$B \to D^{(*)}(1S, 2S)$ form factors and their applications to semi-leptonic and non-leptonic weak decays

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2026 轻强子专题研讨会 @ 商丘师范学院，商丘

2026-05-17

Outline

Introduction

- The ground states $D_{(s)}$ and $D^*_{(s)}$ have been well determined in experiments, their produtions and decays have been extensively studied.
- While the first radially excited states $D_{(s)}(2S)$ and $D^*_{(s)}(2S)$ have not been well confirmed.
- For example both $D_0(2550)$ and $D_J(2580)$ are considered as the candidates of $D(2^1S_0)$ state.
- $D_{s0}(2590)$ observed by LHCb is suggested to a candidate of $D_s(2^1S_0)$ state. Some theoretical works suggest that it may not be a pure state but rather has D^*K component.

Introduction

- $D_1^*(2680)^0, D^*(2650)^0$ and $D_1^*(2600)^0$ observed by LHCb are considered as the candidated of netrual $D(2^3S_1)$. The $D^*(2640)^{\pm}$ discovered by Delphi have masses consistent with the predictions of charged $D(2^3S_1)$, while it has not been confirmed in any other experiments.
- Except for the assignment of $D_s(2^3S_1)$ to $D_{s1}^*(2700)^{\pm}$, $D_s(1^3D_1)$ and a mixture of $D_s(2^3S_1)$ and $D_s(1^3D_1)$ are also been proposed.
- We will assume these several particles discovered in experiments as the corresponding first excited D mesons in the calcualtions:
  $D_0(2550) \to D(2^1S_0),\ D_{s0}(2590)^{\pm} \to D_s(2^1S_0),$
  $D_1^*(2600)^0, D^*(2640)^{\pm} \to D(2^3S_1),\ D_{s1}^*(2700)^{\pm} \to D_s(2^3S_1).$

Introduction

At the end of the twentieth century, Jaus put forward the covariant light-front quark model (CLFQM). The CLFQM has some unique advantages:

- The light-front wave functions descrbing the hadron through quark and gluon degrees of freedom can preserve a Lorentz invariant formalism.
- The final state meson at $q^2 = 0$ is usually relativistic. The CLFQM with relativistic effects involved is suitable to study hadronic transition form factors.

The CLFQM general calculation procedure

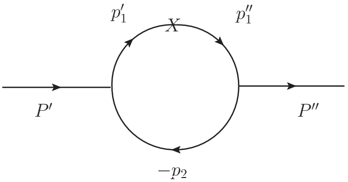
The Bauer-Stech-Wirble (BSW) form factors for $B \to D$ and $B \to D^{*}$ transitions are defined as follows,

$$\begin{aligned}
\left\langle D\left(P''\right)\left|V_{\mu}\right|B\left(P'\right)\right\rangle &= \left(P_{\mu}-\frac{m_B^2-m_D^2}{q^2}q_{\mu}\right)F_1^{BD}\left(q^2\right)+\frac{m_B^2-m_D^2}{q^2}q_{\mu}F_0^{BD}\left(q^2\right),\\
\left\langle D^{*}\left(P'',\varepsilon''\right)\left|V_{\mu}\right|B\left(P'\right)\right\rangle &= -\frac{1}{m_{D^*}+m_B}\epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu}P^{\alpha}q^{\beta}V^{BD^*}\left(q^2\right),\\
\left\langle D^{*}\left(P'',\varepsilon''\right)\left|A_{\mu}\right|B\left(P'\right)\right\rangle &= i\left\{\left(m_{D^*}+m_B\right)\varepsilon''^{*}_{\mu}A_1^{BD^*}\left(q^2\right)-\frac{\varepsilon''^{*}\cdot P}{m_{D^*}+m_B}P_{\mu}A_2^{BD^*}\left(q^2\right)\right.\\
&\quad\left.-2m_{D^*}\frac{\varepsilon''^{*}\cdot P}{q^2}q_{\mu}\left[A_3^{BD^*}\left(q^2\right)-A_0^{BD^*}\left(q^2\right)\right]\right\},
\end{aligned}$$

where $P=P'+P''$, $q=P'-P''$, ϵ'' is the polarization vector. The four-momentum of the initial (final) meson is $P'=p_1'+p_2$ $(P''=p_1''+p_2)$.

The decay amplitude in the lowest order for the transitions $B \to D$ and $B \to D^*$

$$\mathcal{B}_\mu^{BD} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{h_B' h_D''}{N_1' N_1'' N_2} S_\mu^{BD},$$
$$\mathcal{B}_\mu^{BD^*} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{h_B' (i h_{D^*}'')}{N_1' N_1'' N_2} S_{\mu\nu}^{BD^*} \varepsilon''^{*\nu}, \quad (1)$$



- $N_1'^{('')} = p_1'^{('')2} - m_1'^{('')2}, N_2 = p_2^2 - m_2^2$ arise from the quark propagators.

The CLFQM general calculation procedure

The traces S_μ^{BD} and $S_{\mu\nu}^{BD^*}$ can be obtained directly using Lorentz contraction,

$$S_\mu^{BD} = \mathrm{Tr}\left[\gamma_5\left(\not p_1'' + m_1''\right)\gamma_\mu\left(\not p_1' + m_1'\right)\gamma_5\left(-\not p_2 + m_2\right)\right],$$

$$S_{\mu\nu}^{BD^*} = \left(S_V^{BD^*} - S_A^{BD^*}\right)_{\mu\nu}$$

$$= \mathrm{Tr}\left[\left(\gamma_\nu - \frac{1}{W_V''}\left(p_1'' - p_2\right)_\nu\right)\left(\not p_1'' + m_1''\right)\left(\gamma_\mu - \gamma_\mu\gamma_5\right)\left(\not p_1' + m_1'\right)\gamma_5\left(-\not p_2 + m_2\right)\right], \quad (2)$$

The covariant vertex function h_M'' with $M = D(1S,2S), D^*(1S,2S)$ is defined as

$$h_M'' = \left(M''^2 - M_0''^2\right)\sqrt{\frac{x_1 x_2}{N_c}}\frac{1}{\sqrt{2}\widetilde{M}_0''}\varphi,$$

$$M_0''^2 = \left(e_1'' + e_2\right)^2 = \frac{p_\perp'^2 + m_1''^2}{x_1} + \frac{p_\perp^2 + m_2^2}{x_2}, \quad \widetilde{M}_0'' = \sqrt{M_0''^2 - \left(m_1'' - m_2\right)^2}. \quad (3)$$

The CLFQM general calculation procedure

The phenomenological Gaussian-type wave function φ depicts the light-front momentum distribution amplitude for the S-wave mesons,

$$\varphi(1S) = 4\left(\frac{\pi}{\beta^2}\right)^{\frac{3}{4}}\sqrt{\frac{dp_z}{dx_2}}\exp\left(-\frac{p_z^2+p_\perp^2}{2\beta^2}\right),$$

$$\varphi(2S) = 4\left(\frac{\pi}{\beta^2}\right)^{\frac{3}{4}}\sqrt{\frac{dp_z}{dx_2}}\exp\left(-\frac{p_z^2+p_\perp^2}{2\beta^2}\right)(3-2\frac{p_z^2+p_\perp^2}{\beta^2}),$$

- The shape parameter β can be fixed by fitting the corresponding decay constant.

Transition Form Factors

Input parameters:

- The constituent quark masses(GeV): $m_c = 1.4$, $m_s = 0.37$, $m_{u,d} = 0.25$.
- The masses of the initial and the final mesons(GeV): $m_D = 1.86966$,

  $m_{D(2S)} = 2.549$, $m_{D_s} = 1.96835$, $m_{D_s(2S)} = 2.591$,

  $m_{D_s^{\star\pm}} = 2.1122, m_{D^{\star 0}} = 2.0068, m_{D^{\star\pm}} = 2.0102$, $m_B = 5.279$,

  $m_{D_s^{\star\pm}(2S)} = 2.732$, $m_{D^{\star 0}(2S)} = 2.627, m_{D^{\star\pm}(2S)} = 2.637, m_{B_s^0} = 5.4154$.
- The CKM matrix element: $|V_{cb}| = (41.1 \pm 1.2) \times 10^{-3}$, $V_{cd} =$

  0.221 ± 0.004, $V_{cs} = 0.975 \pm 0.006$, $V_{ud} = 0.97373 \pm 0.00031$.

Input parameters:

- The shape parameters fitted by the decay constants:

$$\beta_{D_s^{\star\pm}(1S)} = 0.534^{+0.014}_{-0.014}, \beta_{D^{\star 0}(1S)} = 0.500^{+0.140}_{-0.187}, \beta_D = 0.466^{+0.022}_{-0.021},$$

$$\beta_{D_s^{\star\pm}(2S)} = 0.473^{+0.041}_{-0.041}, \beta_{D^{\star 0}(2S)} = 0.456^{+0.004}_{-0.003}, \beta_{D_s} = 0.600^{+0.026}_{-0.025},$$

$$\beta_{D^{\star\pm}(1S)} = 0.502^{+0.041}_{-0.041}, \beta_{D^{\star\pm}(2S)} = 0.453^{+0.004}_{-0.003},\ \beta_{\bar{B}_s^0} = 0.626^{+0.045}_{-0.045},$$

$$\beta_{D(2S)} = 0.297^{+0.041}_{-0.041}, \beta_{D_s(2S)} = 0.422^{+0.026}_{-0.025}, \beta_B = 0.555^{+0.060}_{-0.060}.$$

- mean life$(10^{-12}s)$: $\tau_{B^0} = (1.519 \pm 0.004)$, $\tau_{B_s^0} = (1.520 \pm 0.005)$,

  $\tau_{B^\pm} = (1.638 \pm 0.004)$.

Transition Form Factors

- All the calculations are carried out within the $q^+ = 0$ reference frame, where the form factors can only be obtained at spacelike momentum transfers $q^2 = -q_\perp^2 \leq 0$,
- The parameterized form factors are extrapolated from the space-like region to the time-like region by using

$$F(q^2) = \frac{F(0)}{(1 - q^2/m^2)\left[1 - a(q^2/m^2) + b(q^2/m^2)^2\right]}. \tag{4}$$

- $F(q^2)$ denotes different form factors, such as $F_1(q^2)$, $F_0(q^2)$, $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$.

Transition Form Factors

- The form factors of the transtions $B_{(s)} \to D_{(s)}(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of $B_{(s)}$ and final state mesons.

	$F(0)$	$F(q^2_{max})$	a	b
F_1^{BD}	$0.66^{+0.00+0.01}_{-0.01-0.01}$	$0.81^{+0.01+0.01}_{-0.00-0.01}$	$0.80^{+0.01+0.04}_{-0.02-0.04}$	$0.86^{+0.01+0.02}_{-0.01-0.02}$
F_0^{BD}	$0.66^{+0.00+0.01}_{-0.01-0.01}$	$0.70^{+0.02+0.01}_{-0.03-0.01}$	$0.46^{+0.14+0.00}_{-0.12-0.01}$	$0.77^{+0.01+0.05}_{-0.01-0.05}$
$F_1^{BD(2S)}$	$0.26^{+0.01+0.01}_{-0.02-0.02}$	$0.34^{+0.02+0.01}_{-0.03-0.03}$	$0.99^{+0.04+0.16}_{-0.10-0.18}$	$0.66^{+0.07+0.17}_{-0.12-0.15}$
$F_0^{BD(2S)}$	$0.26^{+0.01+0.02}_{-0.01-0.02}$	$0.32^{+0.03+0.02}_{-0.00-0.04}$	$0.65^{+0.03+0.05}_{-0.01-0.04}$	$-0.24^{+0.02+0.01}_{-0.03-0.03}$
$F_1^{B_sD_s}$	$0.67^{+0.00+0.01}_{-0.00-0.01}$	$0.81^{+0.00+0.00}_{-0.01-0.01}$	$0.82^{+0.00+0.02}_{-0.01-0.02}$	$0.96^{+0.01+0.03}_{-0.02-0.03}$
$F_0^{B_sD_s}$	$0.67^{+0.00+0.01}_{-0.00-0.01}$	$0.71^{+0.01+0.01}_{-0.02-0.01}$	$0.48^{+0.00+0.08}_{-0.01-0.08}$	$0.85^{+0.02+0.06}_{-0.02-0.06}$
$F_1^{B_sD_s(2S)}$	$0.26^{+0.02+0.02}_{-0.02-0.02}$	$0.29^{+0.02+0.01}_{-0.02-0.02}$	$0.59^{+0.12+0.05}_{-0.16-0.01}$	$0.35^{+0.11+0.04}_{-0.13-0.02}$
$F_0^{B_sD_s(2S)}$	$0.26^{+0.01+0.01}_{-0.02-0.02}$	$0.25^{+0.02+0.01}_{-0.02-0.02}$	$-0.09^{+0.22+0.22}_{-0.14-0.26}$	$-0.07^{+0.13+0.18}_{-0.21-0.08}$

- The form factors of the transtions $B \to D^*(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of B and final state mesons.

F	F(0)	$F(q^2_{max})$	a	b
V^{BD^*}	$0.77^{+0.00+0.01}_{-0.00-0.01}$	$0.94^{+0.00+0.01}_{-0.00-0.01}$	$0.78^{+0.00+0.03}_{-0.01-0.03}$	$0.82^{+0.04+0.14}_{-0.08-0.16}$
$A_0^{BD^*}$	$0.75^{+0.00+0.06}_{-0.01-0.02}$	$0.79^{+0.00+0.05}_{-0.02-0.02}$	$0.17^{+0.01+0.01}_{-0.01-0.00}$	$0.12^{+0.07+0.02}_{-0.06-0.03}$
$A_1^{BD^*}$	$0.67^{+0.00+0.01}_{-0.01-0.11}$	$0.76^{+0.00+0.01}_{-0.01-0.01}$	$0.38^{+0.01+0.01}_{-0.01-0.01}$	$0.19^{+0.05+0.09}_{-0.05-0.09}$
$A_2^{BD^*}$	$0.58^{+0.00+0.00}_{-0.01-0.02}$	$0.69^{+0.01+0.01}_{-0.01-0.02}$	$0.68^{+0.02+0.00}_{-0.02-0.00}$	$0.66^{+0.25+0.10}_{-0.12-0.16}$
$V^{BD^*(2S)}$	$0.19^{+0.05+0.01}_{-0.06-0.01}$	$0.19^{+0.01+0.03}_{-0.04-0.01}$	$0.11^{+0.05+0.03}_{-0.06-0.05}$	$0.34^{+0.00+0.03}_{-0.04-0.06}$
$A_0^{BD^*(2S)}$	$0.27^{+0.04+0.01}_{-0.05-0.00}$	$0.28^{+0.00+0.01}_{-0.02-0.00}$	$0.24^{+0.05+0.00}_{-0.06-0.00}$	$-0.07^{+0.15+0.03}_{-0.29-0.06}$
$A_1^{BD^*(2S)}$	$0.16^{+0.04+0.01}_{-0.05-0.00}$	$0.15^{+0.05+0.02}_{-0.04-0.02}$	$-0.23^{+0.04+0.00}_{-0.04-0.00}$	$0.17^{+0.03+0.01}_{-0.01-0.01}$
$A_2^{BD^*(2S)}$	$-0.05^{+0.04+0.01}_{-0.04-0.00}$	$0.01^{+0.00+0.00}_{-0.00-0.00}$	$-1.10^{+0.02+0.01}_{-0.01-0.01}$	$-0.63^{+0.02+0.01}_{-0.01-0.00}$

Transition Form Factors

- The form factors of the transtions $B_s \to D_s^*(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of B_s and final state mesons.

F	F(0)	$F(q^2_{max})$	a	b
$V^{B_sD_s^*}$	$0.78^{+0.01+0.01}_{-0.01-0.01}$	$0.87^{+0.00+0.00}_{-0.00-0.01}$	$0.86^{+0.01+0.04}_{-0.01-0.04}$	$1.11^{+0.01+0.02}_{-0.01-0.02}$
$A_0^{B_sD_s^*}$	$0.74^{+0.01+0.01}_{-0.01-0.01}$	$0.69^{+0.01+0.01}_{-0.01-0.00}$	$0.23^{+0.01+0.01}_{-0.01-0.01}$	$0.21^{+0.00+0.01}_{-0.00-0.01}$
$A_1^{B_sD_s^*}$	$0.66^{+0.01+0.01}_{-0.02-0.02}$	$0.95^{+0.00+0.00}_{-0.00-0.01}$	$0.81^{+0.01+0.02}_{-0.01-0.02}$	$0.93^{+0.01+0.01}_{-0.01-0.01}$
$A_2^{B_sD_s^*}$	$0.57^{+0.00+0.00}_{-0.01-0.00}$	$0.68^{+0.01+0.00}_{-0.01-0.00}$	$0.80^{+0.00+0.03}_{-0.01-0.03}$	$0.96^{+0.01+0.03}_{-0.01-0.02}$
$V^{B_sD_s^*(2S)}$	$0.26^{+0.03+0.04}_{-0.03-0.04}$	$0.28^{+0.00+0.02}_{-0.09-0.10}$	$0.25^{+0.02+0.04}_{-0.01-0.04}$	$0.30^{+0.03+0.04}_{-0.01-0.00}$
$A_0^{B_sD_s^*(2S)}$	$0.31^{+0.02+0.01}_{-0.03-0.02}$	$0.33^{+0.00+0.08}_{-0.01-0.07}$	$0.21^{+0.01+0.03}_{-0.05-0.00}$	$-0.09^{+0.04+0.10}_{-0.02-0.14}$
$A_1^{B_sD_s^*(2S)}$	$0.21^{+0.02+0.03}_{-0.03-0.03}$	$0.20^{+0.00+0.01}_{-0.07-0.06}$	$-0.16^{+0.20+0.04}_{-0.17-0.02}$	$0.12^{+0.00+0.12}_{-0.01-0.07}$
$A_2^{B_sD_s^*(2S)}$	$-0.01^{+0.02+0.06}_{-0.02-0.06}$	$0.01^{+0.00+0.03}_{-0.01-0.01}$	$-4.03^{+0.51+0.75}_{-0.47-1.38}$	$4.31^{+0.00+2.13}_{-0.00-2.44}$

Transition Form Factors

图: The q^2-dependence of the $B_{(s)} \to D_{(s)}(1S, 2S)$ transtion form factors.

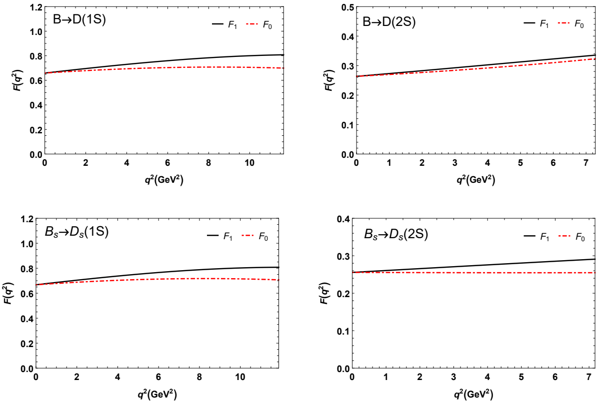
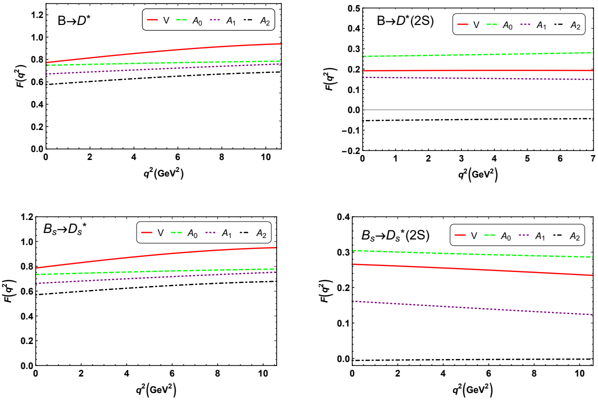


图: The q^2-dependence of the $B_{(s)} \to D^*_{(s)}(1S, 2S)$ transtion form factors.



Semi-leptonic Decays

$$B \to D^{(*)}(1S, 2S)\ell\nu_\ell$$

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

The differential decay widths of the semileptonic $B_{(s)}$ decays can be obtained by the combinations of the helicity amplitudes via the form factors

$$\frac{d\Gamma(B_{(s)} \to D_{(s)}\ell\nu)}{dq^2} = (1-\hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(q^2)}\, G_F^2 |V_{cb}|^2}{384 m_{B_{(s)}}^3 \pi^3} \Bigg\{ (\hat{m}_\ell^2 + 2)\, \lambda(q^2) F_1^2(q^2) + 3\hat{m}_\ell^2 (m_{B_{(s)}}^2 - m_{D_{(s)}}^2)^2 F_0^2(q^2) \Bigg\},$$

$$\frac{d\Gamma_L(B_{(s)} \to D_{(s)}^*\ell\nu)}{dq^2} = (1-\hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(q^2)}\, G_F^2 |V_{cb}|^2}{384 m_{B_{(s)}}^3 \pi^3} \Bigg\{ 3\hat{m}_\ell^2 \lambda(q^2) A_0^2(q^2) + (\hat{m}_\ell^2 + 2) \times \Bigg| \frac{1}{2m_{D_{(s)}^*}} \Big[(m_{B_{(s)}}^2 - m_{D_{(s)}^*}^2 - q^2)(m_{B_{(s)}} + m_{D_{(s)}^*}) A_1(q^2) - \frac{\lambda(q^2)}{m_{B_{(s)}} + m_{D_{(s)}^*}} A_2(q^2) \Big] \Bigg|^2 \Bigg\},$$

$$\frac{d\Gamma^{\pm}(B_{(s)} \to D^*_{(s)}\ell\nu)}{dq^2} = (1-\hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(q^2)}G_F^2|V_{cb}|^2}{384 m_{B_{(s)}}^3 \pi^3} \Bigg\{ (m_\ell^2 + 2q^2)\lambda(q^2) \times \left| \frac{V(q^2)}{m_{B_{(s)}} + m_{D^*_{(s)}}} \mp \frac{(m_{B_{(s)}} + m_{D^*_{(s)}})A_1(q^2)}{\sqrt{\lambda(q^2)}} \right|^2 \Bigg\},$$

where $\lambda(q^2) = (m_{B_{(s)}}^2 + m_{D^*_{(s)}}^2 - q^2)^2 - 4m_{B_{(s)}}^2 m_{D^*_{(s)}}^2$, $\hat{m}_\ell^2 = m_\ell^2/q^2$ and m_ℓ is the mass of the lepton ℓ with $\ell = e, \mu, \tau$. The combined transverse and total differential decay widths are defined as

$$\frac{d\Gamma_T}{dq^2} = \frac{d\Gamma_+}{dq^2} + \frac{d\Gamma_-}{dq^2}, \quad \frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2}. \tag{5}$$

For the $B_{(s)} \to D^*_{(s)}(1S,2S)\ell\nu_\ell$ decays, defining the polarization fraction is important owing to the existence of different polarizations

$$f_L = \frac{\Gamma_L}{\Gamma_L + \Gamma_+ + \Gamma_-}. \tag{6}$$

The analytical expression of the forward-backward asymmetry is defined as

$$A_{FB} = \frac{\int_0^1 \frac{d\Gamma}{dcos\theta} dcos\theta - \int_{-1}^0 \frac{d\Gamma}{dcos\theta} dcos\theta}{\int_{-1}^1 \frac{d\Gamma}{dcos\theta} dcos\theta} = \frac{\int b_\theta(q^2)dq^2}{\Gamma_{B\to D^{(*)}\ell\nu_\ell}}. \tag{7}$$

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays $B \to D(1S)\ell^+\nu_\ell(\%)$.

Modes	$\mathcal{B}(B^0 \to D^-(1S)e^+\nu_e)$	$\mathcal{B}(B^0 \to D^-(1S)\mu^+\nu_\mu)$	$\mathcal{B}(B^0 \to D^-(1S)\tau^+\nu_\tau)$
This work(CLFQM)	$1.96^{+0.06}_{-0.06}(\sim 1.5\sigma)$	$1.96^{+0.06}_{-0.06}(\sim 1.5\sigma)$	$0.51^{+0.02}_{-0.02}(\sim 2.2\sigma)$
PQCD	2.19	2.19	0.82
PQCD+Lattice	1.95	1.95	0.62
CQM	2.74	2.74	0.73
HQET	–	–	0.64
PQCD	2.03	2.03	0.87
CPQCD	–	1.65	0.554
PDG	2.10 ± 0.07	2.10 ± 0.07	0.98 ± 0.21
Modes	$\mathcal{B}(B^+ \to D^0(1S)e^+\nu_e)$	$\mathcal{B}(B^+ \to D^0(1S)\mu^+\nu_\mu)$	$\mathcal{B}(B^+ \to D^0(1S)\tau^+\nu_\tau)$
This work(CLFQM)	$2.11^{+0.06}_{-0.06}(\sim 1.6\sigma)$	$2.11^{+0.06}_{-0.06}(\sim 1.6\sigma)$	$0.55^{+0.02}_{-0.02}(\sim 0.9\sigma)$
RQM	2.53	2.53	0.68
PQCD	2.29	2.29	0.86
PQCD+Lattice	2.10	2.10	0.69
LQCD	–	–	0.65
HQET	–	–	0.66
PQCD	2.19	2.19	0.95
PDG	2.26 ± 0.07	2.26 ± 0.07	0.77 ± 0.25

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays $B_s \to D_s(1S)\ell^+\nu_\ell(\%)$.

Modes	$\mathcal{B}(B_s^0 \to D_s^-(1S)e^+\nu_e)$	$\mathcal{B}(B_s^0 \to D_s^-(1S)\mu^+\nu_\mu)$	$\mathcal{B}(B_s^0 \to D_s^-(1S)\tau^+\nu_\tau)$
This work(CLFQM)	$2.05^{+0.12}_{-0.13}$	$2.04^{+0.11}_{-0.11}(\sim 1.1\sigma)$	$0.53^{+0.04}_{-0.07}$
NCQM	2.32	–	0.67
LCSR	1.817	1.817	0.606
QCDSR	2.03	2.03	–
LCSR	1.0	1.0	0.33
pQCD	2.13	2.13	0.84
RQM	2.1	2.1	0.62
CQM	2.73 – 3.00	2.73 – 3.00	–
pQCD	1.97	1.97	0.72
pQCD+Lattice	1.84	1.84	0.63
LQCD	2.013 – 2.469	2.013 – 2.469	0.619 – 0.724
BS	1.4 – 1.7	1.4 – 1.7	0.47 – 0.55
LQCD	2.31	2.31	0.69
CQM	2.89	2.88	0.78
PDG	–	2.29 ± 0.21	–

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays $B \to D^*(1S)\ell^+\nu_\ell(\%)$.

Modes	$B^0 \to D^{*-}(1S)e^+\nu_e$	$B^0 \to D^{*-}(1S)\mu^+\nu_\mu$	$B^0 \to D^{*-}(1S)\tau^+\nu_\tau$
This work	$4.97^{+0.36}_{-0.16}(\sim 0.3\sigma)$	$4.95^{+0.36}_{-0.16}(\sim 0.3\sigma)$	$1.13^{+0.07}_{-0.04}(\sim 3.1\sigma)$
RQM	6.28	6.28	1.45
PQCD	5.32	5.32	1.53
PQCD+Lattice	4.63	4.63	1.25
PQCD	4.52	4.52	1.36
HQET	–	–	1.29
CQM	6.64	6.64	1.57
CPQCD	–	4.33	1.175
PDG	4.87 ± 0.09	4.87 ± 0.09	1.48 ± 0.09
Modes	$B^+ \to D^{*0}(1S)e^+\nu_e$	$B^+ \to D^{*0}(1S)\mu^+\nu_\mu$	$B^+ \to D^{*0}(1S)\tau^+\nu_\tau$
This work	$5.37^{+0.39}_{-0.17}(\sim 0.3\sigma)$	$5.35^{+0.38}_{-0.17}(\sim 0.3\sigma)$	$1.22^{+0.08}_{-0.04}(\sim 3.1\sigma)$
RQM	6.81	6.77	1.52
PQCD	5.53	5.53	1.60
PQCD+Lattice	4.89	4.89	1.34
PQCD	4.87	4.87	1.47
HQET	–	–	1.34
PDG	5.26 ± 0.10	5.26 ± 0.10	1.88 ± 0.20

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays $B_s \to D_s^*(1S)\ell^+\nu_\ell(\%)$.

Modes	$\mathcal{B}(B_s^0 \to D_s^{*-}(1S)e^+\nu_e)$	$\mathcal{B}(B_s^0 \to D_s^{*-}(1S)\mu^+\nu_\mu)$	$\mathcal{B}(B_s^0 \to D_s^{*-}(1S)\tau^+\nu_\tau)$
This work(CLFQM)	$5.02^{+0.48}_{-0.54}$	$5.00^{+0.48}_{-0.53}(\sim 0.3\sigma)$	$1.13^{+0.09}_{-0.10}$
NCQM	6.26	–	1.53
PQCD	4.76	4.76	1.44
RQM	5.3	5.3	1.3
RQM	2.54	2.54	0.70
CQM	$7.49-7.66$	$7.49-7.66$	–
pQCD	5.04	5.04	1.45
pQCD+Lattice	4.42	4.42	1.20
BS	$5.1-5.8$	$5.1-5.8$	$1.2-1.3$
LQCD	5.25	5.25	1.31
CQM	$1.89-6.61$	$1.89-6.61$	–
CQM	6.42	6.39	1.53
PDG	–	5.2 ± 0.5	–

- The branching ratios of the decays $B \to D^{(*)}(2S)\ell\nu_\ell(10^{-3})$.

Modes	This work	BS	Modes	This work	BS
$\mathcal{B}(B^0 \to D^-(2S)e^+\nu_e)$	$1.44^{+0.00+0.18+0.15}_{-0.00-0.19-0.19}$	0.139	$\mathcal{B}(B^+ \to D^0(2S)e^+\nu_e)$	$1.55^{+0.00+0.19+0.17}_{-0.00-0.21-0.20}$	0.15
$\mathcal{B}(B^0 \to D^-(2S)\mu^+\nu_\mu)$	$1.43^{+0.00+0.18+0.15}_{-0.00-0.19-0.19}$	0.138	$\mathcal{B}(B^+ \to D^0(2S)\mu^+\nu_\mu)$	$1.54^{+0.00+0.19+0.17}_{-0.00-0.21-0.20}$	0.149
$\mathcal{B}(B^0 \to D^-(2S)\tau^+\nu_\tau)$	$0.18^{+0.00+0.03+0.03}_{-0.00-0.03-0.03}$	0.0135	$\mathcal{B}(B^+ \to D^0(2S)\tau^+\nu_\tau)$	$0.19^{+0.00+0.03+0.03}_{-0.00-0.04-0.04}$	0.0149
$\mathcal{B}(B^0 \to D^{*-}(2S)e^+\nu_e)$	$2.67^{+0.00+0.20+0.11}_{-0.00-0.24-0.14}$	0.1942	$\mathcal{B}(B^+ \to D^{*0}(2S)e^+\nu_e)$	$2.93^{+0.00+0.21+0.13}_{-0.00-0.26-0.16}$	0.2123
$\mathcal{B}(B^0 \to D^{*-}(2S)\mu^+\nu_\mu)$	$2.65^{+0.00+0.20+0.11}_{-0.00-0.24-0.14}$	0.1932	$\mathcal{B}(B^+ \to D^{*0}(2S)\mu^+\nu_\mu)$	$2.91^{+0.00+0.21+0.13}_{-0.00-0.26-0.16}$	0.2113
$\mathcal{B}(B^0 \to D^{*-}(2S)\tau^+\nu_\tau)$	$0.19^{+0.00+0.02+0.10}_{-0.00-0.02-0.11}$	0.0137	$\mathcal{B}(B^+ \to D^{*0}(2S)\tau^+\nu_\tau)$	$0.21^{+0.00+0.02+0.11}_{-0.00-0.03-0.11}$	0.0155

- The branching ratios of the decays $B_s \to D_s^{(*)}(2S)\ell\nu_\ell(10^{-3})$.

Modes	This work	BS	RQM	Modes	This work	BS	RQM
$\mathcal{B}(B_s^0 \to D_s^-(2S)e^+\nu_e)$	$1.35^{+0.00+0.25+0.14}_{-0.00-0.23-0.22}$	0.314	2.7	$\mathcal{B}(B_s^0 \to D_s^{*-}(2S)e^+\nu_e)$	$2.25^{+0.01+1.09+0.64}_{-0.01-0.70-0.80}$	0.5873	3.8
$\mathcal{B}(B_s^0 \to D_s^-(2S)\mu^+\nu_\mu)$	$1.34^{+0.00+0.25+0.14}_{-0.00-0.23-0.22}$	0.312	–	$\mathcal{B}(B_s^0 \to D_s^{*-}(2S)\mu^+\nu_\mu)$	$2.23^{+0.01+1.09+0.64}_{-0.01-0.70-0.80}$	0.5842	–
$\mathcal{B}(B_s^0 \to D_s^-(2S)\tau^+\nu_\tau)$	$0.14^{+0.00+0.03+0.02}_{-0.00-0.03-0.03}$	0.0244	0.11	$\mathcal{B}(B_s^0 \to D_s^{*-}(2S)\tau^+\nu_\tau)$	$0.15^{+0.00+0.05+0.06}_{-0.00-0.07-0.06}$	0.0405	0.15

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- For the decays $B_{u,d} \to D\ell'\nu_{\ell'}$ and $B_s \to D_s\ell'\nu_{\ell'}$ with $\ell' = e, \mu$, their branching ratios deviate from the PDG data by about 1.5σ and 1.1σ. For the decays $B_{u,d} \to D^*\ell'\nu_{\ell'}$ and $B_s \to D_s^*\ell'\nu_{\ell'}$, the deviations are about 0.3σ.
- The branching ratios of all the considered decays with $\tau\nu_\tau$ involved exihibit systematically smaller than the experimental values. The deviations between our predictions and the data for the decays $B^0 \to D^{(*)-}\tau^+\nu_\tau$, $B^+ \to D^{*0}\tau^+\nu_\tau$ exceed 2σ or even 3σ.
- The branching ratios of the decays $B \to D^{(*)}(2S)\ell\nu_\ell$ lie in the range $10^{-4} \sim 10^{-3}$, which are about one order larger than those predicted by BS equation. For the decays $B_s \to D_s^{(*)}(2S)\ell\nu_\ell$, our predictions are $3 \sim 5$ times of those given by BS equation, while are consistent with RQM calculations.

Semi-leptonic Decays $B\to D^{(*)}(1S,2S)\ell\nu_\ell$

- Forward-backward asymmetries A_{FB} for the decays $B\to D^{(*)}(1S)\ell\nu_\ell$.

Channels	$B^0\to D^-(1S)e^+\nu_e$	$B^0\to D^-(1S)\mu^+\nu_\mu$	$B^0\to D^-(1S)\tau^+\nu_\tau$
A_{FB}	$4.4^{+0.0+0.0+0.1}_{-0.0-0.0-0.1}\times10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.36^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$
CQM	-11.7×10^{-7}	–	-0.36
PQCD	–	–	0.35
PQCD+Lattice	–	–	0.36
Channels	$B^+\to D^0(1S)e^+\nu_e$	$B^+\to D^0(1S)\mu^+\nu_\mu$	$B^+\to D^0(1S)\tau^+\nu_\tau$
A_{FB}	$4.4^{+0.00+0.00+0.01}_{-0.00-0.00-0.00}\times10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.36^{+0.00+0.04+0.01}_{-0.00-0.05-0.01}$
RQM	-9.8×10^{-7}	-0.013	-0.37
Channels	$B_s^0\to D_s^-(1S)e^+\nu_e$	$B_s^0\to D_s^-(1S)\mu^+\nu_\mu$	$B_s^0\to D_s^-(1S)\tau^+\nu_\tau$
A_{FB}	$4.4^{+0.0+0.0+0.1}_{-0.0-0.0-0.1}\times10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.36^{+0.00+0.04+0.02}_{-0.00-0.02-0.01}$
RQM	-9.7×10^{-7}	-0.013	-0.36
PQCD	–	–	0.36
PQCD+Lattice	–	–	0.36

Channels	$B^0\to D^{*-}(1S)e^+\nu_e$	$B^0\to D^{*-}(1S)\mu^+\nu_\mu$	$B^0\to D^{*-}(1S)\tau^+\nu_\tau$
A_{FB}	$-0.20^{+0.00+0.00+0.00}_{-0.00-0.01-0.01}$	$-0.20^{+0.00+0.00+0.00}_{-0.00-0.01-0.01}$	$-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.04}$
CQM	0.19	–	0.027
PQCD	–	–	-0.085
PQCD+Lattice	–	–	-0.054
HQET	–	–	-0.084
Channels	$B^+\to D^{*0}(1S)e^+\nu_e$	$B^+\to D^{*0}(1S)\mu^+\nu_\mu$	$B^+\to D^{*0}(1S)\tau^+\nu_\tau$
A_{FB}	$-0.20^{+0.00+0.01+0.00}_{-0.00-0.00-0.01}$	$-0.20^{+0.00+0.01+0.00}_{-0.00-0.00-0.01}$	$-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.04}$
RQM	-0.22	-0.23	-0.32
Channels	$B_s^0\to D_s^{*-}(1S)e^+\nu_e$	$B_s^0\to D_s^{*-}(1S)\mu^+\nu_\mu$	$B_s^0\to D_s^{*-}(1S)\tau^+\nu_\tau$
A_{FB}	$-0.20^{+0.00+0.01+0.02}_{-0.00-0.00-0.01}$	$-0.20^{+0.00+0.01+0.02}_{-0.00-0.00-0.01}$	$-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.01}$
RQM	-0.26	-0.27	-0.32
PQCD	–	–	-0.083
PQCD+Lattice	–	–	-0.050

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- Forward-backward asymmetries A_{FB} for the decays $B \to D^{(*)}(2S)\ell\nu_\ell$.

Channel	$B^0 \to D^-(2S)e^+\nu_e$	$B^0 \to D^-(2S)\mu^+\nu_\mu$	$B^0 \to D^-(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$6.6^{+0.0+0.8+0.8}_{-0.0-0.9-0.7} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.35^{+0.00+0.05+0.05}_{-0.00-0.06-0.06}$
Channels	$B^+ \to D^0(2S)e^+\nu_e$	$B^+ \to D^0(2S)\mu^+\nu_\mu$	$B^+ \to D^0(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$6.6^{+0.0+0.8+0.7}_{-0.0-0.9-0.8} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.35^{+0.00+0.05+0.05}_{-0.00-0.06-0.06}$
Channels	$B_s^0 \to D_s^-(2S)e^+\nu_e$	$B_s^0 \to D_s^-(2S)\mu^+\nu_\mu$	$B_s^0 \to D_s^-(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$6.6^{+0.0+1.1+0.6}_{-0.0-1.0-1.0} \times 10^{-7}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	$0.37^{+0.00+0.07+0.06}_{-0.00-0.08-0.08}$
Channels	$B^0 \to D^{*-}(2S)e^+\nu_e$	$B^0 \to D^{*-}(2S)\mu^+\nu_\mu$	$B^0 \to D^{*-}(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.11^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.05^{+0.00+0.00+0.03}_{-0.00-0.02-0.03}$
Channels	$B^+ \to D^{*0}(2S)e^+\nu_e$	$B^+ \to D^{*0}(2S)\mu^+\nu_\mu$	$B^+ \to D^{*0}(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$	$-0.05^{+0.00+0.00+0.03}_{-0.00-0.02-0.03}$
Channels	$B_s^0 \to D_s^{*-}(2S)e^+\nu_e$	$B_s^0 \to D_s^{*-}(2S)\mu^+\nu_\mu$	$B_s^0 \to D_s^{*-}(2S)\tau^+\nu_\tau$
$\mathcal{A}_{\rm FB}$	$-0.13^{+0.00+0.04+0.06}_{-0.00-0.06-0.05}$	$-0.13^{+0.00+0.04+0.06}_{-0.00-0.06-0.05}$	$-0.07^{+0.00+0.02+0.04}_{-0.00-0.06-0.03}$

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The values of $\mathcal{A}_{FB}(B_{(s)} \to D_{(s)}\ell\nu_\ell)$ exhibit a clear hierarchical relationship, While the difference among $\mathcal{A}_{FB}(B_{(s)} \to D^*_{(s)}\ell\nu_\ell)$ is not significant.
- $\mathcal{A}_{\mathsf{FB}}(B_{(s)} \to D_{(s)}\ell\nu_\ell)$ are positive, while $\mathcal{A}_{\mathsf{FB}}(B_{(s)} \to D^*_{(s)}\ell\nu_\ell)$ are negative.
- Our predictions are consistent with those given by the PQCD, PQCD+Lattice and HQET approaches, while have the opposite sign compared to the calculations in CQM and RQM except for those for the decays $B_{(s)} \to D^*_{(s)}\ell\nu_\ell$.
- The forward-backward asymmetries of the decays $B_{(s)} \to D^{(*)}_{(s)}(2S)\ell\nu_\ell$ are similar with those of the corresponding decays $B_{(s)} \to D^{(*)}_{(s)}(1S)\ell\nu_\ell$.

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- The longitudinal polarization fractions f_L for the decays $B_{(s)} \to D^*_{(s)}(nS)\ell\nu_\ell$ in Region 1, Region 2 and total physical region.

Observables	Region 1	Region 2	Total	Observables	Region 1	Region 2	Total
$f_L(B^0 \to D^{*-}(1S)l'^+\nu_{l'})$	0.35	0.20	$0.55^{+0.00+0.00+0.03}_{-0.00-0.04-0.02}$	$f_L(B^0 \to D^{*-}(2S)\ell'^+\nu_{\ell'^+})$	0.50	0.18	$0.68^{+0.00+0.06+0.03}_{-0.00-0.05-0.03}$
$f_L(B^0 \to D^{*-}(1S)\tau^+\nu_\tau)$	0.20	0.25	$0.45^{+0.00+0.00+0.01}_{-0.00-0.03-0.02}$	$f_L(B^0 \to D^{*-}(2S)\tau^+\nu_\tau)$	0.22	0.29	$0.51^{+0.00+0.03+0.01}_{-0.00-0.06-0.02}$
PQCD	–	–	0.42	–	–	–	–
PQCD+Lattice	–	–	0.43	–	–	–	–
SM	–	–	0.457	–	–	–	–
SM	–	–	0.441	–	–	–	–
Belle	–	–	$0.60 \pm 0.08 \pm 0.04$	–	–	–	–
LHCb	–	–	$0.41 \pm 0.06 \pm 0.03$	–	–	–	–
Observables	Region 1	Region 2	Total	Observables	Region 1	Region 2	Total
$f_L(B^+ \to D^{*0}(1S)\ell'^+\nu_{\ell'^+})$	0.35	0.20	$0.55^{+0.00+0.00+0.03}_{-0.00-0.03-0.02}$	$f_L(B^+ \to D^{*0}(2S)\ell'^+\nu_{\ell'^+})$	0.49	0.18	$0.68^{+0.00+0.05+0.03}_{-0.00-0.06-0.03}$
RQM	–	–	0.55	–	–	–	–
$f_L(B^+ \to D^{*0}(1S)\tau^+\nu_\tau)$	0.21	0.25	$0.46^{+0.00+0.00+0.01}_{-0.00-0.03-0.06}$	$f_L(B^+ \to D^{*0}(2S)\tau^+\nu_\tau)$	0.22	0.29	$0.51^{+0.00+0.4+0.03}_{-0.00-0.06-0.03}$
RQM	–	–	0.47	–	–	–	–
Observables	Region 1	Region 2	Total	Observables	Region 1	Region 2	Total
$f_L(B^0_s \to D^{*-}_s(1S)\ell'^+\nu_{\ell'^+})$	0.34	0.20	$0.54^{+0.00+0.04+0.05}_{-0.00-0.03-0.07}$	$f_L(B^0_s \to D^{*-}_s(2S)\ell'^+\nu_{\ell'})$	0.51	0.17	$0.68^{+0.00+0.31+0.15}_{-0.00-0.20-0.22}$
RQM	–	–	0.49	–	–	–	–
$f_L(B^0_s \to D^{*-}_s(1S)\tau^+\nu_\tau)$	0.19	0.25	$0.45^{+0.00+0.00+0.05}_{-0.00-0.02-0.03}$	$f_L(B^0_s \to D^{*-}_s(2S)\tau^+\nu_\tau)$	0.23	0.28	$0.51^{+0.01+0.12+0.11}_{-0.01-0.18-0.11}$
PQCD	–	–	0.42	–	–	–	–
PQCD+Lattice	–	–	0.43	–	–	–	–
RQM	–	–	0.42	–	–	–	–

Semi-leptonic Decays $B \to D^{(*)}(1S, 2S)\ell\nu_\ell$

- Region 1 $m_\ell^2 < q^2 < \frac{(m_B - m_{D^*(nS)})^2 + m_\ell^2}{2}$
  Region 2 $\frac{(m_B - m_{D^*(nS)})^2 + m_\ell^2}{2} < q^2 < (m_B - m_{D^*(nS)})^2$ with $n = 1, 2$.
- For the decays $B_{(s)} \to D^*_{(s)}(nS)\ell'\nu_{\ell'}$, the longitudial polarization fractions from Region 1 are larger than those from Region 2, while it is contrary to the decays $B_{(s)} \to D^*_{(s)}(nS)\tau\nu_\tau$.
- Relations $f_L(B_{(s)} \to D^*_{(s)}(nS)\ell'\nu_{\ell'}) > f_L(B_{(s)} \to D^*_{(s)}(nS)\tau\nu_\tau), f_L(B_{(s)} \to D^*_{(s)}(2S)\ell\nu_\ell) > f_L(B_{(s)} \to D^*_{(s)}(1S)\ell\nu_\ell)$.
- Our predictions for the considered decays are consistent with other theoretical calculations. $f_L(B^0 \to D^{*-}(1S)\tau\nu_\tau)$ is in good agreement with the experimental data given by LHCb, but smaller than Belle measurement.

Non-leptonic Decays

$$B \to D^{(*)}(1S, 2S)M$$

Combining with the form factors, we can obtain the partial widths for our considered non-leptonic decays, which are written as

$$\Gamma\left(B \to DP\right) = \frac{G_F^2 \left(m_B^2 - m_D^2\right)^2 |\vec{p}_c|}{16\pi m_B^2} \left| V_{uq}^* V_{cb} \right|^2 |a_1|^2 f_P^2 F_0^2 \left(m_P^2\right), \quad (8)$$

$$\Gamma\left(B \to D^* P\right) = \frac{G_F^2 |\vec{p}_c|^3}{4\pi} \left| V_{uq}^* V_{cb} \right|^2 |a_1|^2 f_P^2 A_0^2 \left(m_P^2\right), \quad (9)$$

$$\Gamma\left(B \to DV\right) = \frac{G_F^2 |\vec{p}_c|^3}{4\pi} \left| V_{uq}^* V_{cb} \right|^2 |a_1|^2 f_V^2 F_+^2 \left(m_V^2\right), \quad (10)$$

$$\Gamma\left(B \to D^* V\right) = \frac{G_F^2 |\vec{p}_c|}{16\pi m_B^2} \left| V_{uq}^* V_{cb} \right|^2 \left(|H_0|^2 + |H_+|^2 + |H_-|^2\right), \quad (11)$$

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

Here $\vec{p}_c$ is the momentum of either of the two final state meson in the B rest frame and $H_{0,\pm}$ are the helicity amplitudes,

$$
\begin{aligned}
H_0 = & \frac{\mathrm{i} f_V a_1}{2m_{D^*}} \Big[\left(m_B^2 - m_{D^*}^2 - m_V^2\right)\left(m_B + m_{D^*}\right) A_1^{BD^*}\left(m_V^2\right) \\
& - \frac{4m_B^2 p_c^2}{m_B + m_{D^*}} A_2^{BD^*}\left(m_V^2\right) \Big], \qquad (12)
\end{aligned}
$$

$$
H_\pm = \mathrm{i} f_V m_V a_1 \left[-\left(m_B + m_{D^*}\right) A_1^{BD^*}\left(m_V^2\right) \mp \frac{2m_B p_c}{m_B + m_{D^*}} V^{BD^\star}\left(m_V^2\right) \right]. \qquad (13)
$$

The polarization fractions are defined as $f_{L,\parallel,\perp} = \frac{H_{0,\parallel,\perp}}{H_0 + H_\parallel + H_\perp}$, where $H_\parallel$ and $H_\perp$ are parallel and perpendicular amplitudes, respectively, and can be obtained through $H_{\parallel,\perp} = \frac{(H_- \pm H_+)}{\sqrt{2}}$.

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

- The branching ratios of the decays $B \to D(1S, 2S)M$ with $M = \pi, K, \rho, K^*$.

Modes	This work	Bethe-Salpeter	QCDF	PQCD	Exp.	Unit
$\bar{B}^0 \to D^+(1S)\pi^-$	$4.37^{+0.01+0.01+0.01}_{-0.01-0.09-0.01}$	3.24	3.93	2.69	2.52	
$\bar{B}^0 \to D^+(1S)K^-$	$0.34^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$	0.245	0.30	0.243	0.186	(10^{-3})
$\bar{B}^0 \to D^+(1S)\rho^-$	$10.09^{+0.03+0.10+0.25}_{-0.03-0.20-0.28}$	7.91	10.42	6.96	7.6	
$\bar{B}^0 \to D^+(1S)K^{*-}$	$0.56^{+0.00+0.01+0.01}_{-0.00-0.00-0.01}$	0.431	0.53	0.407	0.45	
$\bar{B}^0 \to D^+(2S)\pi^-$	$4.76^{+0.01+0.48+0.33}_{-0.01-0.62-0.57}$	0.458	–	–	–	
$\bar{B}^0 \to D^+(2S)K^-$	$0.37^{+0.00+0.04+0.04}_{-0.00-0.05-0.03}$	0.034	–	–	–	(10^{-4})
$\bar{B}^0 \to D^+(2S)\rho^-$	$10.54^{+0.03+1.06+0.73}_{-0.03-1.37-1.26}$	1.03	–	–	–	
$\bar{B}^0 \to D^+(2S)K^{*-}$	$0.58^{+0.00+0.06+0.04}_{-0.00-0.08-0.07}$	0.0557	–	–	–	
$B^- \to D^0(1S)\pi^-$	$4.71^{+0.01+0.02+0.12}_{-0.01-0.09-0.13}$	3.49	–	5.11	4.68	
$B^- \to D^0(1S)K^-$	$0.37^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$	0.264	–	0.400	0.363	(10^{-3})
$B^- \to D^0(1S)\rho^-$	$10.88^{+0.03+0.00+0.27}_{-0.03-0.20-0.31}$	8.40	–	11.3	13.4	
$B^- \to D^0(1S)K^{*-}$	$0.60^{+0.00+0.00+0.02}_{-0.00-0.01-0.02}$	0.466	–	0.696	0.53	
$B^- \to D^0(2S)\pi^-$	$5.14^{+0.01+0.52+0.36}_{-0.01-0.67-0.61}$	0.488	–	–	–	
$B^- \to D^0(2S)K^-$	$0.40^{+0.00+0.04+0.03}_{-0.00-0.05-0.05}$	0.0362	–	–	–	(10^{-4})
$B^- \to D^0(2S)\rho^-$	$11.36^{+0.03+1.14+0.79}_{-0.03-1.48-1.35}$	1.09	–	–	–	
$B^- \to D^0(2S)K^{*-}$	$0.62^{+0.00+0.06+0.04}_{-0.00-0.08-0.07}$	0.0595	–	–	–	

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

- The branching ratios of the decays $\bar{B}_s^0 \to D_s^+(1S, 2S)(\pi, \rho, K^{(*)})^-$.

Modes	This work	BS	RQM	NCQM	QCDSRs	Instant-BS	PQCD	QCDF	Exp.	Unit
$\bar{B}_s^0 \to D_s^+(1S)\pi^-$	$4.57^{+0.02+0.04+0.07}_{-0.02-0.09-0.09}$	2.92	3.5	5.3	5	2.7	1.7	4.39	2.98 ± 0.14	
$\bar{B}_s^0 \to D_s^+(1S)K^-$	$0.36^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$	0.221	0.28	0.4	0.4	0.21	0.13	0.33	0.225 ± 0.012	(10^{-3})
$\bar{B}_s^0 \to D_s^+(1S)\rho^-$	$10.55^{+0.04+0.09+0.16}_{-0.04-0.22-0.21}$	7.04	9.4	12.6	13	6.4	4.7	–	6.8 ± 1.4	
$\bar{B}_s^0 \to D_s^+(1S)K^{*-}$	$0.59^{+0.00+0.01+0.01}_{-0.00-0.01-0.01}$	0.392	0.47	0.8	0.6	0.38	0.281	–	–	
$\bar{B}_s^0 \to D_s^+(2S)\pi^-$	$4.70^{+0.02+0.61+0.50}_{-0.02-0.68-0.59}$	1.13	7	–	–	–	–	–	–	
$\bar{B}_s^0 \to D_s^+(2S)K^-$	$0.37^{+0.00+0.05+0.04}_{-0.00-0.05-0.05}$	0.084	0.5	–	–	–	–	–	–	(10^{-4})
$\bar{B}_s^0 \to D_s^+(2S)\rho^-$	$10.43^{+0.04+1.36+1.11}_{-0.04-1.52-1.31}$	2.49	17	–	–	–	–	–	–	
$\bar{B}_s^0 \to D_s^+(2S)K^{*-}$	$0.57^{+0.00+0.07+0.06}_{-0.00-0.08-0.07}$	0.134	0.8	–	–	–	–	–	–	

- The branching ratios (10^{-3}) of the decays $\bar{B}_s^0 \to D_s^{*+}(1S, 2S)(\pi, \rho, K^{(*)})^-$.

Modes	This work	RQM	QCDSR	3P QCDSR	PQCD	BS	RIQ	Exp.
$\bar{B}_s^0 \to D_s^{*+}(1S)\pi^-$	$4.04^{+0.02+0.03+0.14}_{-0.02-0.09-0.15}$	2.7	2	2.11	1.89	3.37	–	1.9
$\bar{B}_s^0 \to D_s^{*+}(1S)K^-$	$0.31^{+0.00+0.00+0.01}_{-0.00-0.00-0.01}$	0.21	0.2	0.159	0.164	0.249	–	0.132
$\bar{B}_s^0 \to D_s^{*+}(1S)\rho^-$	$11.22^{+0.05+0.08+0.36}_{-0.05-0.25-0.38}$	8.7	13	–	5.23	7.26	11.73	9.5
$\bar{B}_s^0 \to D_s^{*+}(1S)K^{*-}$	$0.66^{+0.00+0.00+0.02}_{-0.00-0.01-0.02}$	0.48	0.56	0.163	0.322	0.688	0.69	–
$\bar{B}_s^0 \to D_s^{*+}(2S)\pi^-$	$0.61^{+0.00+0.10+0.06}_{-0.00-0.11-0.09}$	0.8	–	–	–	0.108	–	–
$\bar{B}_s^0 \to D_s^{*+}(2S)K^-$	$0.05^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	0.05	–	–	–	0.00777	–	–
$\bar{B}_s^0 \to D_s^{*+}(2S)\rho^-$	$1.51^{+0.00+0.27+0.19}_{-0.00-0.29-0.25}$	2.2	–	–	–	0.0475	1.05	–
$\bar{B}_s^0 \to D_s^{*+}(2S)K^{*-}$	$0.09^{+0.00+0.02+0.01}_{-0.00-0.02-0.01}$	0.08	–	–	–	0.00332	0.06	–

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

- Polarization fractions (10^{-2}) of the decays $B_{(s)} \to D^{*}_{(s)}(1S, 2S)(\rho, K^{*})$.

Channels	$\bar{B}^0 \to D^{*+}(1S)\rho^-$	$\bar{B}^0 \to D^{*+}(1S)K^{*-}$	$\bar{B}^0 \to D^{*+}(2S)\rho^-$	$\bar{B}^0 \to D^{*+}(2S)K^{*-}$
$f_L[\%]$	90.59	88.11	93.52	91.56
PDG	88.5 ± 2.0	92^{+38}_{-32}	-	-
$f_{\|\|}[\%]$	8.27	10.47	6.42	8.36
Channel	$B^- \to D^{*0}(1S)\rho^-$	$B^- \to D^{*0}(1S)K^{*-}$	$B^- \to D^{*0}(2S)\rho^-$	$B^- \to D^{*0}(2S)K^{*-}$
$f_L[\%]$	90.60	88.12	93.25	91.21
PDG	89.2 ± 2.4	86.0 ± 6.7	-	-
$f_{\|\|}[\%]$	8.26	10.46	6.68	8.70
Channels	$\bar{B}^0_s \to D^{*+}_s(1S)\rho^-$	$\bar{B}^0_s \to D^{*+}_s(1S)K^{*-}$	$\bar{B}^0_s \to D^{*+}_s(2S)\rho^-$	$\bar{B}^0_s \to D^{*+}_s(2S)K^{*-}$
$f_L[\%]$	90.68	88.22	92.68	90.52
BS+FA	87.40	84.10	—	—
BS+PQCD	85.40	85.70	—	—
PQCD	87	83	—	—
Bell	105^{+6}_{-11}	—	—	—
$f_{\|\|}[\%]$	8.22	10.41	7.32	9.48
BS+FA	10.40	13.30	—	—
BS+PQCD	11.30	10.40	—	—

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

- Although there exist significant difference between the form factors of the transitions $B_{(s)} \to D^*_{(s)}(1S)$ and $B_{(s)} \to D^*_{(s)}(2S)$, the similar polarization behaviors can be observed in these decays $B_{(s)} \to D^*_{(s)}(1S, 2S)(\rho, K^*)$.
- The longitudinal polarization is dominant, reaching approximately 90%, while the transverse parallel and perpendicular polarization factions are only a few percent or roughly 10%.

Summary

- The form factors of the transitions $B_{(s)} \to D_{(s)}(2S), D^*_{(s)}(2S)$ are much smaller than those of the corrresponding transtions $B_{(s)} \to D_{(s)}(1S), D^*_{(s)}(1S)$ because of the different structures of the wave functions between the ground and radilly excited charmed mesons.
- The branching ratios of the decays $B_{(s)} \to D_{(s)}\ell'\nu_{\ell'}$ and $B_{(s)} \to D^*_{(s)}\ell'\nu_{\ell'}$ are consistent well with the experimental data, while those of the decays $B_{(s)} \to D_{(s)}\tau\nu_\tau$ and $B_{(s)} \to D^*_{(s)}\tau\nu_\tau$ are systematically smaller than the data.
- The branching ratios of the decays $B_{(s)} \to D_{(s)}(2S)\ell\nu_\ell$ and $B_{(s)} \to D^*_{(s)}(2S)\ell\nu_\ell$ lie in the range $10^{-4} \sim 10^{-3}$, which can be observed by current LHCb and Belle II experiments.
- There exist similar forward-backward asymmetry (polarization) characteristics between the decays $B \to D^{(*)}(1S)\ell\nu_\ell$ and $B \to D^{(*)}(2S)\ell\nu_\ell$.

Summary

- Except for the branching ratios of the neutral decays $B^0_{(s)} \to D^{(*)\pm}_{(s)}(1S)(\pi, \rho, K^{(*)})^\mp$, which have some excess compared to the experimental data, overall our predicitions for the decays $B_{(s)} \to D^{(*)}_{(s)}(1S)(\pi, \rho, K^{(*)})$ are consistent with the experimental measurements.
- Most the branching ratios of the decays $B_{(s)} \to D^{(*)}_{(s)}(2S)(\pi, \rho, K^{(*)})$ lie in the range of 10^{-5} to 10^{-4}, which are likely to be detected by the present LHCb and Belle II experiments. Our preditions for these decays are larger than the results given by the BS equation, but agree well with the RQM and RIQ calculations.
- Although there exist obvious difference in the branching ratios between the decays $B_{(s)} \to D^*_{(s)}(1S)(\rho, K^*)$ and $B_{(s)} \to D^*_{(s)}(2S)(\rho, K^*)$, the similar polarization behavious can be observed in these decays $B_{(s)} \to D^*_{(s)}(1S, 2S)(\rho, K^*)$.

Thank you for your attention!

- $f_D = 0.2058 \pm 0.0089$ GeV, $f_{D_s} = 0.2499 \pm 0.0005$ Gev, $f_{D^*} = 0.245^{+0.023}_{-0.022}$ GeV, $f_{D_s^*} = 0.272^{+0.039}_{-0.038}$ GeV.

The effective Hamiltonian for the decays $B_{(s)} \to D_{(s)}^{(*)}(1S, 2S) M$ with $M = \pi, K, \rho, K^*$ can be written as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{uq} \{ C_1 Q_1 + C_2 Q_2 \} , \tag{14}$$

The local tree four-quark operators $Q_{1,2}$ are defined by :

$$Q_1 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha] , \tag{15}$$

$$Q_2 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\beta] , \tag{16}$$

where α and β are color indices.

- As one of the pillars of the SM, the ratio $R(D^{(*)})$ is a powerful test of the LFU,

$$\mathcal{R}\left(D^{(*)}\right)=\frac{Br\left(B\to D^{(*)}\tau\nu\right)}{Br\left(B\to D^{(*)}\ell'\nu_{\ell'}\right)}. \tag{17}$$

	$\mathcal{R}\left(D^{-}(1S)\right)$	$\mathcal{R}\left(D^{0}(1S)\right)$	$\mathcal{R}\left(D^{*-}(1S)\right)$	$\mathcal{R}\left(D^{*0}(1S)\right)$
This work	$0.260\pm0.014(\sim3.1\sigma)$	0.261 ± 0.013	$0.228\pm0.026(\sim2.1\sigma)$	0.228 ± 0.026
RQM	–	0.269	0.231	0.224
PQCD+Lattice	0.318	0.329	0.270	0.274
PQCD	0.429	0.434	0.301	0.302
Belle-II	0.418	–	0.306	–
LHCb	0.249	–	0.402	–
HFLAF	0.347 ± 0.025	-	0.288 ± 0.012	-
	$\mathcal{R}\left(D^{-}(2S)\right)$	$\mathcal{R}\left(D^{0}(2S)\right)$	$\mathcal{R}\left(D^{*-}(2S)\right)$	$\mathcal{R}\left(D^{*0}(2S)\right)$
This work	0.125 ± 0.037	0.123 ± 0.039	0.072 ± 0.0431	0.072 ± 0.043
BS	0.097	0.099	0.071	0.073
	$\mathcal{R}\left(D_s^{-}(1S)\right)$	$\mathcal{R}\left(D_s^{*-}(1S)\right)$	$\mathcal{R}\left(D_s^{-}(2S)\right)$	$\mathcal{R}\left(D_s^{*-}(2S)\right)$
This work	0.259 ± 0.033	0.226 ± 0.031	0.104 ± 0.038	0.067 ± 0.052
RQM	0.276	0.276	–	–
RQM	0.295	0.245	–	–
pQCD+Lattice	0.342	0.271	–	–
BS	–	–	0.078	0.069