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New horizon symmetries, hydrodynamics, and quantum chaos

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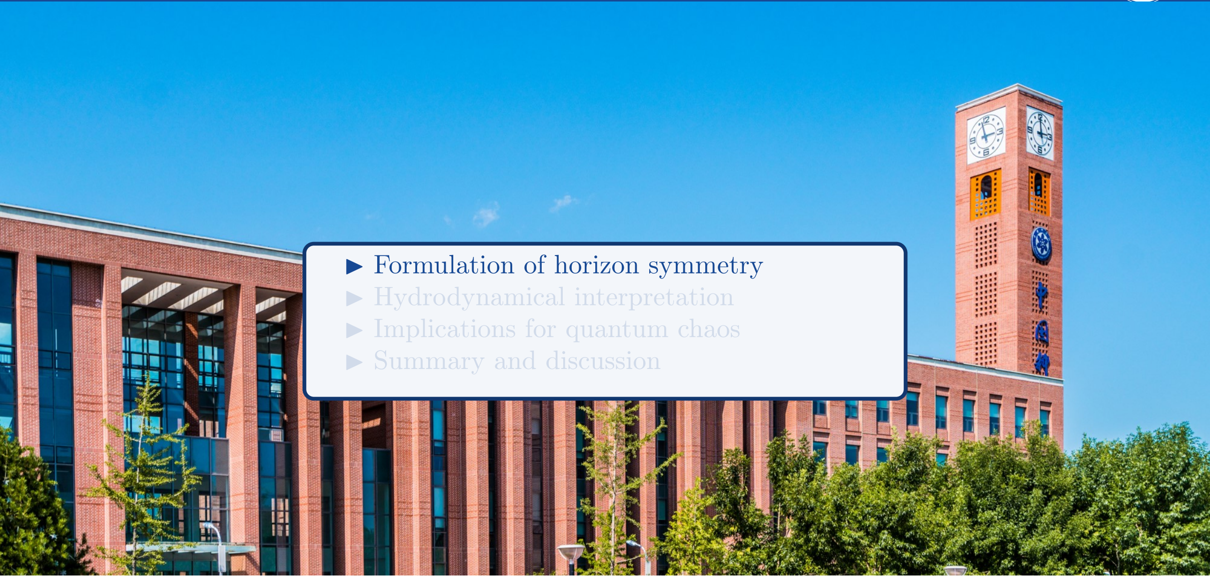

Definition of key null vector

Consider a (d+1)-dimensional black hole spacetime $\mathcal{M}$ with a horizon denoted as $\mathcal{H}$. By definition, $\mathcal{H}$ can be specified by a set of embedding functions $X^A(\sigma^a)$ in $\mathcal{M}$, where $\{\sigma^a\}$ denote intrinsic coordinates on $\mathcal{H}$.

The intrinsic metric on $\mathcal{H}$ can be written as

$$ds^2_{\mathcal{H}} = h_{ab} d\sigma^a d\sigma^b, \quad h_{ab} \triangleq G_{AB}(X)\partial_a X^A \partial_b X^B, \tag{1}$$

where "$\triangleq$" is an equality evaluated at the horizon.

By definition, h_{ab} is degenerate and has a single eigenvector $\hat{\ell}^a$ with zero eigenvalue,

$$h_{ab}\hat{\ell}^b = 0. \tag{2}$$

Through $X^A(\sigma^a)$, $\hat{\ell}^a$ can be pushed forward to a spacetime vector field ℓ^A,

$$\ell^A \triangleq \hat{\ell}^a \partial_a X^A. \tag{3}$$

Properties related to null vector ℓ^A

By definition ℓ^A is null on $\mathcal{H}$ and also normal to $\mathcal{H}$, that is

$$G_{AB}\ell^A\ell^B \triangleq 0, \quad \ell_A \partial_a X^A \triangleq 0. \tag{4}$$

Moreover, ℓ^A must satisfy the corresponding geodesics equation

$$\ell^B \nabla_B \ell^A \triangleq \kappa \ell^A, \tag{5}$$

with some scalar κ related to non-affine parameter. Scalar κ can be fixed to be constant.

The pairs $(\hat{\ell}^a, \kappa)$ is defined as *horizon structure*. $\Longrightarrow$ **Key definition in this work!**

In general, there is no unique way to define horizon structure due to the following freedom to normalize $\hat{\ell}^a$,

$$\hat{\ell}^a \to \hat{\ell}'^a = e^\rho \hat{\ell}^a, \quad \kappa \to e^\rho \left(\kappa + \mathcal{L}_\ell \rho\right) \triangleq e^\rho \left(\kappa + \mathcal{L}_{\hat{\ell}} \rho\right), \tag{6}$$

where ρ is an arbitrary scalar on the horizon. $\Longrightarrow$ **Define the equivalent class.**

Definition of horizon symmetry

Consider an infinitesimal variation of the metric $G_{AB} \to G'_{AB} = G_{AB} + f_{AB} \implies$

$$\delta h_{ab} \triangleq f_{AB} \partial_a X^A \partial_b X^B. \tag{7}$$

The variation f_{AB} preserves the horizon structure if the following two conditions holds:

- The vector $\hat{\ell}^a$ on $\mathcal{H}$ has still zero eigenvalue under the new horizon metric,

$$\delta h_{ab} \hat{\ell}^b \triangleq f_{AB} \partial_a X^A \partial_b X^B \hat{\ell}^b \triangleq 0 \iff f_{AB} \ell^B \triangleq -c \ell_A, \tag{8}$$

  for some infinitesimal function c defined on $\mathcal{H}$.

- Require κ to be unchanged under the new metric,

$$\ell^B \nabla'_B \ell^A \triangleq \kappa \ell^A \iff \ell^B \delta \nabla_B \ell^A \triangleq 0, \; \nabla'_B = \nabla_B + \delta \nabla_B, \tag{9}$$

  where ∇'_A denotes the covariant derivative associated with G'_{AB}.

Equations for horizon symmetry

It is convenient to introduce another spacetime 1-form n_A that is null on $\mathcal{H}$,

$$\ell^A n_A \triangleq -1, \quad G_{AB} n^A n^B \triangleq 0, \quad n^A = G^{AB} n_B. \tag{10}$$

Now suppose the metric variation is generated by a diffeomorphism,

$$f_{AB} = \mathcal{L}_\chi G_{AB}, \tag{11}$$

for some infinitesimal vector field χ^A.

Key point: χ^A generates a horizon symmetry $\Longleftrightarrow$ horizon structure is preserved.

After some tedious algebra, the following *central equations* can be obtained,

$$\begin{cases} \mathcal{L}_\chi \ell^A - G^{AB} \mathcal{L}_\chi \ell_B \triangleq c \ell^A \ , \\ \frac{1}{2} \ell^B \ell^C \mathcal{L}_n \mathcal{L}_\chi G_{BC} + c\kappa \triangleq \mathcal{L}_\ell c \ . \end{cases} \Longrightarrow \textbf{Complicated Equations!} \tag{12}$$

Practical form of constraint equations

Generally, the vector field χ^A can be parameterized as

$$\chi^A \triangleq f\ell^A + Y^A + Zn^A = \tilde{\chi}^A + Zn^A, \quad Y^A n_A \triangleq 0, \quad Y^A \ell_A \triangleq 0, \tag{13}$$

where f, Y^A and Z are functions of the horizon coordinates.

Based on this parametrization, two central equations can be written as

$$\begin{aligned} \mathcal{L}_\ell Z - \kappa Z &\triangleq 0 \\ \mathcal{L}_\ell Y^A - q^{AB}\nabla_B Z + Z\eta_\perp^A &\triangleq a\ell^A \end{aligned} \tag{14}$$

and

$$\begin{aligned} &\mathcal{L}_\ell\left(\mathcal{L}_\ell + \kappa\right) f + \left(\mathcal{L}_\ell + \kappa\right) a + \mathcal{L}_Y \kappa + \eta_\perp^A \nabla_A Z + Z\mathcal{F} \triangleq 0 \\ &\mathcal{F} \triangleq \frac{1}{2}\ell^A \ell^B \mathcal{L}_n \mathcal{L}_n G_{AB} - \eta_\perp^2 - \left(\mathcal{L}_\ell - \kappa\right)\lambda \end{aligned} \tag{15}$$

Comment: $\chi_A \ell^A \triangleq -Z = 0$ corresponds to special case $\Longrightarrow \chi^A$ along the horizon.

Simplifications of the horizon symmetry equations

Further simplifications of the horizon symmetry equations can be made by choosing a convenient set of horizon coordinates

Choose the normalization of $\hat{\ell}^a$ and the intrinsic coordinates σ^a such that

$$\kappa = \kappa_0 = \text{ const } , \quad \hat{\ell}^a = \left(\frac{\partial}{\partial \sigma^0}\right)^a . \tag{16}$$

Also, the pull back 1-form $\hat{n}_a \triangleq \partial_a X^A n_A$ can be chosen as $\hat{n}_a = (-1, 0)$.

The first equation which only involves unknown function Z then becomes,

$$\ell^A \partial_A Z \triangleq \frac{\partial}{\partial \sigma^0} Z = \kappa_0 Z \quad \Longrightarrow \quad Z = \gamma(\vec{\sigma}) e^{\kappa_0 \sigma^0} = \gamma(\vec{\sigma}) e^{-\frac{2\pi}{\beta} u_\mu x^\mu} , \tag{17}$$

where γ is a generic function of the spatial horizon coordinates $\vec{\sigma}$.

New horizon symmetry $\Longrightarrow$ exponentially growing solution $\Longrightarrow$ **key to quantum chaos**

An explicit example

Other equations can not be solved without specifying the concrete horizon metric.

Considering the black brane solution in AdS_{d+1} in the EF coordinates,

$$ds^2 = G_{AB} dx^A dx^B = -2u_\mu dx^\mu dr + \chi_{\mu\nu} dx^\mu dx^\nu, \tag{18}$$

with

$$\begin{aligned} \chi_{\mu\nu} &= -F(r) u_\mu u_\nu + g(r) \Delta_{\mu\nu}, \quad & \Delta_{\mu\nu} &= \eta_{\mu\nu} + u_\mu u_\nu, \\ F(r) &= \frac{r^2}{R^2}\left(1 - \frac{r_0^d}{r^d}\right), & g(r) &= \frac{r^2}{R^2}. \end{aligned} \tag{19}$$

Based on this specific case, the remaining unknown functions Y^i and f can be solved as

$$\begin{aligned} Y^i &= \zeta^i(\vec{\sigma}) + \frac{1}{\kappa_0 g(r_0)} \partial_i \gamma(\vec{\sigma}) e^{-\frac{2\pi}{\beta} u_\mu x^\mu}, \\ f &= \lambda(\vec{\sigma}) + \alpha(\vec{\sigma}) e^{\frac{2\pi}{\beta} u_\mu x^\mu} + \frac{1}{4\kappa_0^2} F''(r_0) \gamma(\vec{\sigma}) e^{-\frac{2\pi}{\beta} u_\mu x^\mu} \end{aligned} \tag{20}$$

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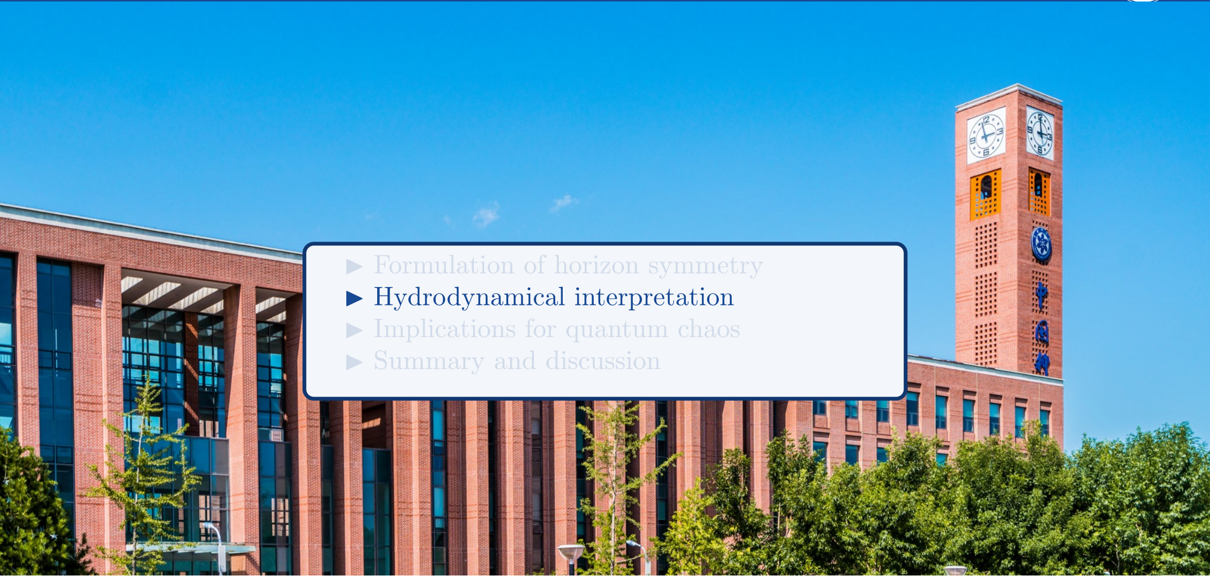

Short review of quantum hydrodynamics

The dynamical variables are maps $\mathcal{Y}^\mu(\sigma^a)$ between $\sigma^a = (\sigma^0, \sigma^i)$ and $y^\mu = (t, y^i)$.

Spatial $\sigma^i \Longrightarrow$ label fluid elements. Temporal $\sigma^0 \Longrightarrow$ internal time of fluid elements.

By using such maps, one can define the basic dynamical variables as

$$\beta^\mu \equiv \beta u^\mu = \frac{\partial \mathcal{Y}^\mu}{\partial \sigma^0}, \quad \eta_{\mu\nu} u^\mu u^\nu = -1. \tag{21}$$

An important element in the formulation of the EFT is the "gauge symmetries"

$$\sigma^0 \to \sigma'^0 = \sigma^0 + \lambda(\vec{\sigma}), \quad \sigma^i \to \sigma'^i = \sigma^i + \zeta^i(\vec{\sigma}), \tag{22}$$

where λ and ζ^i are arbitrary functions of the spatial variable $\vec{\sigma}$.

For maximally chaotic systems, another "shift symmetry" is needed,

$$\sigma^0 \to \sigma'^0 = \sigma^0 + \alpha(\vec{\sigma}) e^{-\kappa_0 \sigma_0} + \tilde{\gamma}(\vec{\sigma}) e^{\kappa_0 \sigma_0}, \quad \sigma^i \to \sigma'^i = \sigma^i. \tag{23}$$

Horizon symmetry as gauge symmetry

Starting from y^μ on the boundary, horizon is reached at (r_0, y^μ). Map $\mathcal{Y}^\mu(\sigma^a)$ is given by

$$\mathcal{Y}^\mu\left(\sigma^a\right) = X^\mu\left(\sigma^a\right) = x^\mu\left(\sigma^a\right). \tag{24}$$

Make a horizon symmetry transformation generated by some infinitesimal vector field χ^A,

$$X^A \rightarrow X'^A\left(\sigma^a\right) = X^A\left(\sigma^a\right) + \chi^A\left(\sigma^a\right), \tag{25}$$

with G_{AB} unchanged. The geodesic starting at y^μ now hits the horizon at $(r_0'(y^\mu), y^\mu)$ where

$$r_0'\left(y^\mu\right) = r_0 + \chi^r\left(\sigma^a\left(y^\mu\right)\right). \tag{26}$$

The new boundary to horizon map is now given by

$$\mathcal{Y}'^\mu\left(\sigma^a\right) = x'^\mu\left(\sigma^a\right) = \mathcal{Y}^\mu\left(\sigma^a\right) + \chi^\mu\left(\sigma^a\right). \tag{27}$$

By using this new map, the new velocity field is then

$$\beta'^\mu = \partial_a \mathcal{Y}'^\mu \hat{\ell}^a = \beta^\mu + \partial_0 \chi^\mu \quad \Longrightarrow \textbf{change local inverse temperature/velocity} \tag{28}$$

An explicit example

The black brane solution in AdS_{d+1}: $ds^2 = G_{AB}dx^A dx^B = 2dvdr - F(r)dv^2 + g(r)d\vec{x}^2$.

From the explicit horizon symmetry transformation in this case

$$\begin{cases} Z = \gamma(\vec{\sigma})e^{2\pi\sigma^0} \\ \hat{Y}^i = \zeta^i(\vec{\sigma}) + \frac{1}{2\pi g\left(r_0\right)}\partial_i\gamma(\vec{\sigma})e^{2\pi\sigma^0} \\ f = \lambda(\vec{\sigma}) + \alpha(\vec{\sigma})e^{-2\pi\sigma^0} + \frac{1}{16\pi^2}F''\left(r_0\right)\gamma(\vec{\sigma})e^{2\pi\sigma^0} \end{cases} \tag{29}$$

In this case, the transformation of the boundary quantities are given by

$$\begin{cases} \beta'^0\left(y^\mu\right) = \beta\left(1 - 2\pi\alpha(\vec{y})e^{-\frac{2\pi}{\beta}t} + \frac{1}{8\pi}F''\left(r_0\right)\gamma(\vec{y})e^{\frac{2\pi}{\beta}t}\right) \Longrightarrow \textbf{shift symmetry} \\ \beta'^i\left(y^\mu\right) = \frac{1}{g\left(r_0\right)}\partial_i\gamma(\vec{y})e^{\frac{2\pi}{\beta}t} \Longrightarrow \textbf{new prediction} \end{cases} \tag{30}$$

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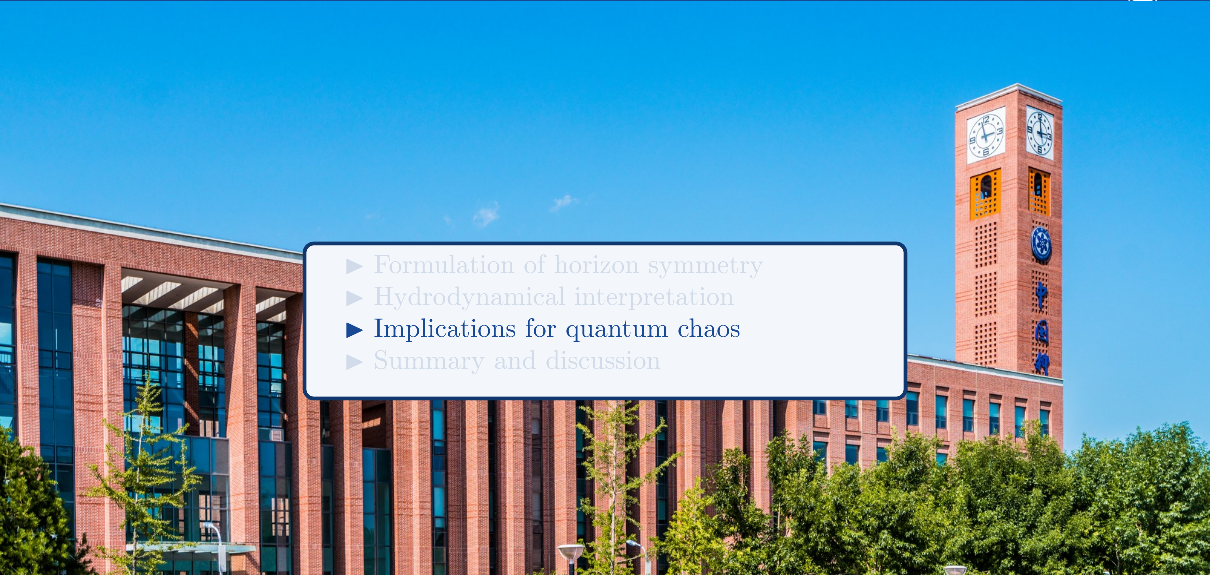

Connection with quantum chaos

Pole skipping phenomenon is often treated as the "smoking gun" of the quantum chaos.

It was previously shown that the exponentially growing part of the symmetry parameterized by $\gamma(\vec{\sigma})$ implies that the momentum space retarded two-point function of the linearized hydrodynamic mode has a pole in the upper half complex ω-plane at

$$\omega = i\frac{2\pi}{\beta}, \quad \vec{k}^2 = -k_c^2, \quad k_c = \frac{2\pi}{\beta}\frac{1}{v_B}, \tag{31}$$

which gives rise to the coordinate space behavior

$$G_{\epsilon\epsilon}(t,\vec{x}) \sim e^{\frac{2\pi}{\beta}\left(t - \frac{|\vec{x}|}{v_B}\right)}, \tag{32}$$

where $\dfrac{2\pi}{\beta}$ is the maximal Lyapunov exponent and v_B is the butterfly velocity.

It is obvious that **the horizon symmetry discussed in this paper gives a natural interpretation of the shift symmetry proposed by hand before**.

Some further comments

Albeit the big success with horizon symmetry, there remains some further comments:

- While the pole in ω given before is universal, the concrete values of k_c and the butterfly velocity v_B depend on the specific theory and can not be determined from symmetries.
- According to the argument based on horizon symmetry, any effort trying to construct EFT related to non-maximally chaotic system by using information coming from single operator insertion will fail.
- While the horizon symmetries allow any choice of $\gamma(\vec{\sigma})$, the explicit profile $\gamma(\vec{\sigma})$ is determined through the shock wave calculation which in turn comes from the specific form of the Einstein equations.
- Only the pole skipping phenomenon associated with the energy density operator is implied by the horizon symmetries. The pole skipping phenomenon in the other channels appears to be unrelated to the horizon symmetries. $\Longrightarrow$ **Some different physics are needed!**

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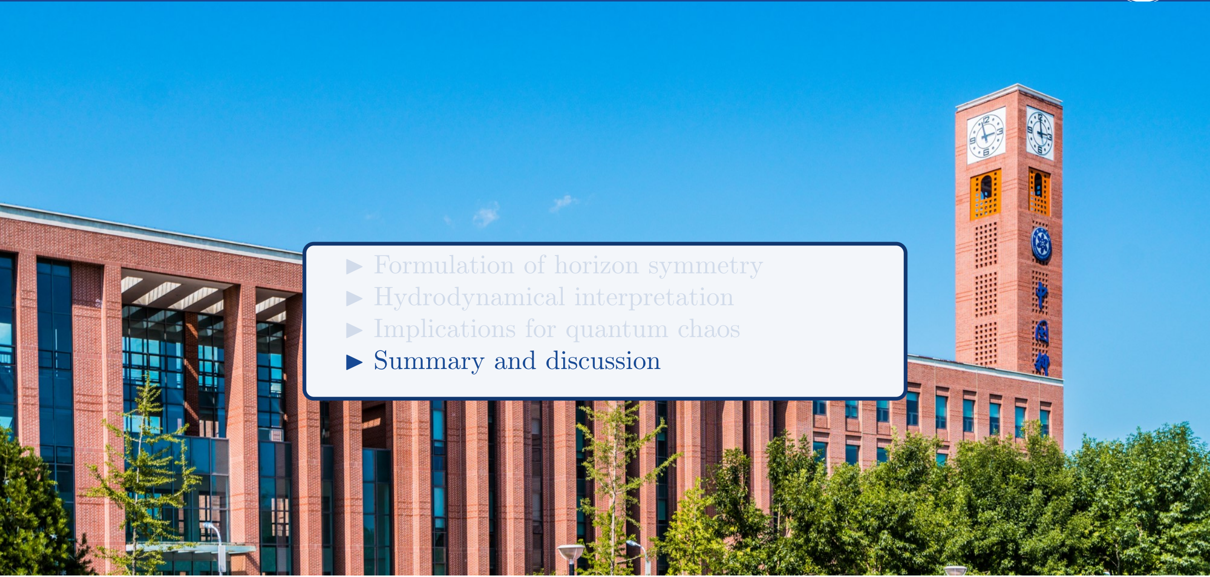

Summary

Here is a brief summary of the main outcomes achieved in this paper:

- The formulation of horizon symmetries presented in previous literature is generalized to include diffeomorphisms that can shift the location of the horizon.
- In the context of the AdS/CFT duality, the horizon symmetries can be interpreted on the boundary as emergent low-energy gauge symmetries.
- A new class of horizon symmetries that extend the so-called shift symmetry are identified, which was previously postulated for effective field theories of maximally chaotic systems.
- The connections of horizon symmetries with the phenomenon of pole skipping are discussed. Some clues related to non-maximally chaotic system are hinted.

Open questions

There remain several questions that merit further investigation:

- Among the most straightforward generalizations to consider are more general black holes, such as charged black holes and those corresponding to far from equilibrium states. It would also be useful to understand what happens to the horizon symmetries when including higher derivative or stringy corrections. $\Longrightarrow$ **Possible way to construct EFT suitable for non-maximally chaotic system?**
- Consider spacetimes that are not asymptotically AdS since much of the discussion can be generalized to the case of other signs of the cosmological constant.
- Extend the EFT for maximally chaotic systems to also include momentum conservation and derive the implications of the transformation of the spatial coordinates due to $\partial_i \gamma(\vec{\sigma})$ on quantum chaos.
- Is it possible to identify gravitational and effective field theory symmetries related to the additional, infinite set of skipped poles found in various different channels and for various fields?