数据分析基础练习

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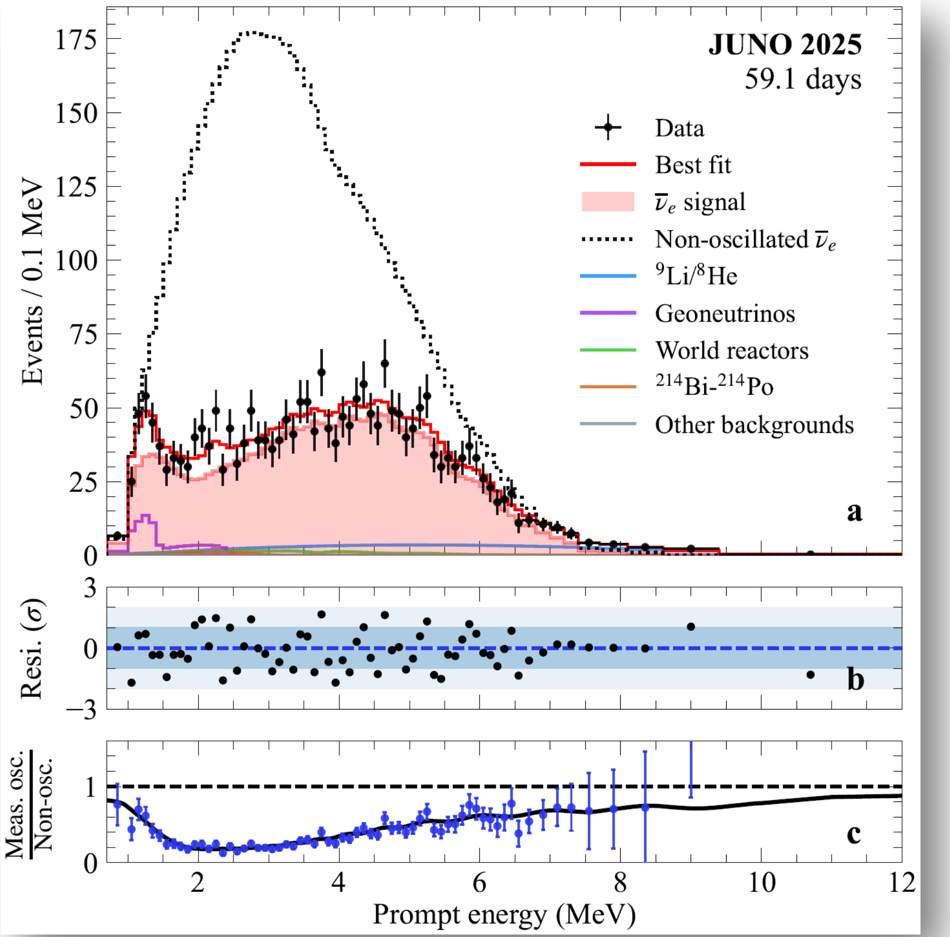
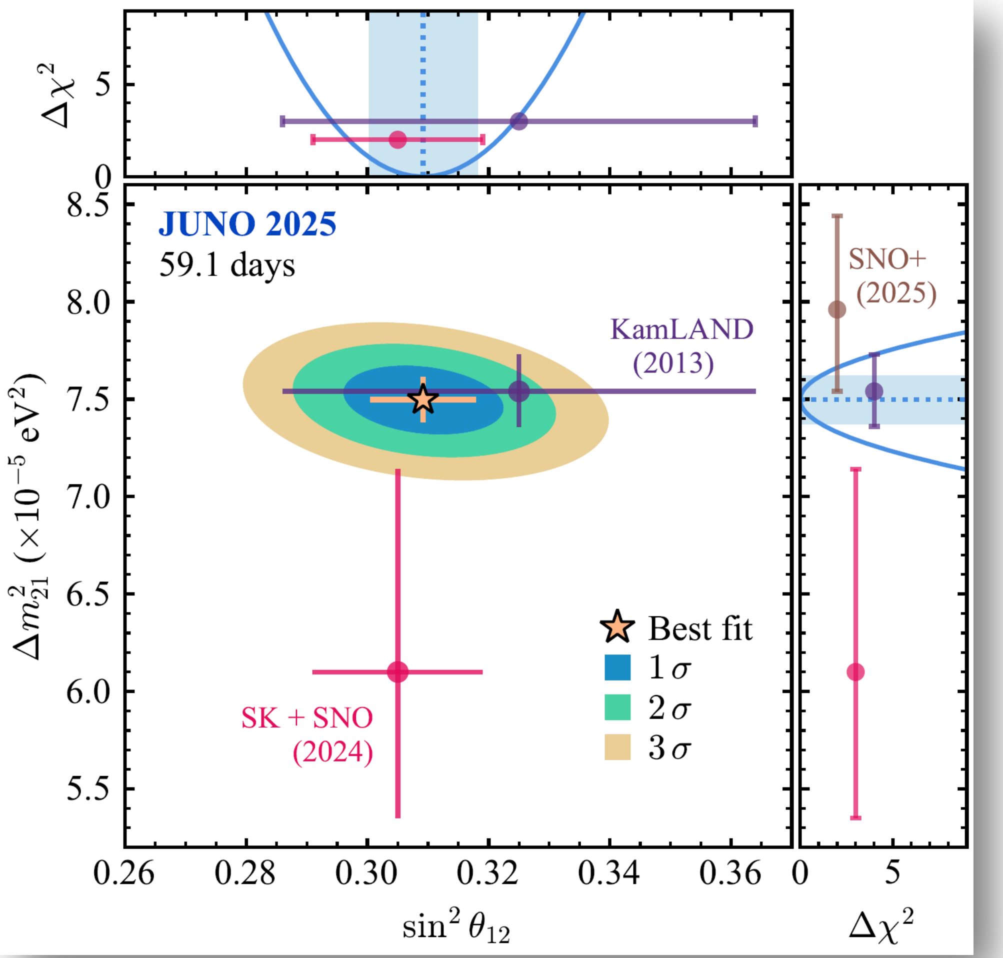
JUNO实验的首个中微子振荡结果





中微子振荡基础

味道本征态 $|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}^{*}|\nu_i\rangle$ 质量本征态

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

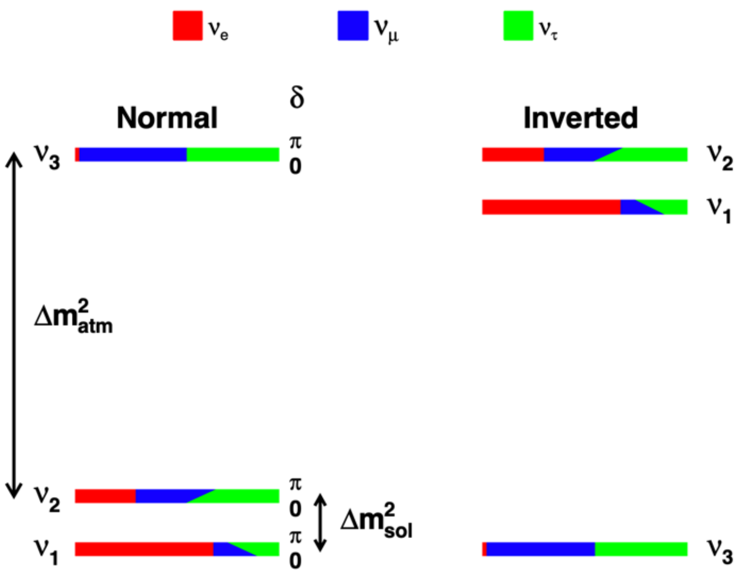
$c_{ij} = \cos\theta_{ij}$ $s_{ij} = \sin\theta_{ij}$

三个混合角: $\theta_{23}, \theta_{13}, \theta_{12}$

$c_{12}^2 \equiv \cos^2\theta_{12} = \frac{|U_{e1}|^2}{1-|U_{e3}|^2}$ $s_{12}^2 \equiv \sin^2\theta_{12} = \frac{|U_{e2}|^2}{1-|U_{e3}|^2}$

$s_{13}^2 \equiv \sin^2\theta_{13} = |U_{e3}|^2$

$s_{23}^2 \equiv \sin^2\theta_{23} = \frac{|U_{\mu 3}|^2}{1-|U_{e3}|^2}$ $c_{23}^2 \equiv \cos^2\theta_{23} = \frac{|U_{\tau 3}|^2}{1-|U_{e3}|^2}$



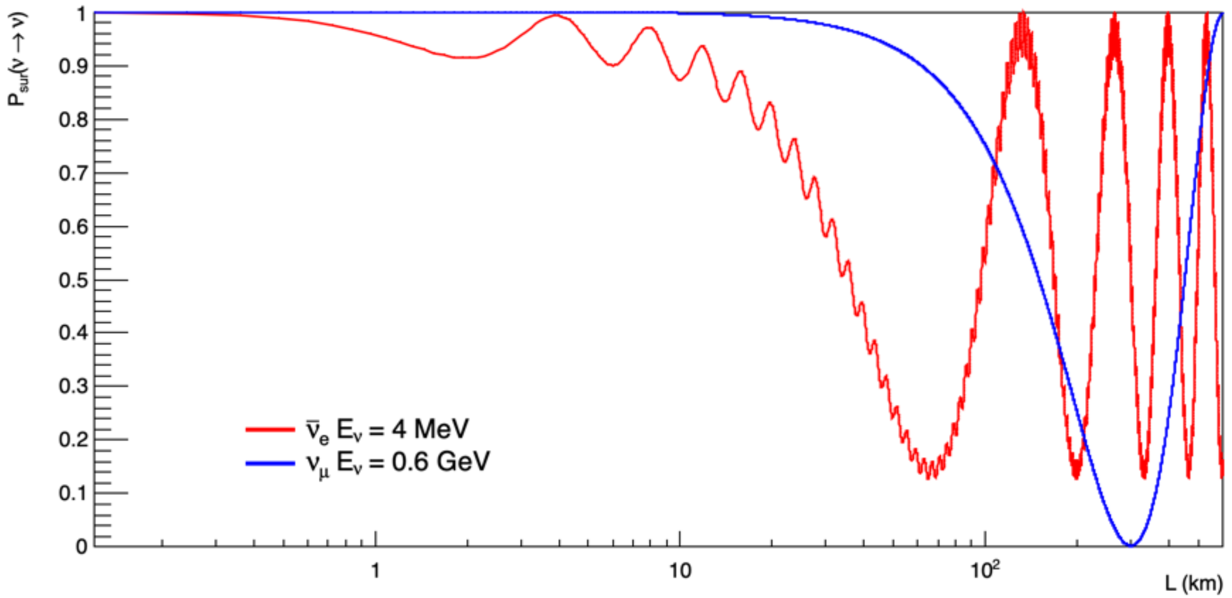
中微子振荡概率

$$U=\begin{pmatrix}U_{e1}&U_{e2}&U_{e3}\\U_{\mu1}&U_{\mu2}&U_{\mu3}\\U_{\tau1}&U_{\tau2}&U_{\tau3}\end{pmatrix}=\begin{pmatrix}c_{12}c_{13}&s_{12}c_{13}&s_{13}e^{-i\delta}\\-s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta}&c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta}&s_{23}c_{13}\\s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta}&-c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta}&c_{23}c_{13}\end{pmatrix}$$

$$P_{\alpha\beta}=\left|\langle\nu_\beta|\nu_\alpha(t)\rangle\right|^2=\delta_{\alpha\beta}-4\sum_{i<j}^{3}Re\left[U_{\alpha i}U_{\beta i}^*U_{\alpha j}^*U_{\beta j}\right]\sin^2\Delta_{ij}+2\sum_{i<j}^{3}Im\left[U_{\alpha i}U_{\beta i}^*U_{\alpha j}^*U_{\beta j}\right]\sin2\Delta_{ij}$$

$$\Delta_{ij}=\Delta m_{ij}^2\frac{L}{4E}$$

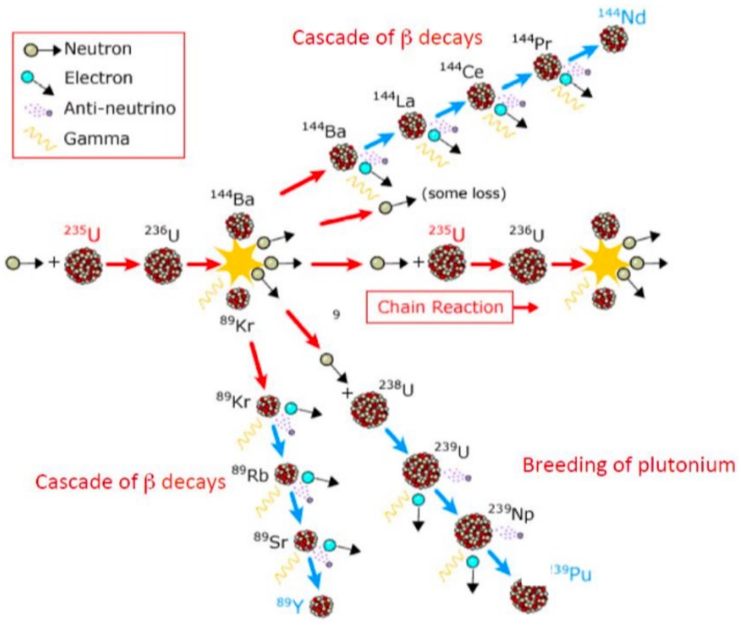
$$P_{\alpha\alpha}=1-4|U_{\alpha1}|^2|U_{\alpha2}|^2sin^2\left(\frac{\Delta m_{21}^2L}{4E}\right)-4|U_{\alpha1}|^2|U_{\alpha3}|^2sin^2\left(\frac{\Delta m_{31}^2L}{4E}\right)-4|U_{\alpha2}|^2|U_{\alpha3}|^2sin^2\left(\frac{\Delta m_{32}^2L}{4E}\right)$$



反应堆中微子的产生

反应堆中微子源：U/Pu的裂变级联过程

- 常见商用反应堆功率：2.9 GW
- $1\ GW = 6.2415 \times 10^{21}\ MeV$
- ~ 200 MeV / fission，　6 $\bar{\nu}_e$ / fission
- ~ 2 x 10^{20} $\bar{\nu}_e$/GW_{th}/Sec (1/5 above IBD threshold)

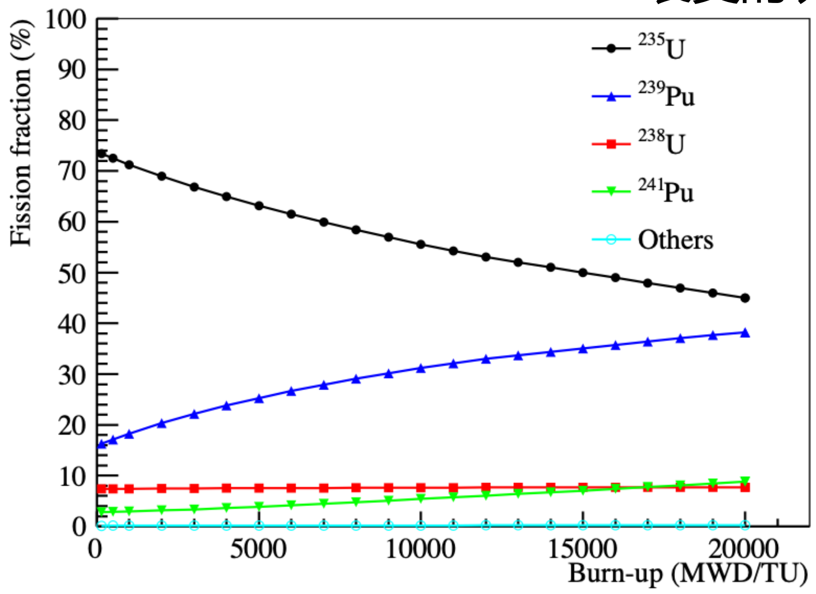


裂变产生的中微子数 $$S(E_\nu) = \sum_i^{isotopes} f_i S_i(E_\nu)$$

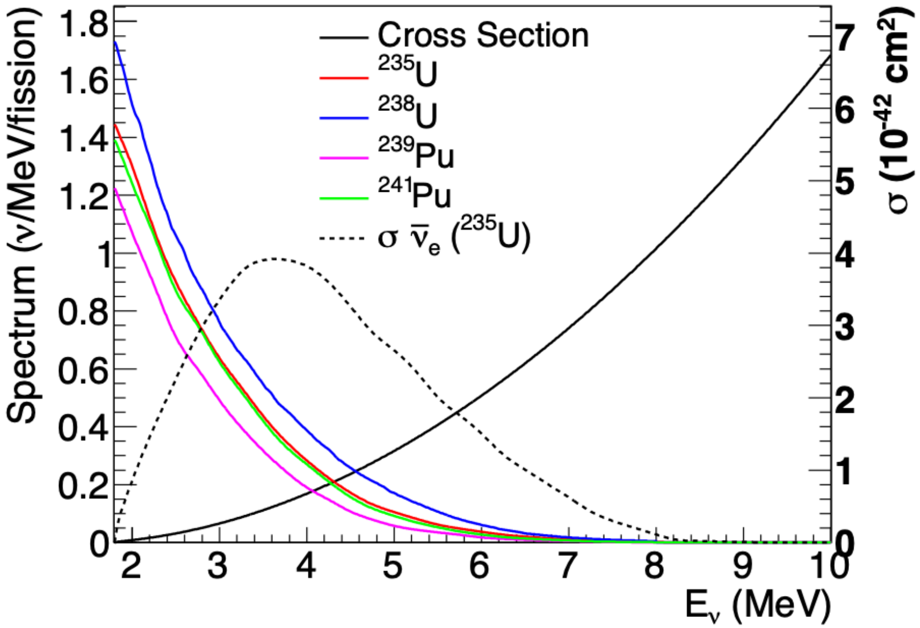
每秒产生的中微子数 $$\boxed{S(E_\nu) = \frac{W_{th}}{\sum_i (f_i/F) e_i} \sum_i^{istopes} (f_i/F) S_i(E_\nu)}$$

裂变的热功率 $$W_{th} = \sum_i f_i e_i\ , \quad F = \sum_i f_i$$

裂变的次数



中微子反应



反贝塔衰变 (IBD) $\bar{\nu}_e + p \rightarrow n + e^+$

- 快信号: $E_{\text{prompt}} \approx E_\nu - 0.78\ MeV$
- 慢信号: nH (2.2 MeV)

$$E_\nu = E_{e^+} + m_n + T_n - m_p \approx E_{prompt} + 0.78\ MeV,$$

1 IBD / (ton•GW•day) @ 1km

作业1： 中微子振荡曲线

- $sin^2\theta_{12} = 0.3$
- $sin^2\theta_{13} = 0.02$
- $\Delta m_{21}^2 = 7.5\times10^{-5}eV^2$
- $\Delta m_{32}^2 = 2.4\times10^{-3}eV^2$
- $L = 52.5\ km$
- $E: 0 - 12\ MeV$

- 画出振荡概率和能量的关系图

作业1: 预测中微子能谱

$$\frac{dN}{dE_{\rm vis}^{\rm obs}} = \frac{N_p T}{4\pi L^2}\int_{E_{\rm thr}}^{\infty} dE_\nu \frac{dN}{dE_\nu} P_{ee}(L, E_\nu) \times \sigma_{\rm IBD}(E_\nu)\, G(E_\nu - 0.8{\rm MeV} - E_{\rm vis}^{\rm obs}, \delta E_{\rm vis})$$

Neutrino Flux per fission:

$$\begin{aligned}\phi(E_\nu) = \ & f_{^{235}U} \exp\left(0.870 - 0.160E_\nu - 0.091E_\nu^2\right) \\ & + f_{^{239}Pu} \exp\left(0.896 - 0.239E_\nu - 0.0981E_\nu^2\right) \\ & + f_{^{238}U} \exp\left(0.976 - 0.162E_\nu - 0.0790E_\nu^2\right) \\ & + f_{^{241}Pu} \exp\left(0.793 - 0.080E_\nu - 0.1085E_\nu^2\right),\end{aligned}$$

Thermal Power: $20\mathrm{GW}_{\mathrm{th}}$

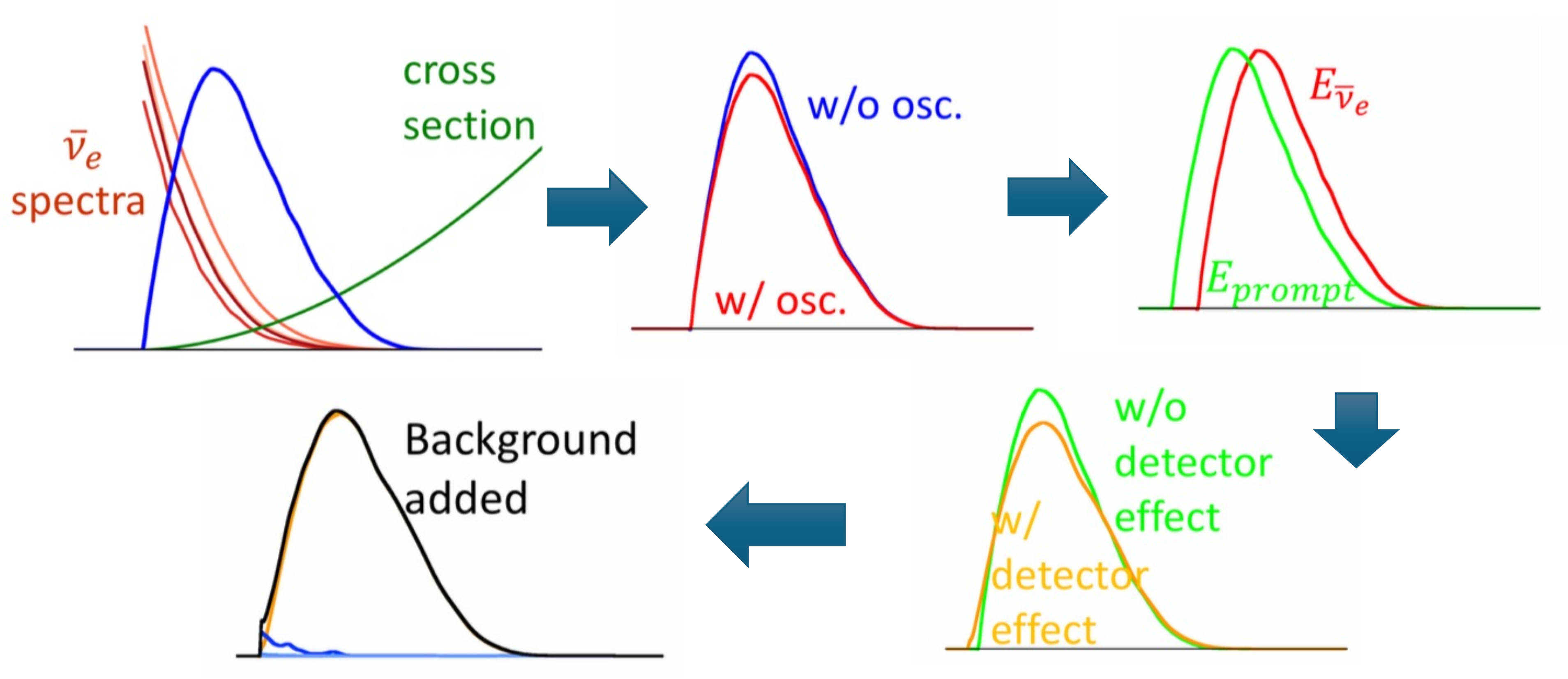
Fission Fraction: $f_{U235} = 0.58$, $f_{Pu239} = 0.29$, $f_{U238} = 0.08$, $f_{Pu241} = 0.05$

Fission Energy: $\varepsilon_{U235} = 202.36$ MeV, $\varepsilon_{Pu239} = 211.12$ MeV,

$\varepsilon_{U238} = 205.99$ MeV, $\varepsilon_{Pu241} = 214.26$ MeV

- IBD 反应截面: $$\sigma_{\mathrm{IBD}} = 0.0952\left(\frac{E_e\, p_e}{1\mathrm{MeV}^2}\right) \times 10^{-42}\,\mathrm{cm}^2,$$
- 2万吨液闪的质子数： $N_p = 1 \text{ x } 10^{33}$
- T = 5年
- 探测效率: 80%
- $E_{vis} \sim E_e + m_e \sim E_v - 0.78$ MeV
- 能量分辨率: $$\frac{\delta E}{E} = \frac{3\%}{\sqrt{E(MeV)}}$$

能谱预测



$\bar{\nu}_e$
spectra
cross
section
w/o osc.
w/ osc.
$E_{\bar{\nu}_e}$
E_{prompt}
w/o
detector
effect
w/
detector
effect
Background
added