QCD phase diagram at high baryon densities

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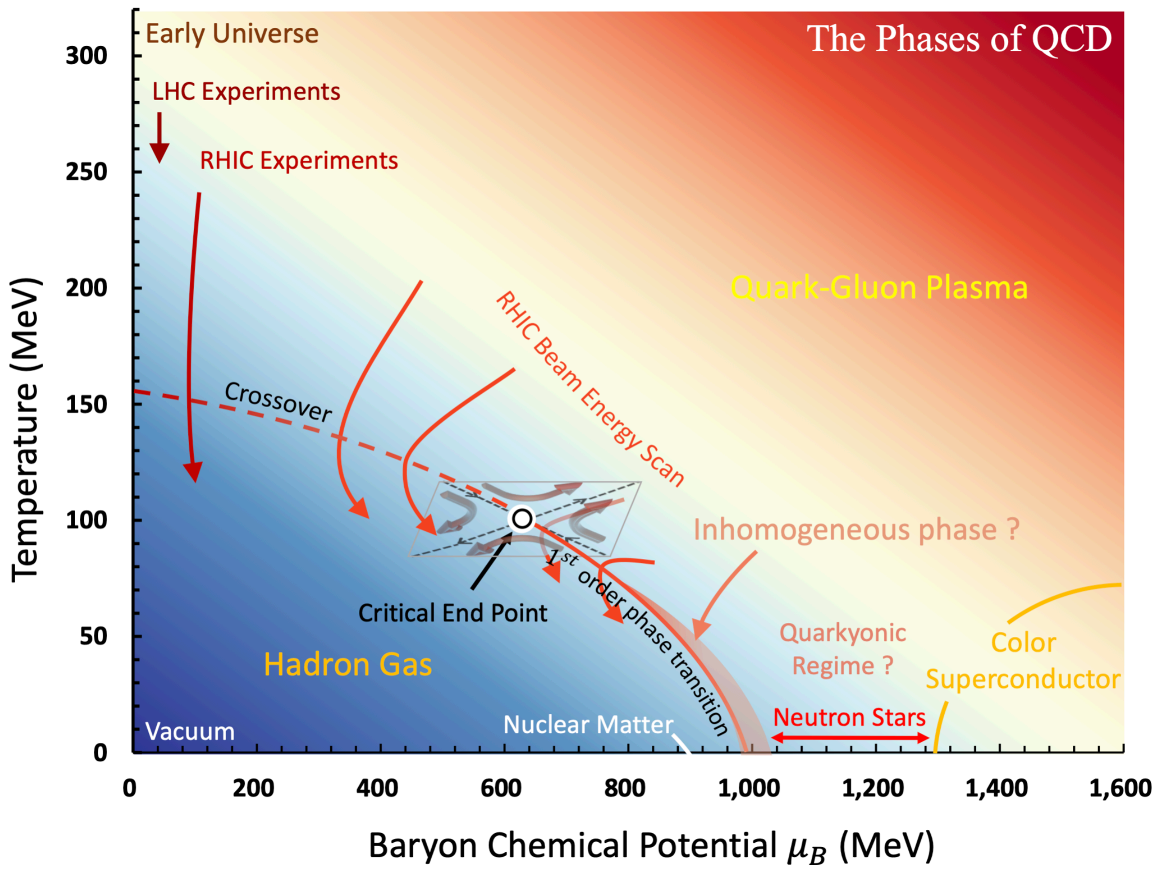
极端核物质前沿研讨会 (Workshop on extreme nuclear matter frontiers), 宜昌, April 24-28, 2026

Many thanks to my collaborators:

Gaoqing Cao, Hao-Lei Chen, Jinhui Chen, Yong-rui Chen, Lipei Du, Chuang Huang, Xu-Guang Huang, Xiaofeng Luo, Guo-Liang Ma, Jan M. Pawlowski, Robert D. Pisarski, Fabian Rennecke, Yang-yang Tan, Zi-ning Wang, Rui Wen, Shi Yin, Chunjian Zhang, Li-jun Zhou, *et al.*

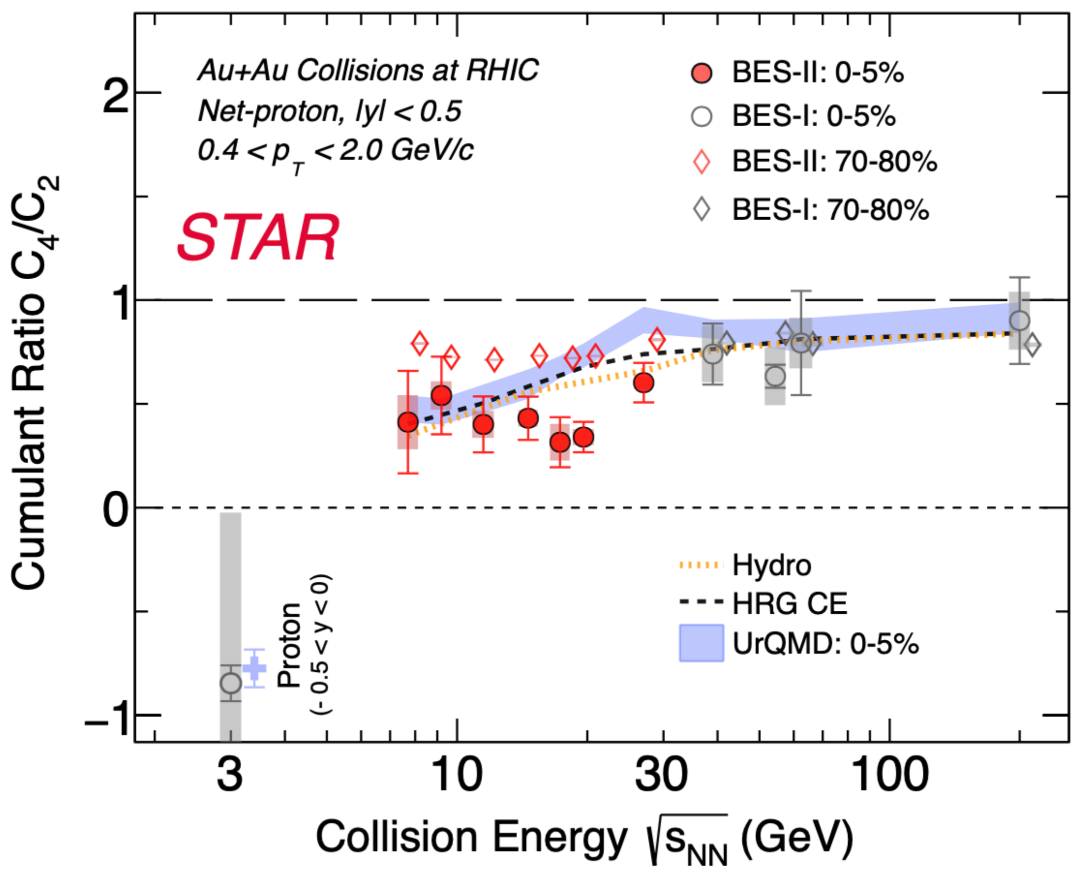
CEP in QCD phase diagram

QCD phase diagram



Non-monotonicity:
M. Stephanov, *PRL* 107 (2011) 052301

Fluctuations measured in BES-II



STAR Collaboration, *PRL* 135 (2025) 142301,
arXiv:2504.00817

- Is there a "peak" structure serving as the smoking gun signal for the critical end point in the QCD phase diagram?

See the talks by Xiaofeng and Shusu

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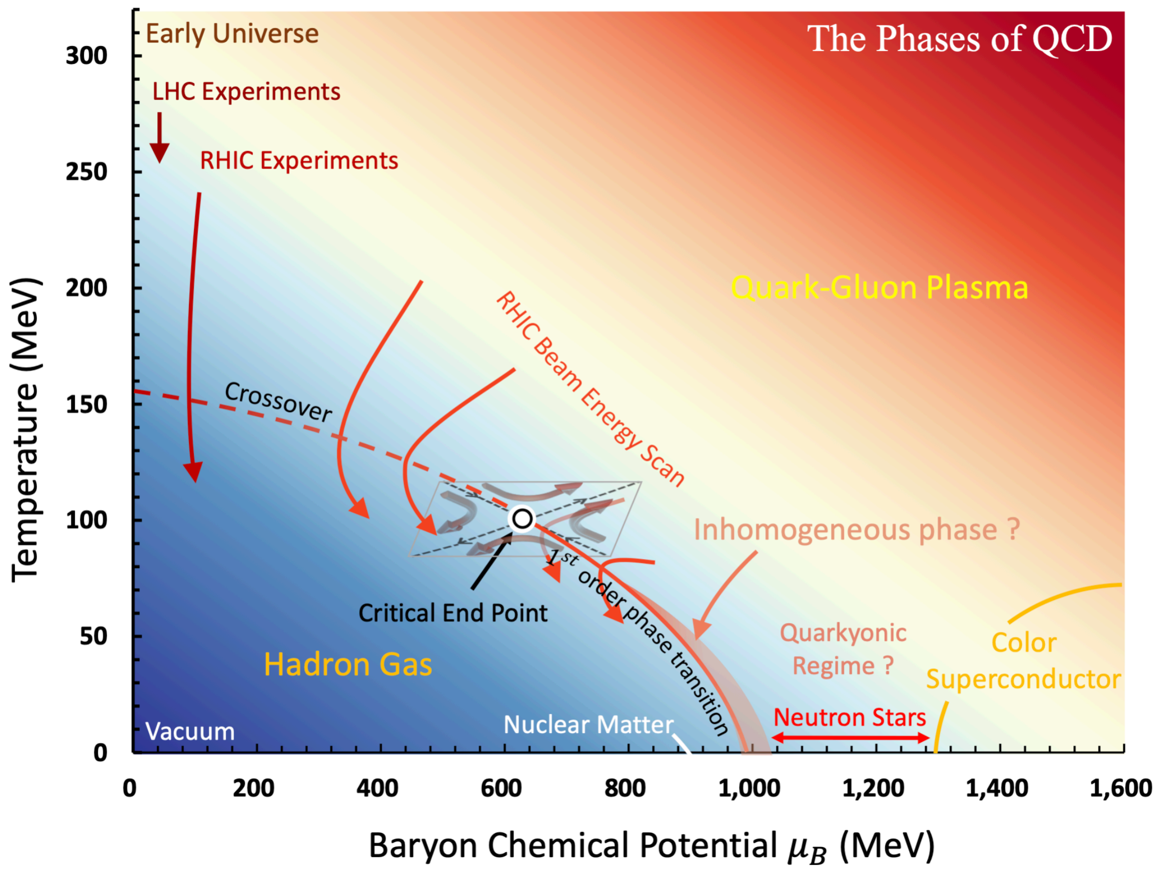
QCD phase diagram



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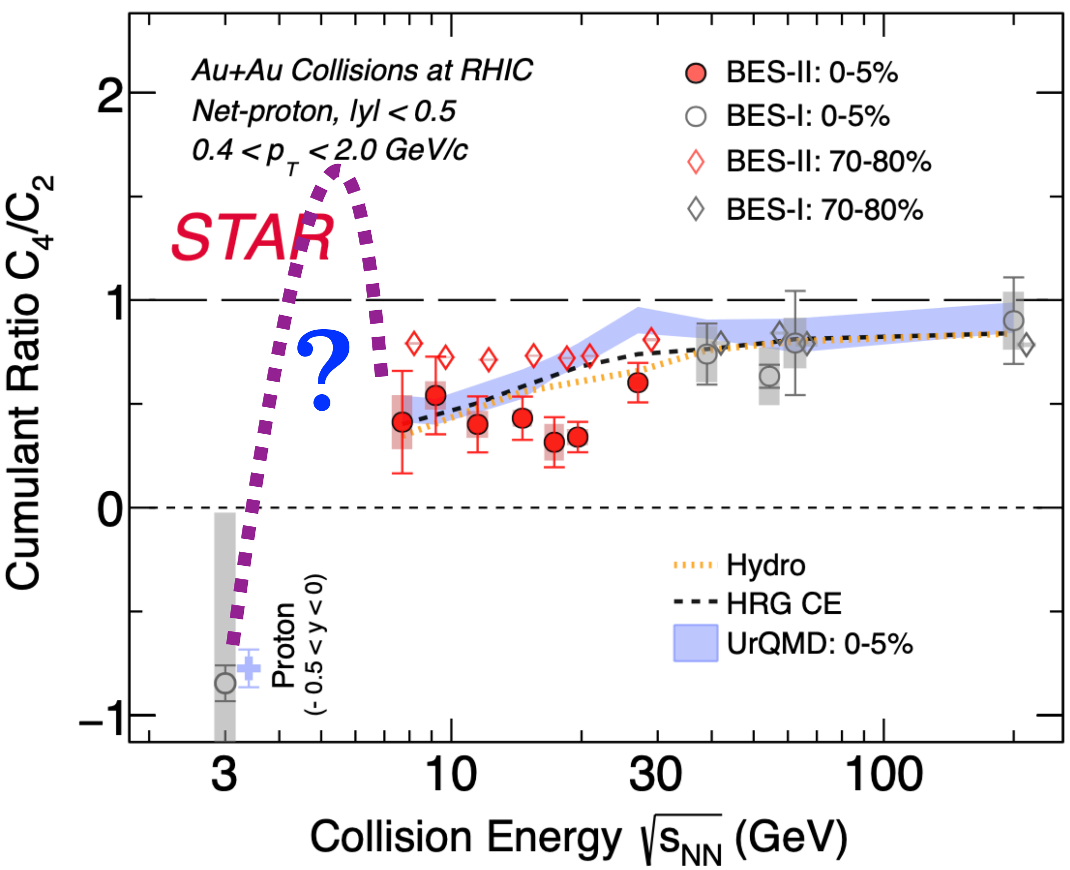

STAR Collaboration, *PRL* 135 (2025) 142301, arXiv:2504.00817

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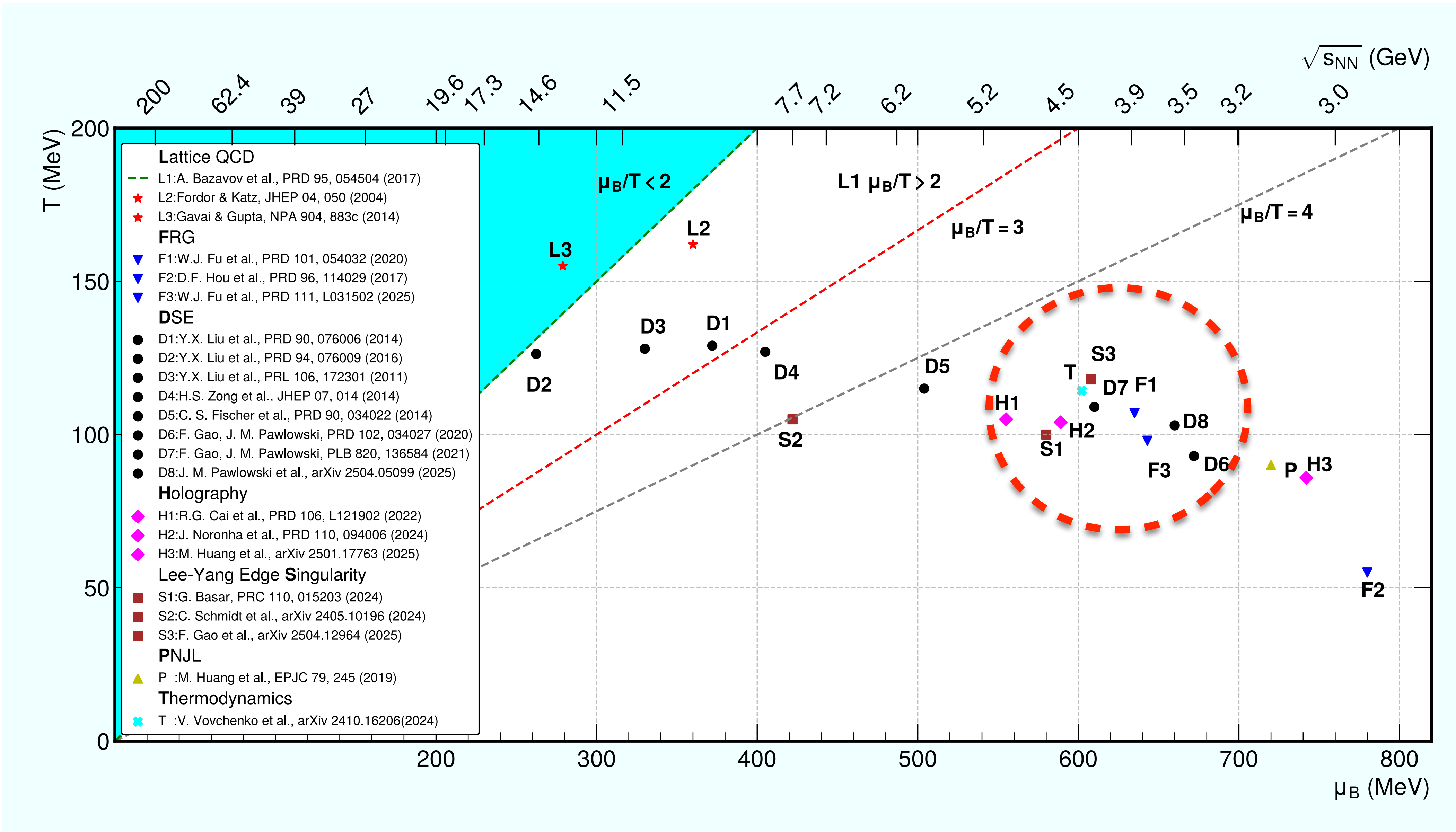
Theoretical predictions of CEP



By courtesy of Xiaofeng

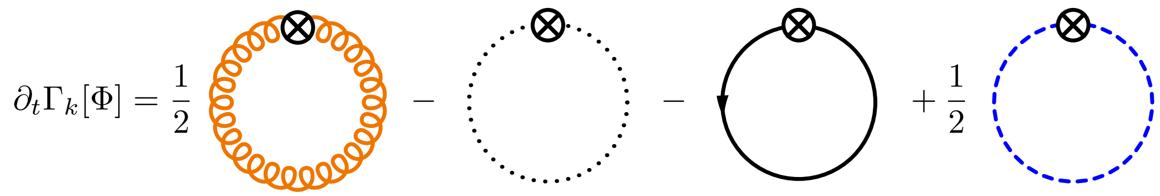
- There have been continuous efforts for more than two decades to predict the location of CEP from theoretical computations, and researchers in China have made significant contributions.

Outline

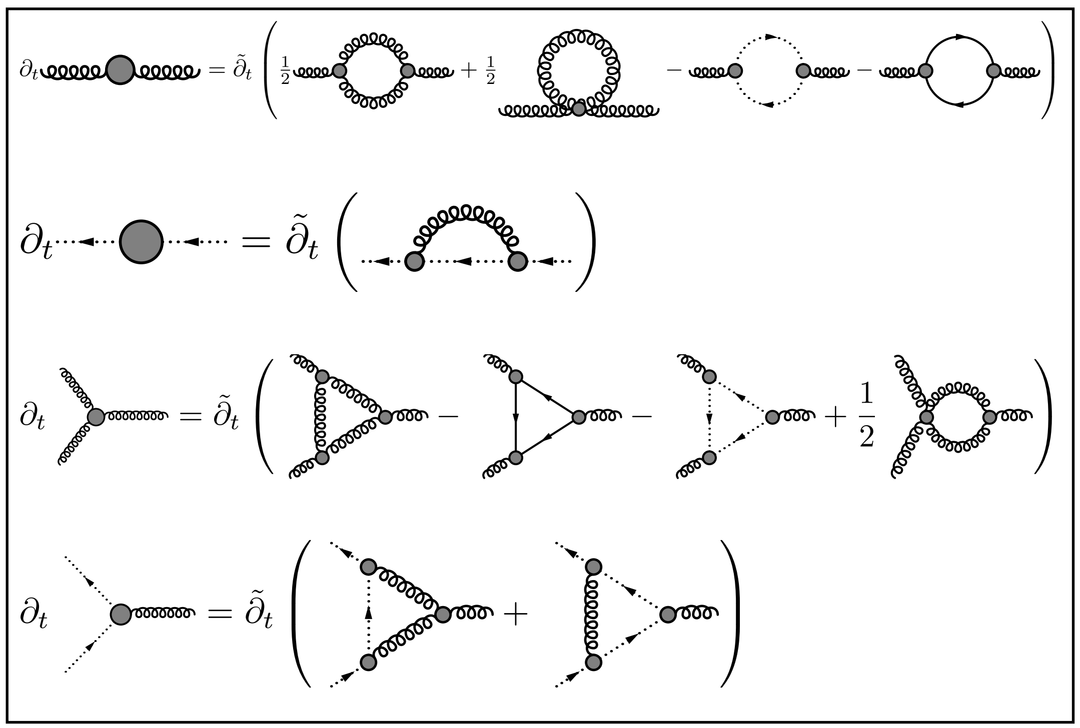
- Introduction
- Recent estimates of the location of CEP
- Baryon (proton) number fluctuations
- Mean pt fluctuations and temperature fluctuations
- Summary and outlook

QCD within fRG

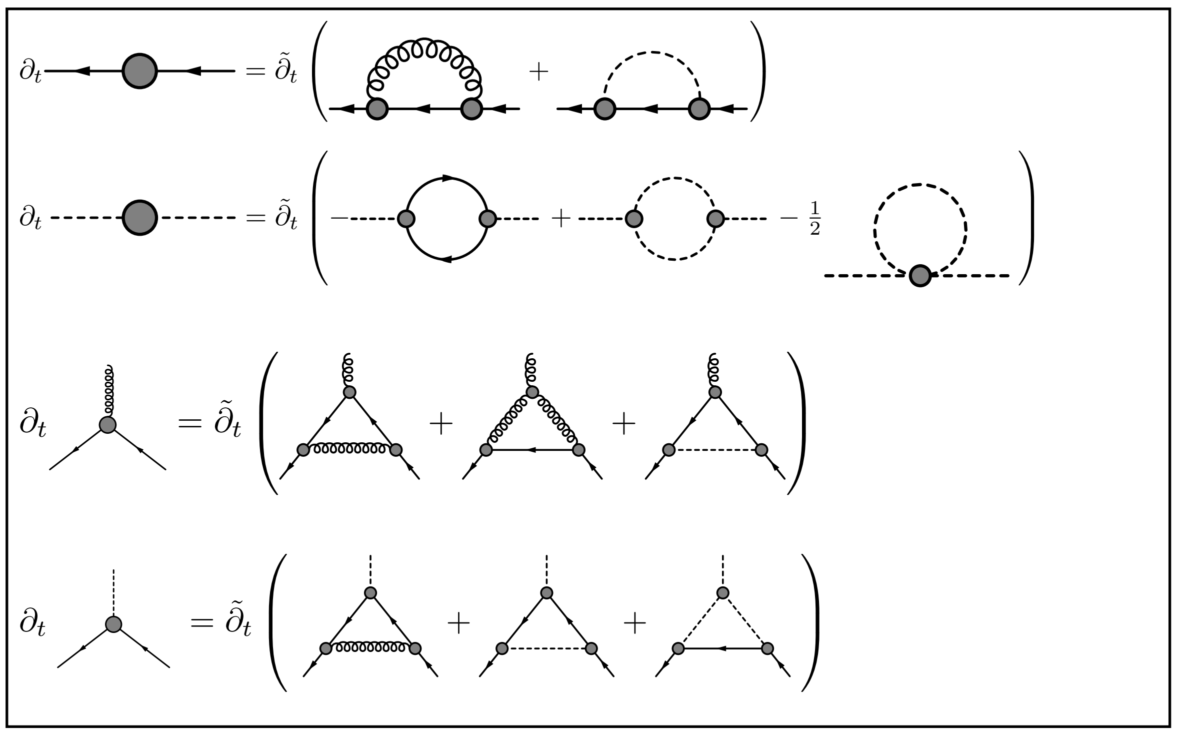
QCD flow equation:



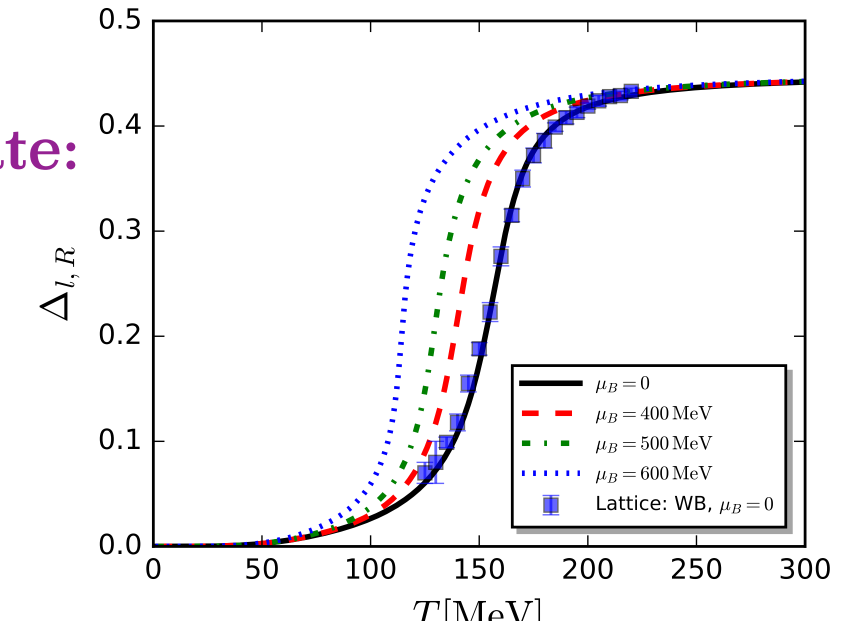
Glue sector:

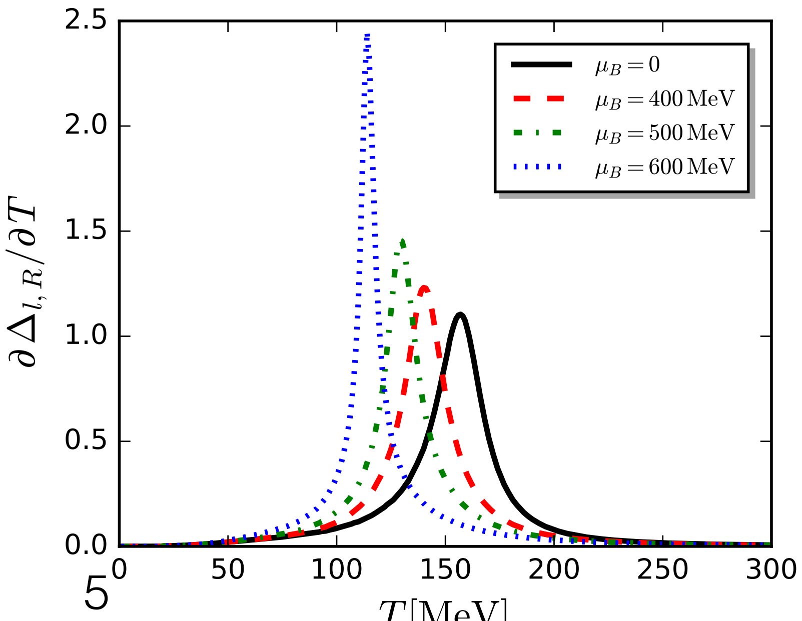


Matter sector:



Quark condensate:



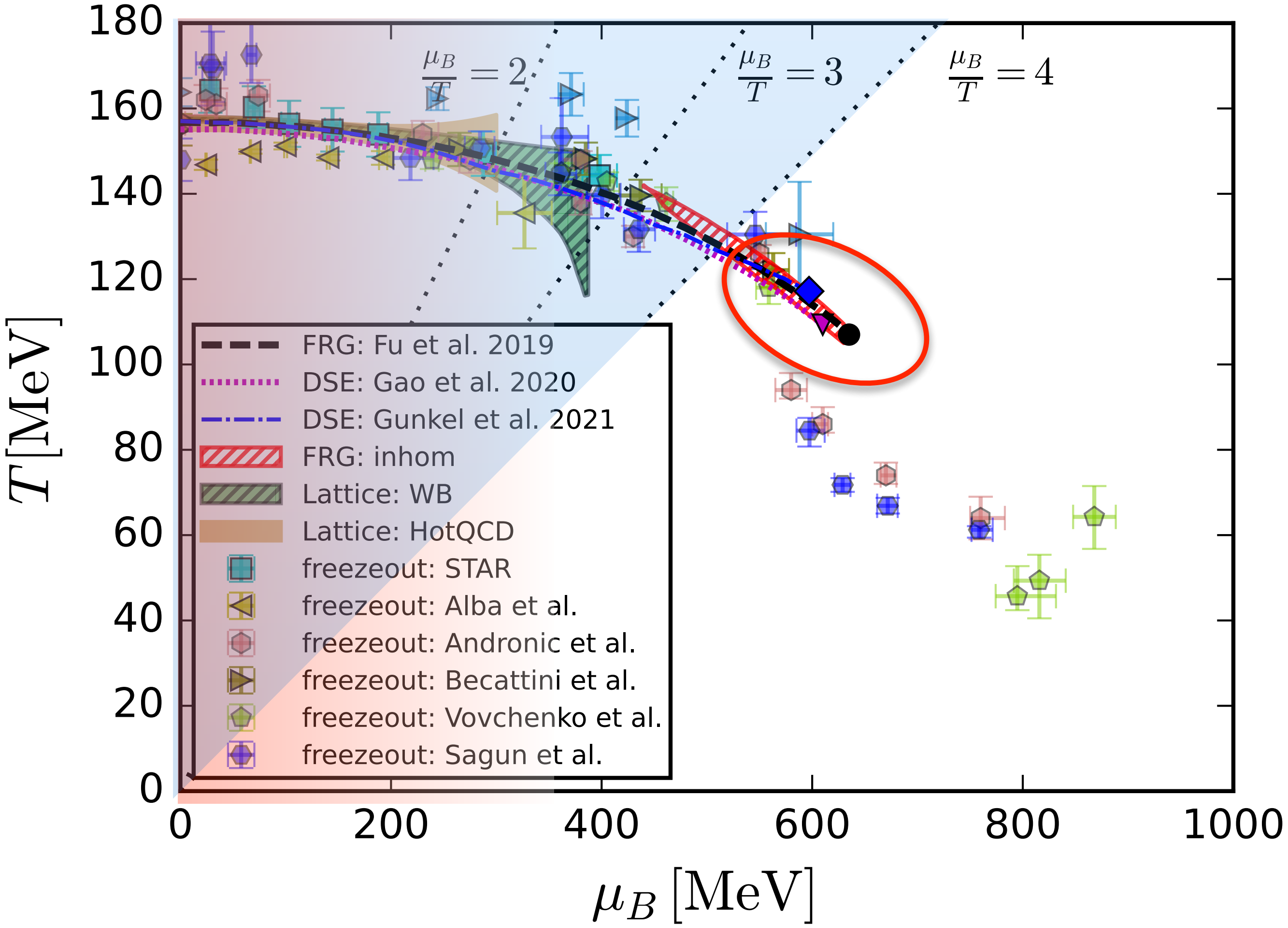


fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

Lattice: Borsanyi *et al.* (WB), *JHEP* 09 (2010) 073

Quantitative errors analysis in fRG: Ihssen, Pawlowski, Sattler, Wink, arXiv:2408.08413

CEP from functional QCD around the year 2020



Passing through strict benchmark tests in comparison to lattice QCD at vanishing and small μ_B.

Regime of quantitative reliability of functional QCD with $\mu_B/T \lesssim 4$.

Estimates of the location of CEP from first-principles functional QCD:

fRG:

● $(T, \mu_B)_{\rm CEP} = (107, 635)$ **MeV**

WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032, arXiv:1909.02991

DSE:

▼ $(T, \mu_B)_{\rm CEP} = (109, 610)$ **MeV**

Gao, Pawlowski, *PLB* 820 (2021) 136584, arXiv:2010.13705

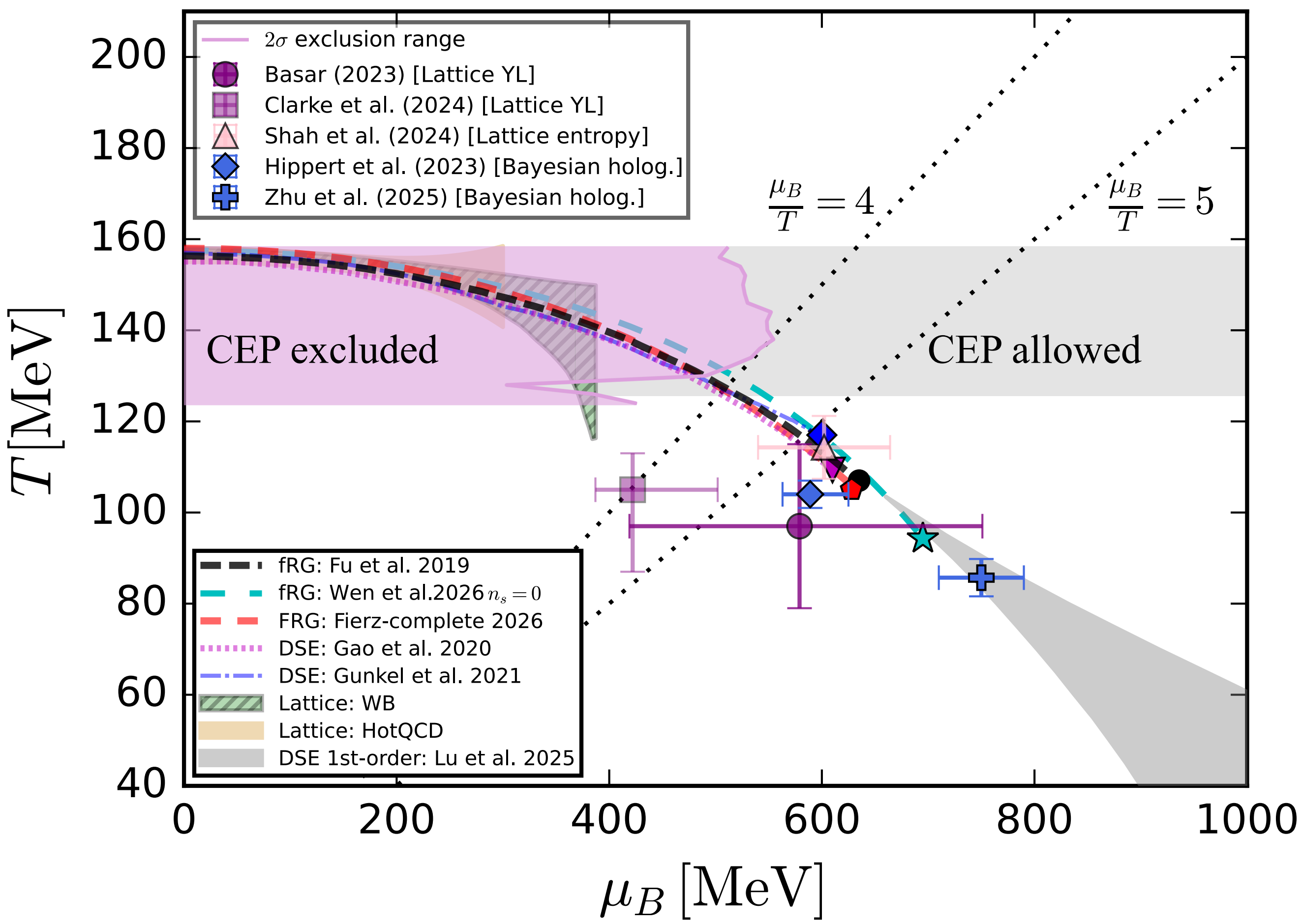
◆ $(T, \mu_B)_{\rm CEP} = (117, 600)$ **MeV**

Gunkel, Fischer, *PRD* 104 (2021) 054022, arXiv:2106.08356

- Recent studies of QCD phase structure from both fRG and DSE have shown convergent estimate for the location of CEP:

$$(T, \mu_B)_{\rm CEP} \approx (110 \pm 10, 630 \pm 30)\ \text{MeV}$$

CEP from different approaches



Functional QCD (recent results):

fRG:
Pawlowski *et al.*, arXiv:2512.20510.
Fu *et al.*, arXiv:2603.13455.
DSE:
Lu *et al.*, *PRD* 113 (2026) 054019, arXiv:2504.05099.
Lu *et al.*, arXiv:2603.09336.

- **Combined results of different approaches indicate the location of CEP is not favored in the region $\mu_B/T \lesssim 4 \sim 5$.**

Lattice extrapolation:

Yang-Lee edge singularities

$$(T, \mu_B)_{\rm CEP} = (97^{+18}_{-18}, 579^{+172}_{-160})\ \text{MeV}$$

Basar, *PRC* 110 (2024) 015203, arXiv:2312.06952.

$$(T, \mu_B)_{\rm CEP} = (105^{+8}_{-18}, 422^{+80}_{-35})\ \text{MeV}$$

Clarke *et al.*, *PRD* 112 (2025) L091504, arXiv:2405.10196.

$T_{\rm CEP} \le 103$ MeV (84 % level) or no CEP

Adam *et al.*, arXiv:2507.13254.

Contours of constant entropy density

$$(T, \mu_B)_{\rm CEP} = (114.3 \pm 6.9, 602.1 \pm 62.1)\ \text{MeV}$$

Shah *et al.*, *PRC* 113 (2026) L012201, arXiv:2410.16206.

No CEP at $\mu_{B_{\rm CEP}} < 450$ MeV at the 2σ level

Borsanyi *et al.*, *PRD* 112 (2025) L111505, arXiv:2502.10267.

Bayesian holography:

$$(T, \mu_B)_{\rm CEP} = (104 \pm 3, 589^{+36}_{-26})\ \text{MeV (95\% level)}$$

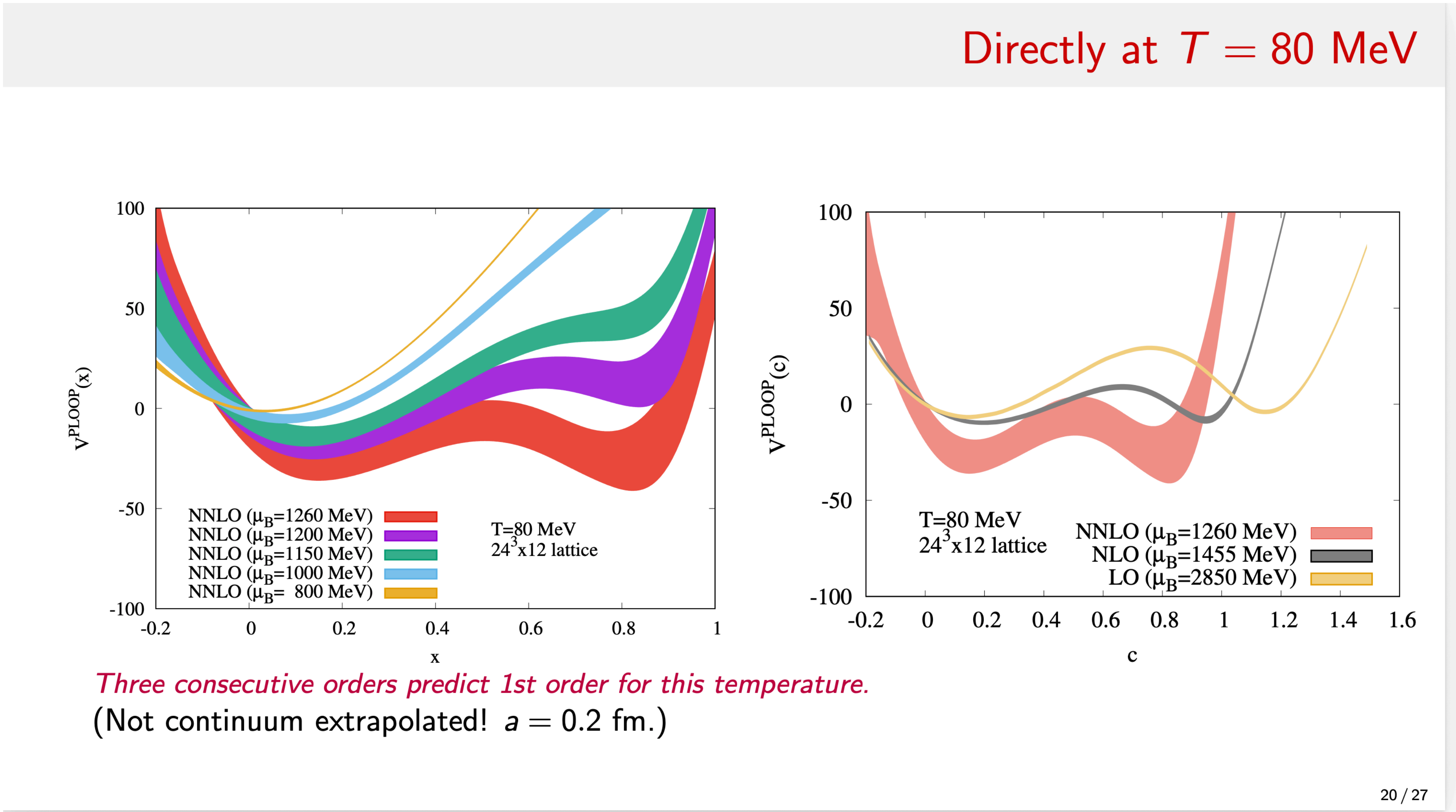
Hippert *et al.*, *PRD* 110 (2024) 094006, arXiv:2309.00579.

$$(T, \mu_B)_{\rm CEP} = (81.6 - 89.8, 710 - 790)\ \text{MeV (95\% level)}$$

Zhu *et al.*, *PRD* 112 (2025) 026019, arXiv:2501.17763.

See the talk by Jana N. Guenther on Mon

First-order phase transition observed in lattice?

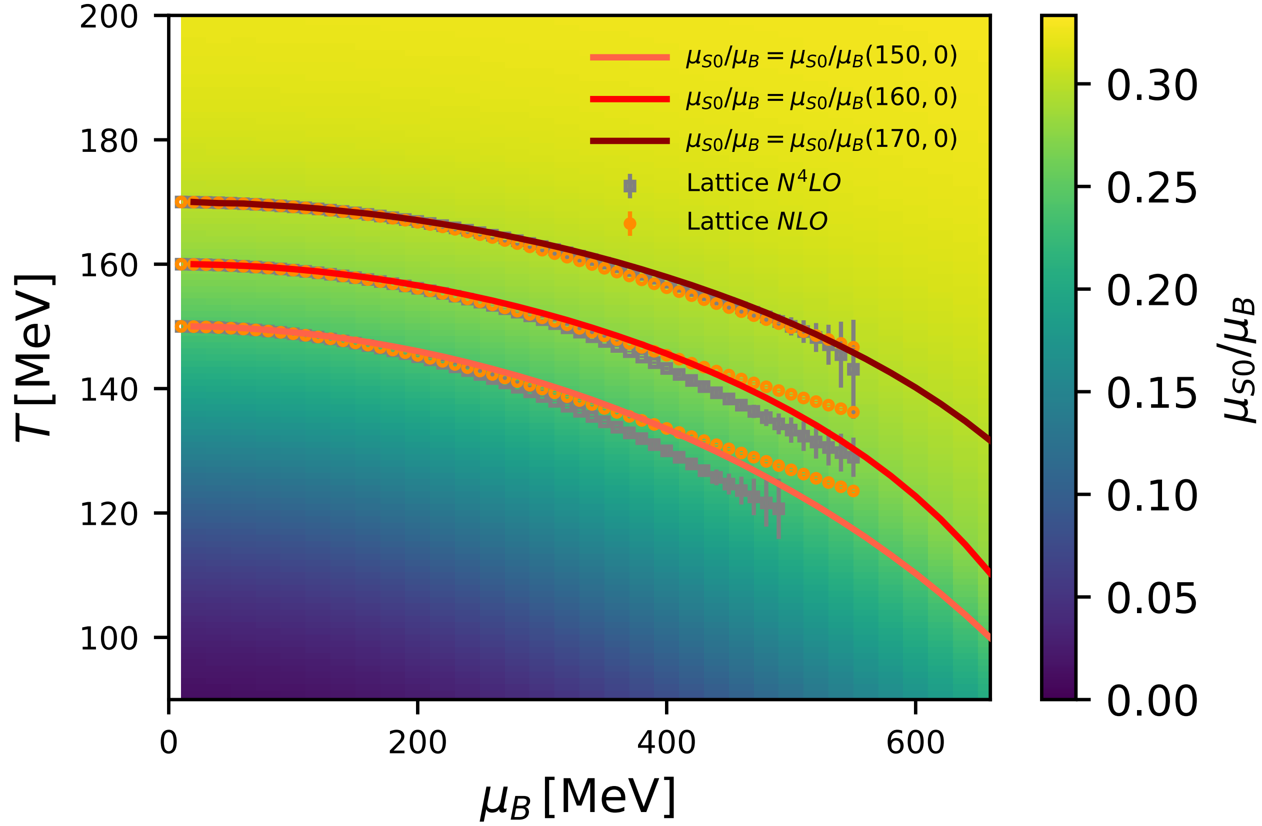


Slide from Szabolcs Borsanyi at CPOD2026

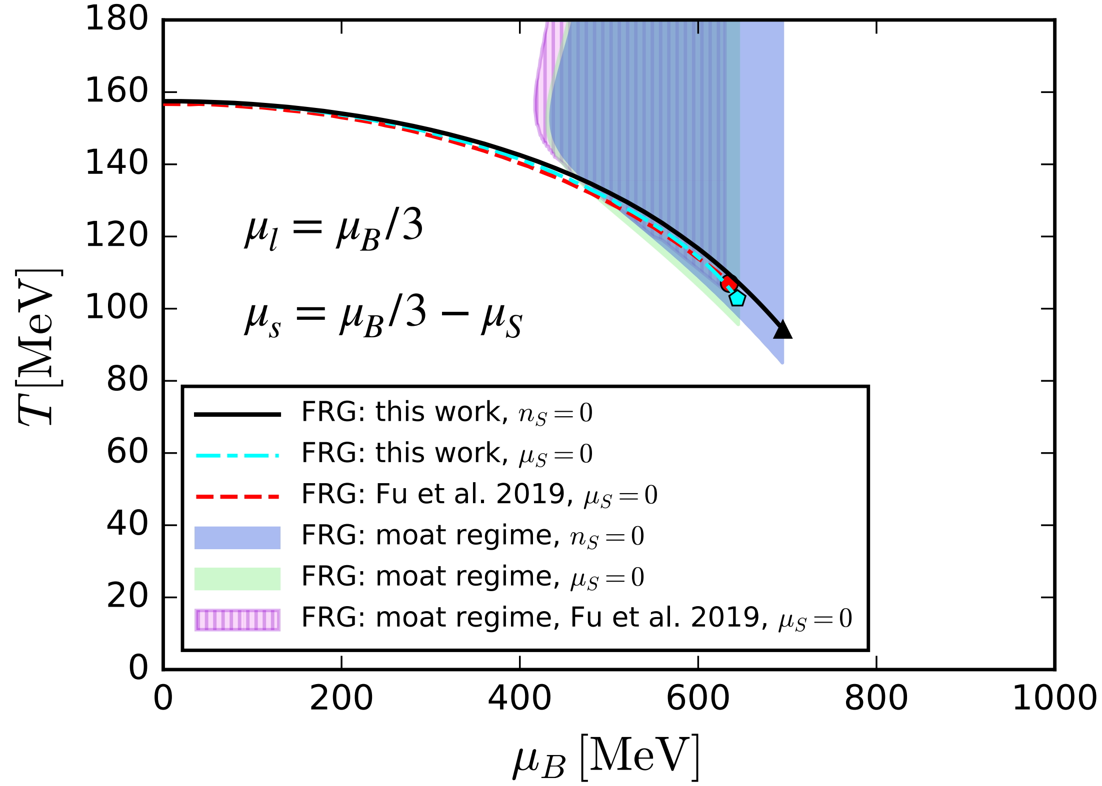
- **Two dips in the Polyakov loop potential are observed by lattice simulations in the region of low temperature and high baryon chemical potential.**

CEP and strangeness neutrality

μ_S with constraint $n_S = 0$



Phase diagram with $n_S = 0$



Curvature of the phase boundary:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \cdots$$

fRG: $\dfrac{\kappa_2^{n_S=0}}{\kappa_2^{\mu_S=0}} = 0.897(20)$ **Lattice:** $\dfrac{\kappa_{2,\text{lat}}^{n_S=0}}{\kappa_{2,\text{lat}}^{\mu_S=0}} = 0.893(35)$

- **Strangeness neutrality flattens a bit the phase boundary and moves CEP to larger μ_B.**

$\mu_S = 0$: $(T, \mu_B)_{\text{CEP}} = (102, 644)$ MeV

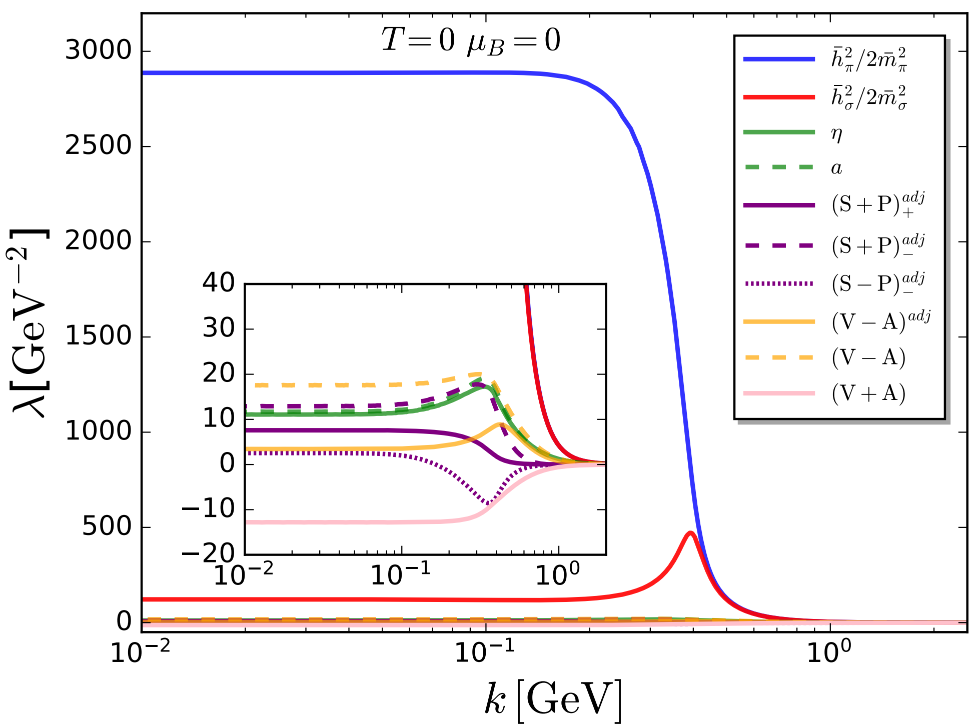
$n_S = 0$: $(T, \mu_B)_{\text{CEP}} = (92, 696)$ MeV

fRG: WF, Huang, Pawlowski, Rennecke, Wen, Yin, arXiv:2603.13455.

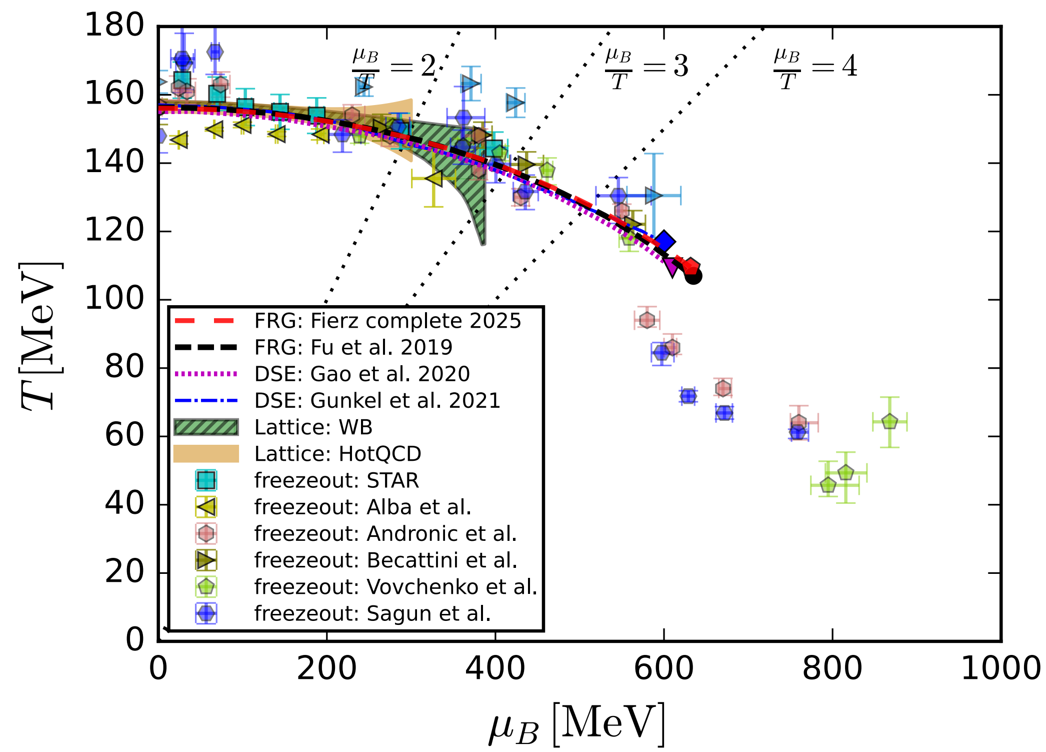
Lattice: Ding *et al.*, *PRD* 109 (2024) 114516, arXiv:2403.09390.
Lattice: Borsanyi *et al.*, arXiv:2510.26455.

CEP and Fierz-complete four-quark basis

Four-quark couplings in the vacuum:

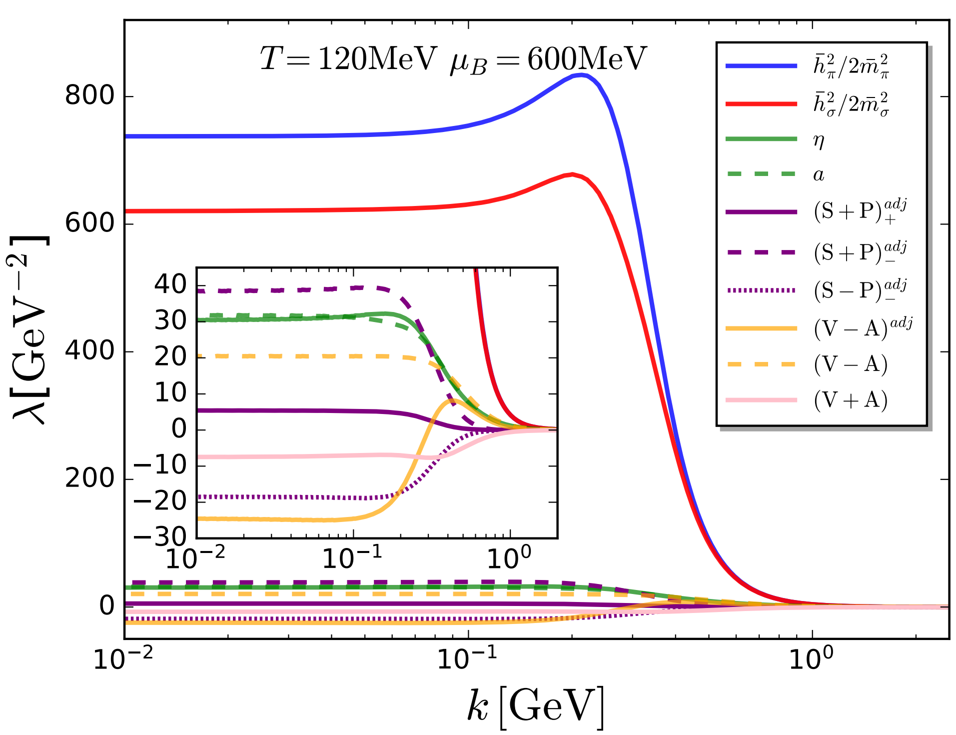


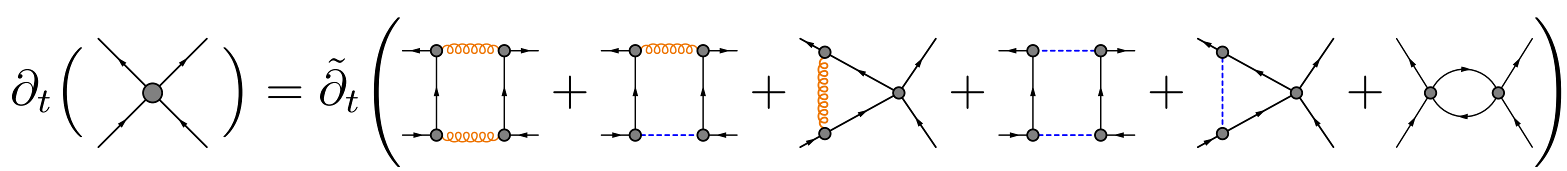
QCD phase diagram with Fierz-complete basis:



Wang, Zhou, Huang, Wen, Yin, WF, in preparation.

Four-quark couplings near CEP:





Scalar-pseudoscalar channel:

$$(T, \mu_B)_{\mathrm{CEP}} = (107, 635)\ \mathrm{MeV}$$

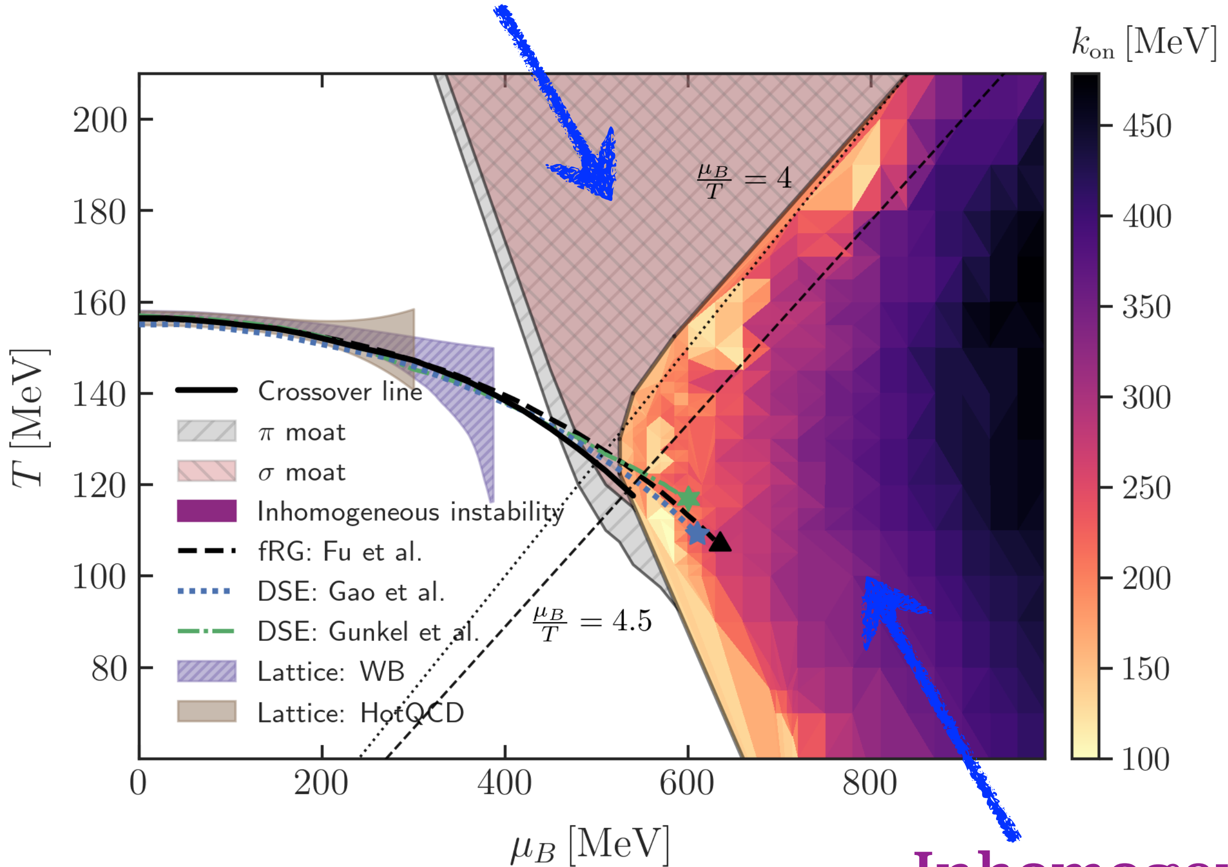
Fierz-complete channels:

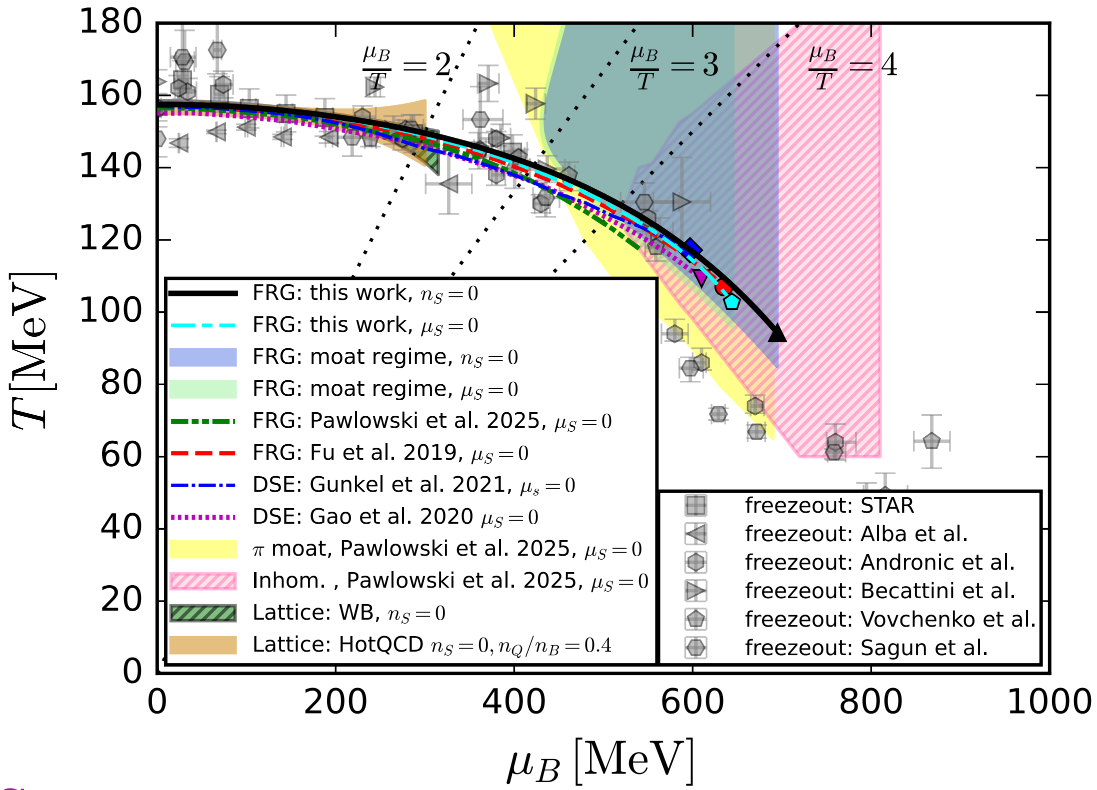
$$(T, \mu_B)_{\mathrm{CEP}} = (109, 631)\ \mathrm{MeV}$$

- **Channels other than the scalar-pseudoscalar channels become more relevant towards CEP, But their influence on CEP is mild.**

CEP and instability

Inhomogeneous instability at intermediate scale (moat)



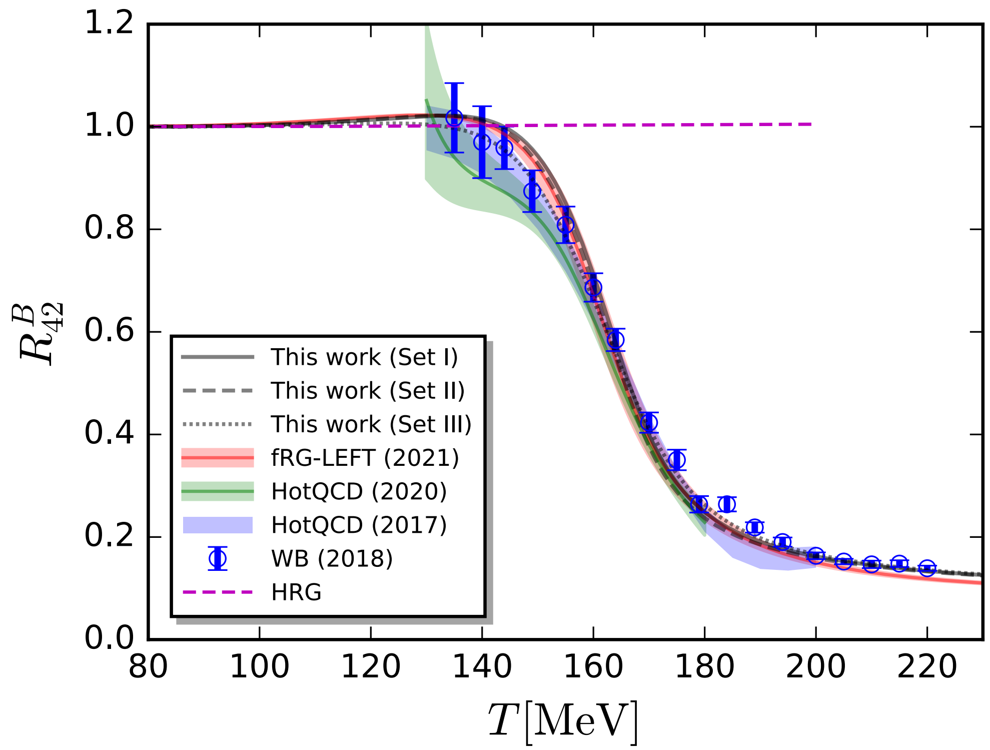


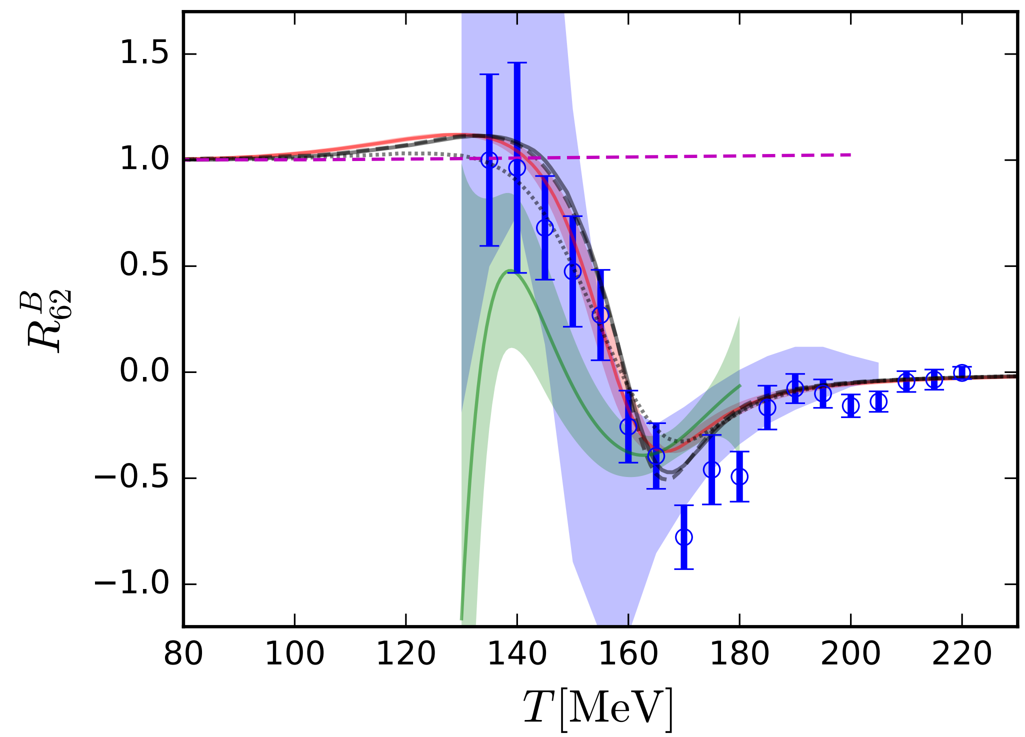
Inhomogeneous instability

Pawlowski, Rennecke, Sattler, arXiv:2512.20510.

- **Moat or inhomogeneous instability may appear in the region of large μ_B.**
- **How do the new massless modes related to the instability affect the CEP? Washing out the CEP? This is still an open question.**

Baryon number fluctuations





$\mu_B = 0$

fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508; WF, Luo, Pawlowski, Rennecke, Wen, Yin, *PRD* 104 (2021) 094047.

HotQCD: A. Bazavov *et al.*, *PRD* 95 (2017), 054504; *PRD* 101 (2020), 074502.

WB: S. Borsanyi *et al.*, *JHEP* 10 (2018) 205.

Baryon number fluctuations

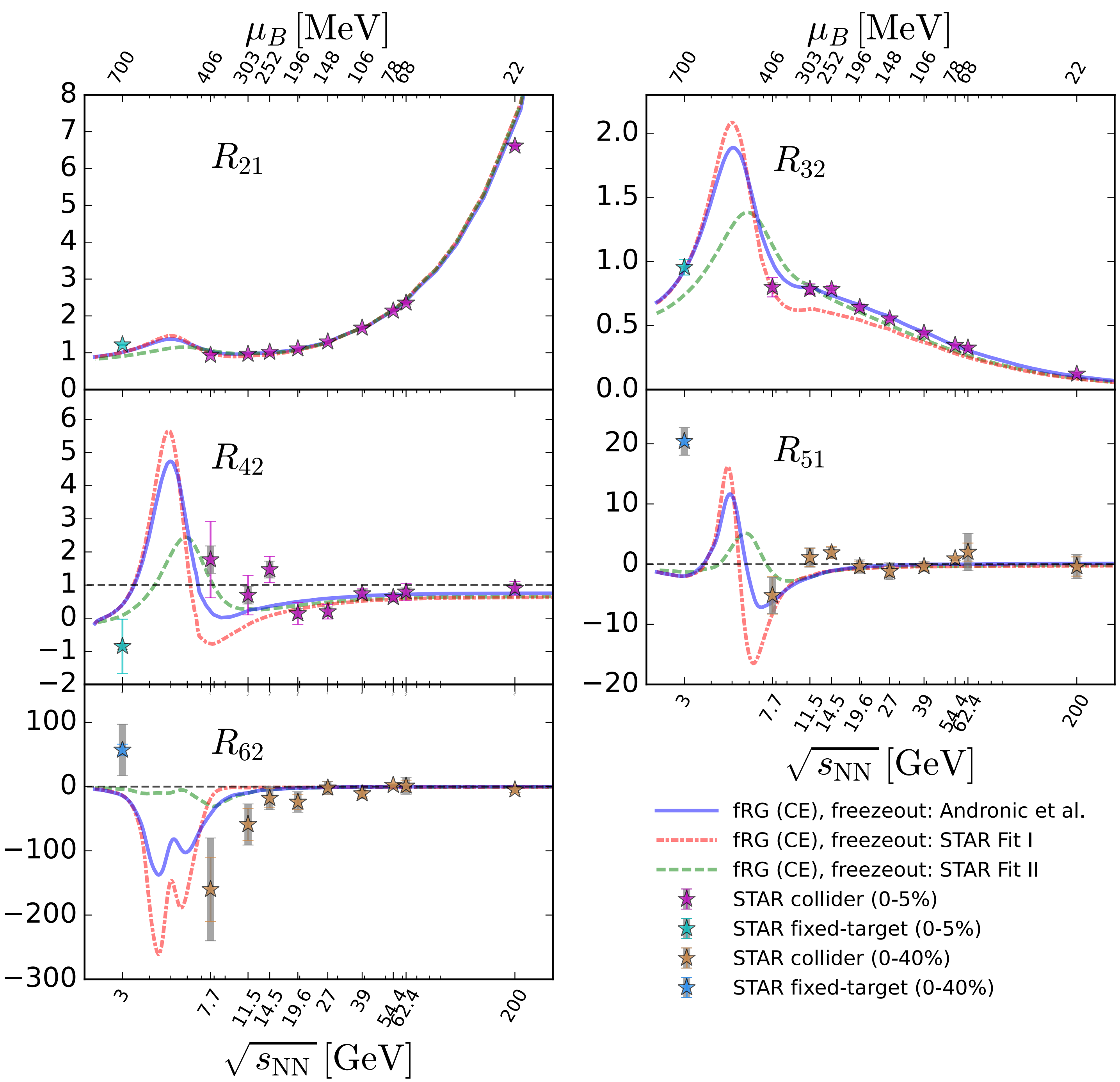
$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4} \qquad\qquad R^B_{nm} = \frac{\chi_n^B}{\chi_m^B}$$

Relation to the cumulants

$$\frac{M}{VT^3} = \chi_1^B\,,\ \frac{\sigma^2}{VT^3} = \chi_2^B\,,\ S = \frac{\chi_3^B}{\chi_2^B\sigma}\,,\ \kappa = \frac{\chi_4^B}{\chi_2^B\sigma^2}\,,$$

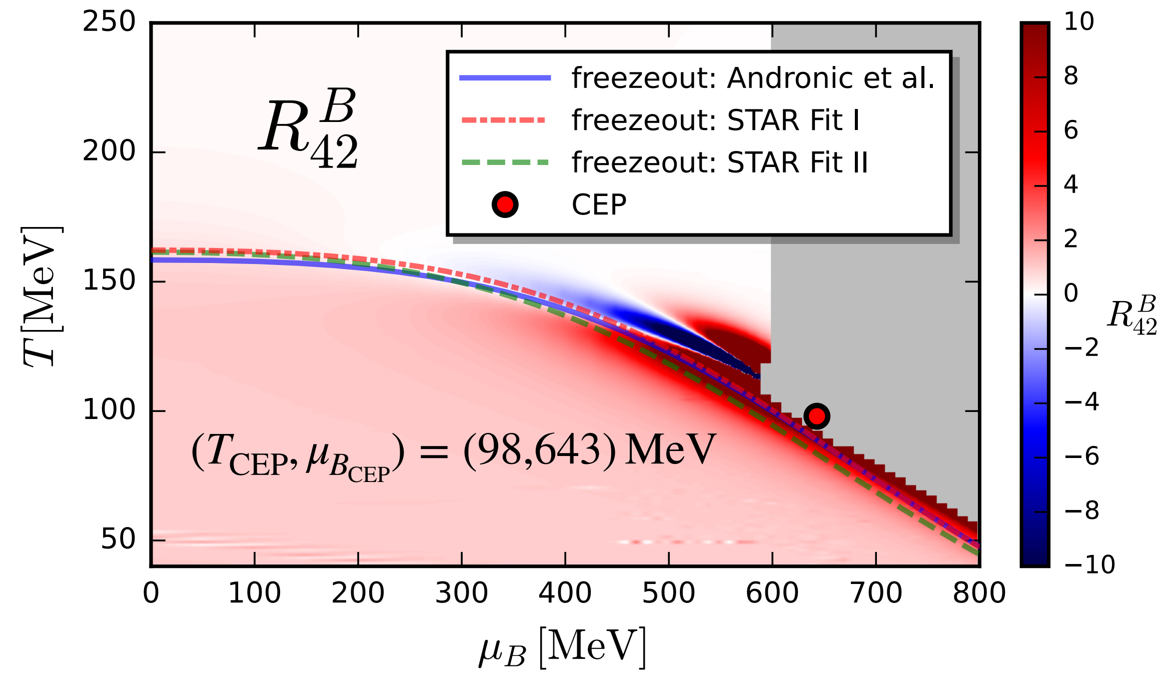
- **In comparison to lattice results and our former results, the improved results of baryon number fluctuations at vanishing chemical potential in the QCD-assisted LEFT are convergent and consistent.**

Canonical fluctuations at the freeze-out



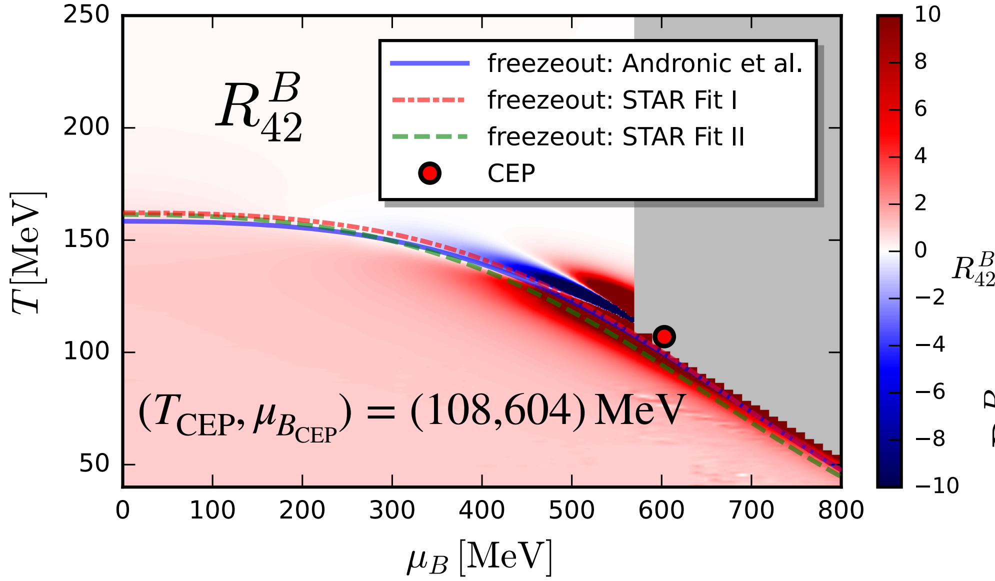
fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508.

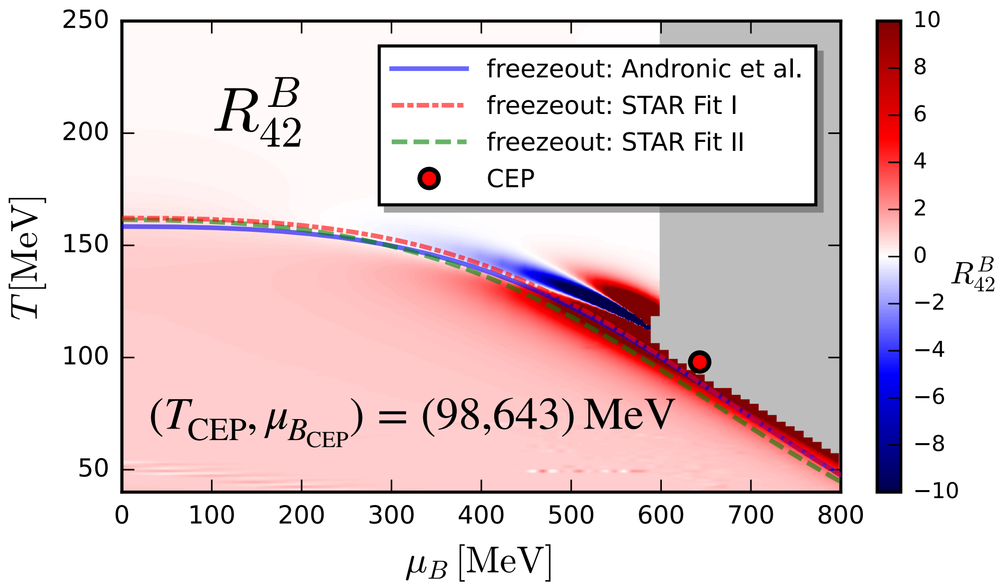
STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301; Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303; Aboona *et al.* (STAR), *PRL* 130 (2023) 082301.

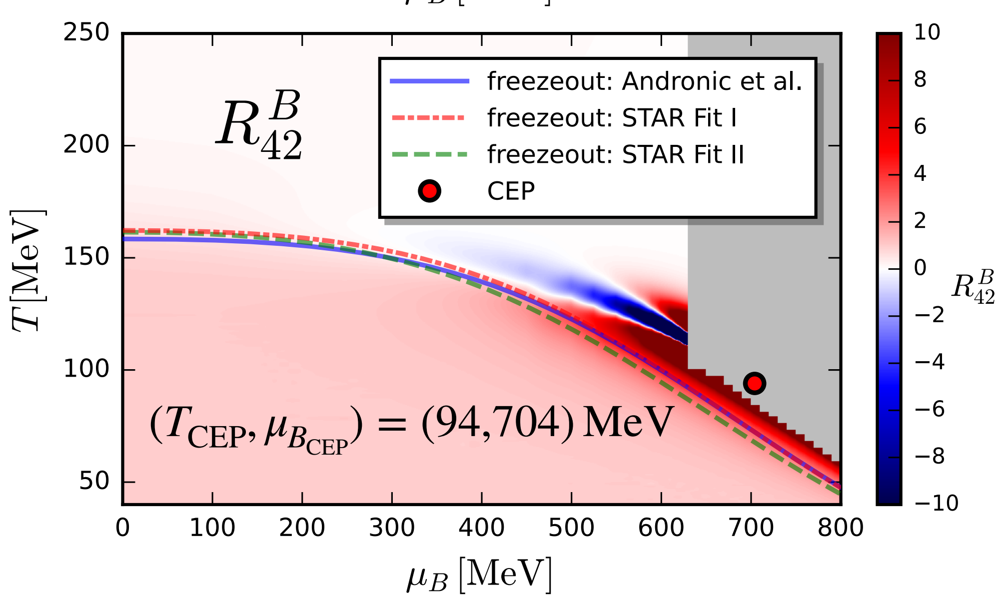


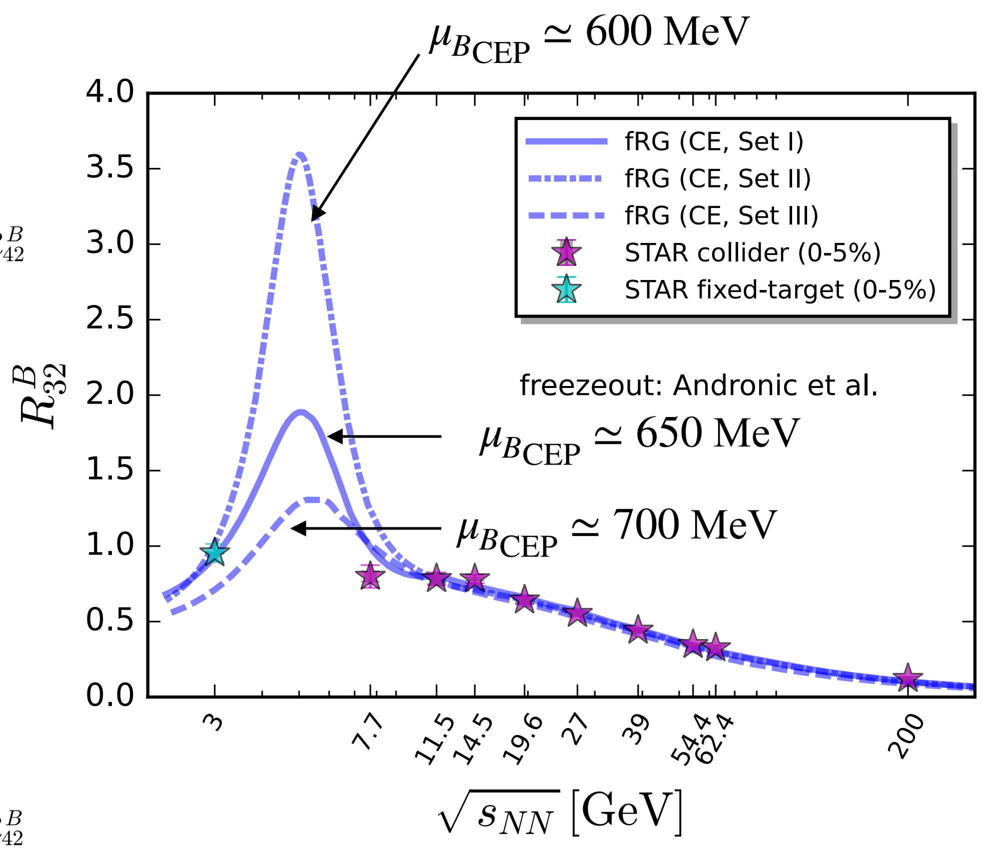
- **Peak structure is found in 3 GeV $\lesssim \sqrt{s_{NN}} \lesssim$ 7.7 GeV.**
- **Position of peak in R_{42} is $\mu_{B_{peak}}$ = 536, 541 and 486 MeV for the three freeze-out curves, significantly smaller than $\mu_{B_{CEP}} = 643$ MeV.**

Dependence on the location of CEP









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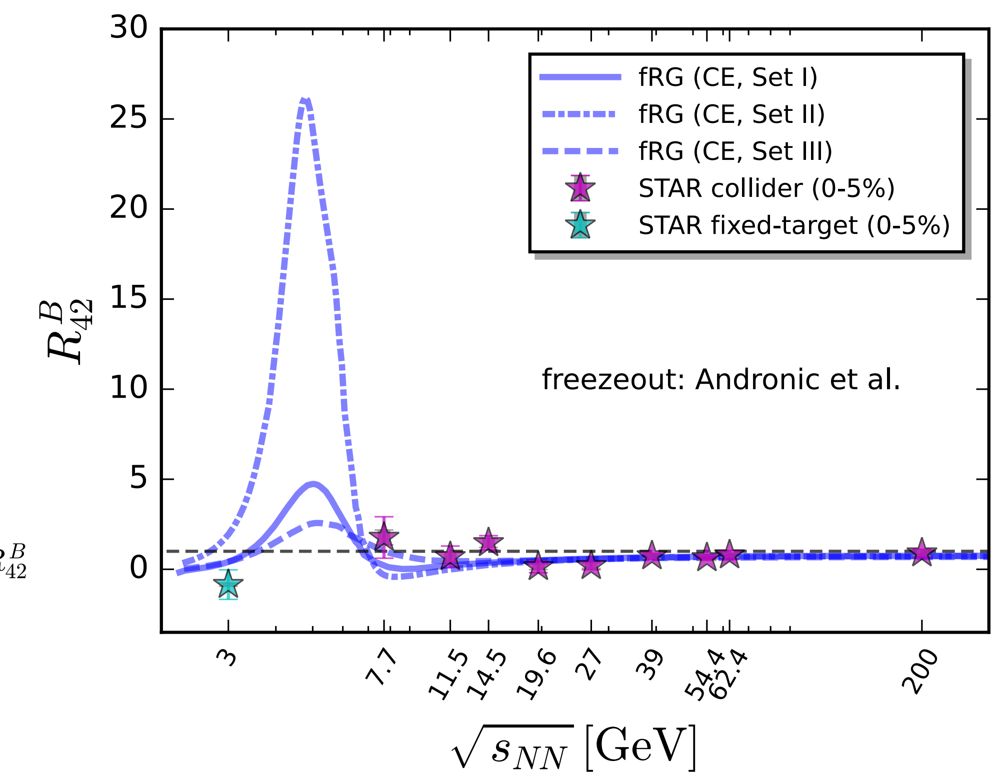
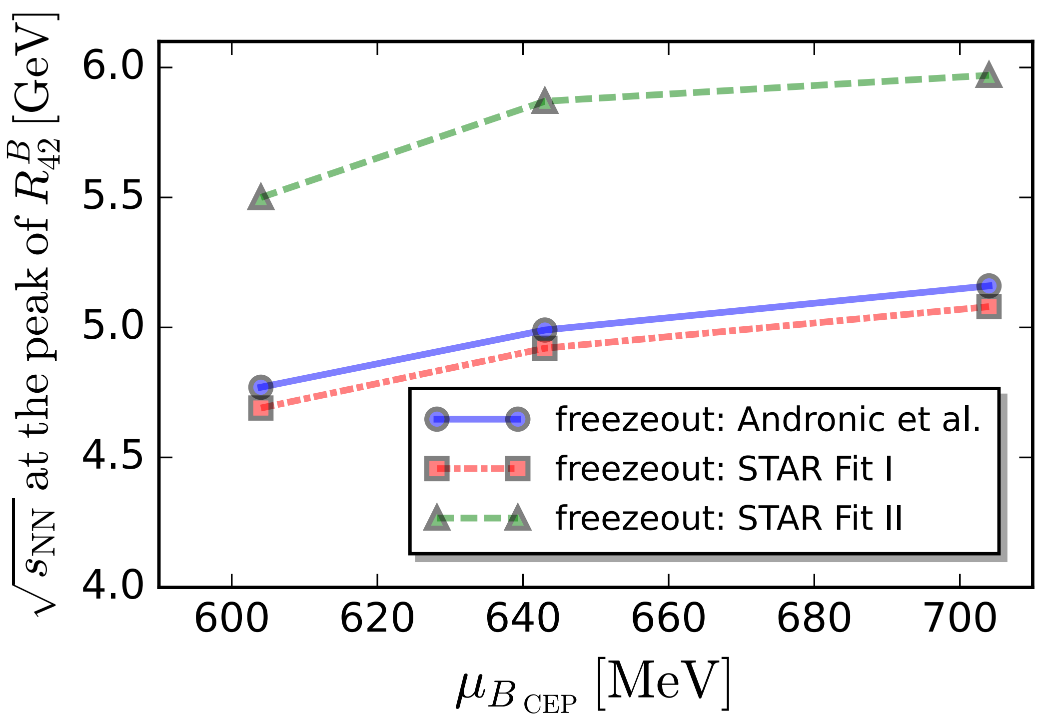
fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502.



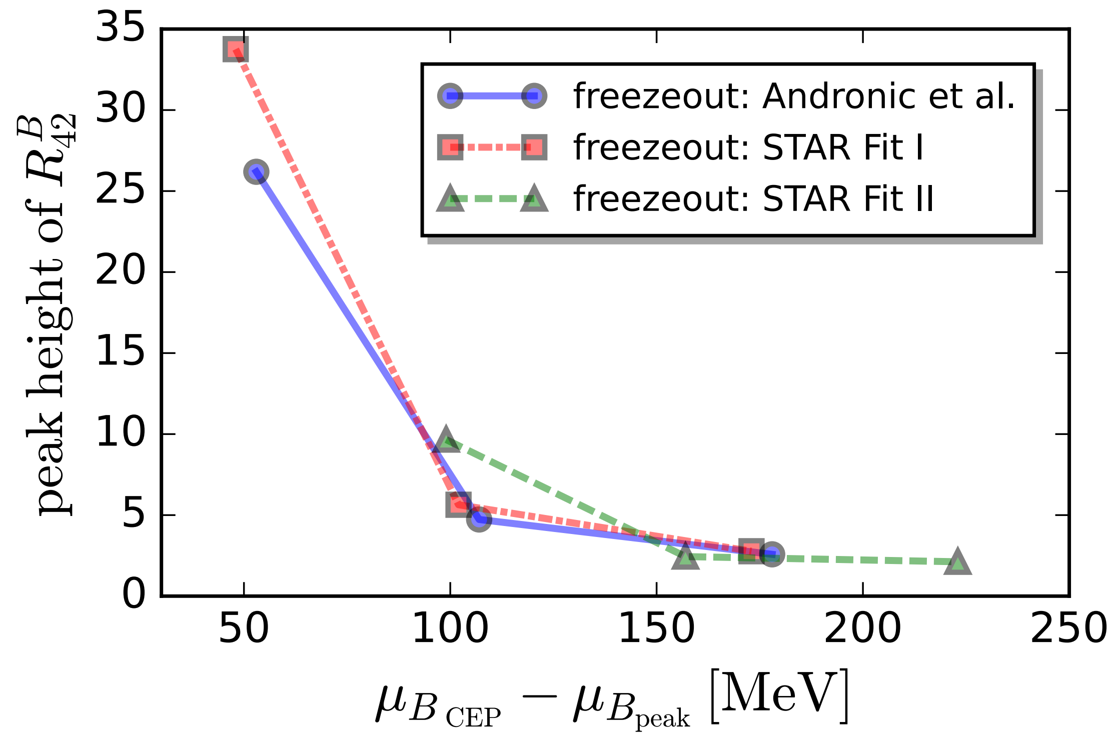
- **Position** of the peak is **insensitive** to the location of CEP.
- **Height** of peak **decreases** as CEP moves towards larger μ_B.

Ripples of the QCD critical point

Position of peak:



Height of peak:

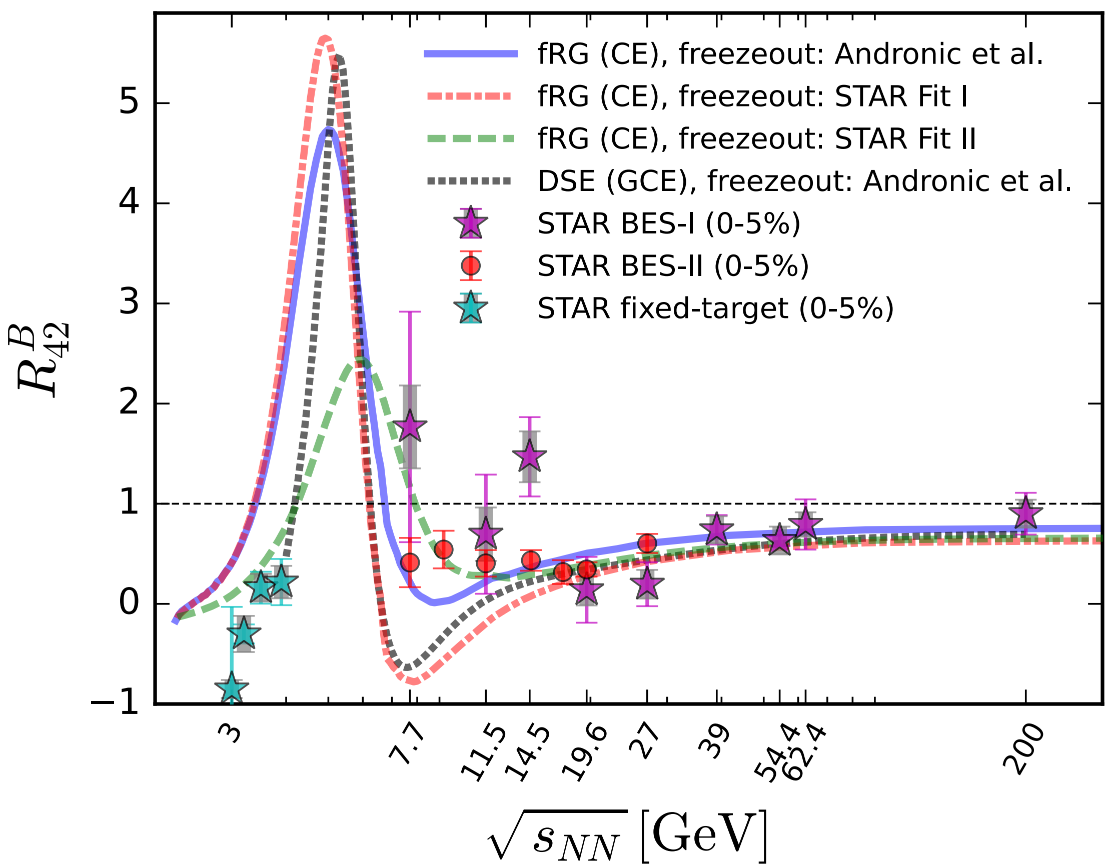


fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502.

- **Note that the ripples of CEP can be far away from the critical region characterized by the universal scaling properties, e.g., critical exponents, the critical slowing down.**
- **But, the information of CEP, such as its location and properties, etc., is still encoded in the ripples.**

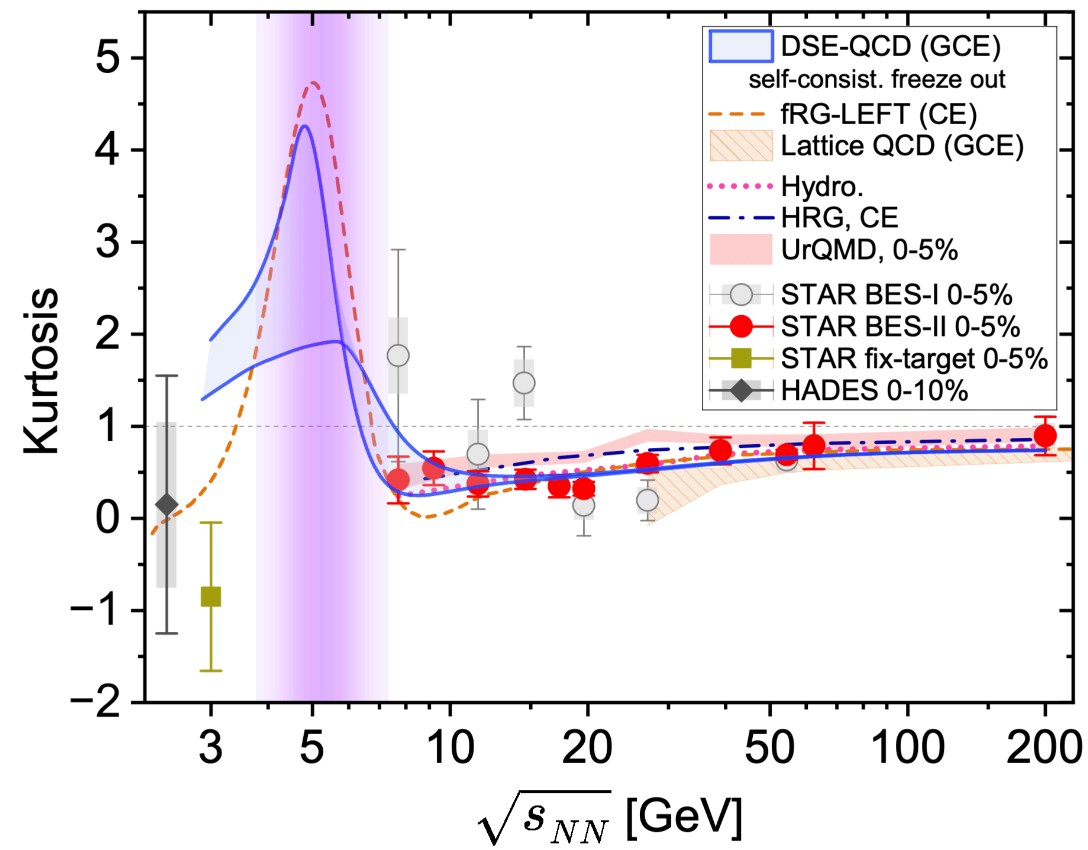
C_4/C_2: Comparison to STAR data

Net baryon (proton) number kurtosis:



STAR: *PRL* 135 (2025) 142301, arXiv:2504.00817.

fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508.

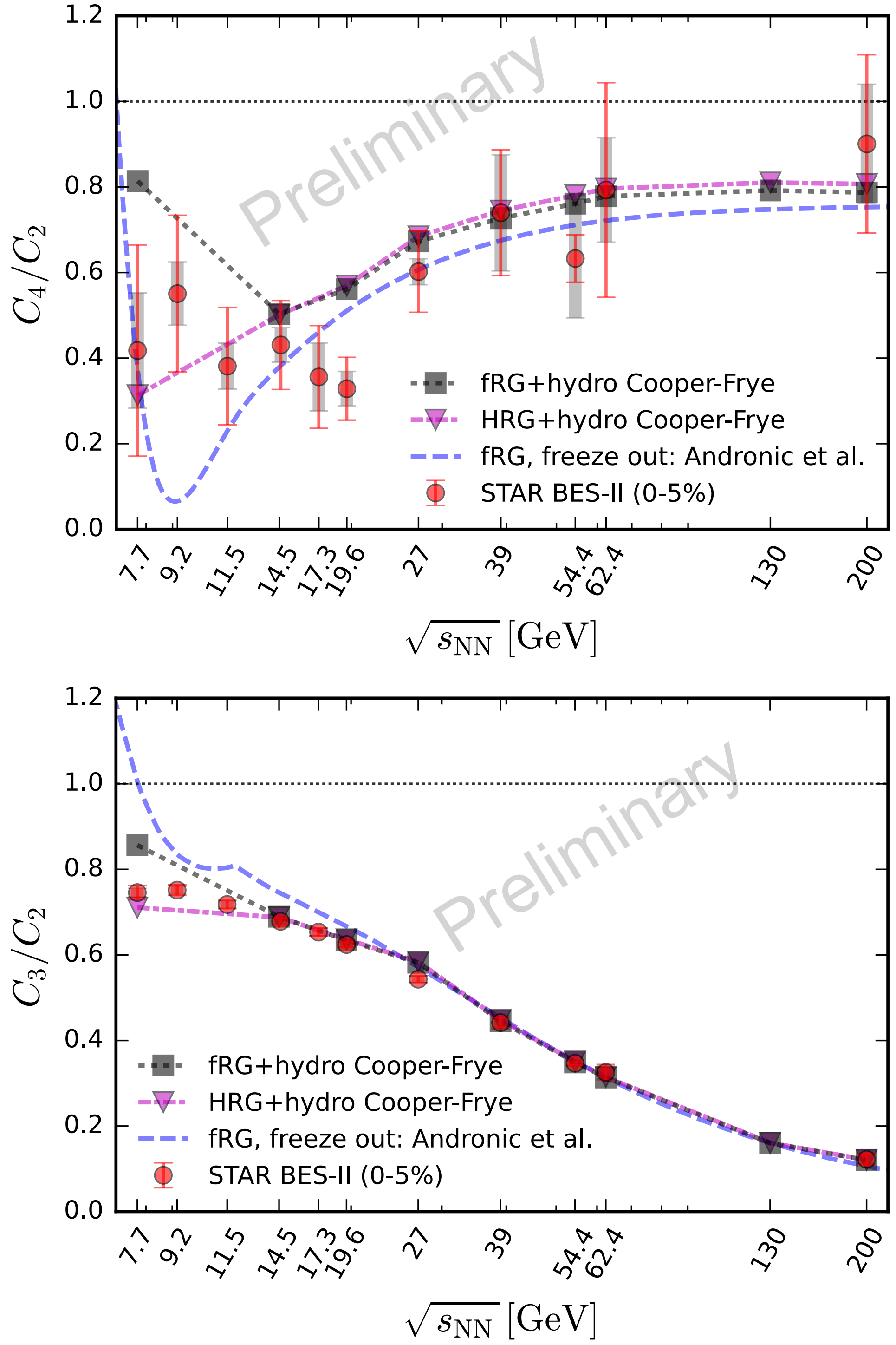


DSE: Lu, Fischer, Gao, Liu, Pawlowski, arXiv:2603.09336; Lu, Gao, Liu, Pawlowski, *PRD* 113 (2026) 054019, arXiv:2504.05099.

- **Theoretical prediction with critical fluctuations (fRG and DSE) is consistent with STAR data.**
- **A peak structure is predicted in the energy regime of fixed-target experiments, i.e. 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV. Experimental search of this peak is very important.**

Critical fluctuations + hydrodynamics

fRG+hydro:



fRG+hydro: Zhao, Yin, Wu, Du, Luo, WF, in preparation.

Effects included:

- **Hypersurface elements** of different T and μ_B obtained from hydrodynamics simulations

$$\delta C_n^{B^\pm,gce}(x_i) = \delta V_i^{\mathrm{eff}} T^3(x_i) \chi_n^{B^\pm}(x_i).$$

- Implementing the baryon number fluctuations calculated from **fRG**

$$\chi_n^{B^\pm} = \frac{\chi_n^{B^\pm,HRG}}{\chi_n^{B,HRG}} \chi_n^{B,fRG}.$$

- Using the Cooper-Frye formula to determine the probability within an **acceptance window** of momenta and rapidity $p_{\mathrm{acc}}(x_i; \Delta p_{\mathrm{acc}})$.

- Using the **isospin randomization** to calculate the (anti)proton cumulants.

- **Global baryon number conservation** via the subensemble acceptance method (SAM).

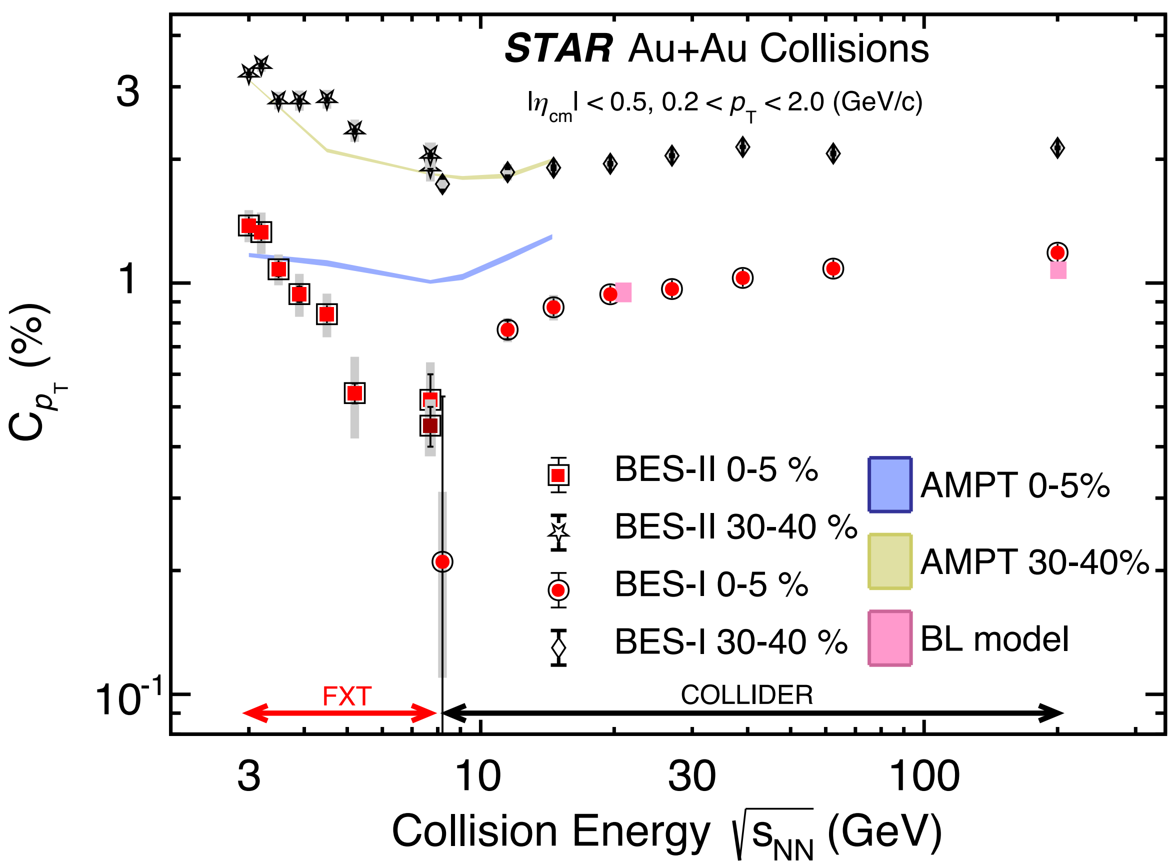
HRG+hydro: Vovchenko, Koch, Shen, *PRC* 105 (2022) 014904, arXiv:2107.00163.

hydro: Du, Gao, Jeon, Gale, *PRC* 109 (2024) 014907, arXiv:2302.13852.

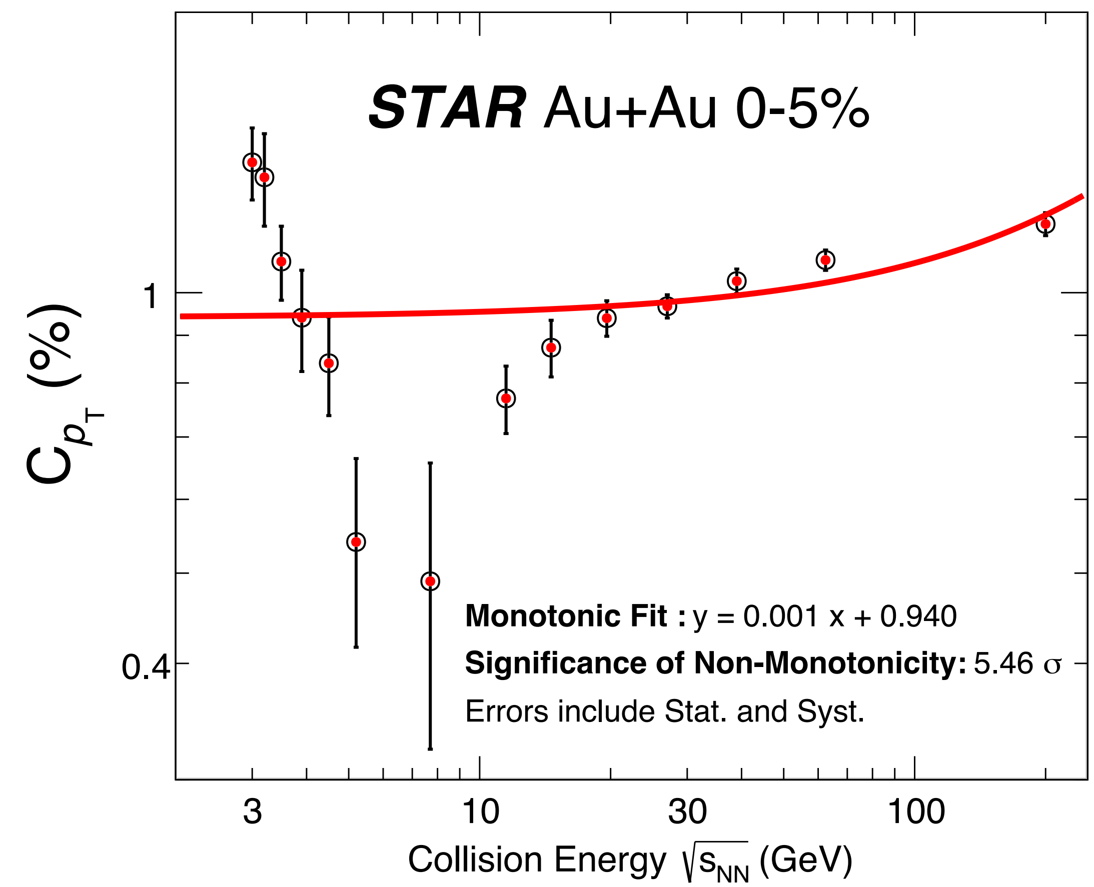
SAM: Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, *PLB* 811 (2020) 135868.

Mean pt fluctuations recently measured By STAR

Variance of mean pt fluctuations vs collision energy:



Rutik Manikandhan, CPOD2026



STAR Collaboration, arXiv:2604.06434.

See the talk by Chunjian

- **Observation of non-monotonic behavior in the variance of mean pt fluctuations in most central collisions with significance $\sim 5\sigma$ by the STAR collaboration.**

- **The mean transverse momentum fluctuations of charged particles can be potentially used to probe the QCD thermodynamics and phase transitions.**

Mean pt and temperature fluctuations

- Derivation of temperature fluctuations

We introduce a new thermodynamic state function

$$\mathrm{d}W = T\mathrm{d}S - p\mathrm{d}V - N_B \mathrm{d}\mu_B$$

With

$$W = \Omega + TS = U - \mu_B N_B$$

$N_{\mathrm{ch}} \sim S$ is fixed, and V and μ_B are also fixed with some acceptance window. So, W is an appropriate state function to describe the experimental observables.

From the state function W, the temperature and its fluctuations can be obtained

$$\frac{\partial w}{\partial s} = T$$

and

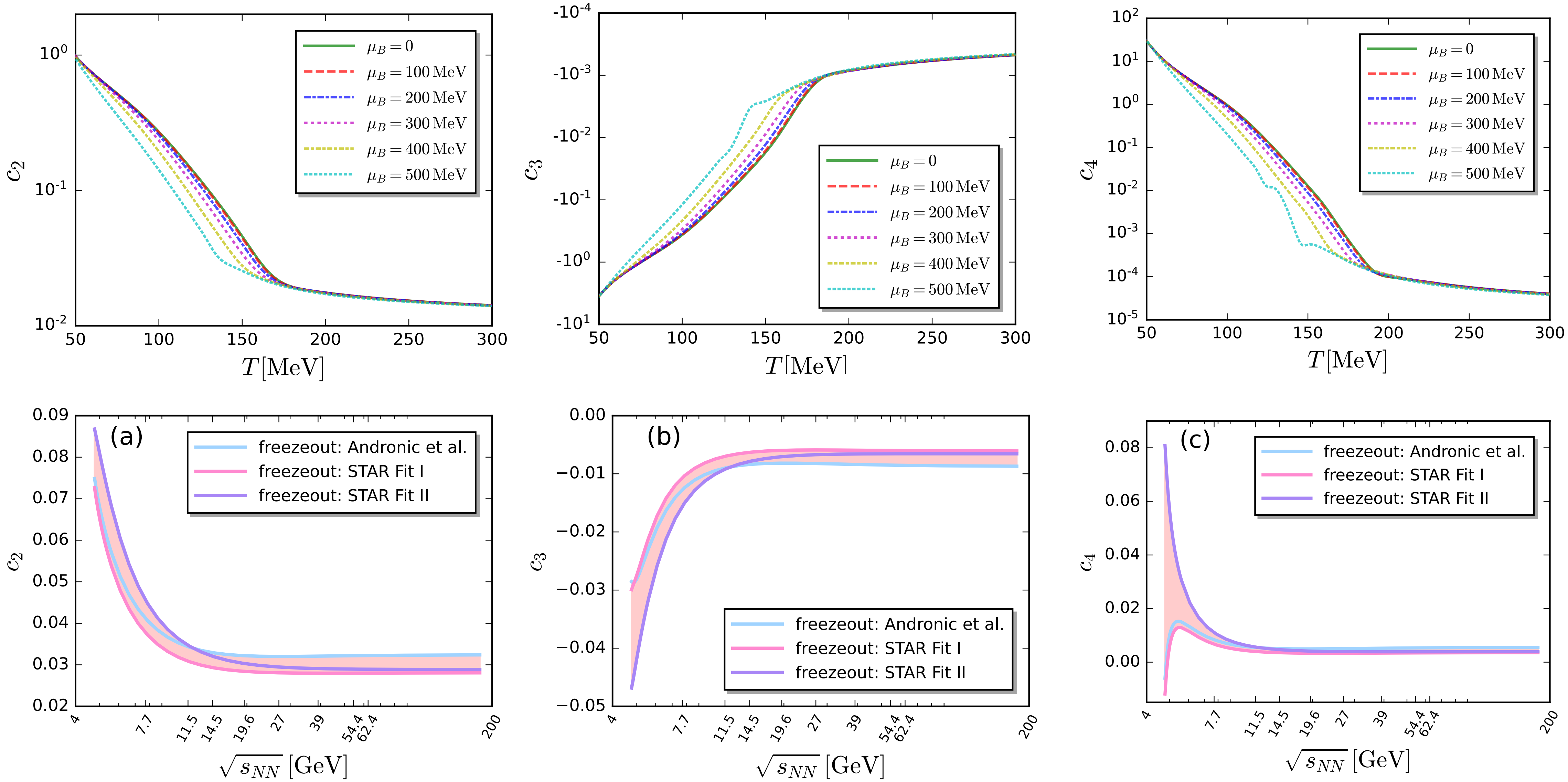
$$\langle (\Delta T)^n \rangle = T^{4n-4} \frac{\partial^n w}{\partial s^n}$$

It is convenient to adopt a dimensionless temperature fluctuation

$$c_n = \frac{\langle (\Delta T)^n \rangle}{T^n}$$

Jinhui Chen, WF, Shi Yin, Chunjian Zhang, arXiv:2504.06886.

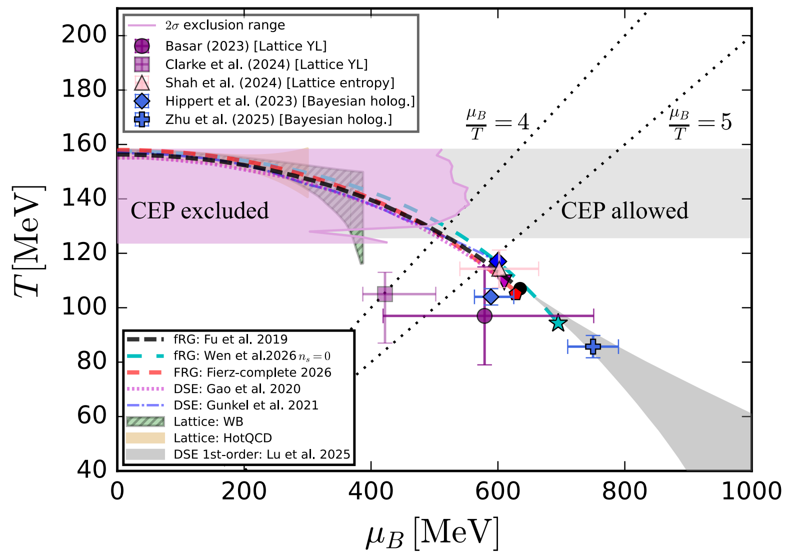
Temperature fluctuations

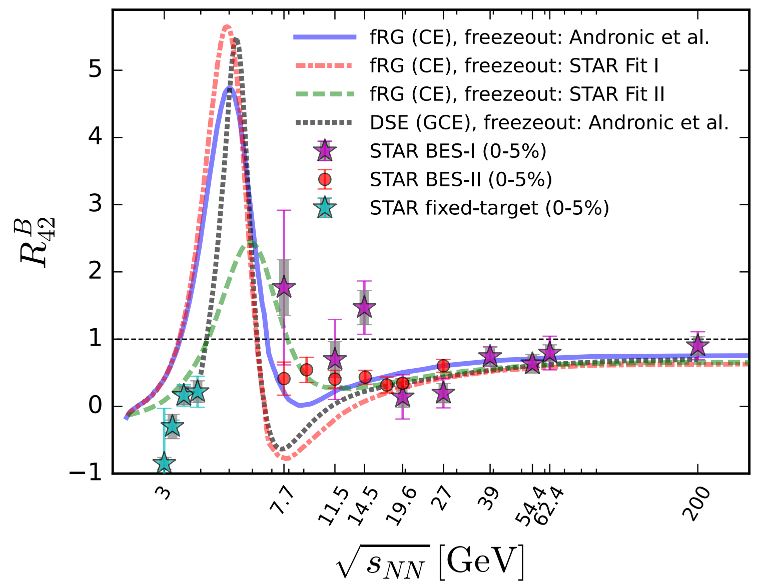


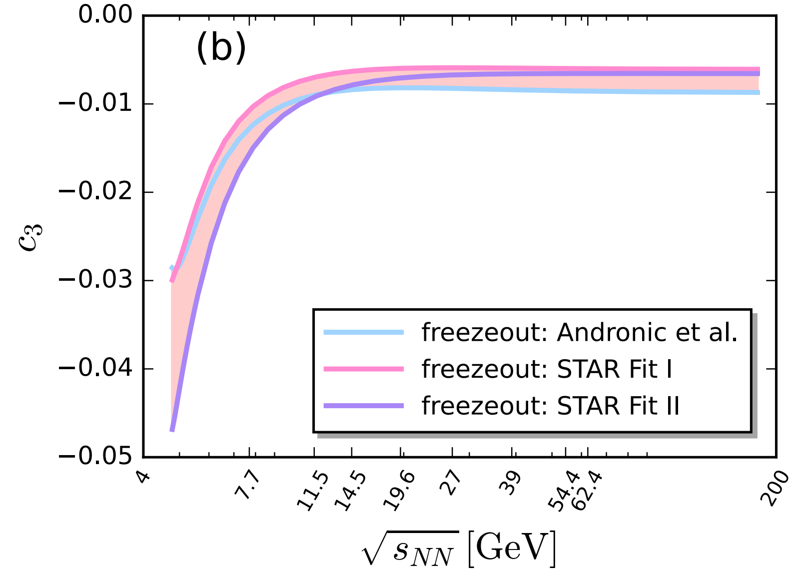
- **T fluctuations are suppressed remarkably as the system transitions from HRG to the QGP.**
- **Skewness of T fluctuations is negative, a smoking-gun signature of the temperature fluctuations.**
- **Due to the fact that the heat capacity of QGP is significantly larger than that of HRG.**

Jinhui Chen, WF, Shi Yin, Chunjian Zhang, arXiv:2504.06886.

Summary and outlook





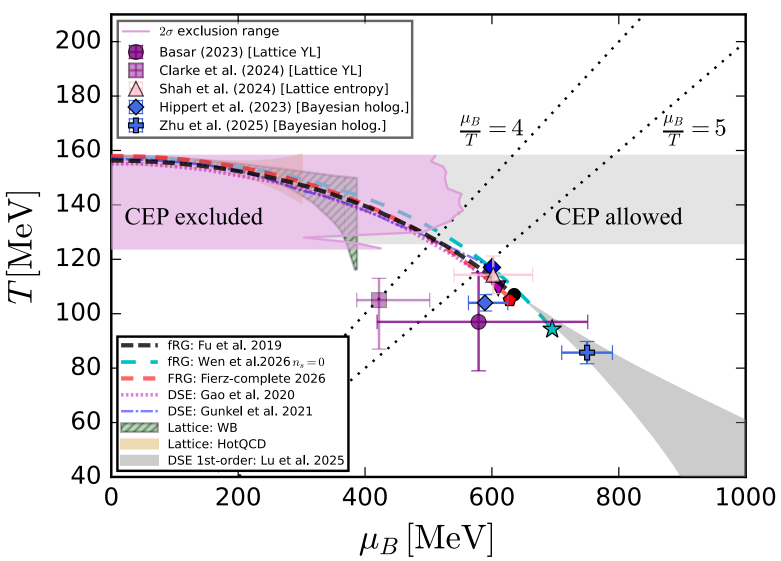


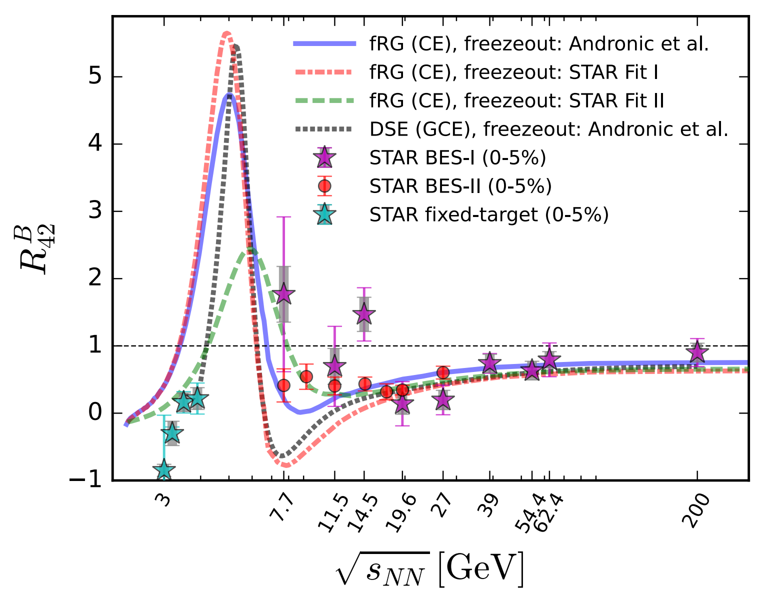
★ Combined results of different approaches, including functional QCD, lattice, holography, indicate that the location of CEP is not favored in the region $\mu_B/T \lesssim 4 \sim 5$.

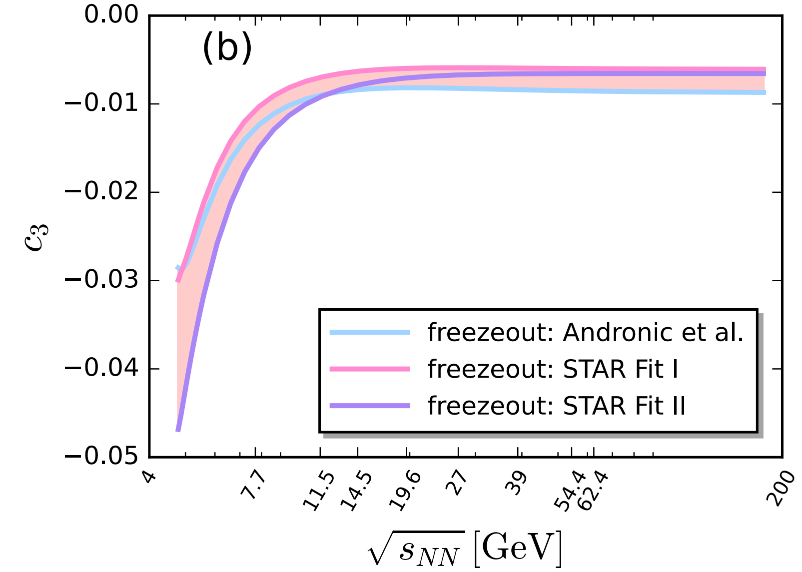
★ A prominent peak structure is predicted in the baryon number fluctuations in the fixed-target energy of 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV, which need to be confirmed in experiments.

★ Skewness of temperature fluctuations is found to be negative, a smoking-gun signature of the temperature fluctuations.

Summary and outlook







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Thank you very much for your attentions!

大连 **2019**

祝庄老师生日快乐！

The 3rd International Workshop on Physics at High Baryon Density (PHD2026, 第三届高重子密度物理国际研讨会)

Nov 10 – 13, 2026
Crowne Plaza Xinghai, Dalian, China
Asia/Shanghai timezone

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Physics Topics :

1) QCD Phase Structure at High Baryon Density

2) Nuclear Matter at High Density and Equation of State

3) Dynamical Evolution of Heavy-ion Collisions

4) Nuclear Matter Under Extreme External Fields

5) Hadron Properties in Nuclear Medium

6) Nuclear Physics in Compact Stars

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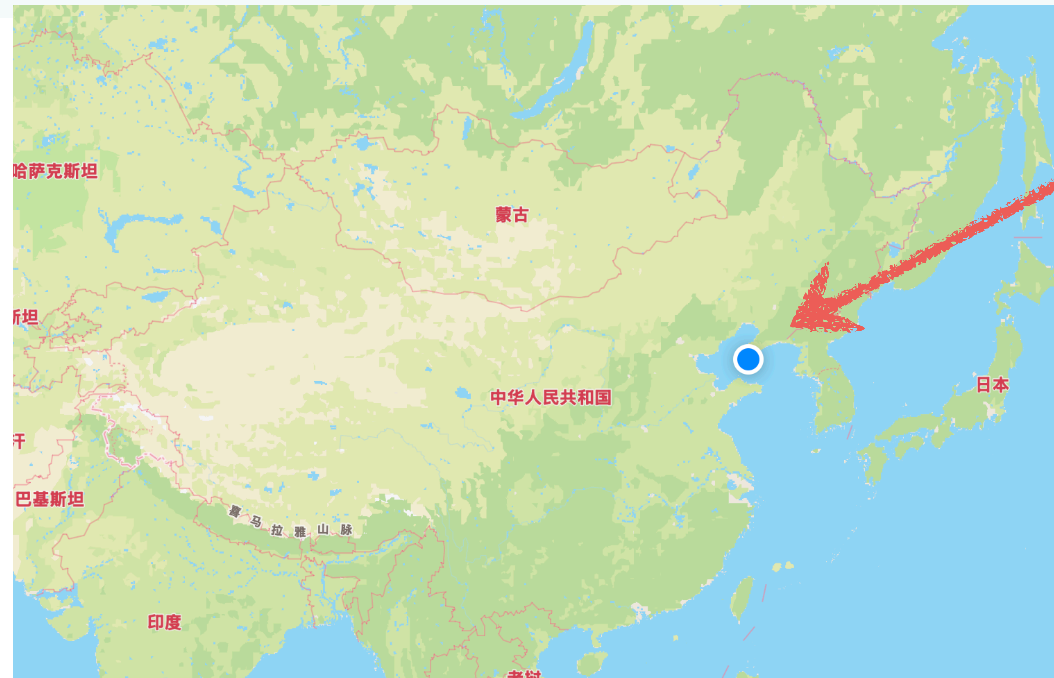
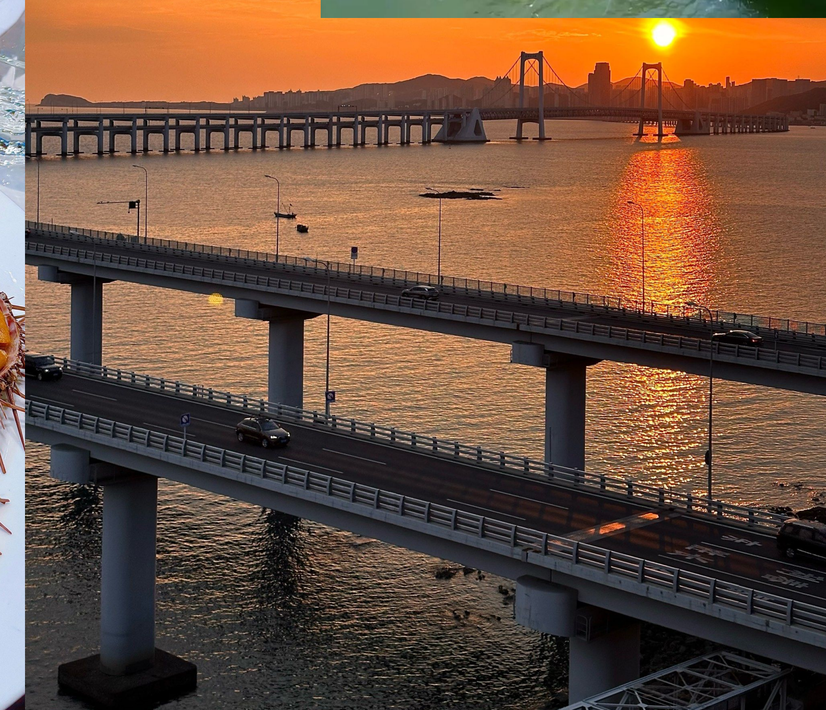


Dalian

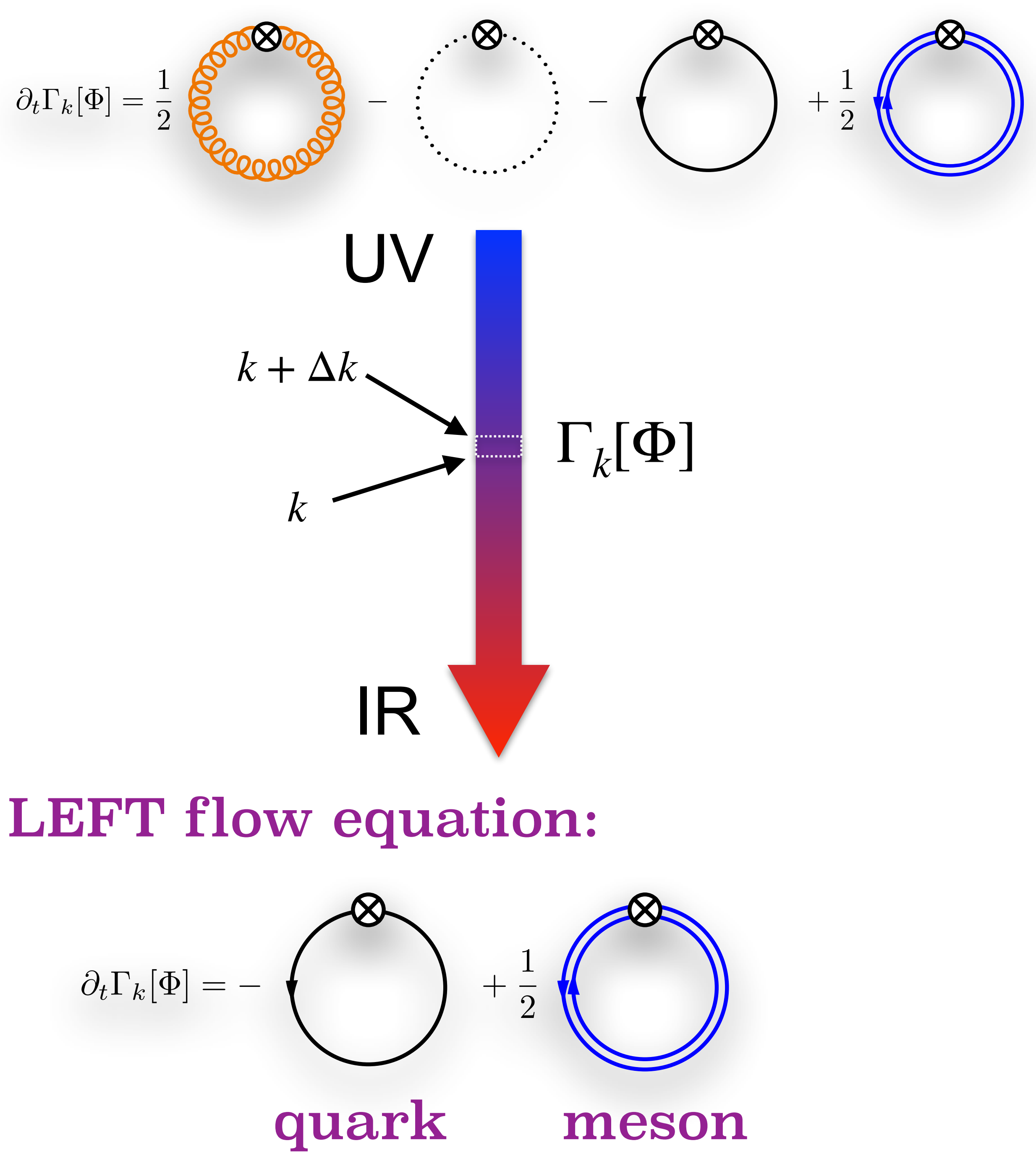


Welcome to PHD2026 in Dalian!

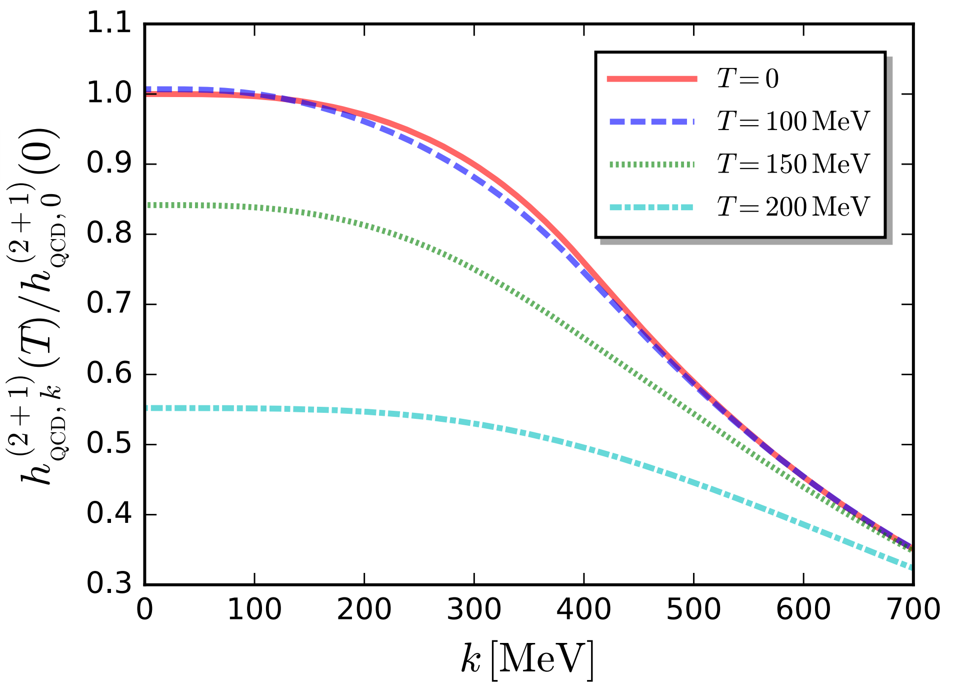
Backup

QCD-assisted LEFT

QCD flow equation:

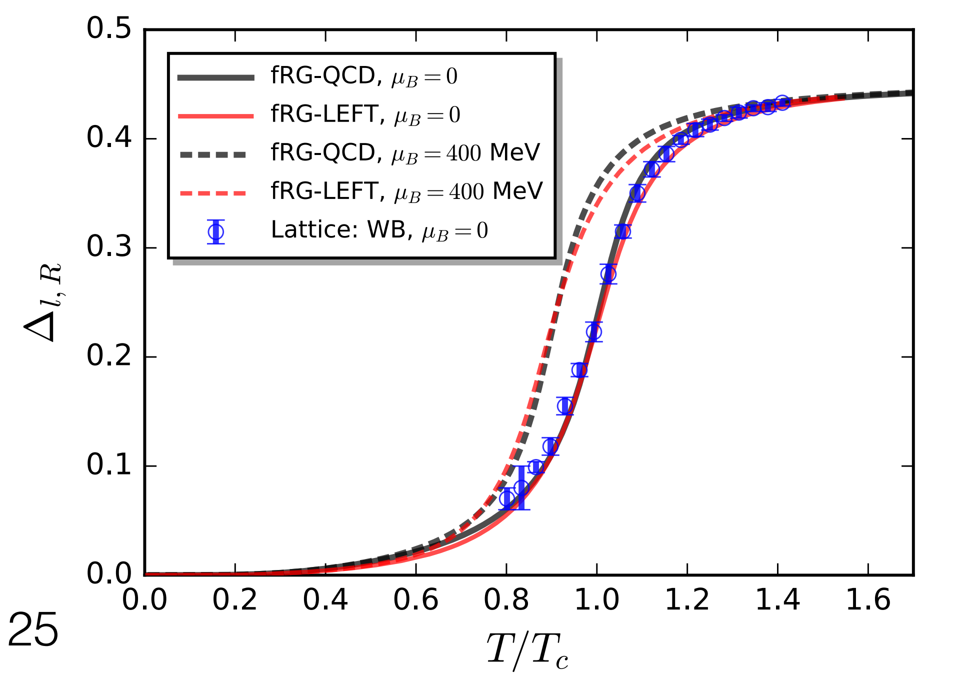


- **Yukawa couplings obtained in QCD inputted in QCD-assisted LEFT**



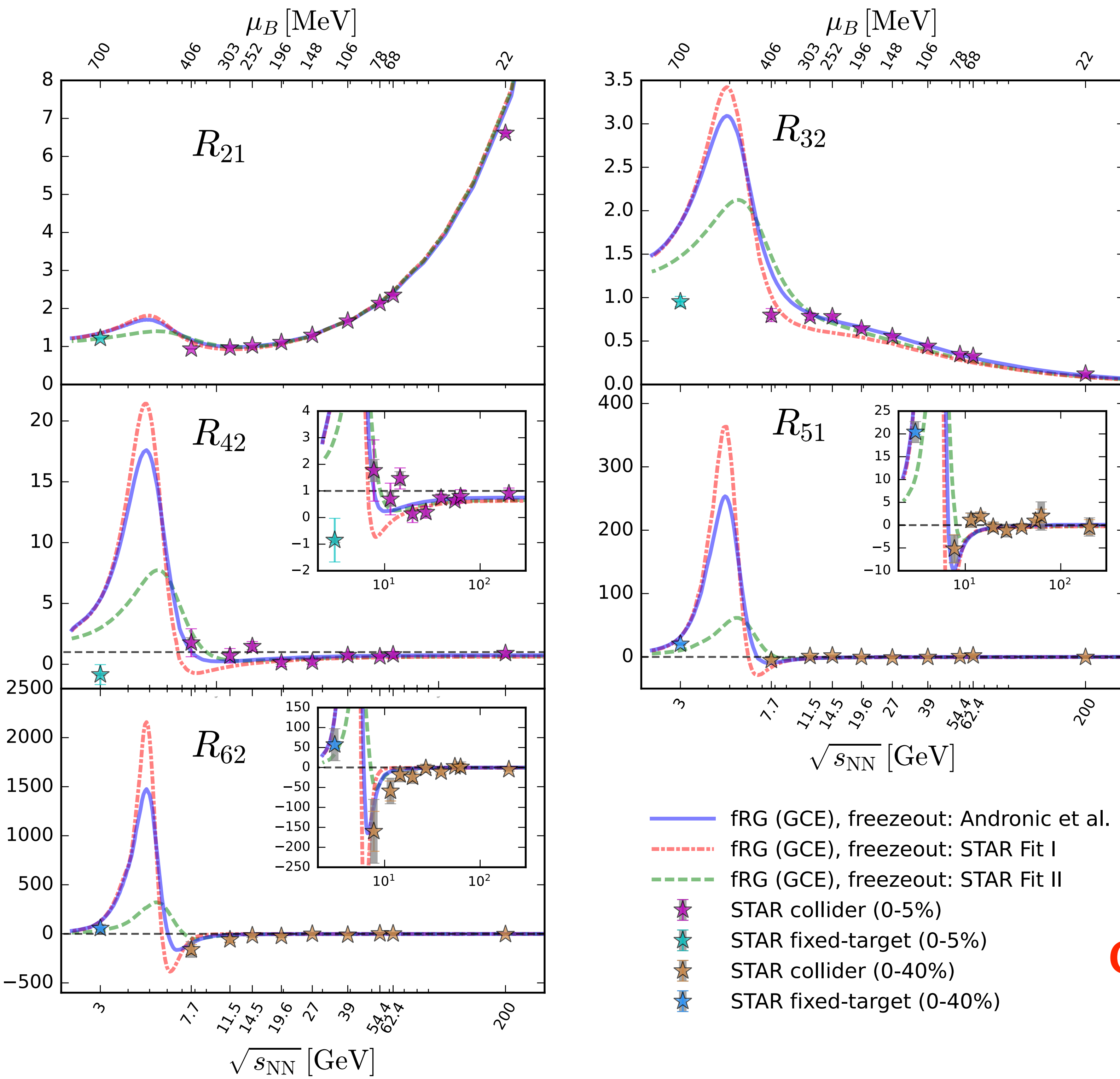
WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

- **Chiral condensates in QCD and QCD-assisted LEFT in agreement**



WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

Grand canonical fluctuations at the freeze-out



STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301; Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303; Aboona *et al.* (STAR), *PRL* 130 (2023) 082301

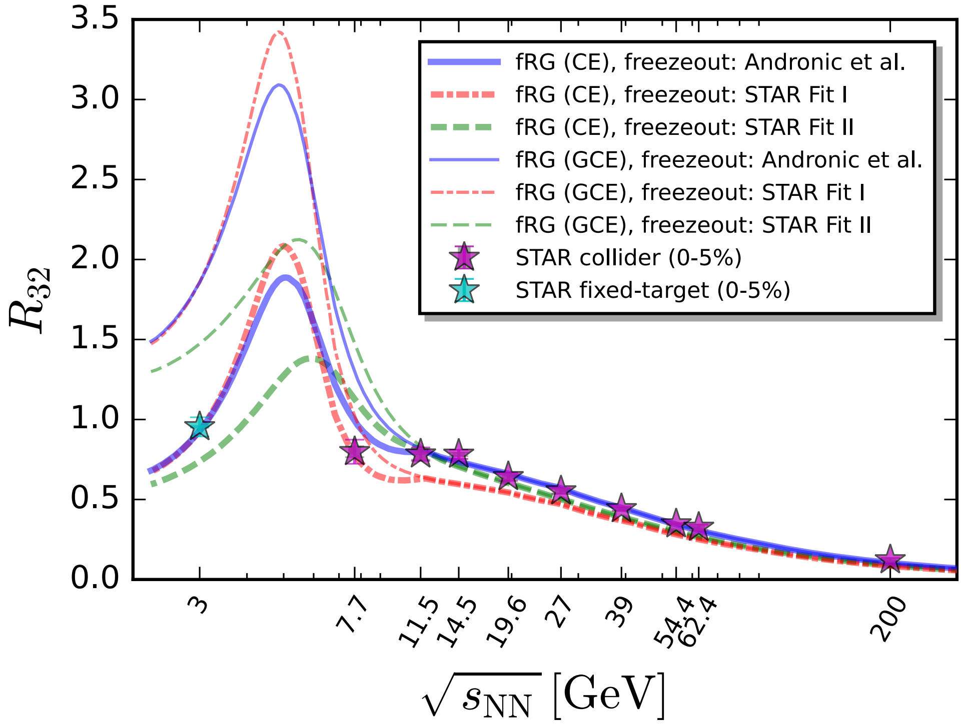
fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508

- **Results in fRG are obtained in the QCD-assisted LEFT with a CEP at $(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (98{,}643)$ MeV.**
- **Peak structure is found in 3 GeV $\lesssim \sqrt{s_{\mathrm{NN}}} \lesssim$ 7.7 GeV.**
- **Agreement between the theory and experiment is worsening with $\sqrt{s_{\mathrm{NN}}} \lesssim$ 11.5 GeV.**
- **Effects of global baryon number conservation in the regime of low collision energy should be taken into account.**

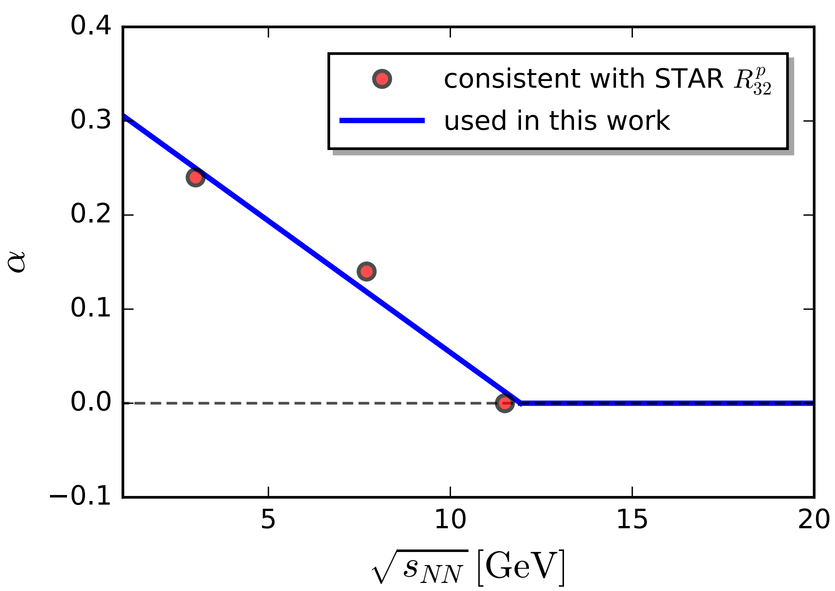
Caveat:

Fluctuations of baryon number in theory are compared with those of proton number in experiments.

Canonical corrections with SAM



- **Experimental data R_{32} is used to constrain the parameter α in the range $\sqrt{s_{NN}} \lesssim 11.5$ GeV.**
- **We choose the simplest linear dependence**



$$\alpha(\bar{s}) = a\left(1-\sqrt{\bar{s}}\right)\theta\left(1-\bar{s}\right)$$

$$a = 0.33\,, \qquad \sqrt{\bar{s}} = \frac{\sqrt{s_{NN}}}{11.9\,\mathrm{GeV}}$$

SAM:

- We adopt the subensemble acceptance method (SAM) to take into account the effects of global baryon number conservation:

$$\alpha = \frac{V_1}{V}$$

V_1: the subensemble volume measured in the acceptance window, V: the volume of the whole system.

- fluctuations with canonical corrections are related to grand canonical fluctuations as follows:

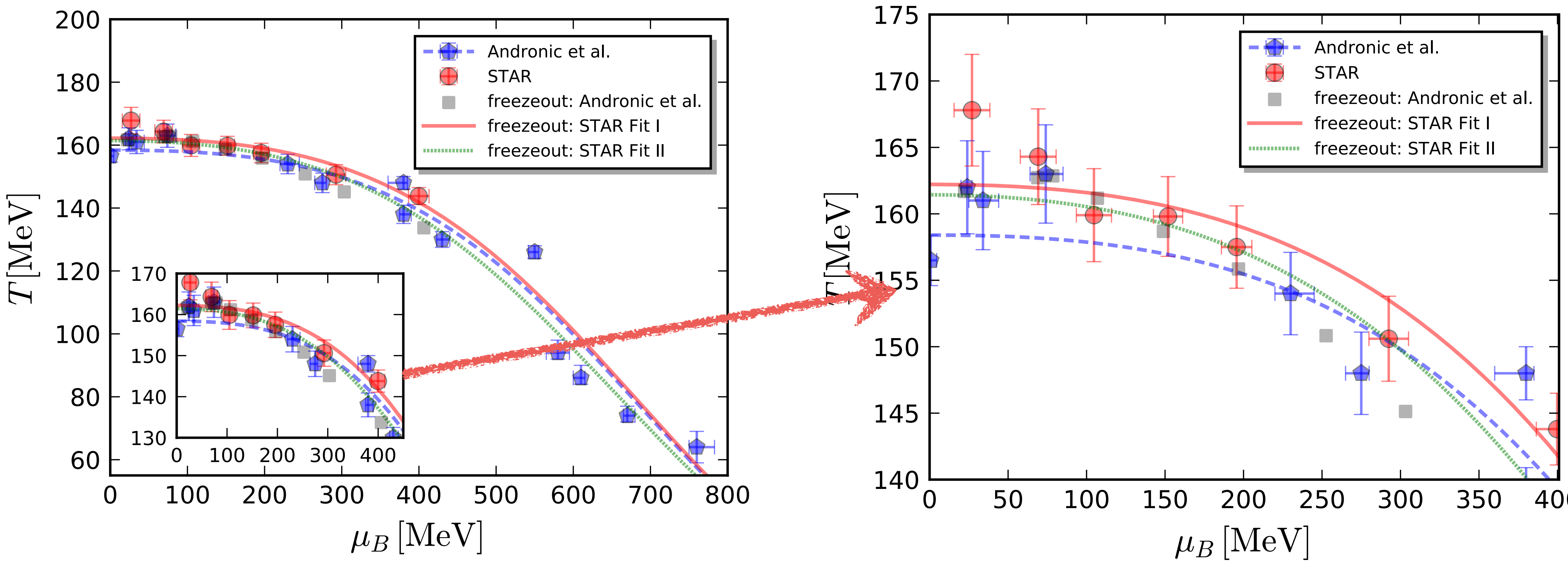
$$\bar{R}^B_{21} = \beta R^B_{21}, \qquad \bar{R}^B_{32} = (1-2\alpha)R^B_{32},$$

$$\bar{R}^B_{42} = (1-3\alpha\beta)R^B_{42} - 3\alpha\beta(R^B_{32})^2$$

$$\beta = 1-\alpha$$

SAM: Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch , *PLB* 811 (2020) 135868

Determination of the freeze-out curve



three freeze-out curves

1. freeze-out: Andronic *et al.*

Andronic, Braun-Munzinger, Redlich, *Nature* 561 (2018) 7723, 321

$$\mu_{B\,\mathrm{CF}} = \frac{a}{1 + 0.288\sqrt{s_{\mathrm{NN}}}}\,,$$

$$T_{\mathrm{CF}} = \frac{T^{(0)}_{\mathrm{CF}}}{1 + \exp\left(2.60 - \ln(\sqrt{s_{\mathrm{NN}}})/0.45\right)}$$

2. freeze-out: STAR Fit I

L. Adamczyk *et al.* (STAR), *PRC* 96 (2017), 044904

all data points

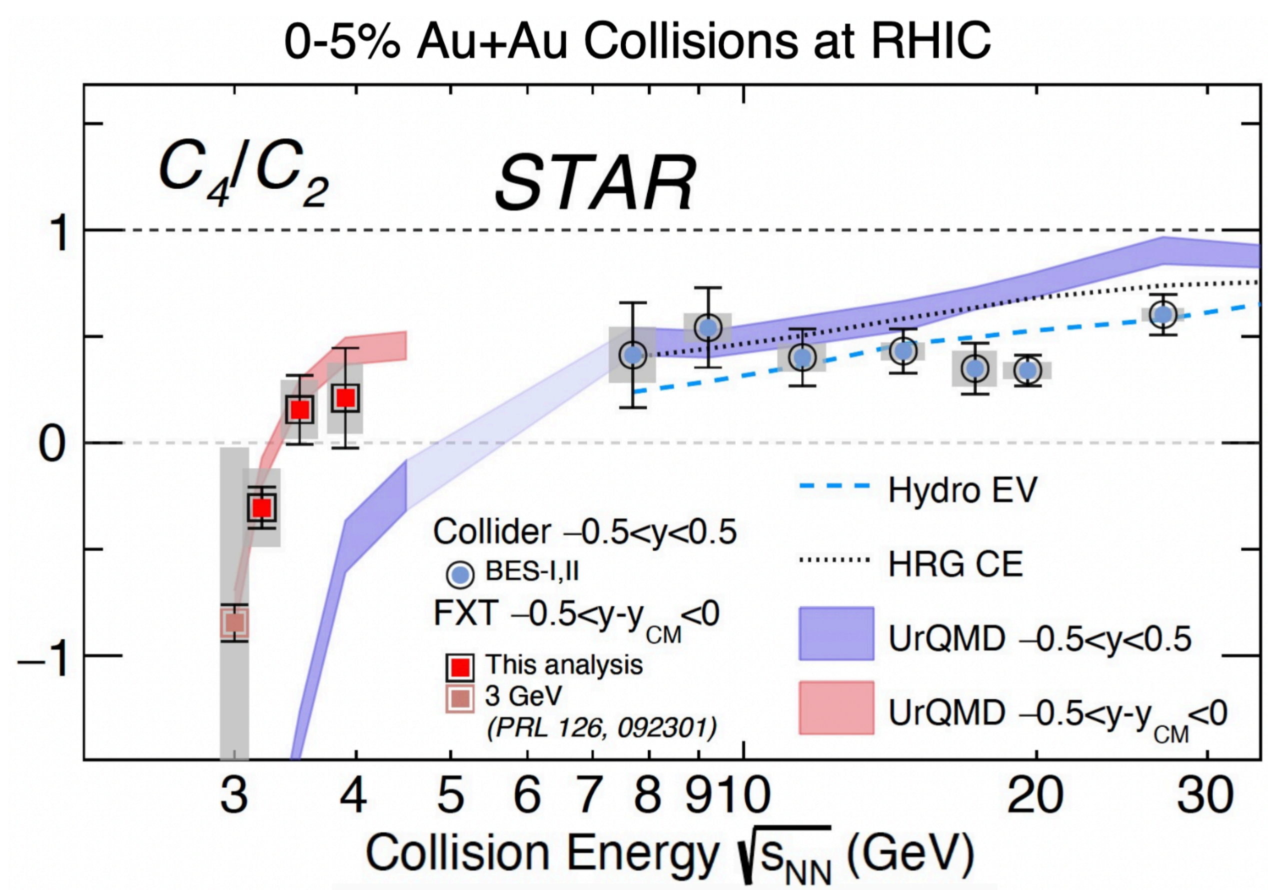
3. freeze-out: STAR Fit II

neglecting first two at low μ_B and the last one

- freeze-out curve should not rise with μ_B
- convexity of the freeze-out curve

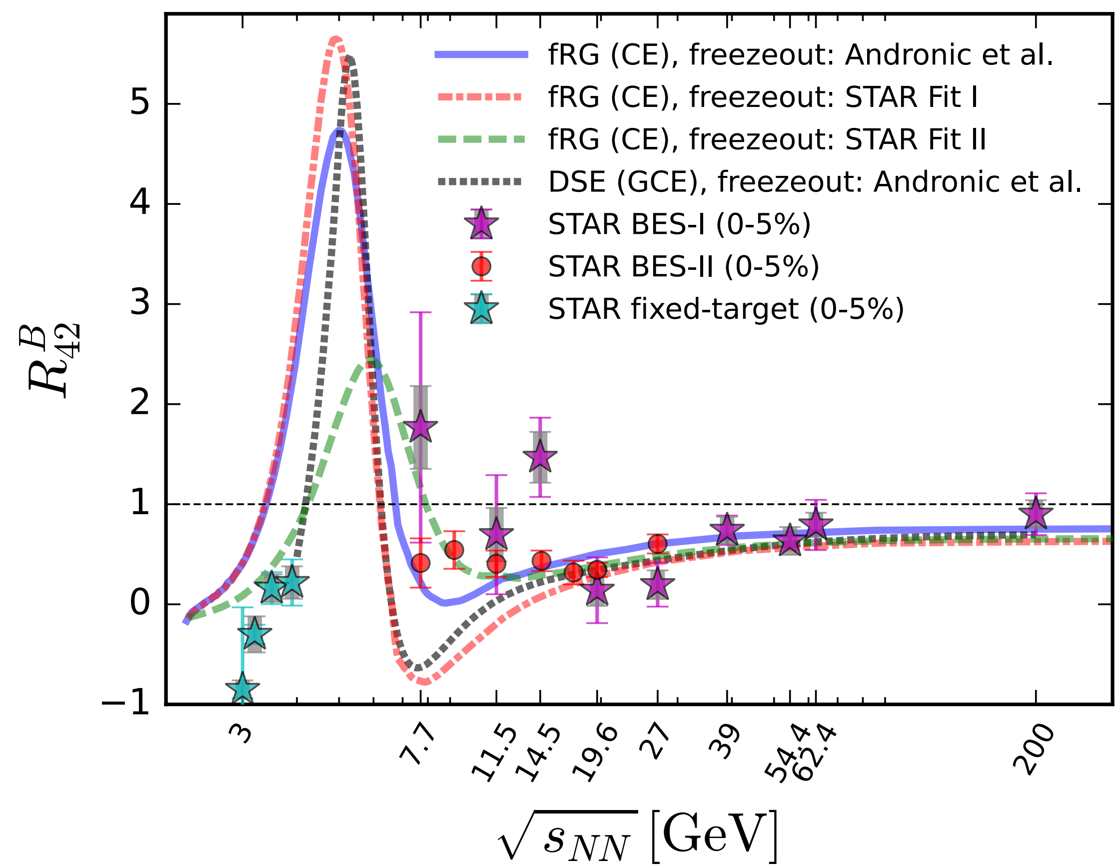
C_4/C_2: Comparison to STAR data

FXT energies at 3.2, 3.5, 3.9 GeV:



STAR: Z. Sweger, Quark Matter 2025
STAR: *PRL* 135 (2025) 142301, arXiv:2504.00817

Net baryon (proton) number kurtosis:

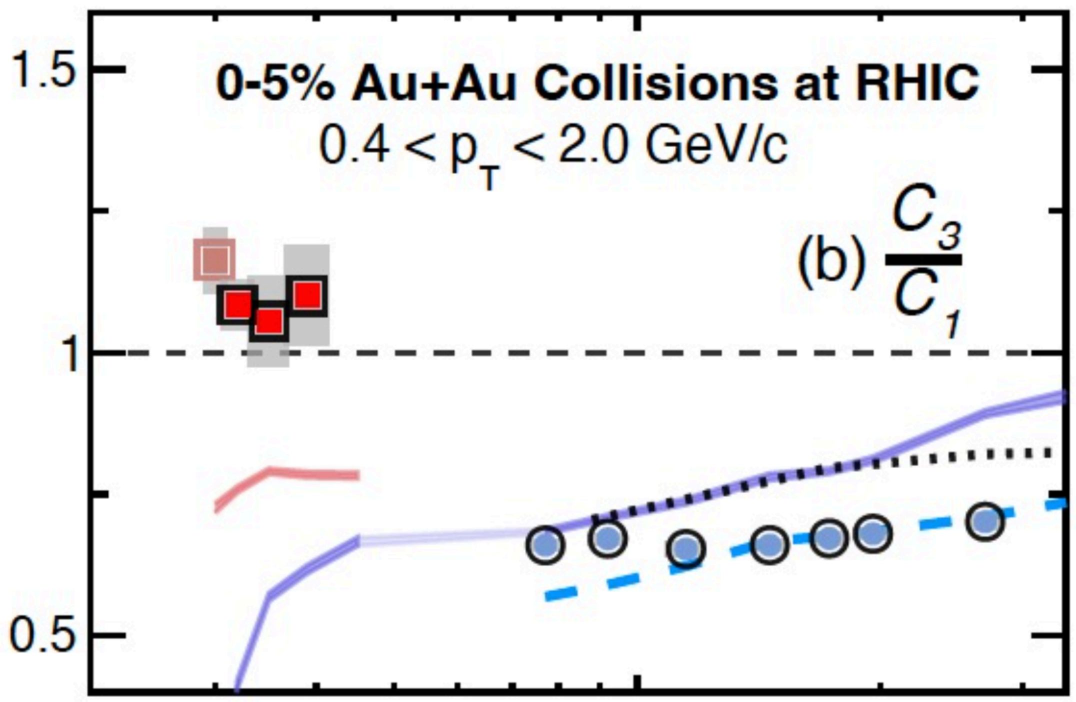


fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508
DSE: Lu, Gao, Liu, Pawlowski, arXiv: 2504.05099

- **Theoretical prediction with critical fluctuations (fRG and DSE) is consistent with STAR data.**
- **A peak structure is predicted in the energy regime of fixed-target experiments, i.e. 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV. Experimental search of this peak is very important.**

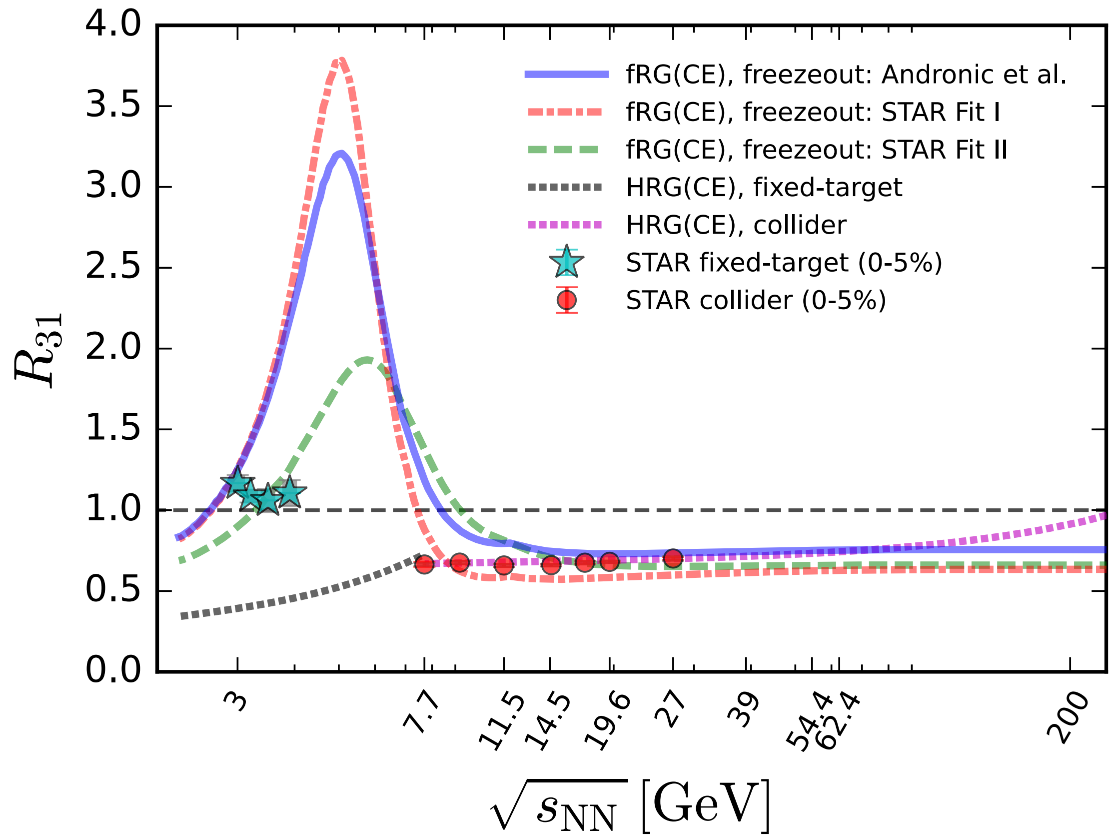
C_3/C_1: Comparison to STAR data

STAR data and UrQMD (baseline):



STAR: Z. Sweger, Quark Matter 2025
STAR: arXiv:2504.00817

fRG (critical) and HRG (baseline):



fRG: WF, Luo, Pawlowski, Rennecke, Yin, *PRD* 111 (2025) L031502, arXiv: 2308.15508; Zhao, Yin, WF, in preparation

- **Significant deviations from both the non-critical baseline results in UrQMD and HRG.**
- **fRG results are in accordance with data.**

Derivation of temperature fluctuations

- Derivation of temperature fluctuations

From the state function W, the temperature and its fluctuations can be obtained

$$\frac{\partial w}{\partial s} = T$$

and

$$\langle (\Delta T)^n \rangle = T^{4n-4} \frac{\partial^n w}{\partial s^n}$$

It is convenient to adopt a dimensionless temperature fluctuation

$$c_n = \frac{\langle (\Delta T)^n \rangle}{T^n}$$

The first three nontrivial orders corresponding to the variance, skewness, and kurtosis of temperature fluctuations, are given by,

$$c_2 = T^2 \left(\frac{\partial^2 p}{\partial T^2} \right)^{-1}$$

$$c_3 = -\, T^5 \left(\frac{\partial^2 p}{\partial T^2} \right)^{-3} \frac{\partial^3 p}{\partial T^3}$$

$$c_4 = T^8 \left[3 \left(\frac{\partial^2 p}{\partial T^2} \right)^{-5} \left(\frac{\partial^3 p}{\partial T^3} \right)^{2} - \left(\frac{\partial^2 p}{\partial T^2} \right)^{-4} \frac{\partial^4 p}{\partial T^4} \right].$$

Jinhui Chen, WF, Shi Yin, Chunjian Zhang, arXiv:2504.06886

Fierz-complete basis of four-quark interactions

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, $SU_A(N_f)$ and $U_A(1)$

$$\mathcal{T}^{(V-A)}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\gamma_\mu T^0 q)^2 - (\bar{q}i\gamma_\mu\gamma_5 T^0 q)^2 \,,$$

$$\mathcal{T}^{(V+A)}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\gamma_\mu T^0 q)^2 + (\bar{q}i\gamma_\mu\gamma_5 T^0 q)^2 \,,$$

$$\mathcal{T}^{(S-P)+}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\,T^0 q)^2 - (\bar{q}\,\gamma_5 T^0 q)^2 + (\bar{q}\,T^a q)^2 - (\bar{q}\,\gamma_5 T^a q)^2 \,,$$

$$\mathcal{T}^{(V-A)^{\mathrm{adj}}}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\gamma_\mu T^0 t^a q)^2 - (\bar{q}i\gamma_\mu\gamma_5 T^0 t^a q)^2 \,,$$

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, $SU_A(N_f)$, breaking $U_A(1)$

$$\mathcal{T}^{(S+P)-}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\,T^0 q)^2 + (\bar{q}\,\gamma_5 T^0 q)^2 - (\bar{q}\,T^a q)^2 - (\bar{q}\,\gamma_5 T^a q)^2 \,,$$

$$\mathcal{T}^{(S+P)^{\mathrm{adj}}_{-}}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\,T^0 t^a q)^2 + (\bar{q}\,\gamma_5 T^0 t^a q)^2 - (\bar{q}\,T^a t^b q)^2 - (\bar{q}\,\gamma_5 T^a t^b q)^2 \,,$$

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, $U_A(1)$, breaking $SU_A(N_f)$

$$\mathcal{T}^{(S-P)-}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\,T^0 q)^2 - (\bar{q}\,\gamma_5 T^0 q)^2 - (\bar{q}\,T^a q)^2 + (\bar{q}\,\gamma_5 T^a q)^2 \,,$$

$$\mathcal{T}^{(S-P)^{\mathrm{adj}}_{-}}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\,T^0 t^a q)^2 - (\bar{q}\,\gamma_5 T^0 t^a q)^2 - (\bar{q}\,T^a t^b q)^2 + (\bar{q}\,\gamma_5 T^a t^b q)^2 \,.$$

Invariant with the transformation of $SU_V(N_f)$, $U_V(1)$, breaking $SU_A(N_f)$, $U_A(1)$

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$$\mathcal{T}^{(S+P)^{\mathrm{adj}}_{+}}_{ijlm}\bar{q}_i q_l \bar{q}_j q_m = (\bar{q}\,T^0 t^a q)^2 + (\bar{q}\,\gamma_5 T^0 t^a q)^2 + (\bar{q}\,T^a t^b q)^2 + (\bar{q}\,\gamma_5 T^a t^b q)^2 \,.$$

Functional renormalization group

Functional integral with an IR regulator

$$Z_k[J] = \int (\mathscr{D}\hat{\Phi}) \exp\left\{ -S[\hat{\Phi}] - \Delta S_k[\hat{\Phi}] + J^a \hat{\Phi}_a \right\}$$

$$W_k[J] = \ln Z_k[J]$$

regulator:

$$\Delta S_k[\varphi] = \frac{1}{2}\int \frac{d^4 q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$

flow of the Schwinger function:

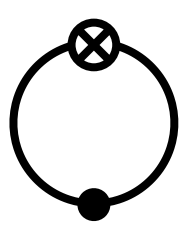
$$\partial_t W_k[J] = -\frac{1}{2}\mathrm{STr}\Big[\big(\partial_t R_k\big) G_k\Big] - \frac{1}{2}\Phi_a \partial_t R_k^{ab} \Phi_b$$

Legendre transformation:

$$\Gamma_k[\Phi] = -W_k[J] + J^a \Phi_a - \Delta S_k[\Phi]$$

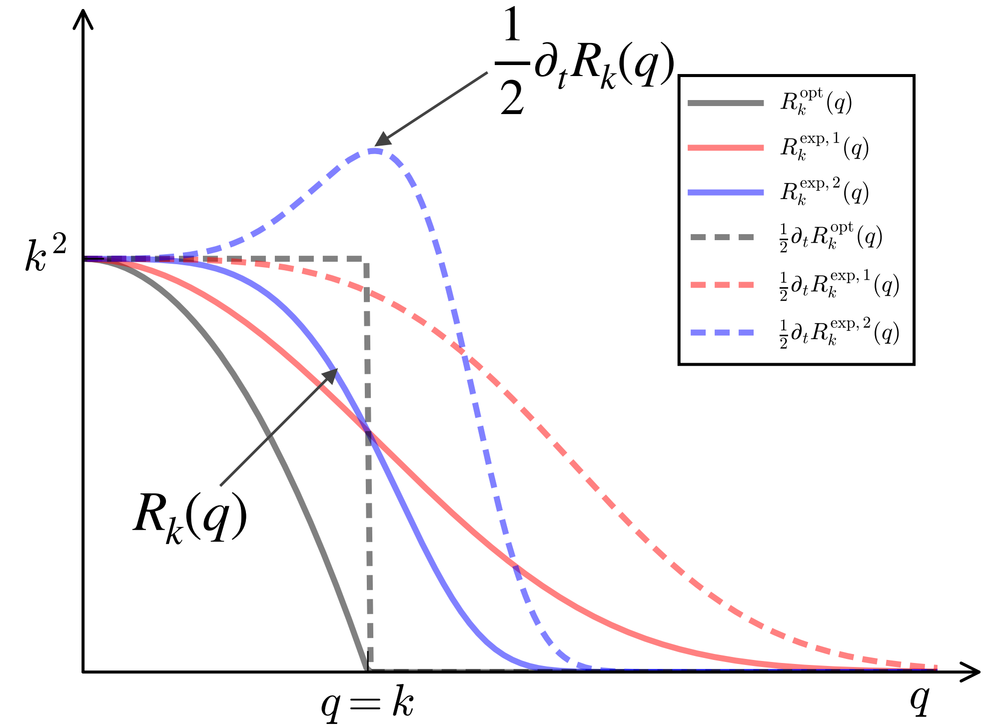
flow of the effective action:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\Big[\big(\partial_t R_k\big) G_k\Big] = \frac{1}{2}$$

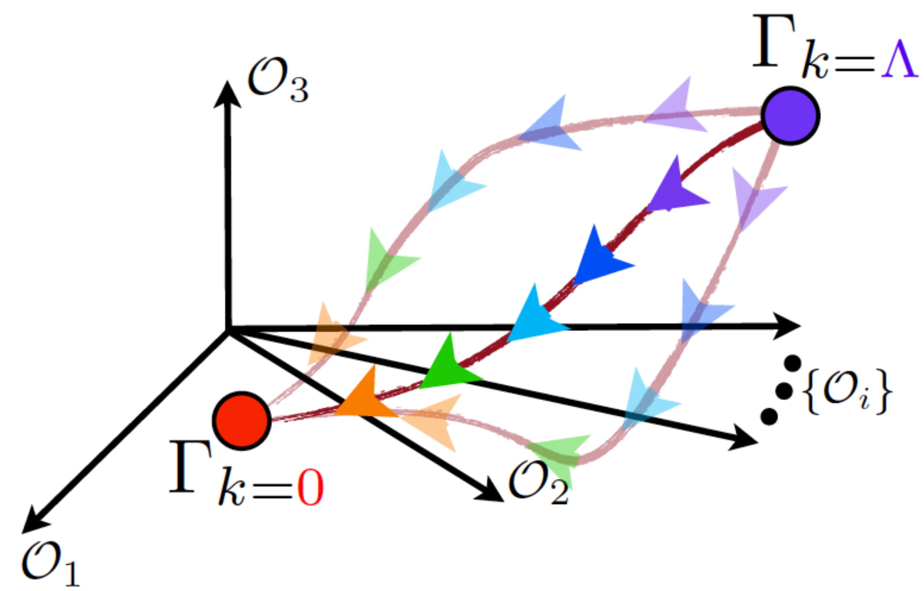


Wetterich formula

C. Wetterich, *PLB*, 301 (1993) 90



$$G_{k,ab} = \gamma^c{}_a \Big(\Gamma_k^{(2)}[\Phi] + \Delta S_k^{(2)}[\Phi]\Big)^{-1}_{cb},$$



Review: WF, *CTP* 74 (2022) 097304, arXiv: 2205.00468 [hep-ph]