2026 轻强子专题研讨会

Investigation of the $D^0 \to K_S{}^0 \pi^0 \eta$ and $K_S{}^0 \pi^0 \pi^0$ decays

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arXiv: 2603.16082

2026.5. 商丘

Outline

1. Introduction
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§1. Introduction

$$a_0(980)$$

$$I^G(J^{PC}) = 1^-(0^{++})$$

VALUE (MeV)
980±20 OUR ESTIMATE
$\eta\pi$ FINAL STATE ONLY

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
50 to 100 OUR ESTIMATE Width determination very model dependent. Peak width in $\eta\pi$ is about 60 MeV, but decay width can be much larger.					

A. Astier, L. Montanet, M. Baubillier and J. Duboc, Phys. Lett. B 25, 294 (1967).
R. Ammar et al., Phys. Rev. Lett. 21, 1832 (1968).
C. Defoix, P. Rivet, J. Siaud, B. Conforto, M. Widgoff and F. Shively, Phys. Lett. B 28, 353 (1968).

$f_0(980)$

$I^G(J^{PC}) = 0^+(0^{++})$

See the review on "Scalar Mesons below 1 GeV."

T-matrix pole $\sqrt{s}$ = (980–1010) − i (20–35) MeV [h]

Mass (Breit-Wigner) = 990 ± 20 MeV [h]

Full width (Breit-Wigner) = 10 to 100 MeV [h]

$f_0(500)$

$I^G(J^{PC}) = 0^+(0^{++})$

also known as σ; was $f_0(600)$

See the review on "Scalar Mesons below 1 GeV."

Mass (T-Matrix Pole $\sqrt{s}$) = (400–550)−i(200–350) MeV

Mass (Breit-Wigner) = 400 to 800 MeV

Full width (Breit-Wigner) = 100 to 800 MeV

$$|a_0(980)\rangle = |us\bar{d}\bar{s}\rangle, \frac{1}{\sqrt{2}}|(u\bar{u} - d\bar{d})s\bar{s}\rangle, |\bar{u}\bar{s}ds\rangle,$$

$$|f_0(980)\rangle = \frac{1}{\sqrt{2}}|(u\bar{u} + d\bar{d})s\bar{s}\rangle,$$

$$|f_0(500)\rangle = |\bar{u}\bar{d}ud\rangle,$$

$$|f_0(980)\rangle = \cos\varphi|s\bar{s}\rangle + \sin\varphi|n\bar{n}\rangle,$$

$$|f_0(500)\rangle = -\sin\varphi|s\bar{s}\rangle + \cos\varphi|n\bar{n}\rangle.$$

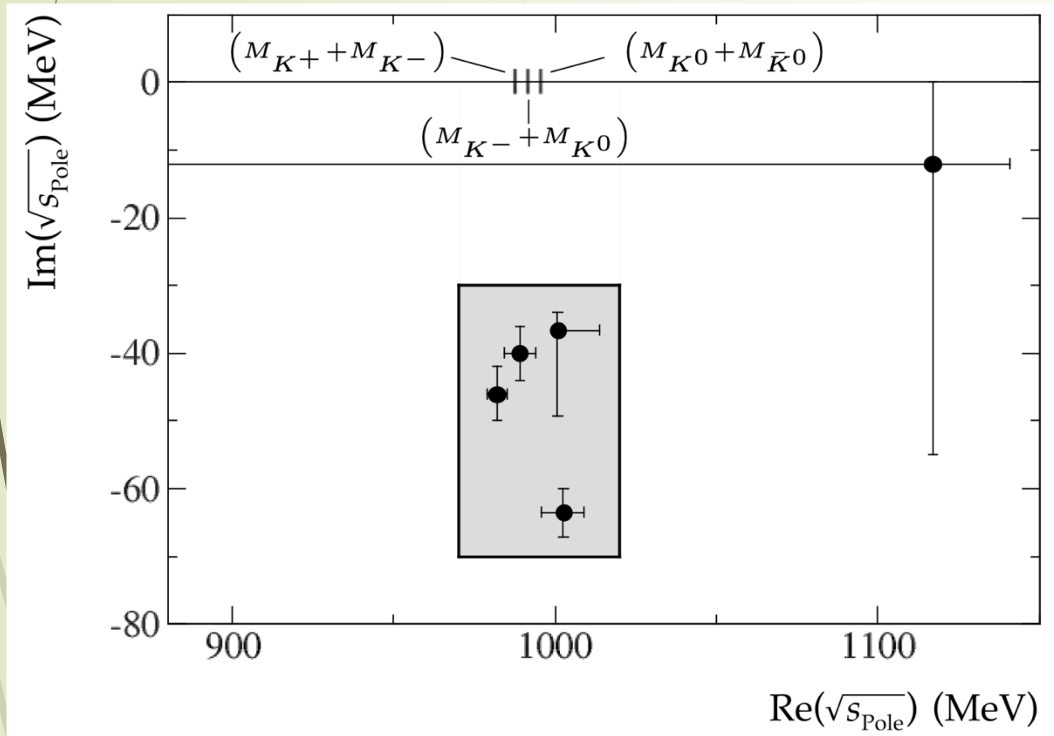
PDG review

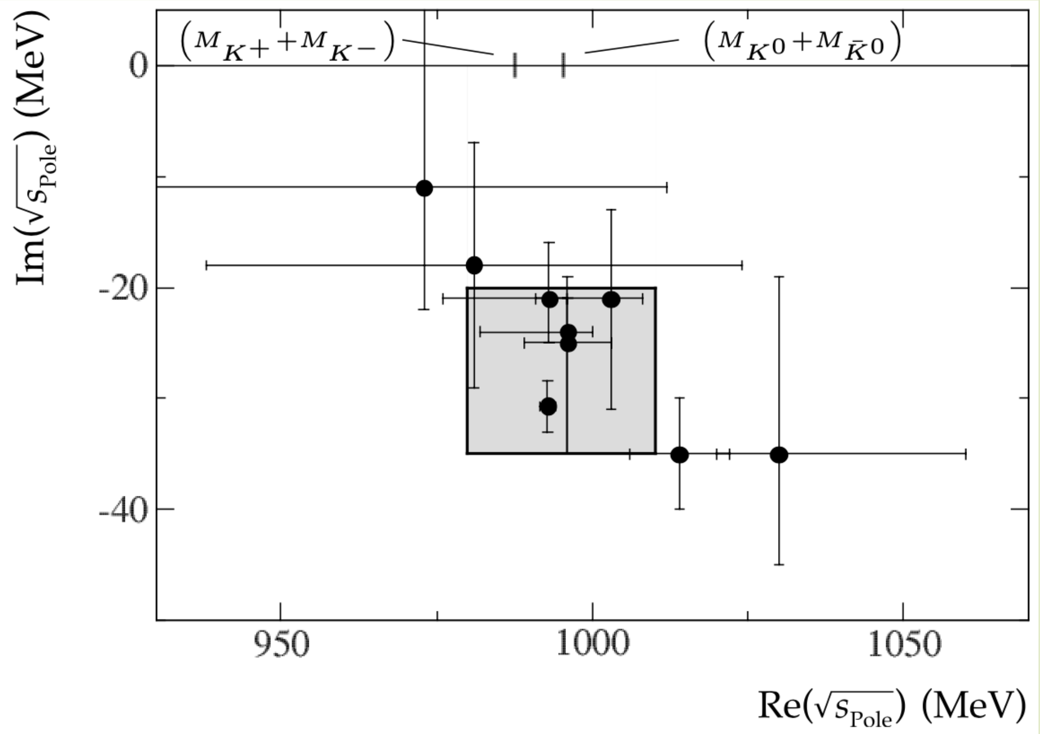
63. Spectroscopy of Light Meson Resonances

PDG review

64. Scalar Mesons below 1 GeV

Revised August 2023 by T. Gutsche (Tübingen U.), C. Hanhart (FZ Jülich), R.E. Mitchell (Indiana U.) and S. Spanier (Tennessee U.).





$$\sqrt{s^{a_0(980)}_{\mathrm{Pole}}} = (970 - 1020) - i(30 - 70)\ \mathrm{MeV}$$

$$\sqrt{s^{f_0(980)}_{\mathrm{Pole}}} = (980 - 1010) - i(20 - 35)\ \mathrm{MeV}$$

$$\sqrt{s^{\sigma}_{\mathrm{Pole}}} = (449^{+22}_{-16}) - i(275 \pm 12)\ \mathrm{MeV}$$

Conventional scalar $q\bar{q}$ states:

S. Godfrey and N. Isgur, Phys. Rev. D 32, 189-231 (1985).
D. Morgan and M. R. Pennington, Phys. Rev. D 48, 1185-1204 (1993).
N. A. Tornqvist and M. Roos, Phys. Rev. Lett. 76, 1575-1578 (1996).

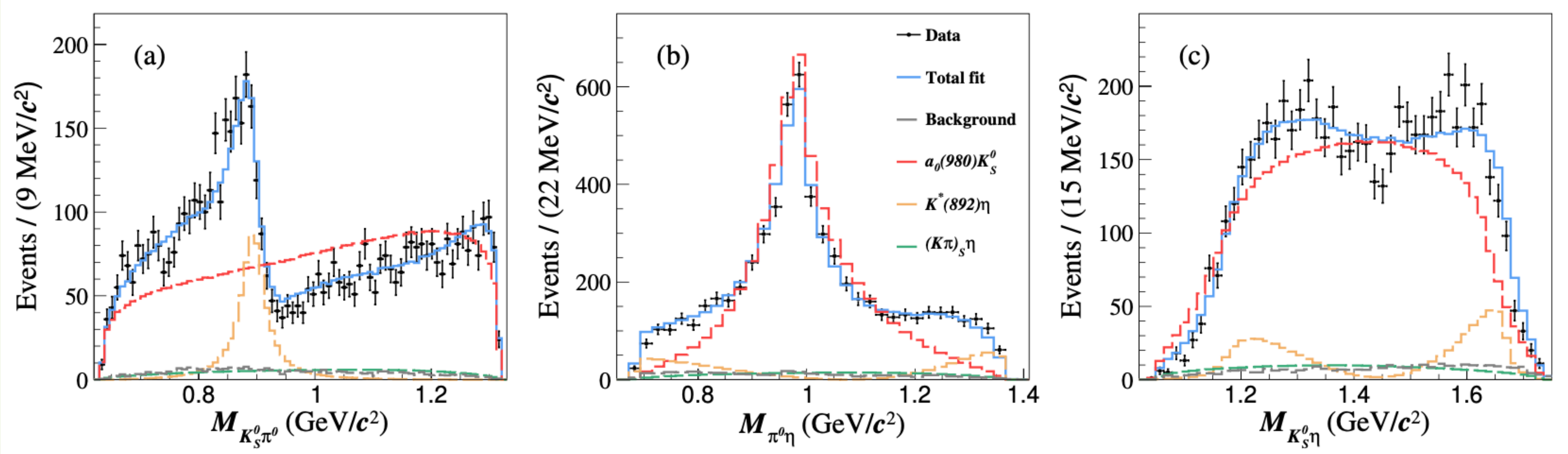
Tetraquarks, composed of four quarks $qq\bar{q}\bar{q}$ states:

R. L. Jaffe, Phys. Rev. D 15, 267 (1977).
J. D. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659 (1982).
N. N. Achasov, Nucl. Phys. A 728, 425-438 (2003).

Molecular state, composed of $K\bar{K}$:

B. S. Zou and D. V. Bugg, Phys. Rev. D 50, 591-594 (1994).
G. Janssen, B. C. Pearce, K. Holinde and J. Speth, Phys. Rev. D 52, 2690-2700 (1995).
J. A. Oller and E. Oset, Nucl. Phys. A 620, 438-456 (1997) [erratum: Nucl. Phys. A 652, 407-409 (1999)].
M. P. Locher, V. E. Markushin and H. Q. Zheng, Eur. Phys. J. C 4, 317-326 (1998).
J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. Lett. 80, 3452-3455 (1998).

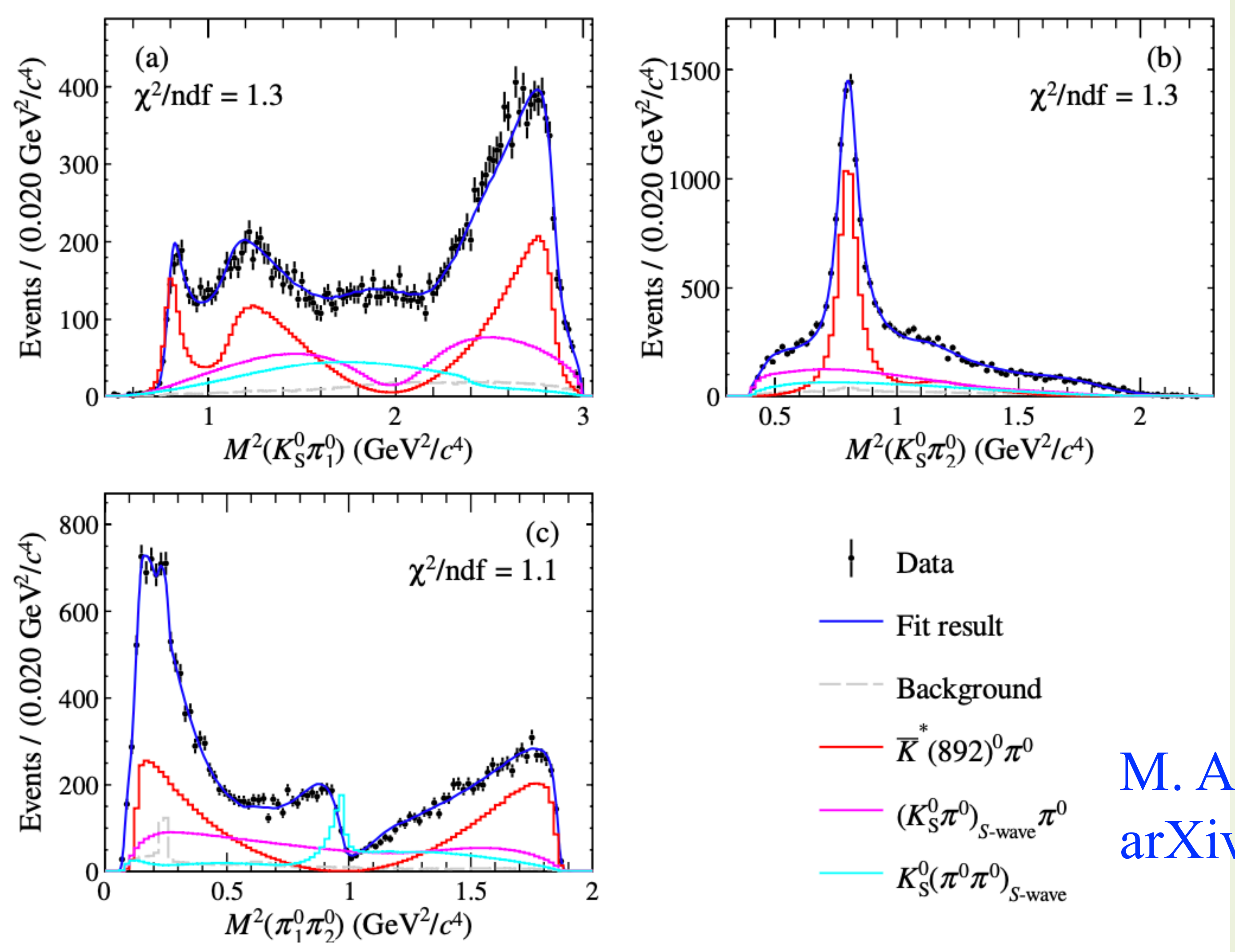
$D^0 \to K_S^0 \pi^0 \eta$



Amplitude	Phase ϕ (rad)	FF (%)	BF (10^{-3})
$D^0 \to K_S^0 a_0(980)^0, a_0(980)^0 \to \pi^0\eta$	0.0(fixed)	$93.2 \pm 3.3 \pm 3.2$	$9.88 \pm 0.37 \pm 0.42$
$D^0 \to K_S^0 a_2(1320)^0, a_2(1320)^0 \to \pi^0\eta$	$-5.41 \pm 0.24 \pm 0.22$	$0.8 \pm 0.1 \pm 0.1$	$0.08 \pm 0.11 \pm 0.11$
$D^0 \to \bar{K}^*(892)^0\eta, \bar{K}^*(892)^0 \to K_S^0\pi^0$	$-2.70 \pm 0.09 \pm 0.06$	$11.5 \pm 0.8 \pm 1.0$	$1.22 \pm 0.09 \pm 0.11$

M. Ablikim et al. [BESIII], arXiv: 2512.23389

$D^0 \to K_S^0 \pi^0 \pi^0$



M. Ablikim et al. [BESIII], arXiv: 2510.25111

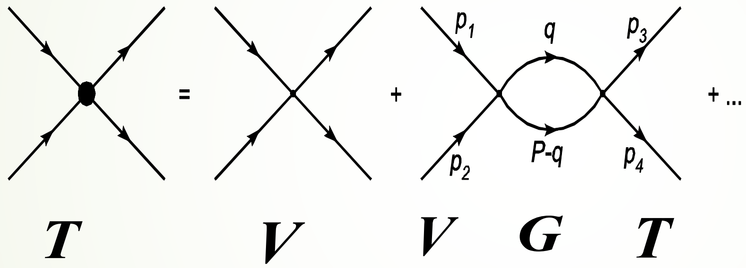
What can we learn from the invariant mass distributions of the experimental measurement?

§2. Formalism

(1) Coupled channel interaction from the chiral unitary approach

- **Chiral Unitary Approach**: solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V\,G\,T,\; T = [1 - V\,G]^{-1}\,V$$



D. L. Yao, L. Y. Dai, H. Q. Zheng and Z. Y. Zhou, Rept. Prog. Phys. 84, 076201 (2021)

where V matrix (potentials) can be evaluated from the interaction Lagrangians.

$I = 0$ sector: $\pi^+\pi^-,\ \pi^0\pi^0,\ K^+K^-,\ K^0\overline{K}^0,\ \eta\eta$

$I = 1$ sector: $K^+K^-,\ K^0\overline{K}^0,\ \pi^0\eta$

$I = 1/2$ sector: $K^+\pi^-,\ K^0\pi^0,\ K^0\eta$

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438
E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99
J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263

G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(P-q)^2-m_{l1}^2+i\varepsilon}\,\frac{1}{q^2-m_{l2}^2+i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

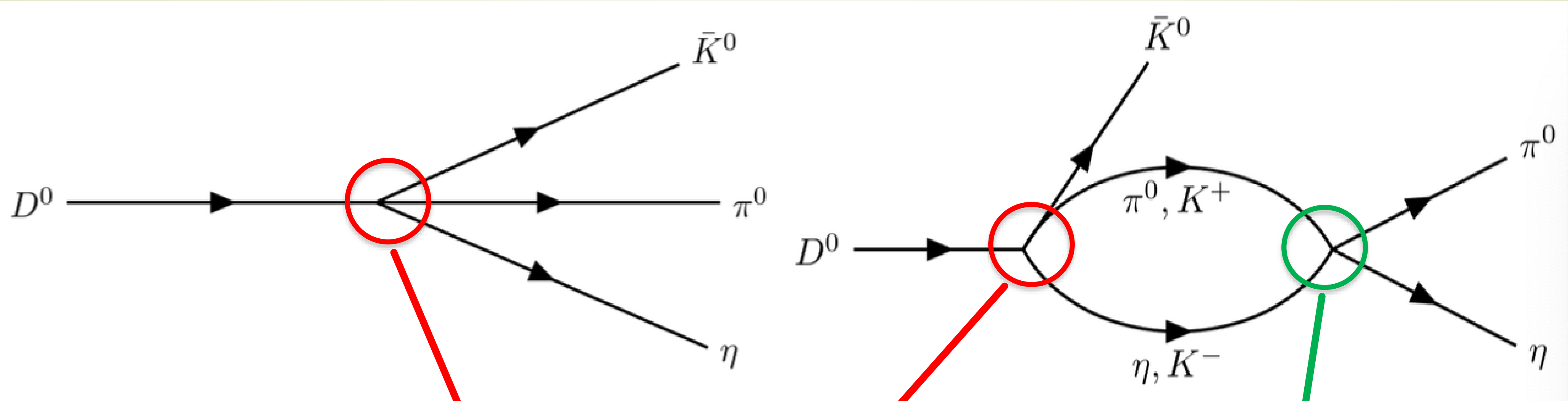
$$\mathrm{Im}\,T_{ij} = T_{in}\,\sigma_{nn}\,T_{nj}^{*}$$

$$\sigma_{nn} \equiv \mathrm{Im}\,G_{nn} = -\frac{q_{cm}}{8\pi\sqrt{s}}\theta(s-(m_1+m_2)^2))$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^{I}(s) + i\,\frac{q_{cm}}{4\pi\sqrt{s}}$$

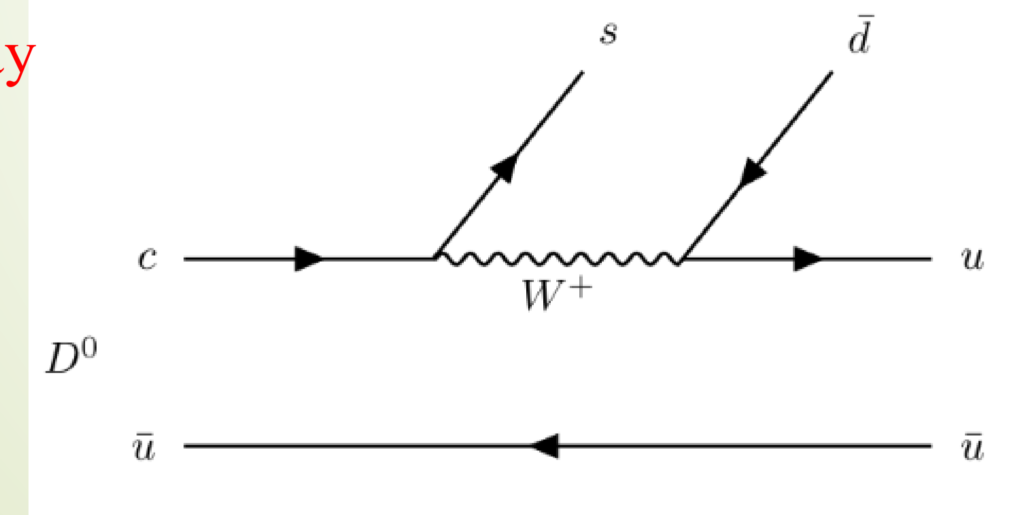
(2) Final state interaction



(a) Tree-level production.

(b) Rescattering of $\pi^0\eta$ and K^+K^-.

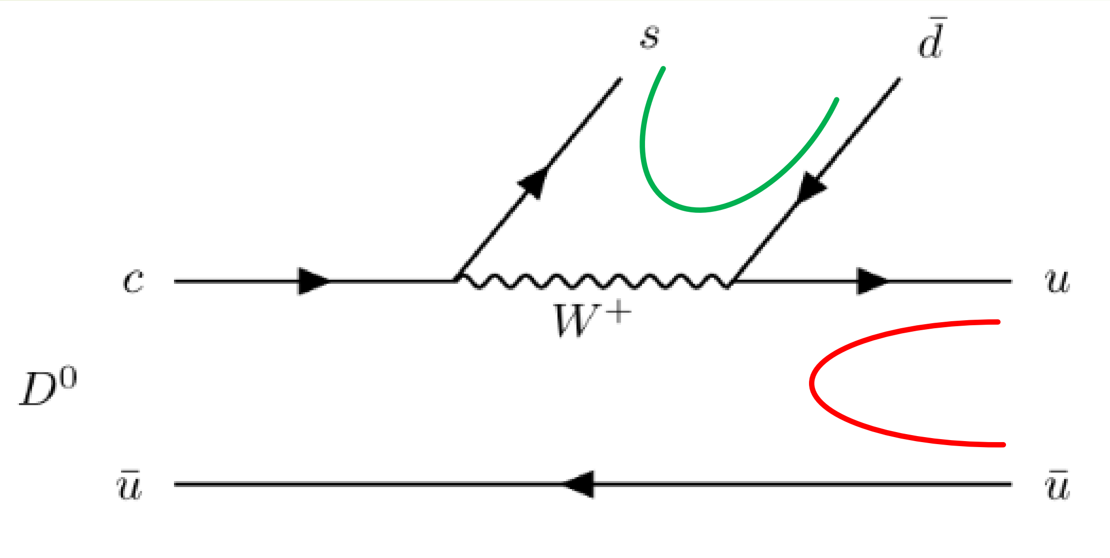
The weak decay process at the quark level



S-wave

$$T = [1 - VG]^{-1} V$$

The final state interaction at the hadron level



$$u(\bar{u}u + \bar{d}d + \bar{s}s)\bar{u} = (M \cdot M)_{11},$$
$$s(\bar{u}u + \bar{d}d + \bar{s}s)\bar{d} = (M \cdot M)_{32}.$$

$$H = V_p V_{cs} V_{ud} \left[\left(s\bar{d} \to \bar{K}^0\right) [M_{11} \to (M \cdot M)_{11}] + \left(u\bar{u} \to \frac{1}{\sqrt{2}}\pi^0\right) [M_{32} \to (M \cdot M)_{32}] \right.$$
$$\left. + \left(u\bar{u} \to \frac{1}{\sqrt{3}}\eta\right) [M_{32} \to (M \cdot M)_{32}] \right]$$

$$M=\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s}\\ d\bar{u} & d\bar{d} & d\bar{s}\\ s\bar{u} & s\bar{d} & s\bar{s}\end{pmatrix}=\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & \pi^+ & K^+\\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & K^0\\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta\end{pmatrix}$$

$$(M\cdot M)_{11}=\frac{1}{2}\pi^0\pi^0+\frac{1}{3}\eta\eta+\frac{2}{\sqrt{6}}\pi^0\eta+\pi^+\pi^-+K^+K^-,$$

$$(M\cdot M)_{32}=K^-\pi^+-\frac{1}{\sqrt{2}}\bar{K}^0\pi^0,$$

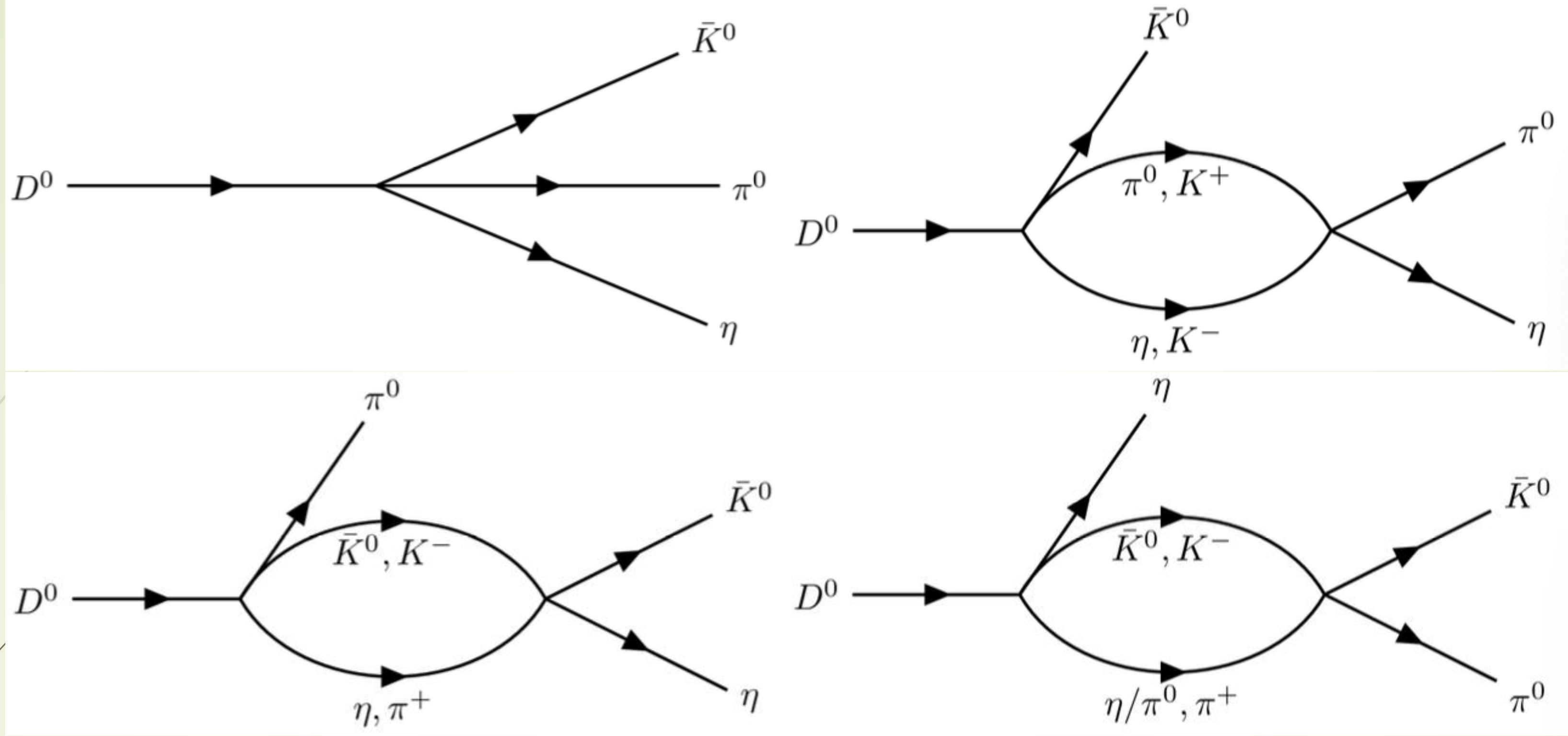
$$H=V_pV_{cs}V_{ud}\Big[\left(s\bar{d}\to\bar{K}^0\right)\left[M_{11}\to(M\cdot M)_{11}\right]+\left(u\bar{u}\to\frac{1}{\sqrt{2}}\pi^0\right)\left[M_{32}\to(M\cdot M)_{32}\right]$$
$$+\left(u\bar{u}\to\frac{1}{\sqrt{3}}\eta\right)\left[M_{32}\to(M\cdot M)_{32}\right]\Big]$$

$D^0\to K_S^0\pi^0\eta$

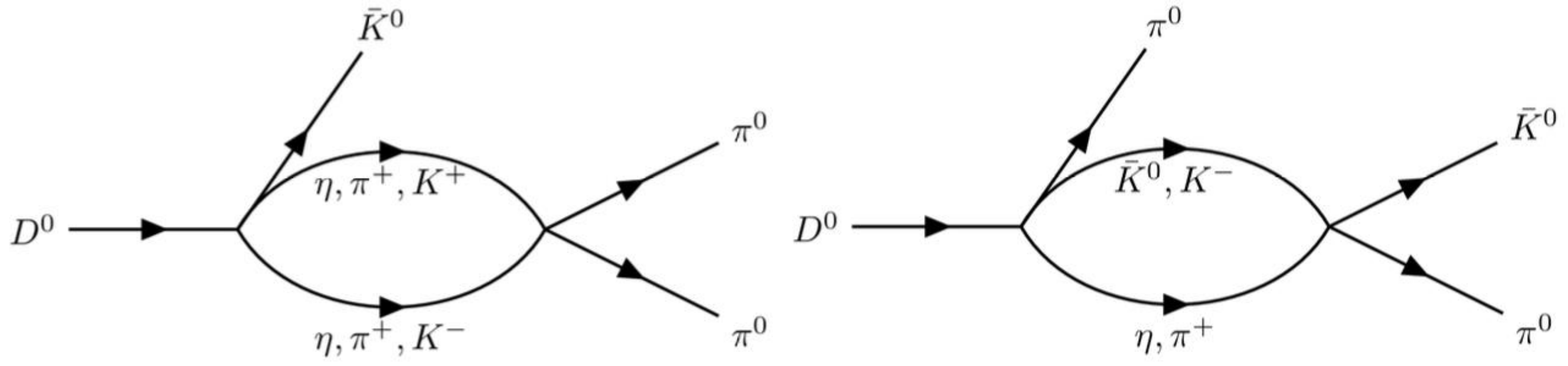
$$=V_pV_{cs}V_{ud}\left[\frac{1}{3}\bar{K}^0\eta\eta+\frac{1}{\sqrt{6}}\bar{K}^0\pi^0\eta+\pi^+\pi^-\bar{K}^0+K^+K^-\bar{K}^0+\frac{1}{\sqrt{2}}\pi^0K^-\pi^++\frac{1}{\sqrt{3}}\eta K^-\pi^+\right]$$

$D^0\to K_S^0\pi^0\pi^0$

$$=C_1\left[\frac{1}{3}\bar{K}^0\eta\eta+\frac{1}{\sqrt{6}}\bar{K}^0\pi^0\eta+\pi^+\pi^-\bar{K}^0+K^+K^-\bar{K}^0+\frac{1}{\sqrt{2}}\pi^0K^-\pi^++\frac{1}{\sqrt{3}}\eta K^-\pi^+\right],$$

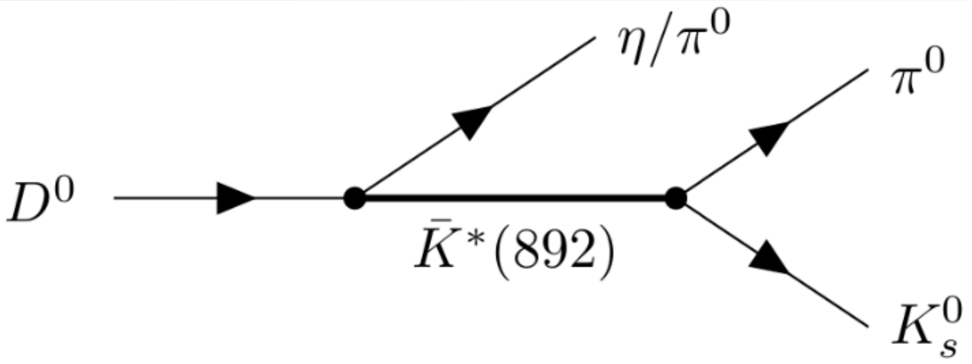


$$
\begin{aligned}
t\left(s_{12}, s_{23}\right)_{D^0\to\bar{K}^0\pi^0\eta} = & \frac{1}{\sqrt{6}}C_1 + \frac{1}{\sqrt{6}}C_1 G_{\pi^0\eta}\left(s_{23}\right) T_{\pi^0\eta\to\pi^0\eta}\left(s_{23}\right) + C_1 G_{K^+K^-}\left(s_{23}\right) T_{K^+K^-\to\pi^0\eta}\left(s_{23}\right) \\
& + \frac{1}{\sqrt{6}}C_1 G_{\bar{K}^0\eta}\left(s_{23}\right) T_{\bar{K}^0\eta\to\bar{K}^0\eta}\left(s_{13}\right) + \frac{1}{\sqrt{2}}C_1 G_{K^-\pi^+}\left(s_{13}\right) T_{K^-\pi^+\to\bar{K}^0\eta}\left(s_{13}\right) \\
& + \frac{1}{3}C_1 G_{\bar{K}^0\eta}\left(s_{12}\right) T_{\bar{K}^0\eta\to\bar{K}^0\pi^0}\left(s_{12}\right) + \frac{1}{\sqrt{6}}C_1 G_{\bar{K}^0\pi^0}\left(s_{12}\right) T_{\bar{K}^0\pi^0\to\bar{K}^0\pi^0}\left(s_{12}\right) \\
& + \frac{1}{\sqrt{3}}C_1 G_{K^-\pi^+}\left(s_{12}\right) T_{K^-\pi^+\to\bar{K}^0\pi^0}\left(s_{12}\right),
\end{aligned}
$$



$$
\begin{aligned}
t\left(s_{12}, s_{23}\right)_{D^0\to\bar{K}^0\pi^0\pi^0} = & \frac{1}{2}\times\frac{1}{3}C_1 G_{\eta\eta}\left(s_{23}\right)T_{\eta\eta\to\pi^0\pi^0}\left(s_{23}\right)+C_1 G_{\pi^+\pi^-}\left(s_{23}\right)T_{\pi^+\pi^-\to\pi^0\pi^0}\left(s_{23}\right)\\
& +C_1 G_{K^+K^-}\left(s_{23}\right)T_{K^+K^-\to\pi^0\pi^0}\left(s_{23}\right)+\frac{1}{\sqrt{6}}C_1 G_{\bar{K}^0\eta}\left(s_{12}\right)T_{\bar{K}^0\eta\to\bar{K}^0\pi^0}\left(s_{12}\right)\\
& +\frac{1}{\sqrt{2}}C_1 G_{K^-\pi^+}\left(s_{12}\right)T_{K^-\pi^+\to\bar{K}^0\pi^0}\left(s_{12}\right).
\end{aligned}
$$

(3) Contributions from P-wave



$$\mathcal{M}_{\bar{K}^*(892)}(s_{12}, s_{23}) = \frac{\mathcal{D} e^{i\alpha_{\bar{K}^*(892)}}}{s_{12} - M^2_{\bar{K}^*(892)} + iM_{\bar{K}^*(892)}\Gamma_{\bar{K}^*(892)^+}} \times \left[\frac{\left(m^2_{D^0} - m^2_{\eta/\pi^0}\right)\left(m^2_{K^0} - m^2_{\pi^0}\right)}{M^2_{\bar{K}^*(892)}} - s_{13} + s_{23} \right]$$

$$t'(s_{12}, s_{23})_{D^0 \to K^0_S \pi^0 \eta} = t(s_{12}, s_{23})_{D^0 \to K^0_S \pi^0 \eta} + \mathcal{M}_{\bar{K}^*(892)}(s_{12}, s_{23})$$

$$t'(s_{12}, s_{23})_{D^0 \to K^0_S \pi^0 \pi^0} = t(s_{12}, s_{23})_{D^0 \to K^0_S \pi^0 \pi^0} + \mathcal{M}_{\bar{K}^*(892)}(s_{12}, s_{23}) + (2 \leftrightarrow 3)$$

$$\frac{d^2\Gamma}{ds_{12} ds_{23}} = \frac{1}{(2\pi)^3} \frac{1}{32 m^3_{D^0}} \left| t'(s_{12}, s_{23})_{D^0 \to K^0_S \pi^0 \eta} \right|^2,$$

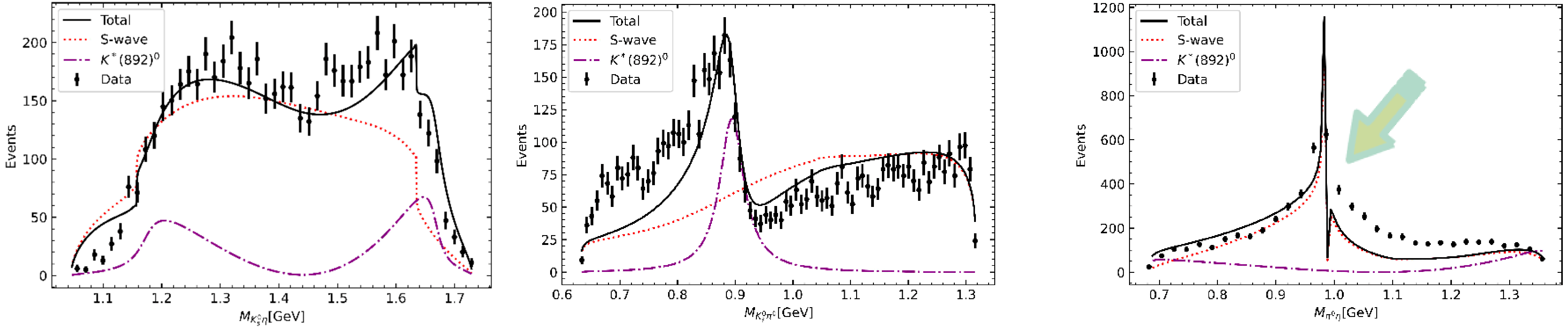
$$\frac{d^2\Gamma}{ds_{12} ds_{23}} = \frac{1}{(2\pi)^3} \frac{1}{32 m^3_{D^0}} \frac{1}{2} \left| t'(s_{12}, s_{23})_{D^0 \to K^0_S \pi^0 \pi^0} \right|^2,$$

$$G(s)T(s) = G(s_{cut})T(s_{cut}) e^{-\alpha(\sqrt{s} - \sqrt{s_{cut}})}, \quad \text{for } \sqrt{s} > \sqrt{s_{cut}}, \quad \sqrt{s_{cut}} = 1.1 \text{ GeV}$$

- **free parameters** S-wave: C_1, α, P-wave: $\mathcal{D}$ $\alpha_{\bar{K}^*(892)}$

(1) $D^0 \to K^0_S \pi^0 \eta$ results (BESIII)

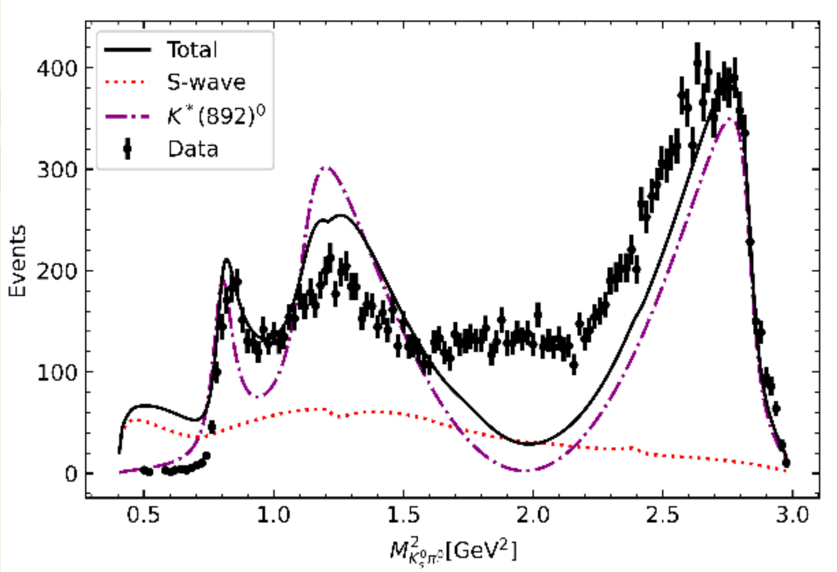
Parameters	C_1	α	D	$\alpha_{\bar{K}^*(892)}$	$\chi^2/dof.$
Fit	3342.04	-3.38	138.31	2.34	11.59

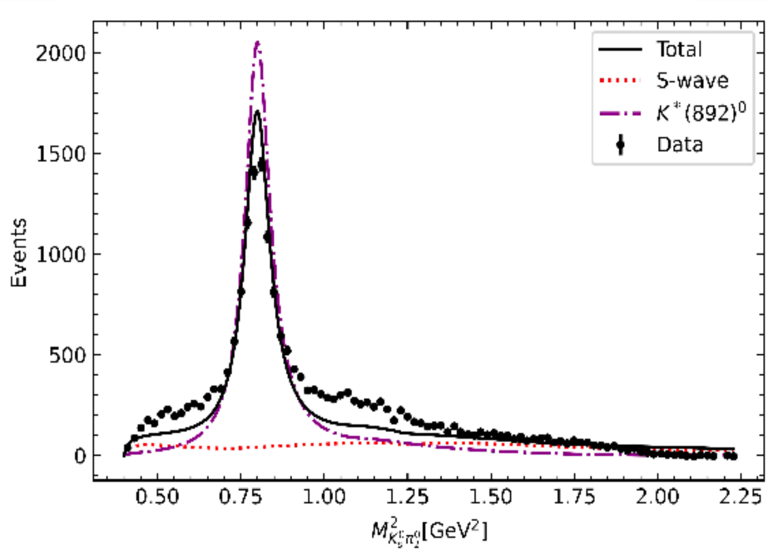


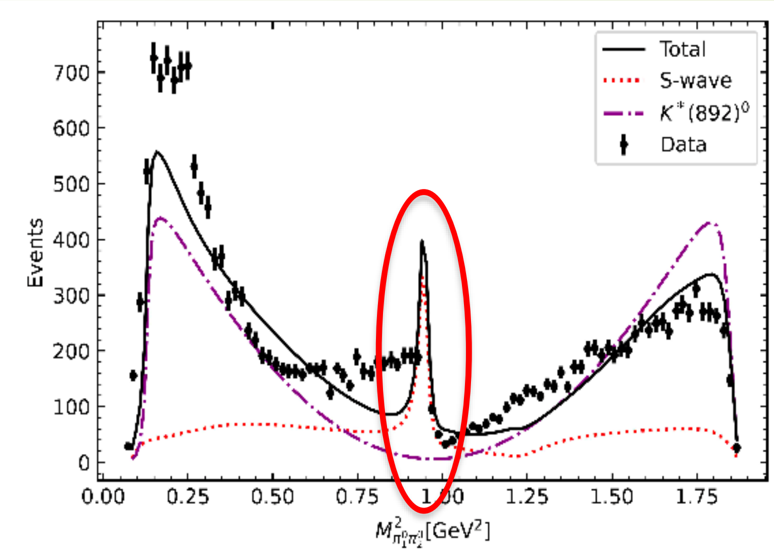
where we have taken into account the bin sizes

(2) $D^0 \rightarrow K^0_S \pi^0 \pi^0$ results (BESIII)

Parameters	C_1	α	D	$\alpha_{\bar{K}^*(892)}$	D_1	$\alpha'_{\bar{K}^*(892)}$	$\chi^2/dof.$
Fit	6040.13	31.36	122.10	1.09	522.41	1.47×10^{-2}	27.94







§4. Summary

- We use the chiral unitary approach to dynamically generate the resonances $a_0(980), f_0(980), f_0(500)$
- Taking the data from BESIII, we fit the invariant mass distributions of the decays $\boldsymbol{D^0} \rightarrow K^0{}_S\, \boldsymbol{\pi}^0\, \boldsymbol{\eta}$, $\boldsymbol{D^0} \rightarrow K^0{}_S\, \boldsymbol{\pi}^0\, \boldsymbol{\pi}^0$ with the final state interaction formalism
- Indicating the molecular nature of these resonances

Thanks for your attention!

感谢大家的聆听！