17 May, 2026

2026年轻强子专题研讨会

Shangqiu, Henan, China

Radiative corrections to inverse beta decay and neutron decay

托马拉克

Sasha Tomalak

PMNS oscillation matrix

neutrinos produced and detected in flavor basis

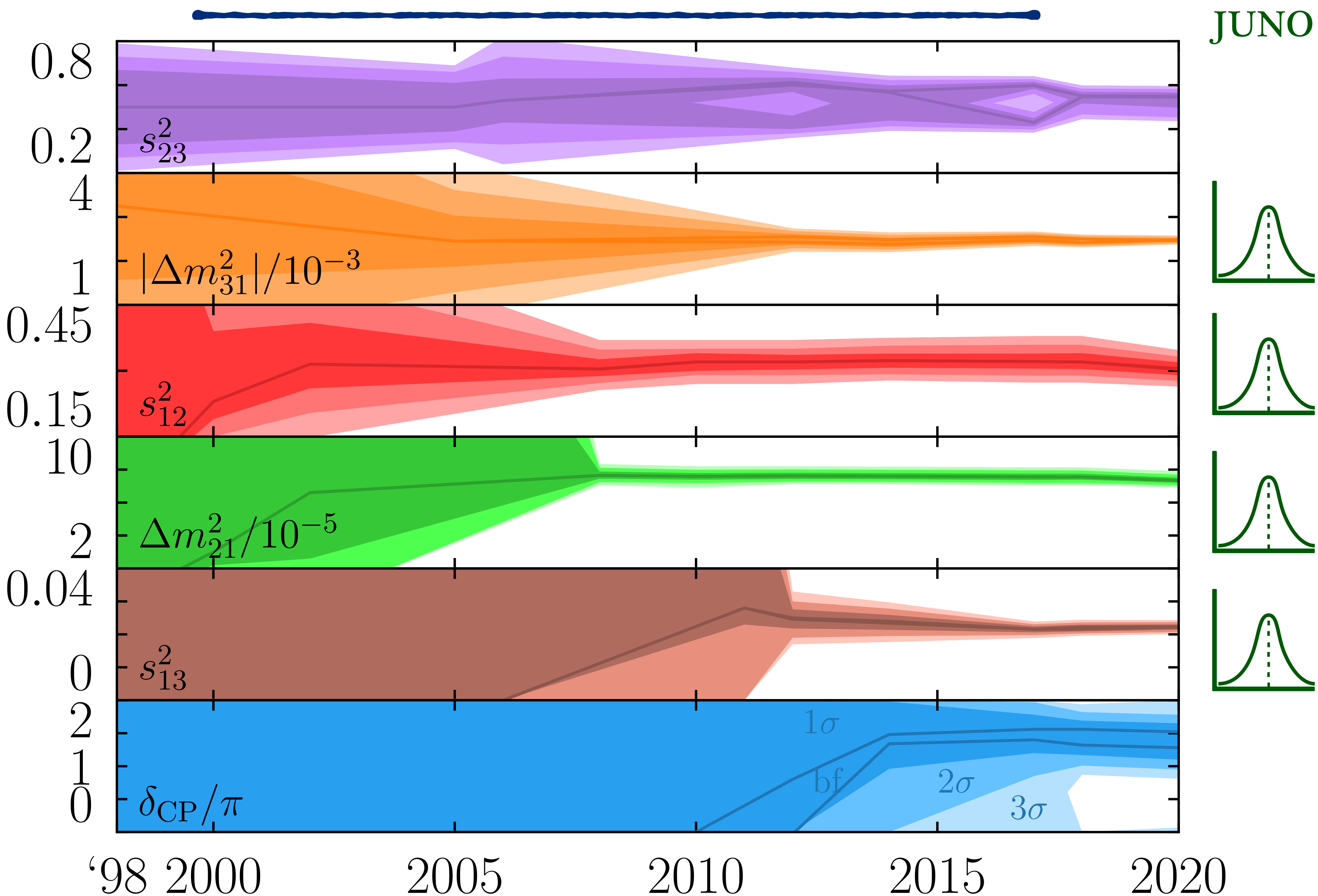
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

neutrinos propagate in mass basis

Pontecorvo-Maki-Nakagawa-Sakata matrix relates two bases

- oscillations are described by PMNS mixing matrix

PMNS oscillation matrix

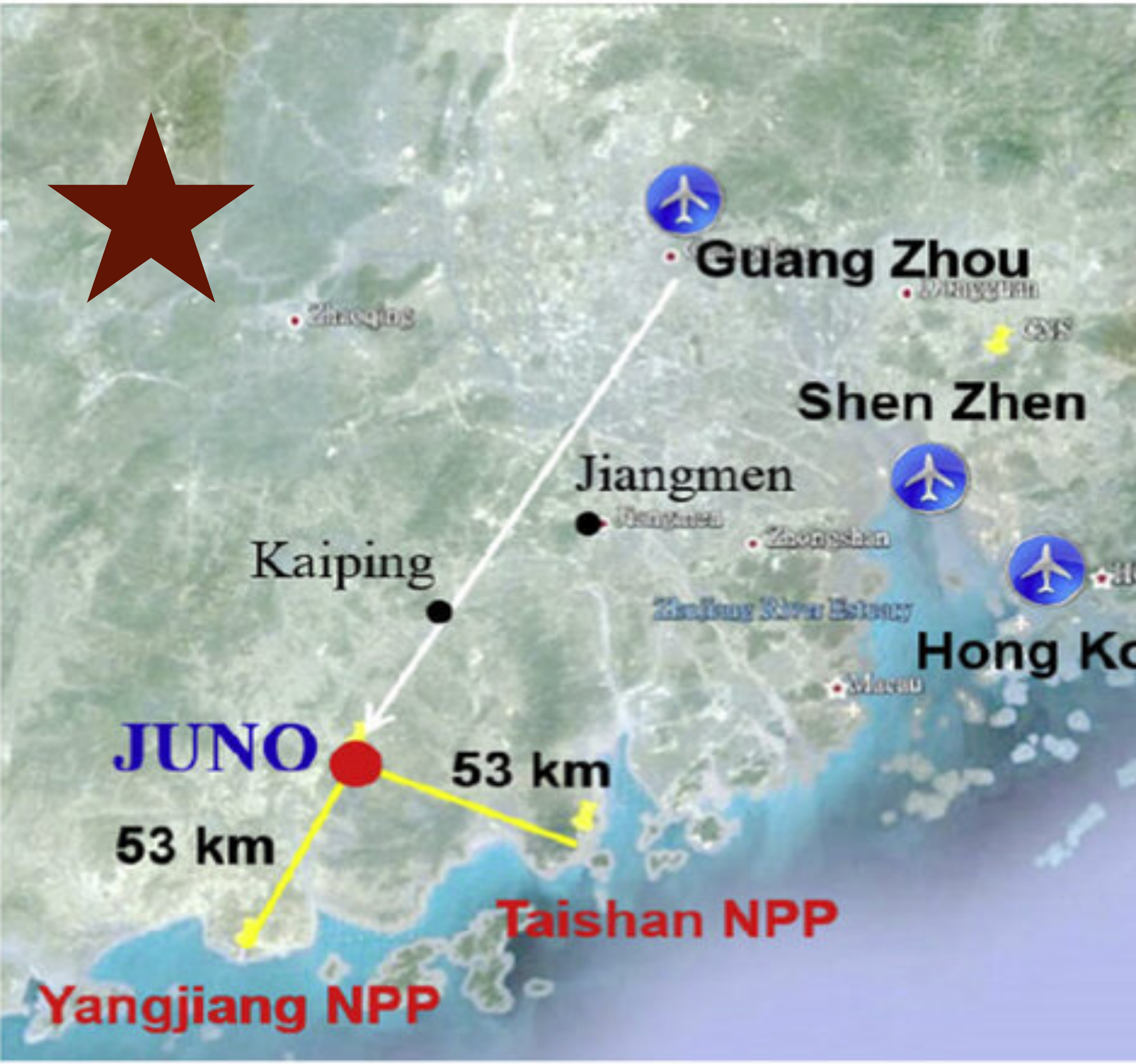


Snowmass 2021 NF01 group report

- tremendous progress in our lifetime

CP violation and mass hierarchy@laboratory

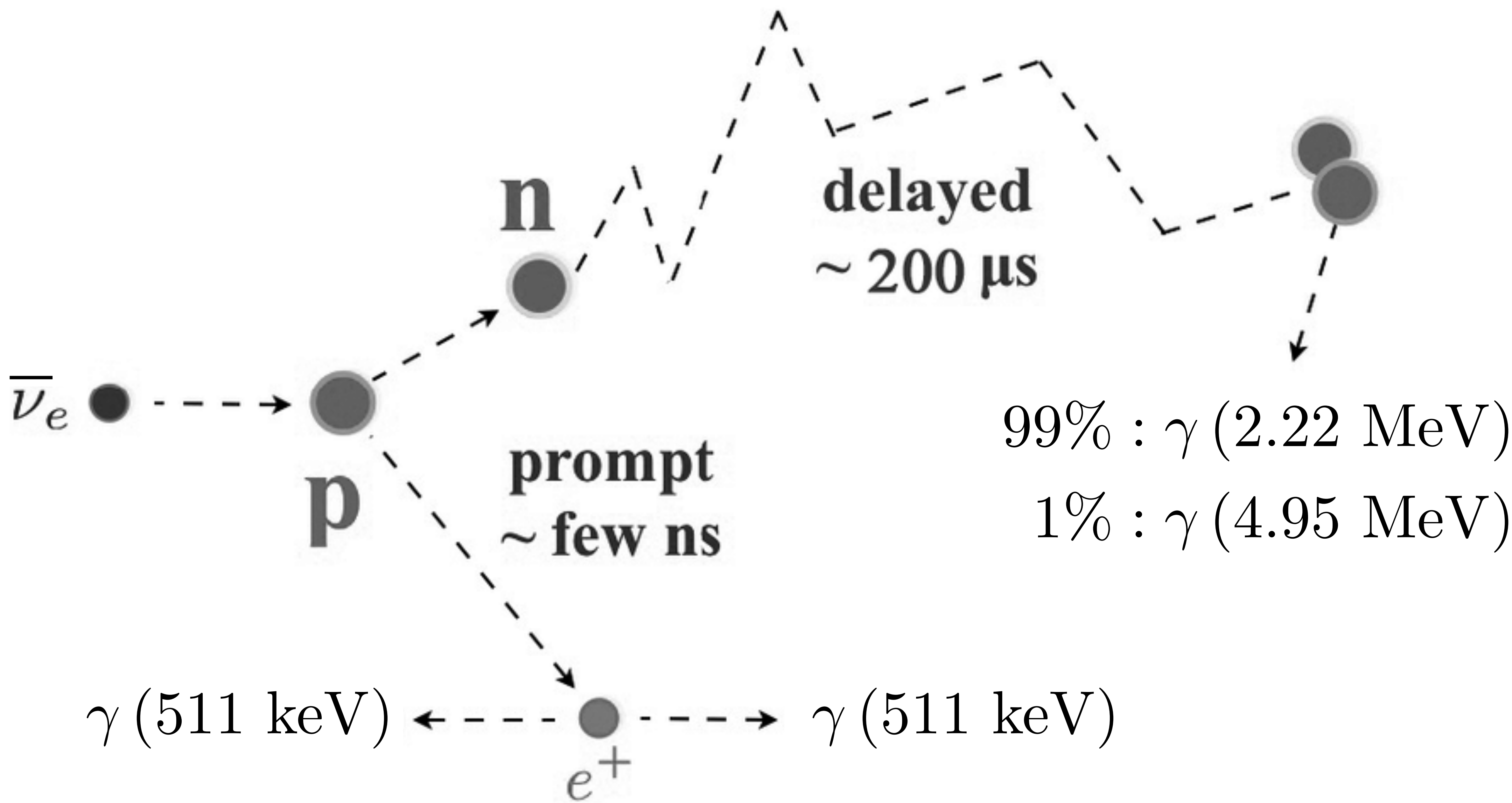
3σ determination of mass hierarchy



- 4 precise oscillation parameters from JUNO !!!

35 m, 20000 t

JUNO detection channel

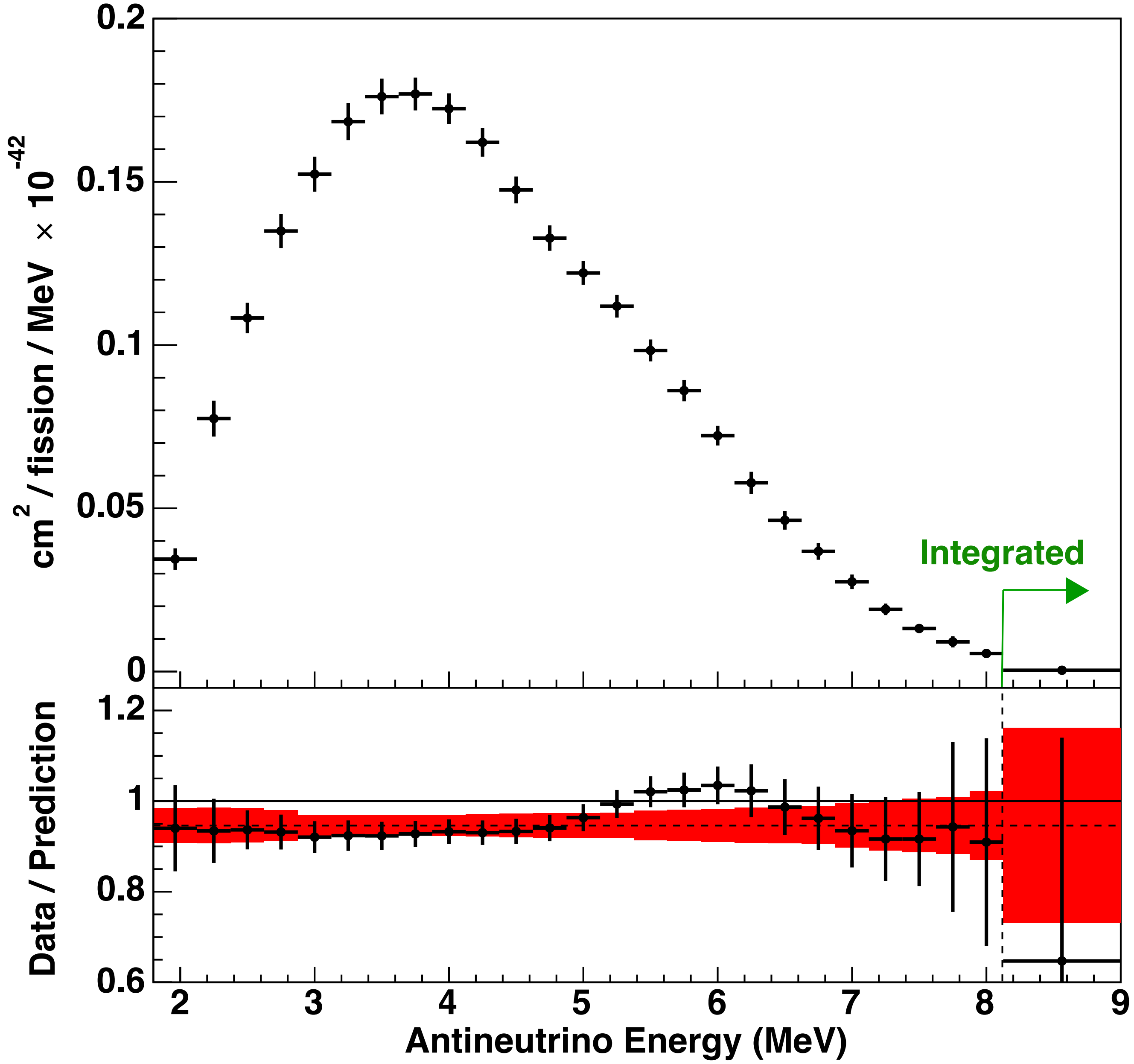


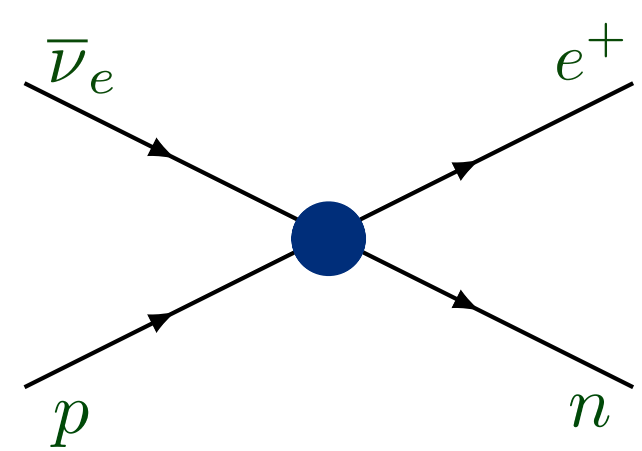
Teng Li et al., ChPC (2017)

- prompt-delayed time signature: background discrimination

IBD with reactor antineutrinos

- neutron is heavier than proton by 1.3 MeV



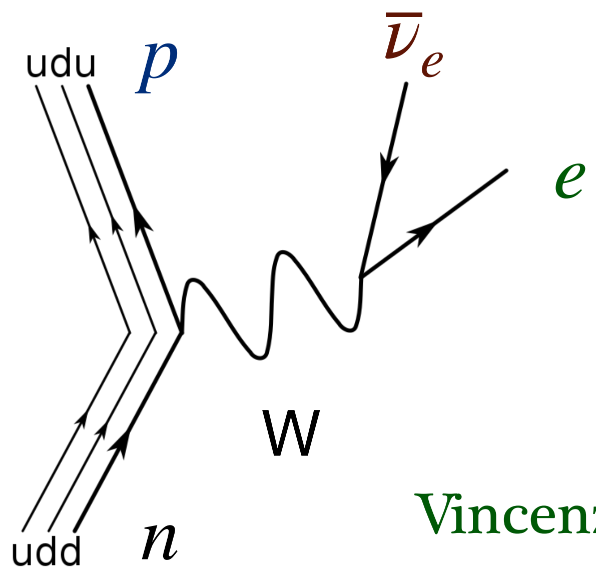


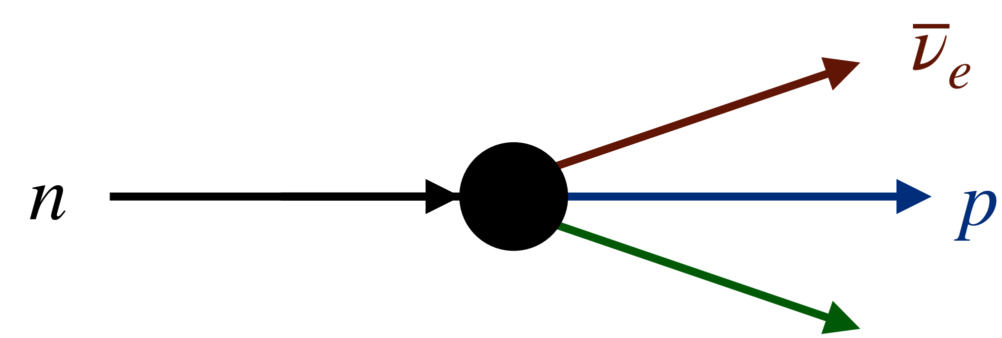
Daya Bay, PRL (2016)

- MeV-scale physics like in neutron decay

Complete EFT approach

- neutron is heavier than proton by 1.3 MeV and can decay





O.T., Few-Body Syst (2023)

Vincenzo Cirigliano, Wouter Dekens, Emanuele Mereghetti, and O.T., PRD (2023) and PRD (2025)

- four-fermion interaction between leptons and heavy nucleons

$$\mathcal{L}_{\rm eff} = -\sqrt{2} \mathrm{G_F} V_{ud} \bar{e} \gamma_\mu \mathrm{P_L} \nu_e \cdot \overline{N} \left(g_V v^\mu - 2 g_A S^\mu \right) \tau^+ N$$

Standard Model -> LEFT -> HBχPT -> $\pi\!\!\!/$EFT

- systematic approach: determine low-energy coupling constants
- first consistent at 0.1%-level QED radiative corrections

Low-energy description

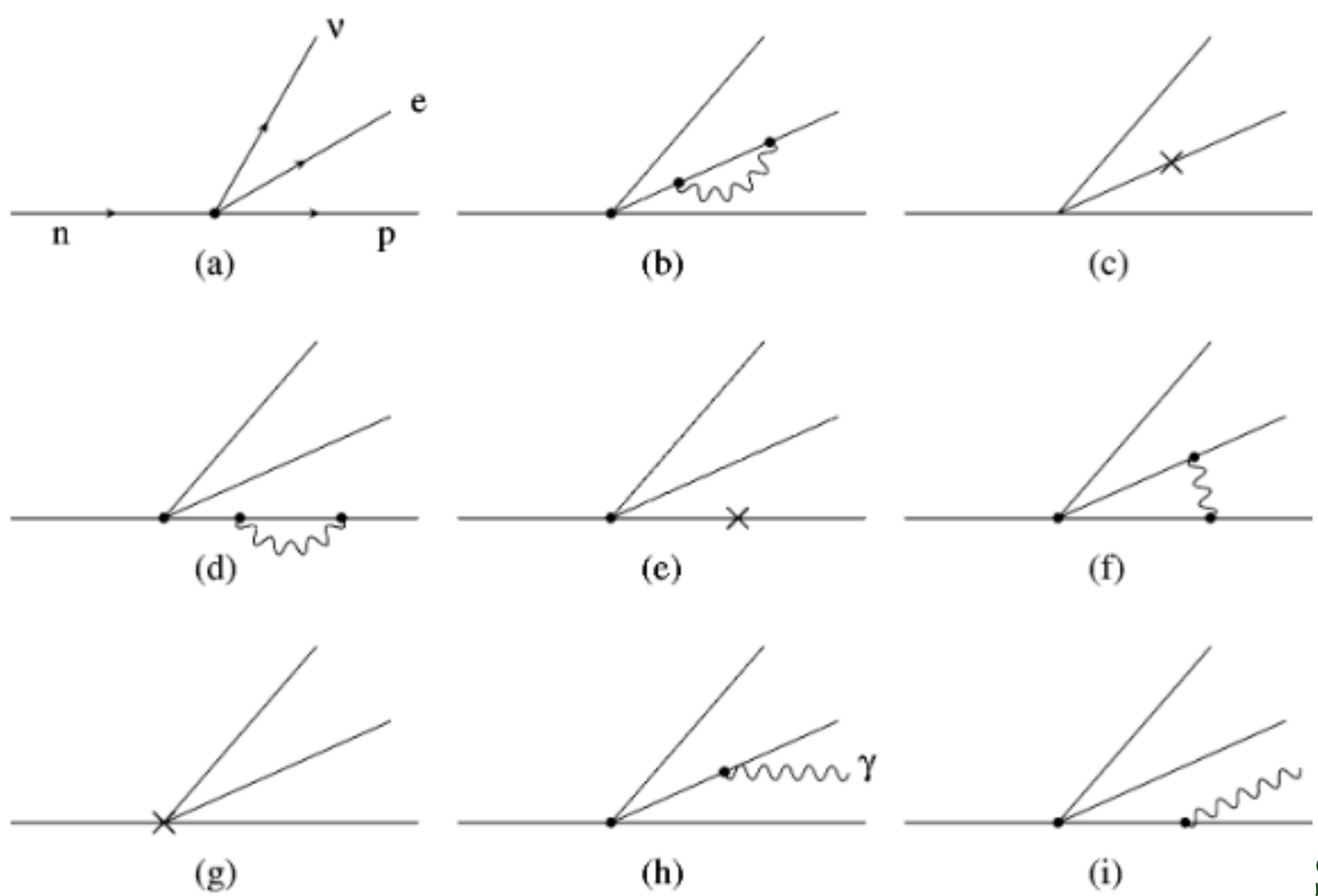
- four-fermion interaction between leptons and heavy nucleons

$$\mathcal{L}_{\text{eff}} = -\sqrt{2} \mathrm{G_F} V_{ud} \bar{e} \gamma_\mu \mathrm{P_L} \nu_e \cdot \overline{N} \left(g_V v^\mu - 2 g_A S^\mu \right) \tau^+ N + \mathrm{O}\left(\frac{m_e}{M_p}, \alpha, \alpha \frac{m_\pi}{M_p}, \alpha \frac{m_e}{m_\pi} \right)$$

$$m_e \sim M_p - M_n$$

A. Sirlin, Phys. Rev. (1967)

- radiative corrections formulated in modern EFT language



vector and axial-vector counterterms (diagrams c, e, g)

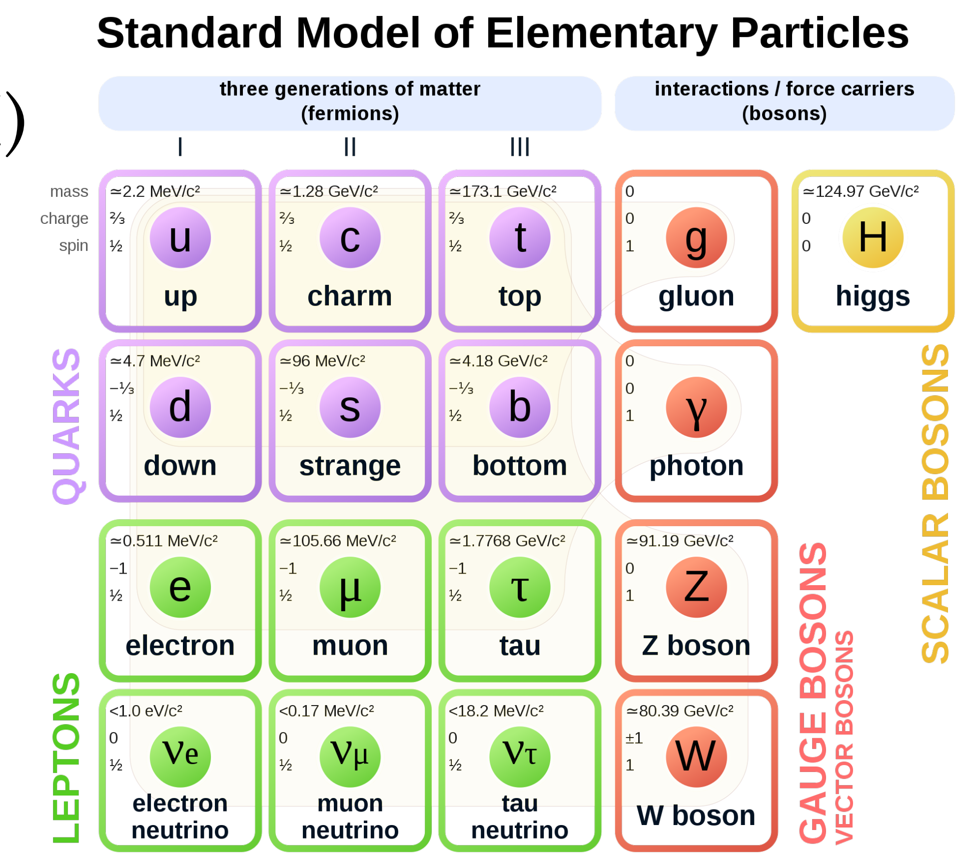
data → counterterms ← Standard Model

S. Ando et al., PLB (2004)

- two coupling constants predict all observables

Effective field theory for IBD and β decay

M_Z full content of Standard Model (SM)



integrate out top, Z, W, h

neutrino: R. J Hill, O.T., PLB (2020)

LEFT: W. Dekens, P. Stoffer, JHEP (2019)

m_b

integrate out GeV particles

m_τ

m_c α_s becomes too strong going to lower energies

matching to ChPT

Vincenzo Cirigliano, Wouter Dekens, Emanuele Mereghetti, and O.T., PRD (2023) and PRD (2025)

m_π dynamical pions

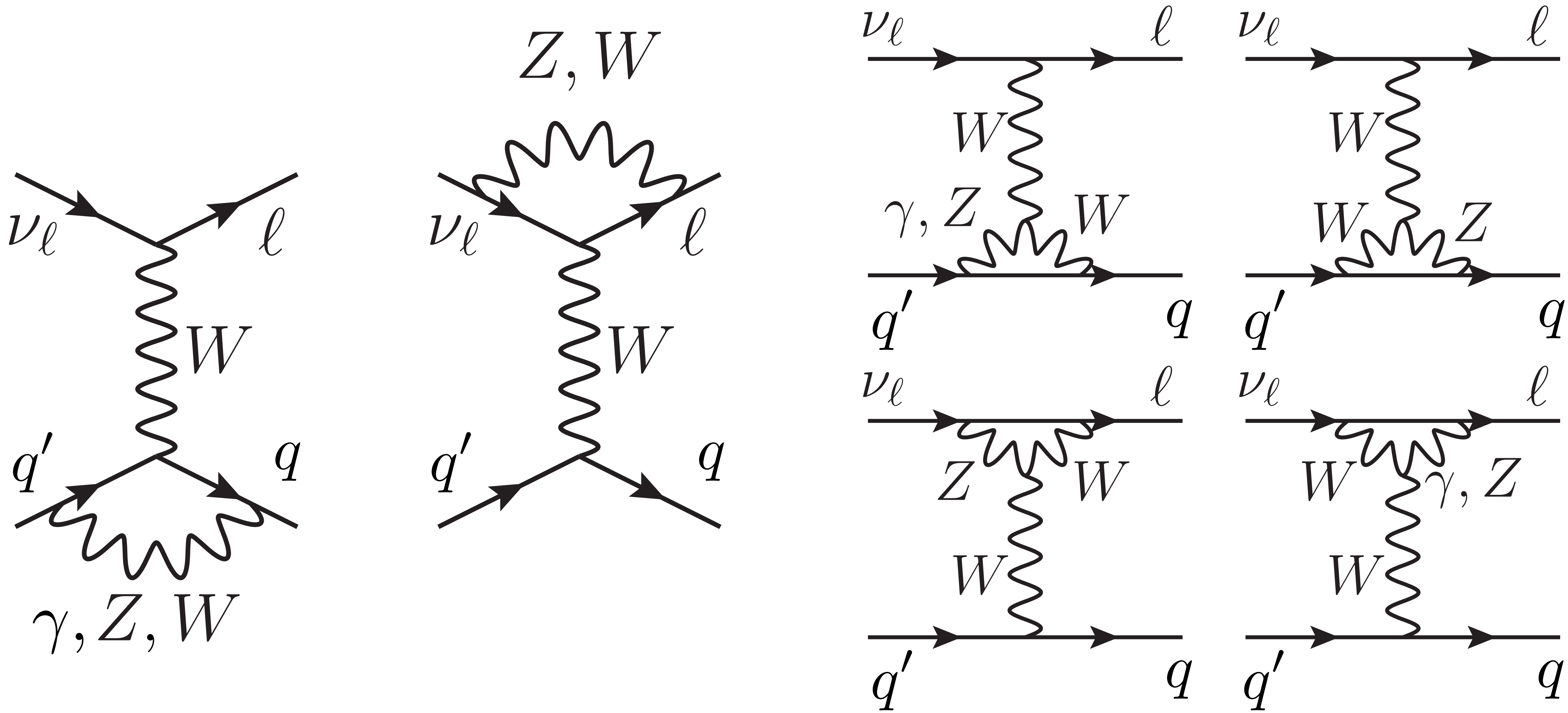
V. Cirigliano, J. de Vries, L. Hayen, E. Mereghetti, and A. Walker-Loud, PRL (2022)

photons, neutrinos, electrons, external nucleons

m_e S. Ando et al., PLB (2004)

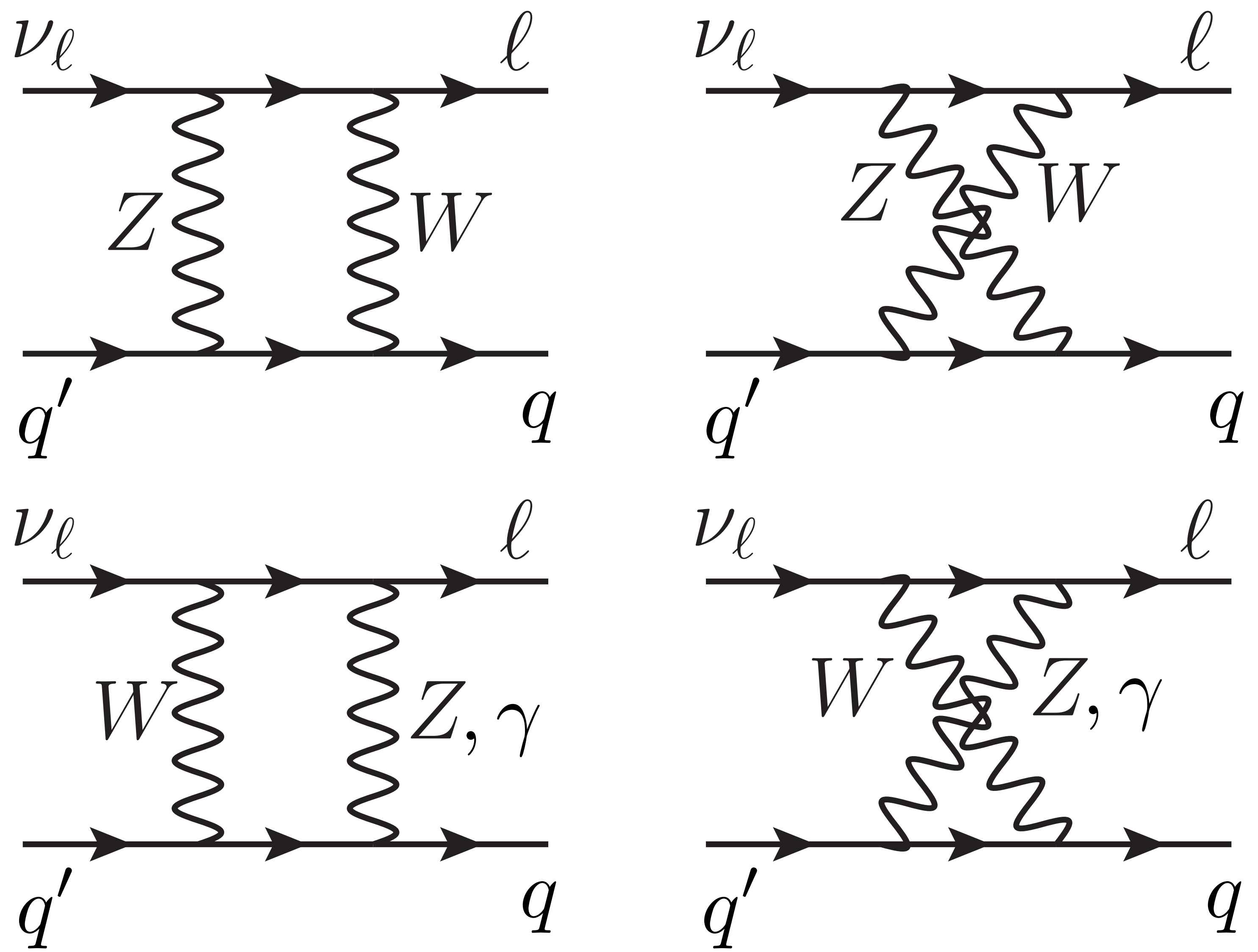
- coupling constants from SM + small QED corrections

Charged current in SM. Vertexes



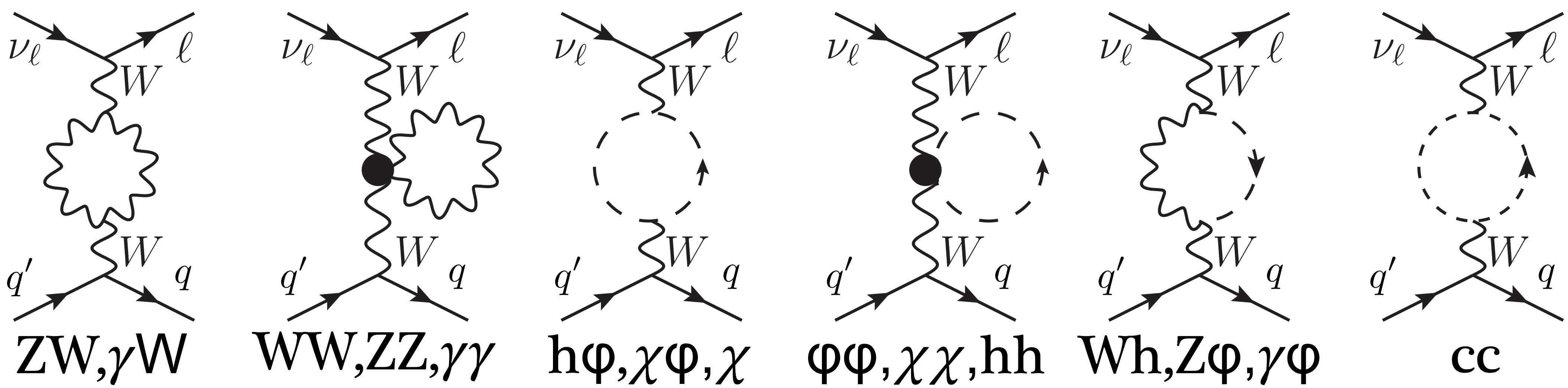
- contribution to effective couplings

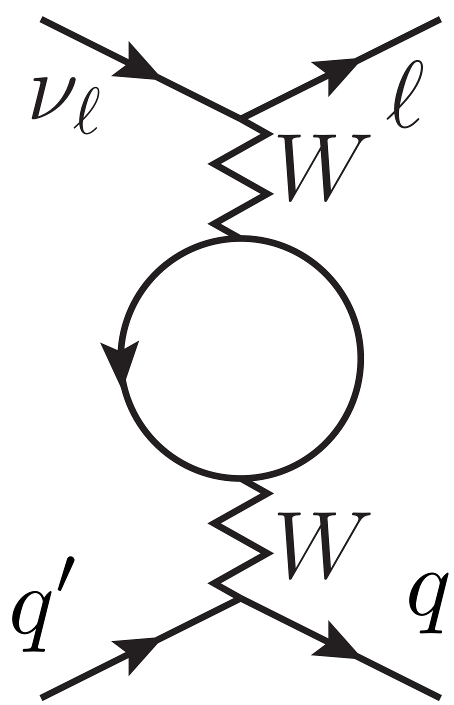
Charged current in SM. Boxes



- contribution to effective couplings

W self energy

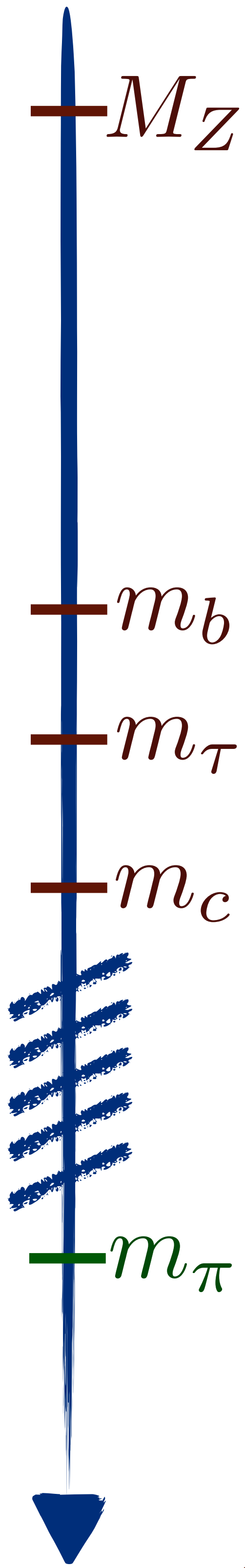




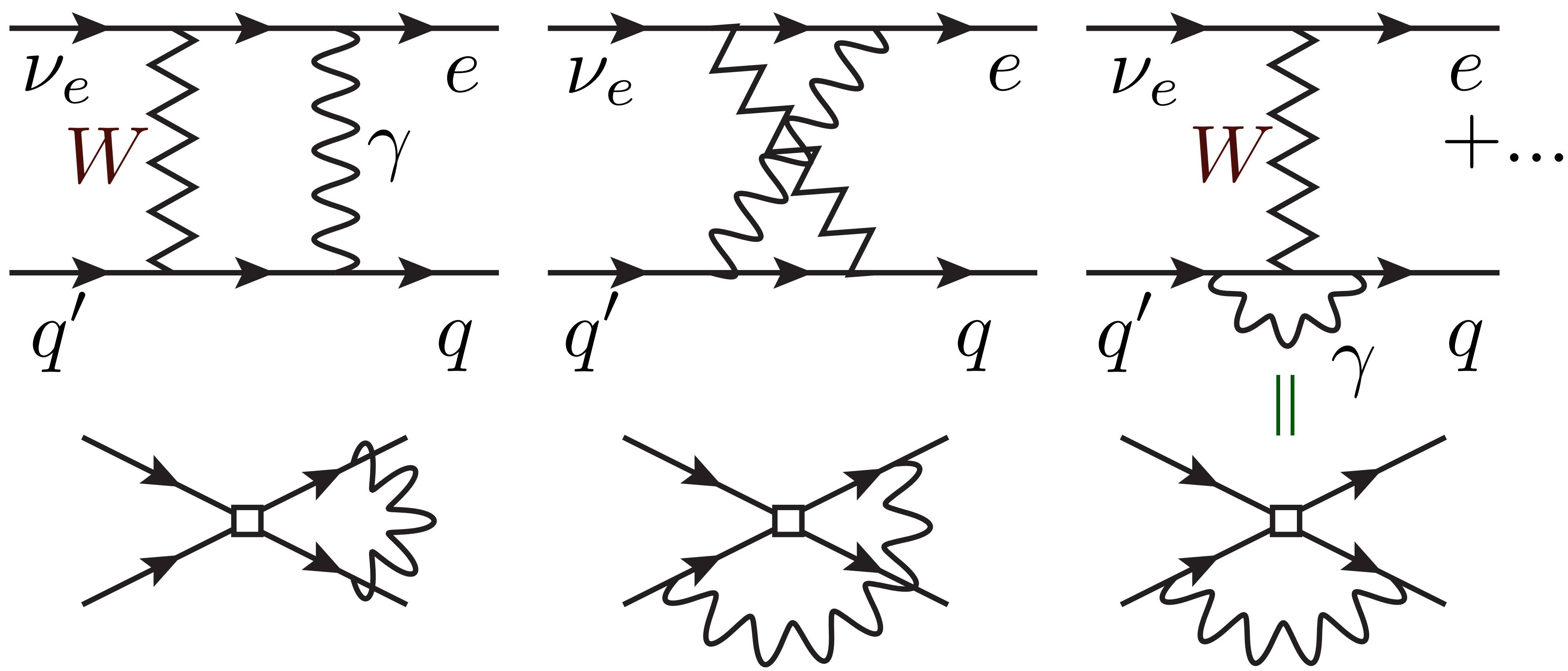
- vanishing contribution to matching besides loops with t quark

- contribution to effective couplings

LEFT Lagrangian



$$\mathcal{L}_{\mathrm{LEFT}} \sim G_F V_{ud} C_\beta^r(a,\mu) \bar{e}_L \gamma_\rho \nu_{eL} \, \bar{u}_L \gamma^\rho d_L$$



scheme independent

scheme dependent for quarks

- NDR scheme for γ_5 for evanescent operators E

$$\gamma^\alpha \gamma^\beta \gamma^\mu \mathrm{P_L} \otimes \gamma_\mu \gamma_\beta \gamma_\alpha \mathrm{P_L} = 4\left(1 + a\left(4-d\right)\right) \gamma^\mu \mathrm{P_L} \otimes \gamma_\mu \mathrm{P_L} + \mathrm{E}\left(a\right)$$

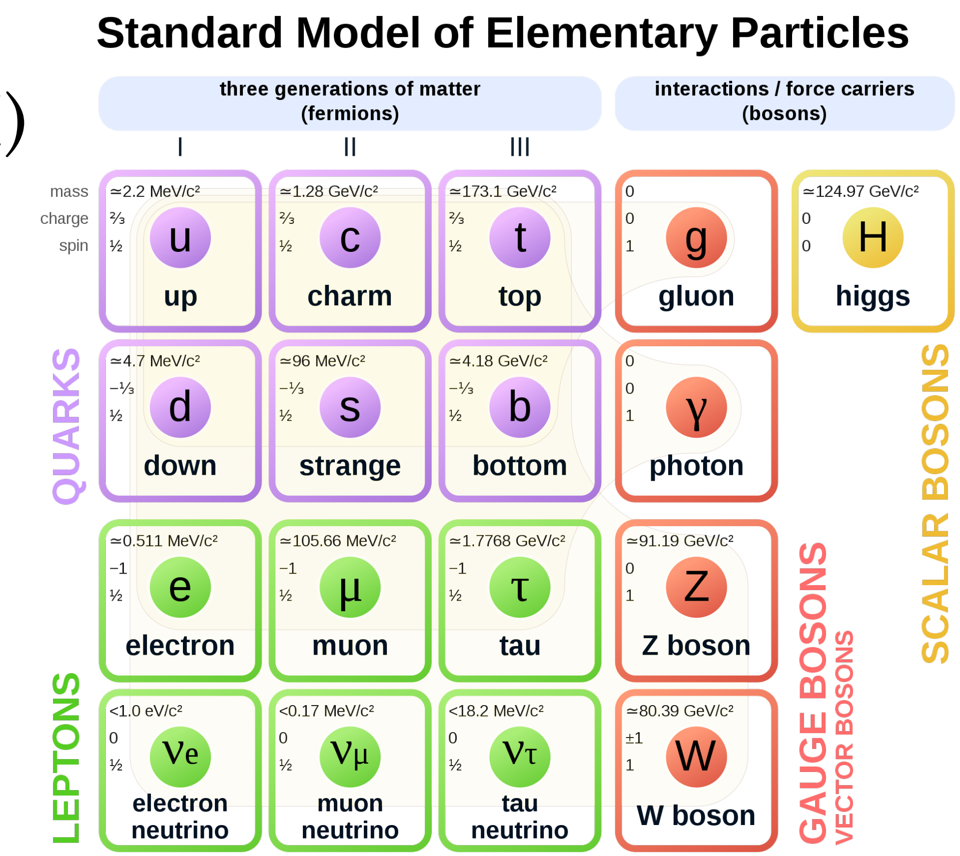
Buras and Weisz, NPB (1990)

- scheme dependence of 1-loop matching and 2-loop running

Effective field theory for IBD and β decay

M_Z full content of Standard Model (SM)

integrate out top, Z, W, h



m_b

m_τ integrate out GeV particles

m_c α_s becomes too strong going to lower energies

matching to ChPT hadron physics →

m_π dynamical pions

m_e photons, neutrinos, electrons, external nucleons

- coupling constants from SM + small QED corrections

HBχPT to $\not\pi$EFT matching

- purely leptonic counterterm

$$\mathcal{L}_{\rm lept}^{\rm CT} = e^2 X_6 \bar{e} \left(i\partial_\mu + e A_\mu \right) \gamma^\mu e$$

- $\not\pi$EFT counterterms from χPT couplings in baryon sector

$$g_V = C_\beta^r \left(1 + e^2 \left[-\frac{X_6}{2} + 2 \left(V_1 + V_2 + V_3 + V_4 \right) - g_9 \right] \right)$$

EW + electromagnetic

- LECs in terms of correlation functions

One-loop result

- clear illustration of leading logarithms

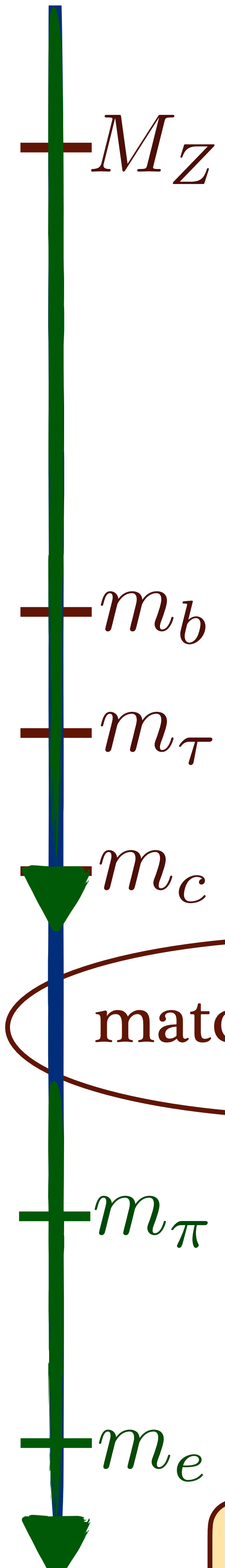
$$g_{\rm V}\left(\mu_\chi\right) = 1 - \frac{\alpha}{2\pi}\left[\frac{5}{8} + \frac{3}{4}\ln\frac{\mu_\chi^2}{\mu_0^2} + \ln\frac{\mu_0^2}{M_Z^2}\right]$$
$$- e^2 \int \frac{i{\rm d}^4 q}{(2\pi)^4}\frac{\nu^2+Q^2}{Q^4}\left[\frac{{\rm T}_3(\nu,Q^2)}{2\nu} - \frac{2}{3}\frac{1}{Q^2+\mu_0^2}\right]$$

- same Regge and resonance inputs as in traditional approach

- updates in elastic; DIS is part of running in LEFT

- agreement with current algebra evaluation

Effect of running



$$\mathcal{L}_{\rm LEFT} \sim G_F V_{ud} C_\beta^r(a,\mu)\bar{e}_L\gamma_\rho\nu_{eL}\,\bar{u}_L\gamma^\rho d_L$$

- renormalization group equations

$$\mu\frac{\mathrm{d}C_\beta^r(a,\mu)}{\mathrm{d}\mu} = \left(\gamma_0\,\frac{\alpha}{\pi} + \gamma_1\,\left(\frac{\alpha}{\pi}\right)^2 + \gamma_{se}\,\frac{\alpha}{\pi}\,\frac{\alpha_s}{4\pi}\right)\;C_\beta^r(a,\mu)$$

$$\mu\frac{\mathrm{d}g_V(\mu)}{\mathrm{d}\mu} = \left(\tilde{\gamma}_0\,\frac{\alpha}{\pi} + \tilde{\gamma}_1\,\left(\frac{\alpha}{\pi}\right)^2\right)\;g_V(\mu)$$

- resummed result with leading $\alpha\alpha_s$ corrections

$$g_V(\mu_\chi = m_e) - 1 = (2.499 \pm 0.013)\;\%$$

full error

- one-loop result without $\alpha\alpha_s$ contributions

$$g_V^{1-\rm loop}(\mu_\chi = m_e) - 1 = (2.430 \pm 0.012)\,\%$$

hadronic error

- cancellation of various contributions -> 0.07 % effect

V_{ud} from neutron decay

- λ from electron spin correlation measurements

$$|V_{ud}|^2 \, \tau_n \, \left(1 + 3\lambda^2\right) \left(1 + \Delta_{\rm TOT}\right) = 5283.321(5) \text{ s}$$

$$\lambda = \frac{g_A}{g_V}$$

$$\Delta_{\rm TOT}^{\rm EFT} = 7.761(27)\% \qquad \text{vs} \qquad \Delta_{\rm TOT} = 7.735(27)\%$$

- extraction with **PDG** lifetime and λ

$$V_{ud}^{\rm n,\ PDG} = 0.97430(2)_{\Delta_f}(13)_{\Delta_R}(82)_{\lambda}(28)_{\tau_n}[88]_{\rm total}$$

- extraction with **UCNτ** lifetime and **PERKEO-III** λ

$$V_{ud}^{\rm n,\ best} = 0.97402(2)_{\Delta_f}(13)_{\Delta_R}(35)_{\lambda}(20)_{\tau_n}[42]_{\rm total}$$

- in agreement with superallowed β decays, similar error

$$|V_{ud}^{0^+ \to 0^+}| = 0.97373(31)$$

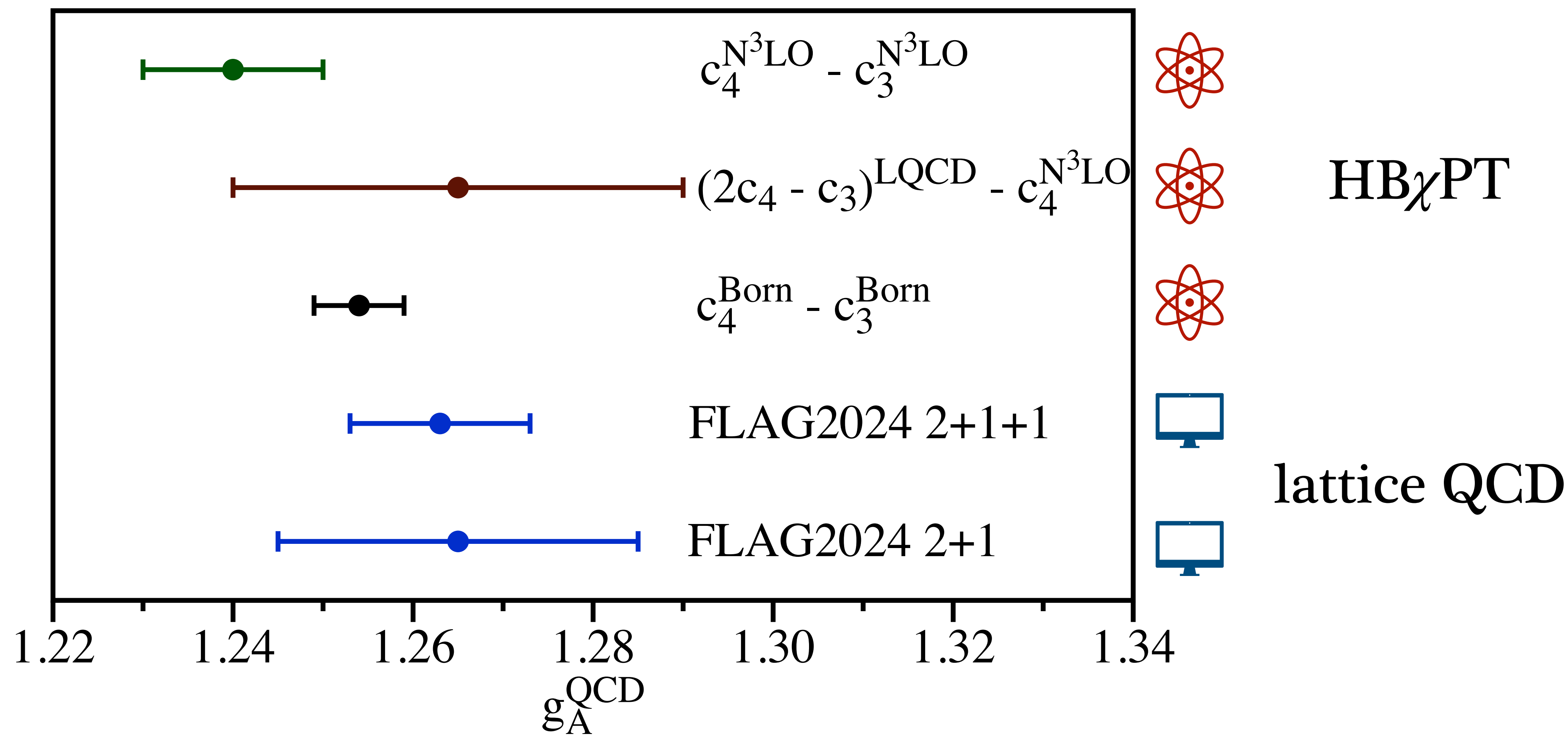
Hardy and Towner, PRC (2020)

- clean neutron lifetime extraction vs superallowed β decays

Nucleon axial-vector charge

- effective field theory approach to low-energy charged currents

Standard Model -> LEFT -> HBχPT -> π̸EFT

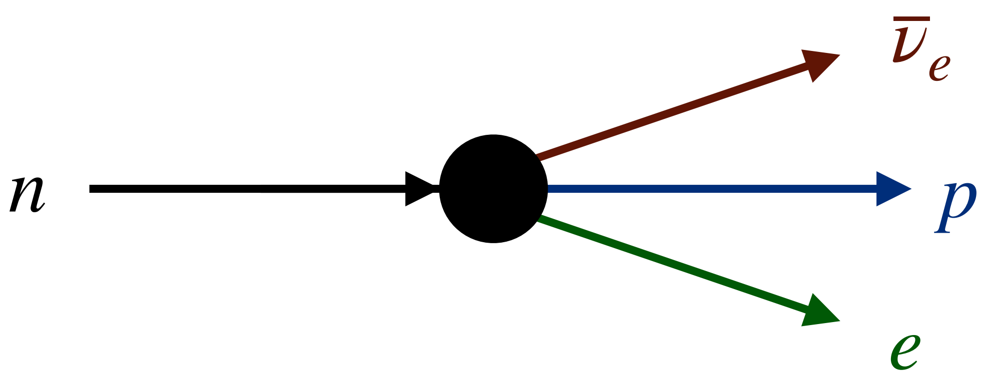


O.T. and Yi-Bo Yang, Universe (2026)
Vincenzo Cirigliano, Wouter Dekens, Emanuele Mereghetti, and O.T., PRD (2024) and PRD (2025)
V. Cirigliano, J. de Vries, L. Hayen, E. Mereghetti, and A. Walker-Loud, PRL (2022)

- first consistent at 0.1%-level QED radiative corrections

Complete EFT approach

- neutron measurements for inverse beta decay



- four-fermion interaction between leptons and heavy nucleons

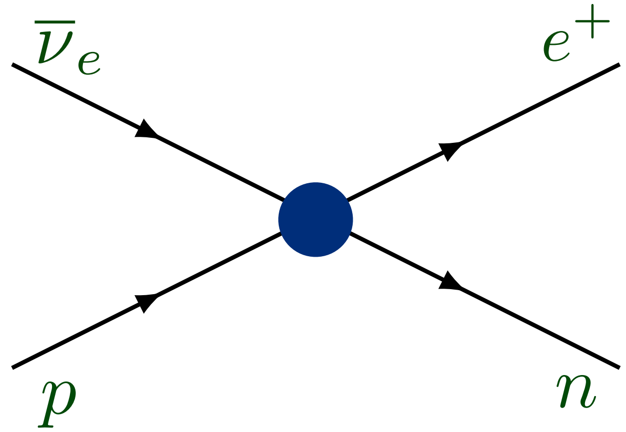
$$\mathcal{L}_{\mathrm{eff}} = -\sqrt{2}\mathrm{G_F} V_{ud} \bar{e}\gamma_\mu \mathrm{P_L} \nu_e \cdot \overline{N}\left(g_V v^\mu - 2 g_A S^\mu\right)\tau^+ N$$

g_V : theoretical prediction from Standard Model

g_A : beta asymmetry in polarized neutron decay

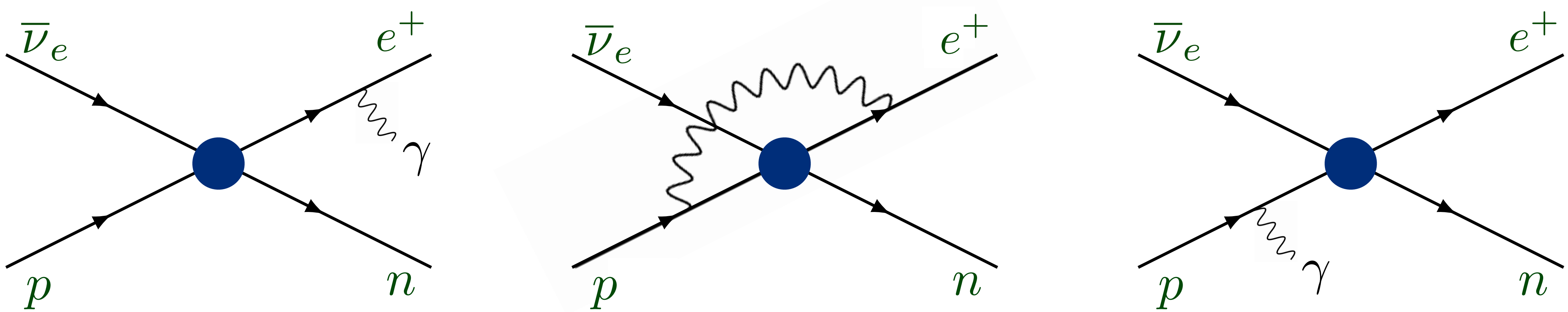
V_{ud}: superallowed transitions

- first consistent at 0.1%-level QED radiative corrections

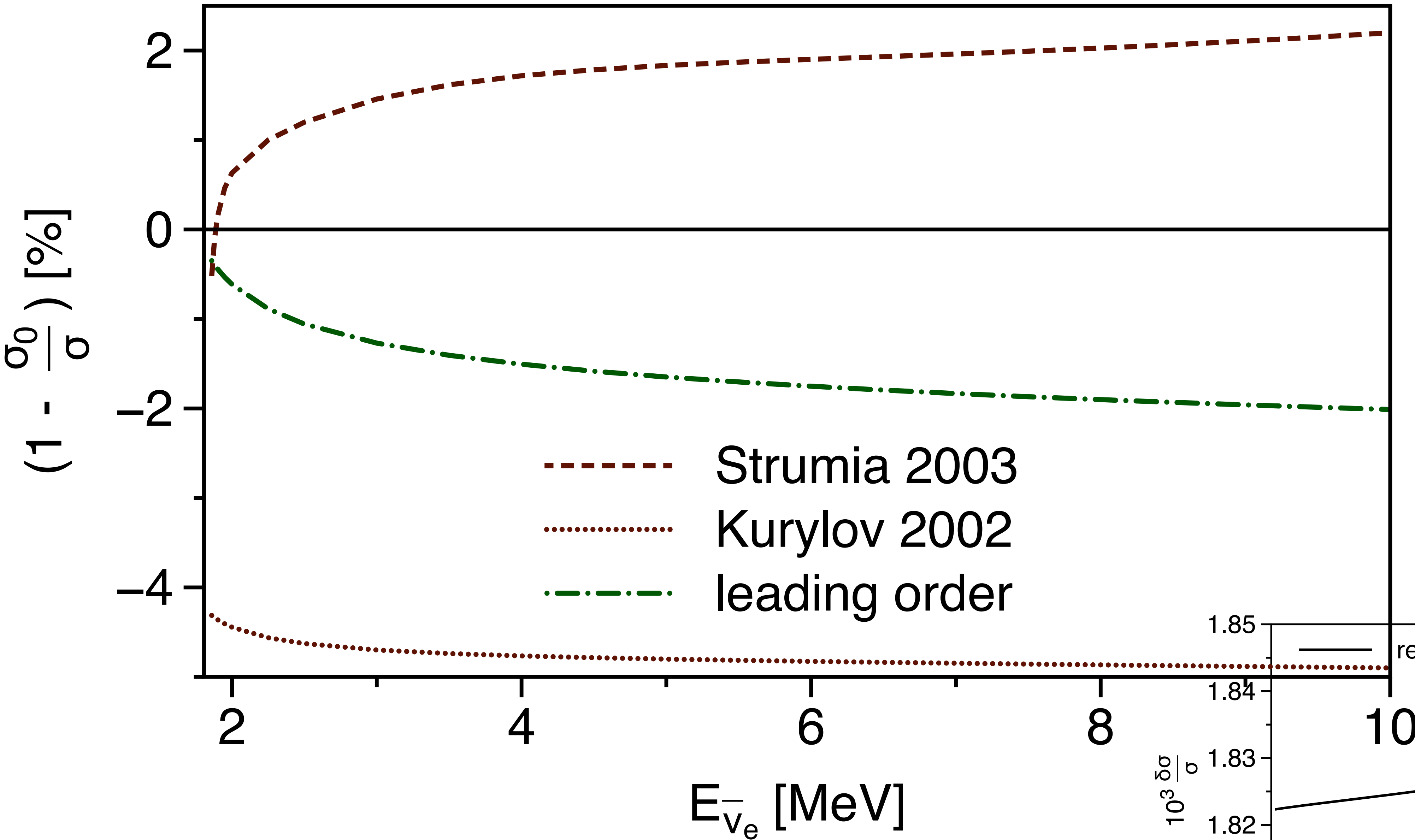


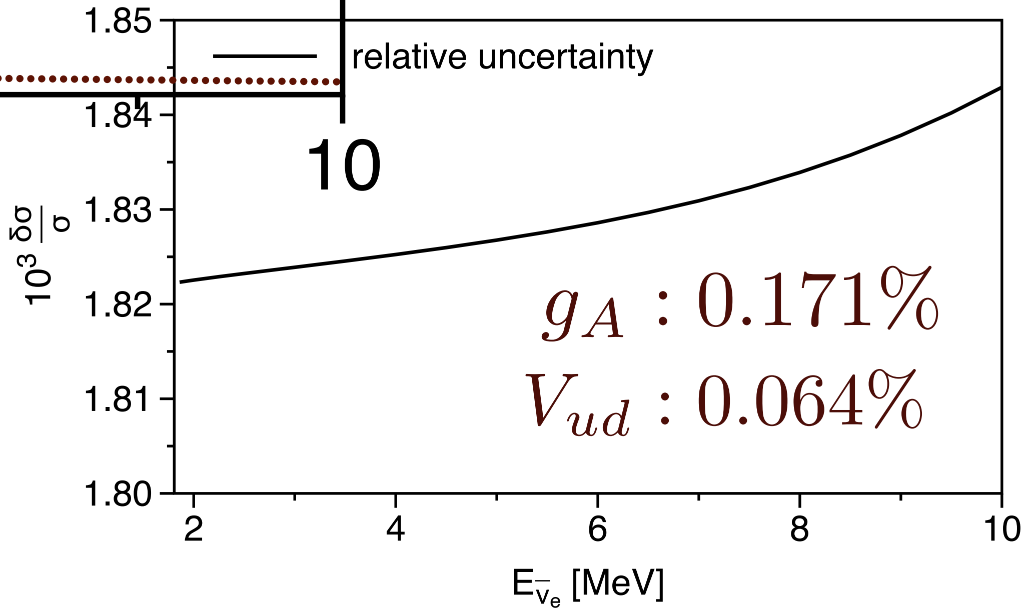
O.T., arXiv:2512.07956
arXiv:2512.07957

Theory of inverse beta decay for reactor antineutrinos



Total cross section



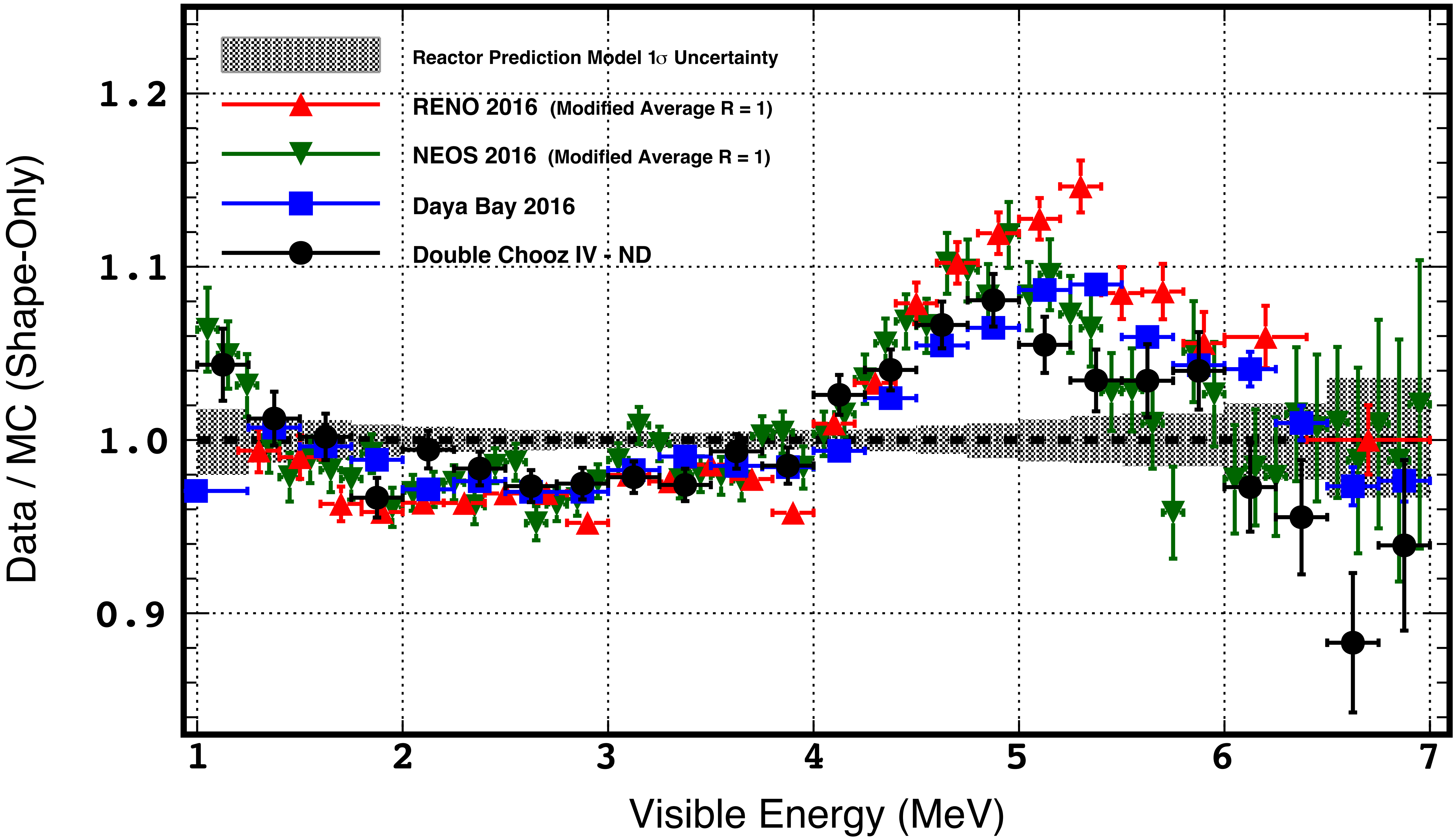


σ_0 - reference

σ - this work

- %-level improvement to previous results; 0.18% uncertainty

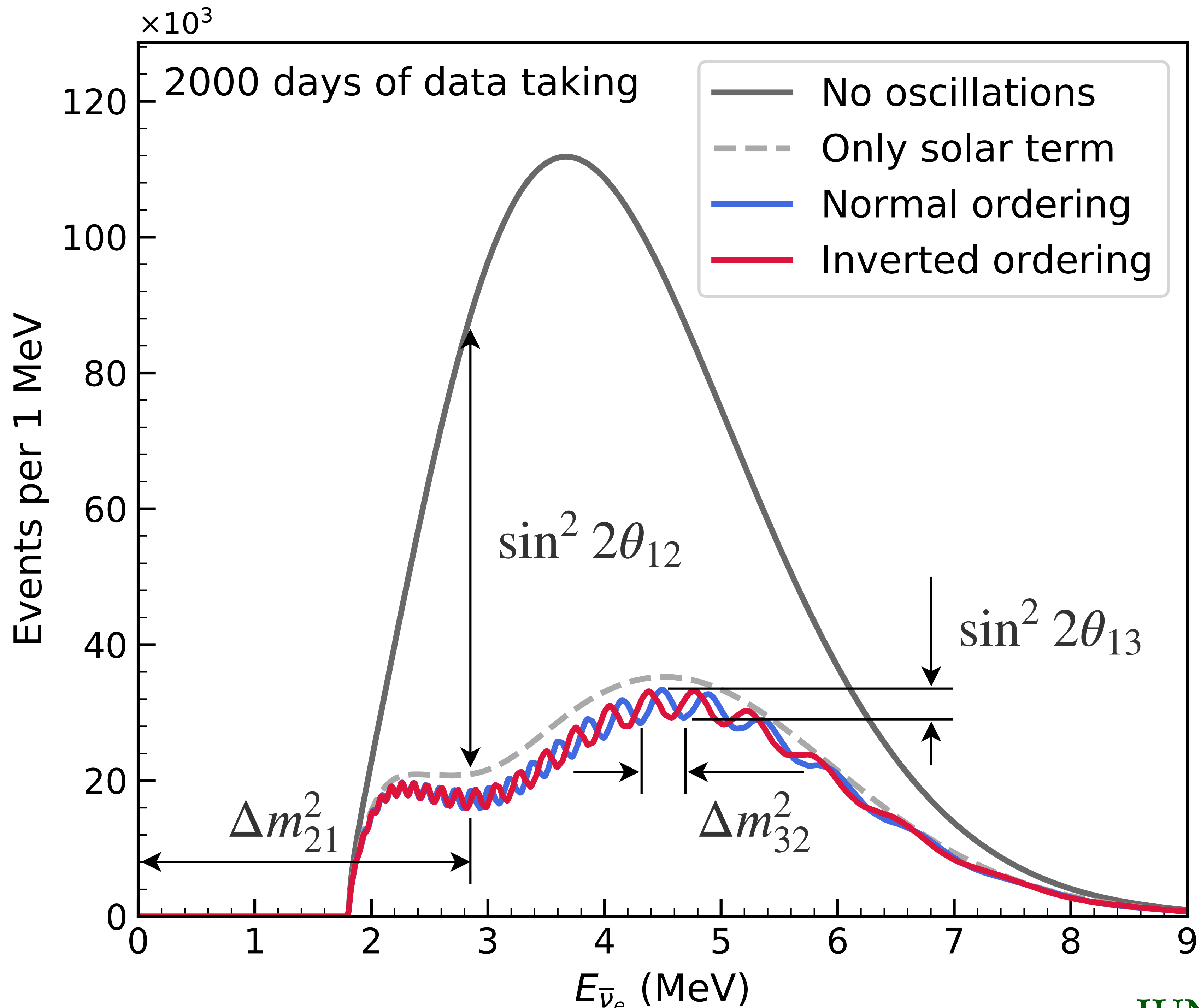
5-MEV flux excess



Double Chooz, Nature Phys (2019)

- update in cross section at 4-6 MeV: shift by 15-20% of bump

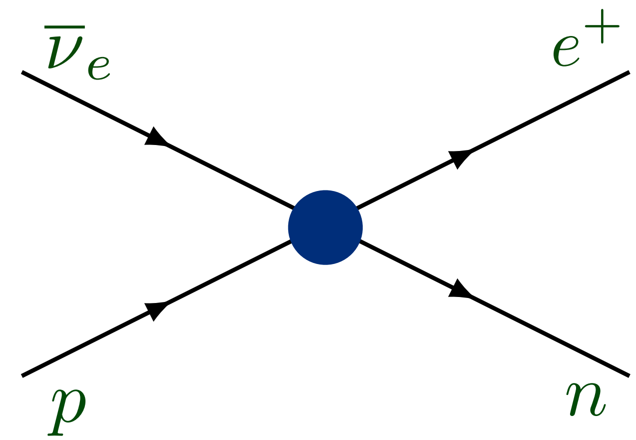
Energy reconstruction is key

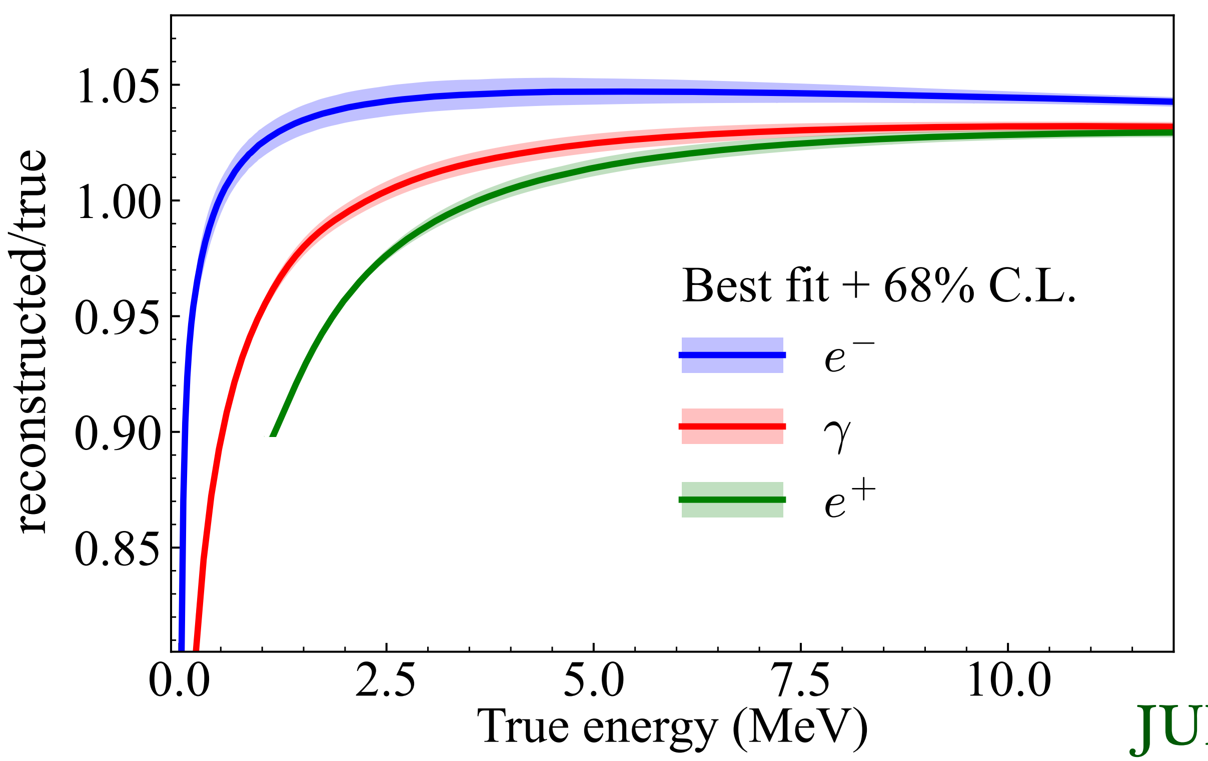


JUNO, PPNP (2021)

- JUNO: 3% energy reconstruction for precision goals

Energy reconstruction is key



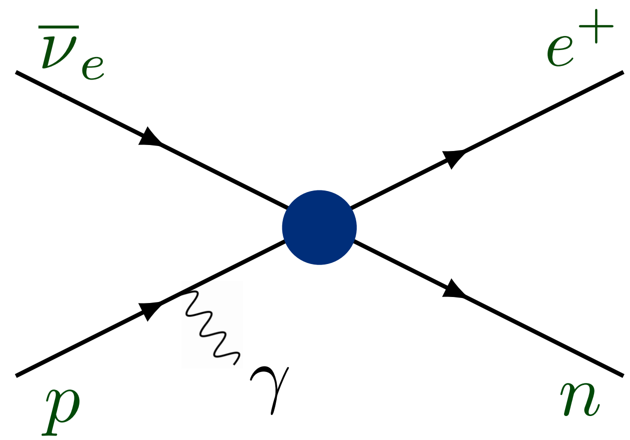


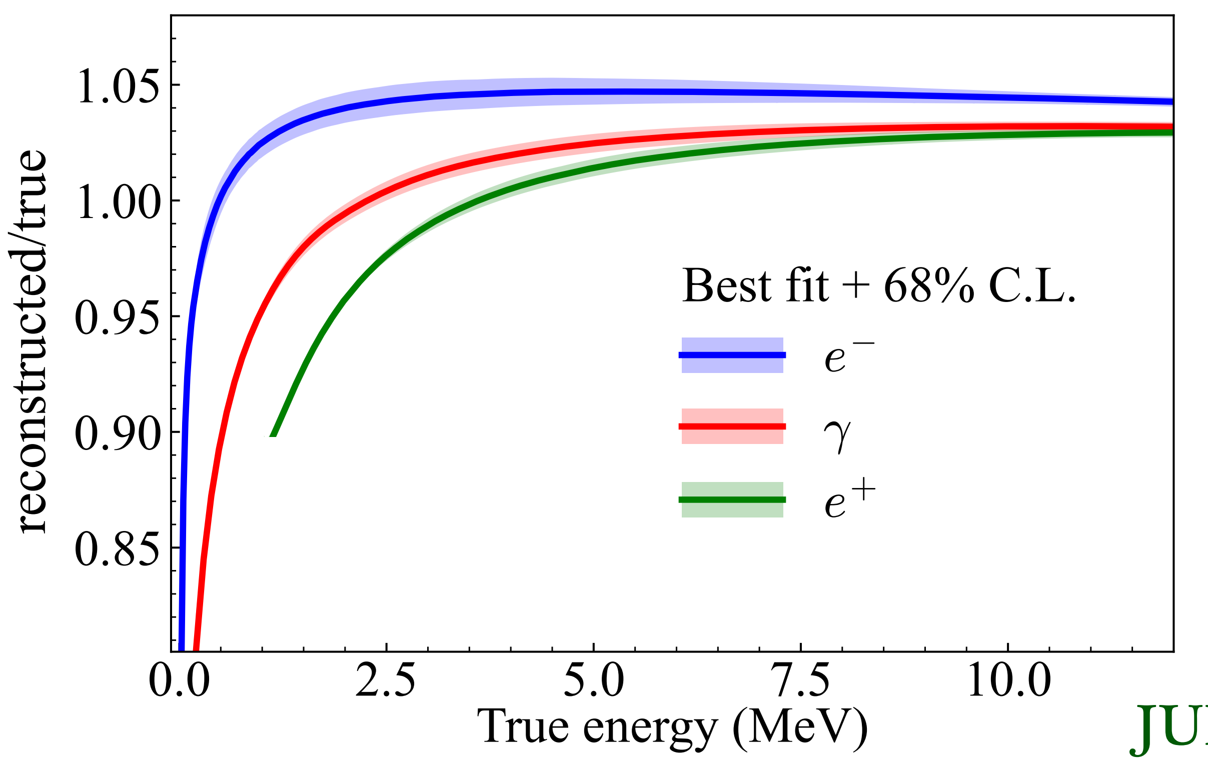
JUNO, arXiv:2511.14590

$$E_{\overline{\nu}_e} \approx m_n - m_p + E_{e^+} + O\left(\frac{1}{M}\right)$$

- control over each particle is necessary to achieve precision

Energy reconstruction is key





JUNO, arXiv:2511.14590

$$E_{\overline{\nu}_e} \approx m_n - m_p + E_{e^+} + E_\gamma + O\left(\frac{1}{M}\right)$$

- control over each particle is necessary to achieve precision

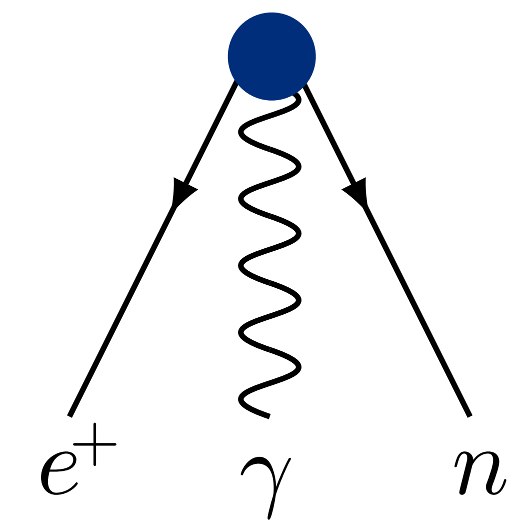
Bremsstrahlung

- new distributions beyond E_{EM} spectrum

E_n, E_γ, θ_n → E_n, θ_n → E_n, E_{EM} → total cross section

E_e, E_γ, θ_e → E_e, θ_e → E_e → total cross section

total cross section



$$E_{\text{EM}} = E_e + E_\gamma$$

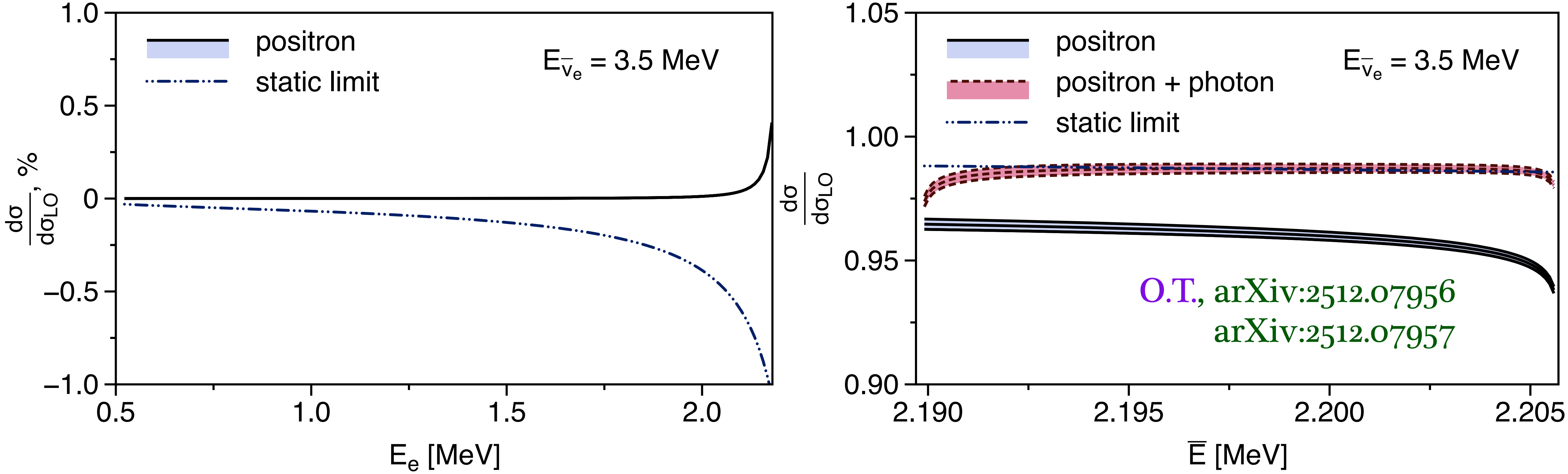
consistency at cross-section level

O.T., arXiv:2512.07956
arXiv:2512.07957

- analytical positron energy spectrum, 2D/3D distributions

Access each particle kinematics

- evaluate radiative process w/o no-recoil approximation

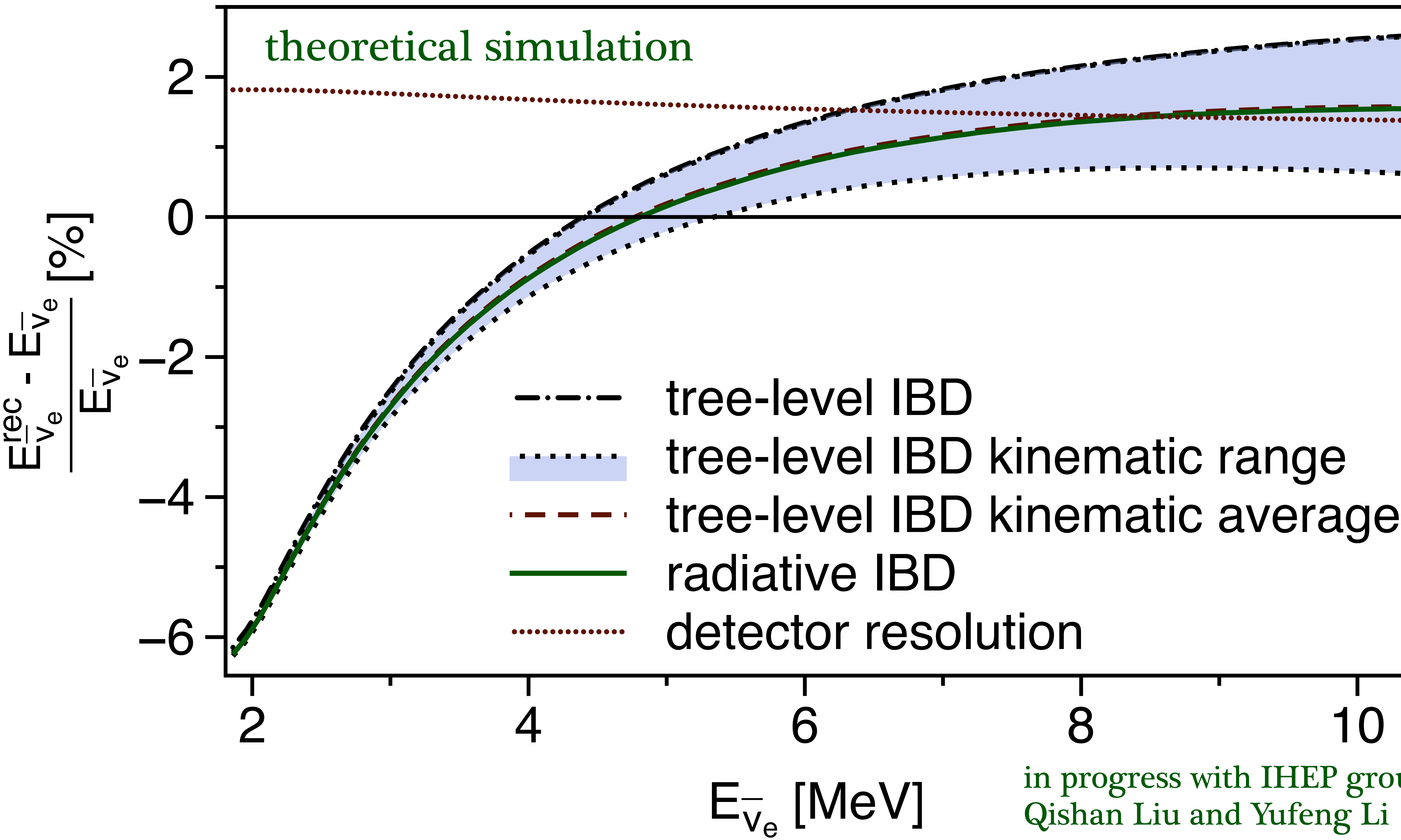


- 1st analytic expression for positron energy spectrum

$$E_{\bar{\nu}_e} \approx m_n - m_p + E_{e^+} + E_\gamma + O\left(\frac{1}{M}\right)$$

- control over each particle is necessary to achieve precision

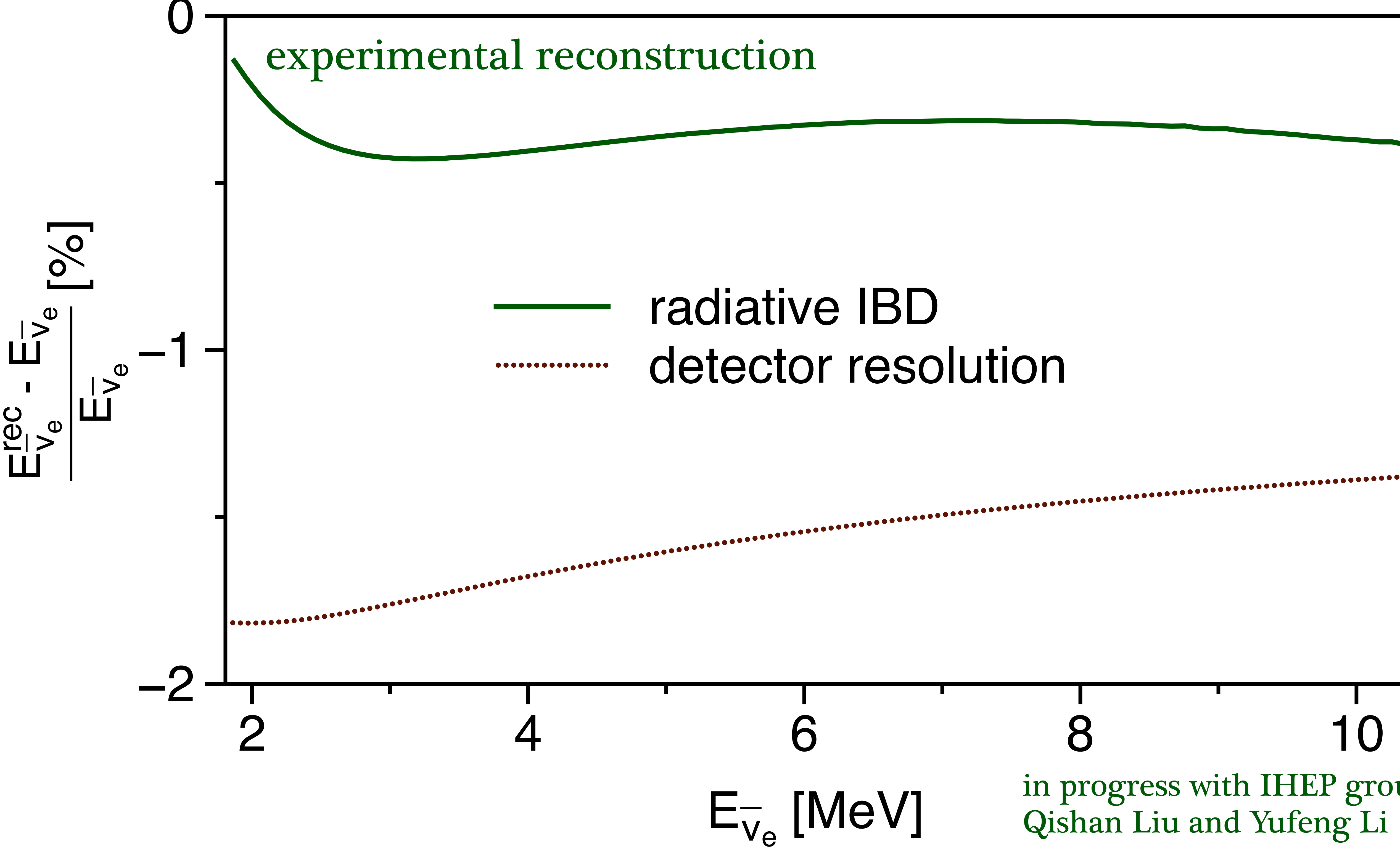
Energy reconstruction is key



in progress with IHEP group
Qishan Liu and Yufeng Li

- tree-level recoil effects are sizable

Energy reconstruction is key



in progress with IHEP group
Qishan Liu and Yufeng Li

- radiative effects ~ 1/4 of current resolution

Update in nuclear/neutron decay

- evaluate radiative process w/o no-recoil approximation

$$m_n = m_p + E_e + E_\gamma + E_\nu + O\left(\frac{1}{M}\right)$$

- V_{ud} from superallowed transitions and neutron decay

$$V_{ud}^{0^+\to 0^+} = 0.97373(31) \quad \longrightarrow \quad V_{ud}^{0^+\to 0^+} = 0.97361(31)$$

Hardy and Towner, PRC (2020)

changes by 1 uncertainty of radiative corrections

- g_A from beta asymmetry in polarized neutron decay

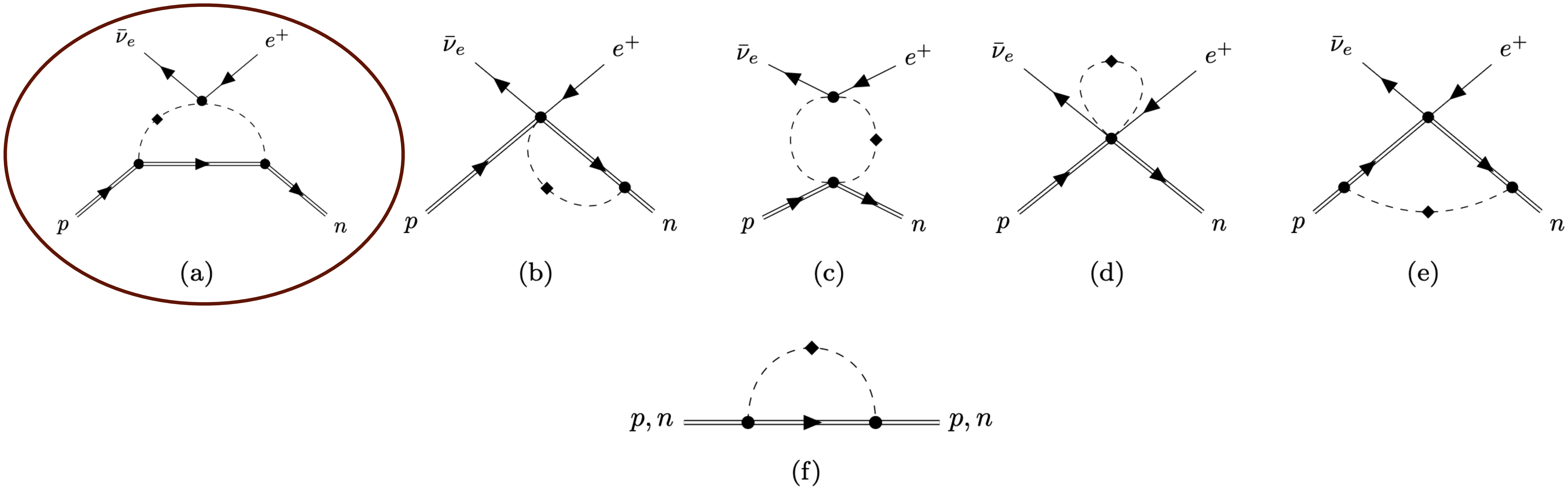
$$A = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

changes by 1/3 of PERKEO-III experimental uncertainty

- numerically quantified shifts in V_{ud} and $g_A \lesssim 1\sigma$

Supernova and πDAR energies

- include pion degrees of freedom to inverse beta decay



- work at leading and next-to-leading order in HBChPT

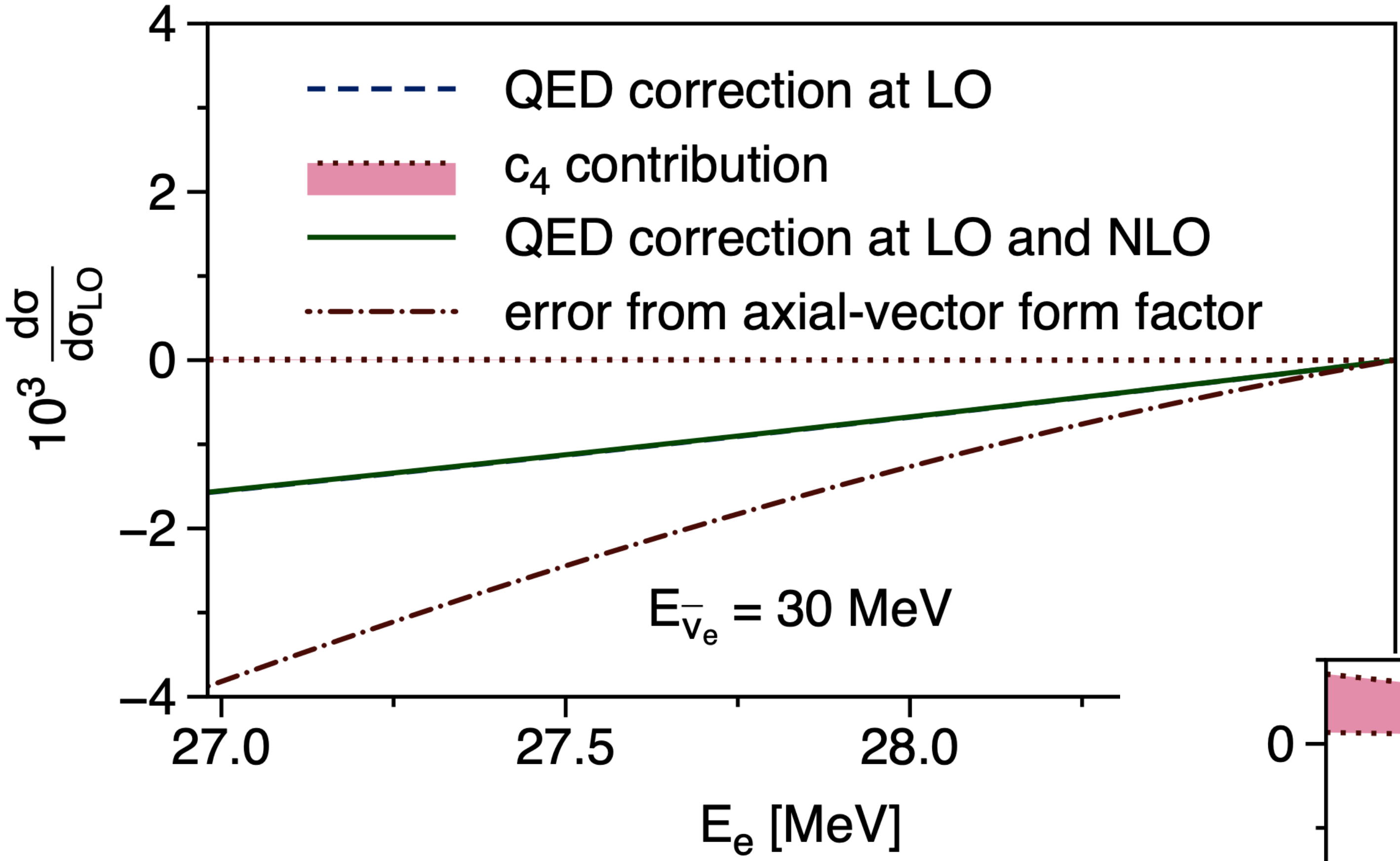
O.T., arXiv:2604.07113

- neglect recoil diagrams without pions

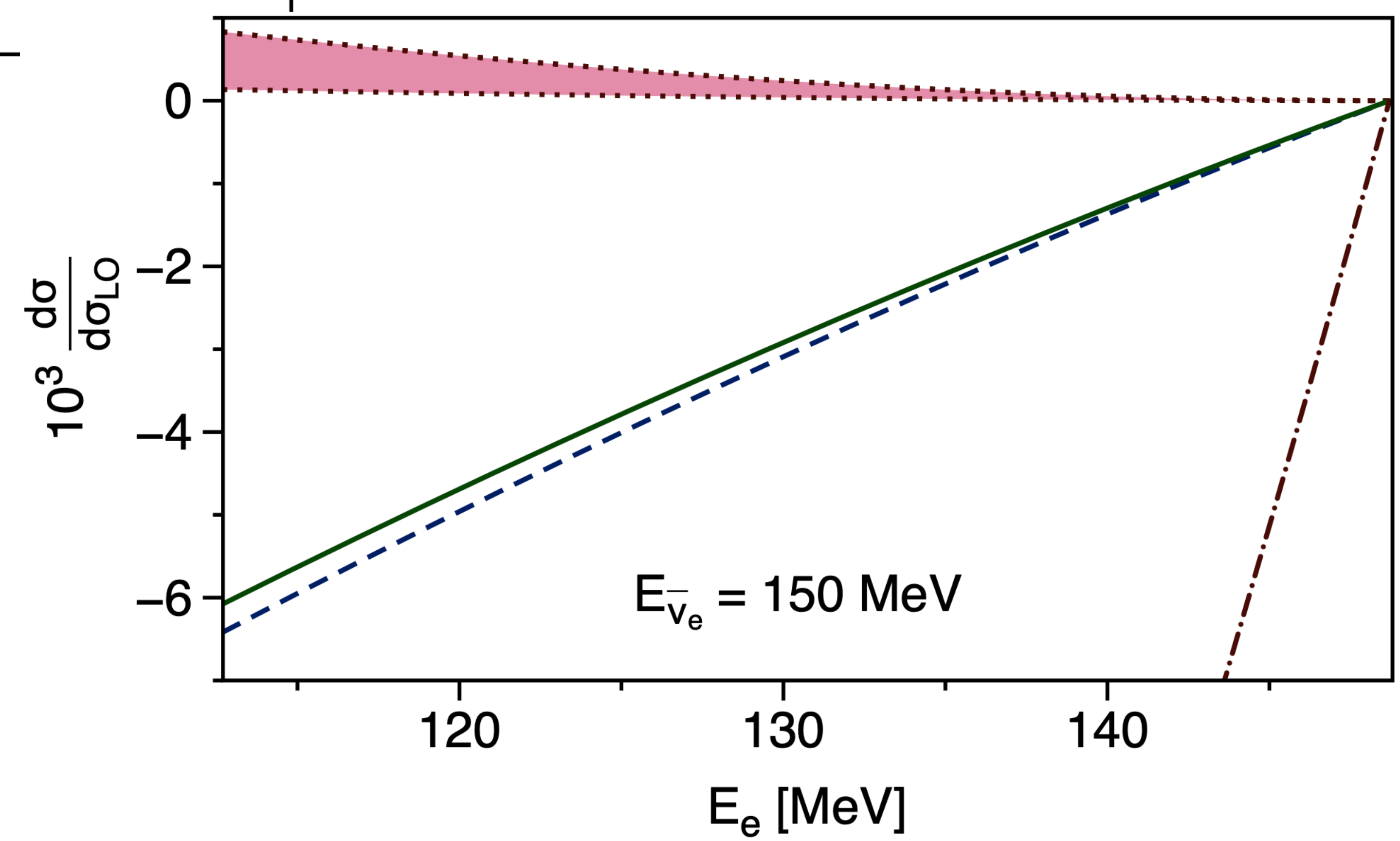
$$\frac{m_e^2}{m_\pi^2} \sim \frac{\alpha}{\pi}\frac{m_e}{m_\pi} << \frac{\alpha}{\pi}\frac{m_e}{E_{\overline{\nu}_e}} \lesssim \frac{\alpha}{\pi}\frac{E_{\overline{\nu}_e}}{m_n} \lesssim \frac{\alpha}{\pi}\frac{E_{\overline{\nu}_e}^2}{m_\pi^2} << \frac{\alpha}{\pi}\frac{m_\pi}{m_n} \lesssim \frac{\alpha}{\pi}\frac{E_{\overline{\nu}_e}}{m_\pi}$$

- extends 0.1%-level predictions to energies above 10 MeV

Supernova and πDAR energies



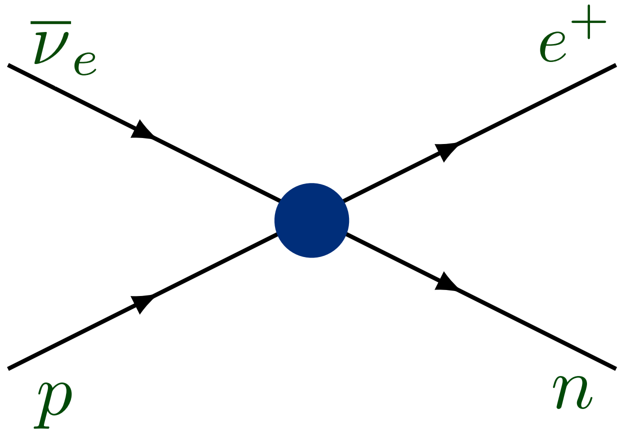
Wilson coefficient c_4 enters at higher energies



O.T., arXiv: 2604.07113

- extends 0.1%-level predictions to energies above 10 MeV

Conclusions



radiative corrections
in EFT framework
+ scheme-dependence

Standard Model -> LEFT -> HBχPT -> $\pi\!\!\!/$EFT

- 1st consistent inclusion of all QCD/EW effects at 0.1% level
- total cross section, energy spectra, 2D/3D distributions in IBD
- precision physics: important to control γ and e^+ kinematics in calibration of energy response in liquid scintillators
- $1\sigma/0.3\sigma$ updates in V_{ud}/g_A from nuclear/neutron decay
- 1st evaluation of pion-induced corrections to inverse beta decay

$$\nu e^- \to \nu e^- \,(\gamma)$$

$$\pi^+ \to \mu^+ \nu_\mu (\gamma) \qquad K^+ \to \mu^+ \nu_\mu (\gamma) \qquad \mu^+ \to e^+ \nu_e \bar{\nu}_\mu (\gamma)$$

Thanks for your attention !!!

$$\bar{\nu}_\ell p \to \ell^+ n \,(\gamma)$$

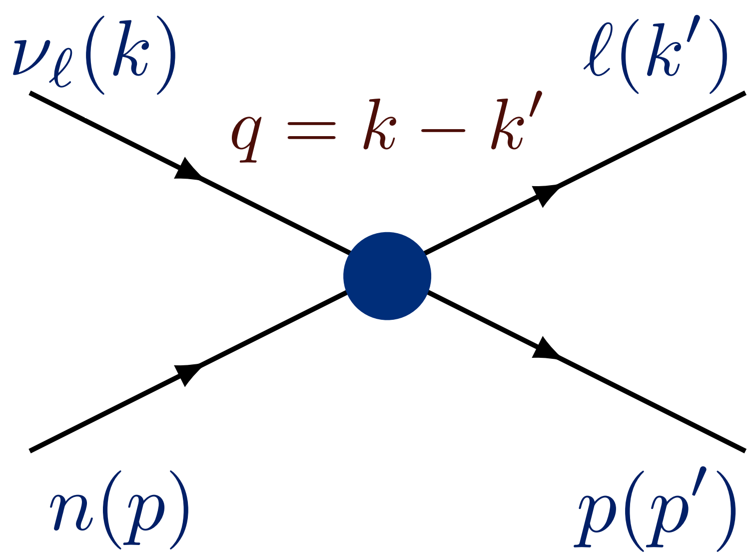
谢谢

$$\nu_\ell n \to \ell^- p \,(\gamma)$$

$$n \to p e^- \overline{\nu}_e$$

$$\nu \; {}^{40}\mathrm{Ar} \to \nu \; {}^{40}\mathrm{Ar} \,(\gamma)$$

Invariant amplitudes



averaged lepton momentum $K_\mu = \frac{k_\mu + k'_\mu}{2}$

averaged nucleon momentum $P_\mu = \frac{p_\mu + p'_\mu}{2}$

- four amplitudes for massless charged lepton

$$T^{m_\ell=0}_{\nu_\ell n \to \ell^- p} = \sqrt{2} G_F V_{ud}\, \bar{\ell}^- \gamma^\mu P_L \nu_\ell\, \bar{p} \left(\gamma_\mu \left(g_M + f_A \gamma_5 \right) - \left(f_2 + f_A^3 \gamma_5 \right) \frac{K_\mu}{M} \right) n$$

- four extra amplitudes for massive charged lepton

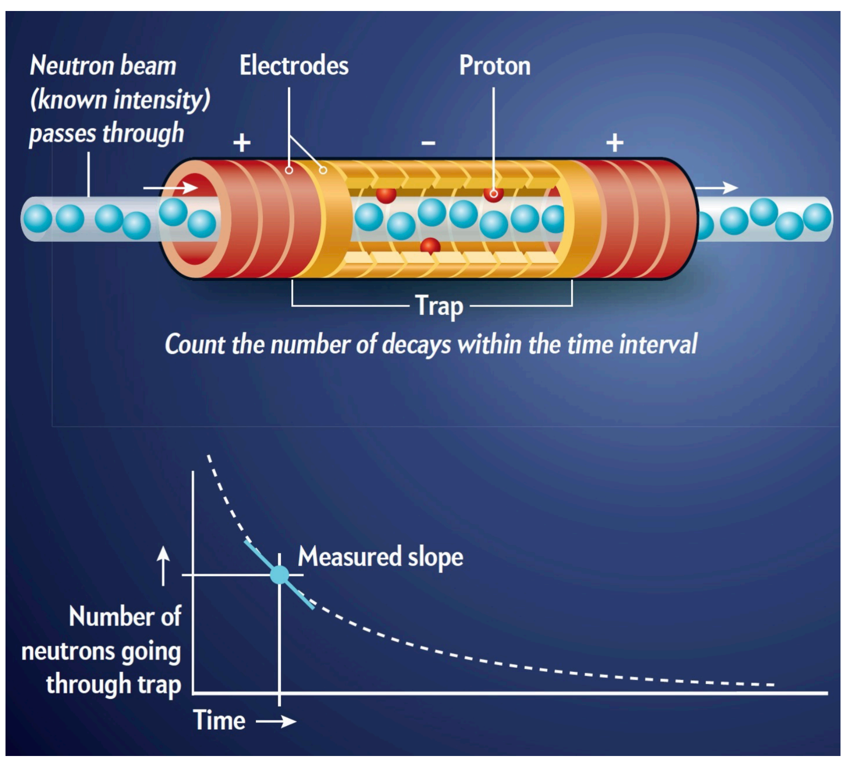
$$T^{m_\ell \neq 0}_{\nu_\ell n \to \ell^- p} = -\sqrt{2} G_F V_{ud} \frac{m_\ell}{M}\, \bar{\ell}^- P_L \nu_\ell\, \bar{p} \left(f_3 + f_P \gamma_5 - \frac{f_R}{4} \frac{P_\mu}{M} \gamma^\mu \gamma_5 \right) n$$

$$+ \sqrt{2} G_F V_{ud} \frac{m_\ell}{M} \frac{f_T}{4}\, \bar{\ell}^- \sigma^{\mu\nu} P_L \nu_\ell\, \bar{p} \sigma_{\mu\nu} n$$

- 8 invariant amplitudes for charged-current elastic scattering

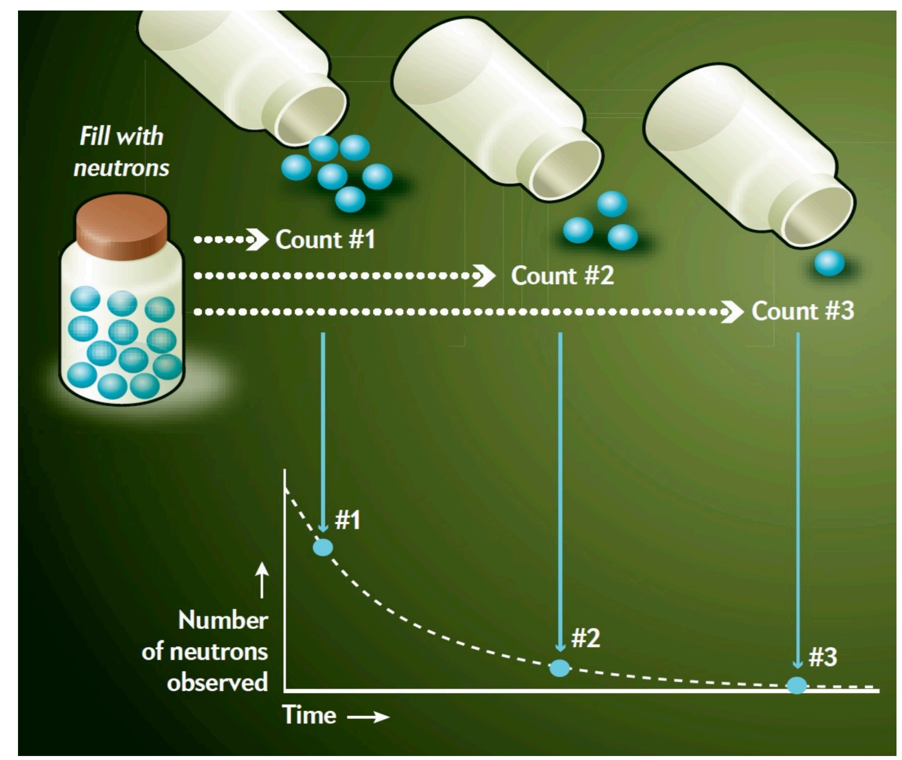
Neutron lifetime measurements

beam method



how many neutrons pass?

bottle method

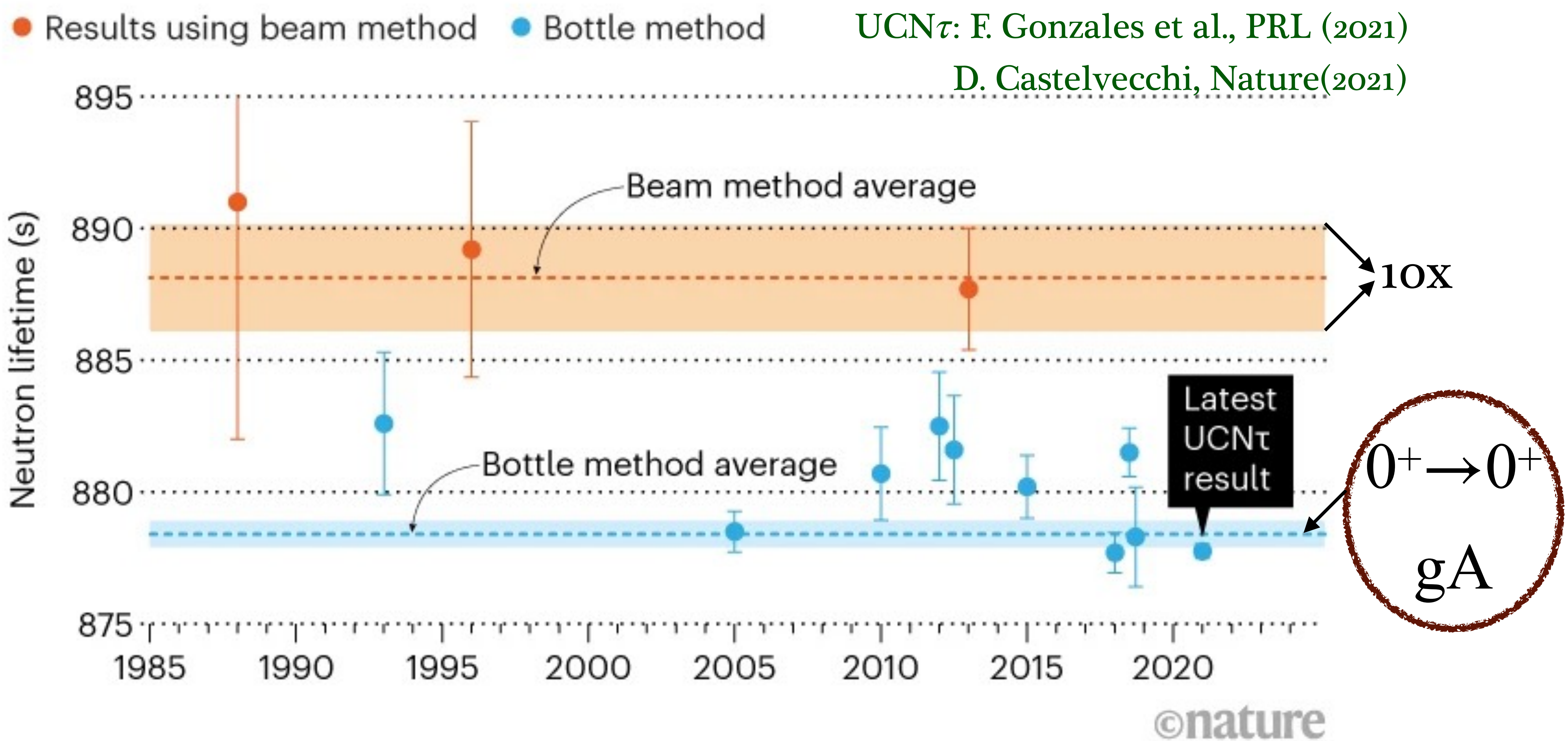


how many neutrons survive?

Neutron lifetime puzzle

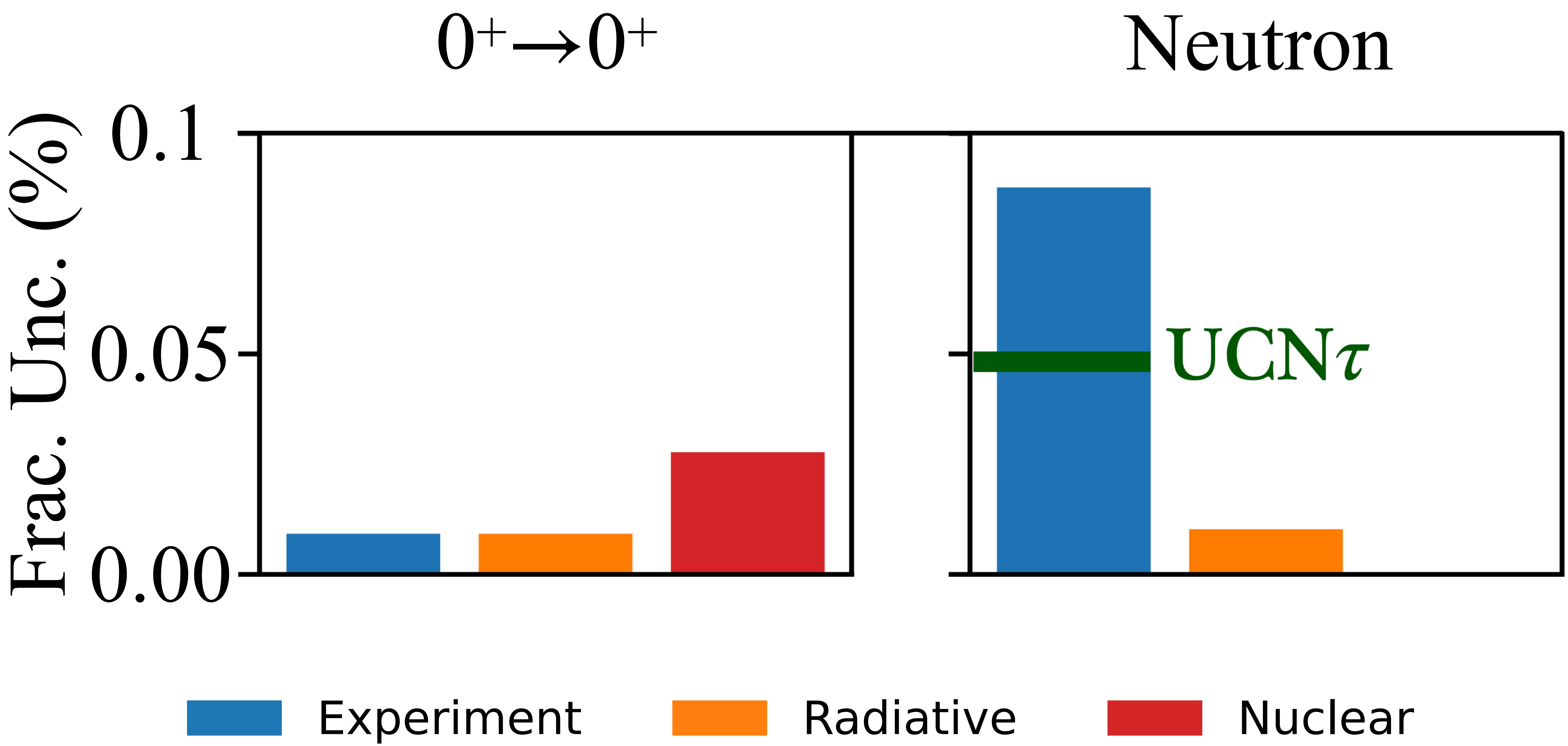
UCNτ: F. Gonzales et al., PRL (2021)

D. Castelvecchi, Nature(2021)



- 8-9 seconds discrepancy beam vs bottle method : 3-5σ
- Vud from $0^+ \to 0^+$ + gA from β asymmetry $\equiv$ bottle
- 0.3 seconds uncertainty of UCNτ@LANL : $(3-4)\times 10^{-4}$

V_{ud} determinations

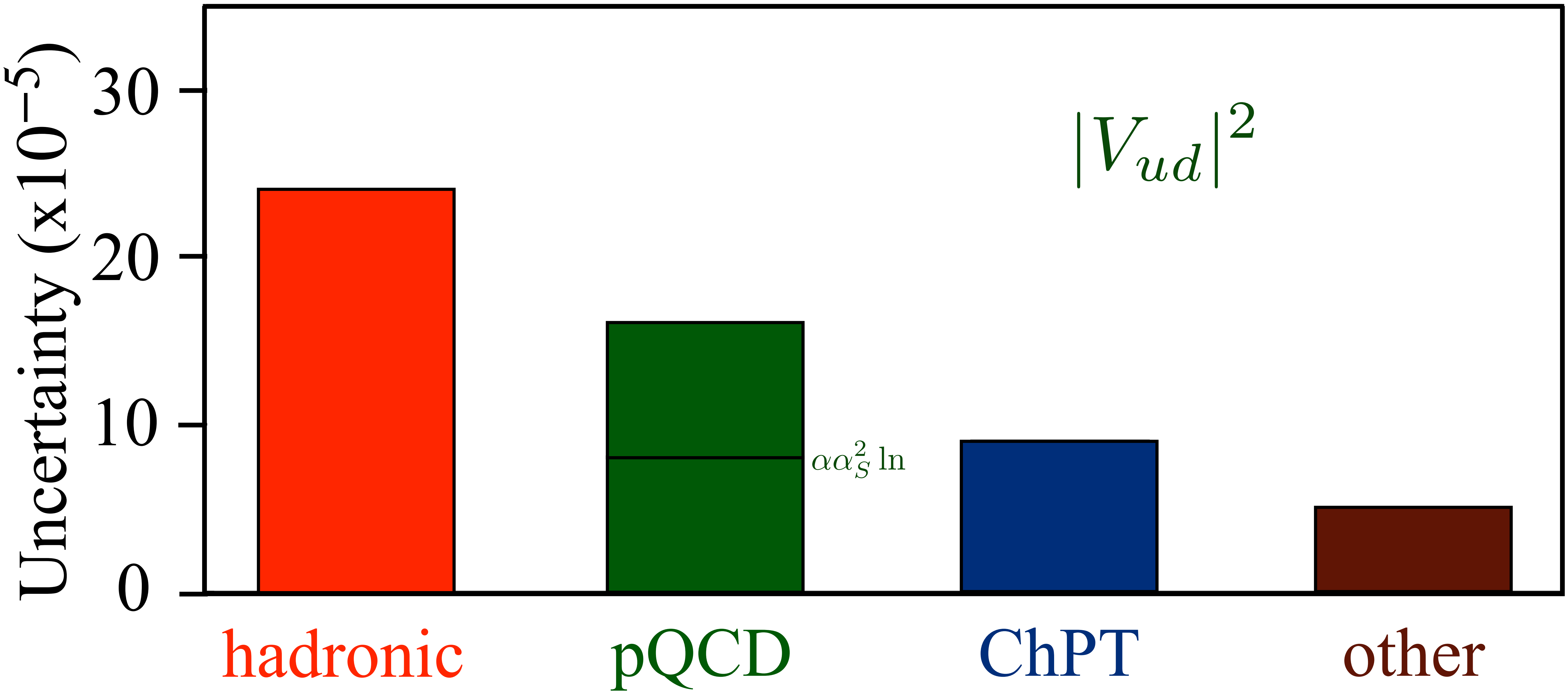


2022 Fundamental Symmetries, Neutrons, and Neutrinos (FSNN) white paper

- neutron decay becomes competitive with $0^+ \to 0^+$

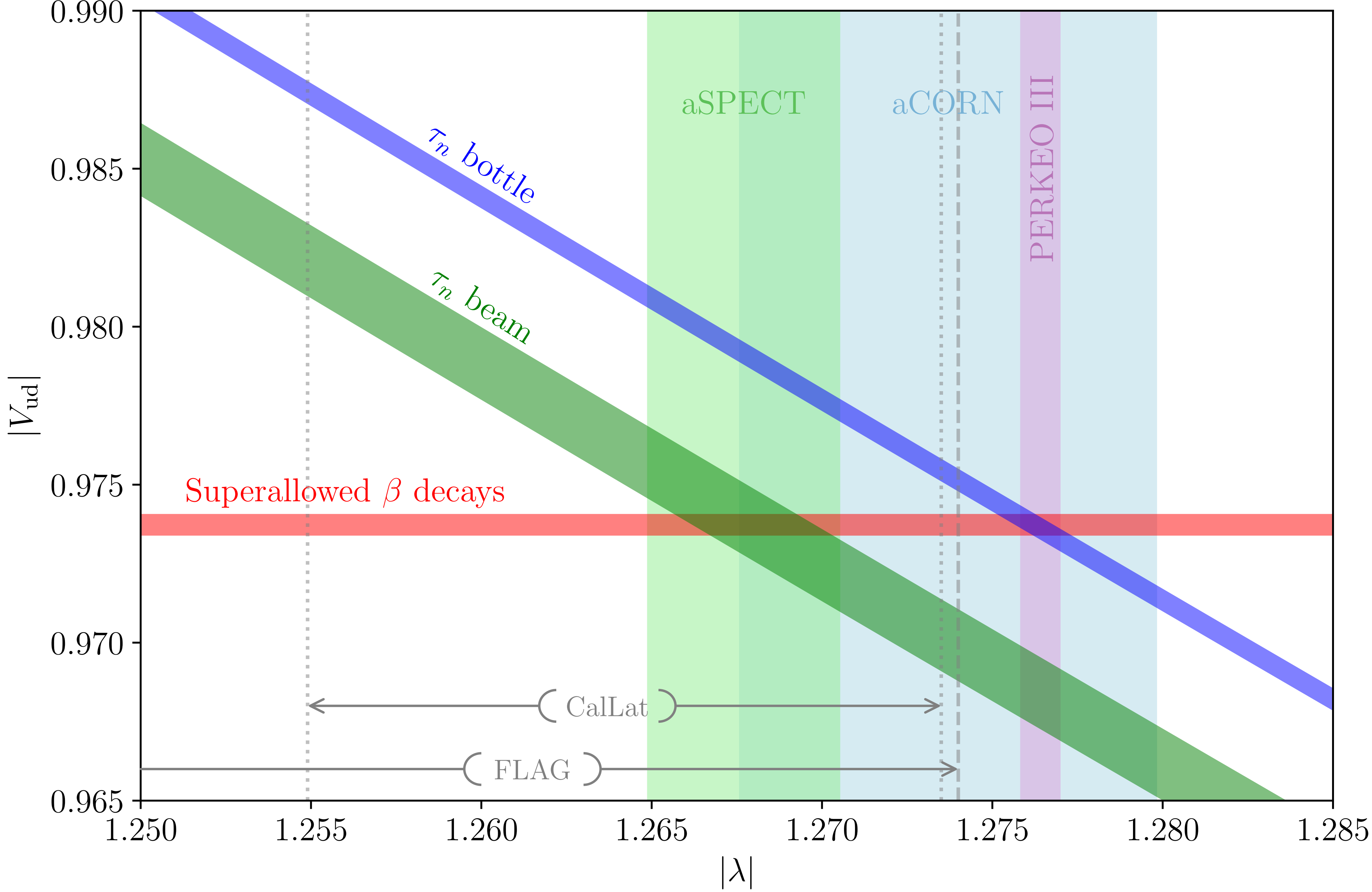
Our work

- detailed error budget and perturbative corrections



- shift of central value by ~1 error of radiative corrections
- quantification of perturbative/nonperturbative uncertainties

Neutron lifetime puzzle



Andrzej Czarnecki, William J. Marciano, Alberto Sirlin, PRL (2018)

Susan Gardner, Mohammadreza Zakeri, Universe (2024)

- tensions in gA data: puzzle is not resolved

Logarithms in neutron decay

M_Z

four-fermion interaction between leptons and heavy nucleons

$$\mathcal{L}_{\rm eff} = -\sqrt{2}\mathrm{G_F} V_{ud}\bar{e}\gamma_\mu \mathrm{P_L}\nu_e \cdot \overline{N}\left(g_V v^\mu - 2 g_A S^\mu\right)\tau^+ N$$

scale separation introduces large logarithms:

hadronic scale

$$\ln \frac{M_Z}{\Lambda_{\rm had}} \qquad \ln \frac{\Lambda_{\rm had}}{m_e}$$

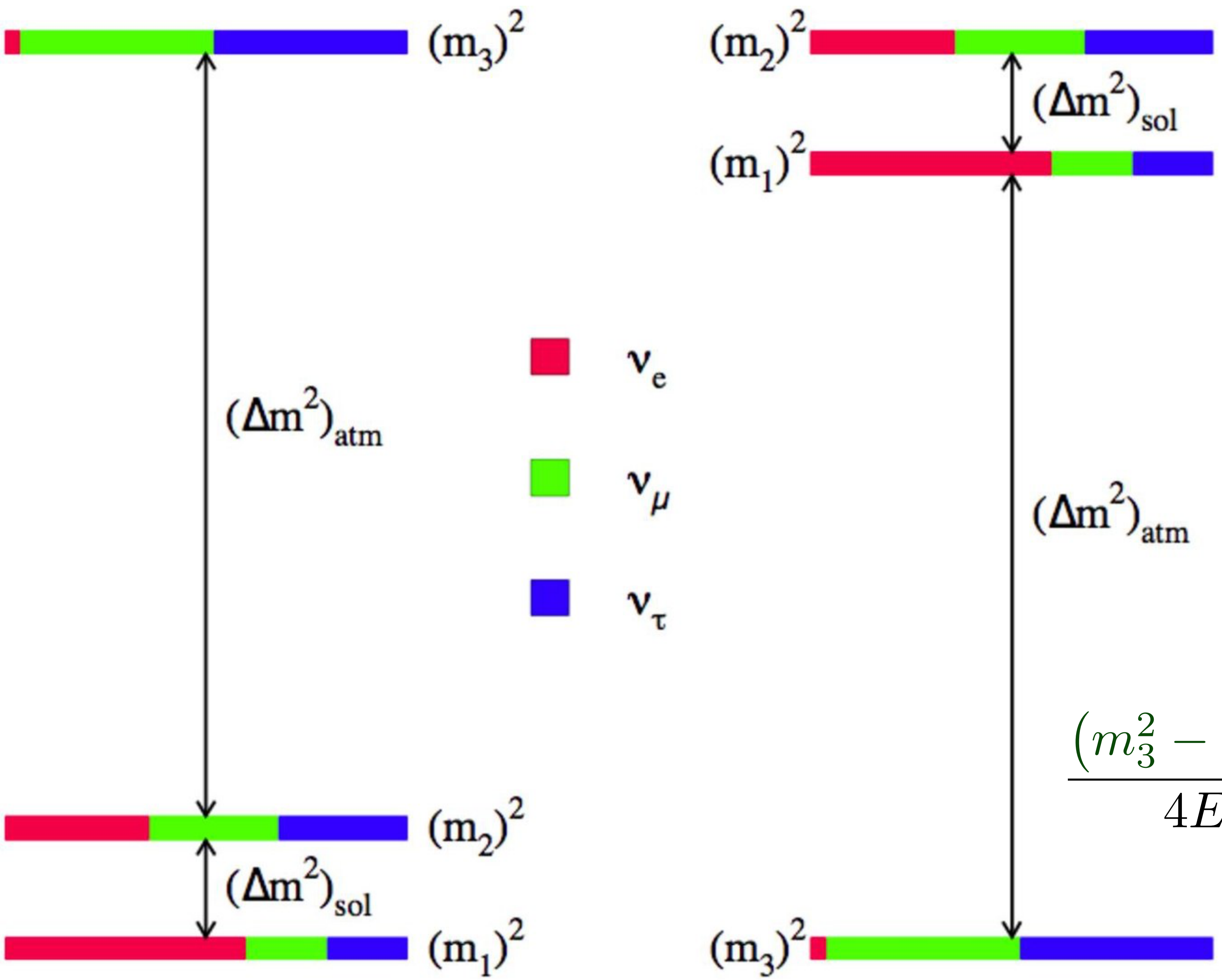
explain ~ half of radiative correction to neutron lifetime

$$\Delta = 7.756(27)\%$$

m_e

- g_V, g_A encode electroweak logarithms and hadronic corrections

Neutrino mass hierarchy



normal hierarchy

inverted hierarchy

forward scattering on atomic electrons

↓

matter effect distinguishes ν and $\overline{\nu}$

↓

$$\frac{(m_3^2 - m_1^2)L}{4E_\nu} \to \frac{(m_3^2 - m_1^2)L}{4E_\nu} \mp \frac{G_F n_e}{\sqrt{2}} L$$

↓

mass ordering

- matter effects can resolve neutrino mass hierarchy