SIDIS过程中高扭度效应的研究

王晓玉

郑州大学

xiaoyuwang@zzu.edu.cn

2026轻强子专题研讨会

中国 商丘

2026年5月14日-2026年5月18日

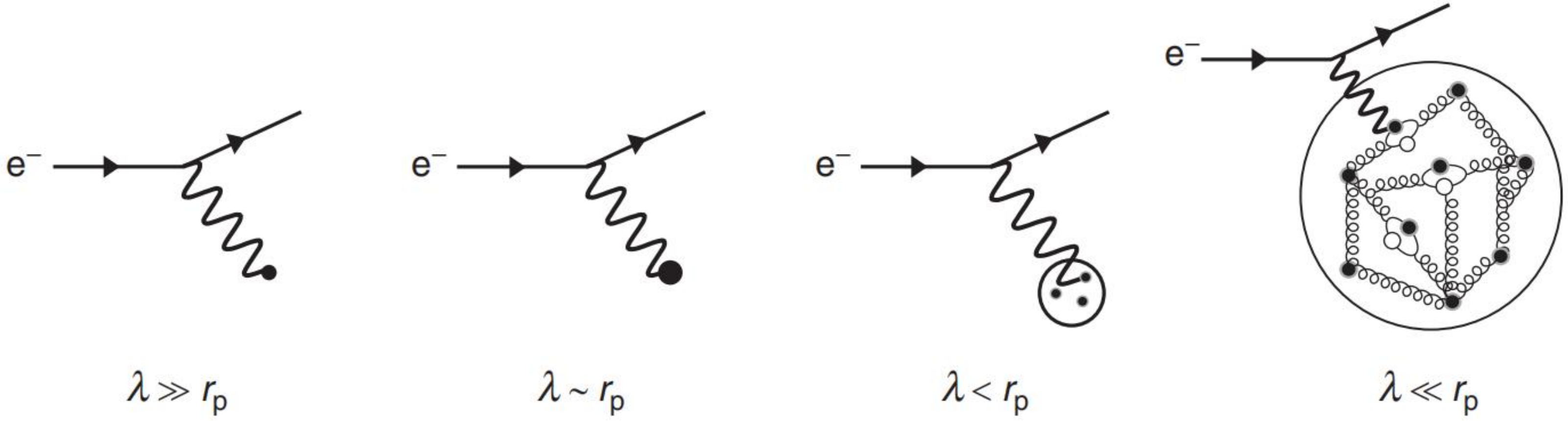
Collaborated With Zhun Lu@SEU, Yongliang Yang@QDU, Yanli Li@ZZU, Keyang She@ZZU, Shichen Xue@HAUT and Wenjuan Mao@ZKNU

Outline

- SIDIS过程与核子结构
- 单强子产生的SIDIS过程中高扭度效应的研究
- 双强子产生的SIDIS过程中高扭度效应的研究
- 未来展望

SIDIS过程与核子结构—从DIS到SIDIS

- 轻子-核子散射是研究核子内部结构的有力工具

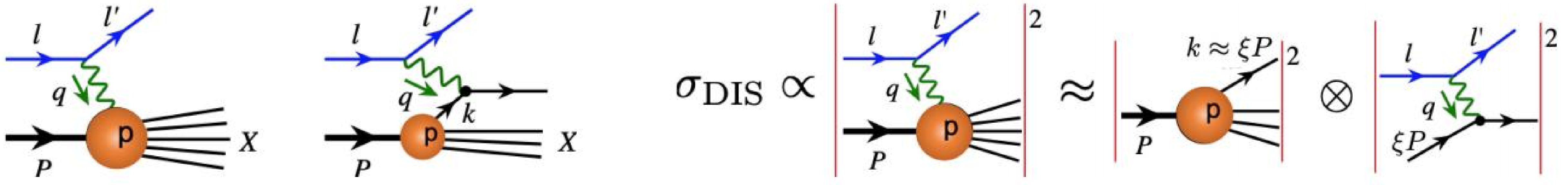


随着能量的提高，对核子结构的研究越深入

- QCD渐近自由导致不同能区的不同行为：高能区，pQCD 可解析计算； 低能区，pQCD 无法解析计算
- QCD色禁闭导致研究核子内夸克-胶子结构面临挑战
- 为了最大化地利用pQCD，发展了因子化(Factorization)定理

SIDIS过程与核子结构—从DIS到SIDIS

- 因子化能标μ_F将强子参与的过程分为长程和短程两部分
  - 高能短程部分利用pQCD 计算；长程NP部分参数化为普适的唯象函数
- 利用普适唯象函数结合微扰 QCD 对强子参与过程进行预言，最大化利用微扰 QCD，以DIS过程为例



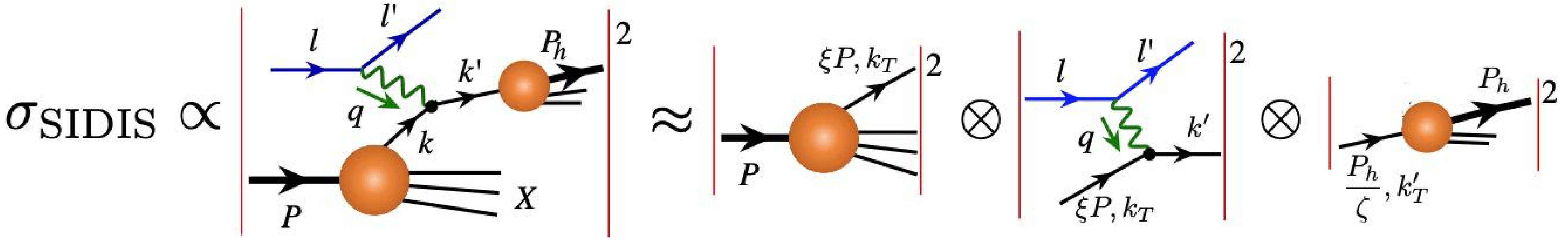
(a) (b) (c)

电子动量转移$Q^2=-q^2$, $Q\gg 1/R$，ep散射认为是电子与Parton散射的非相干叠加，$\sigma_{ep\to e'X}=\sum_i\int d\xi\ f_{i/p}(\xi)\hat{\sigma}_{ei\to e'X}$

- 成功解释了SLAC的实验数据，证明质子内存在spin-1/2的例子，提供了研究质子结构的途径
- $f_{i/p}(\xi)$的概率解释：在动量为P的质子中找到味道为i，动量为 ξP的部分子的几率分布密度
- 定义了部分子分布函数(PDF)

SIDIS过程与核子结构—从DIS到SIDIS

- $\sigma_{ep\to e'X} = \sum_i \int d\xi\ f_{i/p}(\xi)\hat{\sigma}_{ei\to e'X}$，DIS的散射截面对味道求和，无法得知有效的味道依赖的分布信息
- 半单举深度非弹性散射（SIDIS）过程，末态除测量散射轻子外，还测量一个产生强子



- 散射截面可写为$\sigma_{ep\to ehX} \propto \sum_i f_{i/p} \otimes D_{h/i} \otimes \hat{\sigma}_{ei\to ei'}$，较DIS过程而言，多了味道探针，可获取味道信息
- 除味道信息外，还引入了一个软能标
  - 硬能标：$Q \gg \Lambda_{QCD} \sim 1/R$:大动量转移，质子内部探针
  - 软能标：$P_{hT} \ll Q$横向动量，色禁闭强子内部结构
- 因子化定理：共线因子化→ 横动量依赖因子化定理

SIDIS过程与核子结构—从DIS到SIDIS

- 部分子分布函数从一维扩展到了三维
- 给出了表征部分子强子化过程的碎裂函数

Leading Quark TMDPDFs — Nucleon Spin, Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = Boer-Mulders
L		g_1 = Helicity	$h_{1L}^\perp$ = Worm-gear
T	$f_{1T}^\perp$ = Sivers	$g_{1T}^\perp$ = Worm-gear	h_1 = Transversity $h_{1T}^\perp$ = Pretzelosity

Leading Quark TMDFFs — Hadron Spin, Quark Spin

	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons	D_1 = Unpolarized		$H_1^\perp$ = Collins
Polarized Hadrons L		G_1 = Helicity	$H_{1L}^\perp$ =
Polarized Hadrons T	$D_{1T}^\perp$ = Polarizing FF	$G_{1T}^\perp$ =	H_1 = Transversity $H_{1T}^\perp$ =

提供了研究核子内部三维结构以及强子化机制的理论工具！

SIDIS过程与核子结构—从DIS到SIDIS

- twist-3 effect: $\mathcal{O}(1/Q)$ 对SIDIS过程的完整描述是必须的，在中低Q范围，twist-3 effect不可忽略
- Twist-3 effect 的来源有:
  - kinematic power corrections (contractions between the leptonic and hadronic)
  - intrinsic power corrections (quark-quark correlators involving Dirac structure)
  - dynamic power corrections (quark-gluon-quark)

Subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f^\perp,\ g^\perp$	$e\ ,\ h$
	L	$f_L^\perp,\ g_L^\perp$	$e_L,\ h_L$
	T	$f_T,\ f_T^\perp,\ g_T,\ g_T^\perp$	$e_T,\ e_T^\perp,\ h_T,\ h_T^\perp$

Subleading Quark TMDFFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Unpolarized (or Spin 0) Hadrons		$D^\perp,\ G^\perp$	$E\ ,\ H$
Polarized Hadrons	L	$D_L^\perp, G_L^\perp$	E_L, H_L
	T	$D_T, D_T^\perp, G_T, G_T^\perp$	$E_T, E_T^\perp, H_T, H_T^\perp$

Subleading Quark-Gluon-Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$\tilde{f}^\perp,\ \tilde{g}^\perp$	$\tilde{e}\ ,\ \tilde{h}$
	L	$\tilde{f}_L^\perp,\ \tilde{g}_L^\perp$	$\tilde{e}_L,\ \tilde{h}_L$
	T	$\tilde{f}_T,\ \tilde{f}_T^\perp,\ \tilde{g}_T,\ \tilde{g}_T^\perp$	$\tilde{e}_T,\ \tilde{e}_T^\perp,\ \tilde{h}_T,\ \tilde{h}_T^\perp$

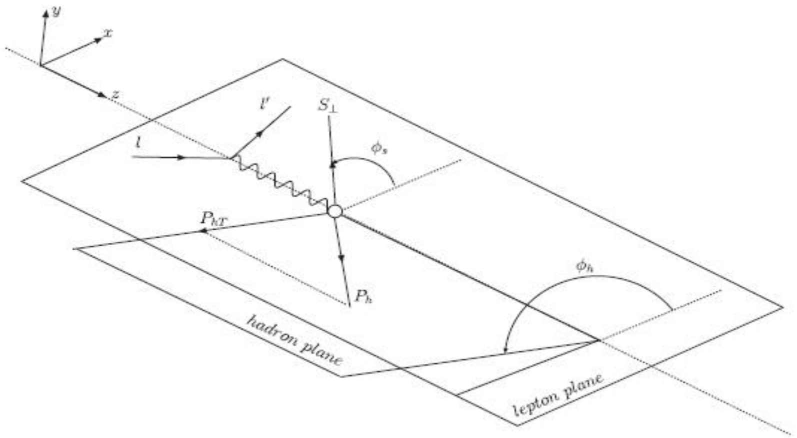
Subleading Quark-Gluon-Quark TMDFFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Unpolarized (or Spin 0) Hadrons		$\tilde{D}^\perp,\ \tilde{G}^\perp$	$\tilde{E}\ ,\ \tilde{H}$
Polarized Hadrons	L	$\tilde{D}_L^\perp, \tilde{G}_L^\perp$	$\tilde{E}_L, \tilde{H}_L$
	T	$\tilde{D}_T, \tilde{D}_T^\perp, \tilde{G}_T, \tilde{G}_T^\perp$	$\tilde{E}_T, \tilde{E}_T^\perp, \tilde{H}_T, \tilde{H}_T^\perp$

Outline

- SIDIS过程与核子结构
- **单强子产生的SIDIS过程中高扭度效应的研究**
- 双强子产生的SIDIS过程中高扭度效应的研究
- 未来展望

SIDIS with $\pi^{\pm,0}$产生的TSSA $A_{UT}^{\sin\phi_S}$



- SIDIS过程的Lorentz不变量

$$x=\frac{Q^2}{2P\cdot q},\qquad y=\frac{P\cdot q}{P\cdot l},\qquad z=\frac{P\cdot P_h}{P\cdot q},$$
$$\gamma=\frac{2Mx}{Q},\qquad Q^2=-q^2,\qquad s=(P+l)^2.$$

- 至twist-3阶，六维(x, y, z, ϕ_h, ϕ_S and P_{hT})散射截面的一般形式为

Bacchetta et al., JHEP0702, 093 (2007)

$$\frac{d^6\sigma}{dx\,dy\,dz\,d\phi_h\,d\phi_S\,dP_{hT}^2}=\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\times\sqrt{2\varepsilon(1+\varepsilon)}\{\sin\phi_S F_{UT}^{\sin\phi_S}(x,z,P_T)+\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}(x,z,P_T)+\text{leading twist terms}\},$$

$$F_{UT}^{\sin\phi_S}=\frac{2M}{Q}\mathcal{C}\left\{\left(xf_TD_1-\frac{M_h}{M}h_1\frac{\tilde{H}}{z}\right)-\frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(xh_TH_1^\perp+\frac{M_h}{M}g_{1T}\frac{\tilde{G}^\perp}{z}\right)-\left(xh_T^\perp H_1^\perp-\frac{M_h}{M}f_{1T}^\perp\frac{\tilde{D}^\perp}{z}\right)\right]\right\},$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)}=\frac{2M}{Q}\mathcal{C}\left\{\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2}{2M^2}\left(xf_T^\perp D_1-\frac{M_h}{M}h_{1T}^\perp\frac{\tilde{H}}{z}\right)-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(xh_TH_1^\perp+\frac{M_h}{M}g_{1T}\frac{\tilde{G}^\perp}{z}\right)+\left(xh_T^\perp H_1^\perp-\frac{M_h}{M}f_{1T}^\perp\frac{\tilde{D}^\perp}{z}\right)\right]\right\},$$

$$\frac{d^4\sigma}{dx\,dy\,dz\,d\phi_S}=\frac{2\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\times\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}(x,z)\,.$$

SIDIS with $\pi^{\pm,0}$产生的TSSA $A_{UT}^{\sin\phi_S}$

- 积分末态横动量后，只有一项“幸存”下来

$$F_{UT}^{\sin\phi_S}(x,z) = \int d^2\boldsymbol{P}_{hT} F_{UT}^{\sin\phi_S}(x,z,P_{hT}) = -\int d^2\boldsymbol{P}_{hT}\frac{2M_h}{Q}\mathcal{C}\left[h_1\frac{\tilde{H}}{z}\right]$$

$$= -x\frac{2M_h}{Q}\sum_q e_q^2\int d^2\boldsymbol{k}_T\, z^2\int d^2\boldsymbol{p}_T h_1^q(x,\boldsymbol{k}_T^2)\frac{\tilde{H}^q(z,\boldsymbol{p}_T^2)}{z}$$

$$= -x\sum_q e_q^2\frac{2M_h}{Q}\boxed{h_1^q(x)}\frac{\tilde{H}^q(z)}{z},$$

$$\tilde{H}^q(z) = z^2\int d^2\boldsymbol{p}_T\tilde{H}^q(z,\boldsymbol{p}_T^2).$$

- 定义物理可观测量

$$A_{UT}^{\sin\phi_S}(x) = \frac{\int dy\int dz\,\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}(1+\frac{\gamma^2}{2x})\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_s}(x,z)}{\int dy\int dz\,\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}(1+\frac{\gamma^2}{2x})F_{UU}(x,z)},$$

$$A_{UT}^{\sin\phi_s}(z) = \frac{\int dx\int dy\,\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}(1+\frac{\gamma^2}{2x})\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_s}(x,z)}{\int dx\int dy\,\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}(1+\frac{\gamma^2}{2x})F_{UU}(x,z)}.$$

SIDIS with $\pi^{\pm,0}$产生的TSSA $A_{UT}^{\sin\phi_S}$

- Twist-3的碎裂函数$\widetilde{H}(z)$与$\widehat{H}(z,z_1)$有如下关系

$$\tilde{H}^{h/q}(z) = 2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q,\Im}(z, z_1) .$$

- $\widehat{H}(z,z_1)$可参数化，在$Q^2 = 1\ GeV^2$

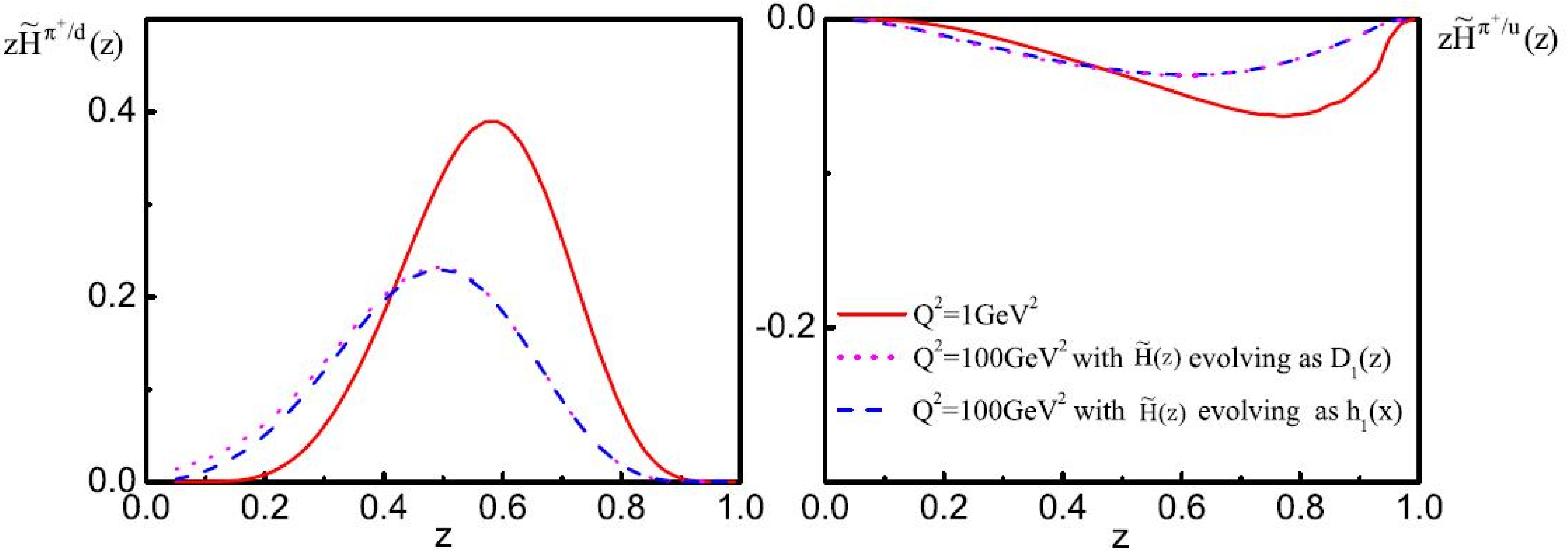
K. Kanazawa, Y. Koike, A. Metz, and D. Pitonyak, Phys. Rev. D 89, 111501 (2014)

$$\frac{\hat{H}_{FU}^{\pi^+/(u,\bar{d}),\Im}(z,z_1)}{D_1^{\pi^+/(u,\bar{d})}(z) D_1^{\pi^+/(u,\bar{d})}(z/z_1)} = \frac{N_{\text{fav}}}{2 I_{\text{fav}} J_{\text{fav}}} z^{\alpha_{\text{fav}}} (z/z_1)^{\alpha'_{\text{fav}}} \times (1-z)^{\beta_{\text{fav}}} (1-z/z_1)^{\beta'_{\text{fav}}} ,$$

- EIC、CLAS等的能区较大，DGLAP演化是必要的
  - 对$\widetilde{H}(z)$进行了一阶DGLAP演化
  - 演化核选择了两种：与D_1一致，与h_1一致

SIDIS with $\pi^{\pm,0}$产生的TSSA $A_{UT}^{\sin\phi_S}$

- $z\widetilde{H}(z)$演化结果如图 X. Wang and Z. Lu, Phys. Rev. D 93, 074009 (2016)



Result of $z\widetilde{H}^{\pi^+/d}$(left panel) $z\widetilde{H}^{\pi^+/u}$(right panel) at the initial scale $Q^2 = 1\ \mathrm{GeV}^2$(solid lines) and the evolved results at $Q^2 = 100\ \mathrm{GeV}^2$ Dotted lines: evolving as D_1, dashed lines: evolving as h_1

- 不同演化核在小z区域有影响，演化会极大的改变twsit 3碎裂函数对z依赖的线形

SIDIS with $\pi^{\pm,0}$产生的TSSA $A_{UT}^{\sin\phi_S}$

- 在EIC的运动学范围

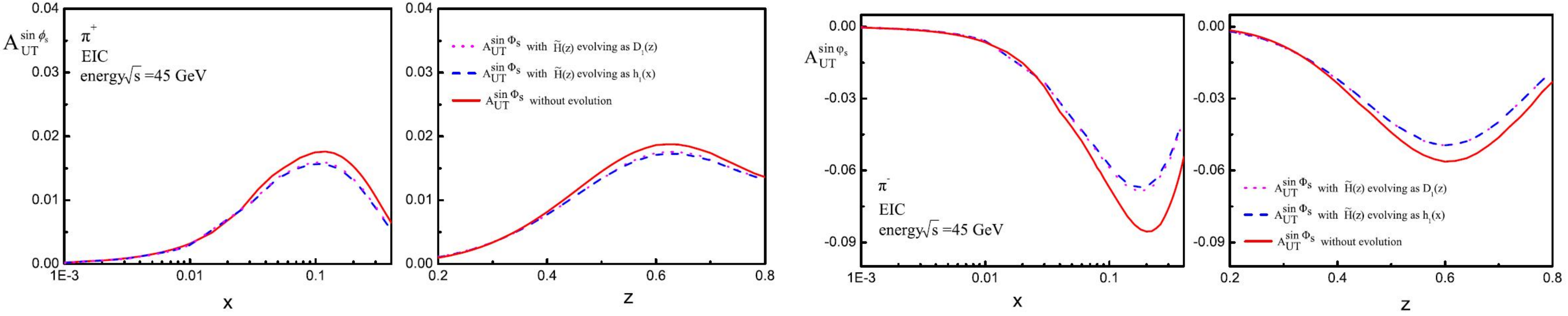
$$Q^2 > 1\ \text{GeV}^2, \quad 0.001 < x < 0.4, \quad 0.01 < y < 0.95,$$

$$0.2 < z < 0.8, \quad \sqrt{s} = 45\ \text{GeV}, \quad W > 5\ \text{GeV},$$

$$W^2 = (P+q)^2 \approx \frac{1-x}{x} Q^2$$

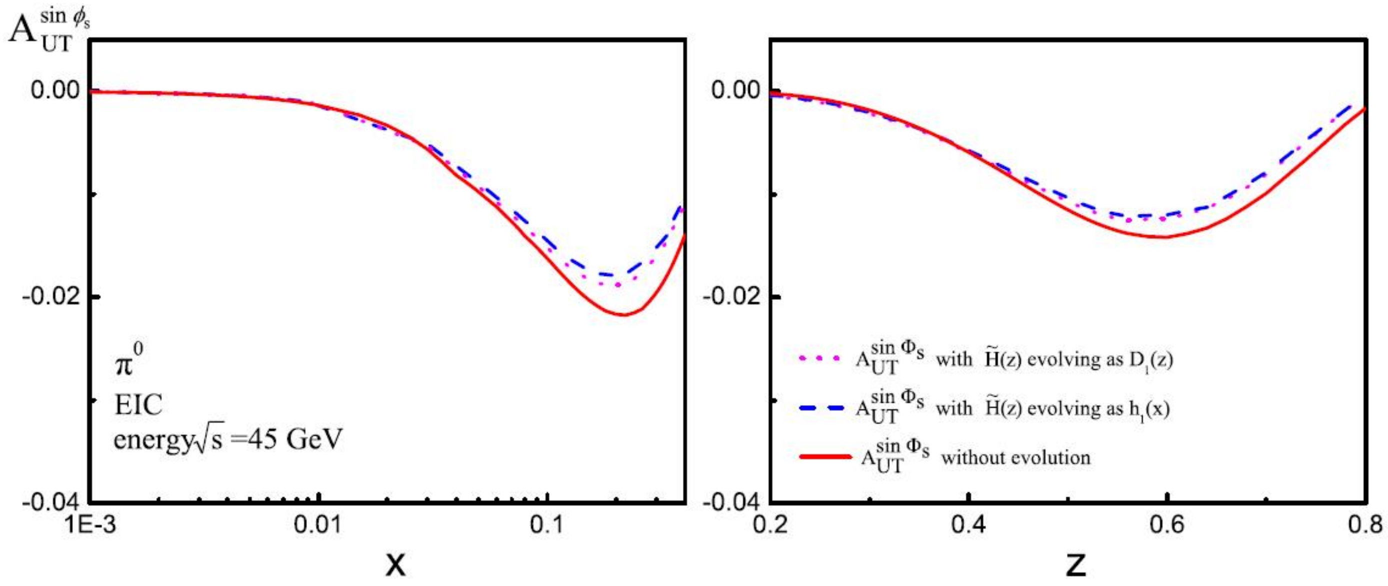
A. Accardi et al., arXiv:1212.1701

- 数值计算预言了TSSA



SIDIS with $\pi^{\pm,0}$产生的TSSA $A_{UT}^{\sin\phi_S}$

- 数值计算预言了TSSA

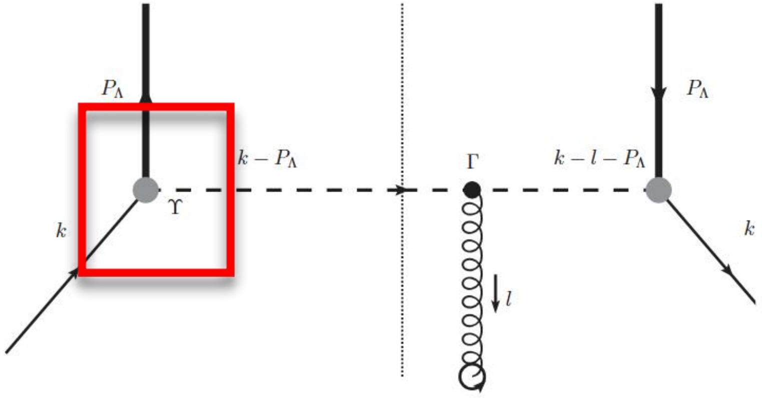


- 数值计算表明$\pi^{\pm,0}$产生的SIDIS过程中的不对称度都可测
- DGLAP演化对物理可观测量的影响较大

- 积分掉末态强子的横向动量，提供了较为“干净”的过程探测twist-3效应
- 提供了提取transversity的新途径

SIDIS with Λ 产生的TSSA $A_{UT}^{\sin\phi_S}$

- 采用旁观夸克模型对Λ碎裂函数进行模型计算
- Quark-gluon-quark关联函数求迹可得到twist-3碎裂函数$\tilde{H}(z)$



$$\frac{z}{4M_\Lambda} Tr[(\tilde{\Delta}_{A\alpha}(z,k_T;S_\Lambda)+\tilde{\Delta}_{A\alpha}(z,k_T;-S_\Lambda))\sigma^{\alpha-}] = \tilde{H}(z,k_T)+i\tilde{E}(z,k_T)$$

$$\tilde{\Delta}_A^\alpha(z,k_T;S_\Lambda) = \sum_X \frac{1}{2z}\int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3}\int e^{ik\cdot\xi}\langle 0|\int_{\pm\infty^+}^{\xi^+} d\eta^+ \mathcal{U}^{\xi_T}_{(\infty^+,\eta^+)}$$

$$\times gF_\perp^{-\alpha}(\eta)\mathcal{U}^{\xi_T}_{(\eta^+,\xi^+)}\psi(\xi)|P_\Lambda,S_\Lambda;X\rangle\langle P_\Lambda,S_\Lambda;X|\bar{\psi}(0)\mathcal{U}^{0_T}_{(0^+,\infty^+)}\mathcal{U}^{\infty^+}_{(0_T,\xi_T)}|0\rangle\Big|_{\substack{\eta^+=\xi^+=0\\ \eta_T=\xi_T}}.$$

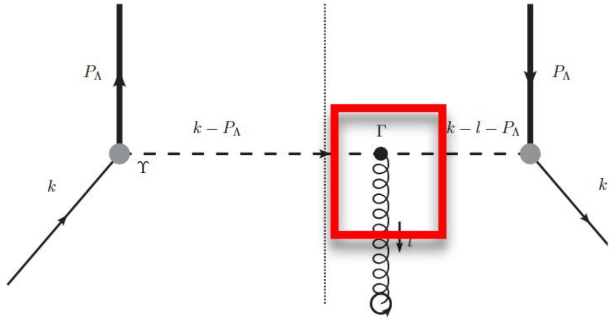
- 利用diquark spectator model写出quark-diquark-hyperon顶角
- 轴矢量双夸克极化态求和 $d_{\mu\nu}=\sum_\lambda \varepsilon_\mu^{*(\lambda)}\varepsilon_\nu^{(\lambda)} = -g_{\mu\nu}+\frac{P_{\Lambda\mu}P_{\Lambda\nu}}{M_\Lambda^2}$
- 标量和轴矢量顶角$\Upsilon_s=g_s(k^2)$、$\Upsilon_v^\mu=\frac{g_v(k^2)}{\sqrt{3}}\gamma_5(\gamma^\mu+\frac{P_\Lambda^\mu}{M_\Lambda})$
- $g_s=g_v=g_{gh}$ as Gaussian form $g_{gh}(k^2)\mapsto \frac{g_D}{z}e^{-\frac{k^2}{\Lambda^2}}$

SIDIS with Λ 产生的TSSA $A_{UT}^{\sin\phi_S}$

- 关联函数可得

$$\tilde{\Delta}^{\alpha}_{A\,s}(z,k_T,S_\Lambda) = -i\frac{C_F\alpha_S}{2(2\pi)^2(1-z)P_\Lambda^-}\frac{1}{k^2-m^2}\int\frac{d^4l}{(2\pi)^4} \times \frac{(l^- g_T^{\alpha\rho} - l^\alpha n_+^\rho)(\not{k}-\not{l}+m)\bar{\Upsilon}_s U(P_\Lambda,S_\Lambda)\bar{U}(P_\Lambda,S_\Lambda)\Upsilon_s(\not{k}+m)}{((k-l)^2-m^2)(l^2-i\epsilon)((k-l-P_\Lambda)^2-m_s^2)(-l^- - i\epsilon)}\bar{\Gamma}_\rho$$

$$\tilde{\Delta}^{\alpha}_{A\,v}(z,k_T,S_\Lambda) = i\frac{C_F\alpha_S}{2(2\pi)^2(1-z)P_\Lambda^-}\frac{1}{k^2-m^2}\int\frac{d^4l}{(2\pi)^4} \times \frac{(l^- g_T^{\alpha\rho} - l^\alpha n_+^\rho)(\not{k}-\not{l}+m)\bar{\Upsilon}_v^\nu U(P_\Lambda,S_\Lambda)\bar{U}(P_\Lambda,S_\Lambda)\Upsilon_v^\mu(\not{k}+m)}{((k-l)^2-m^2)(l^2-i\epsilon)((k-l-P_\Lambda)^2-m_s^2)(-l^- - i\epsilon)}d^{i\mu}d^{j\nu}\bar{\Gamma}_{ij\rho}$$



- The vertex between gluon and scalar diquark (Γ_s) and axial vector diquark (Γ_v)

$$\Gamma_s^{\rho,a} = T^a(2k-2P-l)^\rho$$

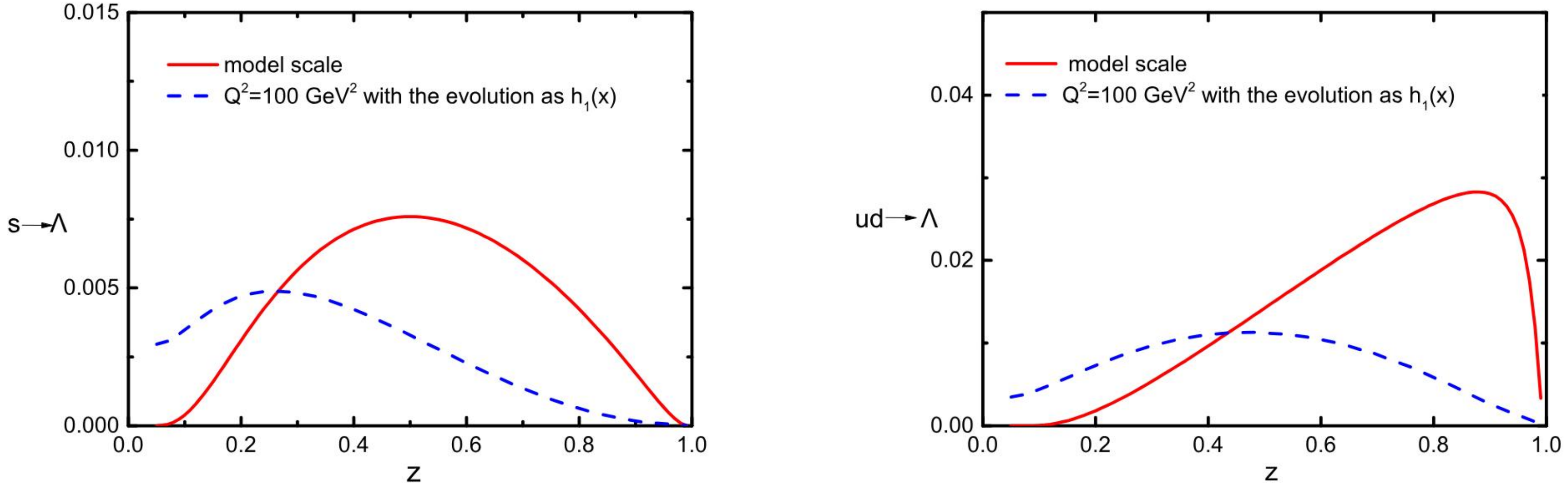
$$\Gamma_v^{\rho\mu\nu,a} = -\;T^a[(2k-2P-l)^\rho g^{\mu\nu} - (k-P-l)^\mu g^{\nu\rho} - (k-P)^\nu g^{\rho\mu}];$$

- SU(6) 味道自旋对称性下，轻味夸克的碎裂函数为 $D^{u\to\Lambda} = D^{d\to\Lambda} = \frac{1}{4}D^{(s)} + \frac{3}{4}D^{(v)}$， $D^{s\to\Lambda} = D^{(s)}$，

SIDIS with Λ 产生的TSSA $A_{UT}^{\sin\phi_S}$

- Twist-3碎裂函数为

Y. Yang, X. Wang and Z. Lu,Phys. Rev. D 103, 114011 (2021)



- 物理可观测量

$$A_{UT}^{\sin\phi_S}(x) = \frac{\int dy \int dz \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}(1+\frac{\gamma^2}{2x})\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_S}(x,z)}{\int dy \int dz \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}(1+\frac{\gamma^2}{2x})F_{UU}(x,z)},$$

$$A_{UT}^{\sin\phi_S}(z) = \frac{\int dx \int dy \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}(1+\frac{\gamma^2}{2x})\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_S}(x,z)}{\int dx \int dy \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}(1+\frac{\gamma^2}{2x})F_{UU}(x,z)}.$$

SIDIS with Λ 产生的TSSA $A_{UT}^{\sin\phi_S}$

- 运动学范围

- For EIC

$$0.001 < x < 0.4, \quad 0.01 < y < 0.95, \quad 0.2 < z < 0.8,$$
$$Q^2 > 1\text{GeV}^2, \quad \sqrt{s} = 45\ \text{GeV}, \quad W > 5\ \text{GeV}.$$

- For EicC

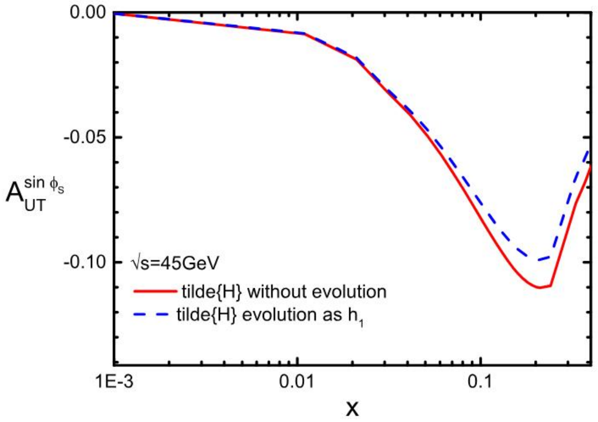
$$0.005 < x < 0.5, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.7,$$
$$Q^2 > 1\text{GeV}^2, \quad \sqrt{s} = 16.7\ \text{GeV}, \quad W > 2\text{GeV}.$$

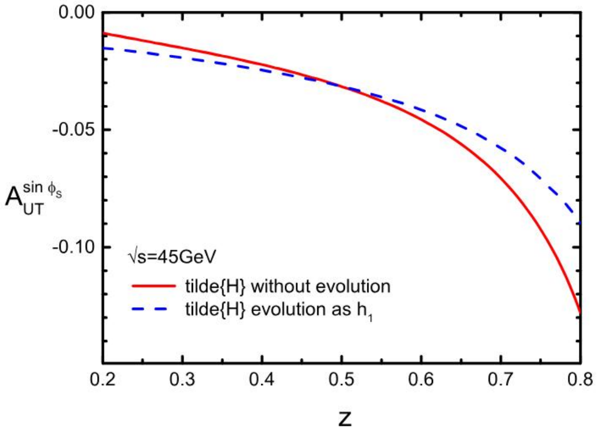
- For COMPASS

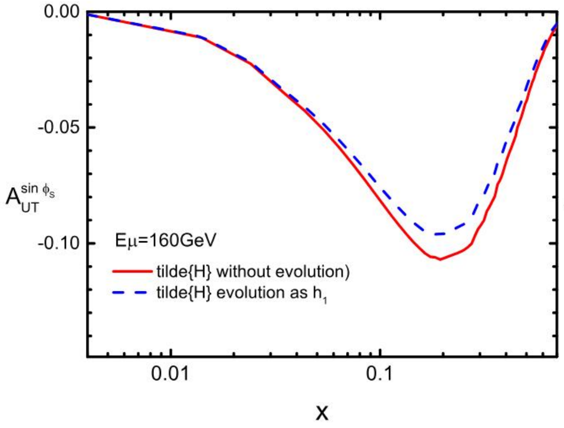
$$Q^2 > 1\text{GeV}^2, \quad 0.004 < x < 0.7, \quad 0.1 < y < 0.9,$$
$$z > 0.2, \quad W > 5\ \text{GeV}, \quad E_h > 1.5\ \text{GeV}.$$

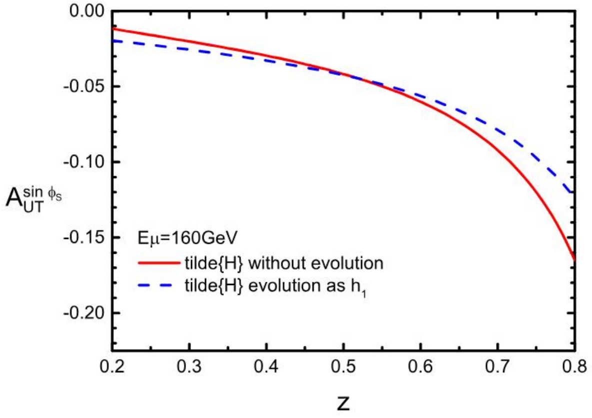
SIDIS with Λ 产生的TSSA $A_{UT}^{\sin\phi_S}$

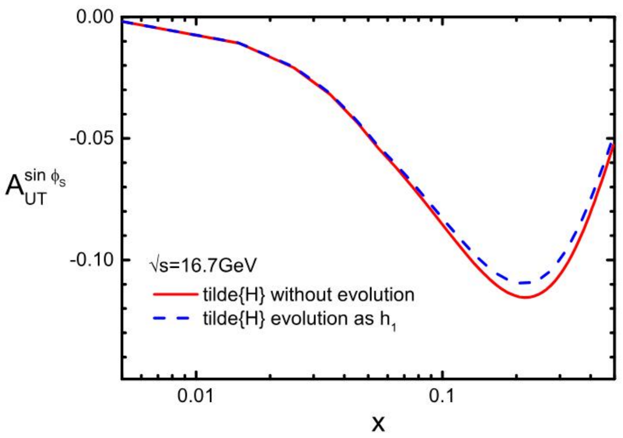
- 数值计算结果

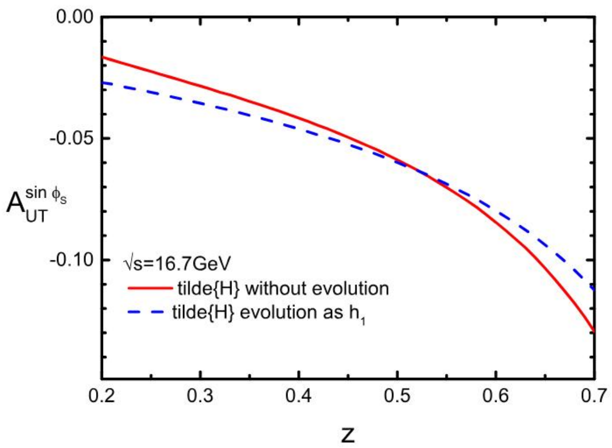












- 数值计算表明不对称度可测
- DGLAP演化对物理可观测量的影响较大
- 给出了研究transversity味道依赖性的途径

SIDIS with $\pi^{\pm,0}$产生的DSA $A_{LT}^{\cos\phi_S}$

郑州大学

- 引入束流纵向极化自由度，TSSA变成了双自旋依赖不对称度

$$\frac{d^6\sigma}{dxdydzd\phi_h d\phi_S dP_{hT}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right) \times |S_T|\lambda_e\{\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}(x,z,P_{hT}) + \sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}(x,z,P_{hT}) + \text{leading twist terms}\},$$

- 积分掉末态横向动量后，只有$F_{LT}^{\cos\phi_S}$幸存

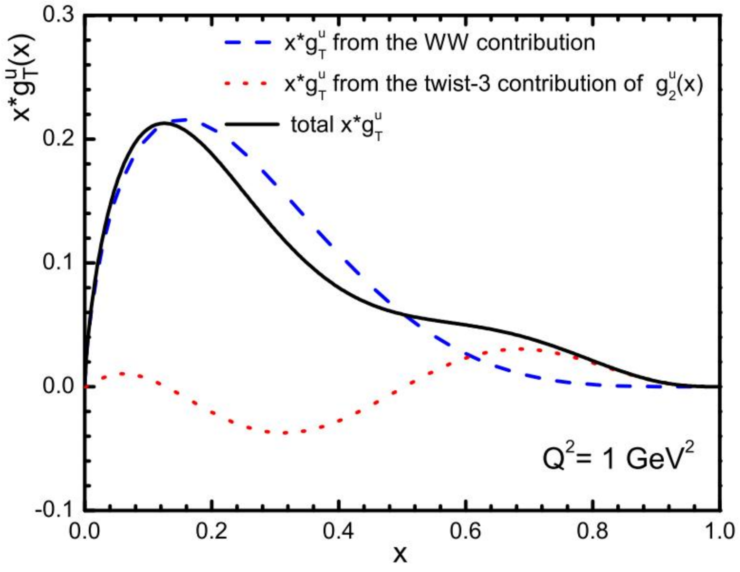
$$F_{LT}^{\cos\phi_S}(x,z) = \int d^2\boldsymbol{P}_{hT}F_{LT}^{\cos\phi_S}(x,z,P_{hT}) = -x\sum_q e_q^2\frac{2M}{Q}\left(xg_T^q(x)D_1^q(z) + \frac{M_h}{M}h_1^q(x)\frac{\tilde{E}^q(z)}{z}\right).$$

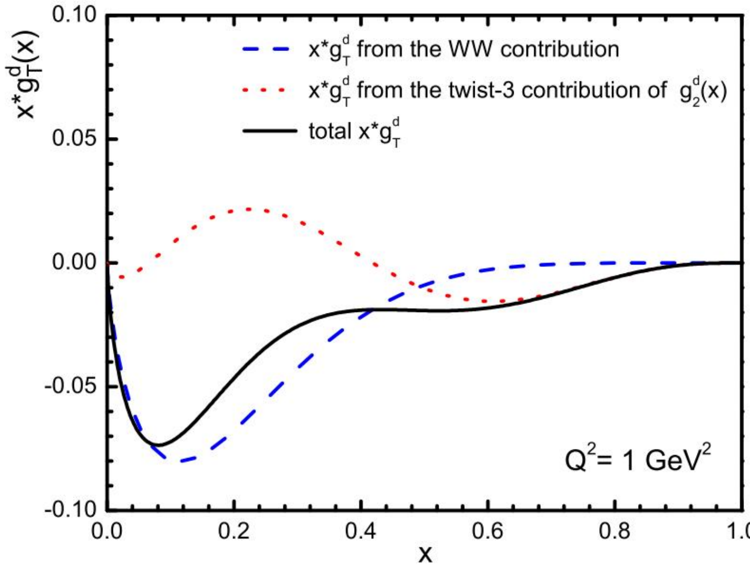
SIDIS with $\pi^{\pm,0}$产生的DSA $A_{LT}^{\cos\phi_S}$

- g_T是在DIS过程中最早引入的twsit-3结构函数
- 用极化结构函数g_1和g_2的线性组合 $\frac{1}{2}\sum_q e_q^2 g_T^q(x) = g_1(x) + g_2(x)\,,$
- g_2与核子的横向极化相关的结构函数， $g_2(x) = g_2^{\mathrm{WW}}(x) + g_2^{\mathrm{tw-3}}(x)\,.$
- 下图给出了纯twsit-3和WW近似得到的g_T数值结果

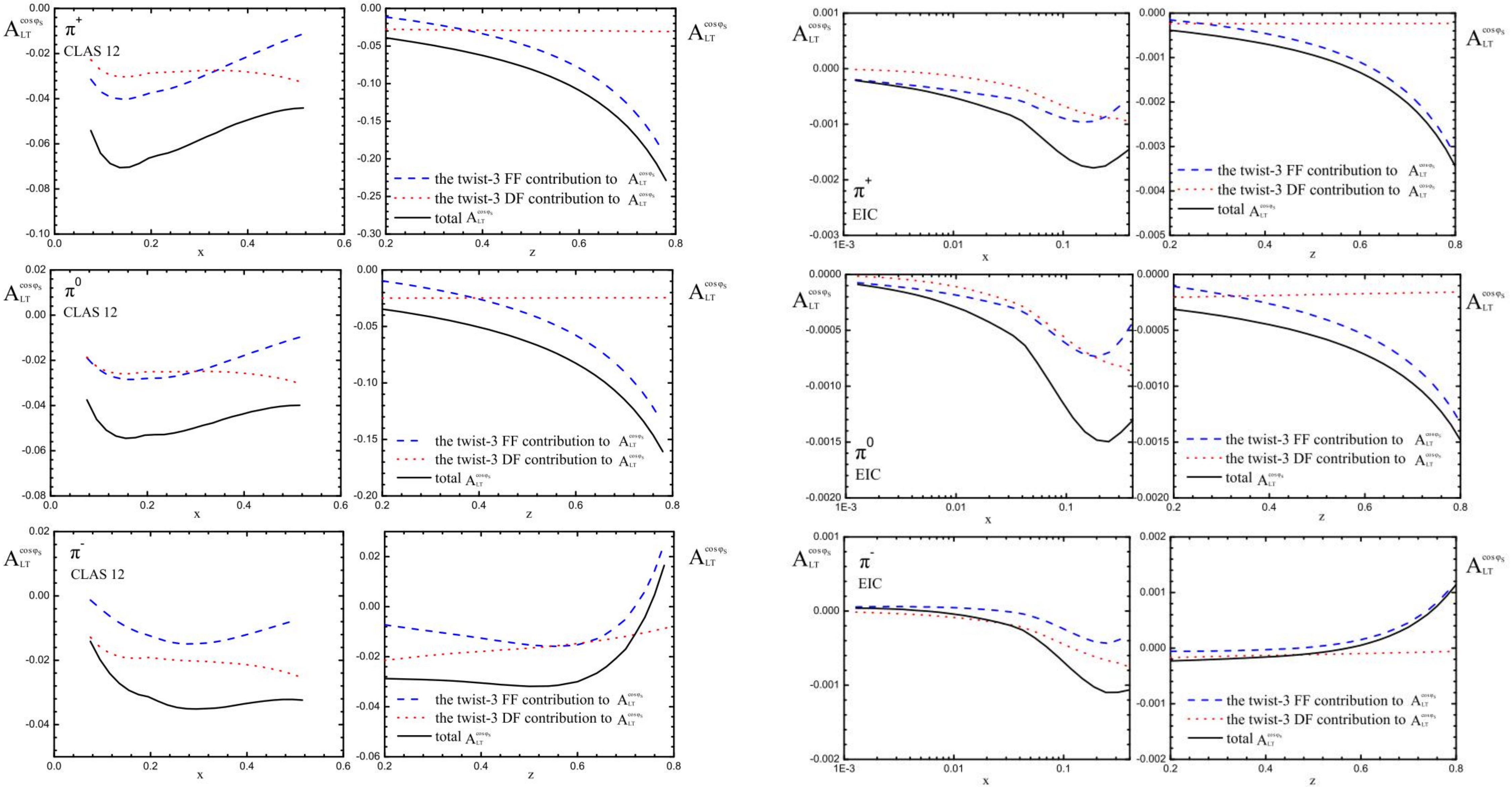
X. Wang and Z. Lu, Phys. Rev. D 93, 074009 (2016)

$$F_{LT}^{\cos\phi_S}(x,z) = \int d^2\boldsymbol{P}_{hT} F_{LT}^{\cos\phi_S}(x,z,P_{hT})$$
$$= -x\sum_q e_q^2 \frac{2M}{Q}\left(x g_T^q(x) D_1^q(z) + \frac{M_h}{M} h_1^q(x) \frac{\tilde{E}^q(z)}{z}\right).$$

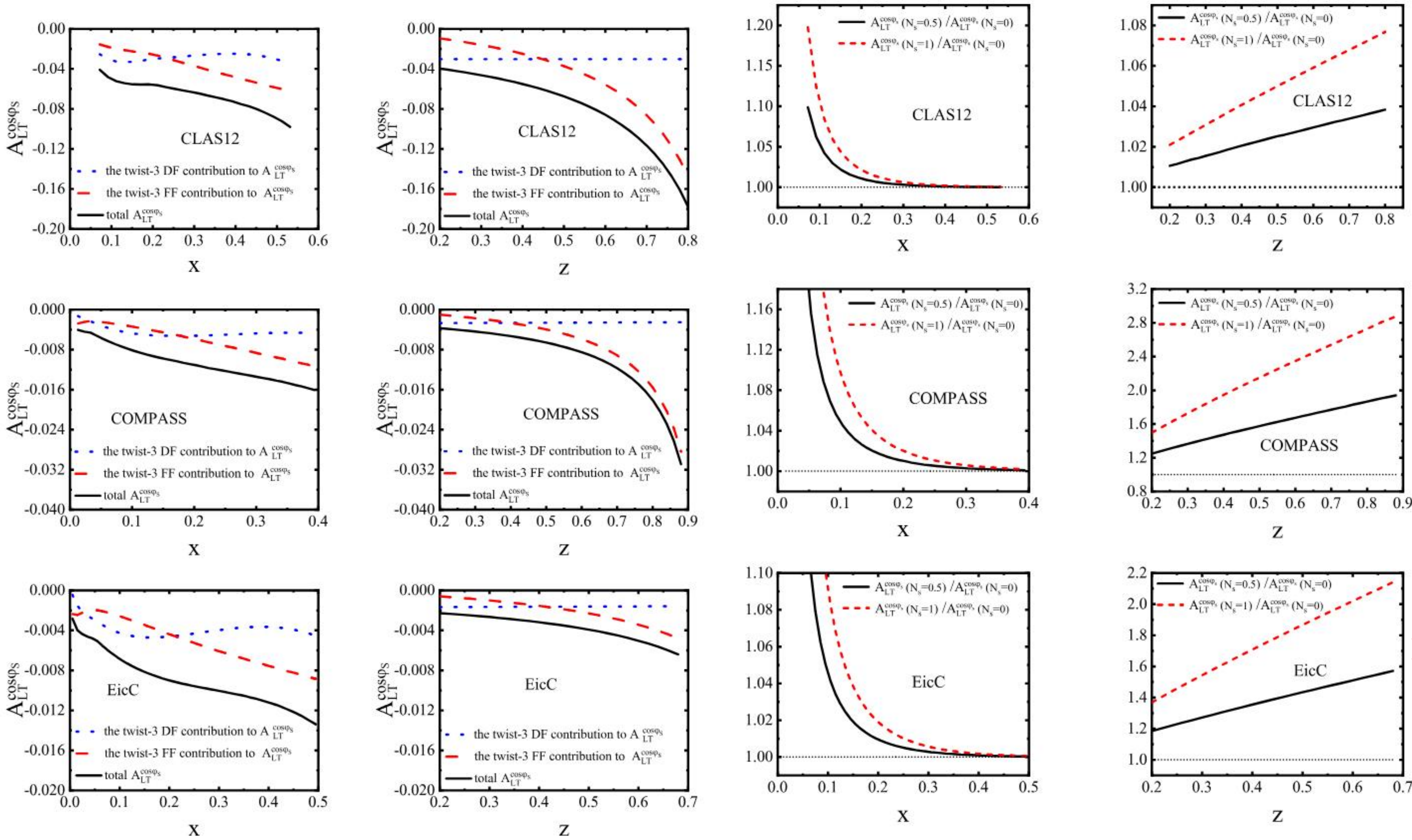




SIDIS with $\pi^{\pm,0}$产生的DSA $A_{LT}^{\cos\phi_S}$



SIDIS with Λ 产生的DSA $A_{LT}^{\cos\phi_s}$



- 不对称度可测
- 大z区域由twist-3碎裂函数主要贡献
- 给出了twist-3 碎裂函数$\tilde{E}$及其味道依赖性的途径

K. She, H. Li and X. Wang Symmetry 18 (2026) 1, 44

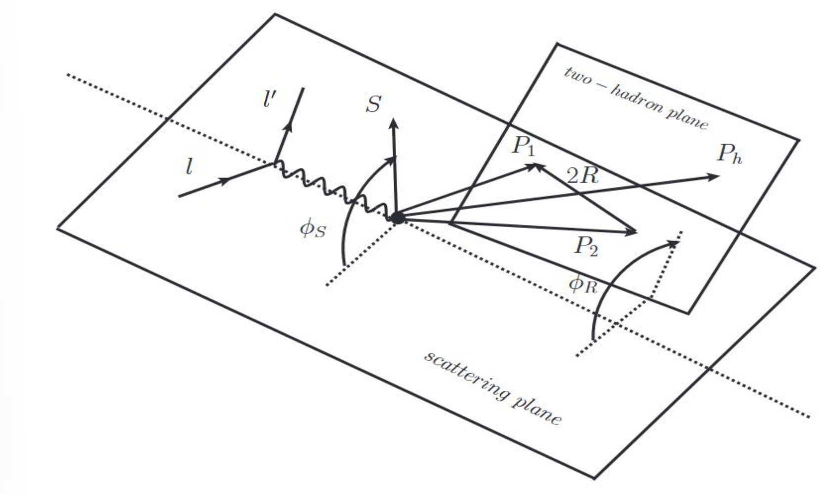
Outline

- SIDIS过程与核子结构
- 单强子产生的SIDIS过程中高扭度效应的研究
- **双强子产生的SIDIS过程中高扭度效应的研究**
- 未来展望

SIDIS with $\pi\pi$ 产生

- 双强子产生的半单举深度非弹性散射 (*SIDIS*) 过程 $l^{\rightarrow}(\ell) + p(P) \longrightarrow l(\ell') + h^{+}(P_1) + h^{-}(P_2) + X$
- 定义 Lorentz Invariants

$$x = \frac{k^+}{P^+}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad z_i = \frac{P_i^-}{k^-},$$
$$z = \frac{P_h^-}{k^-} = z_1 + z_2, \quad Q^2 = -q^2,$$
$$s = (P+l)^2, \quad \zeta = \frac{2R^-}{P_h^-}.$$



双强子产生的 SIDIS 过程示意图

- 积分掉末态强子的横向动量的微分散射截面可表示为:

$$\frac{d^7\sigma}{dxdydzd\zeta dM_h^2 d\phi_R d\phi_S} = \sum_q \frac{\alpha^2 y e_q^2}{32 z Q^4} L_{\mu\nu} 2M W_q^{\mu\nu}$$

$$L_{\mu\nu} = \frac{Q^2}{y}\Big[-2A(y)g_\perp^{\mu\nu} + 4B(y)(\hat{t}^\mu\hat{t}^\nu + \hat{x}^\mu\hat{x}^\nu + \frac{1}{2}g_\perp^{\mu\nu}) + V(y)\hat{t}^{\{\mu}\,{}^{\nu\}}\hat{x}$$
$$+ 2\mathrm{i}\lambda_l C(y)\epsilon_\perp^{\mu\nu} - \mathrm{i}\lambda_l W(y)\hat{t}^{\,[\mu}\epsilon_\perp^{\nu]\rho}\hat{x}_\rho,$$

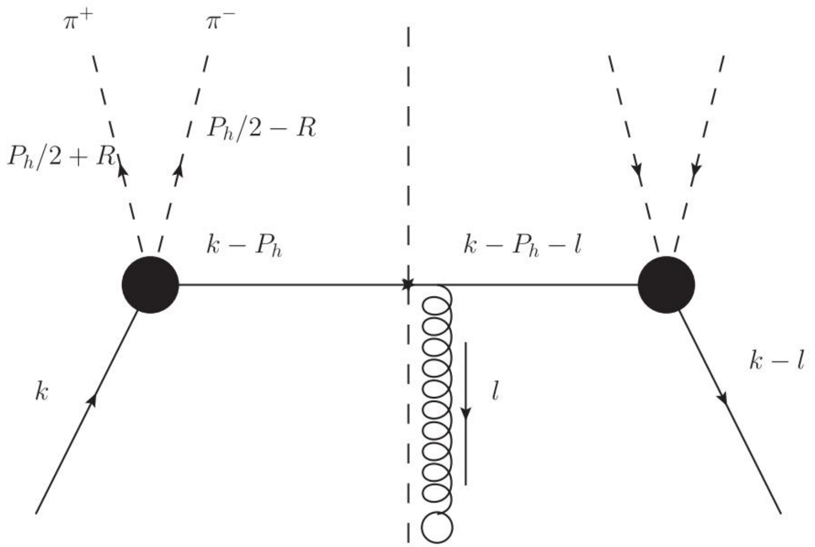
SIDIS with $\pi\pi$ 产生

- 强子张量

$$
\begin{aligned}
2M\,W^{\mu\nu} = \frac{16z}{4\pi}\Bigg[& -g_\perp^{\mu\nu} f_1 D_1 + \mathrm{i}\,\epsilon_\perp^{\mu\nu} S_L\, g_1 D_1 - \frac{R_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho} S_{\perp\rho} + S_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho} R_{T\perp\rho}}{2M_h} h_1 H_1^{\sphericalangle} \\
& + S_L \frac{2\,\hat{t}^{\{\mu}\epsilon_\perp^{\nu\}\rho} R_{T\perp\rho}}{Q} \left(\frac{M}{M_h} x\, h_L H_1^{\sphericalangle} + g_1 \frac{\widetilde{G}^{\sphericalangle}}{z} \right) \\
& + \frac{2M_h\,\hat{t}^{\{\mu}\epsilon_\perp^{\nu\}\rho} S_{\perp\rho}}{Q} \left[h_1 \left(\frac{\widetilde{H}}{z} + \frac{\vec{R}_T^2}{M_h^2} H_1^{\sphericalangle\, o\,(1)} \right) - \frac{M}{M_h} x\, f_T D_1 \right] \\
& + \mathrm{i}\, S_L \frac{2\hat{t}^{\{\mu}\epsilon_\perp^{\nu\}\rho} R_{T\perp\rho}}{Q} \left(g_1 \frac{\widetilde{D}^{\sphericalangle}}{z} - \frac{M}{M_h} x\, e_L H_1^{\sphericalangle} \right) \\
& + \frac{2\hat{t}^{\{\mu} R_{T\perp}^{\nu\}}}{Q} \left(f_1 \frac{\widetilde{D}^{\sphericalangle}}{z} + \frac{M}{M_h} x\, h\, H_1^{\sphericalangle} \right) - \mathrm{i} \frac{2\hat{t}^{\{\mu} R_{T\perp}^{\nu\}}}{Q} \left(\frac{M}{M_h} x e H_1^{\sphericalangle} + f_1 \frac{\widetilde{G}^{\sphericalangle}}{z} \right) \\
& + \mathrm{i} \frac{2M\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho} S_{\perp\rho}}{Q} \left(x\, g_T D_1 + \frac{M_h}{M} h_1 \frac{\widetilde{E}}{z} \right) \Bigg].
\end{aligned}
$$

SIDIS with $\pi\pi$ 产生

- Model



$$\Delta(z,R) = z^2 \sum\!\!\!\!\!\!\int_X \int \frac{d\xi^+}{2\pi} e^{ik\cdot\xi} \langle 0|U^+_{[0,\xi]}\psi(\xi)|P_h,R;X\rangle\langle X;P_h,R|\bar{\psi}(0)|0\rangle|_{\xi^-=\vec{\xi}_T=0}$$
$$= \frac{1}{16\pi}\left\{D_1 \not{n} + H_1^{\sphericalangle} \frac{i}{M_h[\not{R}_T, \not{n}_-]}\right\},$$

$$\tilde{\Delta}^\alpha_A(z,k_T,R) = \frac{1}{2z}\sum_X \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3} e^{ik\cdot\xi}\langle 0| \int_{\pm\infty^+}^{\xi^+} d\eta^+ \mathcal{U}^{\xi_T}_{(\infty^+,\xi^+)}$$
$$\times gF_\perp^{-\alpha}\mathcal{U}^{\xi_T}_{(\eta^+,\xi^+)}\psi(\xi)|P_h,R;X\rangle\langle P_h,R;X|\bar{\psi}(0)\mathcal{U}^{0_T}_{(0^+,\infty^+)}\mathcal{U}^{\infty^+}_{(0_T,\xi_T)}|0\rangle|_{\eta^+=\xi^+=0,\eta_T=\xi_T}.$$

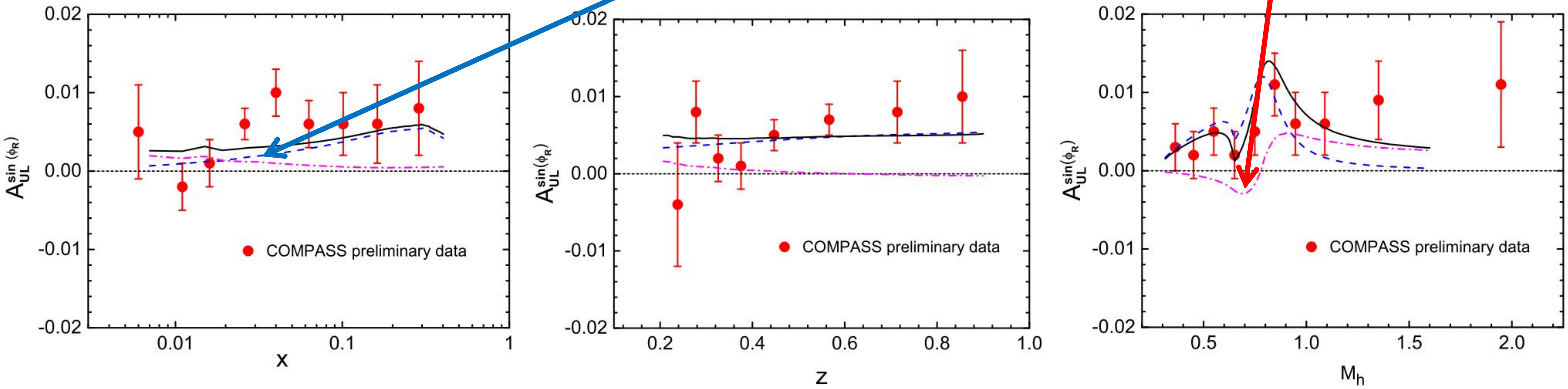
W. Yang, X. Wang and Z. Lu
Phys. Rev. D 99, 054003 (2019)

$$\tilde{\Delta}^\alpha_A(k,P_h,R) = i\frac{C_F\alpha_s}{2(2\pi)^2(1-z)P_h^-}\frac{1}{k^2-m^2}\int\frac{d^4l}{(2\pi)^4}(l^- g_T^{\alpha\mu} - l_T^\alpha g^{-\mu})$$
$$\times \frac{(\not{k}-\not{l}+m)(F^{s\star}e^{-\frac{k^2}{\Lambda_s^2}} + F^{p\star}e^{-\frac{k^2}{\Lambda_p^2}}\not{R})(\not{k}-\not{P}_h-\not{l}+m_s)\gamma_\mu(\not{k}-\not{P}_h+m_s)(F^s e^{-\frac{k^2}{\Lambda_s^2}} + F^p e^{-\frac{k^2}{\Lambda_p^2}}\not{R})(\not{k}+m)}{(-l^-\pm i\epsilon)((k-l)^2-m^2-i\epsilon)((k-P_h-l)^2-m_s^2-i\epsilon)(l^2-i\epsilon)},$$

SIDIS with $\pi\pi$ 产生的TSA $A_{UL}^{\sin\phi_R}$

- 物理可观测量

$$A_{UL}^{\sin\phi_R}(x,z,M_h^2) = -\frac{\sum_a e_a^2 \frac{|\vec{R}|}{Q}\left[\frac{|M|}{M_h} x h_L^a(x) H_{1,ot}^{\sphericalangle,a}(z,M_h^2) + \frac{1}{z} g_1(x)\tilde{G}_{ot}^{\sphericalangle}(z,M_h^2)\right]}{\sum_a e_a^2 f_1^a(x) D_{1,oo}^a(z,M_h^2)}.$$

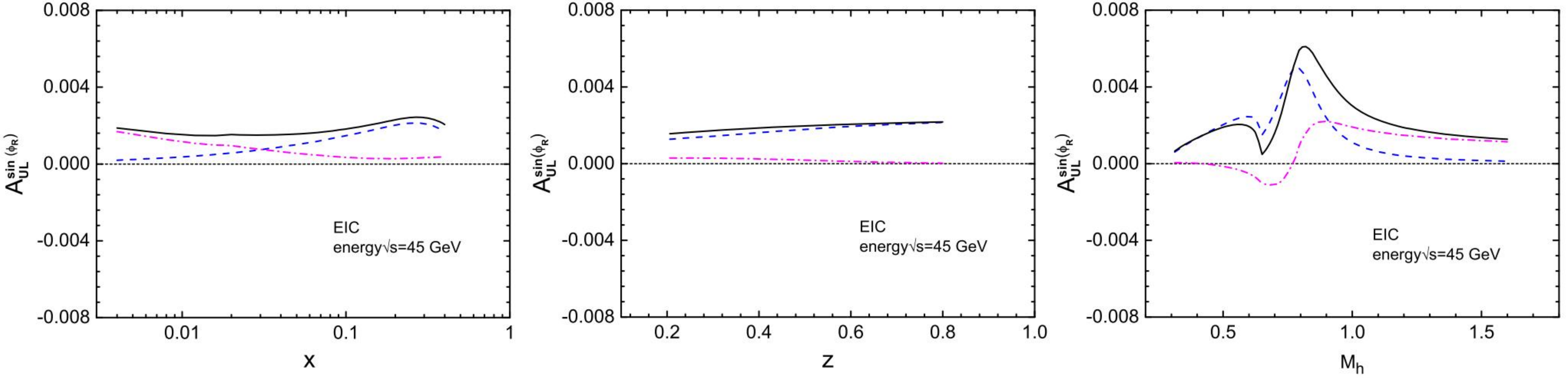


W. Yang, X. Wang and Z. Lu
Phys. Rev. D 99, 054003 (2019)

Phys.Lett.B 845 (2023) 138155、2512.22311

SIDIS with $\pi\pi$ 产生

- 物理可观测量



- 模型计算了DiFFs
- 给出了能够符合COMPASS实验数据的理论描述
- Twist-3 FF对靶自旋不对称度有较大的影响——需考虑超出WW近似项

SIDIS with $\pi\pi$ 产生的BSA $A_{LU}^{\sin\phi_R}$

- 双强子产生的SIDIS过程的微分散射截面

$$\frac{d^6\sigma_{UU}}{d\cos\theta\, dM_h^2\, d\phi_R\, dz\, dx\, dy} = \frac{\alpha^2}{Q^2 y}\left(1 - y + \frac{y^2}{2}\right)\sum_q e_q^2 f_1^q(x)\, D_1^q(z, M_h^2, \cos\theta),$$

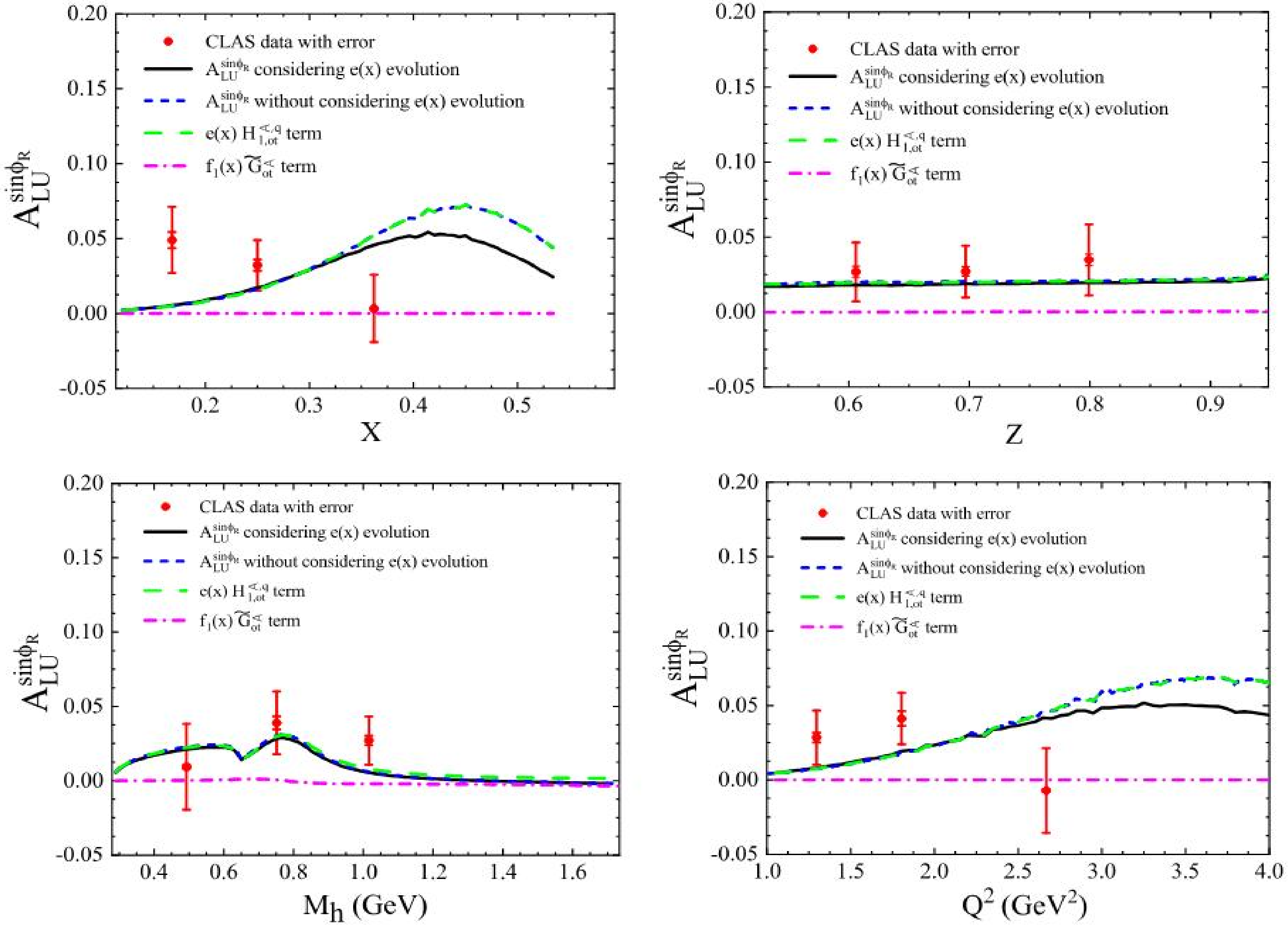
$$\frac{d^6\sigma_{LU}}{d\cos\theta\, dM_h^2\, d\phi_R\, dz\, dx\, dy} = -\frac{\alpha^2}{Q^2 y} 2y\sqrt{1-y}\sum_q e_q^2 \frac{M}{Q}\frac{|\boldsymbol{R}|}{M_h}\sin\phi_R \times\left[x\, e^q(x)\, H_1^{\sphericalangle,q}(z, M_h^2, \cos\theta) + \frac{M_h}{Mz} f_1^q(x)\, \widetilde{G}^{\sphericalangle,q}(z, M_h^2, \cos\theta)\right].$$

- $A_{LU}^{\sin\phi_R}$ 束流单自旋不对称度

$$A_{LU}^{\sin\phi_R}(x, z, M_h^2; Q, y) = -\frac{W(y)}{A(y)}\frac{M}{Q}\frac{|\boldsymbol{R}|}{M_h}\frac{\sum_q e_q^2\left[x e^q(x) H_{1,ot}^{\sphericalangle,q}(z, M_h^2) + \frac{M_h}{zM} f_1^q(x, Q^2)\widetilde{G}_{ot}^{\sphericalangle,q}(z, M_h^2)\right]}{\sum_q e_q^2 f_1^q(x, Q^2) D_{1,oo}^q(z, M_h^2)}.$$

SIDIS with $\pi\pi$ 产生的BSA $A_{LU}^{\sin\phi_R}$

CLAS运动学区域：

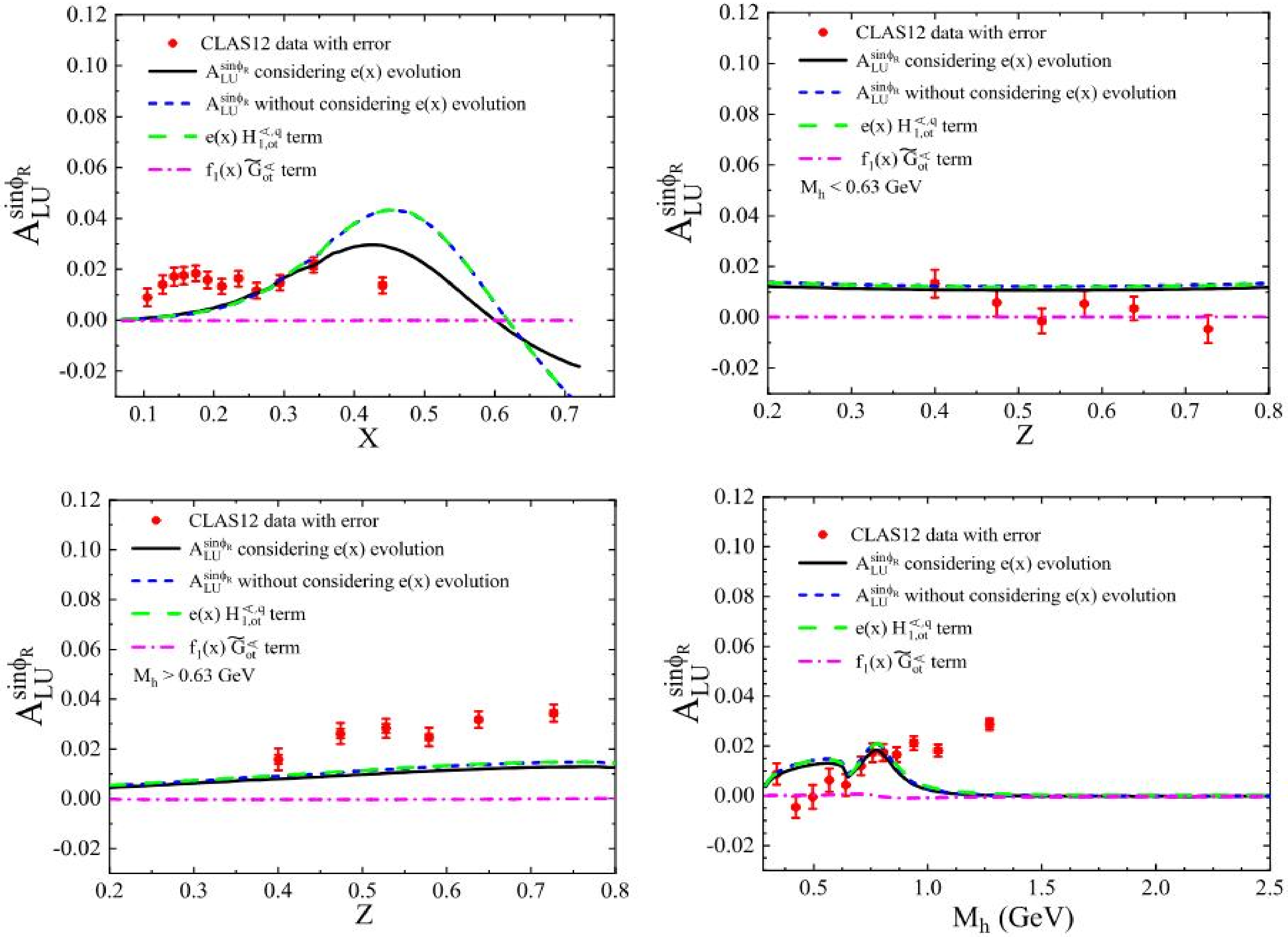


Y. Li, K. She, H. Li, X. Wang, D. M. Li and Z. Lu, Phys. Rev. D 112, 076015 (2025)

SIDIS with $\pi\pi$ 产生的BSA $A_{LU}^{\sin\phi_R}$

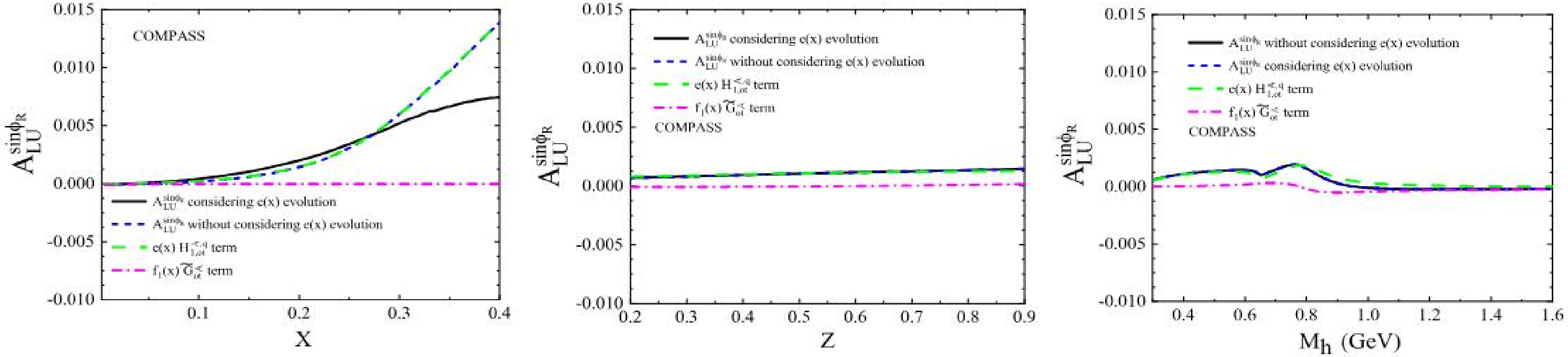
郑州大学

CLAS12运动学区域：

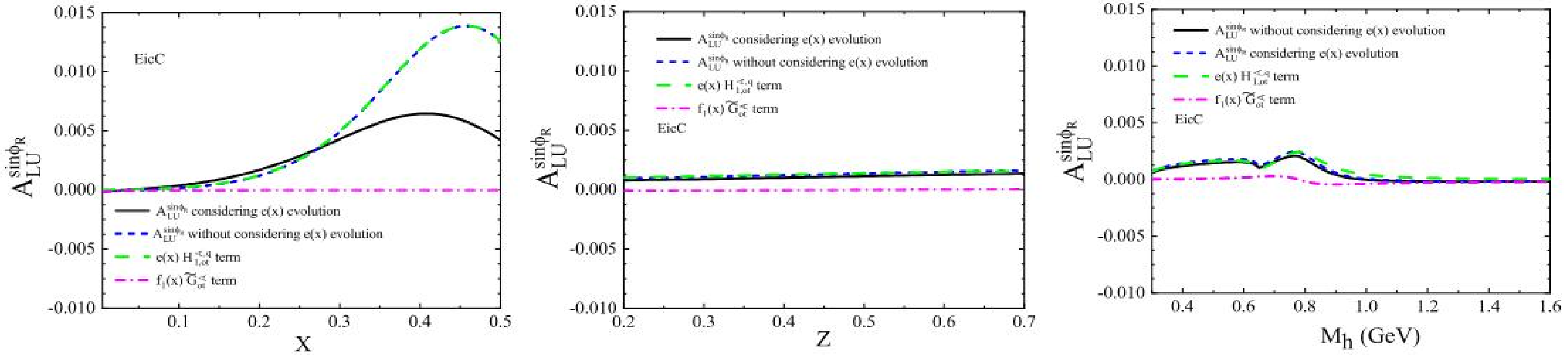


SIDIS with $\pi\pi$ 产生的BSA $A_{LU}^{\sin\phi_R}$

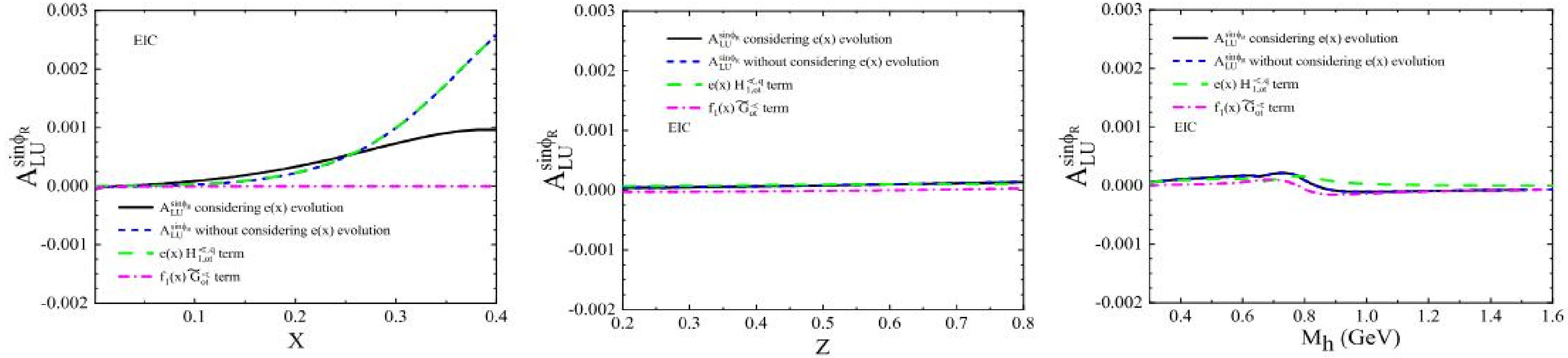
COMPASS运动学区域：



EicC运动学区域：

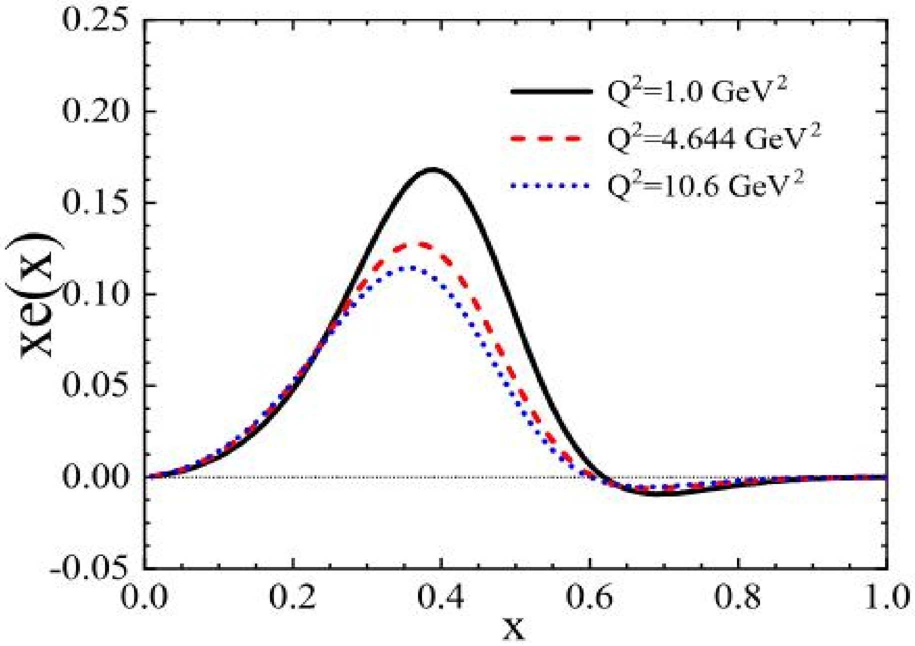


EIC运动学区域：



SIDIS with $\pi\pi$ 产生的BSA $A_{LU}^{\sin\phi_R}$

➢ $e(x)$在不同能标下随x的变化（DGLAP演化）



- 包含核子中夸克-胶子-夸克关联的重要信息。
- 有助于理解夸克和胶子对于核子质量的贡献。
- 手征性为奇的分布函数。
- 通过模型计算，如旁观者模型、袋模型等计算$e(x)$。
- 通过实验测量来参数化获取$e(x)$共线分布函数。

在小x区域，$e(x)$随着Q增大而增大；在大x区域，$e(x)$随着Q增大而减小。

SIDIS with $\pi\pi$ 产生的BSA $A_{LU}^{\sin\phi_R}$

➢ 总结

- 理论计算了不同运动学区域的束流单自旋不对称度。
- 对$e(x)$进行了DGLAP演化。$e(x)$的能标演化影响较大
- 计算了 $e(x)H_{1,ot}^{\sphericalangle,q}$ 耦合项和 $f_1\,\widetilde{G}_{ot}^{\sphericalangle}$ 耦合项对不对称度 $A_{LU}^{\sin\phi_R}$ 贡献。

主要贡献来源

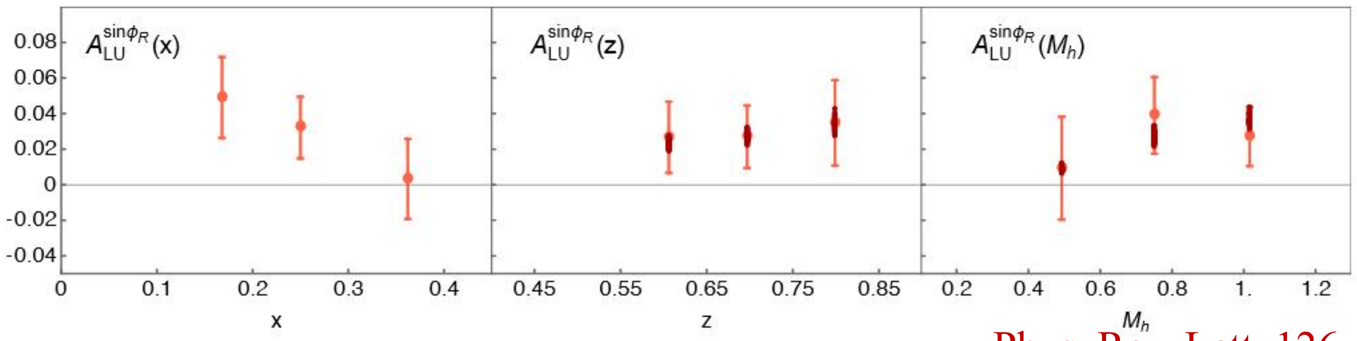
忽略不计

- 为提取$e(x)$提供理论依据，为揭示强子化机制和核子内部部分子相关信息的分布提供理论基础，有助于理解QCD非微扰性质。

SIDIS with $\pi\pi$ 产生的BSA $A_{LU}^{\sin\phi_R}$

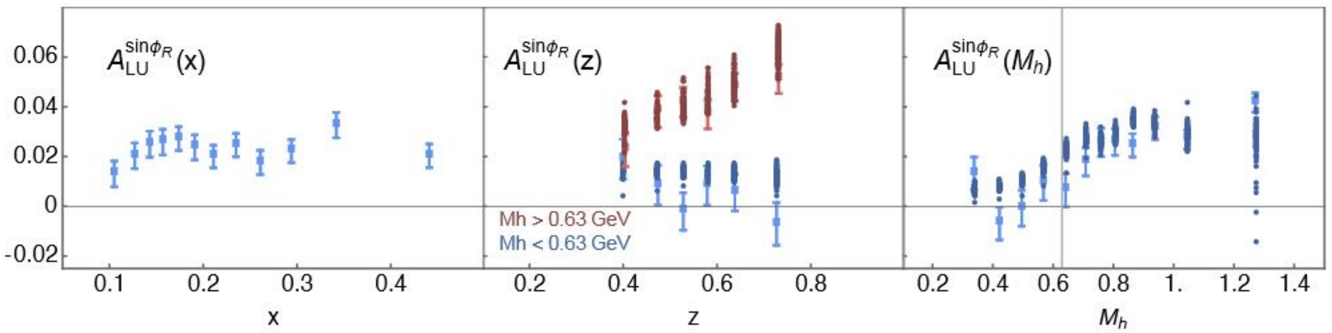
- $\boldsymbol{e(x)}$**提取(point-by-point) (from CLAS data)** ——Phys.Rev.D 106 (2022) 1, 014027

CLAS data:



——Phys. Rev. Lett. 126, 062002 (2021)

CLAS12 data:



——Phys. Rev. Lett. 126, 152501 (2021)

Outline

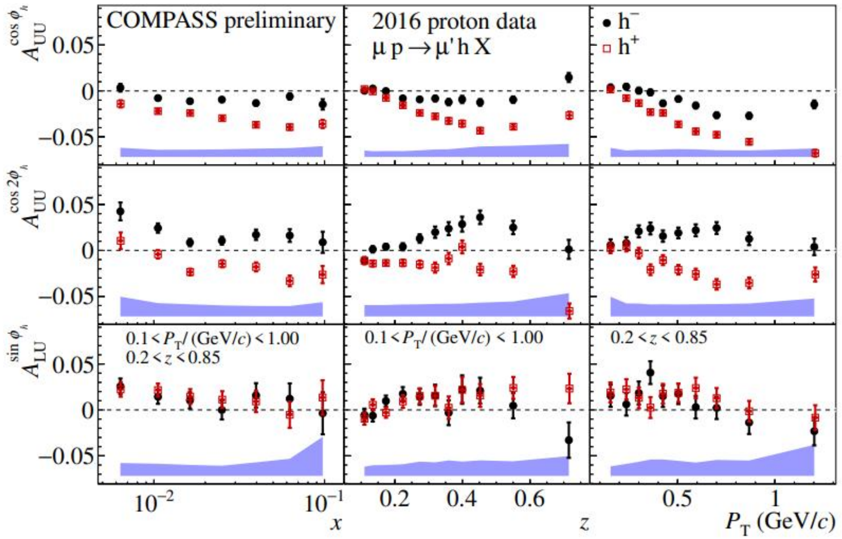
- SIDIS过程与核子结构
- 单强子产生的SIDIS过程中高扭度效应的研究
- 双强子产生的SIDIS过程中高扭度效应的研究
- 未来展望

What’s Next?

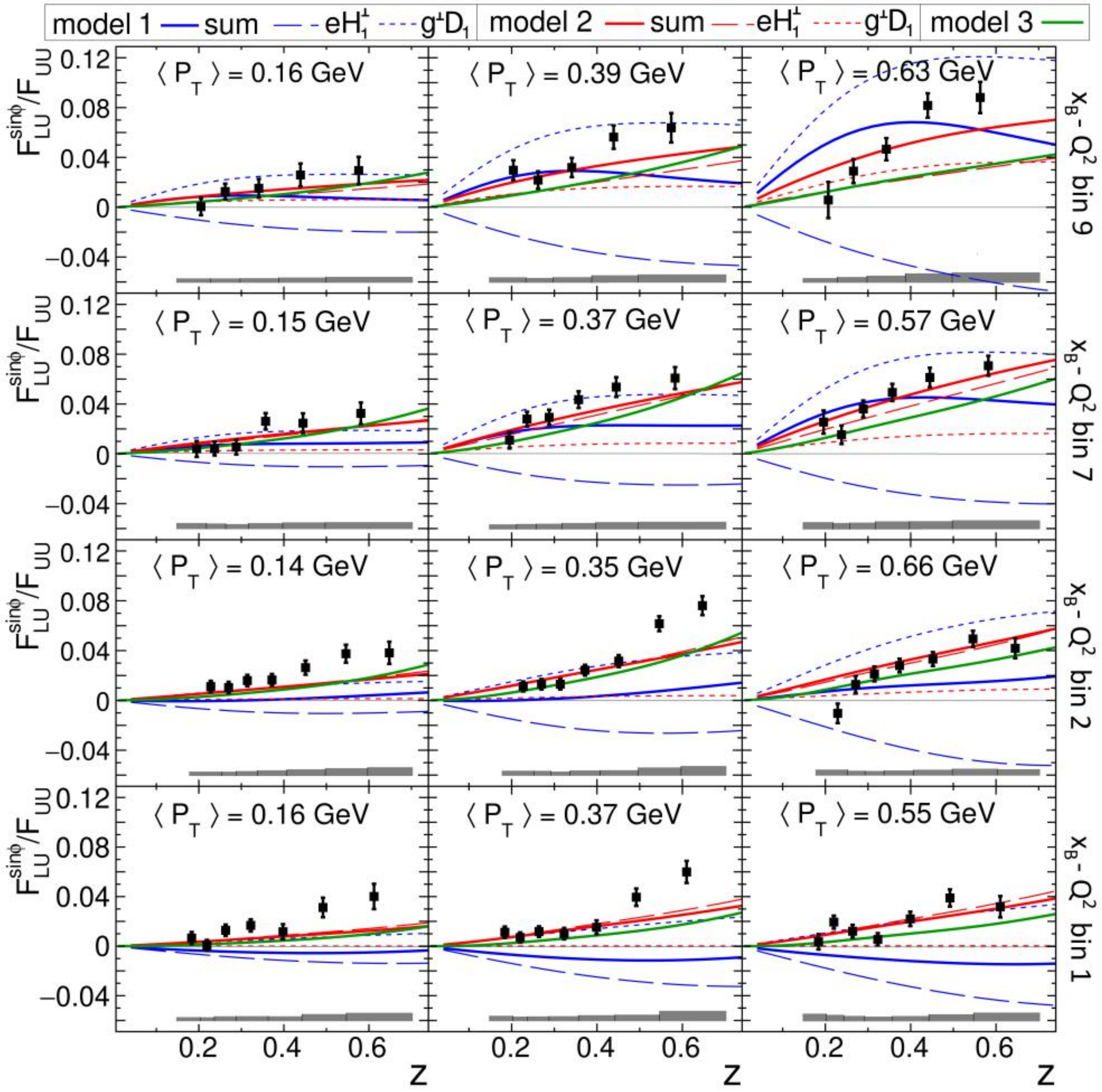
- 引入横动量

$$F_{LU}^{\sin\phi} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_T}{M_h}\left(x_B e H_1^{\perp} + \frac{M_h}{M} f_1 \frac{\tilde{G}^{\perp}}{z}\right) + \frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M}\left(x_B g^{\perp} D_1 + \frac{M_h}{M} h_1^{\perp}\frac{\tilde{E}}{z}\right)\right],$$

COMPASS, preliminary data PoS EPS-HEP2025 (2026) 259



CLAS, Phys.Rev.Lett. 128 (2022) 6, 062005

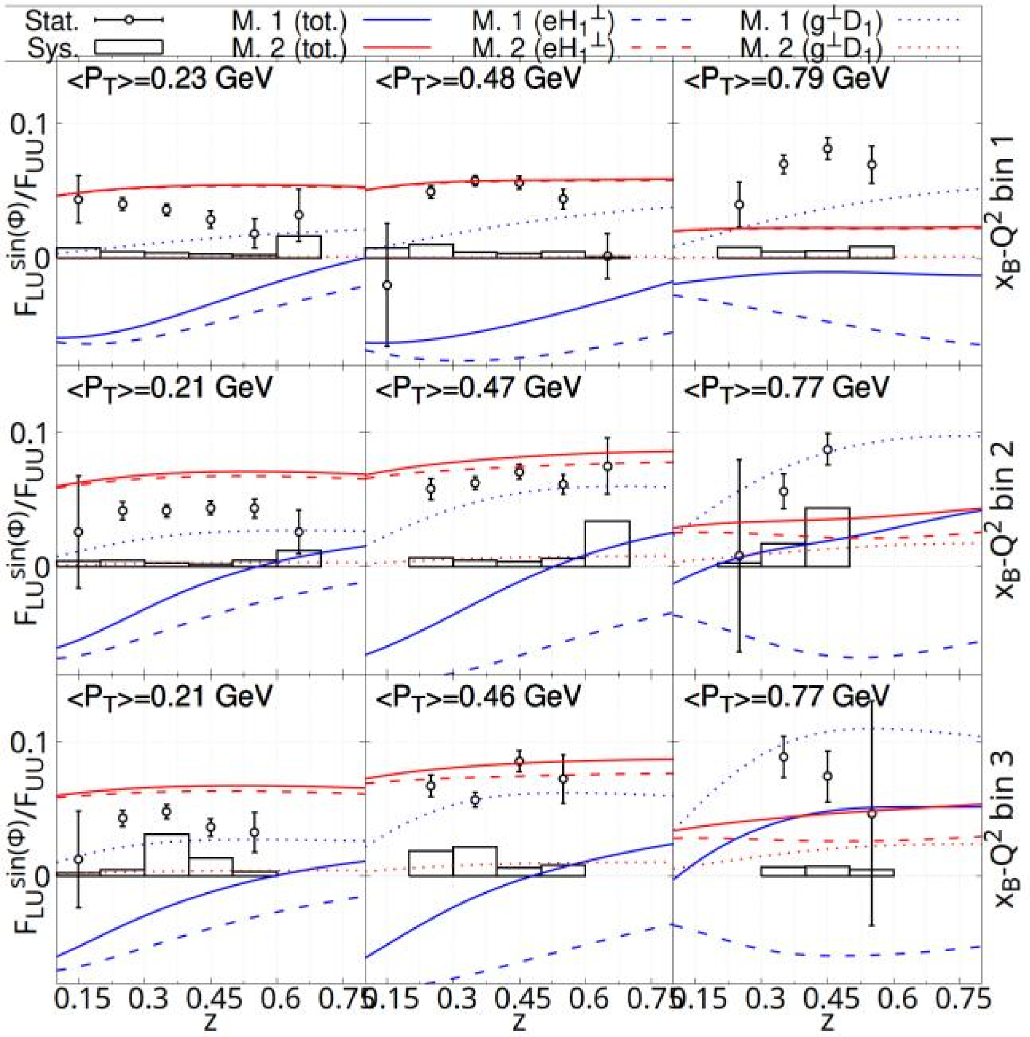


What’s Next?

- 更多的末态粒子
- 更多的维度

$$F_{LU}^{\sin\phi} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_T}{M_h}\left(x_B e H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M}\left(x_B g^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z}\right)\right],$$

CLAS, Phys. Rev. C 112, 055202 (2025)



What’s Next?

郑州大学

- 更高的扭度

PHYSICAL REVIEW D **104**, 094043 (2021)

Leading and higher twist transverse momentum dependent parton distribution functions in the spectator model

Xiao Liu,[1] Wenjuan Mao,[2,*] Xiaoyu Wang,[3,†] and Bo-Qiang Ma[1,4,5,‡]

[1]*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*
[2]*School of Physics and Telecommunication Engineering, Zhoukou Normal University, Zhoukou 466000, Henan, China*
[3]*School of Physics and Microelectronics, Zhengzhou University, Zhengzhou, Henan 450001, China*
[4]*Center for High Energy Physics, Peking University, Beijing 100871, China*
[5]*Collaborative Innovation Center of Quantum Matter, Beijing, China*

(Received 17 December 2020; revised 11 October 2021; accepted 25 October 2021; published 29 November 2021)

Transverse momentum dependent parton distribution functions, also abbreviated as TMDs, offer a three-dimensional picture of hadrons by taking the intrinsic transverse momentum of the parton into consideration. Hence, they are very important for us to understand the structure of hadrons. In this article, we calculate and summarize all TMDs of quark through the spectator model, from twist-2 to twist-4. Especially, we give complete results of TMDs at twist-4. We adopt a general analytical framework to calculate TMDs, with both scalar and axial-vector spectators being considered. All TMDs are calculated analytically in the light-cone coordinate, and single gluon rescattering is considered to generate T-odd TMDs. T-even TMDs are also calculated to this level, maybe for the first time. Different from the traditional point of view, the twist-4 TMDs can contribute to some physical observables like azimuthal asymmetries. An approximate formula of the Sivers asymmetry, including twist-4 TMDs, is given.

DOI: 10.1103/PhysRevD.104.094043

未来展望

- 唯象研究了单强子、双强子SIDIS过程中twist-3物理可观测量
  - 理论框架积分掉末态横向动量，引入末态横向动量后，twist-3效应的来源更复杂，主要贡献来源更难辨别，On the way……

- 为更进一步深入理解twist-3效应，可在初态更干净的BESIII和Belle II能区对双强子碎裂函数进行测量
  - 事实上，Belle II之前曾测量过$H_{1,ot}^{\sphericalangle}$相关的方位角依赖不对称度，2017年给出了第一次非极化双强子碎裂函数的测量，但仍需更多精度更高的数据……

- 理论层面，仍在努力向更高的twist进行扩展……