Two-loop calculations of nucleon sigma term in BChPT

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arXiv: 2502.19168 [hep-ph, JHEP 04, 192 (2025)]
arXiv: 2508.11435 [hep-ph]

2026年轻强子专题研讨会

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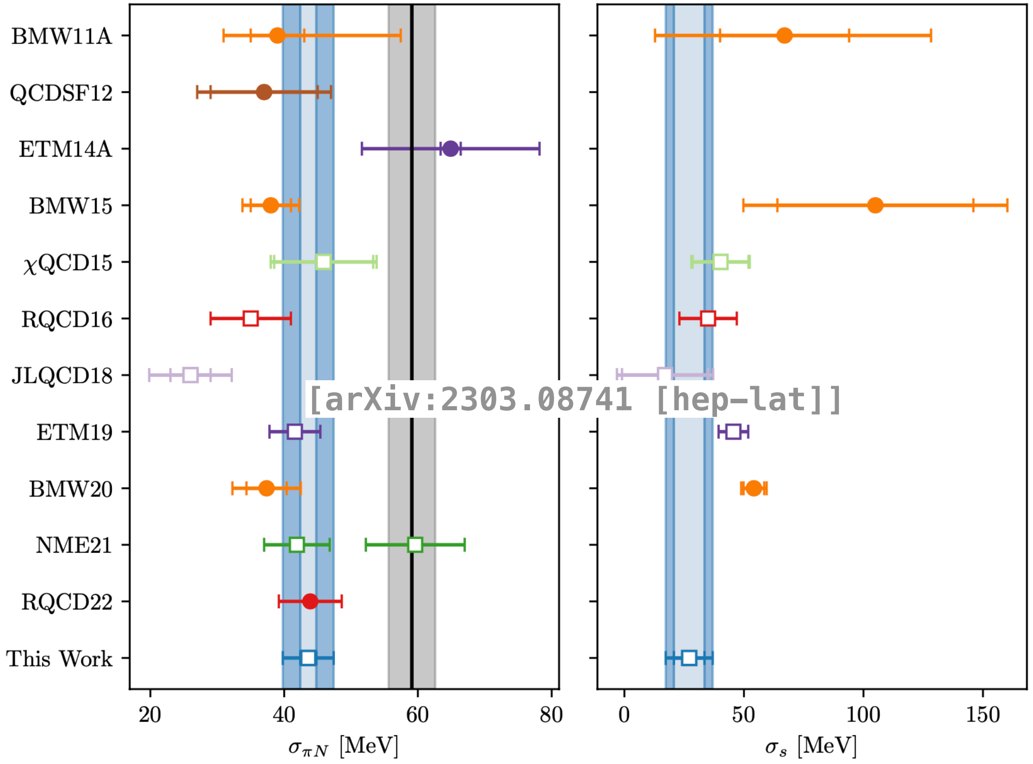
Outline

- Introduction
- Leading two-loop calculation of the nucleon mass
- Two-loop extraction of pion-nucleon sigma term
- Summary and outlook

Introduction

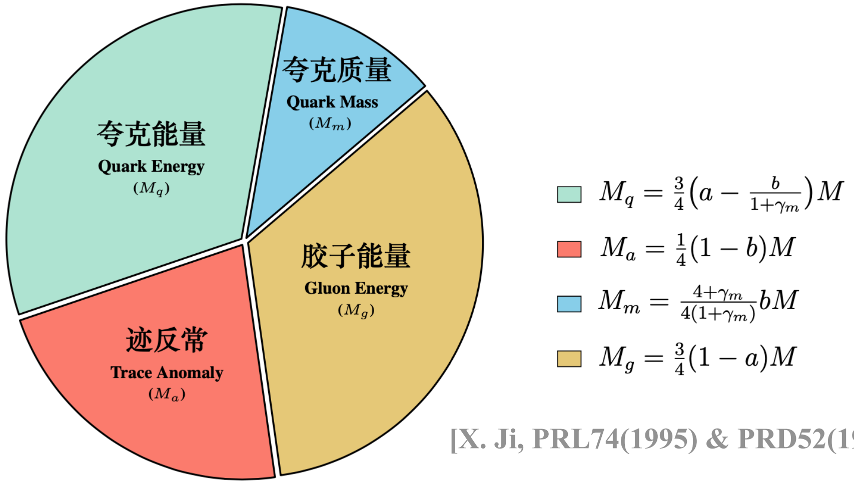
Nucleon mass physics

- High precision determination of the nucleon mass
  - The proton mass decomposition
  - Sigma term: tension between lattice QCD and phenomenological results
  - WIMP dark matter: scalar coupling of the nucleon to dark matter
  - …



Feynman—Hellmann Theorem

$$\sigma_{\pi N} = \frac{\hat{m}}{2m_N}\langle N\,|\,\bar{u}u + \bar{d}d\,|\,N\rangle = M_\pi^2 \frac{\partial m_N}{\partial M_\pi^2}$$

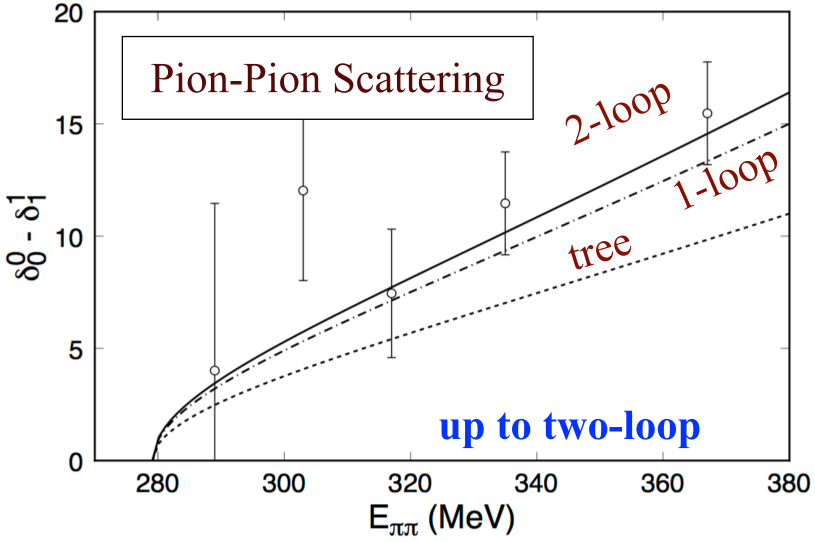


[X. Ji, PRL74(1995) & PRD52(1995)]

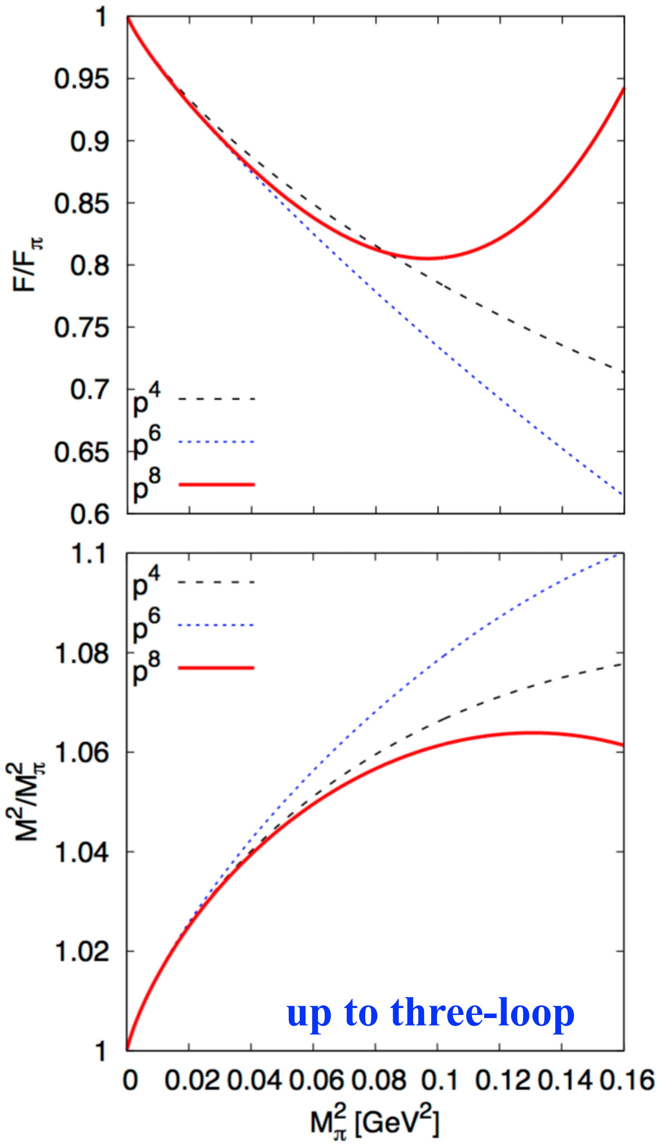
Status of ChPT beyond one-loop

- **Pure meson sector:** a great triumph has been achieved.
  - ➡ Two-loop calculations become standard and good convergence can be seen.
    - Pion mass
    - Pion decay constant
    - Electromagnetic form factor
    - $\pi\pi$ scattering
    - πK scattering
    - $K_{\ell 4}$
    - …

[Bijnens, et al., PLB (1996)]



[Bijnens and Truedsson, JHEP (2017)]



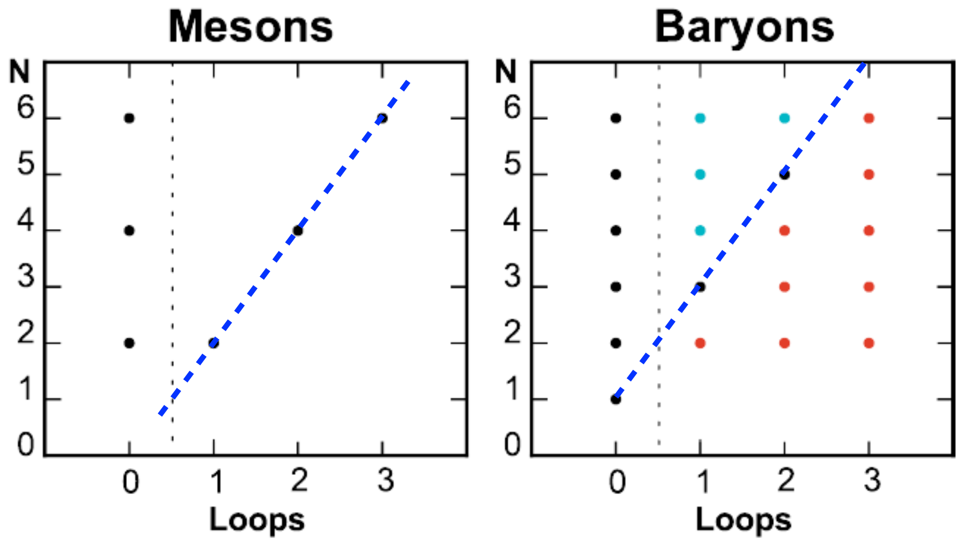
Status of ChPT beyond one-loop

- **Baryon sector:** multi-loop calculation is much more complex than pure meson sector
  - Dirac algebra & axial coupling between baryons and Goldstone bosons

    $\longrightarrow$ *More diagrams and master integrals*
  - Unequal mass

    $\longrightarrow$ *Hard to compute multi-loop integrals*
  - The introduction of non-zero baryon mass in the chiral limit

    $\longrightarrow$ *the notable Power Counting Breaking (PCB) problem!*

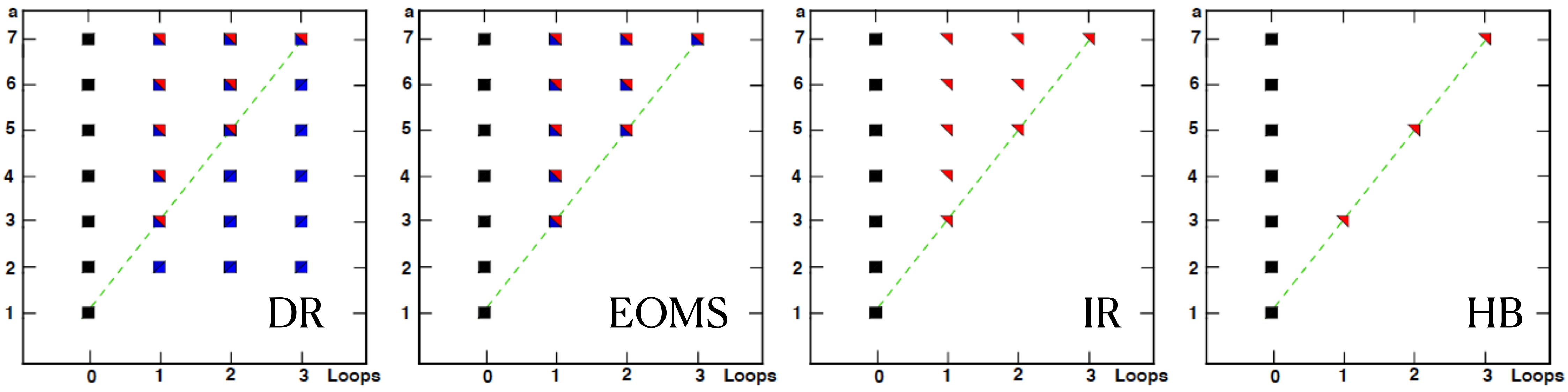


Naive power counting rule !!!

Red dots denote PCB terms!!!

Solutions to the PCB problem

- **Various approaches**
  - Heavy Baryon (HB) formalism [Jenkins and Manohar, PLB255'91]
  - Infrared Regularisation (IR) prescription [T.Becher and H.Leutwyler, Eur.Phys.J.C9'99]
  - Extended-On-Mass-Shell (EOMS) scheme [T.Fuchs, J.Gegelia, G.Japaridze and S.Scherer,PRD68'03]



philosophy: HB & IR: throwing away EOMS: reorganisation

Leading two-loop calculation of the nucleon mass

Status of the nucleon mass at two loop

- **Previous works on the nucleon mass in ChPT**
  - 1999: HB formalism up to $\mathcal{O}(p^5)$ [McGovern & Birse, PLB446(1999)]
  - 2007/2008: IR prescription up to $\mathcal{O}(p^6)$ [truncated]

    [Schindeler, Djukanovic, Gegelia & Scherer, PLB649(2007), NPA803(2008)]
  - 2024: EOMS up to $\mathcal{O}(p^6)$ [truncated]

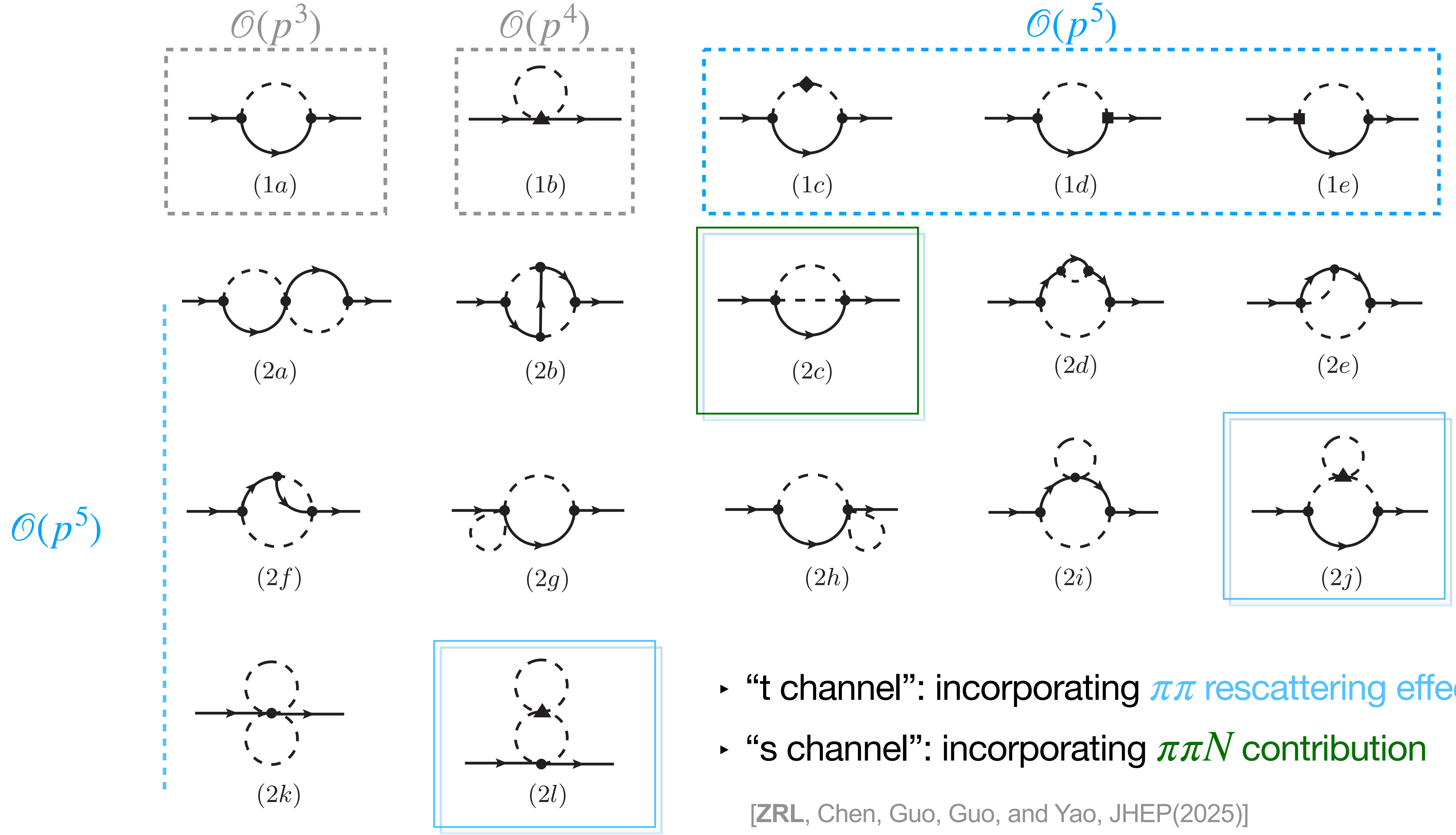
    [Conrad, Gasparyan & Epelbaum, PoS CD2021, 074 (2024), EPJ Web Conf. 303, 02001(2024)]

    [Chen, Hu, Mo & Jia, arXiv:2406.040124 [hep-ph]]

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4 + k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln^2\frac{M}{\mu} + k_8 M^6 \ln\frac{M}{\mu} + k_9 M^6$$

- **Our work: full EOMS chiral result up to $\mathcal{O}(p^5)$**
  - All the necessary analytical two-loop structure
  - Avoid too many unknown LECs

The nucleon mass at two-loop order



- "t channel": incorporating $\pi\pi$ rescattering effect
- "s channel": incorporating $\pi\pi N$ contribution

[**ZRL**, Chen, Guo, Guo, and Yao, JHEP(2025)]

Two-loop nucleon mass in BChPT

- **Chiral expression up to $\mathcal{O}(p^5)$**
  - Tree + one loop + two loop $m_N = -4c_1M^2 - 2e_mM^4 + \Delta m_N^{(1)} + \Delta m_N^{(2)}$
  - One-loop: $\Delta m_N^{(1)} = m_N^{(1a)} + m_N^{(1b)} + m_N^{(1c)} + m_N^{(1d)} + m_N^{(1e)}$,
  - Two-loop: $\Delta m_N^{(2)} = m_N^{(2a)} + m_N^{(2b)} + \cdots + m_N^{(2l)} + m_N^{(2')} + m_N^{\text{sub.}}$.

$$m_N^{(1a)} = \frac{3g^2 m\,\kappa}{2iF^2}\left[J_{01} + M^2 J_{11}\right]$$

$$m_N^{(2a)} = \frac{3g^2\kappa^2}{16mF^4}\Big[8m^2M^4 I_{11011} + 8m^2M^2(I_{01011} + I_{10011}) + 2m^2\big[4I_{00011} + I_{11(-1)01} + I_{11(-1)10} - I_{110(-1)1} - I_{1101(-1)}\big] + 2(M^2 - 2m^2)I_{11000} - I_{110(-1)0} - I_{1100(-1)}\Big],$$

✓ $J_{\nu_1\nu_2}$ and $I_{\nu_1\nu_2\nu_3\nu_4\nu_5}$ are one- and two-loop Feynman integrals.

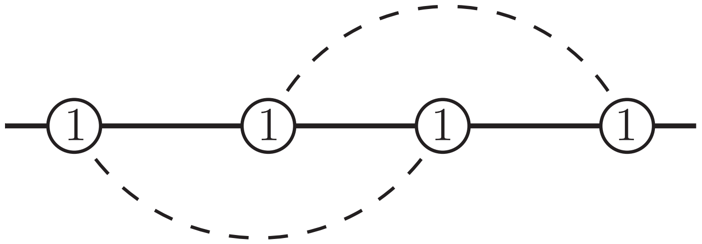
Two-loop Feynman integrals

- **Definition of two-loop integrals**

$$I_{\nu_1\nu_2\nu_3\nu_4\nu_5} = \int\int \frac{\mathrm{d}^d\ell_1}{(i\pi^{d/2})} \frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})} \frac{1}{\mathcal{D}_1^{\nu_1}\mathcal{D}_2^{\nu_2}\mathcal{D}_3^{\nu_3}\mathcal{D}_4^{\nu_4}\mathcal{D}_5^{\nu_5}}$$

  - ν_i integers; scalar integrals: all $\nu_i \geq 0$; tensor integrals: $\exists \nu_i \leq 0$
  - Number of irreducible scalar product: $N = L \times E + L(L+1)/2$

$$\mathcal{D}_1 = \ell_1^2 - M^2$$
$$\mathcal{D}_2 = \ell_2^2 - M^2$$
$$\mathcal{D}_3 = (p + \ell_1 + \ell_2)^2 - m^2$$
$$\mathcal{D}_4 = (p + \ell_1)^2 - m^2$$
$$\mathcal{D}_5 = (p + \ell_2)^2 - m^2$$



Feynman integrals

Calculation of Feynman Integrals

In our work

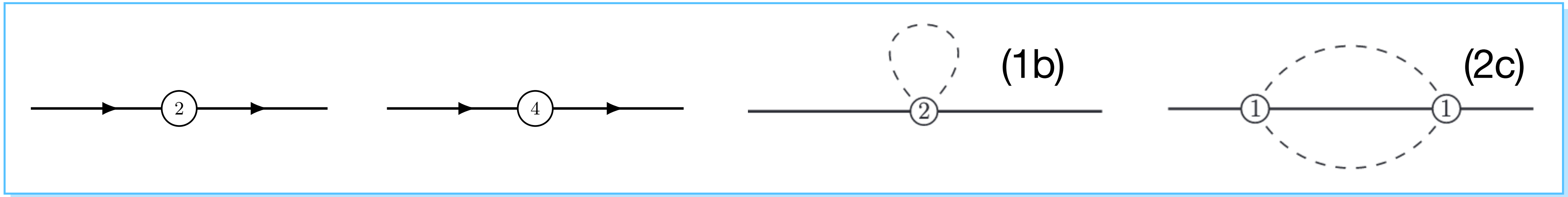
- Numerical ways: AMFLOW, AmpRed, FIESTA, pySecdec
  Liu,Ma,2022 Chen,2024
- Analytic ways: Differential Equations with UT-Basis, Mellin Barnes⋯

Analytic calculation of the high order corrections have many benefits. We can obtain the numerical effect of high order corrections very fast, expand them at any kinematic point, and integrate them to obtain more interesting results.

Slide by L. B. Chen

Renormalization of non-renormalizable EFT

- **Non-local UV divergences**
  - Pedagogic example with $g_A = 0$



- One-loop: $$m_N^{(1b)} = -\frac{3(d(c_3-2c_1)+c_2)M^4}{F^2 d}\left[-\frac{1}{\Lambda\epsilon}+\frac{1}{\Lambda}\log\frac{M^2}{\mu^2}+\mathcal{O}(\epsilon)\right]$$
- Two-loop: $$m_N^{(2c)} = \frac{3\left(6M^4m+8M^2m^3-3m^5\right)}{32F^4\Lambda^2\epsilon^2}+\frac{132M^4m-16M^2m^3-9m^5}{128F^4\Lambda^2\epsilon}$$ $$+\frac{3m^3\left(3m^2-8M^2\right)}{16F^4\Lambda^2\epsilon}\log\frac{m^2}{\mu^2}-\frac{9M^4m}{8F^4\Lambda^2\epsilon}\log\frac{M^2}{\mu^2}+\text{finite}+\mathcal{O}(\epsilon)$$

✓ Local UV div.: tree-level counter term

✓ Non-local UV div.: one loop diagrams as sub diagrams

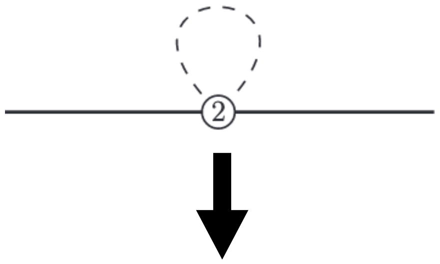
PCB renormalisation

- **EOMS scheme at two-loop:** absorption of PCB terms by LECs

Two-loop

non-local PCB term

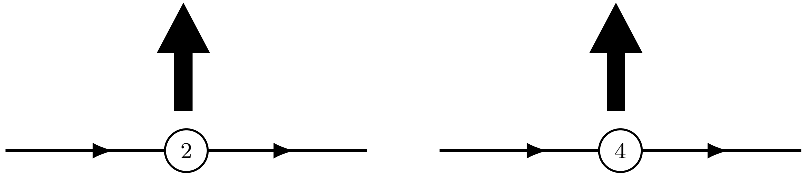
One-loop



$$X^r = \widetilde{X} + \delta_X^{(1)}\ , \quad X \in \{c_1, c_2, c_3\}$$

2-loop local PCB term

$$X^r = \widetilde{X} + \delta_X^{(21)} + \delta_X^{(22)}\ , \quad X \in \{m, c_1, e_m\}$$



Tree

- $\delta_X^{(21)}$: cancel 1-loop local PCB term
- $\delta_X^{(22)}$: cancel 2-loop local PCB term

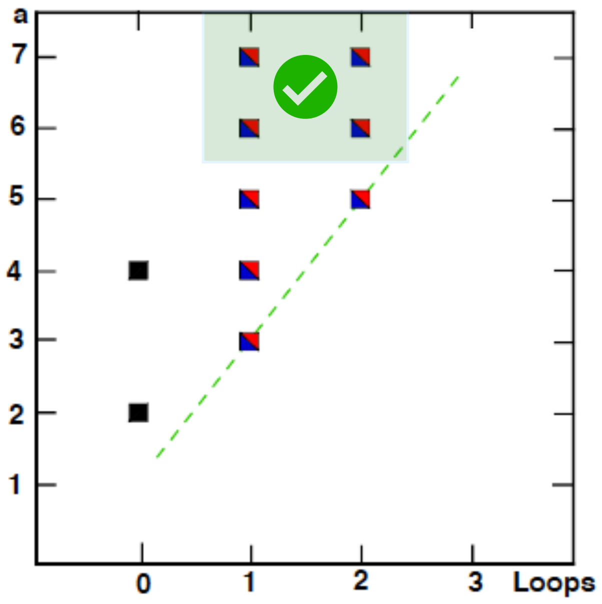
Final two-loop representation of nucleon mass

- **The renormalised nucleon mass up to $\mathcal{O}(p^5)$**

$$m_N = \widetilde{m} \underbrace{- 4\widetilde{c}_1 M^2}_{\mathcal{O}(p^2)} + \underbrace{\bar{m}_N^{(1a)}}_{\mathcal{O}(p^3)} \underbrace{- 2\widetilde{e}_m M^4 + \bar{m}_N^{(1b)}}_{\mathcal{O}(p^4)} + \underbrace{\bar{m}_N^{(1c)} + 2\bar{m}_N^{(1d)} + \bar{m}_N^{2-\text{loop}} + \bar{m}_N^{\text{sub-diag.}}}_{\mathcal{O}(p^5)}$$

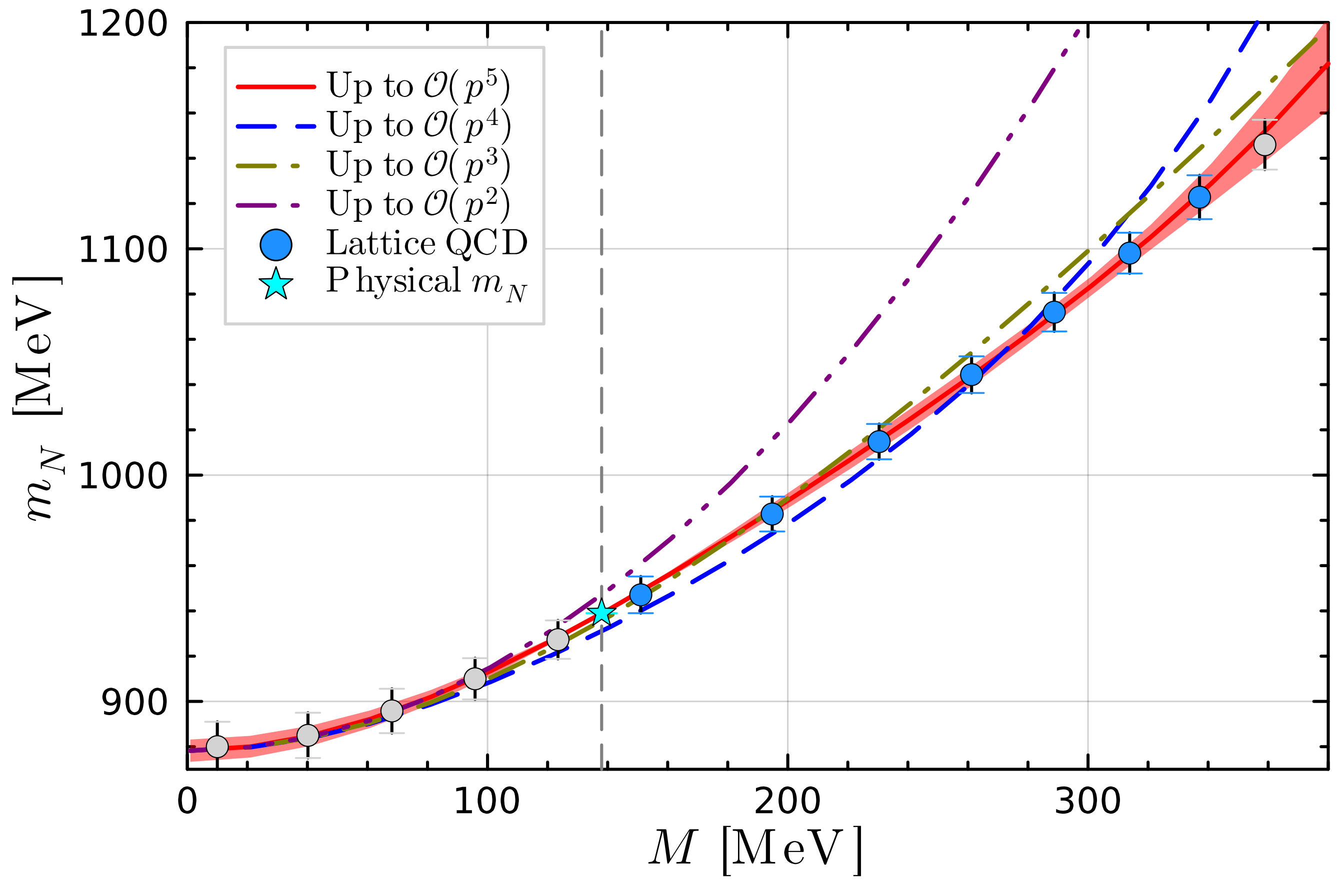
$$m_N^{\text{sub-diag.}} \sim \sum_X \left\{ \left(-\frac{\beta_X^{(1)}}{\epsilon \Lambda} \right) \times [\text{linear term in } \epsilon \text{ from one loops}] \right\}$$

- **Merits**
  - Faithful structure: respect original analytic property
  - Controllable accuracy: possess correct power counting rule
  - Renormalisation scale independent



Chiral extrapolation of nucleon mass

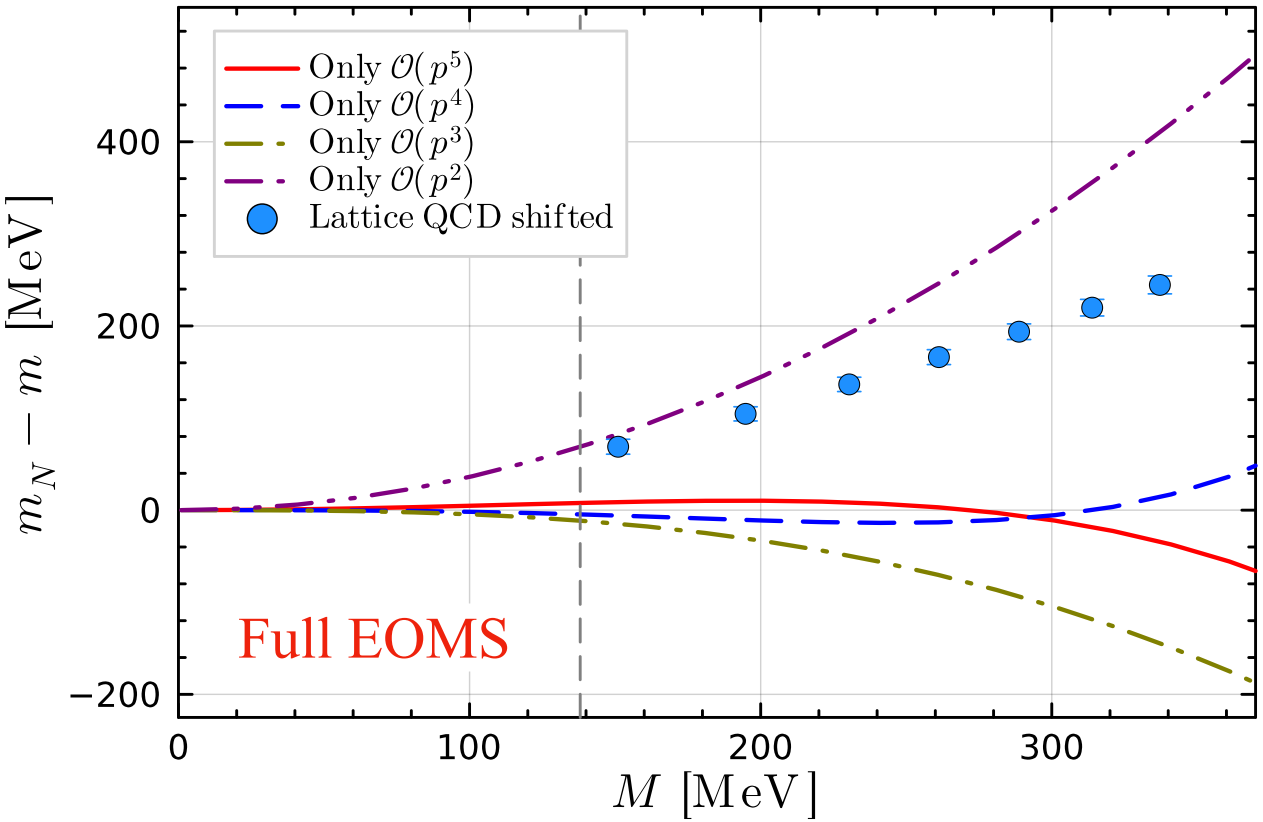
- Full EOMS



- Original analyticity guarantees the successfulness of chiral extrapolation
- Relativistic renormalisation scheme possesses good convergency

Convergency property

- Assess the convergency of the chiral expansion



  - Two-loop contribution is small, approximately 10 MeV, for pion mass < 300 MeV;

Two-loop extraction of pion-nucleon sigma term

Two-loop sigma term in BχPT

- **Feynman-Hellmann theorem**

[**ZRL**, Chen, Guo, Guo, and Yao, arXiv:2508.11435 [hep-ph]]

- **Chiral expression up to $\mathcal{O}(p^5)$**
  - Tree + one loop + two loop $\sigma_{\pi N} = -4c_1M^2 - 4e_mM^4 + \Delta\sigma_{\pi N}^{(1)} + \Delta\sigma_{\pi N}^{(2)}$
  - One-loop: $\Delta\sigma_{\pi N}^{(1)} = \sigma_{\pi N}^{(1a)} + \sigma_{\pi N}^{(1b)} + \sigma_{\pi N}^{(1c)} + \sigma_{\pi N}^{(1d)} + \sigma_{\pi N}^{(1e)}$,
  - Two-loop: $\Delta\sigma_{\pi N}^{(2)} = \sigma_{\pi N}^{(2a)} + \sigma_{\pi N}^{(2b)} + \cdots + \sigma_{\pi N}^{(2l)} + \sigma_{\pi N}^{(2')} + \sigma_{\pi N}^{\text{sub.}}$.

$$\sigma_{\pi N}^{(1a)} = -\frac{3ig_A^2M^2m(J_{11} + M^2J_{21})}{2F^2}$$

$$\sigma_{\pi N}^{(2a)} = \frac{3g_A^2M^2m}{2F^4}(M^4I_{12011} + M^4I_{21011} + M^2I_{02011} + 2M^2I_{11011} + M^2I_{20011} + I_{10011} + I_{01011})$$

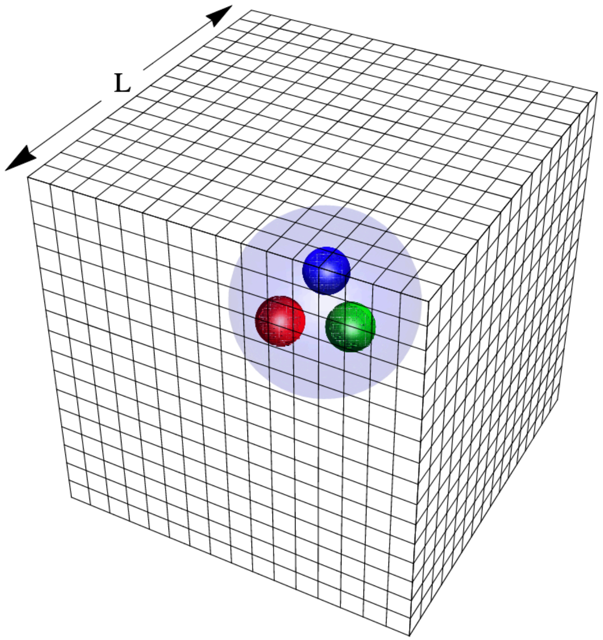
✓ $J_{\nu_1\nu_2}$ and $I_{\nu_1\nu_2\nu_3\nu_4\nu_5}$ are one- and two-loop Feynman integrals.

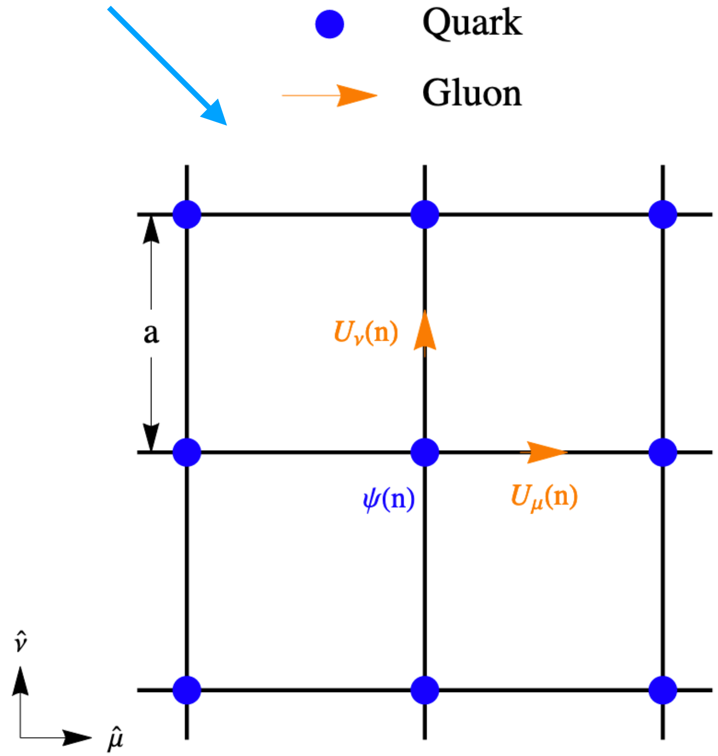
Sigma term on the lattice

- Lattice artifacts: finite-volume & lattice spacing corrections

Continuous Infinite volume *Discrete Finite volume*

$$\sigma_{\pi N} \implies \sigma_{\pi N} + \Delta_L \sigma_{\pi N} + b_\pi \frac{a}{\sqrt{t_0}} M_\pi^2,$$





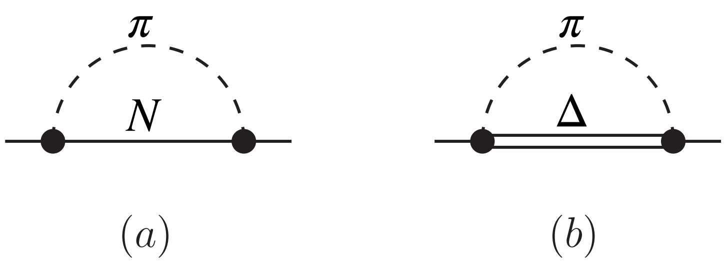
Sigma term on the lattice

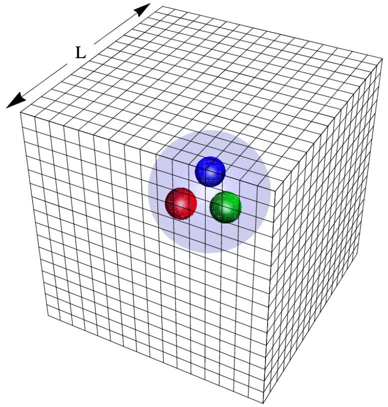
- Our treatment of FV corrections

[Liang, Yao, JHEP(2022)]

✓ ChPT at finite volume:

$$\Delta_L \sigma_{\pi N} = \Delta_L \sigma_{\pi N}^{(N)}(M_\pi; L) + \Delta_L \sigma_{\pi N}^{(\Delta)}(M_\pi; L)$$





✓ Our full expression (N for example):

$$\Delta_L \sigma_{\pi N}^{(N)}(M_\pi; L) = \sum_{n_s} \vartheta(n_s) \int_0^1 \mathrm{d}x_1 \frac{3g_A^2 m M_\pi^2}{32F^2\pi^2} \left[2K_0\left(\sqrt{\mathcal{M}_N^2}\right) + \frac{L^2 M_\pi^2 n_s (x_1 - 1)}{\sqrt{\mathcal{M}_N^2}} K_1\left(\sqrt{\mathcal{M}_N^2}\right) \right]$$

Ensemble	H102	N101	H105	C101	S400	N451	D450	D452	N203	S201	N200	D200	E250	N302	J303	E300
BChPT w/o Δ	−4.72	−0.94	−6.17	−1.49	−8.22	−1.70	−0.68	−1.11	−2.86	−16.3	−3.90	−1.73	−0.52	−9.07	−3.80	−1.06
BChPT w Δ	−9.17	−1.62	−11.7	−2.52	−16.5	−3.03	−1.11	−1.75	−5.37	−33.3	−7.25	−2.89	−0.76	−18.3	−6.94	−1.69

✓ Mainz 23 uses the expression at infinite L and treat b_L free:

$$\Delta_L \sigma_{\pi N}^{(N)}(M_\pi; L) \xrightarrow[L\to\infty]{} b_L \left(\frac{M_\pi^3}{M_\pi L} - \frac{M_\pi^3}{2} \right) \exp(-M_\pi L) \ , \quad b_L = 9g_A^2/(8\pi F^2) \ .$$

Sigma term on the lattice

- **Direct approach: Mainz 23** [Agadjanov, et al., PRL(2023)]
  - Effective scalar form factor (ESFF)

$$G_S^{\rm eff}(t, t_s) \equiv \mathrm{Re}\frac{C_3(t, t_s; \mathbf{0})}{C_2(t_s; \mathbf{0})}$$

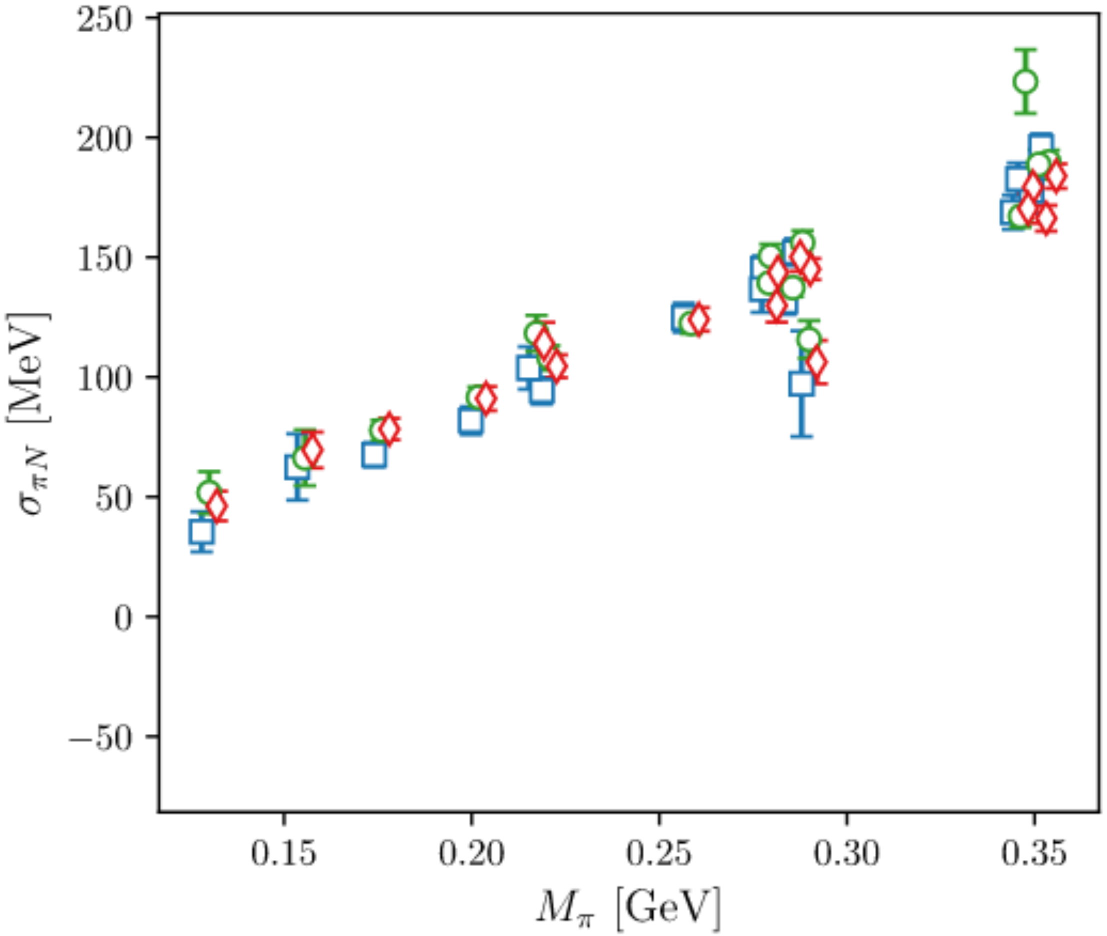
  - Summation method: summed correlator (SC)

$$S(t_s) = a\sum_{t=a}^{t_s-a} G_S^{\rm eff}(t, t_s) \xrightarrow{t_s \gg \Delta^{-1}} b_1 + (t_s - a)G_S$$

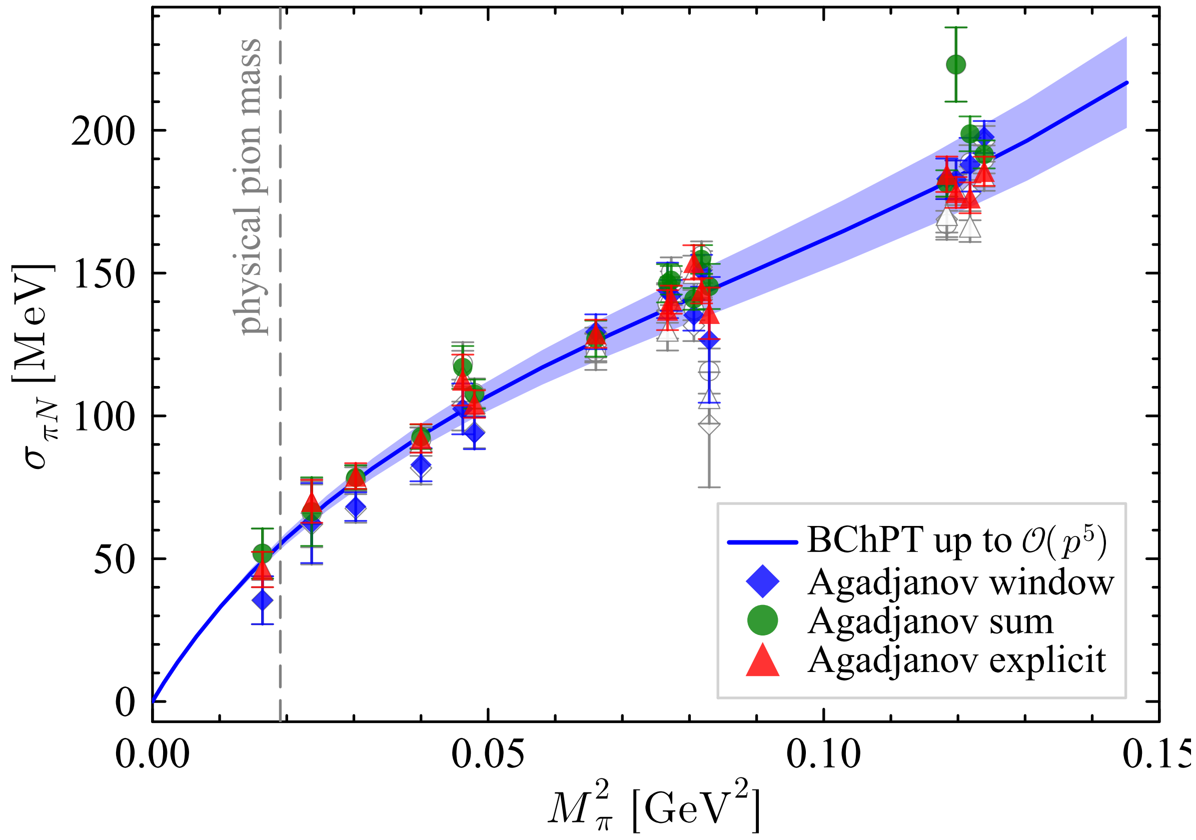
Δ is the energy gap between the g.s. and 1st e.s

- **Excited-state analysis**

✓ "window": window average of the SC
✓ "sum": explicit two-state fit to the SC
✓ "Explicit": explicit two-state fit to the ESFF



Chiral extrapolation of nucleon sigma term



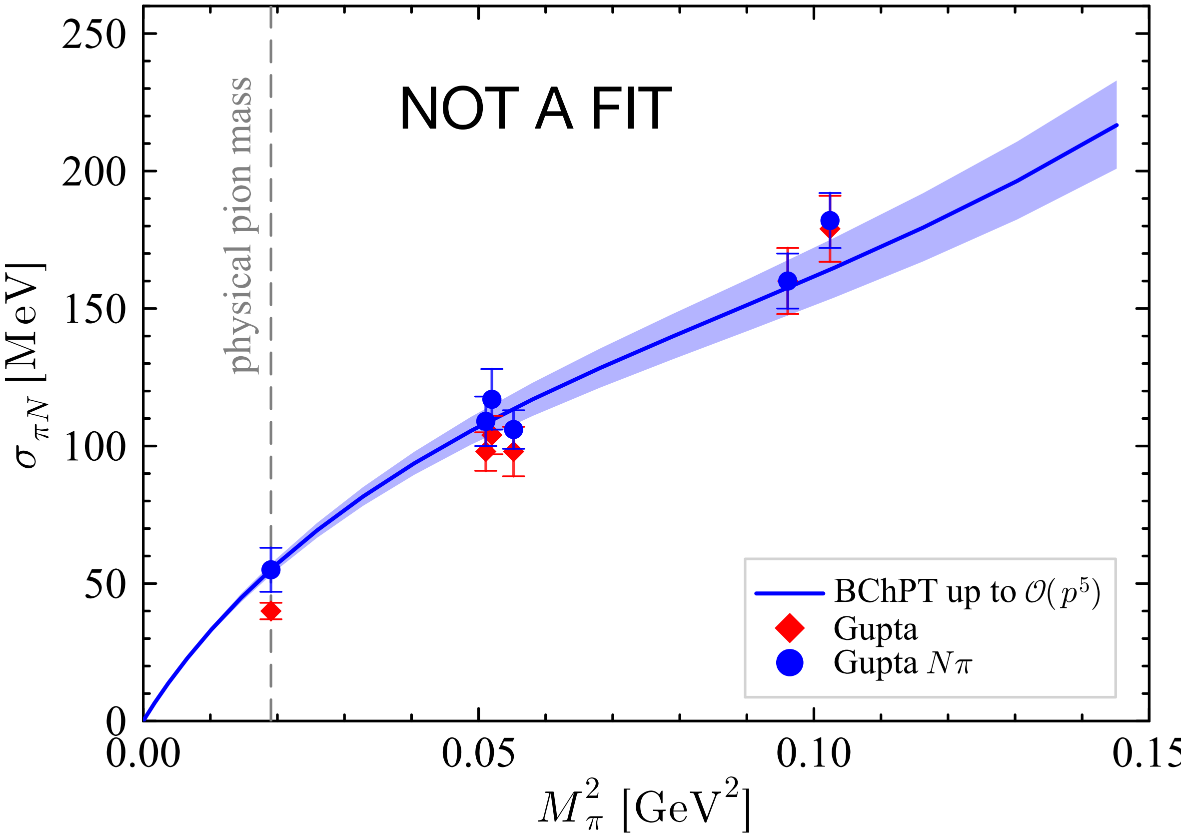
Fit procedure

$$\chi^2 = \chi^2_{m_N} + \omega_i \cdot \{\chi^2_{\text{win}}, \chi^2_{\text{sum}}, \chi^2_{\text{exp}}\}$$

- Constrained by physical m_N
- Three ESC approaches with weighting $\omega_i = \{1/2, 0, 1/2\}$
- Subtracting Finite volume corrections and lattice spacing effects

LECs	Values	Correlation matrix		
		$\tilde{m}$	$\tilde{c}_1$	$\tilde{e}_m$
$\tilde{m}$ [MeV]	863.7 ± 2.2	1.000	0.948	−0.640
$\tilde{c}_1$ [GeV^{-1}]	-1.07 ± 0.02		1.000	−0.703
$\tilde{e}_m$ [GeV^{-3}]	-5.64 ± 0.22			1.000
χ^2/d.o.f.	$16.49/(16+1-4) \simeq 1.27$			

Chiral extrapolation of nucleon sigma term



- Sigma term (M_{π^+}): $\sigma_{\pi N} = 56.1(2.6)$ MeV
- Sigma term (M_{π^0}): $\bar{\sigma}_{\pi N} = 53.3(2.4)$ MeV

In remarkable consistency with the dataset in which the ESC is properly accounted for.

Two-loop extraction of $\sigma_{\pi N}$

- Precise determination of $\sigma_{\pi N}$ in the physical world ($M_\pi = M_\pi^+$)

$$\sigma_{\pi N} = 56.1 \pm (2.0)_{\rm stat} \pm (1.5)_{\rm sys_1} \pm (0.6)_{\rm sys_2} \mathrm{MeV} = 56.1(2.6)\ \mathrm{MeV}$$

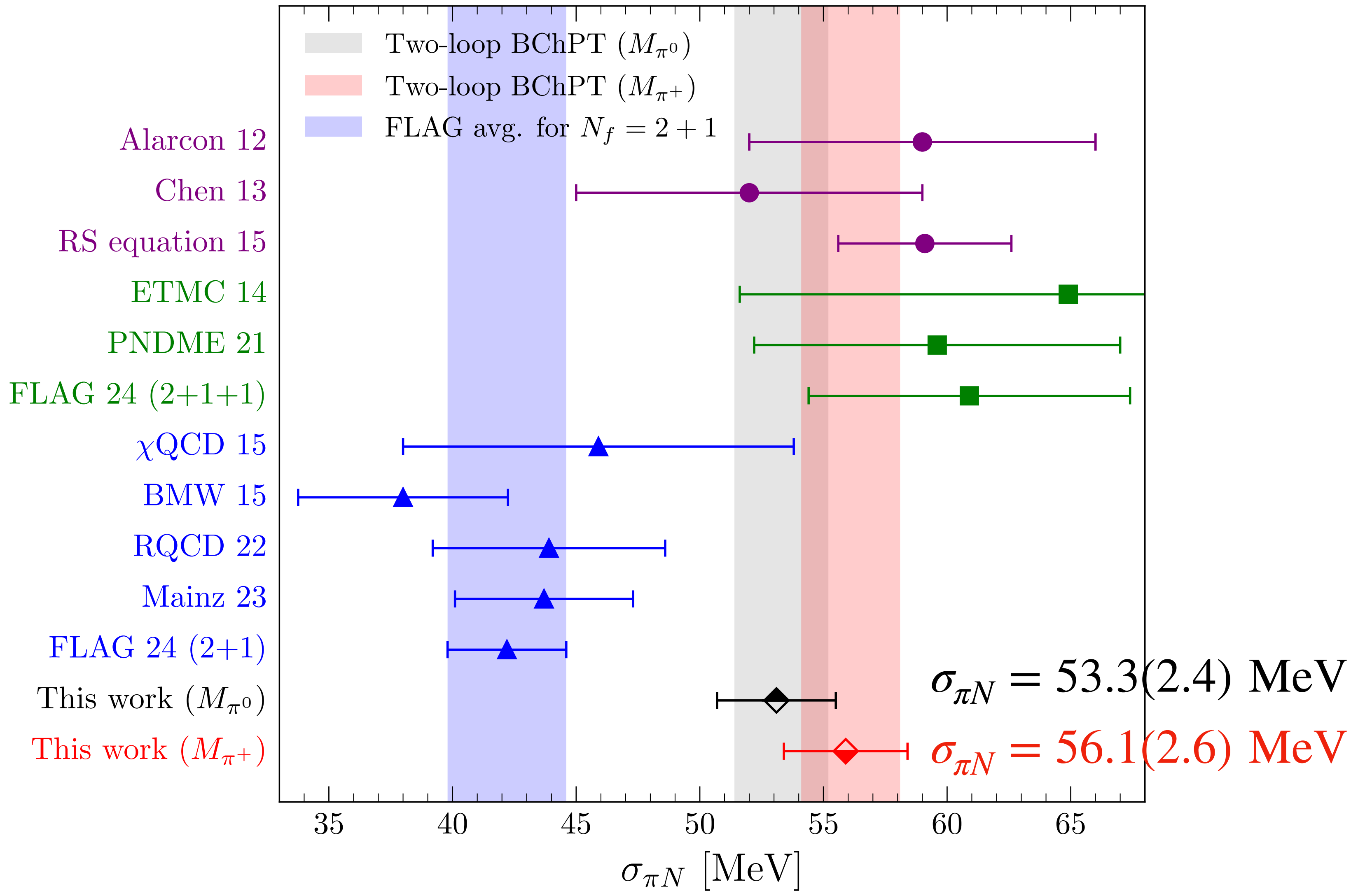
- Error budget

  (i) stat: propagated from the 1σ errors of the fitted parameters $(\tilde{m}, \tilde{c}_1, \tilde{e}_m)$;

  (ii) sys_1: arising from the errors in the one-loop renormalized LECs (c_2, c_3) involved in the $\mathcal{O}(p^4)$ chiral loop contribution;

  (iii) sys_2: due to truncation of the chiral expansion at $\mathcal{O}(p^5)$.

Comparison



Resolving the tension between lattice QCD and phenomenology!

Summary and outlook

Summary and outlook

- **Nucleon mass and sigma term in full-EOMS BChPT at two-loop order**
  - **Validity of EOMS at two-loop level:** the notable PCB issue addressed via dimensional counting analysis.
  - **Convergency:** the $\mathcal{O}(p^5)$ contribution from the full EOMS is small around 10 MeV, implying that the chiral series in the full EOMS scheme converges very well.

$$m_N = \{878.2 + \underbrace{68.8}_{\mathcal{O}(p^2)} + \underbrace{[-11.4]}_{\mathcal{O}(p^3)} + \underbrace{[-4.6]}_{\mathcal{O}(p^4)} + \underbrace{7.9}_{\mathcal{O}(p^5)}\}\ \text{MeV} = 938.9\ \text{MeV}$$

  - **Sigma term:** resolve long-standing tension between lattice QCD and dispersive determinations

$$\sigma_{\pi N} = 56.1(2.6)\ \text{MeV}$$

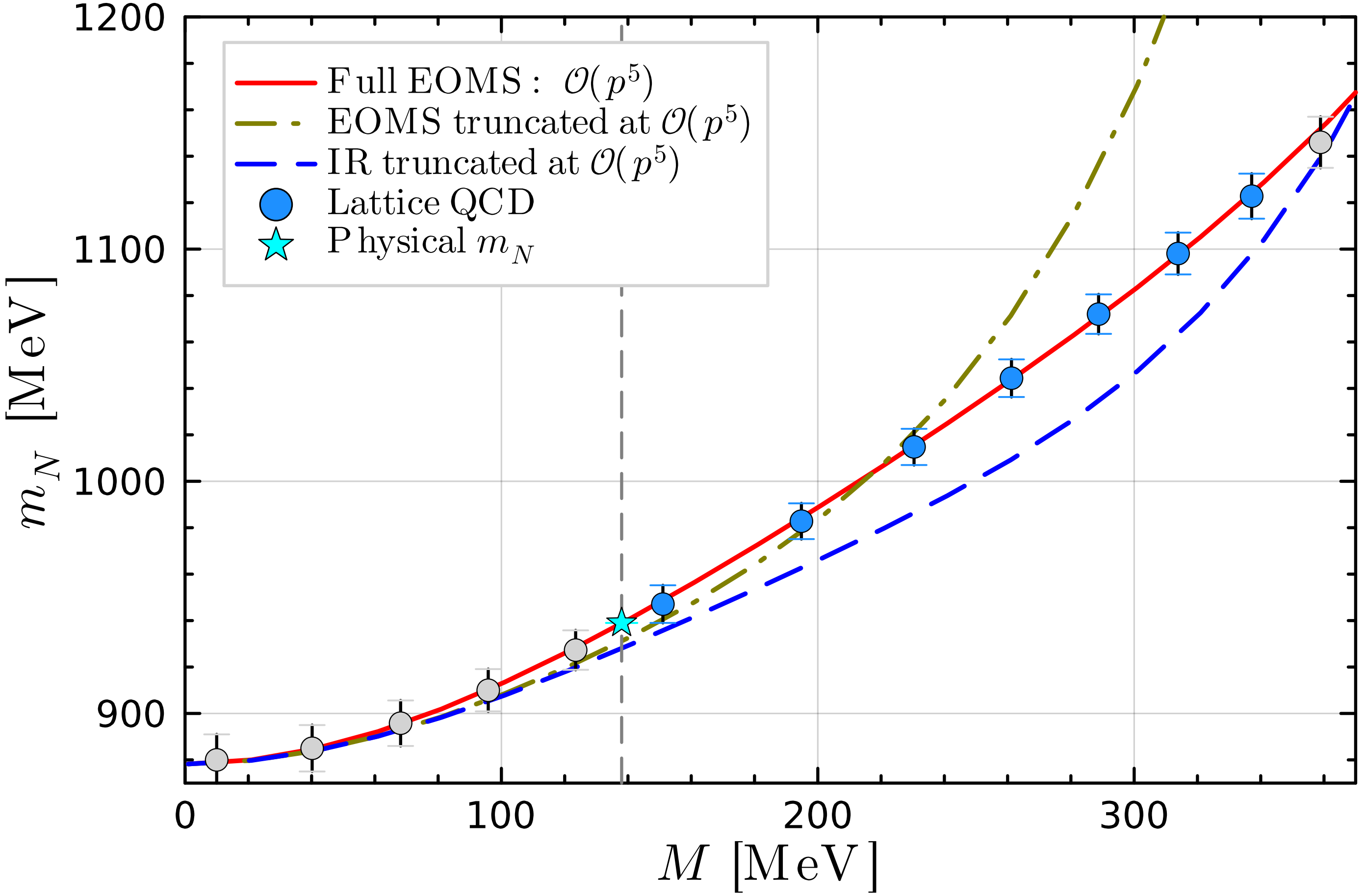
- **Outlook**
  - Provide reliable and high-order chiral expression for chiral extrapolation of lattice QCD data
  - Step into a promising era of two-loop BChPT, where the nucleon property can be explored with high precision!

Thanks!

Backup

Comparison of various schemes

- Full EOMS, EOMS truncated, IR truncated



  - IR-truncated: the discarded terms due to the $\mathcal{O}(p^5)$ truncation contribute sizeably.
  - EOMS-truncated: starts to deviate at pion mass ~220MeV.

Alternative two-loop representation

- **EOMS truncated at** $\mathcal{O}(p^5)$

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4 + k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5$$

$$k_1^{\text{EOMS}} = -4c_1 \,,$$

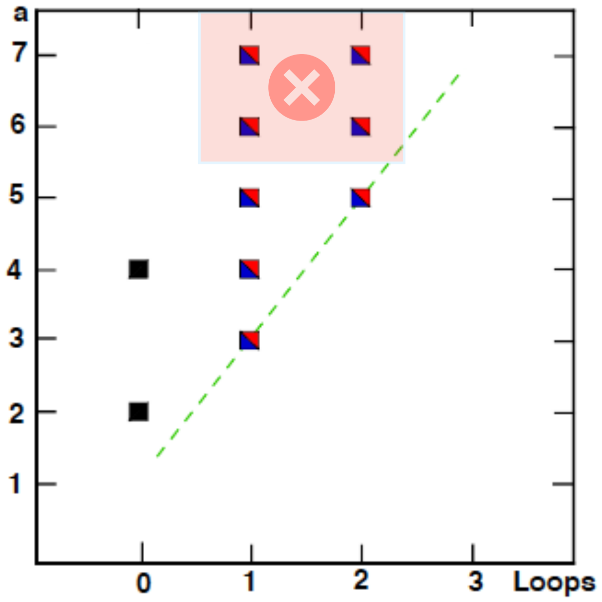
$$k_2^{\text{EOMS}} = -\frac{3g^2\pi}{2F^2\Lambda} \,,$$

$$k_3^{\text{EOMS}} = -\frac{3g^2}{2F^2 m\Lambda} + \frac{3(8c_1 - c_2 - 4c_3)}{2F^2\Lambda} \,,$$

$$k_4^{\text{EOMS}} = -2e_m + \frac{3g^2}{2F^2 m\Lambda} + \frac{3c_2}{8F^2\Lambda} \,,$$

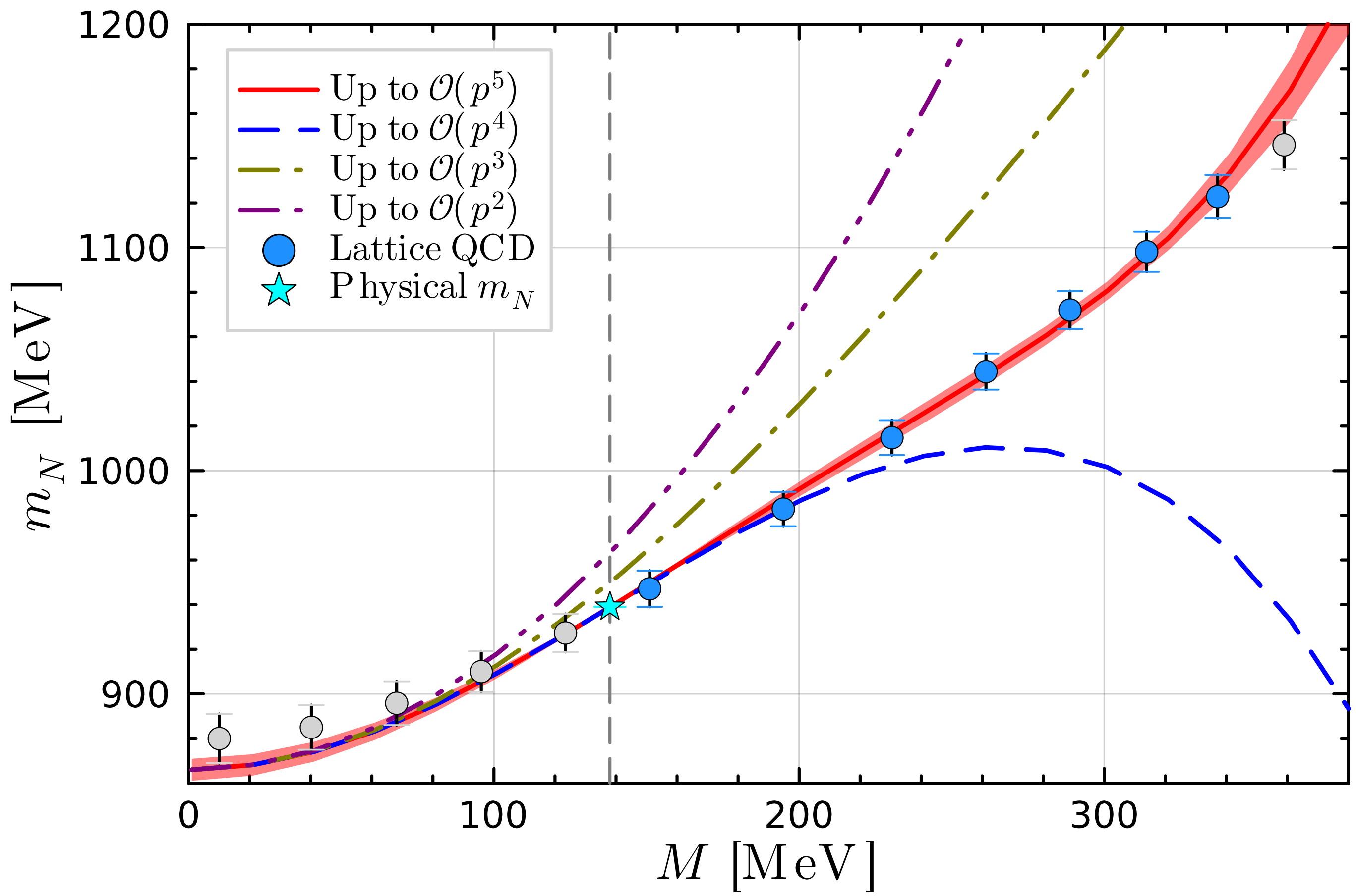
$$k_5^{\text{EOMS}} = \frac{3g^2(16g^2 - 3)\pi}{4F^4\Lambda^2} \,,$$

$$k_6^{\text{EOMS}} = \frac{19\pi g^4}{4F^4\Lambda^2} + \frac{6\pi g(d_{18} - 2d_{16})}{F^2\Lambda} + g^2\left[\frac{6\pi}{F^4\Lambda^2} + \frac{3\pi\left(F^2 + 8m^2(2\ell_4 - 3\ell_3)\right)}{16F^4\Lambda m^2}\right] .$$



Chiral extrapolation

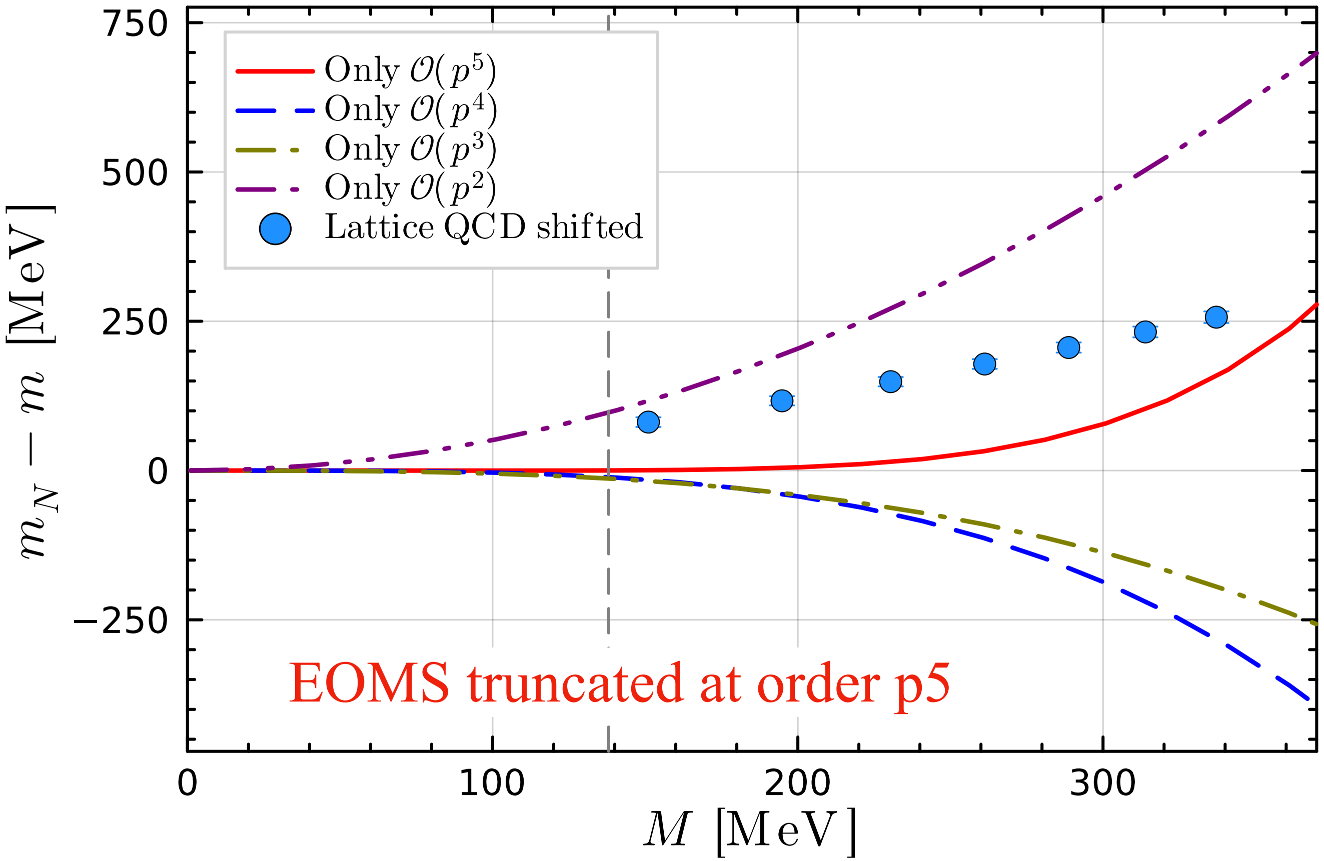
- EOMS truncated at order p5



  - Agree well in the fitting range, but deviate beyond
  - Weird behaviour of the pion mass dependence.

Convergency property

- Assess the convergency of the chiral expansion



  - Worse than full EOMS

Two-loop result

- **IR truncated at $\mathcal{O}(p^5)$**

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln \frac{M}{\mu} + k_4 M^4 + k_5 M^5 \ln \frac{M}{\mu} + k_6 M^5 \ .$$

$$k_1^{\rm IR} = -4c_1 \ ,$$

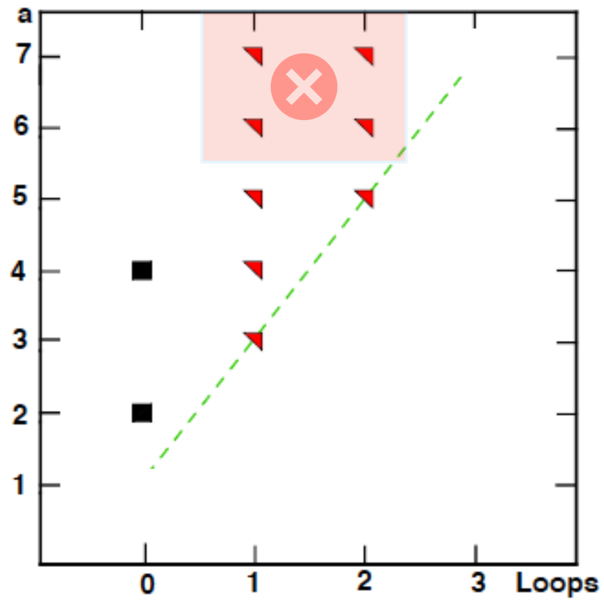
$$k_2^{\rm IR} = -\frac{3g^2\pi}{2F^2\Lambda} \ ,$$

$$k_3^{\rm IR} = -\frac{3g^2}{2F^2 m\Lambda} + \frac{3(8c_1 - c_2 - 4c_3)}{2F^2\Lambda} \ ,$$

$$k_4^{\rm IR} = -2e_m - \frac{3g^2}{4F^2 m\Lambda} + \frac{3c_2}{8F^2\Lambda} \ ,$$

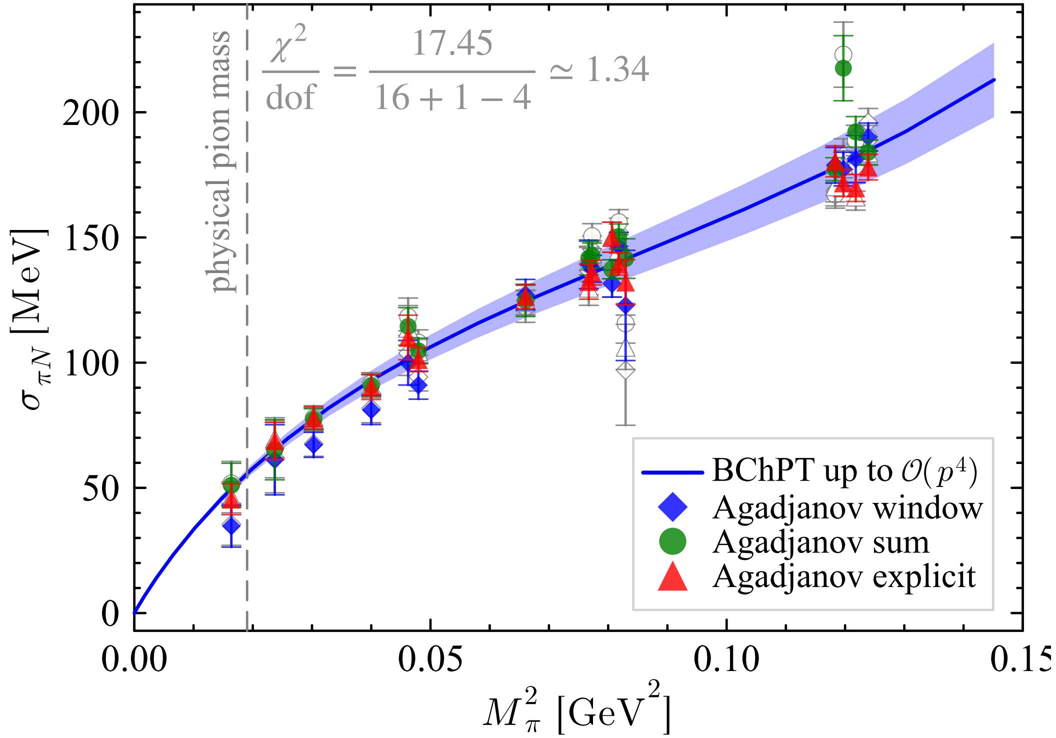
$$k_5^{\rm IR} = \frac{3g^2(16g^2-3)\pi}{4F^4\Lambda^2} \ ,$$

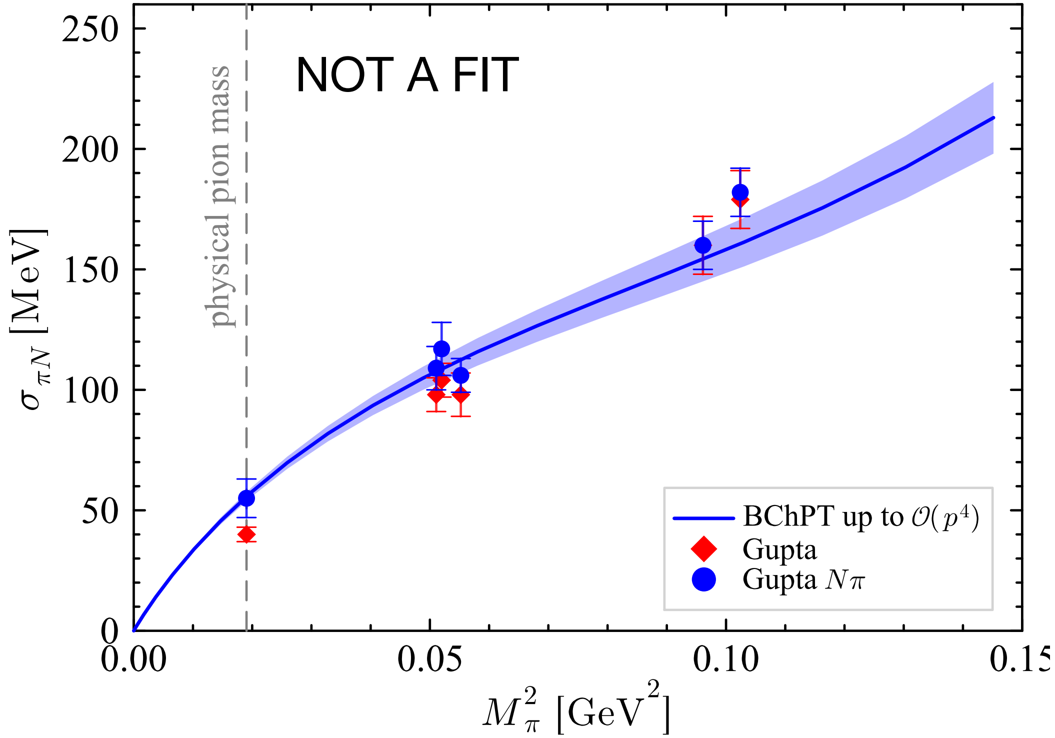
$$k_6^{\rm IR} = \frac{3\pi g^4}{F^4\Lambda^2} + \frac{6\pi g(d_{18} - 2d_{16})}{F^2\Lambda} + \frac{3\pi g^2\left(F^2 + 8m^2(2\ell_4 - 3\ell_3)\right)}{16F^4\Lambda m^2} \ .$$



Chiral extrapolation of nucleon sigma term

- Comparative fit: up to $\mathcal{O}(p^4)$



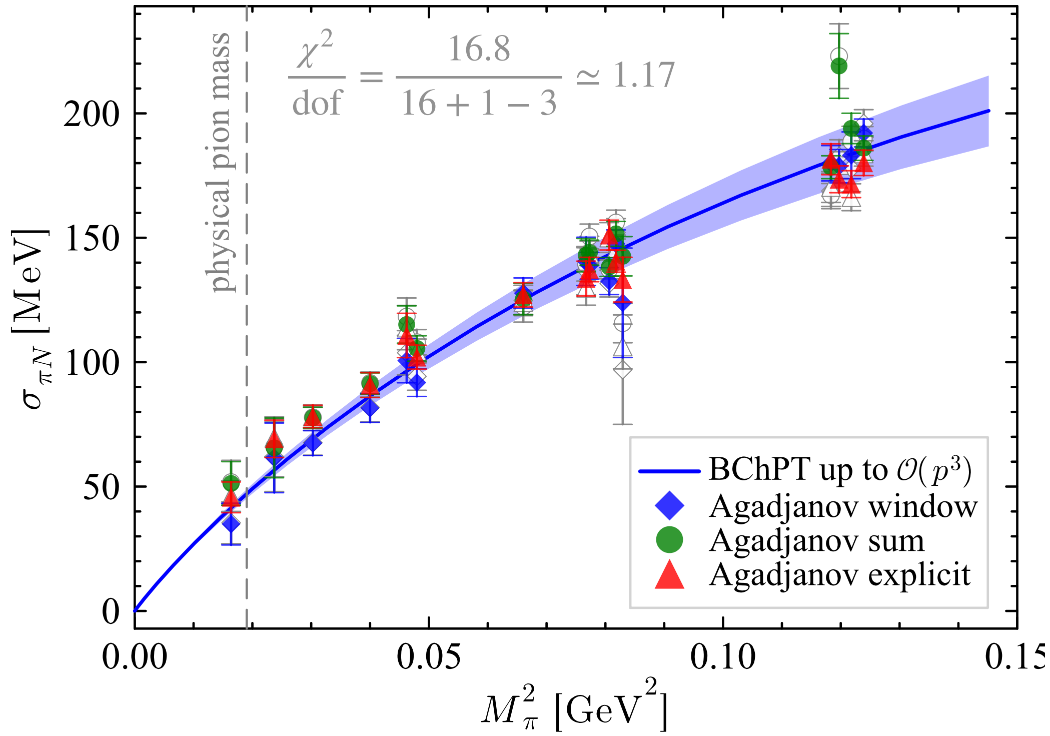


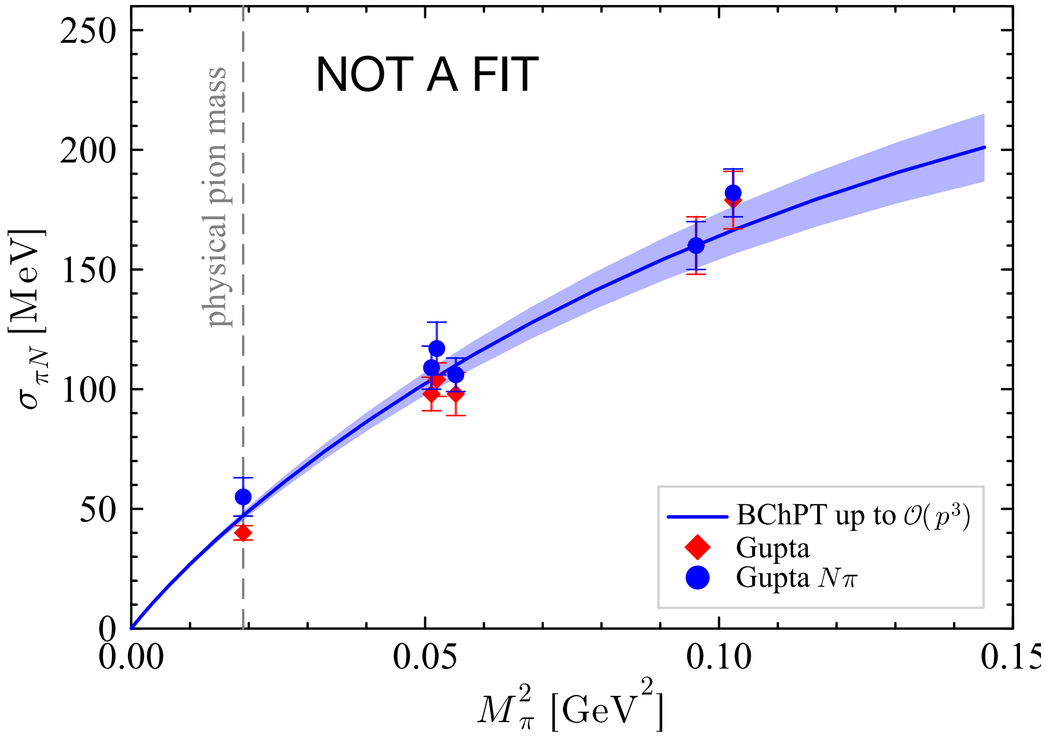
- Parameters: $\tilde{m} = 872.7(2.0)$ MeV, $\tilde{c}_1 = -1.09(2)$ GeV^{-1}, $\tilde{e}_m = -1.39(21)$ GeV^{-3}
- Sigma term (M_{π^+}): $\sigma_{\pi N} = 56.9(2.0)$ MeV
- Sigma term (M_{π^0}): $\bar{\sigma}_{\pi N} = 54.1(1.9)$ MeV

The e_m term mimics part of the two-loop contribution through parameter adjustment

Chiral extrapolation of nucleon sigma term

- Comparative fit: up to $\mathcal{O}(p^3)$





- Parameters: $\tilde{m} = 886.1(2.1)$ MeV, $\tilde{c}_1 = -0.84(3)$ GeV^{-1}
- Sigma term (M_{π^+}): $\sigma_{\pi N} = 48.7(2.1)$ MeV
- Sigma term (M_{π^0}): $\bar{\sigma}_{\pi N} = 46.0(1.9)$ MeV

The $\mathcal{O}(p^3)$ fit indeed yields small nucleon sigma term!