

$f_0(980)$ 结构的能标敏感性研究

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Overview

I: The controversy of $f_0(980)$ structure

II: 2π DAs and H_{14} decays

i: From π -LCDAs to 2π DAs

ii: Energy dependent structure of $f_0(980)/[\pi\pi]_S$ in B_s/D_s decays

III: Summary and Prospect

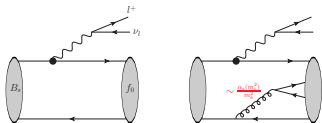
The controversy of $f_0(980)$ structure

- Experimental identification is particularly challenging (large width)
- From the quantum theory, it is a superposition of all possible Fock states

$$|f_0(980)\rangle, \quad |[\pi\pi]_S\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \dots$$

- **Hadron spectroscopy** provides clear evidence for the complex config
 - conventional resonant with possible gluonball component
[J.D. Weinstein and N. Isgur, PRL 48 (1982) 659]
 - exotic multiparticle state (compact tetraquark state, molecular state)
[Y.R. Liu, H.X. Chen, W. Chen, X. Liu and S.L. Zhu, PPNP 107 (2019) 237-320]
[F.K. Guo, C. Hanhart, U.G. Meissner, Q. Wang, Q. Zhao and B.S. Zou, RMP 90 (2018) 015004]
- **The underlying partonic dynamics** can not be extracted directly from spectral analysis, even though it reveals these exotic configurations
- **Semileptonic B, D decays** are powerful probe of the underlying structure
- **LCDAs, scale-dependent functions with a strict twist expansion**

- Semileptonic B, D decays are powerful probe of the underlying structure
- color transparency mechanism in $B_{(s)} \rightarrow f_0 l^+ \nu_l$ decays



† high Fock states' contribution is doubly suppressed by α_s and $\mathcal{O}(1/Q^2)$

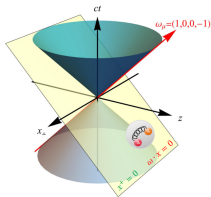
- the mechanism fails in $D_s \rightarrow f_0 l^+ \nu_l$ decays

- the cascade decay analyses of $D_s \rightarrow (f_0 \rightarrow \pi\pi) e \nu$ under $q\bar{q}$ ansatz can explain data
- $D_s \rightarrow f_0$ FFs & Flatté resonant model & fine tuning of the mixing angle
 [SC and S. L. Zhang, EPJC 84 (2024) 379]
 [D.D. Hu, X.G. Wu, H.J. Tian, T. Zhong, and H.B. Fu, PRD 112 (2025) 056023]
 - the scale dependence have revealed in $D_s \rightarrow f_0$ form factor \Downarrow
 - the seemingly agreement in D_{14} decays is attributed to the cascade framework
 - the Flatté parameterization disrupts the assessment of color transparency \Uparrow
- LCDAs, scale-dependent functions with a strict twist expansion

- From π -LCDAs to 2π DAs

[SC, PRD 112 (2025) L111301]

- Energy dependent structure of $f_0(980)/[\pi\pi]_S$ in B_s/D_s decays [SC, L.Y. Dai, J.M. Shen and S.L. Zhang, PRD 113 (2026) L031901]



- The Lorentz and gauge invariant ME

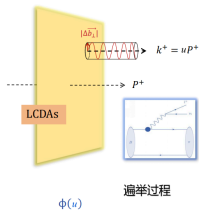
$$\langle 0 | \bar{u}(x) \gamma_\mu \gamma_5 d(-x) | \pi^-(p) \rangle = f_\pi \int_0^1 du e^{i(2u-1)p \cdot x} \left[i p_\mu \left(\phi(u, \mu) + \frac{x^2}{4} \phi_1^4(u, \mu) \right) - \dots \right]$$

$$\langle 0 | \bar{u}(x) \gamma_\mu \gamma_5 d(-x) | \rho^-(p) \rangle = f_\rho m_\rho \int_0^1 du e^{i(2u-1)p \cdot x} \left[p_\mu \frac{\epsilon(\lambda) \cdot x}{p \cdot x} \left(\phi_{\parallel}(u, \mu) - \phi_{\perp}^3 \right) - \dots \right]$$

$$\langle 0 | \bar{q}(x) \gamma_\mu \gamma_5 q(-x) | f_0(p) \rangle = p_\mu \int_0^1 du e^{i(2u-1)p \cdot x} [\phi(u, \mu) + \dots]$$

- LCDAs are dimensionless functions of u and renormalization scale μ
- the probability amplitudes to **find the meson** in a state with minimal number of constituents and have small transversal separation of order $1/\mu$

- longitudinal \otimes transversal dofs
- TMD (μ dependence) by *RGE*
- LMD by collinear subgroup of conformal group \Rightarrow collinear twist \Rightarrow the Gegenbauer polynomials



- LCDAs of pion achieved **great success** in describing large Q^2 processes.

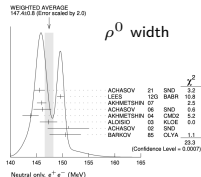
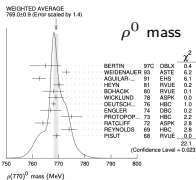
△ the establishment and development of the pQCD factorization $F_\pi(Q^2)$

$\int_0^1 du_i \phi(u_1, \tilde{Q}_1) T_H(u_i, Q) \phi(u_2, \tilde{Q}_2)$	1980s	[Lepage & Brodsky 1980, Efremov & Radyushkin 1980]
$\int_b^{\bar{b}} du_1 \int_{a/u_1}^{1-a/u_1} \phi(u_1, k_1\tau) \bar{T}_H(u_i, Q) \phi(u_2, k_2\tau)$	1990s	[Huang, Shen & Kroll 1991, Huang, Shen & Ma 1994]
$\psi(u_1, \mu_{r_1}) T_H(u_i, b_i, Q) S_i(u_i) e^{-S(u_i, b_i, Q)} \psi(u_2, \mu_{r_2})$	2000s	[Botts & Sterman 1989, Li & Sterman 1992, Li 1999]
$\sum_{t_i} \psi^{t_1}(u_1, \mu_{r_1}) T_H^{t_i, LO+NLO}(u_i, b_i, Q) S_i(u_i) e^{-S(u_i, b, Q)} \psi^{t_2}(u_2, \mu_{r_2})$	2010s	[Li, Shen, Wang & Zou 2011, SC, Fan & Xiao 2014]
$\sum_{t_i} \psi^{t_1}(u_1, b_1, \mu_{r_1}) T_H^{t_i, LO+NLO}(u_i, b_i, Q) S_i(u_i) e^{-S(u_i, b, Q)} \psi^{t_2}(u_2, b_2, \mu_{r_2})$	2020s	[Chai & SC 2025] [Chen ² , Feng & Jia 2024], [Ji, Shi, Wang ³ & Yu 2025]

- LCDAs of proton serves as the fundamental input to explain ep scattering
[Chen², Feng, Hu, Jia 2025], [Huang, Shi, Wang, Zhao 2025], [Yu, SC, Han, Li, Yu 2025]
- The non-perturbative input for HFP theoretical studies that determines the precision and accuracy predicted the CPVs in the $B \rightarrow \pi\pi, K\pi$ decays and et.al.,
- ...

The limitations of the single particle picture

- V, S meson LCDAs reveal certain limitations in the precision testing era
 - △ the probability amplitudes to **find the light meson** in a state with . . .
 - △ the $\pi\pi$ mass in $[0.554, 0.996]$ GeV is selected to identify candidates for the $\rho(770)$



- A second-best approach frequently employed in phenomenology is the **cascade decay framework**

$$\mathcal{M}(B^0 \rightarrow \pi^0 \pi^- l^+ \nu_l) = \mathcal{M}(B^0 \rightarrow \rho^- l^+ \nu_l) \text{BW}(s) \mathcal{M}(\rho \rightarrow \pi\pi)$$

- **model dependence, large uncertainties in f_0 involved processes**
- How to accurately describe the **width effects** of unstable intermediate particles, the contributions and **interference effects** of different partial waves, and the **QCD backgrounds** from non-resonant states

2π DAs provide a general description of $\pi\pi$ spectral

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1) \pi^b(k_2) | \bar{q}_f(zn) \gamma_\mu \tau q_f(0) | 0 \rangle = \kappa_{ab} k_\mu \int du e^{iuz(k \cdot n)} \Phi_{\parallel}^{ab, ff'}(u, \zeta, k^2, \mu)$$

$$\langle 0 | \bar{q}(x) \gamma_\mu \gamma_5 q(-x) | f_0(p) \rangle = p_\mu \int_0^1 du e^{i(2u-1)p \cdot x} \phi(\mu, u)$$

- 2π DAs is decomposed in terms of $C_n^{3/2}(2u-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{J=1}(u, \zeta, k^2, \mu) = 6u(1-u) \sum_{n=0, \text{even}}^{\infty} \sum_{l=1, \text{odd}}^{n+1} B_{n\ell}^{J=1}(k^2, \mu) C_n^{3/2}(2u-1) C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{J=0}(u, \zeta, k^2, \mu) = 6u(1-u) \sum_{n=1, \text{odd}}^{\infty} \sum_{l=0, \text{even}}^{n+1} B_{n\ell}^{J=0}(k^2, \mu) C_n^{3/2}(2u-1) C_\ell^{1/2}(2\zeta-1)$$

- Evolution from $4m_\pi^2$ to large k^2** via the Watson theorem of $\pi\pi$ scattering amplitudes

$$B_{n\ell}^J(k^2) = B_{n\ell}^J(0) \text{Exp} \left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^m}{dk^{2m}} \ln B_{n\ell}^J(0) + \frac{k^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_\ell^J(s)}{s^N(s-k^2-i0)} \right]$$

\triangle 2π DAs in a wide range of energies is given by δ_ℓ^J and a few subtraction constants

2πDAs

- The subtraction constants of $B_{n\ell}(k^2)$ at low k^2 (around the threshold)

(nl)	$B_{n\ell}^{\parallel}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2} \ln B_{n\ell}^{\parallel}(0)$	$B_{n\ell}^{\perp}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2} \ln B_{n\ell}^{\perp}(0)$
(01)	1	0	1.46 \rightarrow 1.80	1	0	0.68 \rightarrow 0.60
(21)	-0.113 \rightarrow 0.218	-0.340	0.481	0.113 \rightarrow 0.185	-0.538	-0.153
(23)	0.147 \rightarrow -0.038	0	0.368	0.113 \rightarrow 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^{π}, a_2^{ρ} [SC 2019, 2023]

- 2πDAs were introduced at leading twist [Polyakov 1999, Diehl 1998]

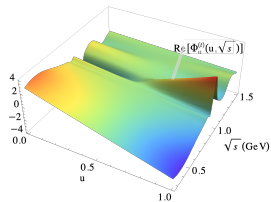
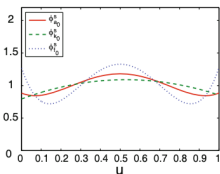
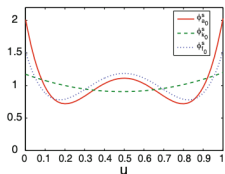
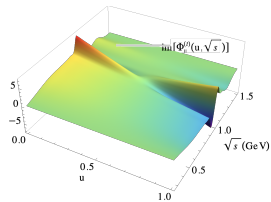
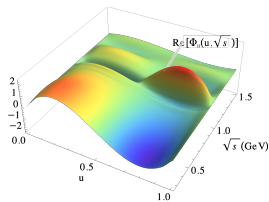
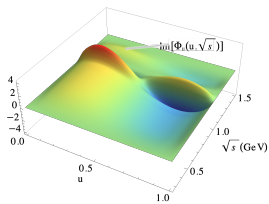
$$\langle \pi^a(k_1) \pi^b(k_2) | \bar{q}_f(zn) \gamma_{\mu} \tau q_f(0) | 0 \rangle = \kappa_{ab} k_{\mu} \int dx e^{iuz(k \cdot n)} \Phi_{\parallel}^{ab,ff'}(u, \zeta, k^2)$$

- improved to twist-three level recently [SC 2025]

$$\langle \pi(k_1) \pi(k_2) | \bar{q}(0) q(x) | 0 \rangle = \int du e^{i\bar{u}k \cdot x} \frac{ik^2(k \cdot x)}{2f_{2\pi}^{\perp}} \Phi_{\parallel}^{(s)},$$

$$\langle \pi(k_1) \pi(k_2) | \bar{q}(0) \sigma^{\mu\nu} q(x) | 0 \rangle = -\frac{i}{f_{2\pi}^{\perp}} \int du e^{i\bar{u}k \cdot x} \left[\frac{k_{\mu} \bar{k}_{\nu} - k_{\nu} \bar{k}_{\mu}}{2\zeta - 1} \Phi_{\perp} - k^2 \frac{k_{\mu} x_{\nu} - k_{\nu} x_{\mu}}{k \cdot x} \Phi_{\parallel}^{(t)} \right].$$

2π DAs of isospin scalar two-pion system



[Han, et.al., Eur. Phys. J. A (2013) 49: 78]

[Lü, et.al., Phys. Rev. D 75. 056001(2007)]

- asymmetry of the twist-3 2π DAs to the parton momentum fraction u
- symmetric for f_0 obtained under the single particle approximation
- where QCD sum rules dictate that the asymmetric component vanishes

$H \rightarrow \pi\pi$ form factors in H_{I4} decays

- **QCD dynamics of H_{I4} decays is incorporated in $H \rightarrow \pi\pi$ form factors** instead of the $H \rightarrow f_0$ ffs followed by $f_0 \rightarrow \pi\pi$ in the cascade decay

$$\begin{aligned}
 i\langle \pi^+(k_1)\pi^-(k_2) | \bar{s}\gamma_\nu(1 - \gamma_5)c | D_s(p) \rangle = & F_\perp(q^2, k^2, \zeta) \frac{2}{\sqrt{k^2}\sqrt{\lambda_B}} i\epsilon_{\nu\alpha\beta\gamma} q^\alpha k^\beta \bar{k}^\gamma \\
 & + F_t(q^2, k^2, \zeta) \frac{q_\nu}{\sqrt{q^2}} + F_0(q^2, k^2, \zeta) \frac{2\sqrt{q^2}}{\sqrt{\lambda_B}} \left(k_\nu - \frac{k \cdot q}{q^2} q_\nu \right) \\
 & + F_\parallel(q^2, k^2, \zeta) \frac{1}{\sqrt{k^2}} \left(\bar{k}_\nu - \frac{4(q \cdot k)(q \cdot \bar{k})}{\lambda_B} k_\nu + \frac{4k^2(q \cdot \bar{k})}{\lambda_B} q_\nu \right)
 \end{aligned}$$

- $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function, $q \cdot k = (m_B^2 - q^2 - k^2)/2$,
 $q \cdot \bar{k} = \sqrt{\lambda}\beta_\pi(k^2) \cos \theta_\pi/2 = \sqrt{\lambda}(2\zeta - 1)$, $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$

- **QCDF** (QCD factorization) in the large two-pion mass
 - [P. Böer, T. Feldmann and D. van Dyk, JHEP 02 (2017) 133] $T_I \propto F_{B \rightarrow \pi} \otimes \phi_\pi$.
- **SU(3) flavor symmetry/breaking** with the intermediate resonance
 - [R.M. Wang, Y.G. Xu, J.H. Sheng, X.D. Cheng and et.al., 2301.00090, PRD 112 (2025) 033002]

$H \rightarrow \pi\pi$ form factors

- **LQCD** (Lattice QCD) in the ρ resonance region with a simple BW model
 - [L. Leskovec and et.al, PRL 134 (2025) 161901, **Editors' Suggestion**]
- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large q^2 by taking dispersive methods in terms of Omnés functions
 - [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89 (2014) 053015]in the full phase-space by a novel parameterization with unitarity
 - [F. Herren, B. Kubis and R. van Tonder, PRD 112 (2025) 014037, **Editors' Suggestion**]
- **LCSRs** (Light-cone sum rules) in the small and intermediate q^2
 - [SC, A. Khodjamirian and J. Virto, JHEP 05 (2017) 157] **B-meson LCSRs**, [S. Descotes-Genon, A. Khodjamirian, J. Virto and K.K. Vos, JHEP 12 (2019) 083, 06 (2023) 034] **$B \rightarrow K\pi$**
 - [C. Hambrock and A. Khodjamirian, NPB 905 (2016) 379-390] **2π DAs LCSRs of $F_{\parallel, \perp}$**
 - [SC, A. Khodjamirian and J. Virto, PRD(R) 96 (2017) 051901] **timelike-helicity FF F_t and F_0**
 - [SC, PRD 99 (2019) 053005] **2π DAs updates and $B \rightarrow [\pi\pi]_{S,P}$ FFs**
 - [SC and J.M Shen, EPJC 6 (2020) 554, SC and S.L Zhang, EPJC 84 (2024) 379] **Pheno**
 - [SC, PRD 112 (2025) L111301] first study of **twist-three 2π DAs and $|V_{ub}|$ extraction**
 - [SC, L.Y. Dai, J.M. Shen and S.L. Zhang, PRD 113 (2026) L031901] **$D_s \rightarrow [\pi\pi]_S e\nu$ this talk**

$D_s \rightarrow [\pi\pi]_S e^+ \nu_e$ decay

- $D_s \rightarrow f_0 e^+ \nu$ [CLEO '09], $D_{(s)} \rightarrow a_0 e^+ \nu$ [BESIII '18, '21], $D^+ \rightarrow f_0/\sigma e^+ \nu$ [BESIII '19]
- $D_s \rightarrow f_0 (\rightarrow \pi^0 \pi^0, K_S K_S) e^+ \nu$ [BESIII 22], $D_s \rightarrow f_0 (\rightarrow \pi^+ \pi^-) e^+ \nu$ [BESIII 23]

$$\mathcal{B}(D_s \rightarrow f_0 (\rightarrow \pi^0 \pi^0) e^+ \nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$

$$\mathcal{B}(D_s \rightarrow f_0 (\rightarrow \pi^+ \pi^-) e^+ \nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$

$$f_+^0(0) |V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- single particle (narrow width limit) $D_s \rightarrow f_0 e^+ \nu$

$$\frac{d\Gamma(D_s^+ \rightarrow f_0 l^+ \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2, \quad D_s \rightarrow f_0 \text{ FF}$$

- improvement with the width effect by resonant models $D_s \rightarrow [f_0 \rightarrow] \pi\pi e^+ \nu$

$$\frac{d\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dsdq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2) g_1 \beta_\pi(s)}{|m_S^2 - s + i(g_1 \beta_\pi(s) + g_2 \beta_K(s))|^2}, \quad \text{BESIII}$$

- calculate directly the signal channel $D_s \rightarrow [\pi\pi]_S e^+ \nu$

$$\frac{d^2\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dsdq^2} = \frac{G_F^2 |V_{cs}|^2 \beta_{\pi\pi}(s) \sqrt{\lambda_{D_s} q^2}}{3(4\pi)^5 m_{D_s}^3} \sum_{\ell=0}^{\infty} |F_0^{(\ell)}(q^2, s)|^2, \quad D_s \rightarrow \pi\pi \text{ FF}$$

$D_s \rightarrow f_0$ FFs and cascade decay $D_s \rightarrow (f_0 \rightarrow) \pi\pi e^+ \nu$

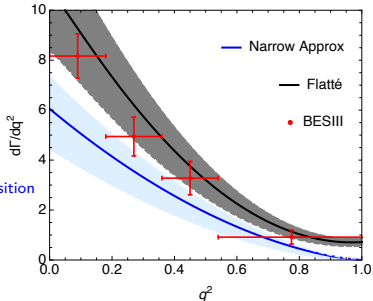
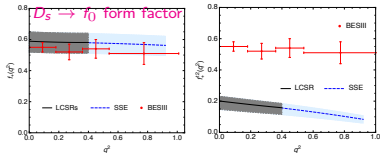
- $\{M^2, s_0\} = \{5.0 \pm 0.5, 6.0 \pm 0.5\} \text{GeV}^2$

this work	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	0.86/0.90	0.30 ± 0.03

- the BESIII result in the $\pi^+ \pi^-$ system $f_+(0) = 0.518 \pm 0.018 \pm 0.036$ [BESIII 23]

different input of the decay constant $\tilde{f}_{f_0} = 335 \text{ MeV}$, much larger than 180 MeV in LCSRs(10)
we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$\bar{s}s - \bar{n}n$ mixing scenario of f_0 with $\theta = 20^\circ \pm 10^\circ$



- Twist-3 LCDAs give dominate contribution in $D_s \rightarrow f_0$ transition

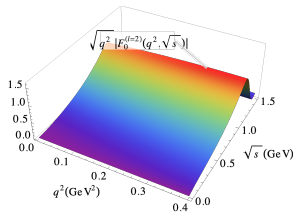
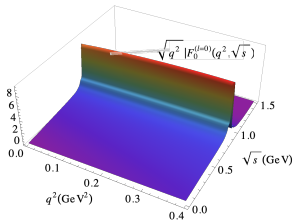
- the uncertainty estimation is conservative
- without NLO correction
- we need a model independent calculation
- for the QCD understanding
- and the future partial-wave measurement

Differential decay width of $D_s^+ \rightarrow (f_0 \rightarrow) [\pi\pi]_S e^+ \nu_e$

$D_s \rightarrow [\pi\pi]_S$ FFs and $D_s \rightarrow [\pi\pi]_S e^+ \nu$ decay

- The LCSRs ℓ' -wave $D_s \rightarrow [\pi\pi]_S$ form factors ($\ell' = \text{even} \ \& \ \ell' \leq n + 1$)

$$\sqrt{q^2} F_0^{(\ell')} (q^2, k^2) = \frac{m_c(m_c + m_s) \sqrt{q^2} \sqrt{\lambda_{D_s}}}{m_{D_s}^2 f_{D_s}} \sum_{n=1, \text{odd}}^{\infty} \frac{\beta_{\pi}(k^2)}{\sqrt{2\ell' + 1}} J_n^0(q^2, k^2, M^2, s_0) B_{n\ell, \parallel}^{l=0}(k^2) I_{\ell\ell'}$$

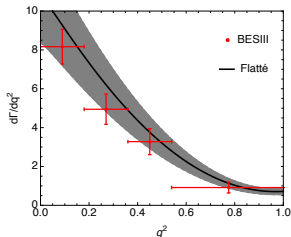
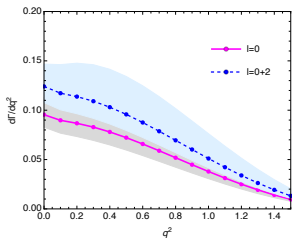


- Twist-2 and twist-3 contributions to $D_s \rightarrow \pi\pi, f_0$ form factors at $q^2 = 0$
under the $q\bar{q}$ approximation

Form Factors	Twist-2	Twist-3	Total
$\sqrt{q^2} F_0^{(l=0)}(0) = \sqrt{q^2} F_t^{(l=0)}(0)$	$0.20_{-0.02}^{+0.02} - i0.24_{-0.02}^{+0.02}$	$-0.41_{-0.05}^{+0.04} + i0.51_{-0.04}^{+0.02}$	$-0.21_{-0.01}^{+0.02} + i0.27_{-0.02}^{+0.03}$
$\sqrt{q^2} F_0^{(l=2)}(0) = \sqrt{q^2} F_t^{(l=2)}(0)$	$0.27_{-0.02}^{+0.03} + i0.21_{-0.01}^{+0.02}$	$-0.55_{-0.03}^{+0.02} - i0.41_{-0.04}^{+0.05}$	$-0.28_{-0.02}^{+0.02} - i0.20_{-0.01}^{+0.02}$
$f_+(0) = f_0(0)$	$0.20_{-0.05}^{+0.03}$	$0.41_{-0.06}^{+0.04}$	$0.61_{-0.07}^{+0.05}$

$D_s \rightarrow [\pi\pi]_S$ FFs and $D_s \rightarrow [\pi\pi]_S e^+ \nu$ decay

- The differential decay width on momentum transfers



$D_s \rightarrow [\pi\pi]_S e^+ \nu_e$	$D_s \rightarrow [f_0 \rightarrow \pi\pi] e^+ \nu_e$ [23]	Data [25]
$0.81^{+0.34}_{-0.14}$	$18.8^{+4.5}_{-3.8}$	17.2 ± 1.6

- non- $q\bar{q}$ Fock states are the dominate component of $[\pi\pi]_S$ in charm decays
- in consistent with the assessment of color transparency
- much different in B decays non-asymptotic leading twist dominated [SC 2025]
- go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g, q\bar{q}q\bar{q}$)

Summary and Prospect

- 2π DAs provide a general description of the $\pi\pi$ mass spectrum
- 2π DAs are a crucial input for the study of H_{I4} decays
- 2π DAs have been studied at the three-twist level, explaining the multi-particle picture of scalar mesons
- wishlists related to the (future) colliders
 - possible anomalies in the FCNC processes of the $D \rightarrow \pi\pi I^+ I^-$ decay
 - the processes e^+e^- annihilation $\gamma^* \rightarrow \pi\pi\gamma$ and $\gamma^*\gamma \rightarrow \pi\pi$

Thank you for your patience.