

2026

April 24-28

极端核物质前沿研讨会

Workshop on extreme nuclear matter frontiers

Learning the heavy-quark potential in the QCD matter

施舒哲 清华大学



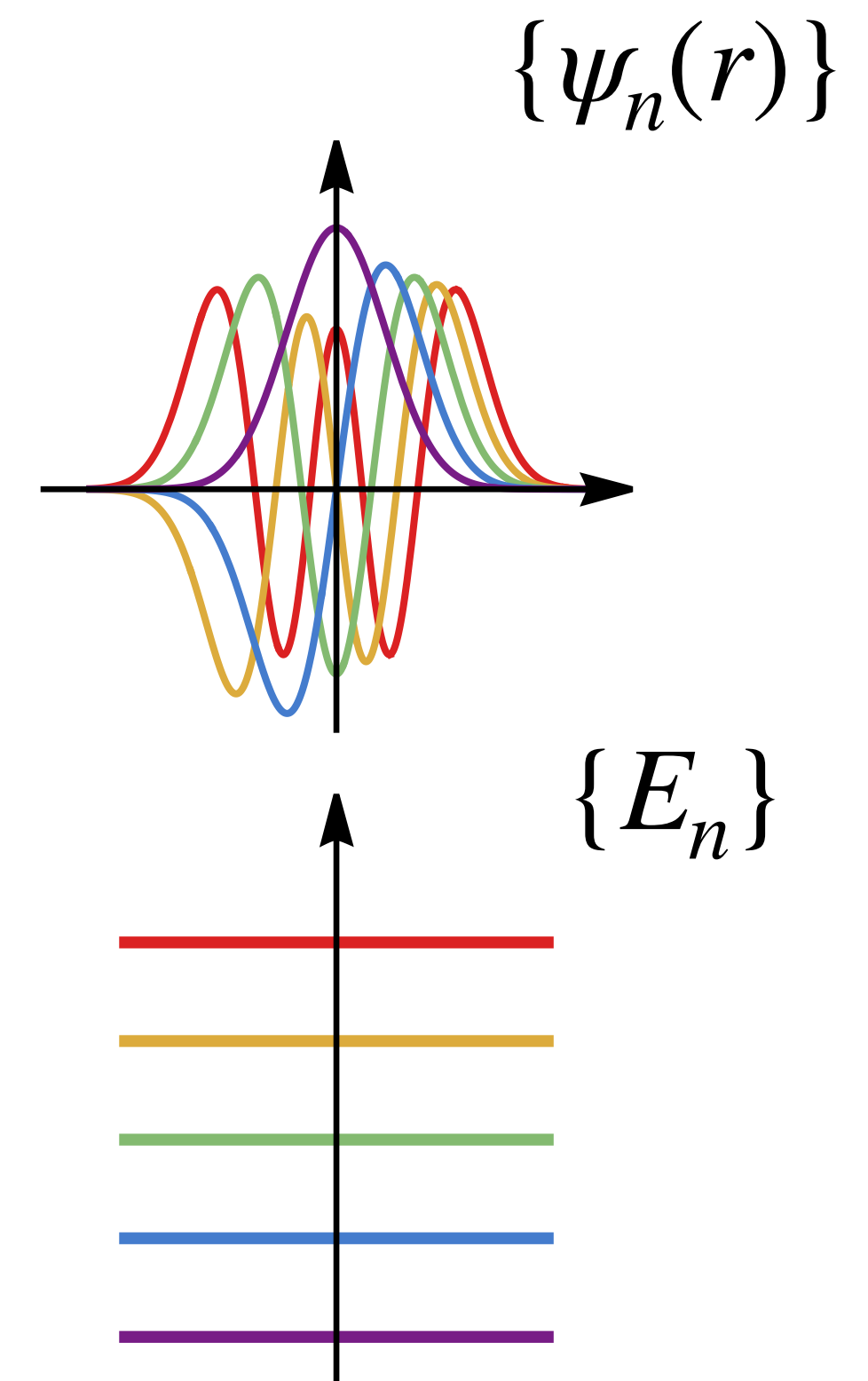
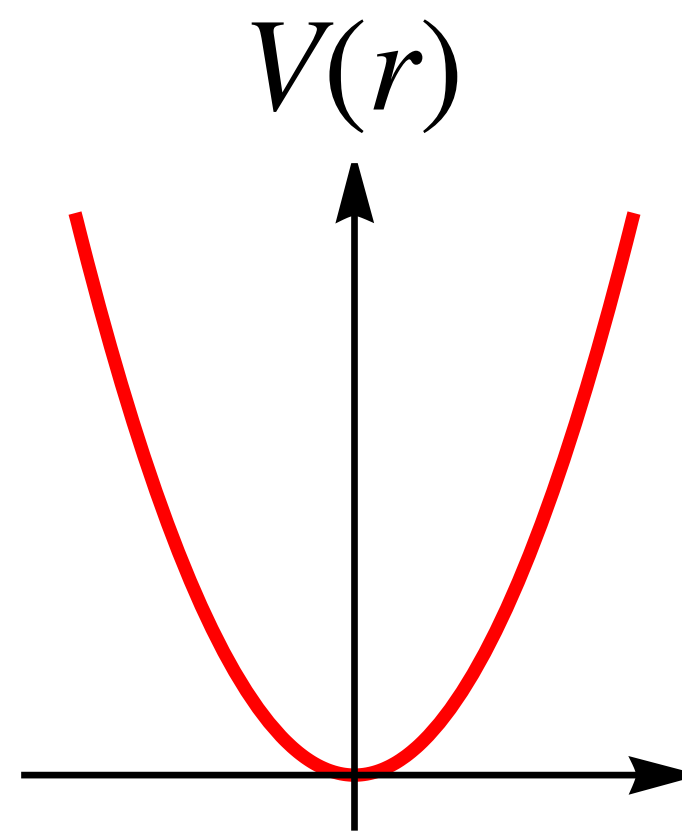
清华大学

Tsinghua University

SS, 周凯, 赵佳星, Mukherjee, **Zhuang**, PhysRevD.105.014017

1. Schrödinger Equation

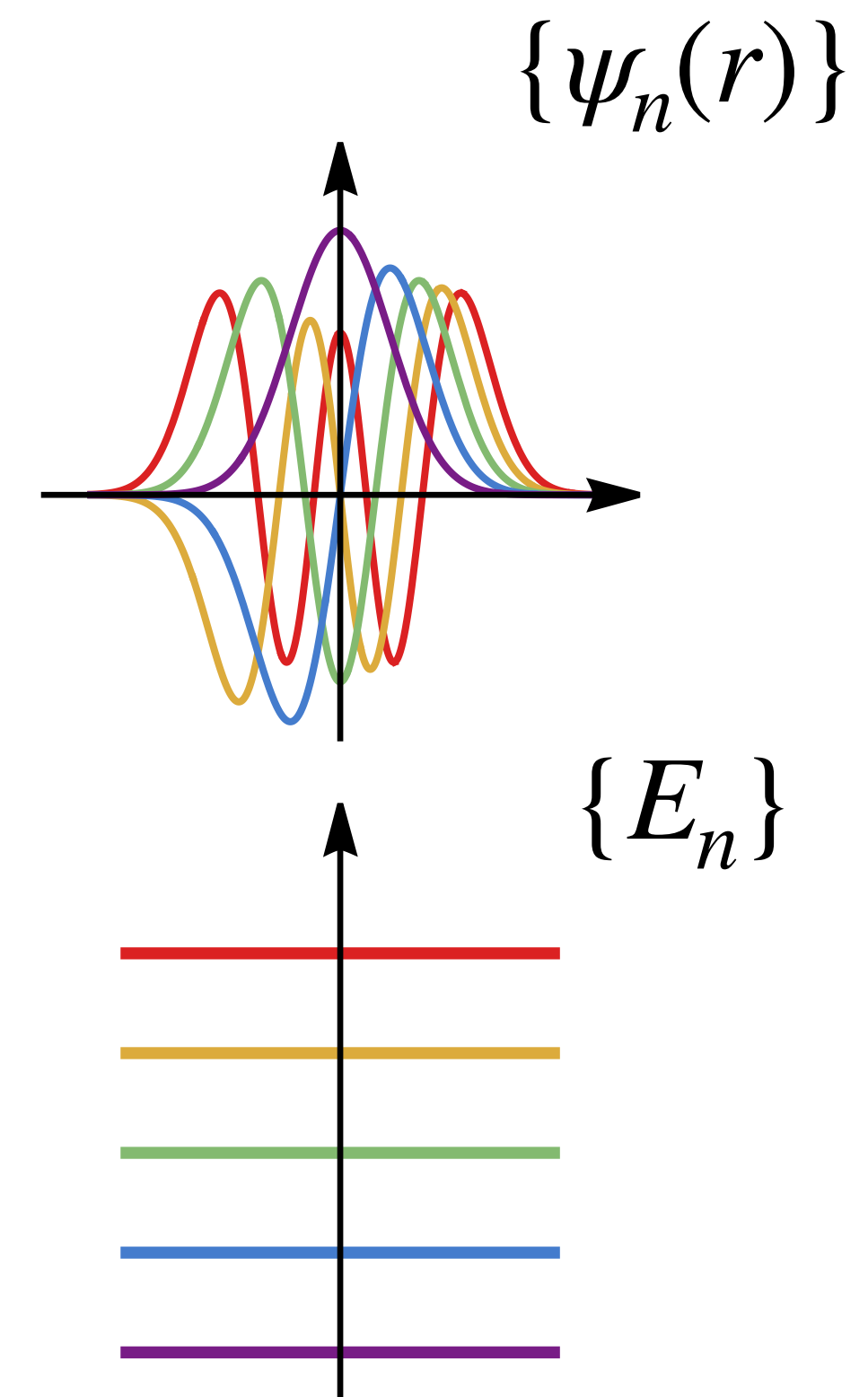
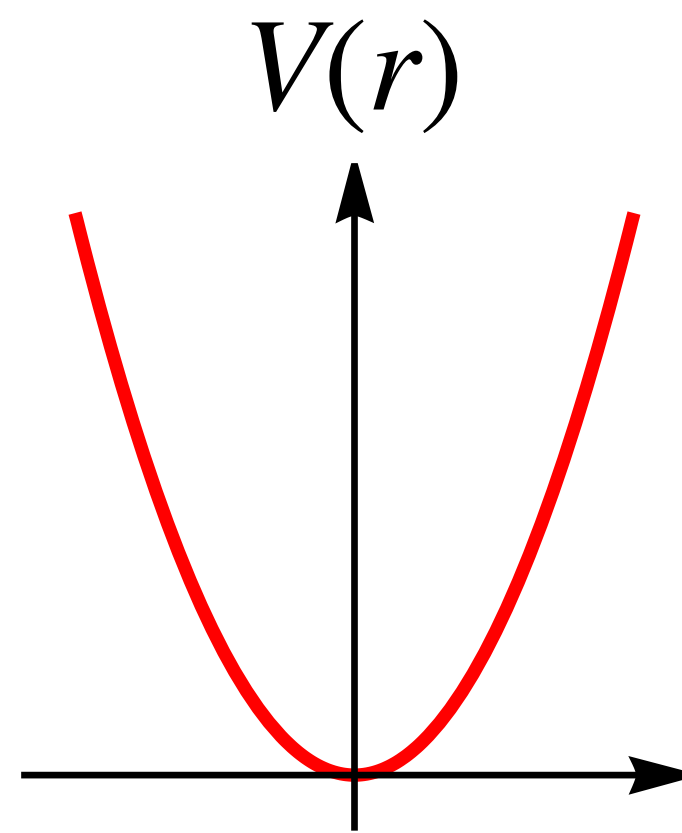
$$\hat{H} \psi_n = -\frac{\nabla^2}{2m} \psi_n + V(r) \psi_n = E_n \psi_n$$



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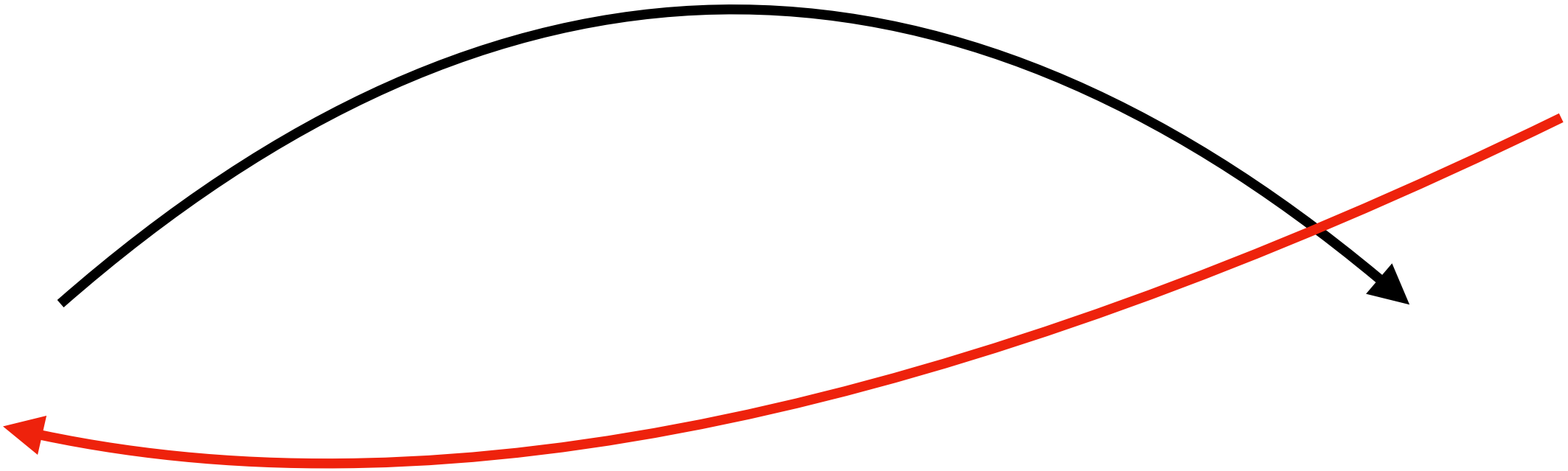
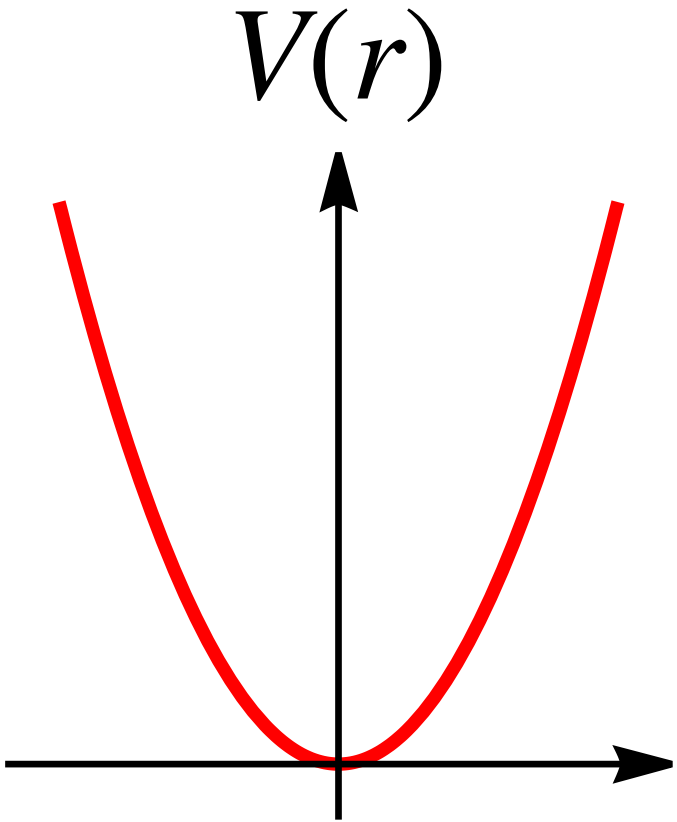
$V(r)$ known $\Rightarrow \{E_n, \psi_n(r)\}$:
numerical methods established.



1. Schrödinger Equation

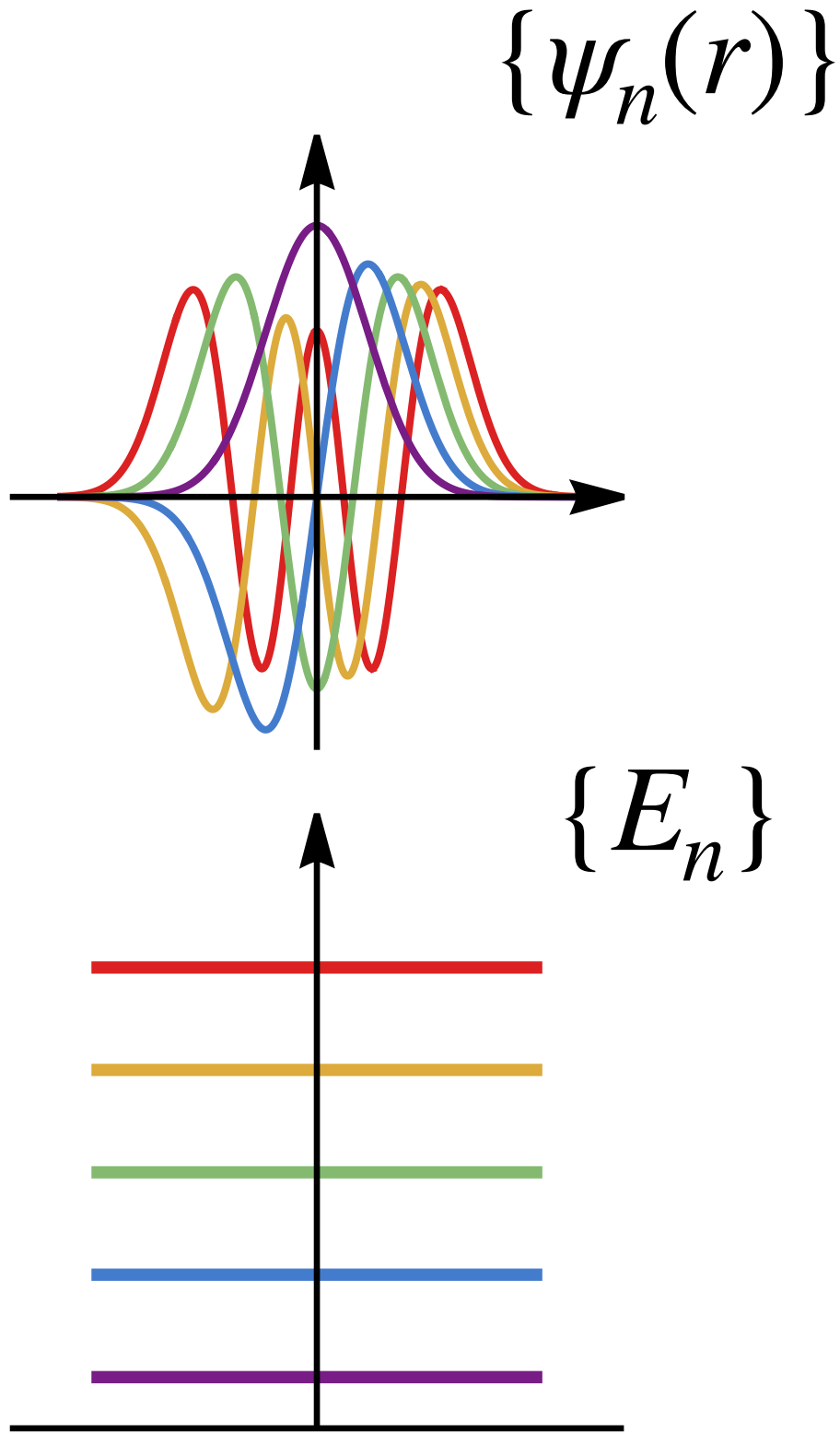
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$\psi_n(r)$ known $\Rightarrow V(r)$:

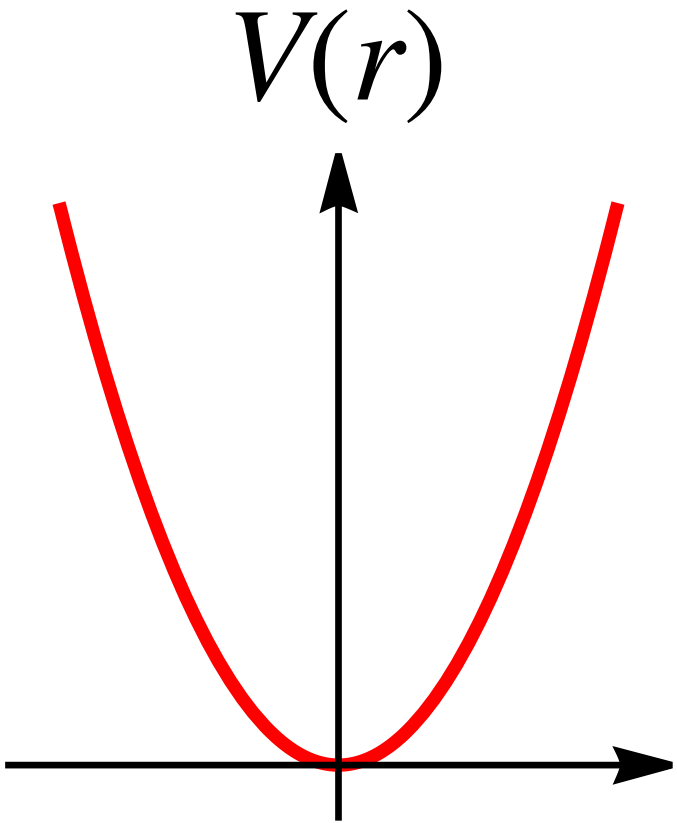
$$\frac{\nabla^2 \psi_n}{2m \psi_n} = V(r) - E_n$$



1. Schrödinger Equation

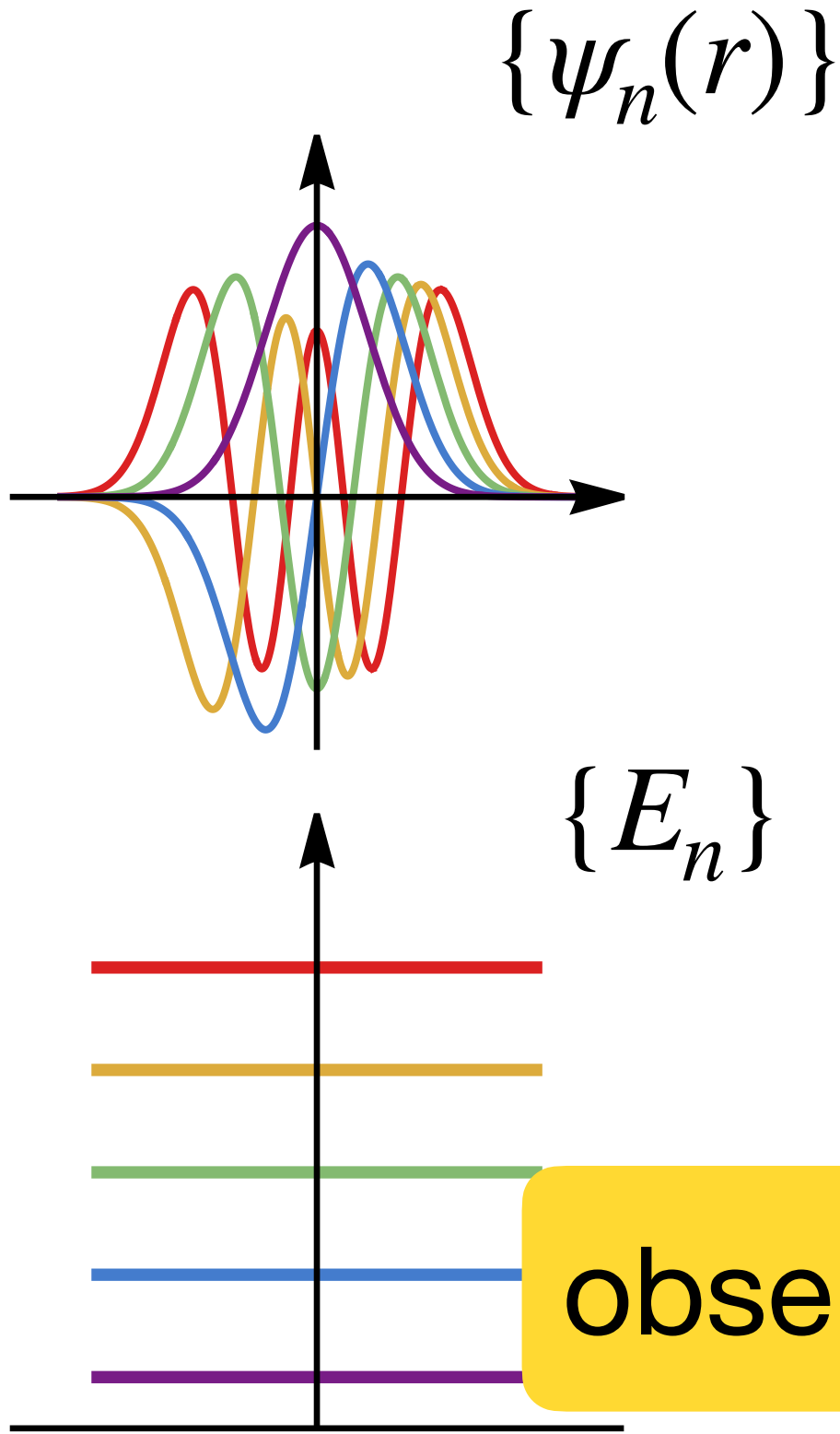
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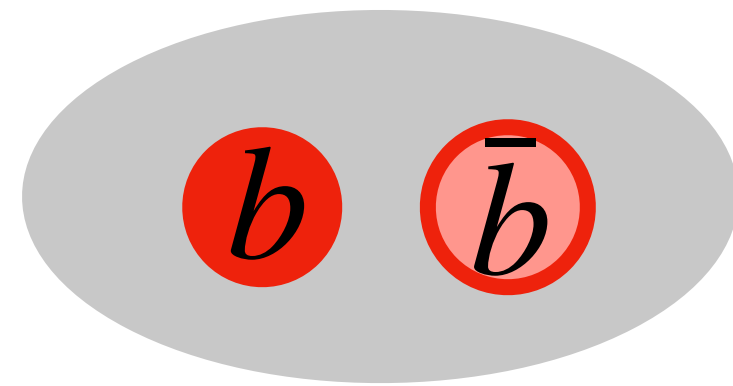
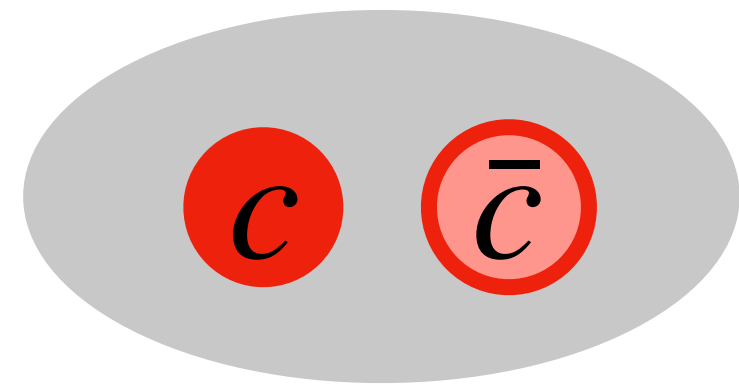
information of interest

$\{E_n\}$ known $\Rightarrow V(r)$: ???



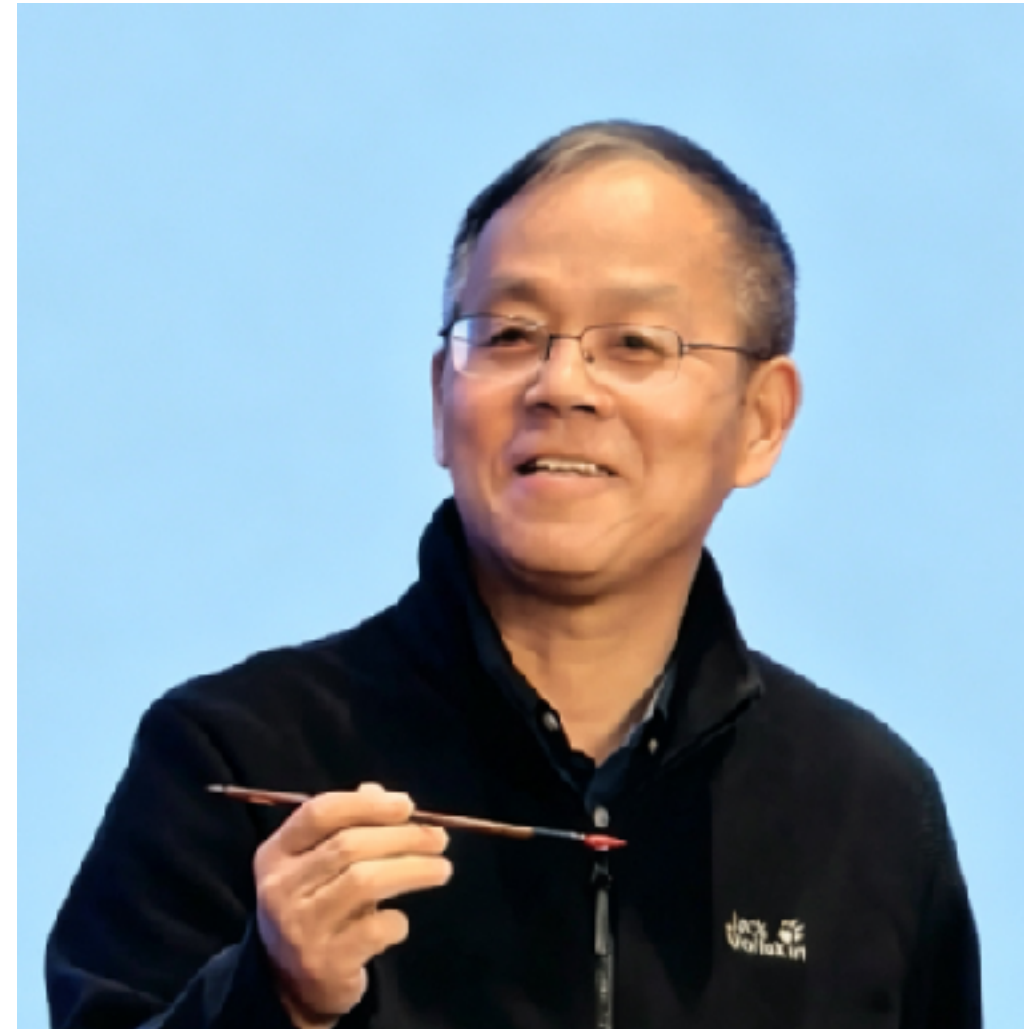
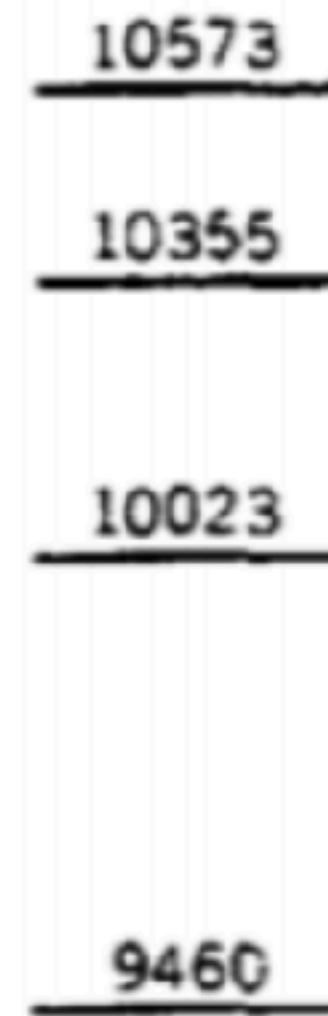
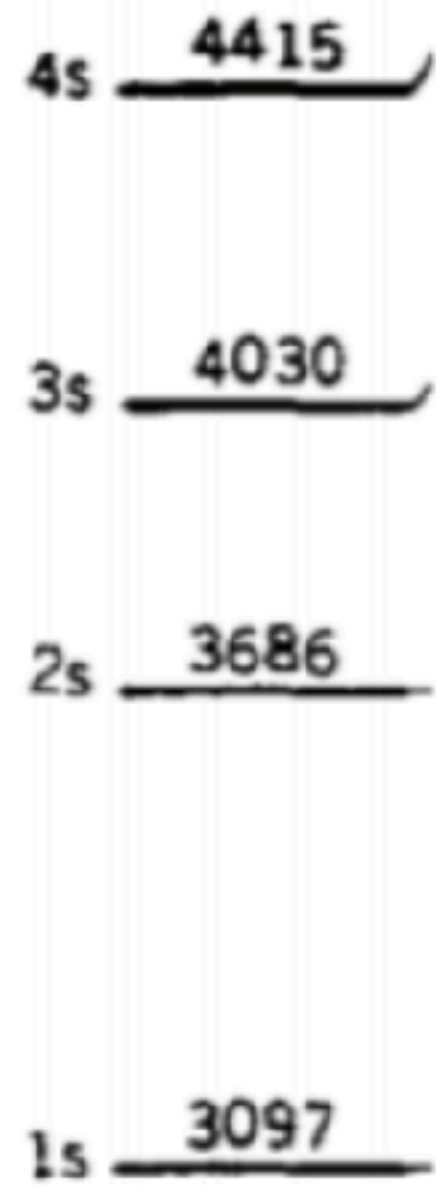
observations

Realistic Example: Quarkonium Spectrum



Charmonium levels

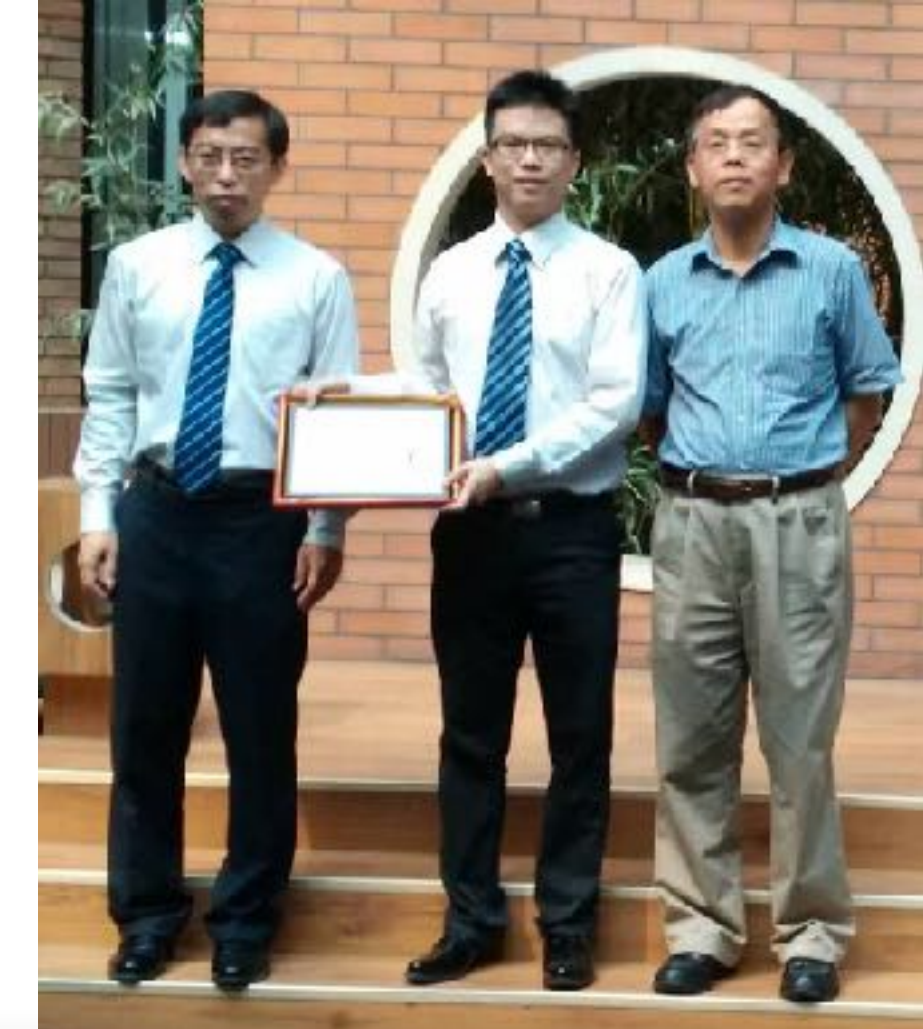
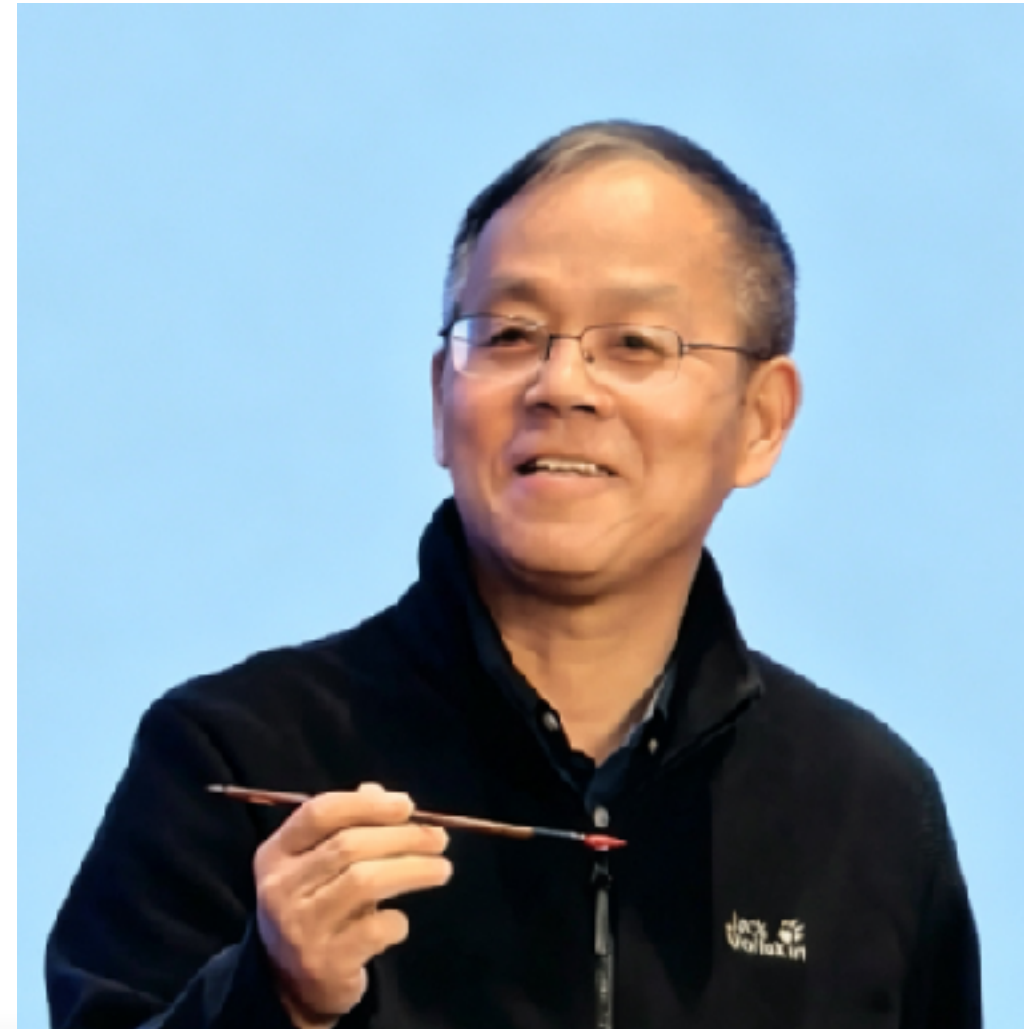
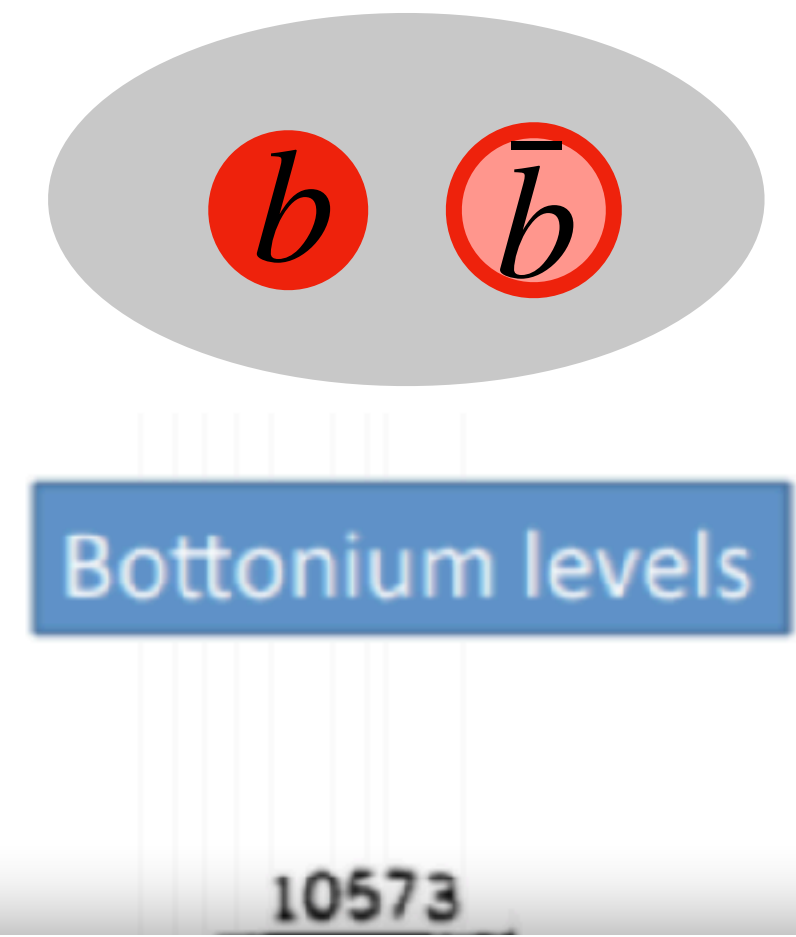
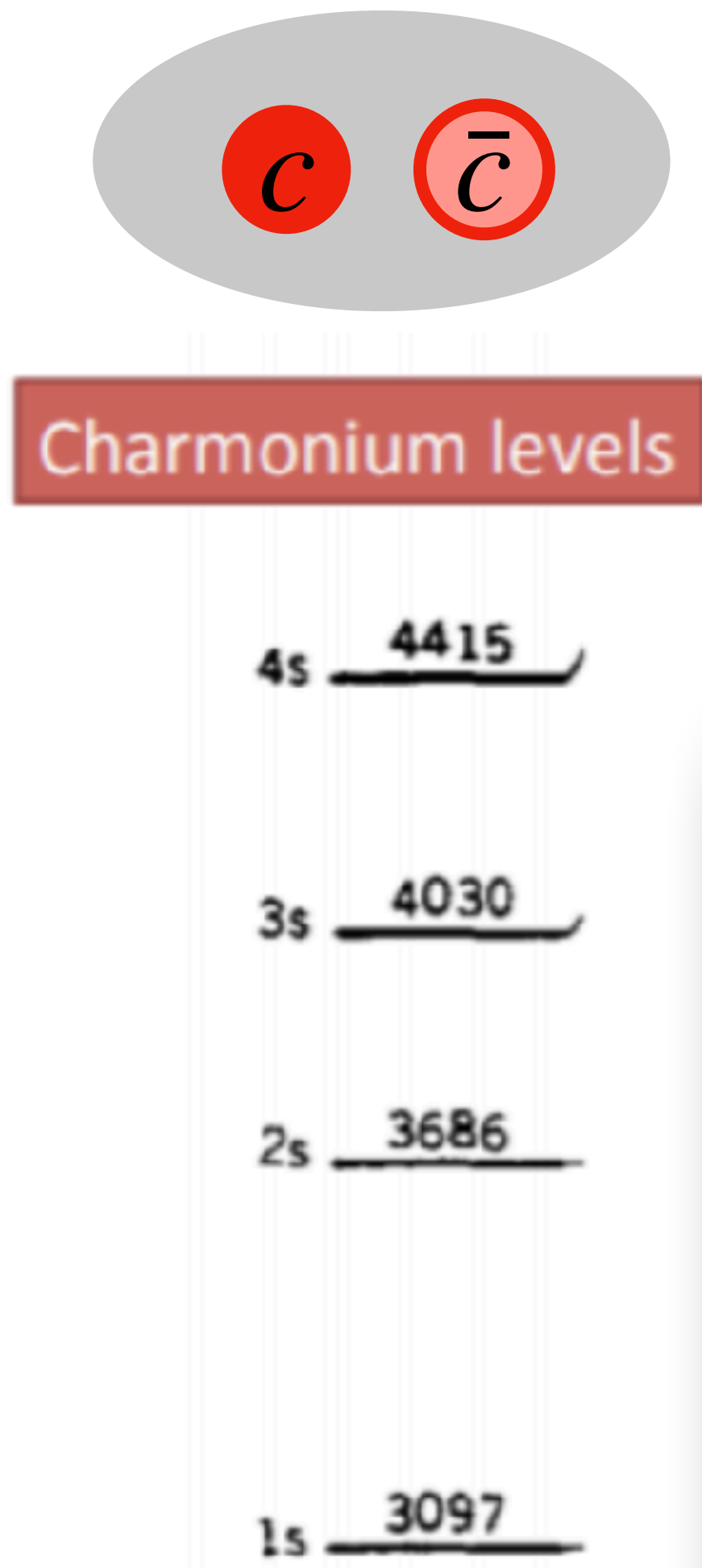
Bottomonium levels



Cornell Potential


$$V(r) = -\frac{\alpha}{r} + \sigma r$$

Realistic Example: Quarkonium Spectrum



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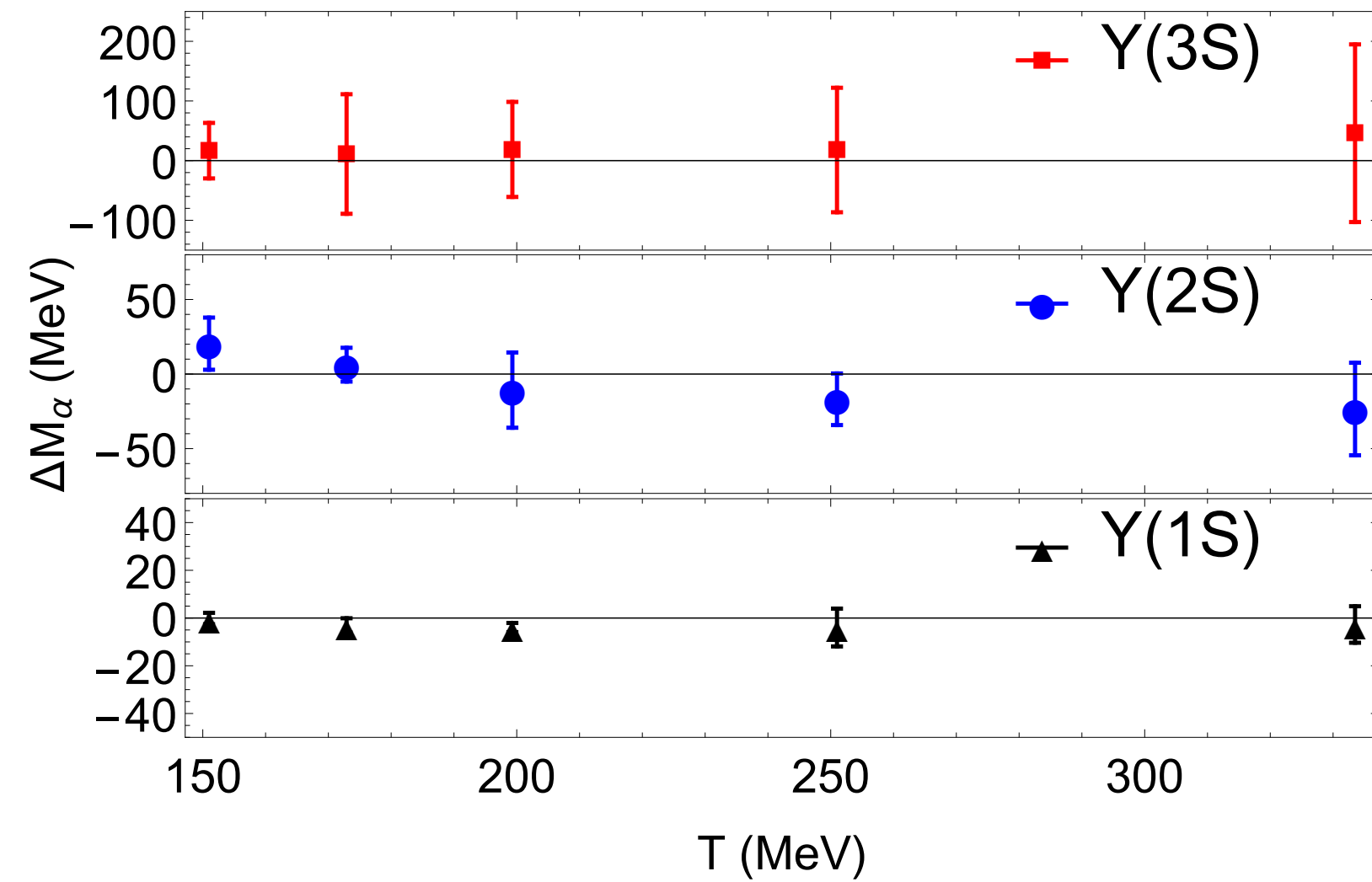
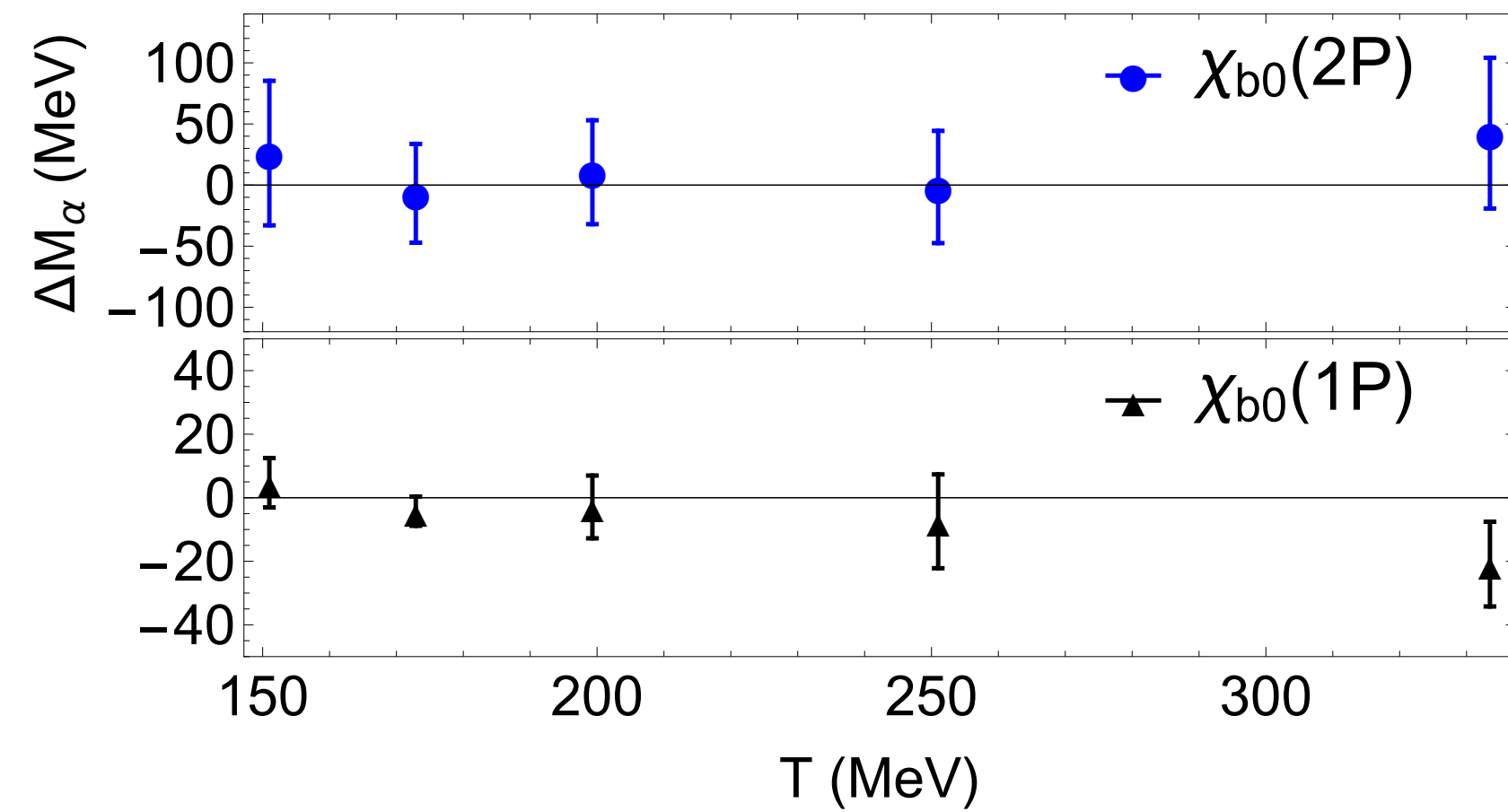


Relativistic correction to charmonium dissociation temperature

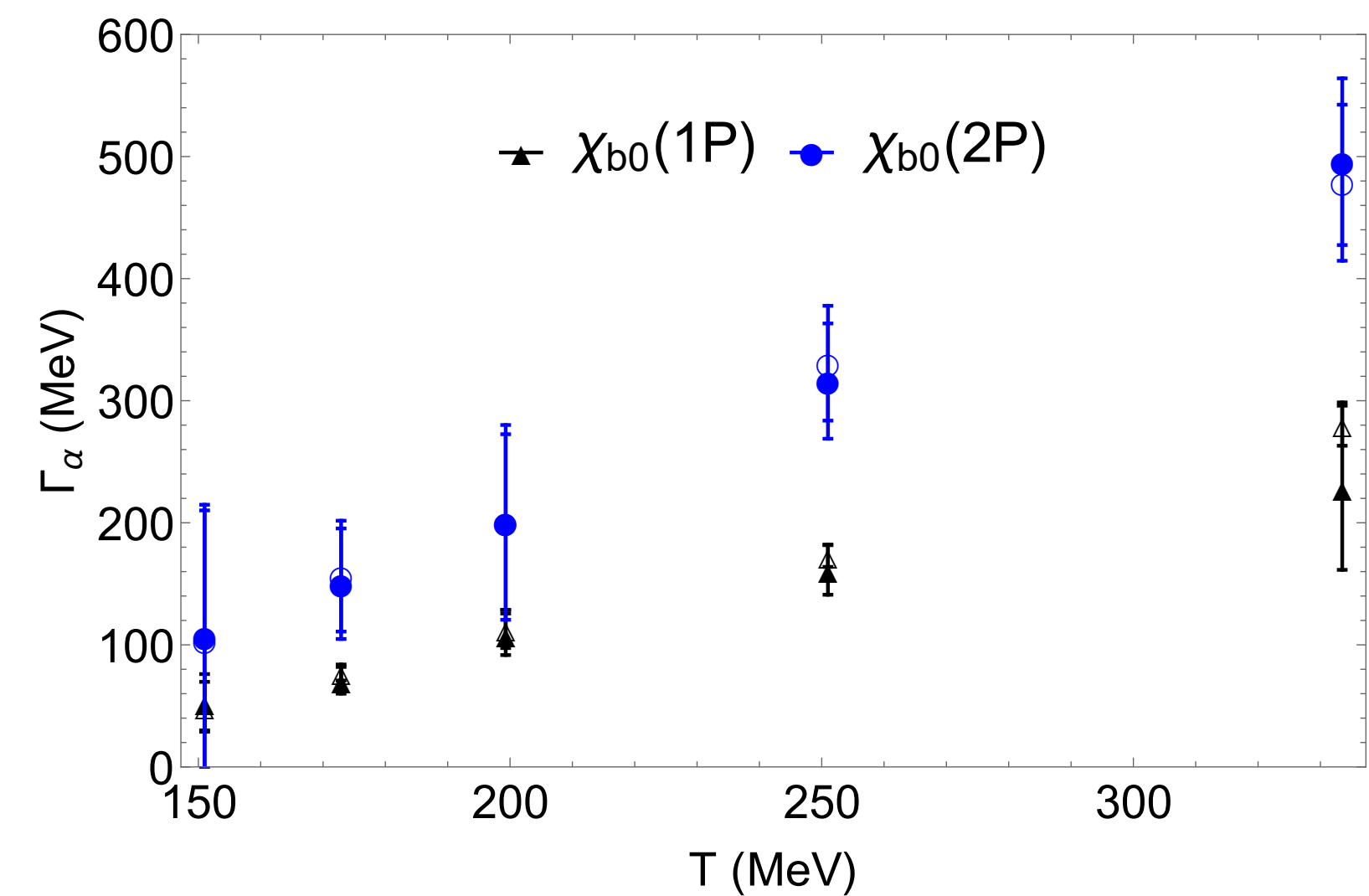
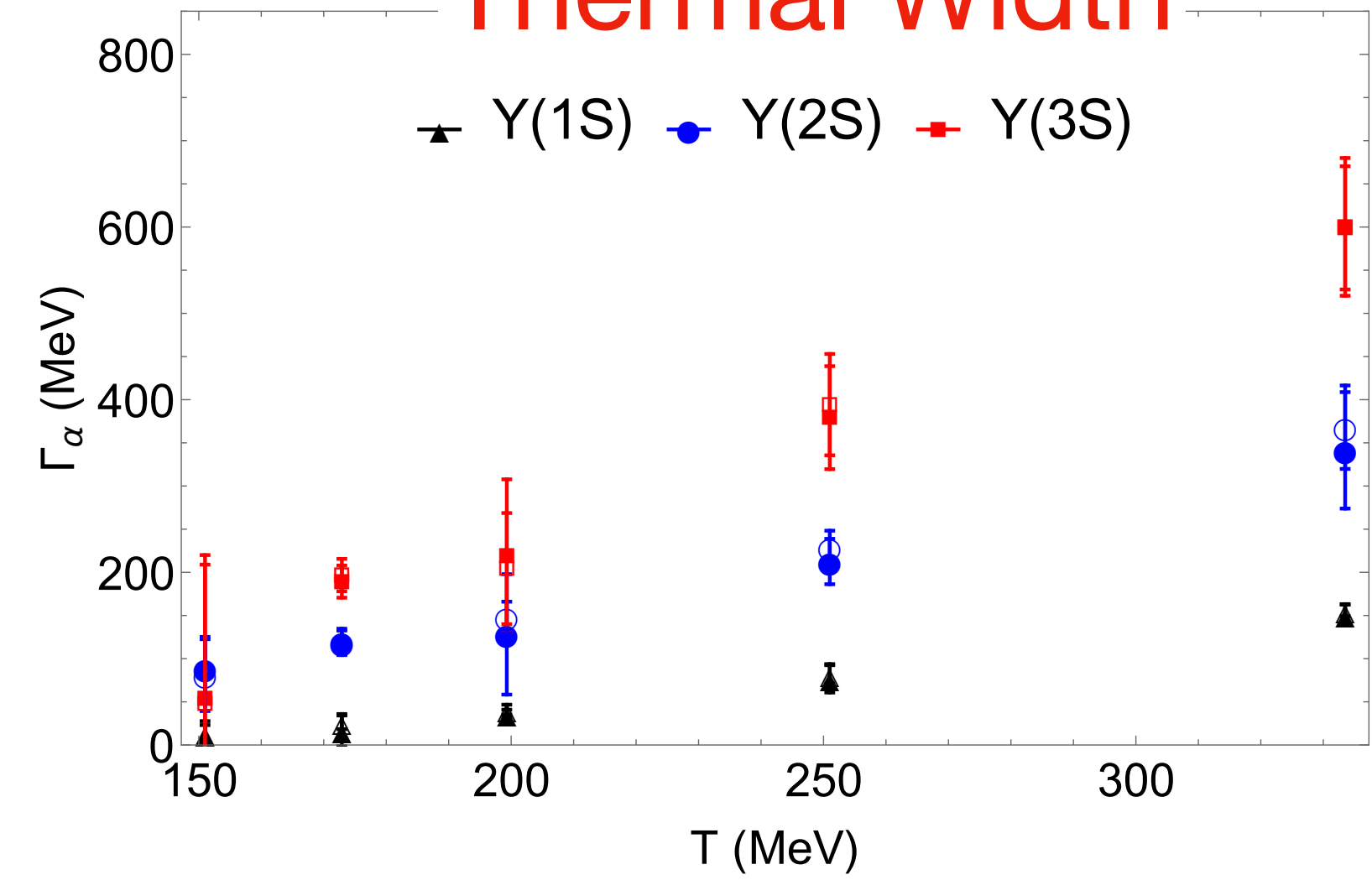
Xingyu Guo, Shuzhe Shi, Pengfei Zhuang*

Physics Department, Tsinghua University, Beijing 100084, China

Mass

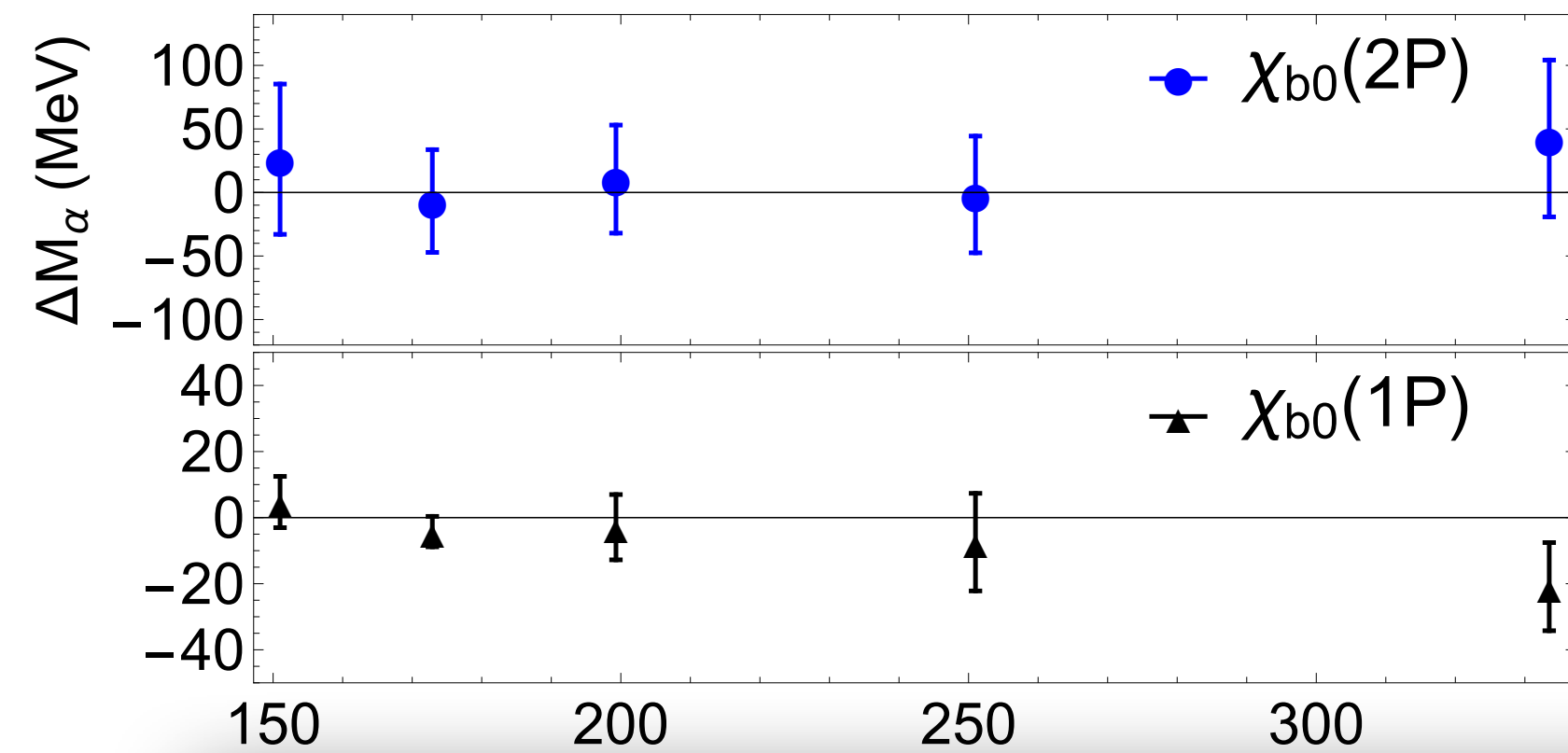


Thermal Width

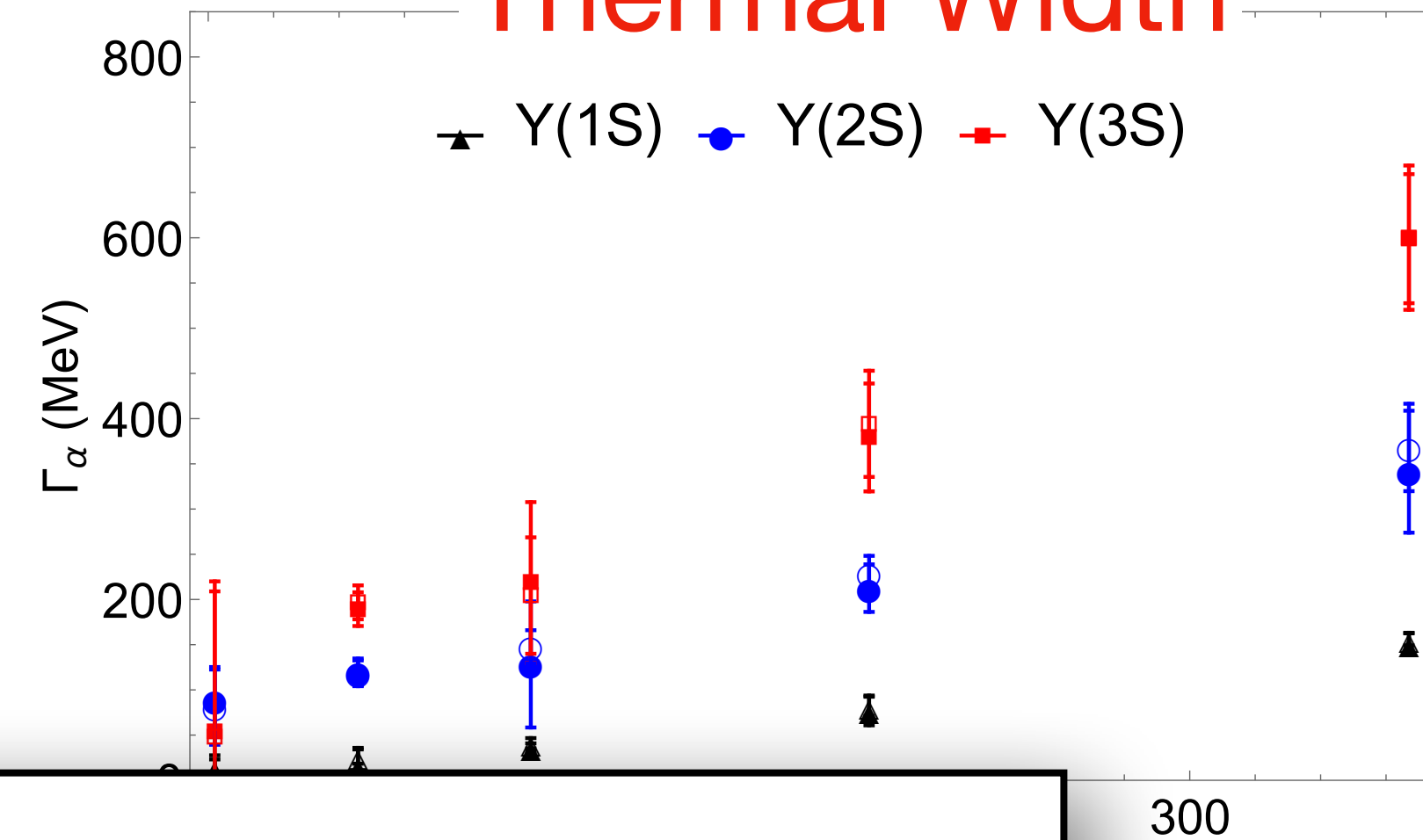


Bottomonium mass and thermal width, lattice QCD with finite m_Q

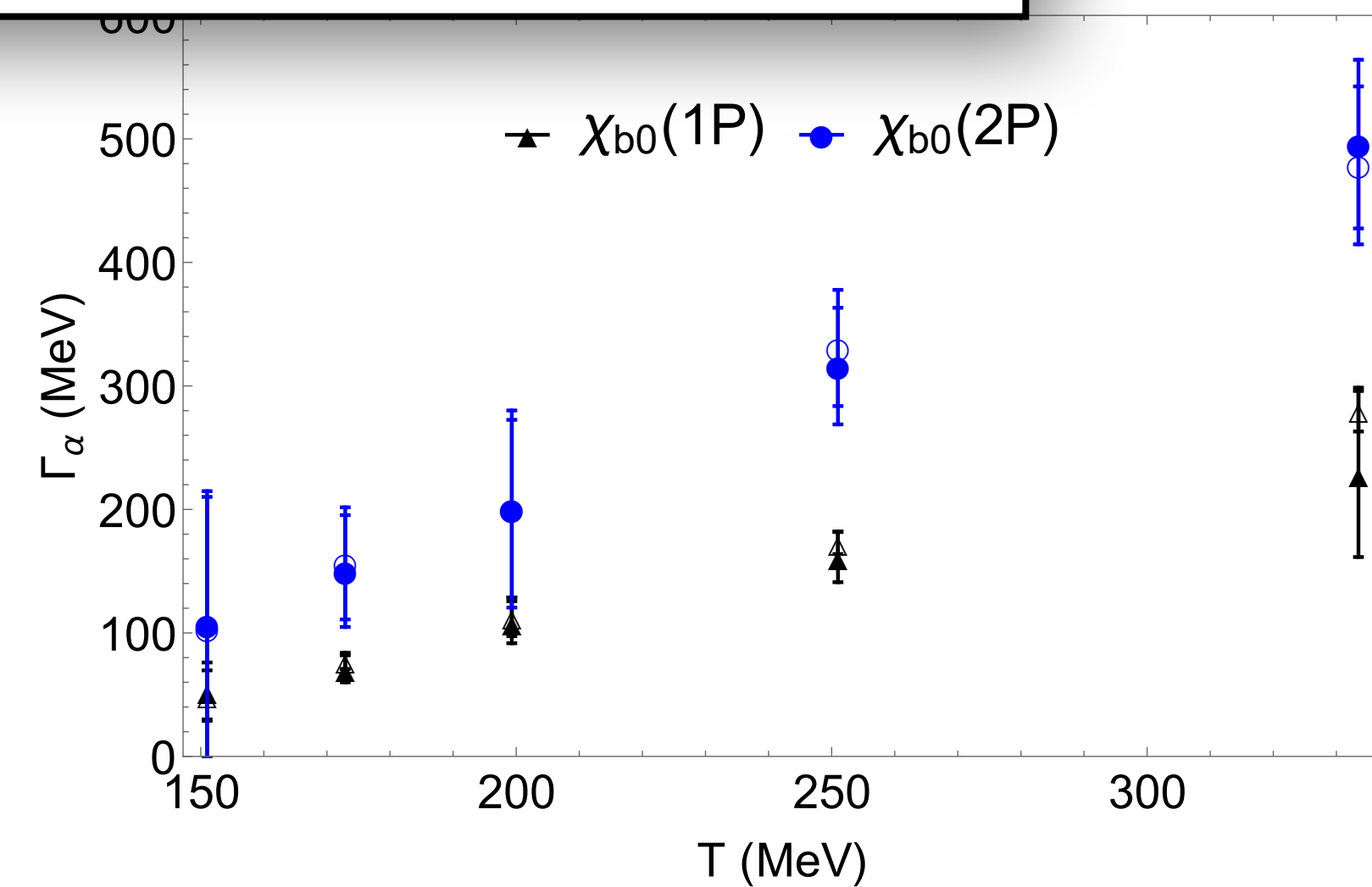
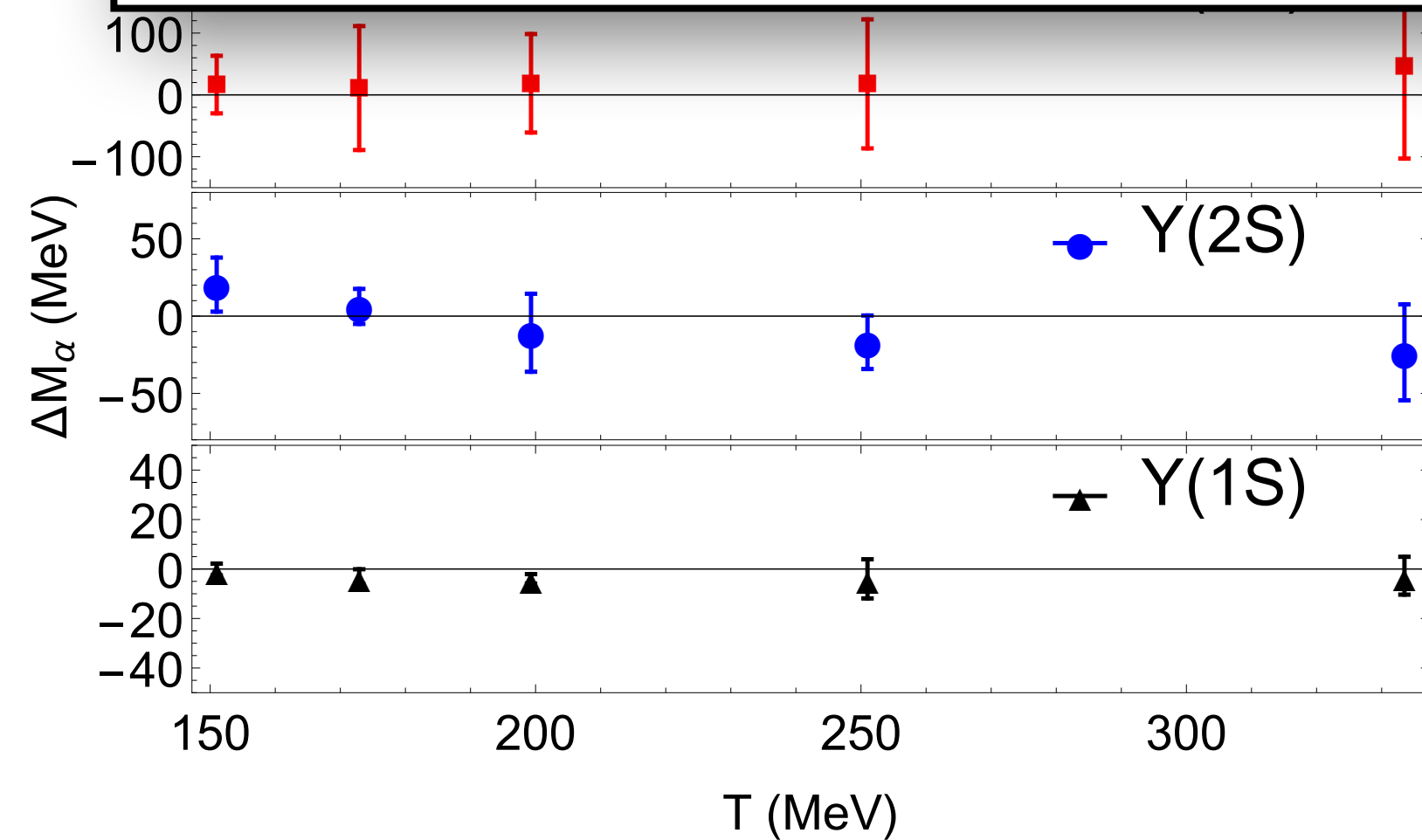
Mass



Thermal Width



can not be understood by existing modeling of potential!



How to learn $V(r)$ from $\{E_n\}$?

- parameterize the potential $V(r | \boldsymbol{\theta})$, **scan** the whole $\boldsymbol{\theta}$ -space,

$$\text{minimize } \chi^2 \equiv \sum_i \left(\frac{E_{\boldsymbol{\theta},i} - E_i}{\delta E_i} \right)^2$$

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- a **gradient-descent** based method:
 - goal -- find the $\boldsymbol{\theta}$ -point that $\nabla_{\boldsymbol{\theta}} \chi^2 = 0$
 - update $\boldsymbol{\theta}$ iteratively according to $\Delta \boldsymbol{\theta} \propto \nabla_{\boldsymbol{\theta}} \chi^2$

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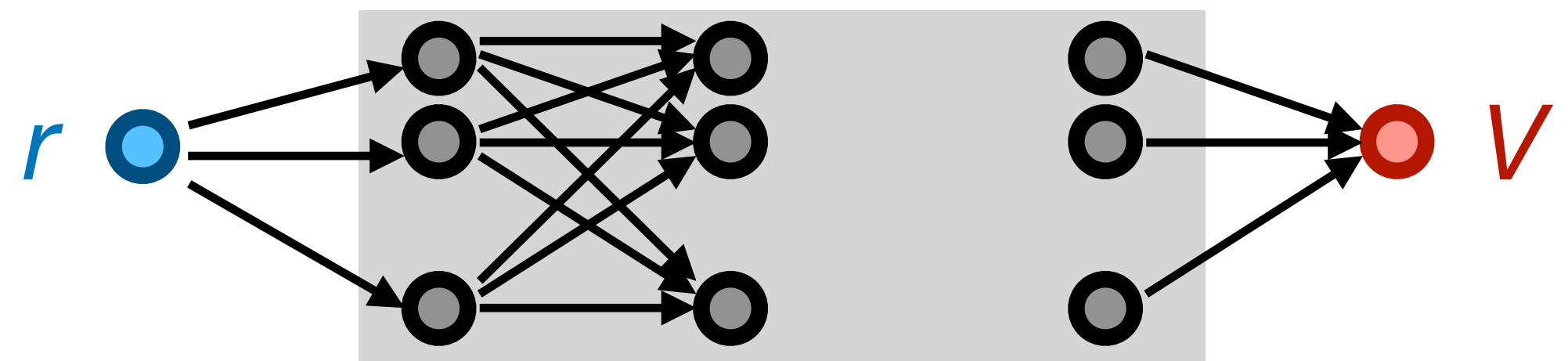
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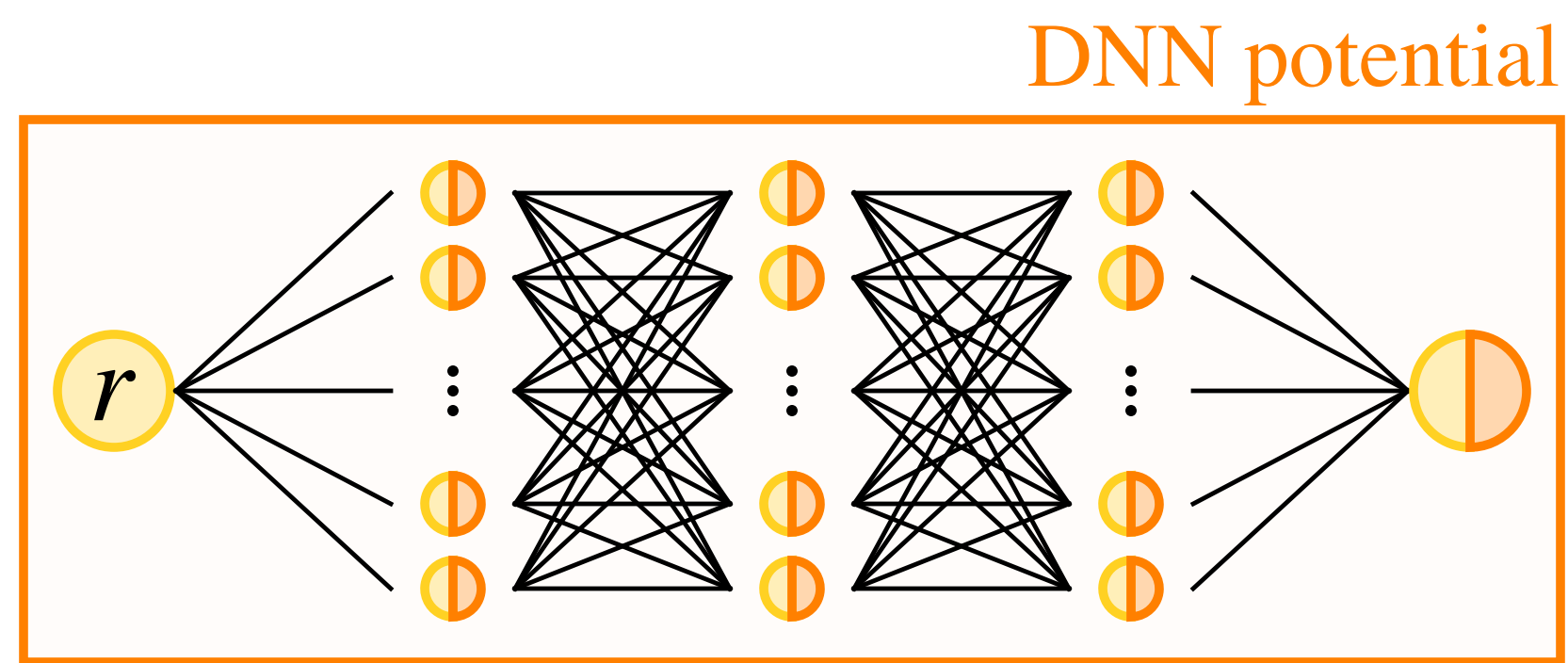
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$$V(r) \approx V_{\text{DNN}}(r | \text{parameters})$$

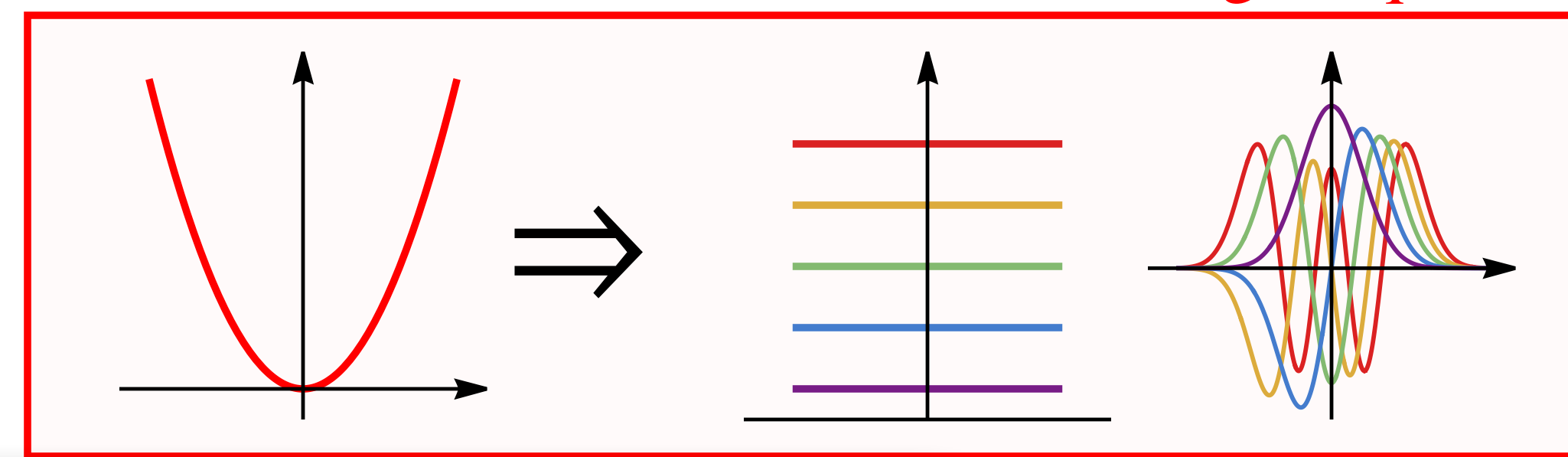


How to learn $V(r)$ from $\{E_n\}$ using DNN?



$$W_{i,j}^{(l)}, b_i^{(l)}$$

$V(r)$



Schrödinger equation

$$\hat{H} \psi_n = -\frac{\nabla^2}{2m} \psi_n + V(r) \psi_n = E_n \psi_n \quad E_n, \psi_n(r)$$

update

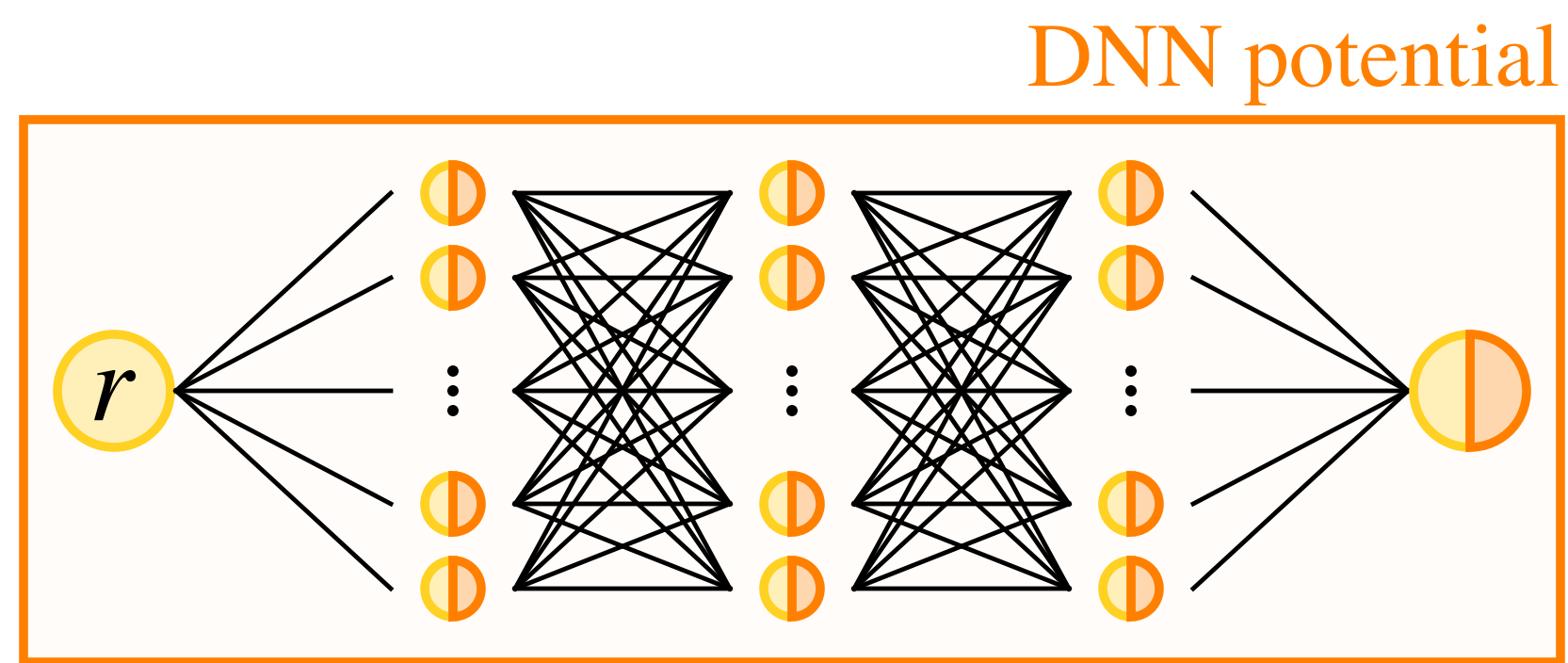
$$\delta E_n = \langle \psi_n | \delta V(r) | \psi_n \rangle$$

$$\Delta W_{i,j}^{(l)} \sim -\frac{\partial \chi^2}{\partial W_{i,j}^{(l)}}, \quad \Delta b_i^{(l)} \sim -\frac{\partial \chi^2}{\partial b_i^{(l)}}$$

$$\frac{\delta \chi^2}{\delta V(r)}$$

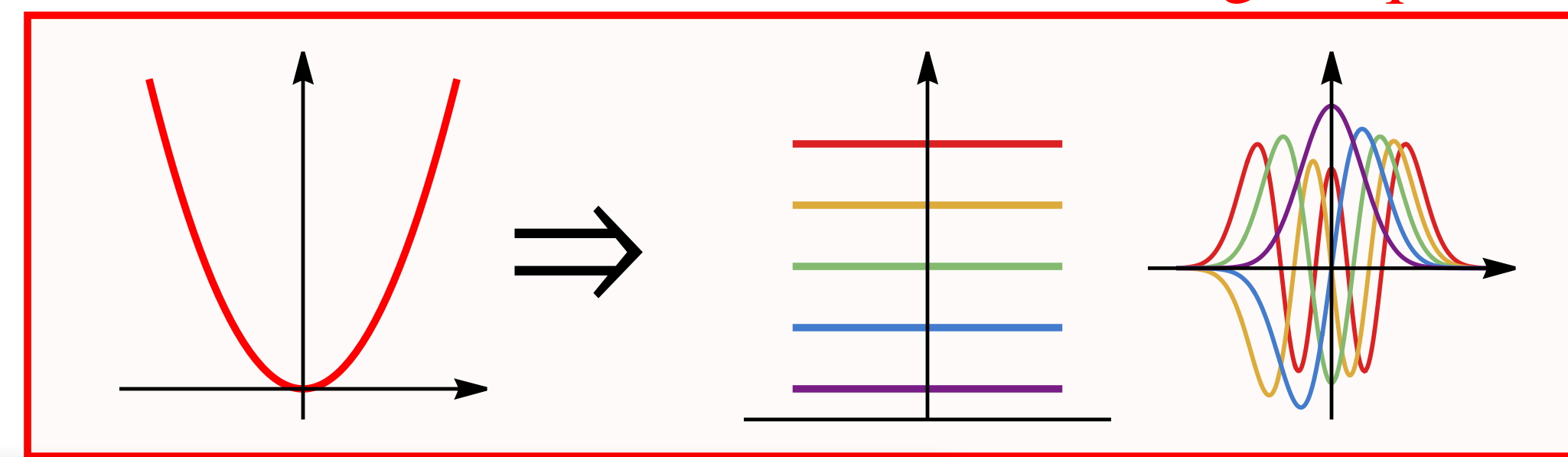
$$\chi^2 = \sum_n \frac{(E_n - E_n^{\text{tgt}})^2}{\Delta_n^2}$$

How to learn $V(r)$ from $\{E_n\}$ using DNN?



$$W_{i,j}^{(l)}, b_i^{(l)}$$

$V(r)$ →



Schrödinger equation

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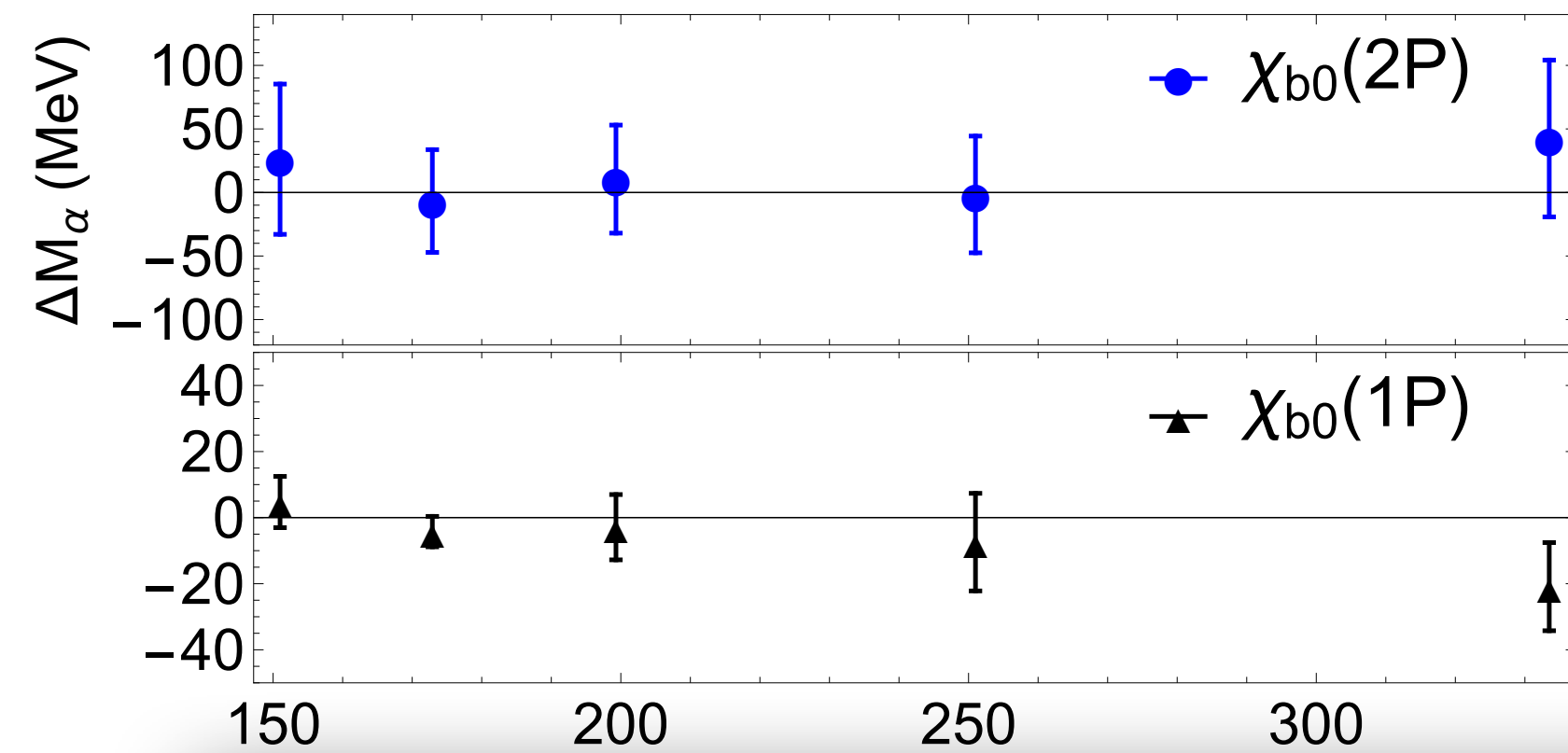
$$\frac{\delta \chi^2}{\delta V(r)}$$

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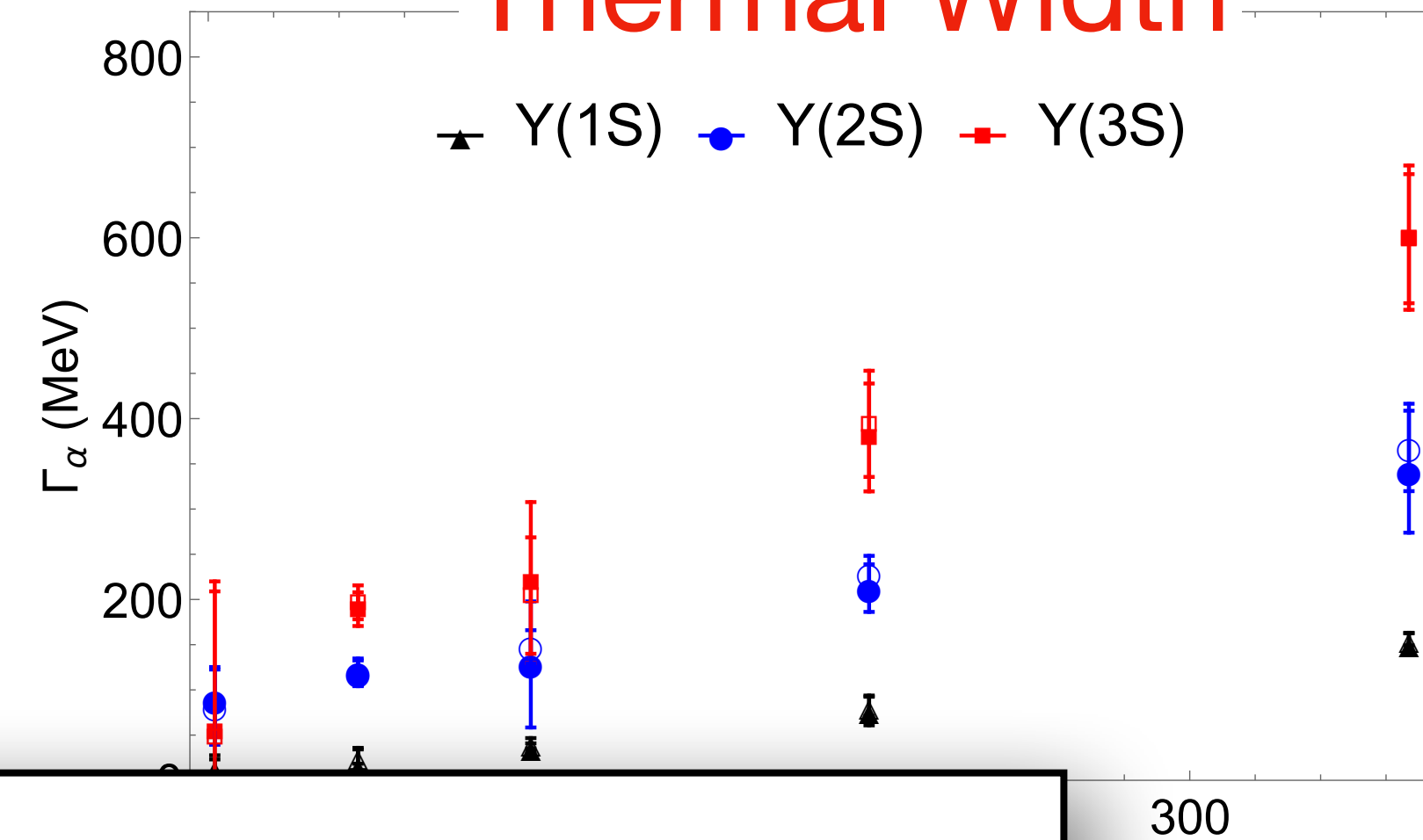
uncertainty: $P(V_\theta) dV = \text{Posterior}(\theta | \text{data}) d^N \theta$

Bottomonium mass and thermal width, lattice QCD with finite m_Q

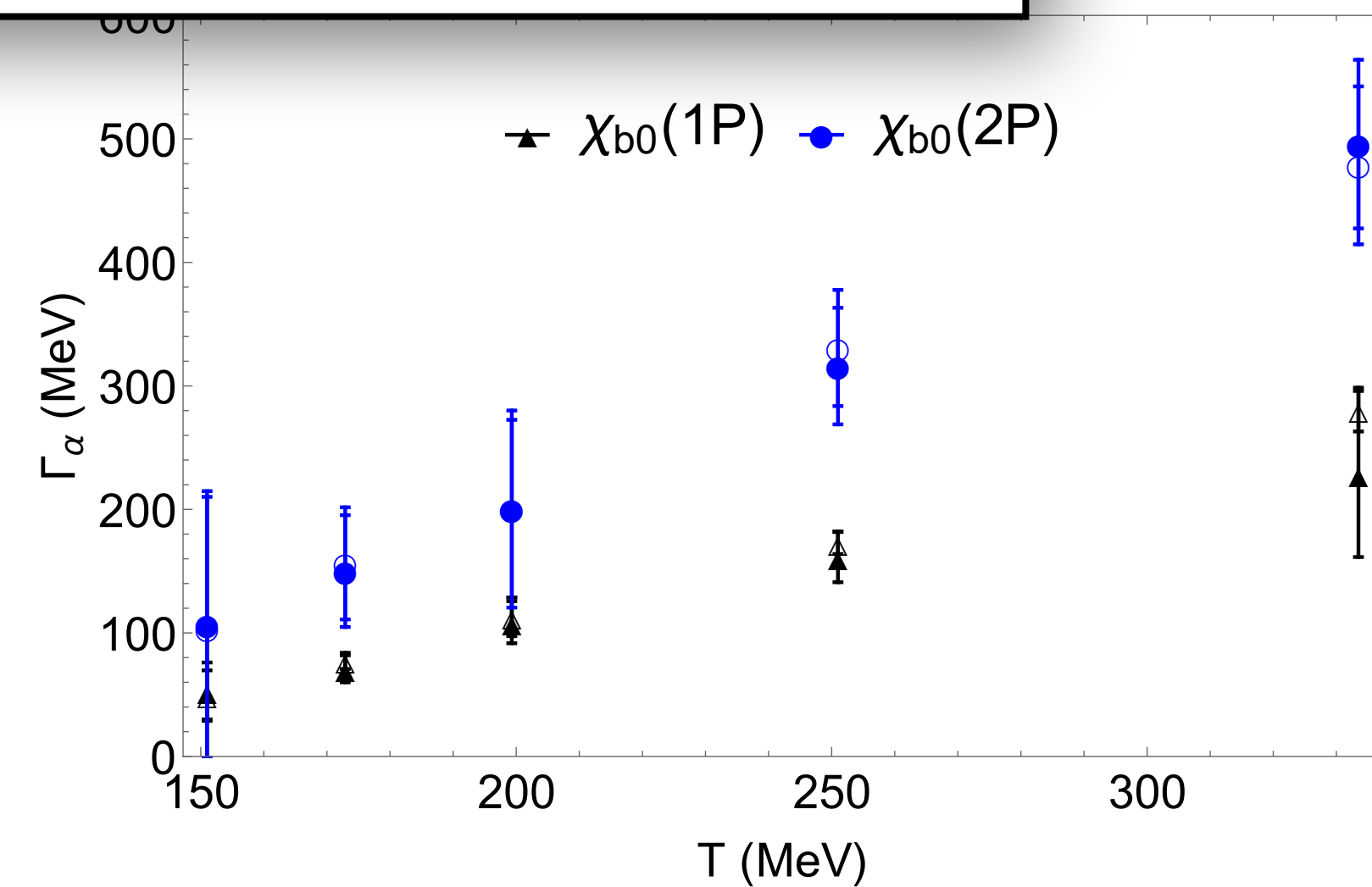
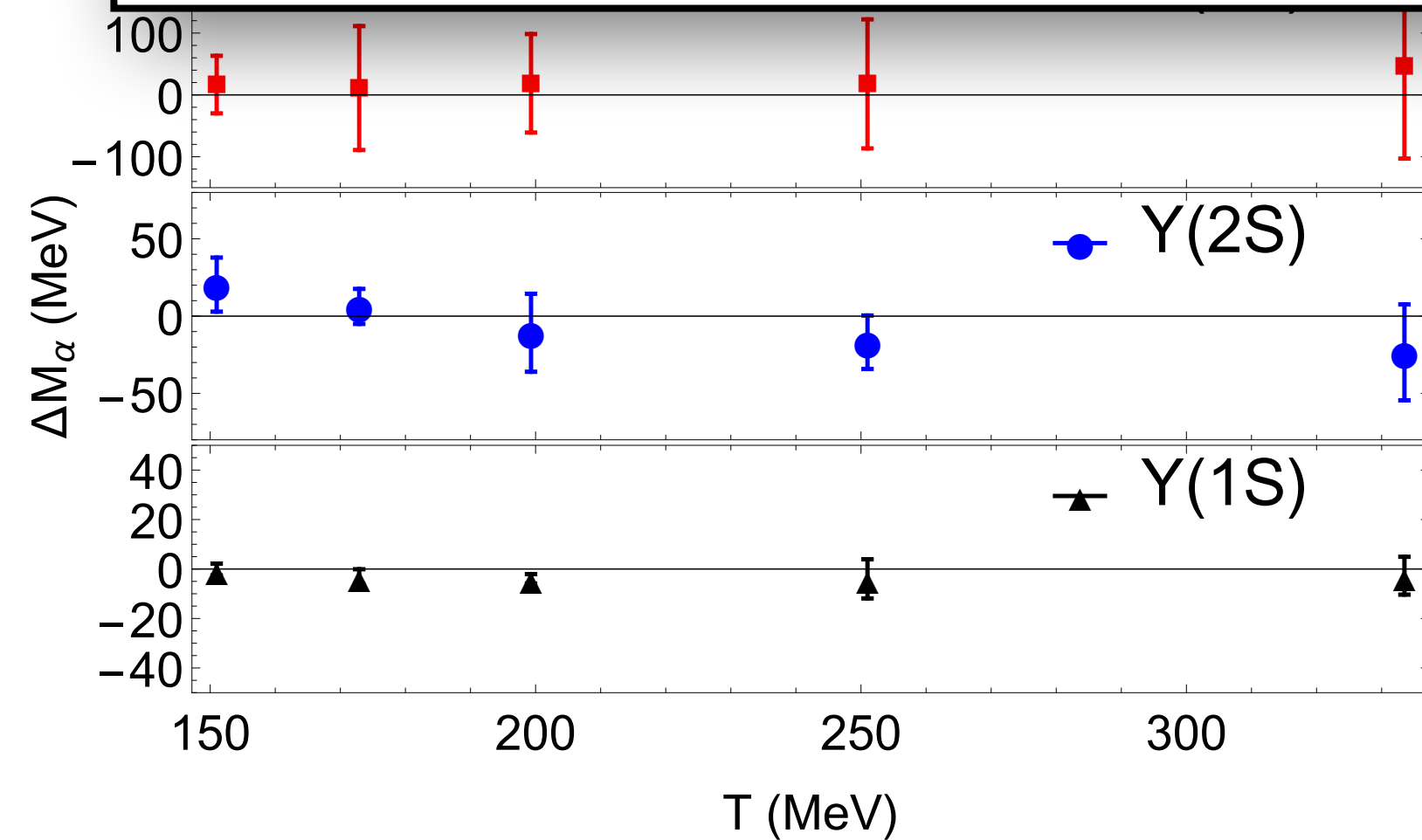
Mass



Thermal Width

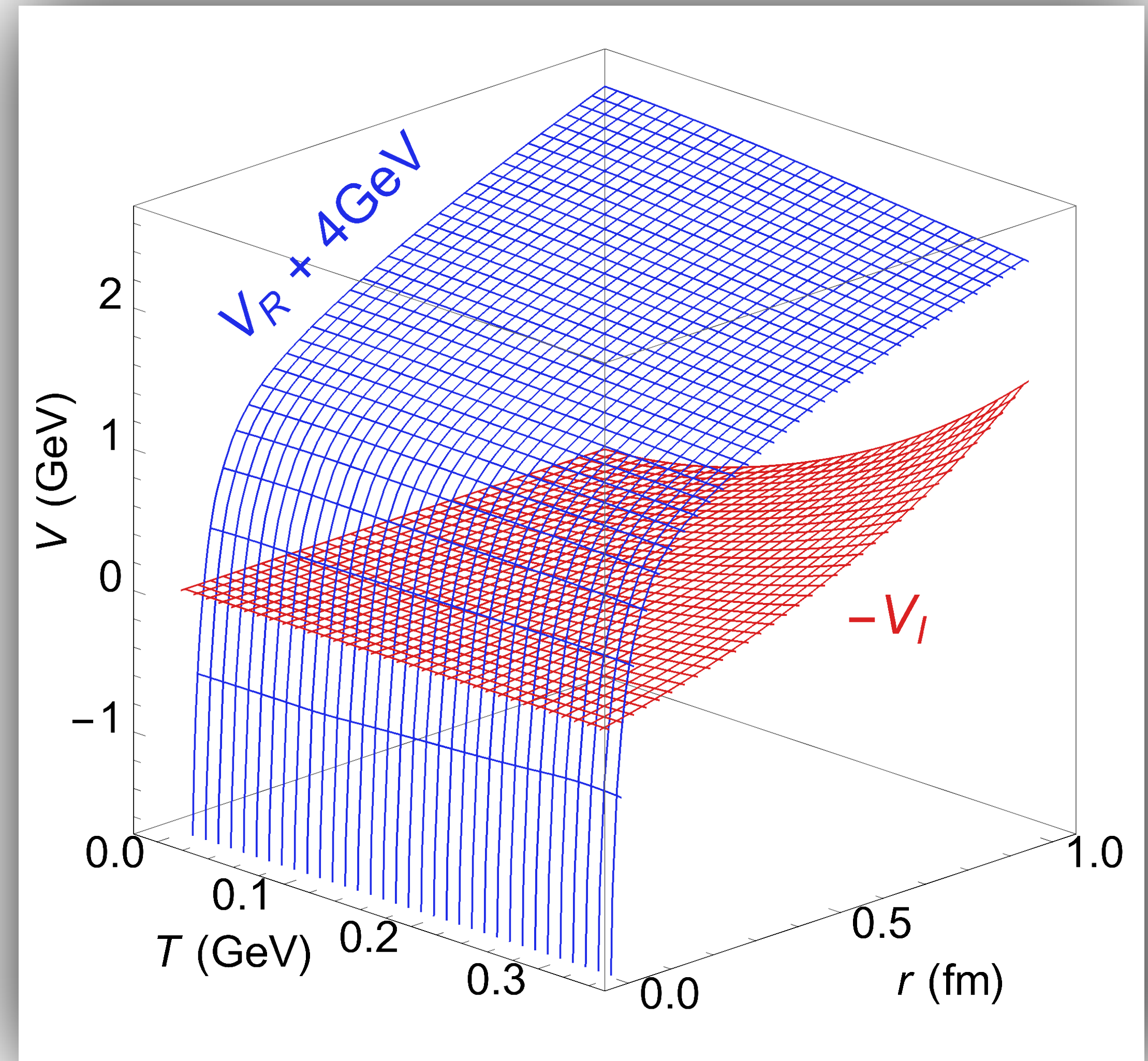


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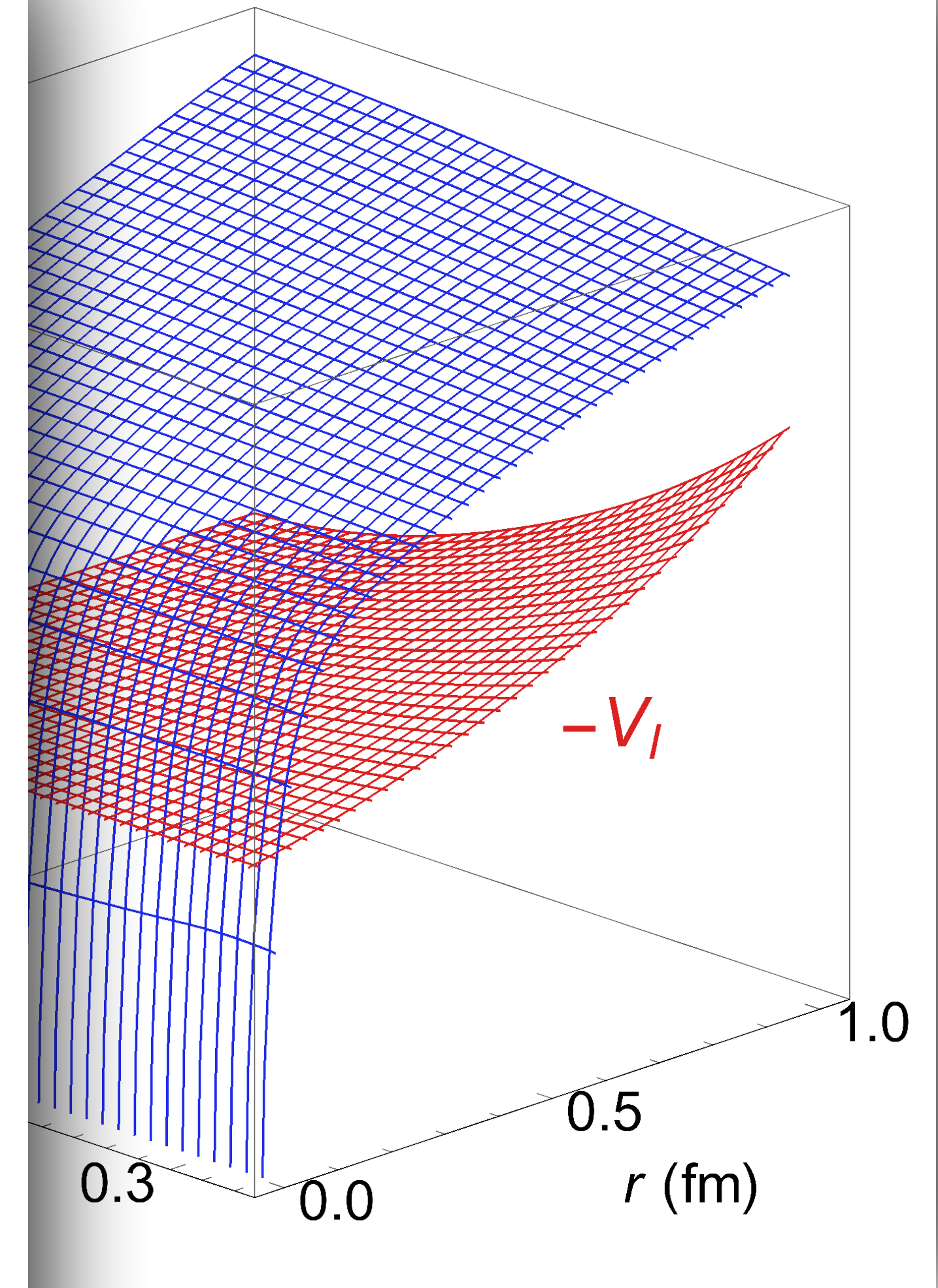
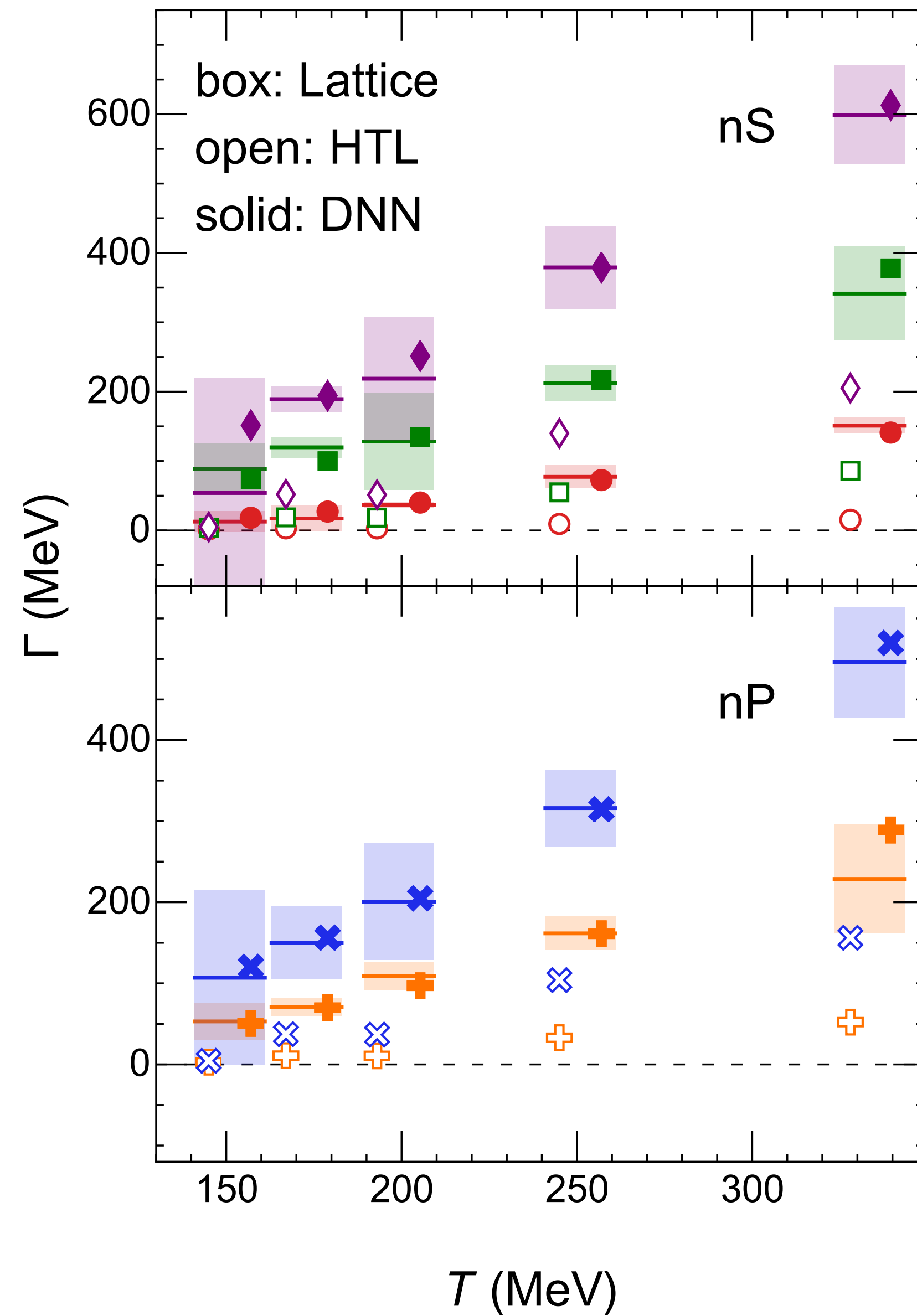
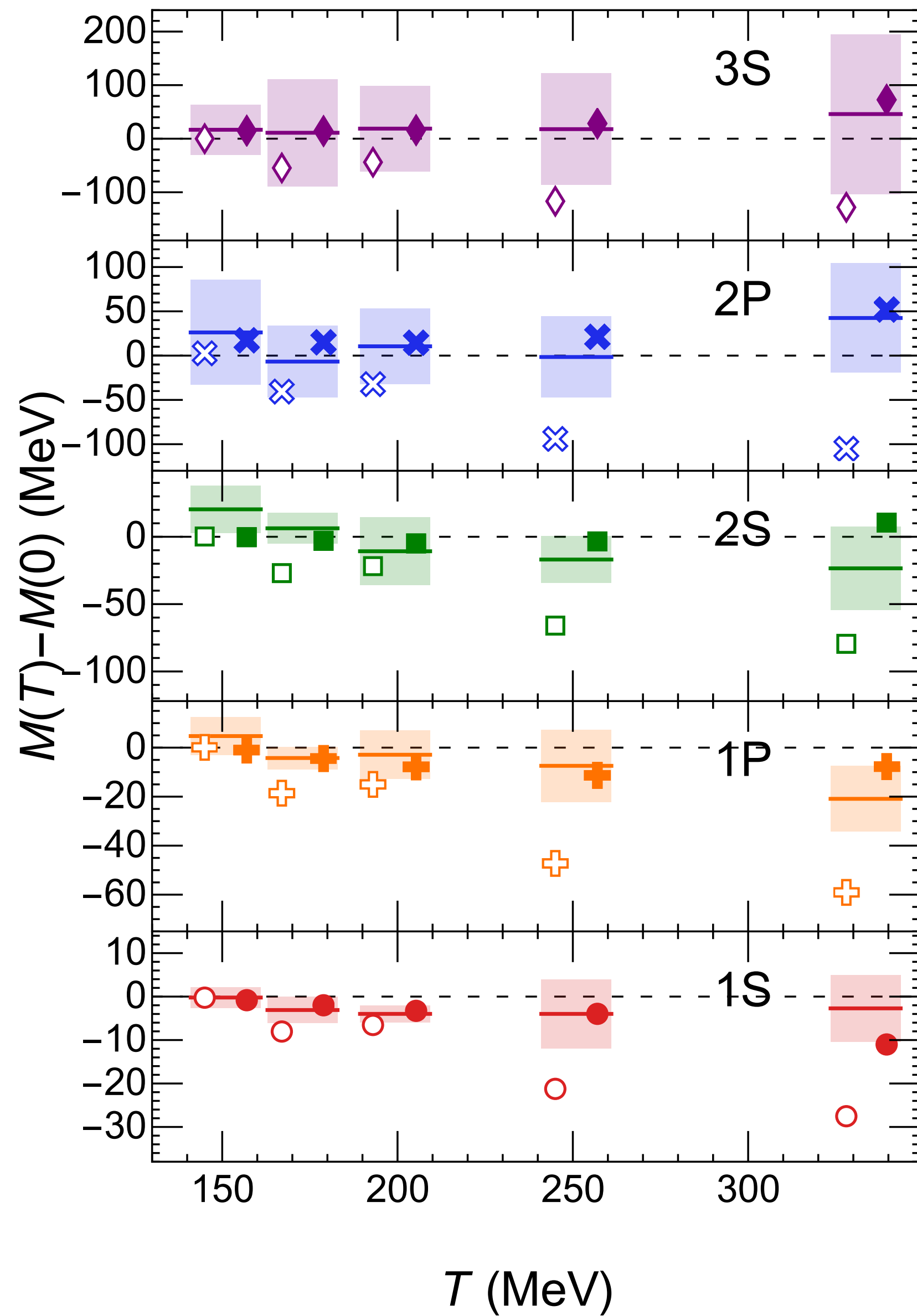
Finite Temperature Heavy-Quark Potential

07



Finite Temperature Heavy-Quark Potential

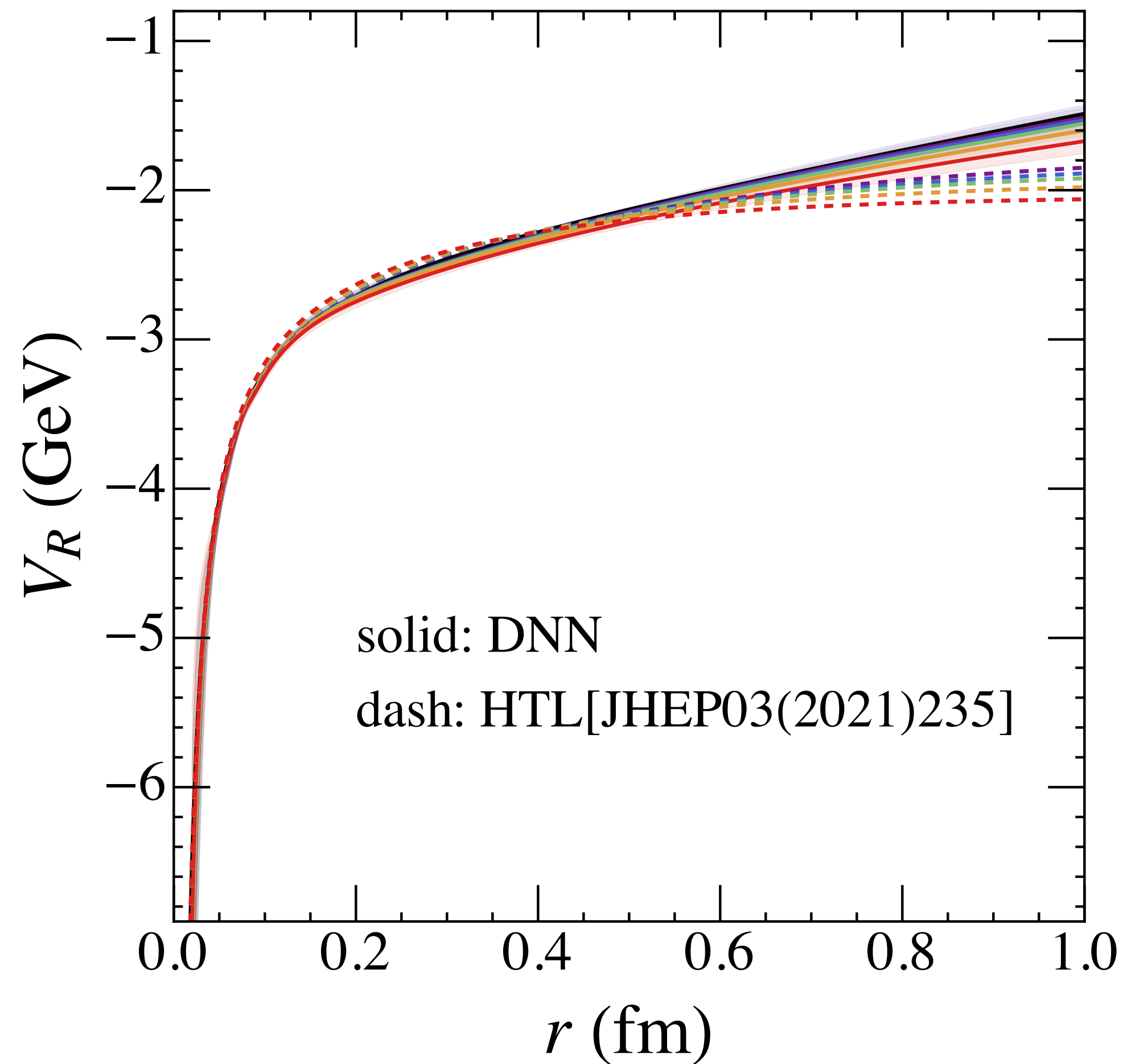
07



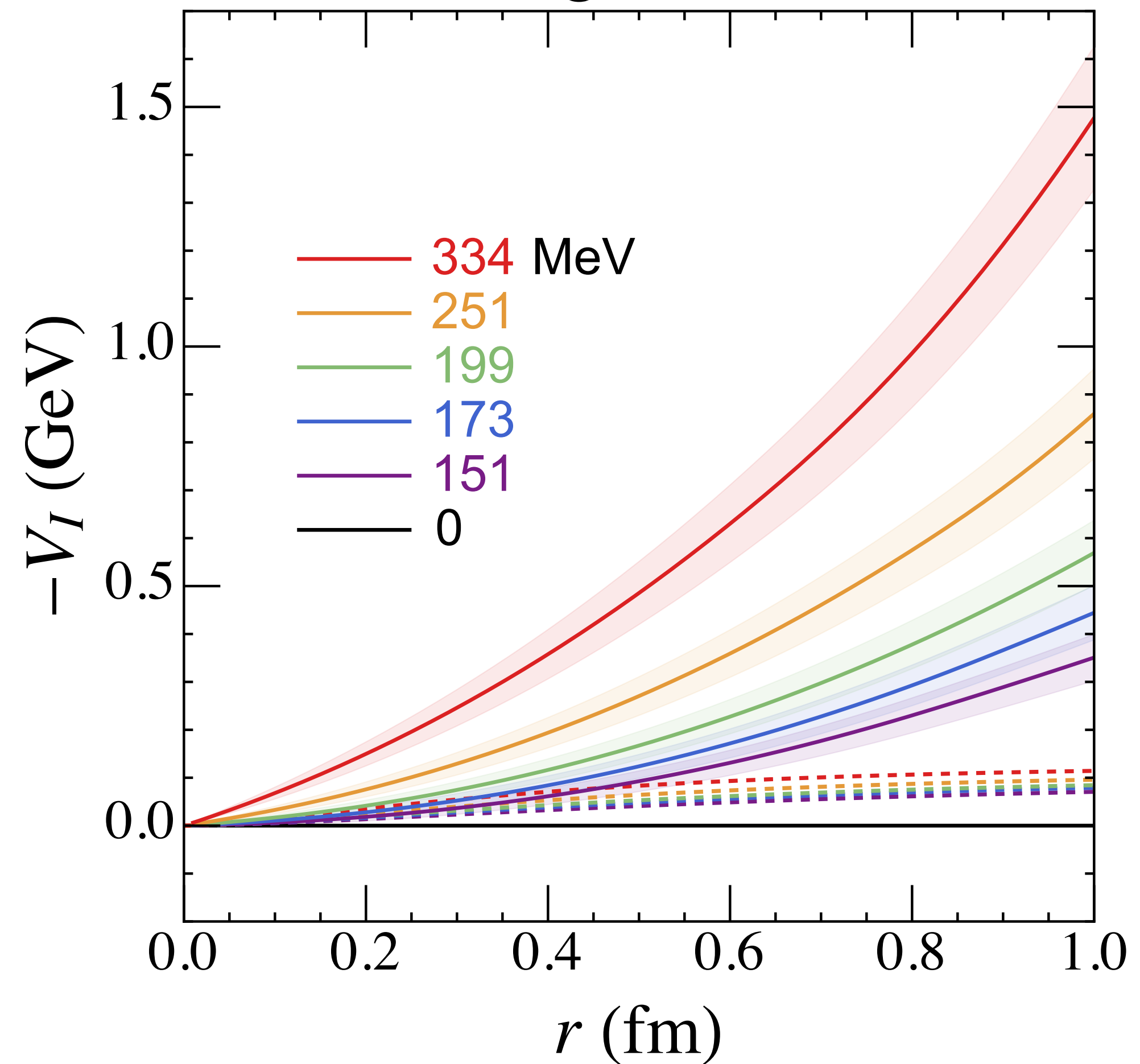
What physics we have learned from $V_{\text{DNN}}(T, r)$?

--- compare with HTL potential model

less screening



more singlet \rightarrow octet



PHYSICAL REVIEW D **105**, 054513 (2022)

Static quark-antiquark interactions at nonzero temperature from lattice QCD

Dibyendu Bala,¹ Olaf Kaczmarek,¹ Rasmus Larsen,² Swagato Mukherjee,³ Gaurang Parkar,² Peter Petreczky,³
Alexander Rothkopf,² and Johannes Heinrich Weber⁴


(HotQCD Collaboration)

¹*Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany*

²*Faculty of Science and Technology, University of Stavanger, NO-4036 Stavanger, Norway*

³*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*

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PHYSICAL REVIEW D **105**, 014017 (2022)

Heavy quark potential in the quark-gluon plasma: Deep neural network meets lattice quantum chromodynamics


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¹*Department of Physics, McGill University, Montreal, Quebec H3A 2T8, Canada*

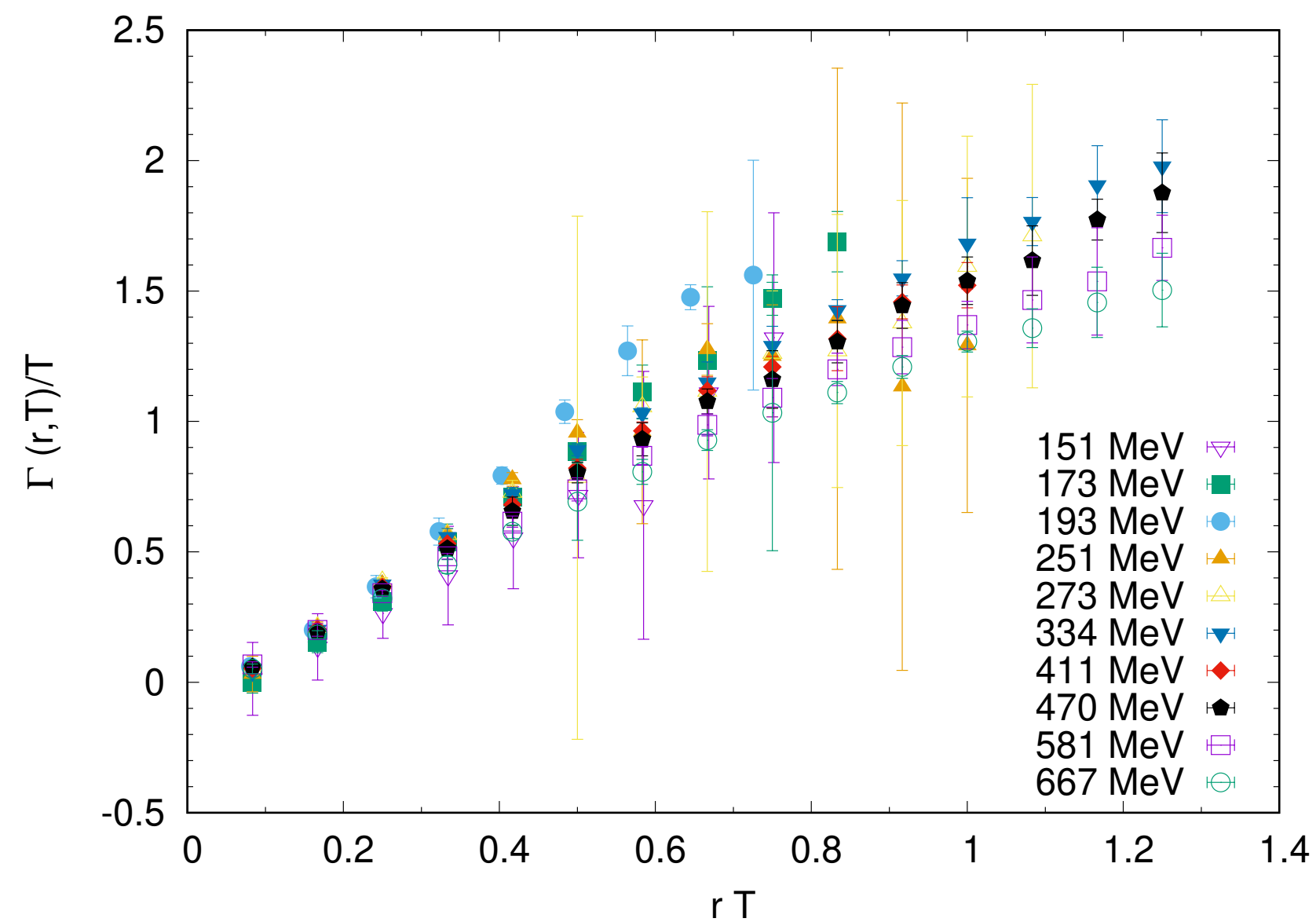
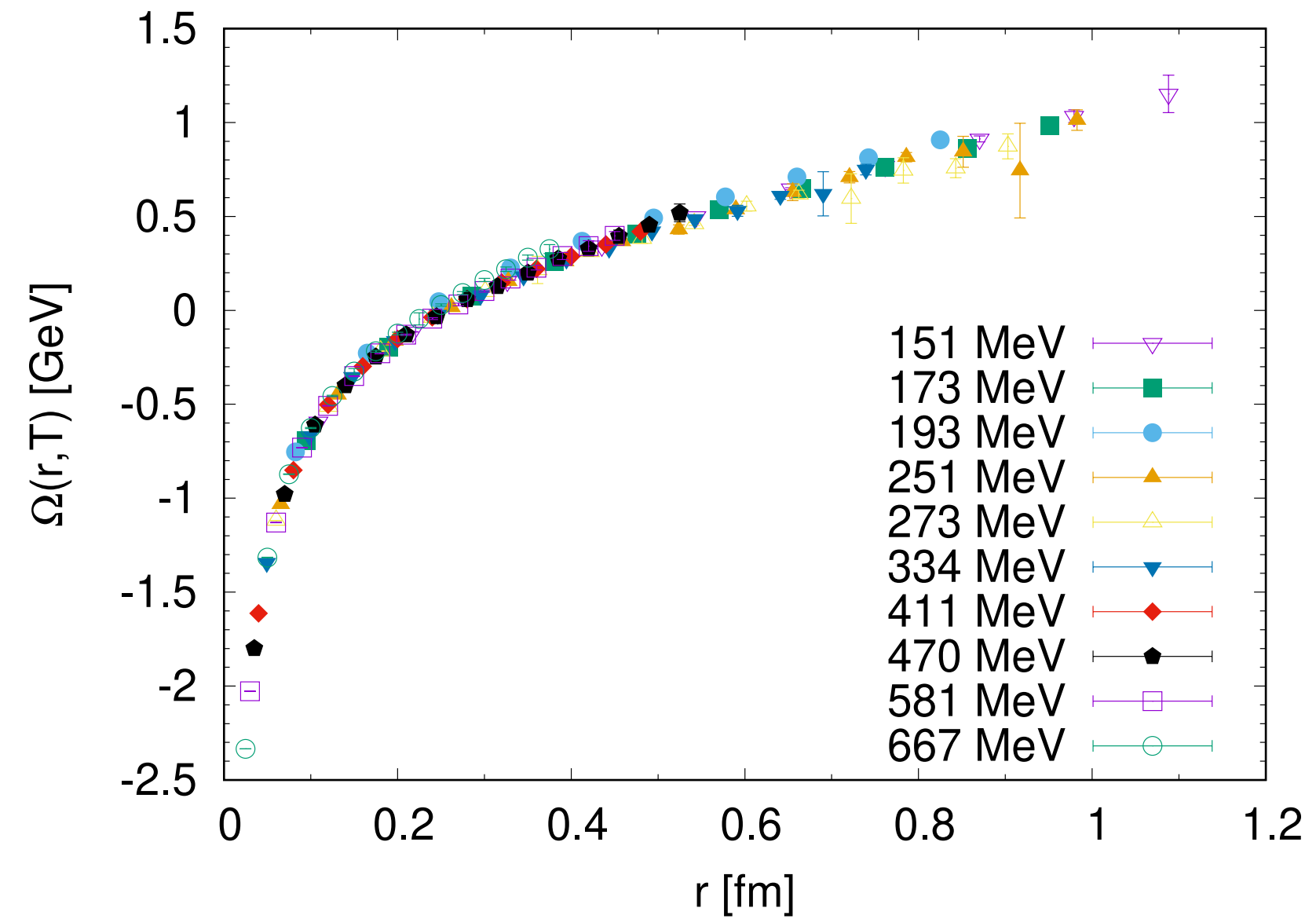
²*Frankfurt Institute for Advanced Studies, Ruth Moufang Strasse 1, D-60438,
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What physics we have learned from $V_{\text{DNN}}(T, r)$?



PHYSICAL REVIEW D **105**, 054513 (2022)

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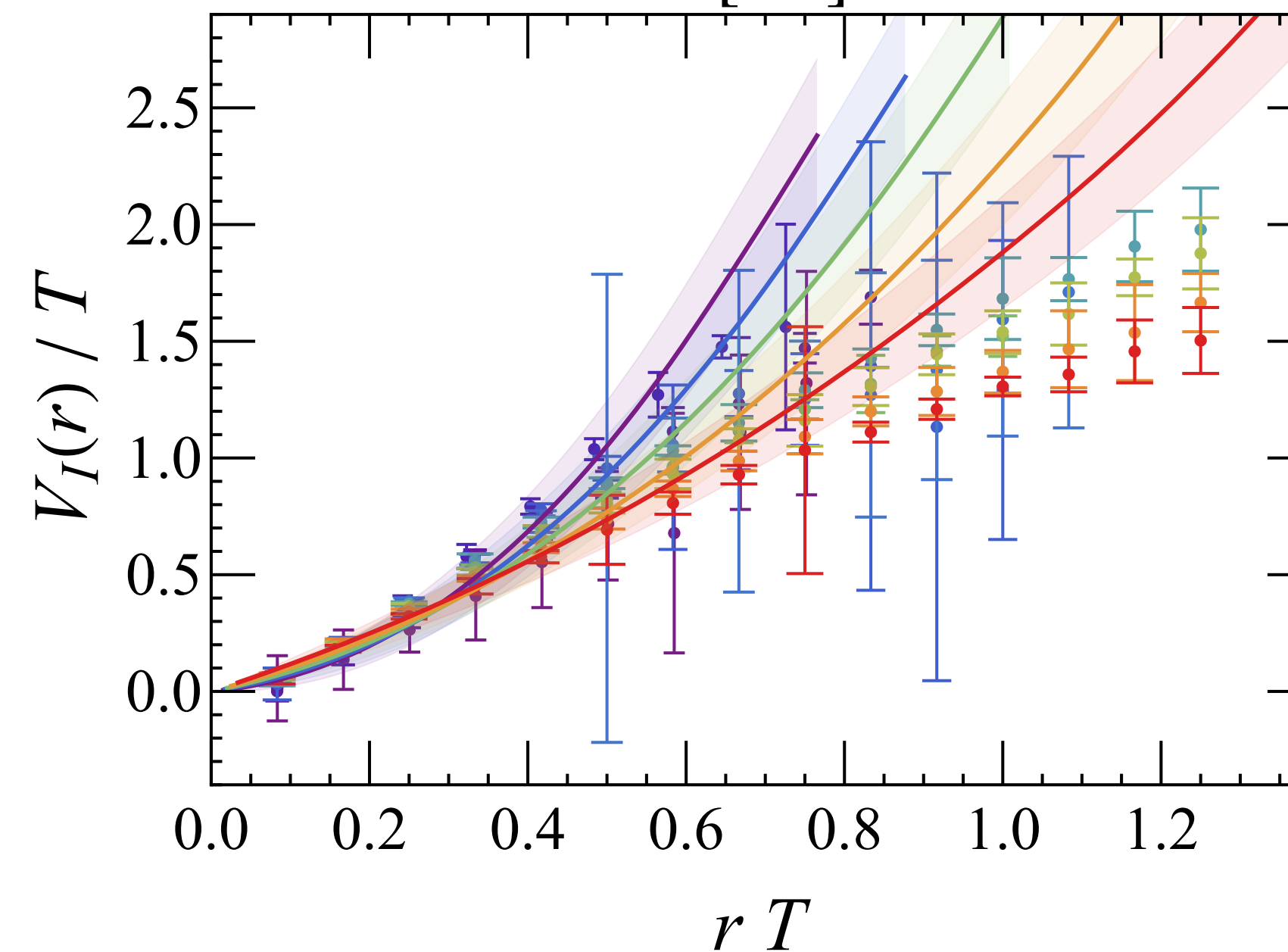
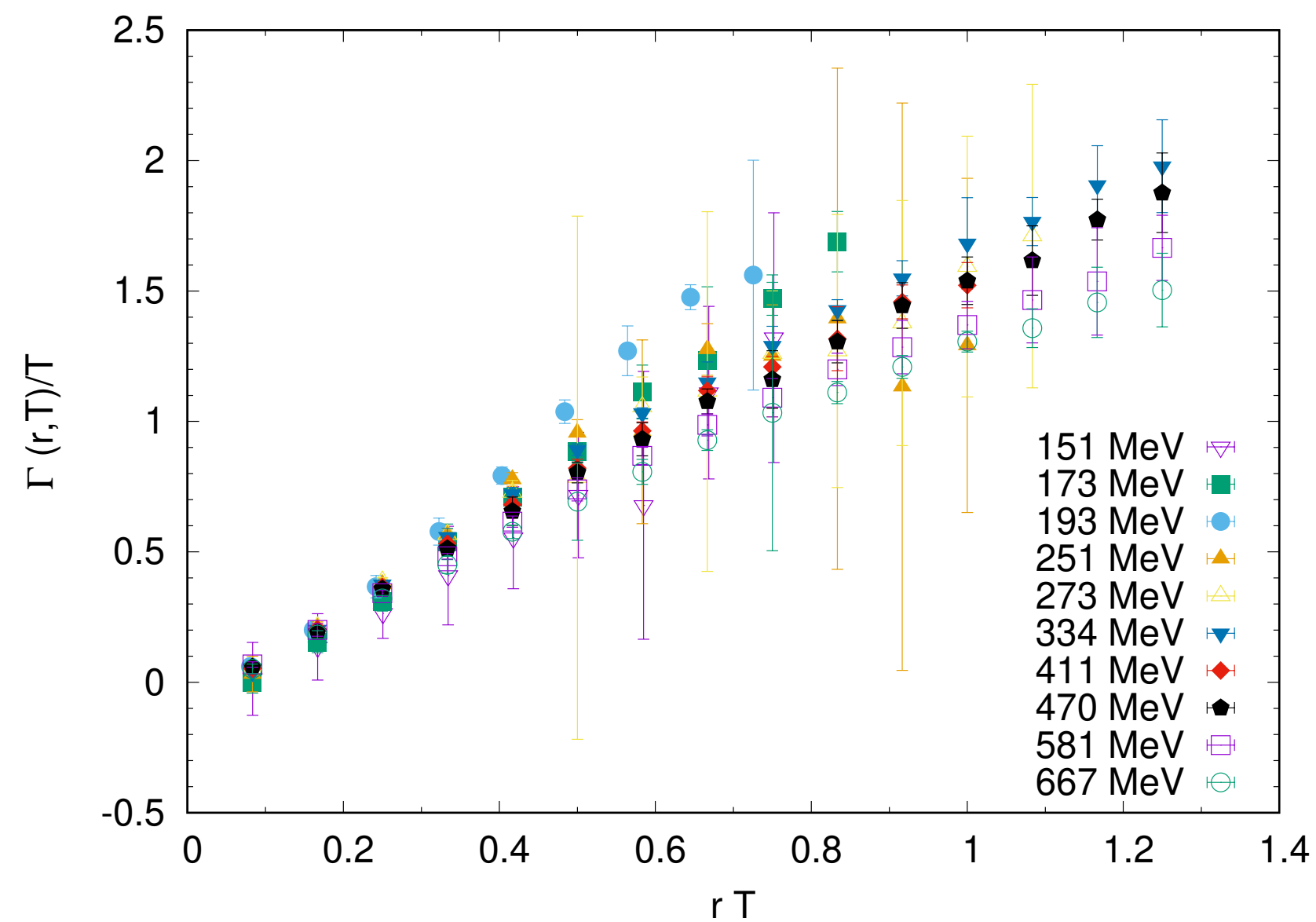
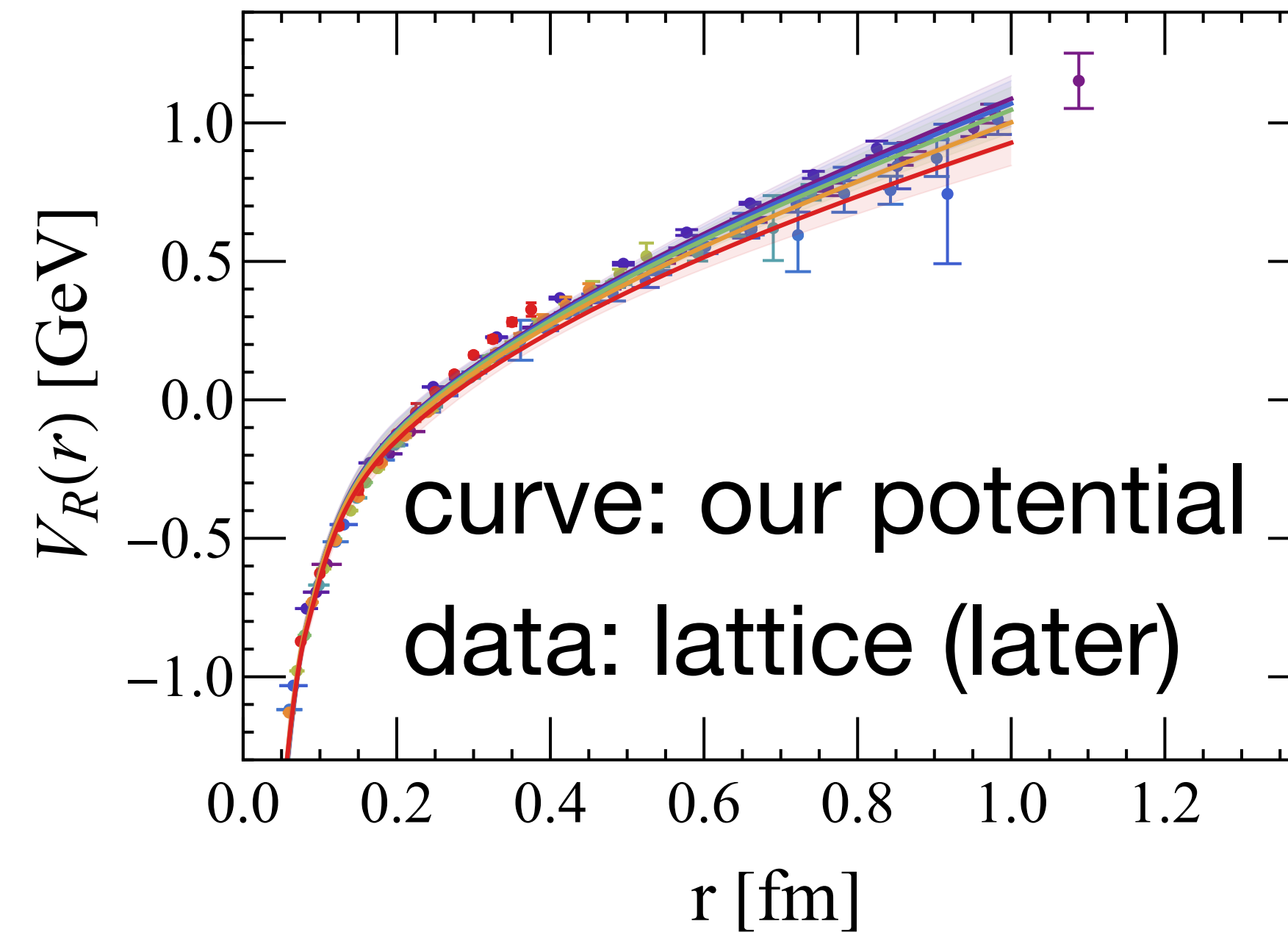
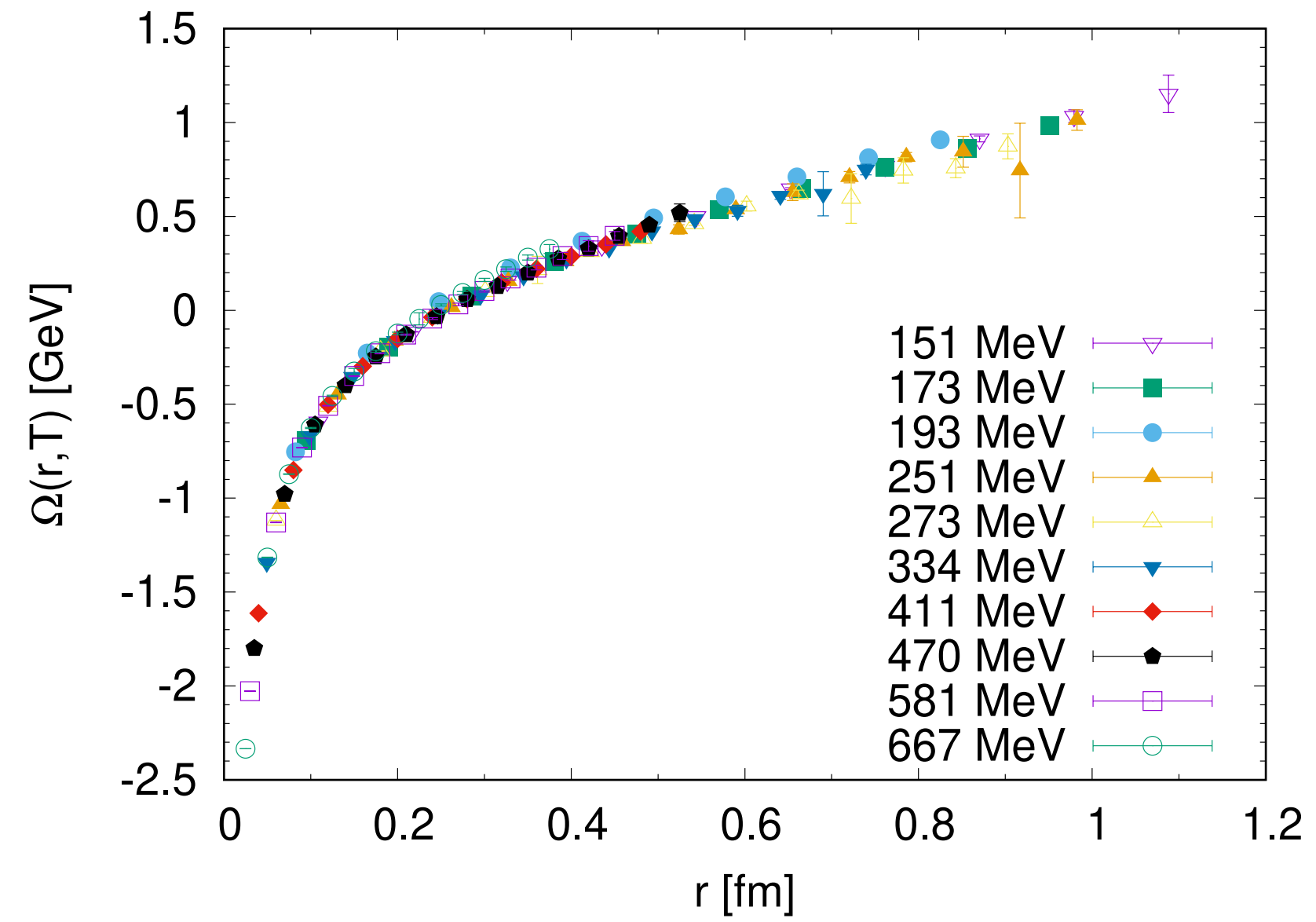
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What physics we have learned from $V_{\text{DNN}}(T, r)$?



Summary

- Extracted HF complex $V(T, r)$ from LQCD results of bottomonium m and Γ .
- New algorithm applicable in solving other inverse problems:
 - Neutron Star Mass-Radius relation \Leftrightarrow QCD EoS
 - Spectral function \Leftrightarrow imaginary time correlation