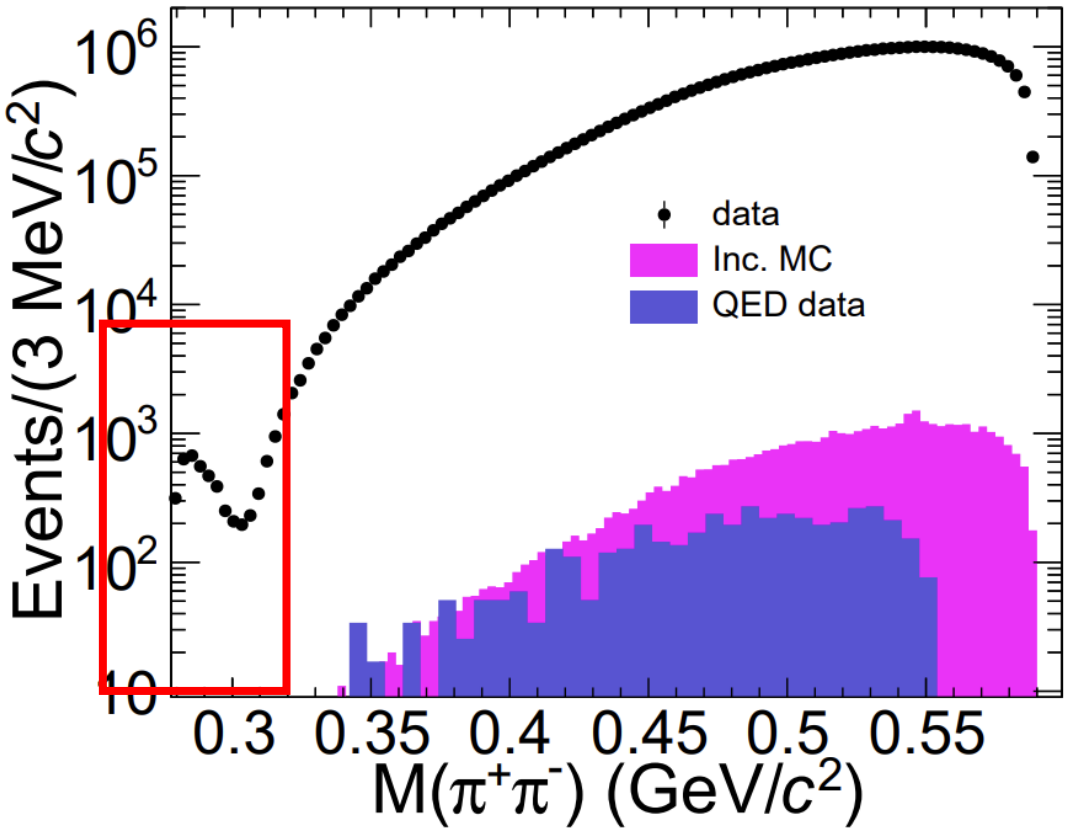


World's Largest τ-charm Data Sets in e^+e^- Annihilation

$\pi^+\pi^-$ enhancement

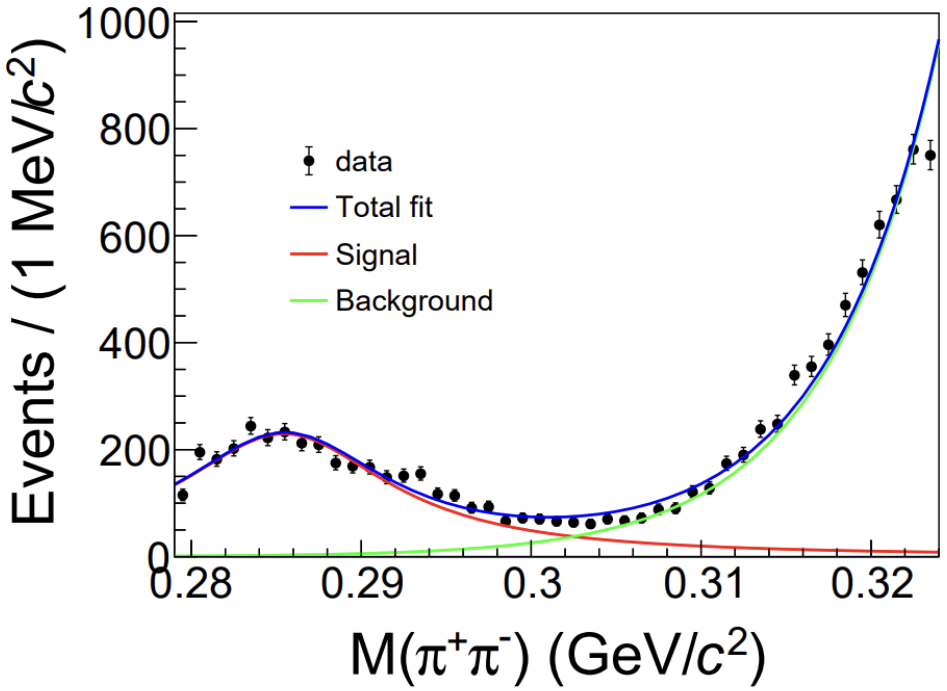
- $\psi(3686) \to \pi^+\pi^- J/\psi, J/\psi \to l^+l^-$
- Selected 3.7×10^7 events
- 99.8% purity

Phys. Rev. Lett. 136, 141902



Clear $\pi^+\pi^-$ threshold enhancement

Resonance model



- Take $\pi^+\pi^-$ threshold as a resonance
- (Breit-Wigner ⊗ Resolution) × Efficiency
- $M = (285.5 \pm 2.6)\ \mathrm{MeV}/c^2, \Gamma = (16.4 \pm 0.8)\ \mathrm{MeV}$
- **t= 4 × 10^{-23} s, much shorter than DIRAC !**

Theoretical model-ChPT

- ChPT:
  - ✓Heavy quark symmetry
  - ✓Chiral symmetry for $\pi\pi$ production

- Lagrangian:

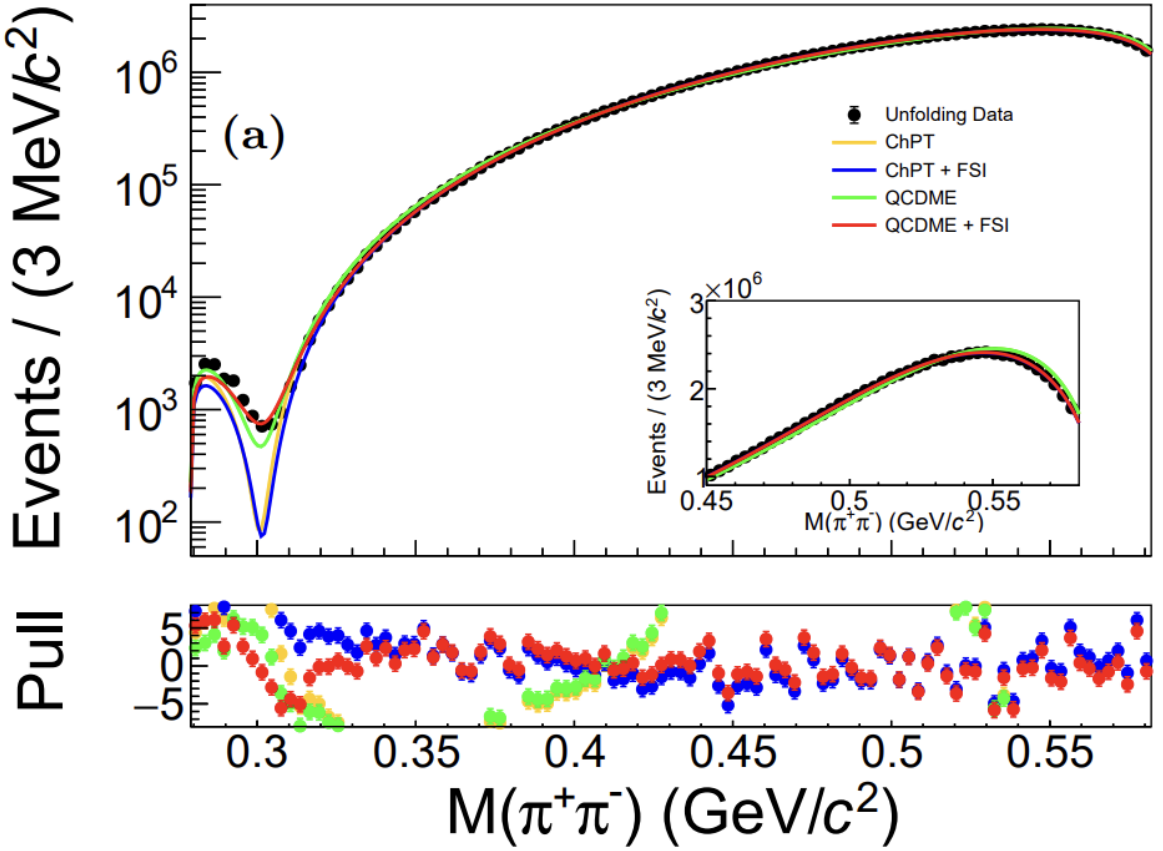
S-wave

$$\mathcal{L}_0 = gA_\mu^{(v)} B^{(v)\mu *}\ \mathrm{Tr}[(\partial_\nu U)(\partial^\nu U)^\dagger] + g_1 A_\mu^{(v)} B^{(v)\mu *}\ \mathrm{Tr}[(v\cdot\partial U)(v\cdot\partial U)^\dagger]$$
$$+ g_2 A_\mu^{(v)} B_\nu^{(v)*}\ \mathrm{Tr}[(\partial^\mu U)(\partial^\nu U)^\dagger + (\partial^\mu U)^\dagger(\partial^\nu U)] + \ \mathrm{h.c.}$$

D-wave

- Decay width:

$$\frac{d\Gamma}{dm_{\pi\pi}\ d\cos\theta_{\pi+}} = \frac{m_{\pi\pi}\sigma_\pi|\mathbf{q}|}{64\pi^3 m_\psi^2}|\mathcal{M}_S + \mathcal{M}_D|^2$$



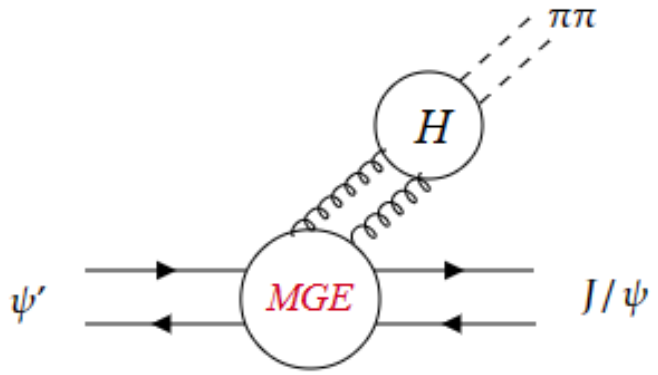
ChPT only can provide enhancement, but not full description

Theoretical model-QCDME

- Amplitude:

$$\mathcal{M} = i\frac{g_E^2}{6}\sum_{KL}\frac{\langle\Phi_F|\, r_{m_1}\,|KL\rangle\,\langle KL|\, r_{m_2}\,|\Phi_I\rangle}{E_I - E_{KL}}\langle\pi\pi|\, E^a_{-m_1}E^a_{-m_2}\,|0\rangle$$

MGE factor H factor



- Decay width:

[Front.Phys.China 1 (2006) 19-37]

$$\frac{d\Gamma}{dm_{\pi\pi}d\cos\theta_{\pi+}} = \cos^2\theta_{\text{mix}}\frac{dG_\alpha}{dm_{\pi\pi}d\cos\theta_{\pi+}} + \sin^2\theta_{\text{mix}}\frac{dH_\beta}{dm_{\pi\pi}d\cos\theta_{\pi+}}$$

✓ $2S \to 1S + \pi\pi$:

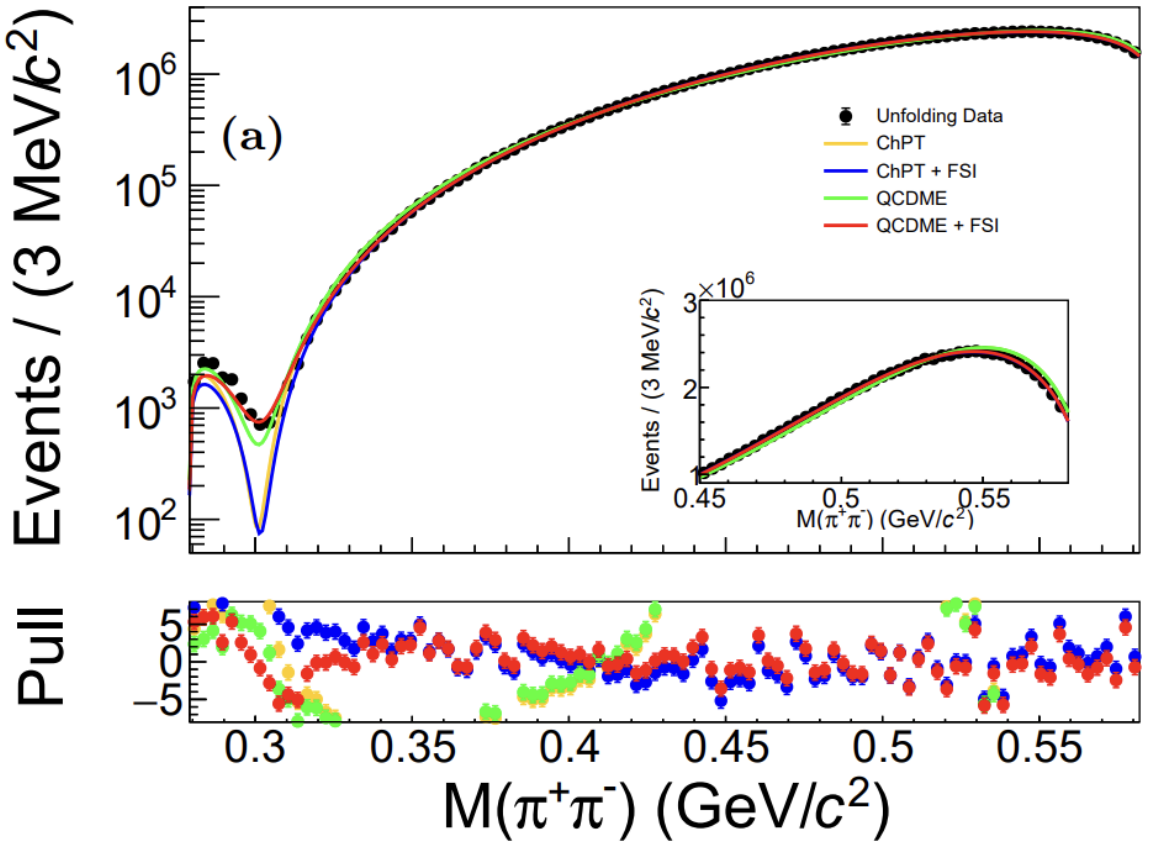
$$\frac{dG_\alpha}{dm_{\pi\pi}d\cos\theta_{\pi+}} = \frac{m_{\pi\pi}|\mathbf{q}|\sigma_\pi}{8\pi^3}\frac{m_{J/\psi}}{m_\psi^2}\left\{\frac{1}{2}(m_{\pi\pi}^2 - 2m_\pi^2) + \alpha\left[m_{\pi\pi}^2 + |\mathbf{q}|^2 - |\mathbf{q}|^2(1 - \frac{4m_\pi^2}{m_{\pi\pi}^2})\cos^2\theta_{\pi+}\right]\right\}^2$$

✓ $1D \to 1S + \pi\pi$:

$$\frac{dH_\beta}{dm_{\pi\pi}d\cos\theta_{\pi+}} = \frac{m_{\pi\pi}|\mathbf{q}|\sigma_\pi}{450\pi^3}\frac{m_{J/\psi}}{m_\psi^2}\beta^2\left\{(m_{\pi\pi}^2 - 4m_\pi^2)^2 + (m_{\pi\pi}^2 - 4m_\pi^2)|\mathbf{q}|^2 + |\mathbf{q}|^4\right.$$
$$\left. - (2|\mathbf{q}|^2 + m_{\pi\pi}^2 + 8m_\pi^2)(m_{\pi\pi}^2 - 4m_\pi^2)\frac{|\mathbf{q}|^2}{m_{\pi\pi}^2}\cos^2\theta_{\pi+} + (m_{\pi\pi}^2 - 4m_\pi^2)^2\frac{|\mathbf{q}|^4}{m_{\pi\pi}^4}\cos^4\theta_{\pi+}\right\}$$

Theoretical model-QCDME

- QCDME:
  - ✓Heavy quarkonium multipole expansion
  - ✓Mixture of 2S-1D $c\bar{c}$ charmonia.
  - ✓Chiral symmetry of $\pi\pi$ production.



The description is reasonable for low $\pi^+\pi^-$ mass

Theoretical model-FSI

- Final state interaction
  - ✓Chiral unitary approach: a good description of meson-meson interaction.

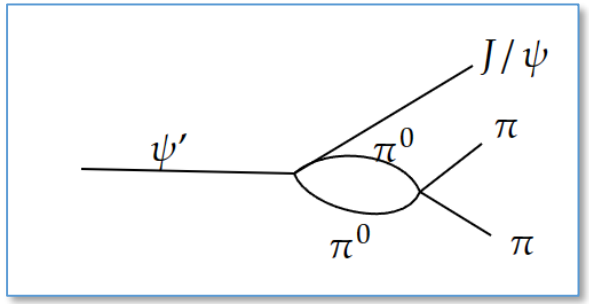
$$M_S = M_{S0} \cdot (1 + G \cdot t\_\pi\pi)$$

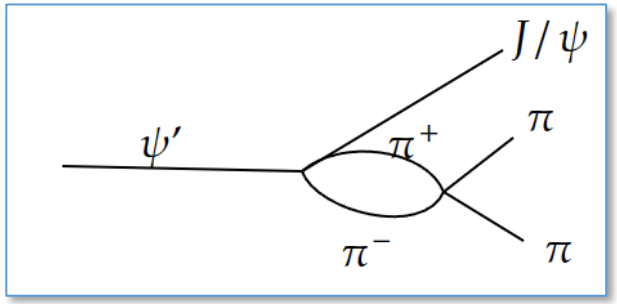
  - ✓For the $t_{\pi\pi}$:

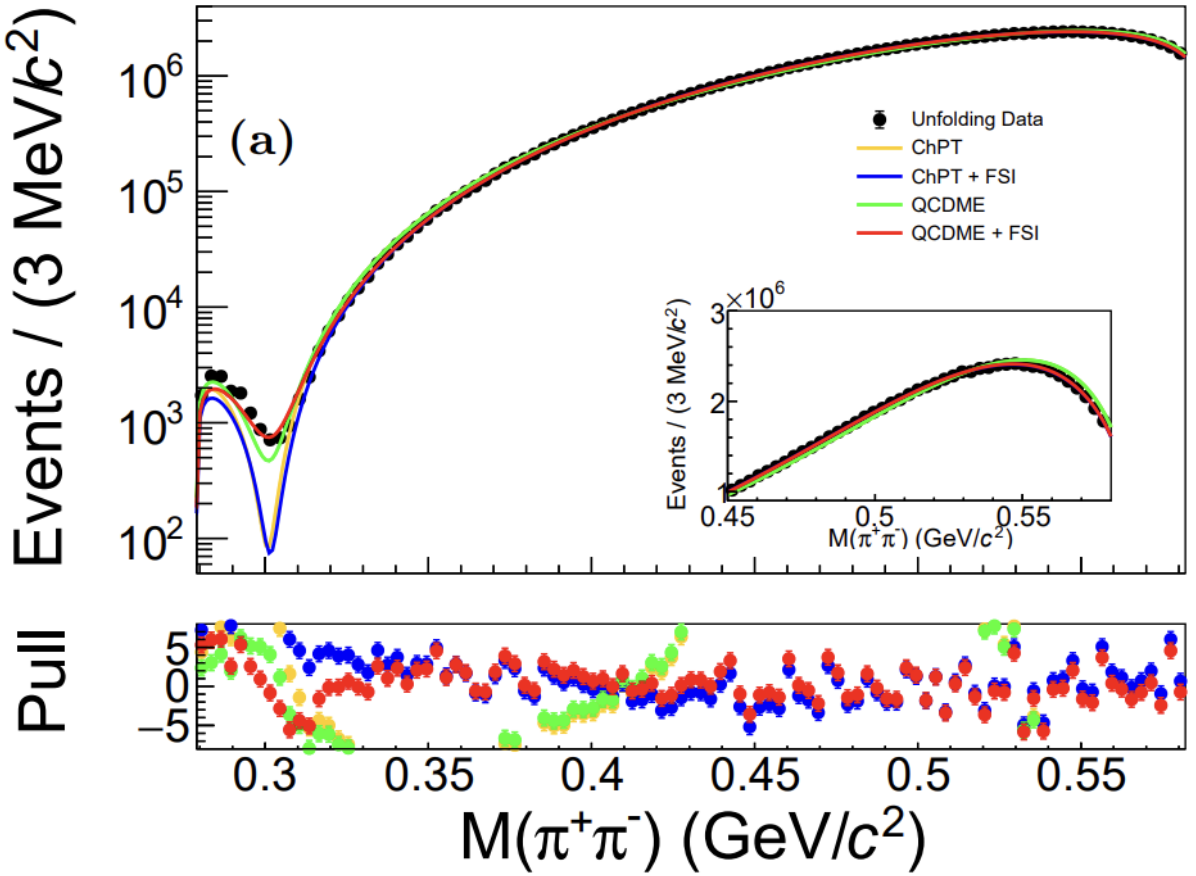
$$t_{\pi\pi} = \frac{1}{1/V_0 - G}$$

  - ✓And for G is to regularize the loop:

$$G = -\frac{1}{16\pi^2}\left[\gamma + \sigma \log\frac{\sigma - 1}{\sigma + 1}\right]$$

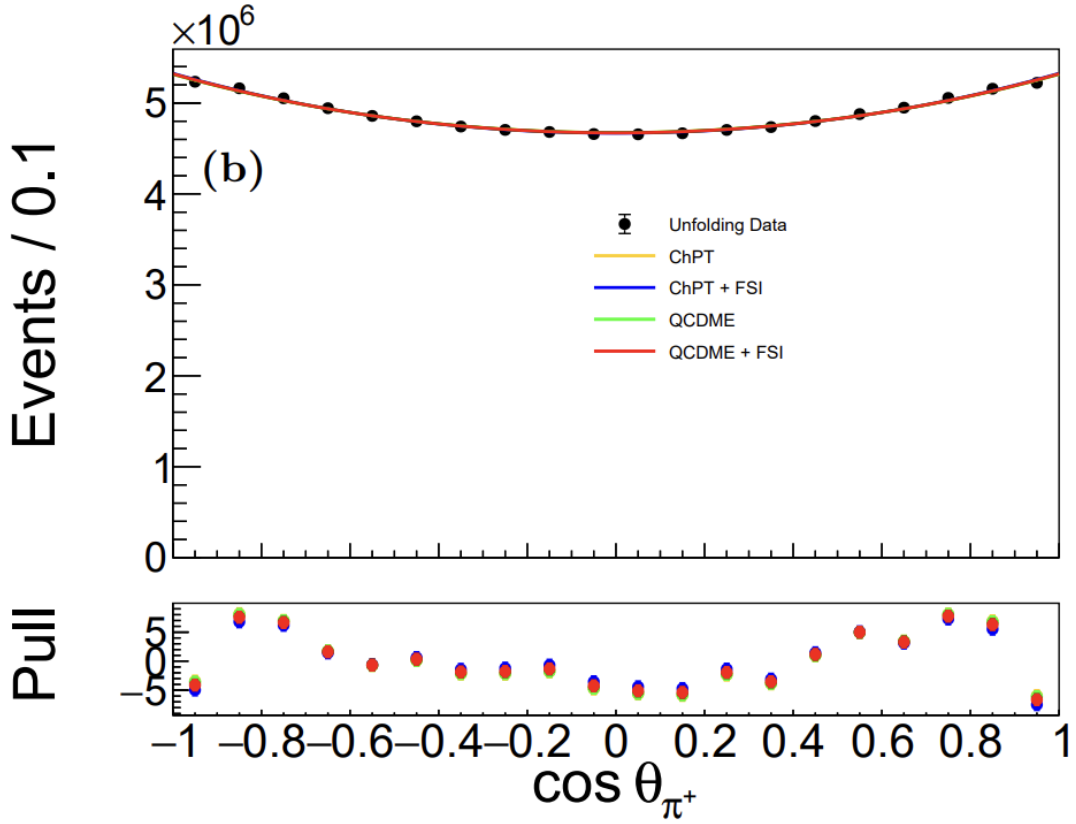


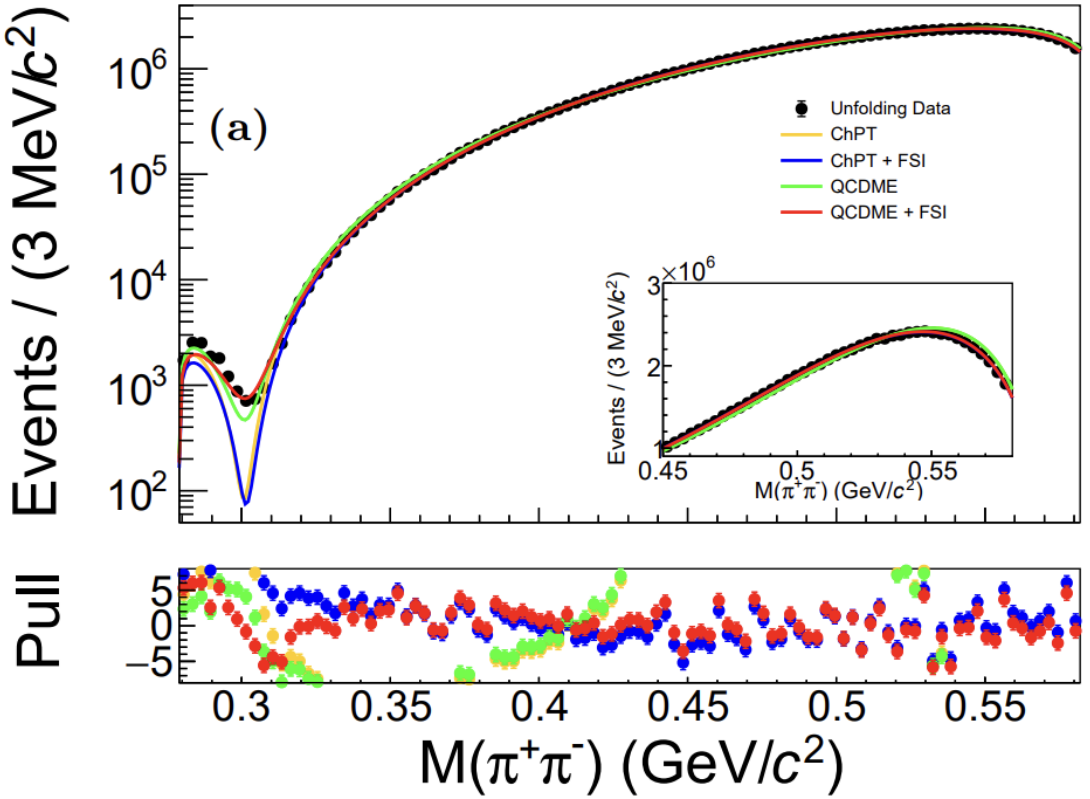




[XH Liu et al, Eur.Phys.J.C 73 (2013) 1, 2284]

Comparison

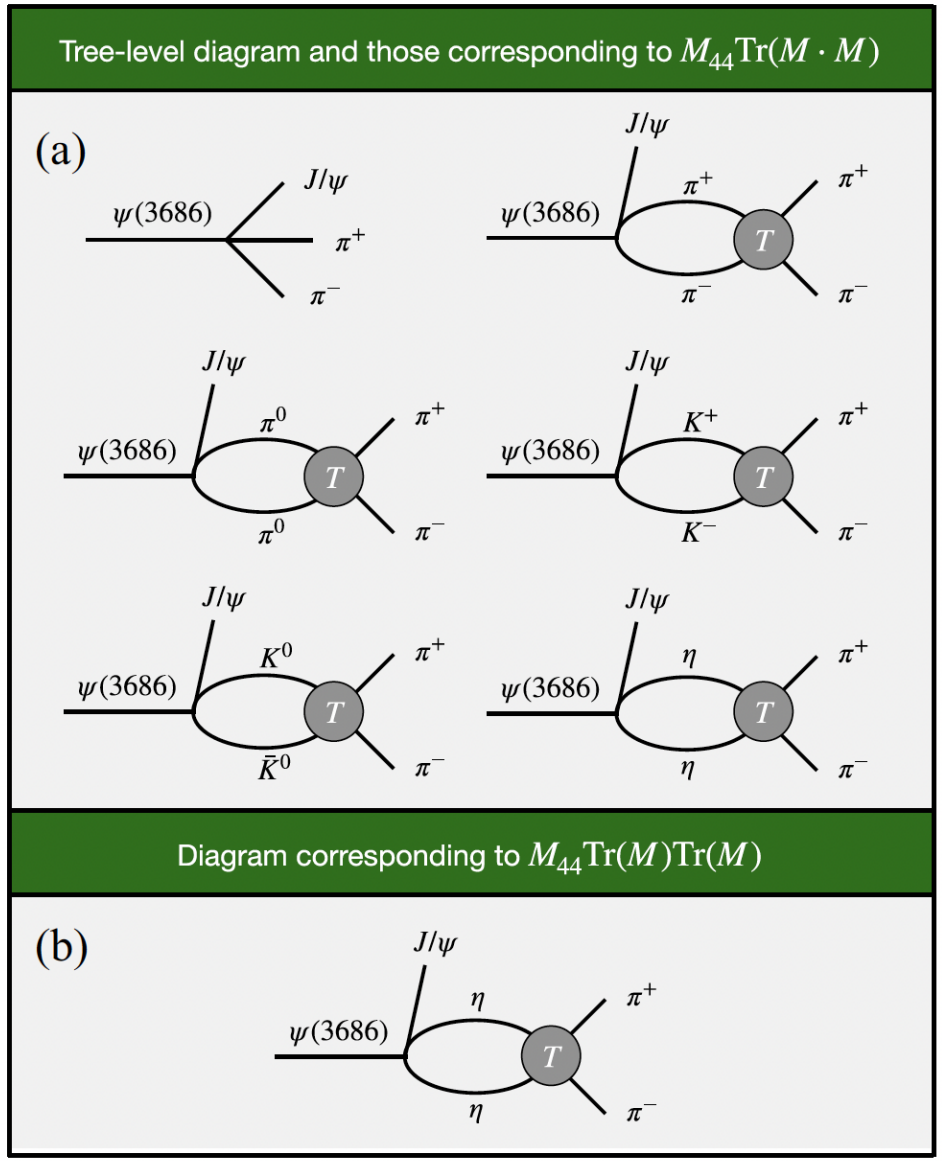


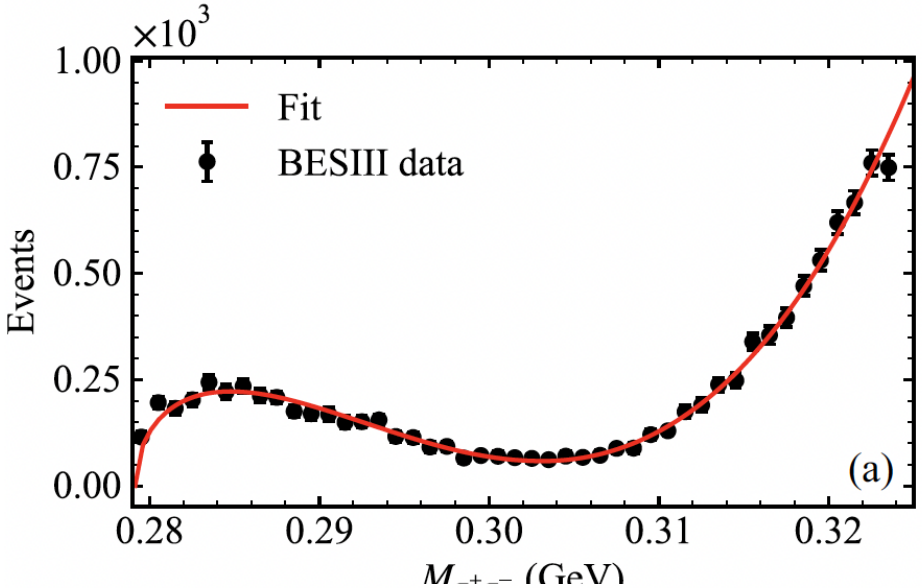


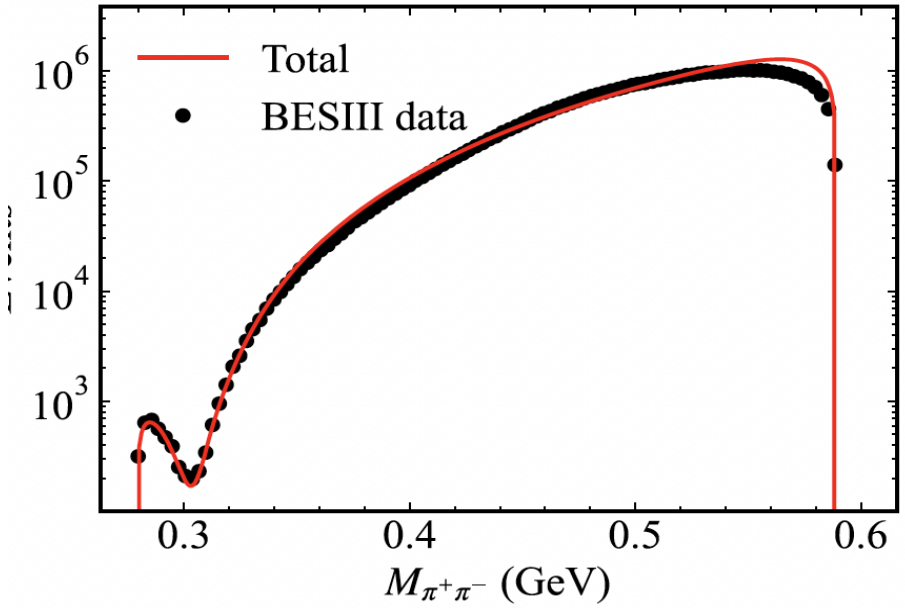
- FSI is necessary for high invariant mass spectrum
- QCDME : $\psi(3686)$ is the mixture of S-wave and D-wave charmonium state !
- Chiral symmetry provide a reasonable description for $\pi\pi$ threshold enhancement

Theorists' work

ZY Wang, Z Liu ,X Liu PRD 113, L031502 (2026)

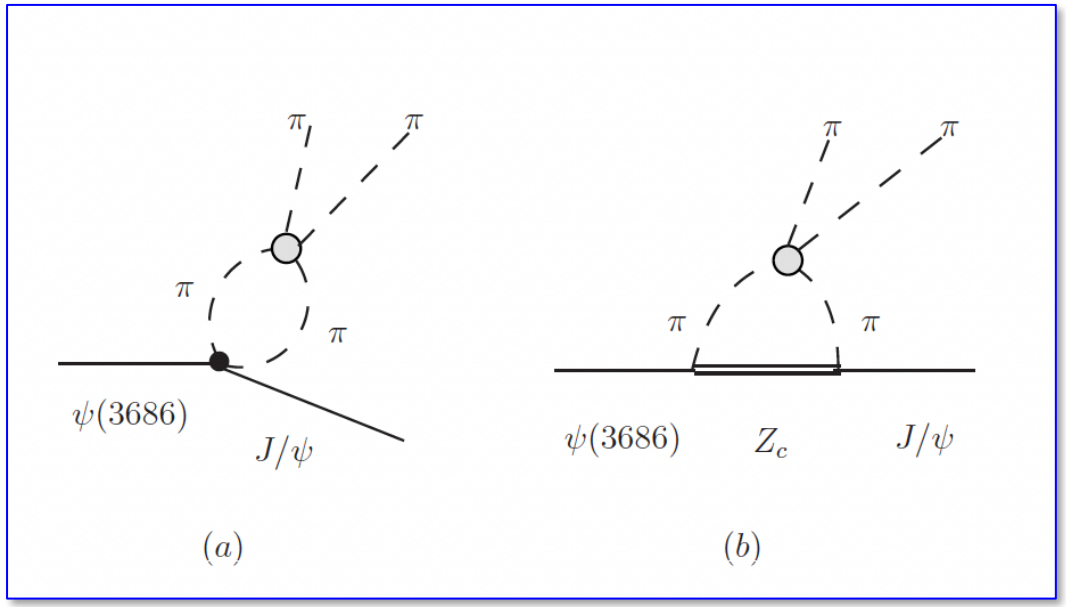


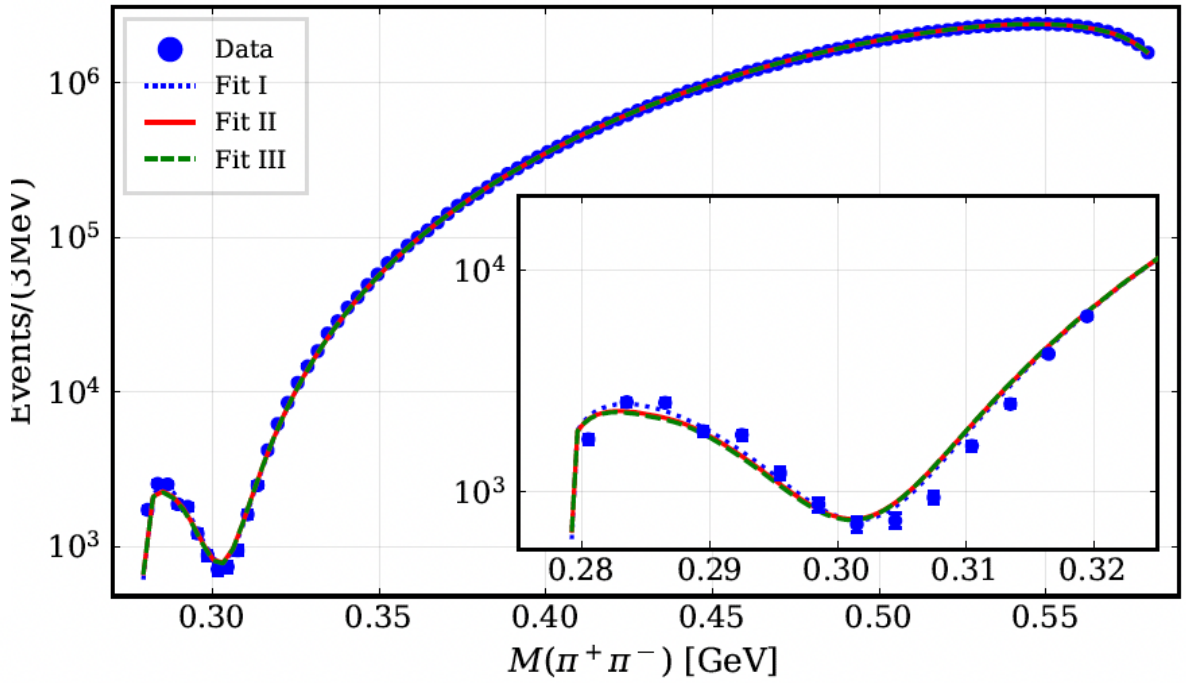




Theorists' work

YH Chen, XK Dong, FK Guo .., PRD 113, 054018 (2026)





$$\mathscr{L}_{\psi'\psi\Phi\Phi} = g_1\langle\psi'^\alpha\psi_\alpha^\dagger\rangle\langle u_\mu u^\mu\rangle + h_1\langle\psi'^\alpha\psi_\alpha^\dagger\rangle\langle u_\mu u_\nu\rangle v^\mu v^\nu + j_1\langle\psi'^\mu\psi_\nu^\dagger\rangle\langle u_\mu u^\nu\rangle + c_m\langle\psi'^\mu\psi_\mu^\dagger\rangle\langle\chi_+\rangle + \text{H.c.}\,,$$

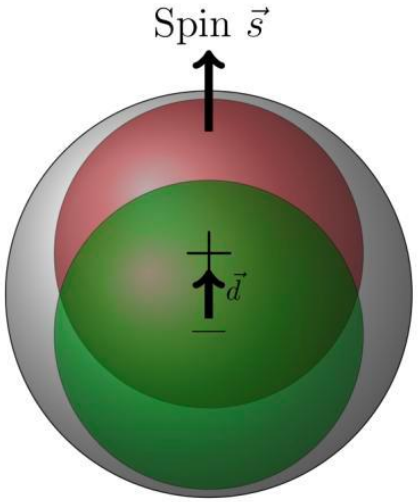
$$\mathscr{L}_{Z_c\psi'\pi} = C_{Z_c\psi'\pi}\psi'^i\langle Z_c^{i\dagger}u_\mu v^\mu\rangle + \text{H.c.}\,,$$

$$\mathscr{L}_{Z_c\psi\pi} = C_{Z_c\psi\pi}\psi^i\langle Z_c^{i\dagger}u_\mu v^\mu\rangle + \text{H.c.}\,.$$

Provide a good description of data

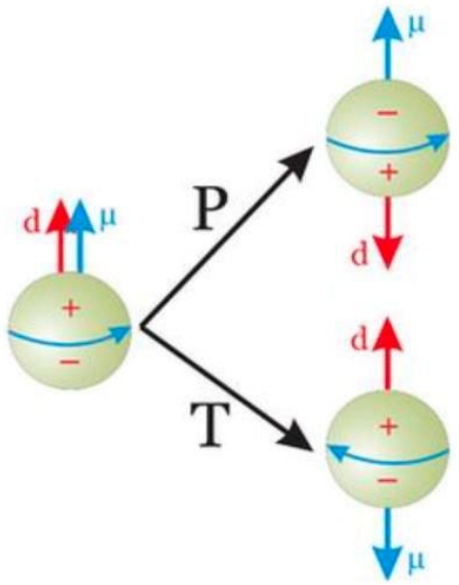
Chromo-EDM in $\psi(3686) \rightarrow \pi^+\pi^- J/\psi$

- The Electric Dipole Moment (EDM) of a fundamental particle or a subatomic system is a measure of the asymmetric charge distribution within the particle volume.



- If the particle is not charged, then a non zero EDM implies that the center of charge of the particle is displaced from its center of mass.

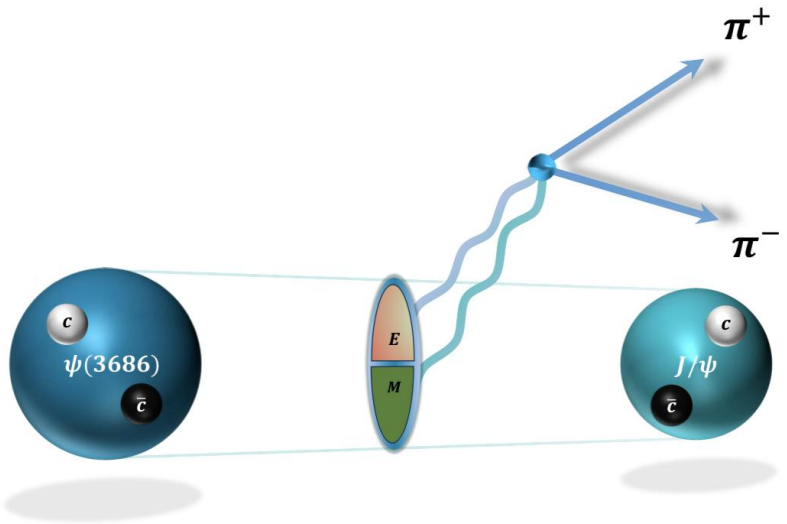
- EDM exists relates to violations of time reversal and parity symmetry.

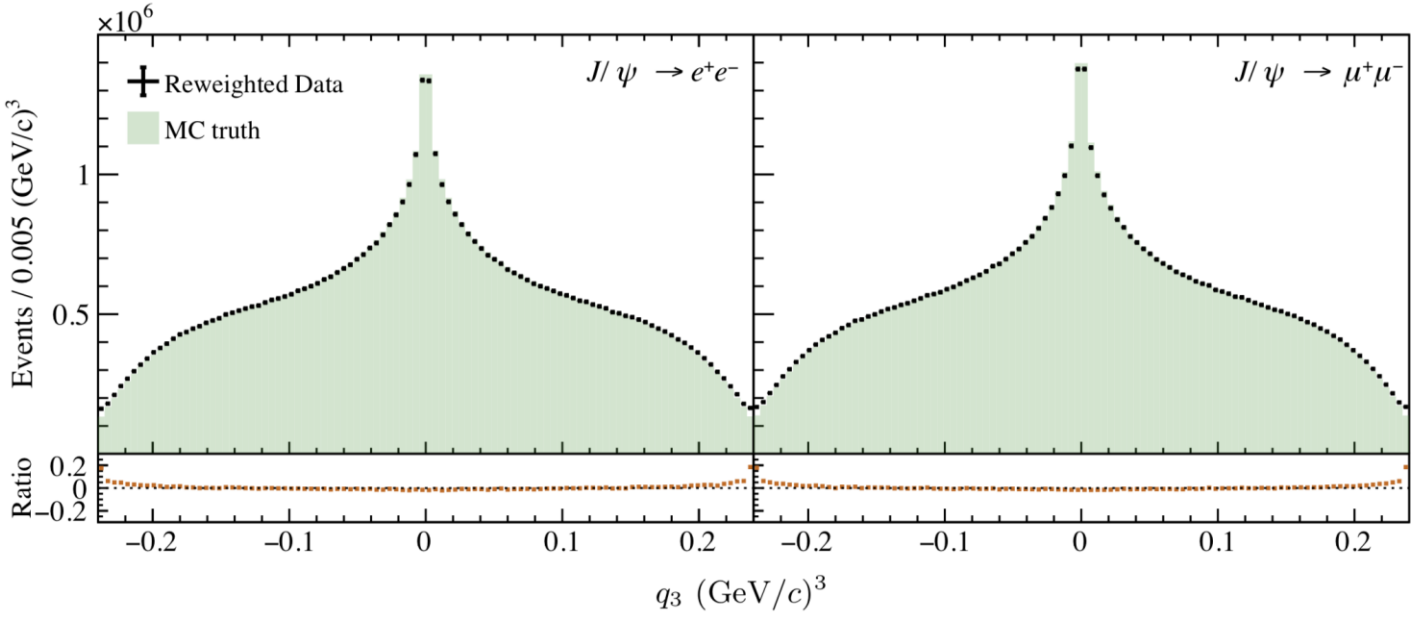


Chromo-EDM in $\psi(3686) \to \pi^+\pi^- J/\psi$

Based on QCDME, an extra contribution introduced:

$$\mathcal{L}_{CEDM} = -i/2 d'_c \bar{\psi}_c \sigma^{\mu\nu} \gamma_5 \lambda_a / 2 \psi_c G^a_{\mu\nu}$$





BESIII: arXiv: 2507.20618, accepted by PRL

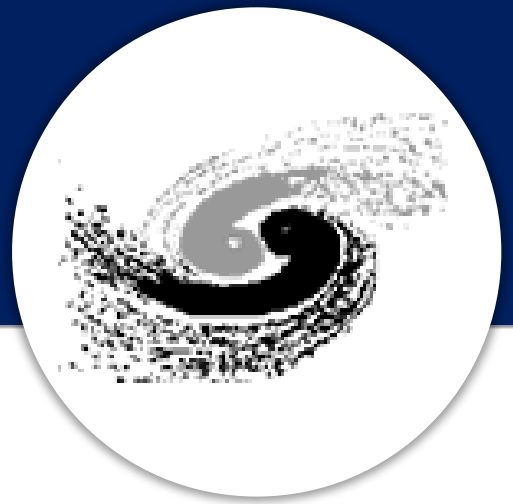
Parameter	$J/\psi \to e^+e^-$	$J/\psi \to \mu^+\mu^-$
A_{CP} $(\times 10^{-4})$	$3.3 \pm 2.9 \pm 0.4$	$-1.2 \pm 2.4 \pm 0.1$
$\langle A_{CP} \rangle$ $(\times 10^{-4})$	$0.6 \pm 1.8 \pm 0.1$	
d'_c [CK] $(e\cdot\text{cm})$	$(2.6 \pm 7.8 \pm 0.4 \pm 0.6) \times 10^{-16}$	
d'_c [Cornell] $(e\cdot\text{cm})$	$(3.5 \pm 10.5 \pm 0.6 \pm 0.5) \times 10^{-16}$	
$\lvert d'_c \rvert$ $(e\cdot\text{cm})$	$< 2.1 \times 10^{-15}$	

Summary

- Observed $\pi^+\pi^-$ threshold enhancement structure
  - ✓ Mass: (285.5 ± 2.6) MeV/c^2, Width: (16.4 ± 0.8) MeV

- Theoretical models
  - ✓ ChPT (QCDME) +FSI : reasonable description of $\pi^+\pi^-$ mass spectrum
  - ✓ QCDME implies $\psi(3686)$ is the mixture of S-wave and D-wave charmonium state
  - ✓ Inspired theoretical discussions on this anomalous enhancement

- Precise test of CEDM of charm quark
- Prospects: $\psi(3686) \to \pi^0\pi^0 J/\psi$ is in progress, released soon !



Thanks for listening!