Recent developments in spin hydrodynamics in relativistic heavy-ion collisions

浦实
中国科学技术大学

极端核物质前沿研讨会，宜昌

（Workshop on extreme nuclear matter frontiers）

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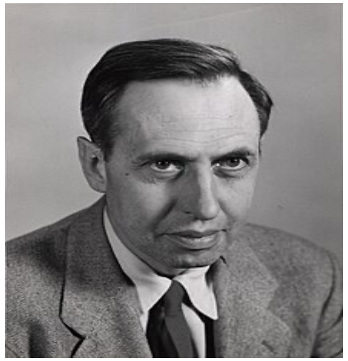
宜昌

Discovery of spin in physics

This year marks the 101th anniversary of the discovery of spin and the 21th anniversary of the proposal for spin polarization in relativistic heavy-ion collisions.

Wolfgang Pauli

Ralph Kronig



George Uhlenbeck

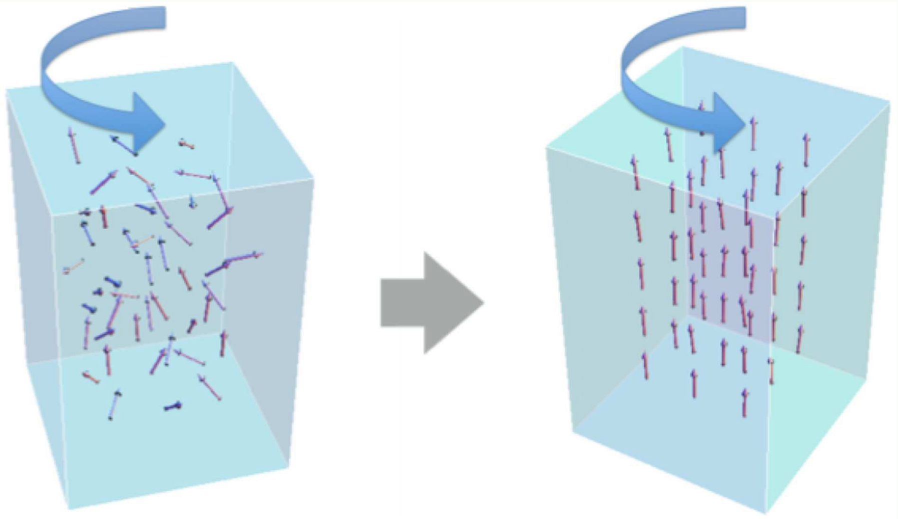
Samuel Goudsmit

Llewellyn H. Thomas

Wolfgang Pauli

- **1924**: two valued -ness not describable classically
- **1925**: self-rotation of the electron
- **1925**: Published the first paper on spin
- **1926**: Thomas procession
- **1927**: spin in quantum mechanics

Barnet and Einstein-de Hass effects



Barnett effect:

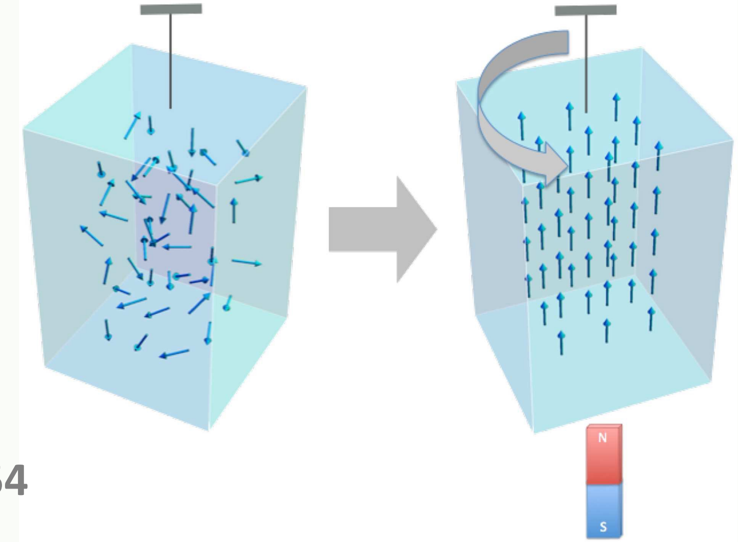
Rotation $\Longrightarrow$ Magnetization

Barnett, Magnetization by rotation, Phys Rev. (1915) 6:239–70.

Einstein-de Haas effect:

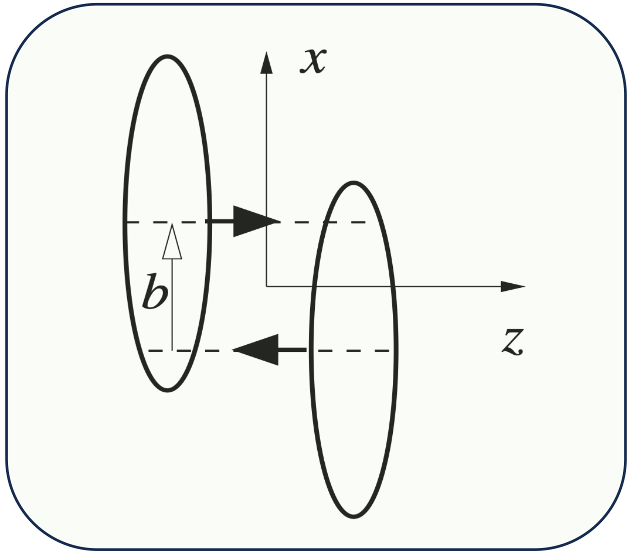
Magnetization $\Longrightarrow$ Rotation

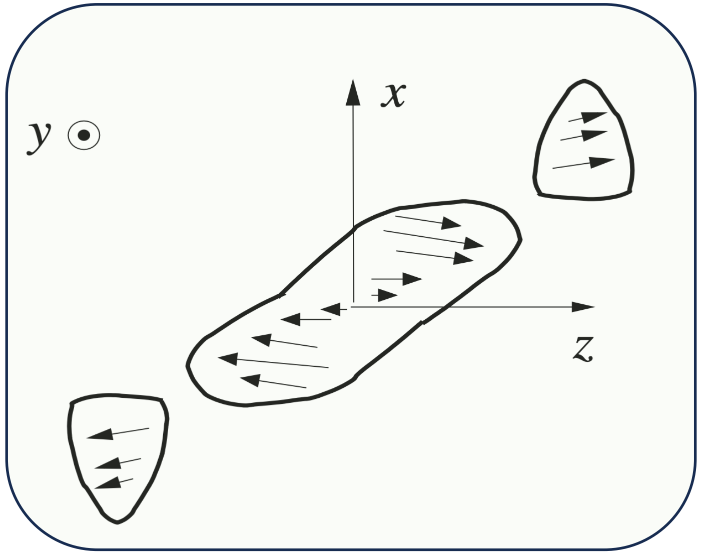
Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents. Verh Dtsch Phys Ges. (1915) 17:152.

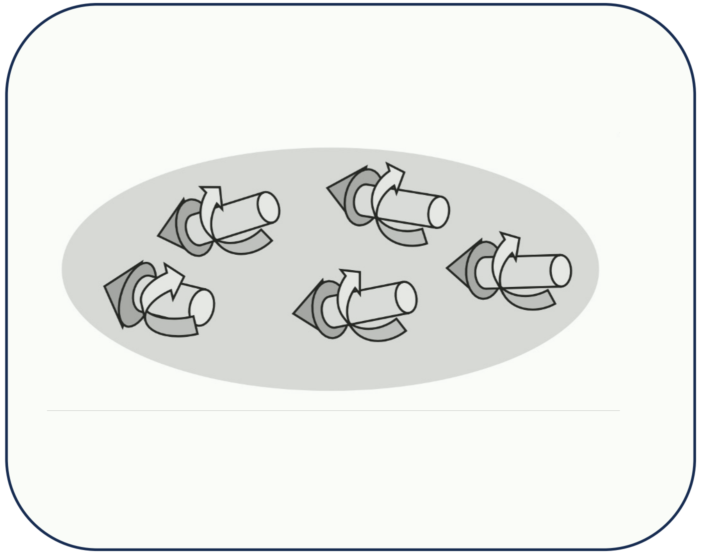


Figures: from paper doi: 10.3389/fphy.2015.00054

Early Pioneer work on spin polarization in heavy ion collisions







梁作堂 院士

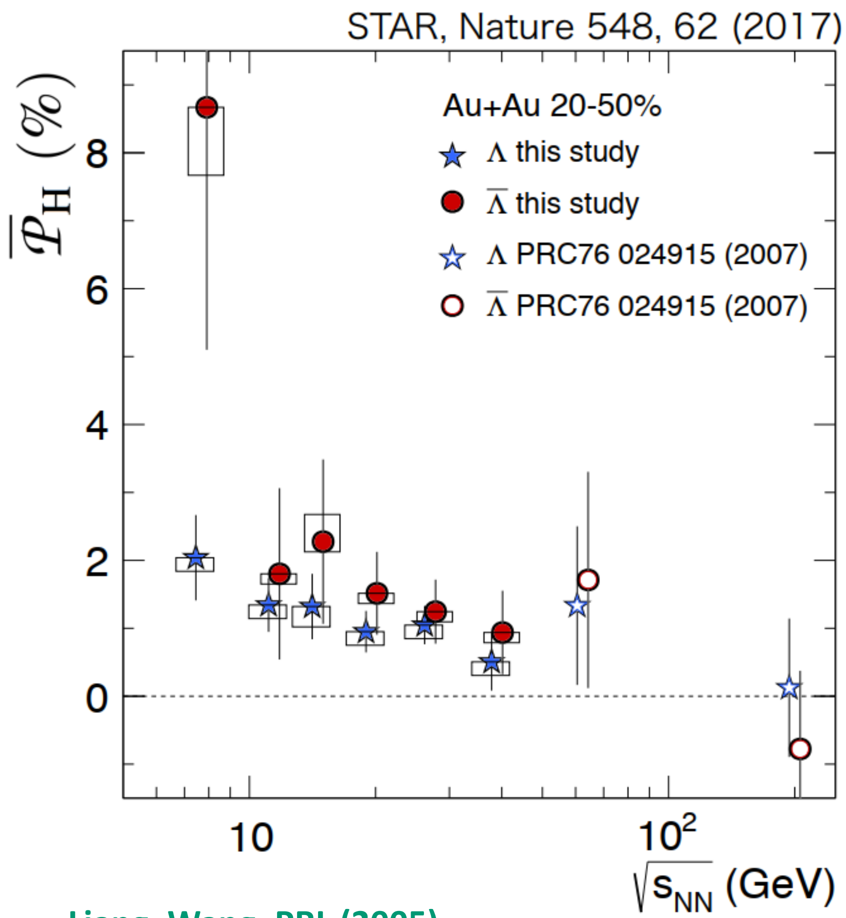
王新年 教授

- Huge global orbital angular momenta ($L \sim 10^5 \hbar$) are produced in HIC.
- Global orbital angular momentum leads to the polarizations of Λ hyperons and spin alignment of vector mesons through spin-orbital coupling.

Liang, Wang, PRL (2005); PLB (2005);
Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Global polarization for Λ and $\bar{\Lambda}$ hyperons

沿着初始角动量方向的整体极化



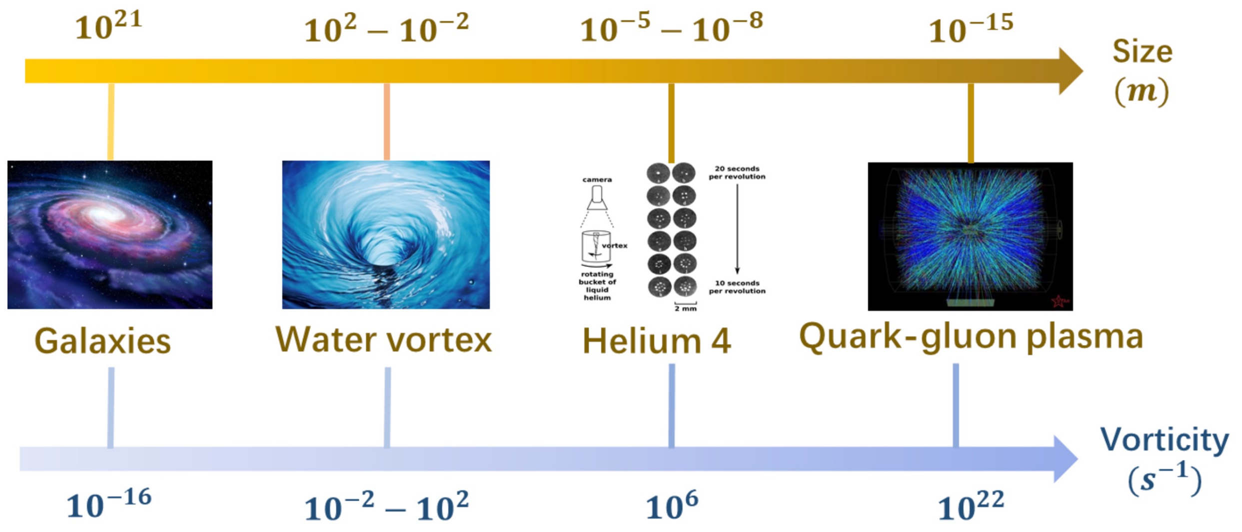
Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)

- **Estimation given by**

Becattini, Karpenko, Lisa, et. al, PRC (2017)

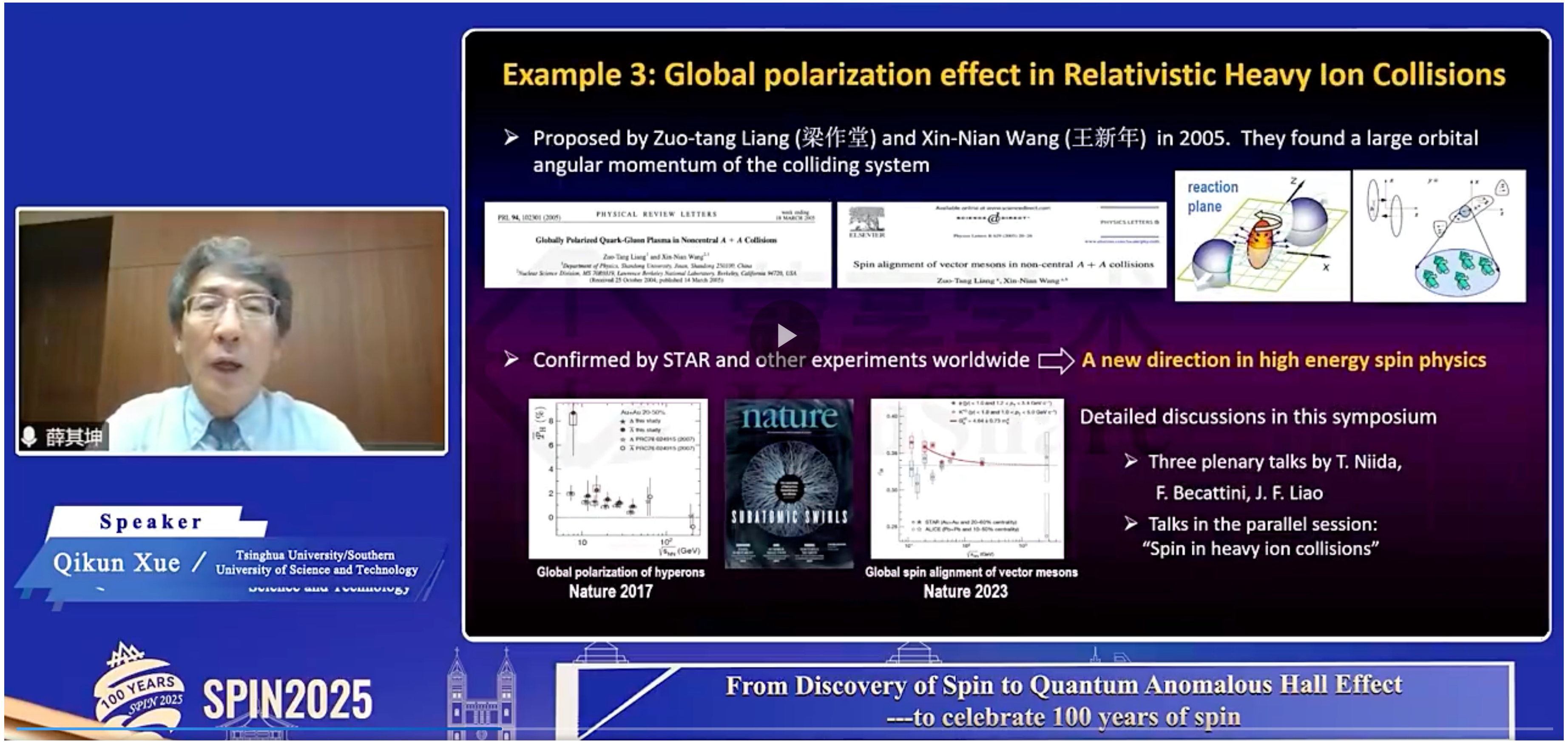
$$\mathbf{P}_{\Lambda} \simeq \frac{\omega}{2T} + \frac{\mu_{\Lambda}\mathbf{B}}{T}$$
$$\mathbf{P}_{\bar{\Lambda}} \simeq \frac{\omega}{2T} - \frac{\mu_{\Lambda}\mathbf{B}}{T}$$

- **ω = (9 ± 1)x10^{21}/s, greater than previously observed in any system.**
- **QGP is most vortical fluid so far.**



Spin in high energy physics in China

One of the most important discoveries in spin physics in China.
相对论重离子碰撞中的整体极化现象是中国高能自旋物理中最重要的发现之一。



Outline

- Extension of Bargmann-Michel-Telegdi equation
- New attractors in spin hydrodynamics (New)
- Summary

Recent developments on spin hydrodynamics

- Extension of Bargmann-Michel-Telegdi equation

A key question:

What is the evolution equation for spin?

How is it connected to well-known spin phenomena?

S. Fang, Kenji Fukushima, SP, D. L. Wang, accepted by PRL

Original Bargmann-Michel-Telegdi (BMT) equation

$$\dot{s}^\mu = \gamma \Delta^{\mu\rho} F_{\rho\nu} s^\nu - u^\mu s^\nu \dot{u}_\nu$$

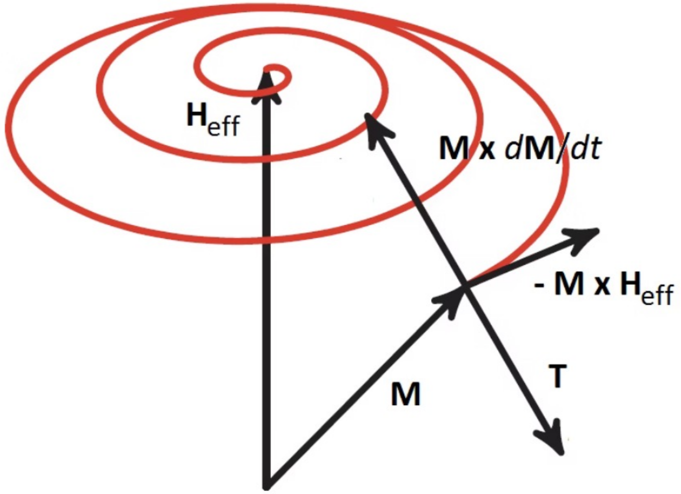
$\dot{a} = da/dt$

$\Delta^{\mu\rho} = g^{\mu\rho} - u^\mu u^\rho$

Spin-EM fields coupling

Thomas procession

The key to get the correct relativistic correction for hydrogen



+ possible relaxation (dissipative) effects

Landau-Lifshitz term (1935)

The key to make the final polarization (magnetization) along the direction of magnetic fields.

Textbook: Jackson, classical electrodynamics

Spin evolution in relativistic heavy ion collisions

What is the BMT equation for a relativistic many-body system in the presence of rotation (vorticity)?

Our strategy:

Conservation equations

Energy momentum

$$\partial_\mu \Theta^{\mu\nu} = 0$$

Charge number

$$\partial_\mu j^\mu = 0$$

Total angular momentum

$$\partial_\lambda J^{\lambda\mu\nu} = 0$$

+

Second law of thermodynamics

$$\partial_\mu \mathcal{S}^\mu \geq 0$$

Spin tensor

s^μ → Rest frame → $(0, \boldsymbol{s})$

Two different choices for spin tensor operators:

$$\Sigma^{\lambda\mu\nu} = \frac{i}{8}\bar{\psi}\gamma^\lambda[\gamma^\mu, \gamma^\nu]\psi$$

Anti-symmetric on $\mu\nu$,
NOT Hermitian
d.o.f for spin tensor is 6.

Commonly used in our field

↓ **Hermitian,**
Also can be derived by using EoM for feilds

$$\Sigma^{\lambda\mu\nu} = \frac{i}{8}\bar{\psi}\{\gamma^\lambda, [\gamma^\mu, \gamma^\nu]\}\psi$$

Total anti-symmetric,
Hermitian
d.o.f for spin tensor is 3.

Commonly used in many other fields
See cosmology textbook by Weinberger

Total anti-symmetry spin tensor

We introduce the total anti-symmetric spin tensor in spin hydrodynamics:

$$\Sigma^{\lambda\mu\nu} = u^{\lambda} S^{\mu\nu} + u^{\mu} S^{\nu\lambda} + u^{\nu} S^{\lambda\mu} + \mathcal{O}(\partial^1)$$

Spin density (tensor) $S^{\mu\nu} = -S^{\nu\mu}$ $S^{\mu\nu} u_{\nu} = 0$

Frenkel-Mathisson-Pirani condition (1926)

Spin density (vector) $s^{\mu} := -\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} u_{\nu} S_{\rho\sigma}$

Rest frame → $(0, \boldsymbol{s})$

Entropy production rate

$$
\begin{aligned}
\partial_\mu \mathcal{S}^\mu \quad = \quad & (h^\mu - \mathcal{H}\nu^\mu)(\partial_\mu \beta + \beta \dot{u}_\mu) \\
& + \beta \pi^{\mu\nu} \partial_{<\mu} u_{\nu>} + \phi^{\mu\nu}(2\beta\omega_{\mu\nu} + \partial_{[\mu}\beta u_{\nu]}) \\
& + 2\beta\omega_{\mu\nu} S^{\lambda\mu} \partial_\lambda u^\nu + q^\mu (\partial_\mu \beta - \beta \dot{u}_\mu) \\
& + \mathcal{O}(\partial^3)
\end{aligned}
$$

Most of the terms can easily be written as squared terms, but …

It is challenging to ensure that the entropy increases with the total antisymmetric spin tensor!

Hongo, Huang, Kaminski, Stephanov, Yee, JHEP 2022

Cao, Hattori, Hongo, Huang, Taya, PRD 2022

…..

Spin correction

Entropy production rate

$$\partial_\mu(\mathcal{S}^\mu + \delta\mathcal{S}^\mu) = (h^\mu - \mathcal{H}\nu^\mu + h_s^\mu)(\partial_\mu\beta + \beta\dot{u}_\mu) + \beta(\pi^{\mu\nu} + \pi_s^{\mu\nu})\partial_{(\mu}u_{\nu)} + (\phi^{\mu\nu} + \phi_s^{\mu\nu})(2\beta\omega_{\mu\nu} + \partial_{[\mu}\beta u_{\nu]}) + \mathcal{O}(\partial^3)$$

Second law of thermodynamics

New spin corrections

Normal dissipative terms from spin hydro

Heat flow	$h^\mu - \mathcal{H}\nu^\mu + h_s^\mu$	$=$	$-\sigma\Delta^{\mu\nu}(\partial_\nu\beta + \beta\dot{u}_\nu)\,,$
Viscous tensor	$\pi^{\mu\nu} + \pi_s^{\mu\nu}$	$=$	$\zeta\Delta^{\mu\nu}(\partial\cdot u) + \eta\partial^{<\mu}u^{\nu>}\,,$
Anti-symmetric part of energy Momentum tensor	$\phi^{\mu\nu} + \phi_s^{\mu\nu}$	$=$	$\gamma_\phi\Delta^{\mu\rho}\Delta^{\nu\sigma}(2\beta\omega_{\rho\sigma} - \Omega_{\rho\sigma})\,,$

Non-relativistic limit

In non-relativistic limit,

Charge current

$$
\begin{aligned}
\boldsymbol{j} \; = & -\frac{1}{2}(\nabla \times \boldsymbol{s}) \\
& +\frac{1}{2}(\dot{\boldsymbol{s}} \times \boldsymbol{v}) + (1+\xi)(\boldsymbol{s} \times \dot{\boldsymbol{v}}) \\
& +\frac{\xi}{T}(\boldsymbol{s} \times \nabla T) \\
& +\frac{\xi}{T}\dot{T}(\boldsymbol{s} \times \boldsymbol{v}) \\
& +\mathcal{O}(v^2)
\end{aligned}
$$

$-\frac{1}{2}(\nabla \times \boldsymbol{s})$ → **inverse spin Hall effect**

Sinova, Valenzuela, Wunderlich, Back, Jungwirth, Rev. Mod. Phys. 87, 1213 (2015)

$+\frac{\xi}{T}(\boldsymbol{s} \times \nabla T)$ → **anomalous Hall effect**

Nagaosa, Sinova, Onoda, MacDonald, Ong, Rev. Mod. Phys. 82, 1539 (2010).

Extension of BMT equations – Thomas procession

Original BMT equation:

$$\dot{s}^{\mu} = \gamma \Delta^{\mu\rho} F_{\rho\nu} s^{\nu} \boxed{- u^{\mu} s^{\nu} \dot{u}_{\nu}}$$

Thomas procession

+ possible relaxation (dissipative) terms

Extension of BMT equation:

$$\begin{aligned}\dot{s}^{\mu} \quad = \quad & \boxed{-u^{\mu} s^{\nu} \dot{u}_{\nu}} \\ & +(\varepsilon^{\mu\nu\rho\sigma} s_{\nu} u_{\rho} - 2\beta\gamma_{\phi} g^{\mu\sigma})(2\omega_{\sigma} - \mathfrak{w}_{\sigma}) \\ & -s_{\nu}\partial^{<\mu} u^{\nu>} - \left(\frac{1}{3} + 2v_{n}^{2}\right) s^{\mu} (\partial \cdot u) \,,\end{aligned}$$

Thomas procession

Extension of BMT equations – Spin-EM coupling

Original BMT equation:

$$\dot{s}^{\mu} = \gamma\Delta^{\mu\rho}F_{\rho\nu}s^{\nu} - u^{\mu}s^{\nu}\dot{u}_{\nu}$$

$$H = -\gamma \boldsymbol{B}\cdot\boldsymbol{s}$$

$$\frac{\partial \boldsymbol{s}}{\partial t} = -\gamma\boldsymbol{B}\times\boldsymbol{s}$$

Spin-EM fields coupling

+ possible relaxation (dissipative) terms

Extension of BMT equation:

Spin-vorticial fields coupling

$$H_{\omega} = -\boldsymbol{\omega}\cdot\boldsymbol{s}$$

$$\begin{aligned}\dot{s}^{\mu} =& -u^{\mu}s^{\nu}\dot{u}_{\nu} \\ &+(\varepsilon^{\mu\nu\rho\sigma}s_{\nu}u_{\rho} - 2\beta\gamma_{\phi}g^{\mu\sigma})(2\omega_{\sigma} - \mathfrak{w}_{\sigma}) \\ &-s_{\nu}\partial^{<\mu}u^{\nu>} - \left(\frac{1}{3}+2v_n^2\right)s^{\mu}(\partial\cdot u)\,,\end{aligned}$$

Extension of BMT equations – Killing condition

Original BMT equation:

$$\dot{s}^\mu = \gamma \Delta^{\mu\rho} F_{\rho\nu} s^\nu - u^\mu s^\nu \dot{u}_\nu$$

+ possible relaxation (dissipative) terms

Extension of BMT equation:

Thermal vorticity combined with spin chemical potential (Killing condition)

$$\begin{aligned}\dot{s}^\mu &= -u^\mu s^\nu \dot{u}_\nu \\ &\quad + (\varepsilon^{\mu\nu\rho\sigma} s_\nu u_\rho - 2\beta\gamma_\phi g^{\mu\sigma})(2\omega_\sigma - \mathfrak{w}_\sigma) \\ &\quad - s_\nu \partial^{<\mu} u^{\nu>} - \left(\frac{1}{3} + 2v_n^2\right) s^\mu (\partial \cdot u)\,,\end{aligned}$$

Extension of BMT equations – Dissipative effects

Original BMT equation:

$$\dot{s}^{\mu} = \gamma\Delta^{\mu\rho}F_{\rho\nu}s^{\nu} - u^{\mu}s^{\nu}\dot{u}_{\nu}$$

+ possible relaxation (dissipative) terms

Extension of BMT equation:

$$\begin{aligned}\dot{s}^{\mu} &= -u^{\mu}s^{\nu}\dot{u}_{\nu} \\ &\quad +(\varepsilon^{\mu\nu\rho\sigma}s_{\nu}u_{\rho} - 2\beta\gamma_{\phi}g^{\mu\sigma})(2\omega_{\sigma} - \mathfrak{w}_{\sigma}) \\ &\quad -s_{\nu}\partial^{<\mu}u^{\nu>} - \left(\frac{1}{3} + 2v_n^2\right)s^{\mu}(\partial\cdot u),\end{aligned}$$

Spin coupled to shear tensor, bulk pressure and other dissipative effects

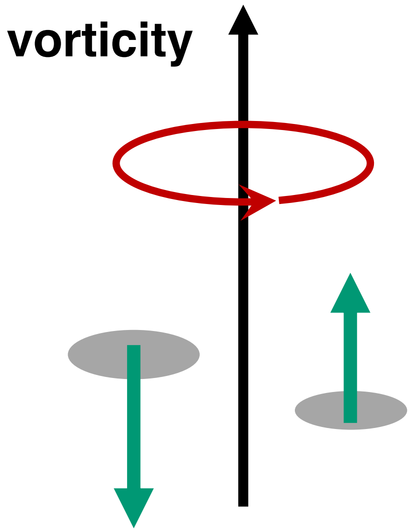
Equilibrium

In global equilibrium,

$$\begin{aligned}\dot{s}^{\mu} &= -u^{\mu}s^{\nu}\dot{u}_{\nu} \\ &+(\varepsilon^{\mu\nu\rho\sigma}s_{\nu}u_{\rho}-2\beta\gamma_{\phi}g^{\mu\sigma})(2\omega_{\sigma}-\mathfrak{w}_{\sigma}) \\ &-s_{\nu}\partial^{<\mu}u^{\nu>}-\left(\frac{1}{3}+2v_{n}^{2}\right)s^{\mu}(\partial\cdot u),\end{aligned}$$

$$\Rightarrow \quad \mathfrak{w}^{\mu}s^{\nu}-\mathfrak{w}^{\nu}s^{\mu}=0$$

Spin is parallel to vorticity in equilibrium.



What if we adopt the commonly-used spin tensor?

$$\Sigma^{\lambda\mu\nu} = \frac{i}{8}\bar{\psi}\gamma^{\lambda}[\gamma^{\mu},\gamma^{\nu}]\psi$$

Anti-symmetric on $\mu\nu$,
NOT Hermitian
d.o.f for spin tensor is 6.

Commonly-used in our field

When the commonly used spin tensor is adopted, the spin-vorticity term is absent. Less is more?

$$\dot{s}^{\mu} = -u^{\mu}s^{\nu}\dot{u}_{\nu} + 2\varepsilon^{\mu\nu\rho\sigma}s_{\nu}u_{\rho}\omega_{\sigma} + \ldots$$

Thomas procession

Spin-vorticial fields coupling

$$H_{\omega} = -\boldsymbol{\omega}\cdot\boldsymbol{s}$$

Our work

$$\Sigma^{\lambda\mu\nu} = \frac{i}{8}\bar{\psi}\{\gamma^{\lambda},[\gamma^{\mu},\gamma^{\nu}]\}\psi$$

Total anti-symmetric,
Hermitian
d.o.f for spin tensor is 3.

Commonly used in many other fields
See cosmology textbook by Weinberger

Recent developments on spin hydrodynamics

- Attractors in spin hydrodynamics

Gui-Hui Li, Xiang Ren, D. L. Wang, SP, arXiv: 2603.27182

Two issues in spin hydrodynamics (1)

Energy-momentum conservation
Total angular momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\lambda J^{\lambda\mu\nu} = 0$$

$$J^{\lambda\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu} + \Sigma^{\lambda\mu\nu}$$

Orbital angular momentum | Spin

$$\partial_\lambda \Sigma^{\lambda\mu\nu} = -2(T^{\mu\nu} - T^{\nu\mu})$$

Spin (density) is not a conserved quantity.
(a) The contribution of spin density on the freeze-out hypersurface in the final state may be negligible.
(b) Introducing spin density into hydrodynamics is unphysical and lacks clear physical meaning.

Typical decaying behavior as function of proper time	
Spin density	exponential X power-law
number density	power-law

Two issues in spin hydrodynamics (2)

- **The initial conditions for spin density are not well established.**

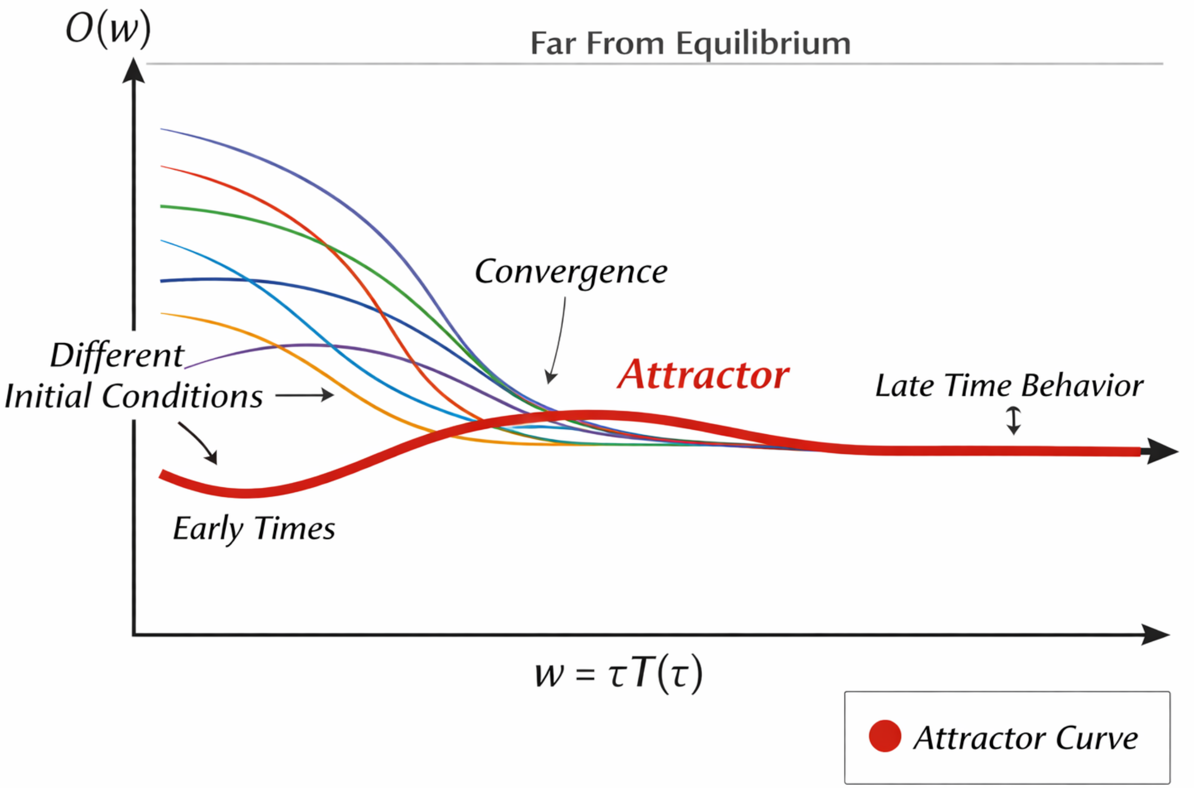
- **We do not yet know how to estimate the magnitude of the spin density in the initial state.**

- **But, what if spin hydrodynamics have attractors?**

Attractors in relativistic hydrodynamics

- **The attractor tells us that the system can become effectively describable by a few macroscopic variables much earlier than full local equilibrium would suggest.**

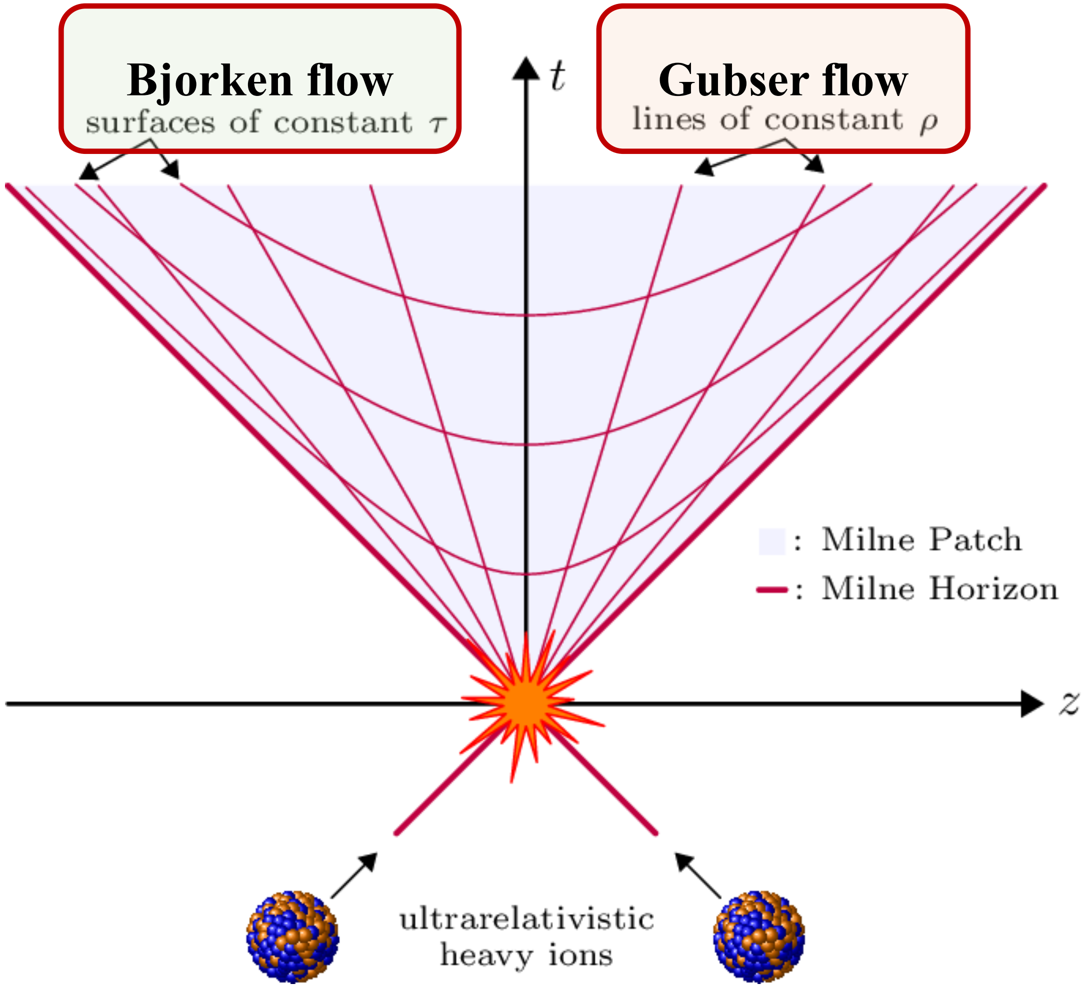
- **The attractor framework provides a theoretical explanation:**
  - **nonhydrodynamic modes decay,**
  - **solutions are pulled toward the attractor,**
  - **once near the attractor, the evolution is effectively universal,**
  - **full microscopic memory becomes much less important.**



Heller, Spalinski, PRL (2015) PRL (2020); Romatschke PRL (2018); Romatschke, Yan, PLB (2018); Denicol, Noronha, PRD (2018); Strickland, JHEP (2018); Almaalol, Kurkela, Strickland PRL (2020), …..

Introduction to Gubser flow

- **Gubser flow:** a 1+1 dimensional analytic solution for relativistic hydrodynamics with **transverse flows**



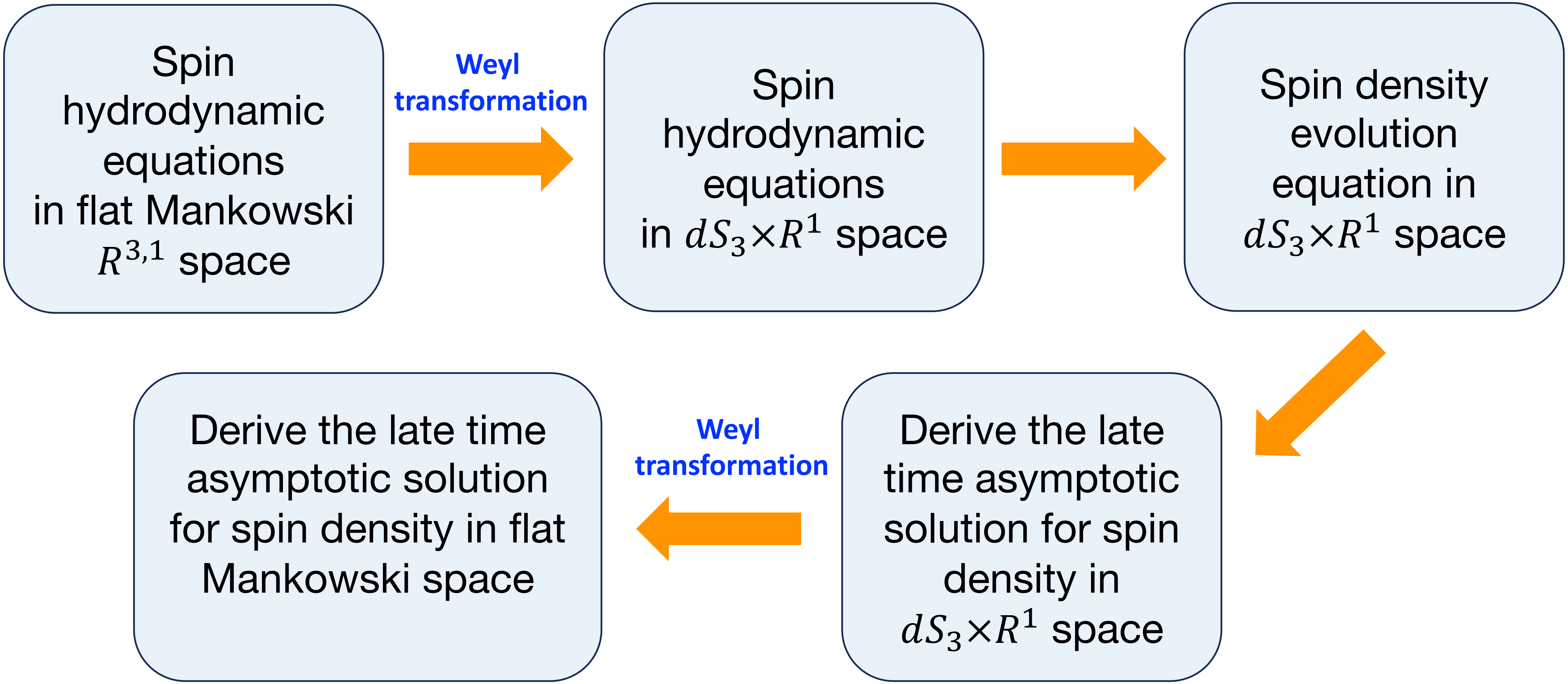
- **Motivation:**
  Extended Bjorken flow with radial flow

- **Basic idea:**
  Search for the analytic solution as $u^{\mu} = (1,0,0,0)$ in special coordinates?

- **Answer:**
  Taking conformal (Weyl) transformation

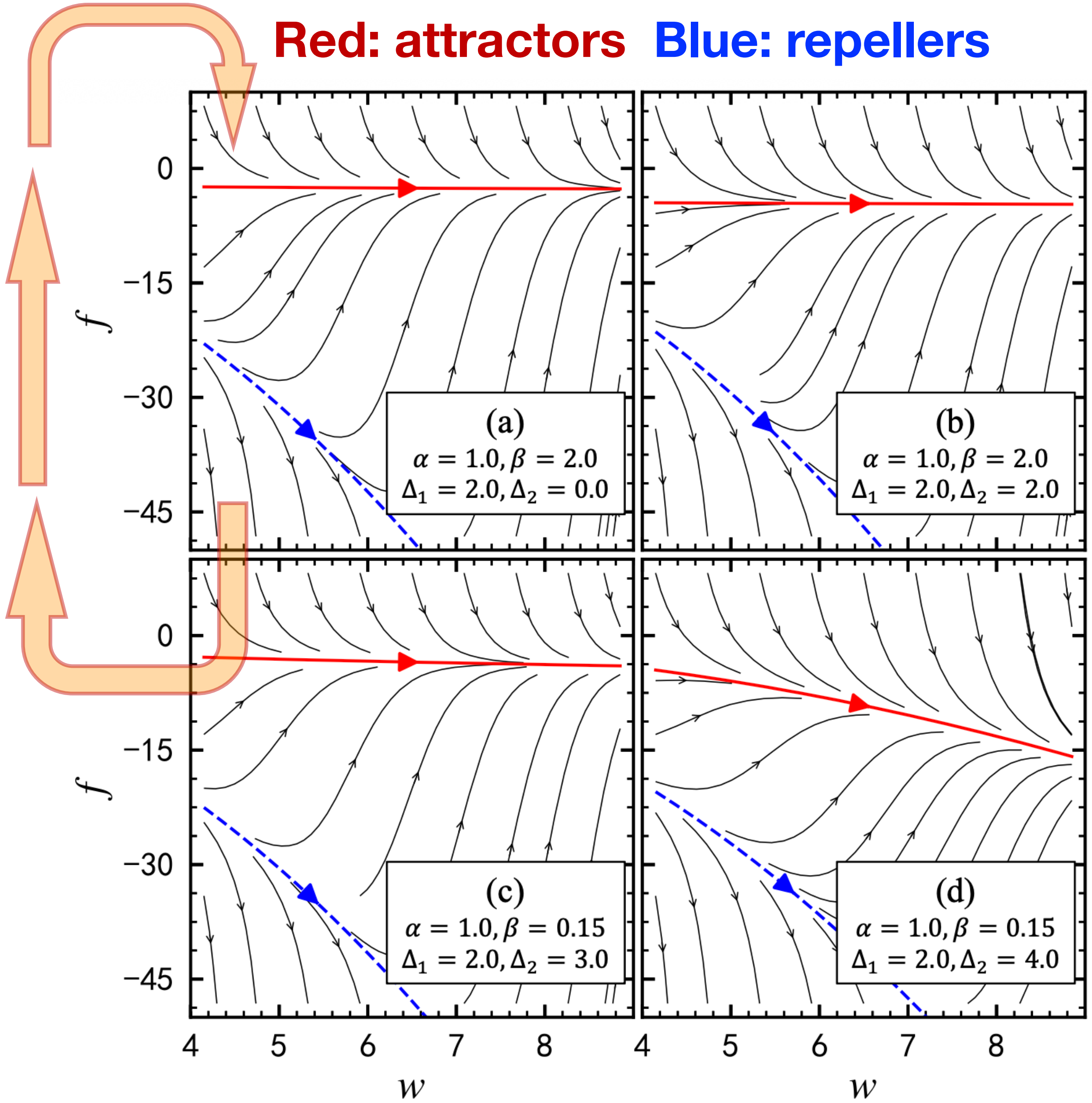
Gubser, PRD (2010); Gubser, Yarom, NPB (2011)

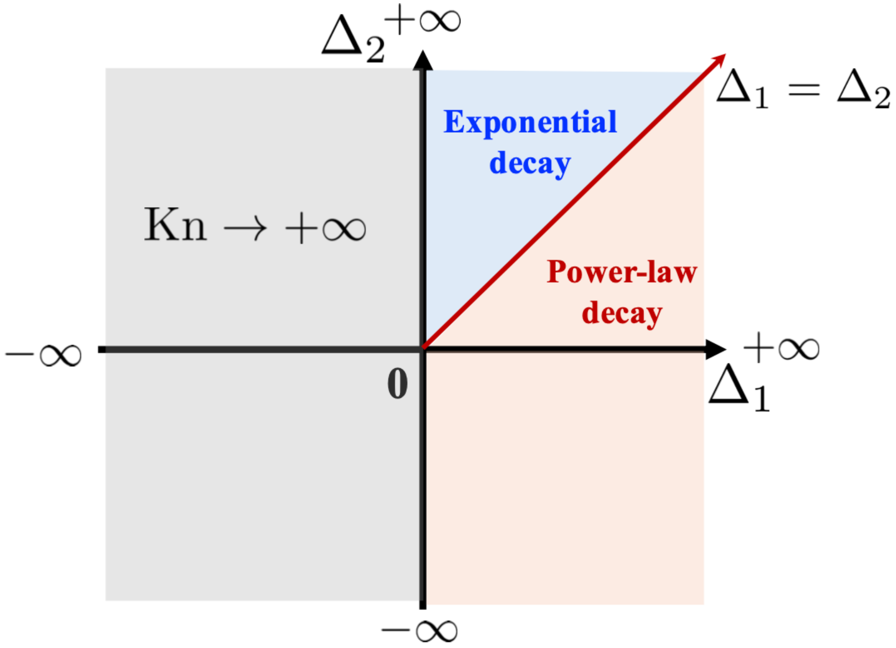
Workflow


Spin hydrodynamic equations in flat Mankowski $R^{3,1}$ space
Weyl transformation
Spin hydrodynamic equations in $dS_3 \times R^1$ space
Spin density evolution equation in $dS_3 \times R^1$ space
Derive the late time asymptotic solution for spin density in $dS_3 \times R^1$ space
Weyl transformation
Derive the late time asymptotic solution for spin density in flat Mankowski space

Attractors in spin hydrodynamics in Gubser flow

Red: attractors **Blue: repellers**





$$L \gg \tau,\ S^{tx}, S^{ty}, S^{zx}, S^{zy} \sim \frac{1}{\tau L^2}$$

$$\tau \gg L,\ S^{tx}, S^{ty}, S^{zx}, S^{zy} \sim \frac{L^4}{\tau^7}.$$

L: size of the system

Gui-Hui Li, Xiang Ren, D. L. Wang, SP, arXiv: 2603.27182

Summary

Summary

- **Spin hydrodynamics: extension of BMT equation**

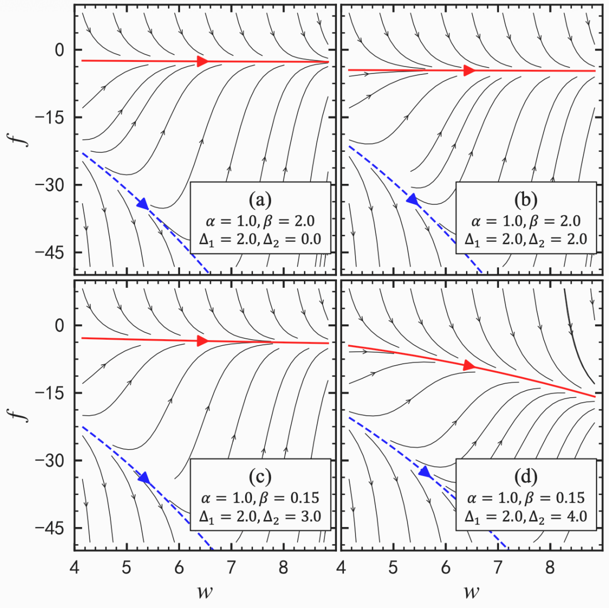
Thomas procession

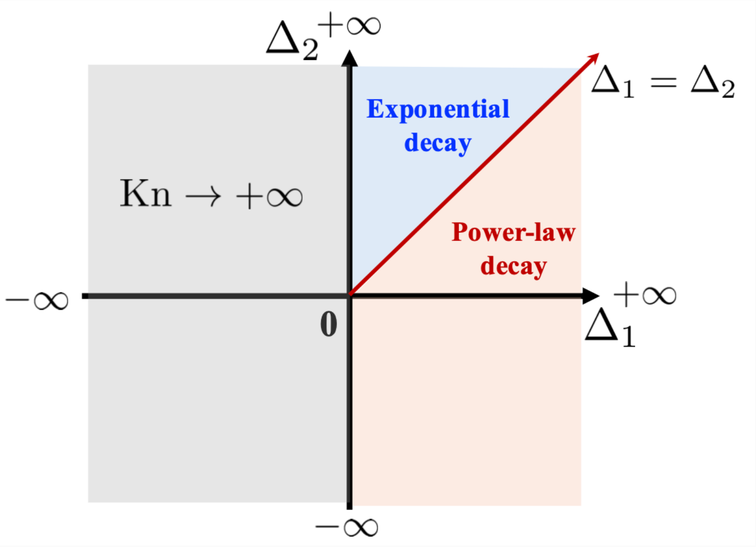
Spin-vorticial fields coupling

Killing condition

$$\dot{s}^{\mu} = -u^{\mu} s^{\nu} \dot{u}_{\nu} + \left(\varepsilon^{\mu\nu\rho\sigma} s_{\nu} u_{\rho} - 2\beta\gamma_{\phi} g^{\mu\sigma}\right)\left(2\omega_{\sigma} - \varpi_{\sigma}\right) - s_{\nu}\partial^{<\mu}u^{\nu>} - \left(\frac{1}{3} + 2v_n^2\right) s^{\mu}(\partial \cdot u)\,,$$

Spin coupled to shear tensor, bulk pressure and other dissipative effects



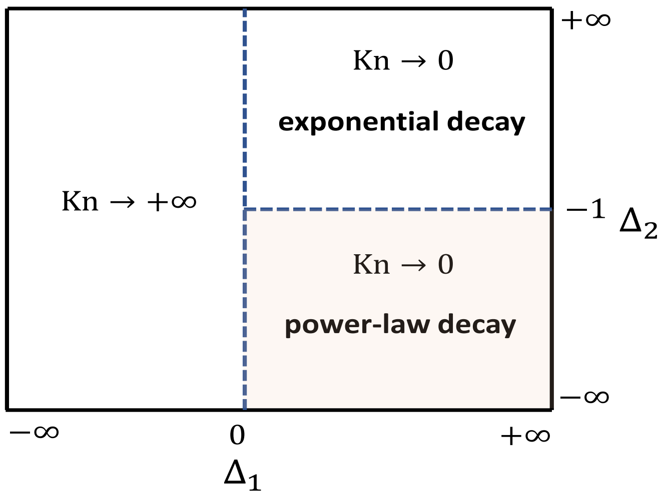


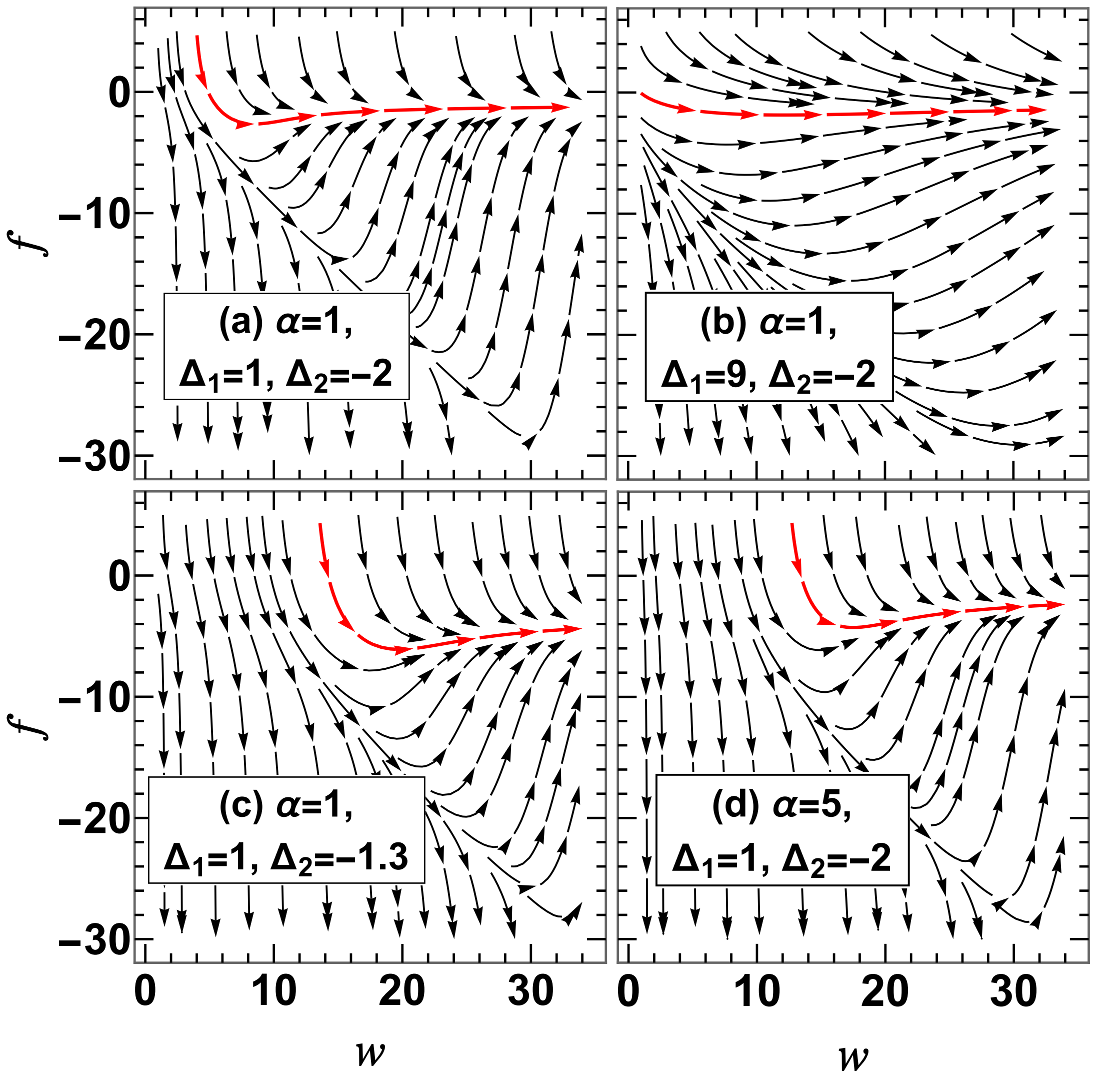
We derive the attractors and repellers in spin hydrodynamics in Gubser flow.

感谢各位专家！
欢迎批评指正！

Test for spin hydrodynamics in Bjorken flow

$w \to +\infty$	
$\Delta_2 > 0$	$S_{(1),(2)} \propto e^{-w/(2\Delta_1)}$
$\Delta_2 = 0$	$S_{(1),(2)} \propto e^{-w(1\pm\sqrt{1-32\alpha})/(2\Delta_1)}$
$-1 < \Delta_2 < 0$	$S_{(1)} \propto e^{-w/\Delta_1}$, $S_{(2)} \propto \exp\left[-\frac{8\alpha w^{1+\Delta_2}}{\Delta_1(1+\Delta_2)}\right]$
$\Delta_2 = -1$	$S_{(1)} \propto e^{-w/\Delta_1}$, $S_{(2)} \sim w^{-(1+8\alpha)/\Delta_1}$
$\Delta_2 < -1$	$S_{(1)} \propto e^{-w/\Delta_1}$, $S_{(2)} \sim w^{-1/\Delta_1}$





$$f(w) \equiv \frac{\tau}{S^{xy}} \frac{dS^{xy}}{d\tau},$$

- **We observe the late time attractors in spin hydrodynamics in Bjorken flow.**
- **We also find the power law decaying of spin density.**

D.L. Wang, Y. Li, SP, PRD (2025)