

# Production of light and hyper-nuclei in high energy heavy-ion collisions

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References: Phys. Rev. C **112**, 034908, 2025; Nucl. Sci. Tech. **36**, 185, 2025;  
Phys. Rev. C **109**, 034907, 2024; Phys. Rev. C **105**, 054908, 2022;  
Phys. Rev. C **103**, 064908, 2021; arXiv: 2603.21164.

# Outline

## I. Introduction

## II. An analytical coalescence model

Formalism of two-body coalescence

General formalism of  $N$ -body coalescence

## III. Applications in heavy-ion collisions at RHIC & LHC

Coalescence factor  $B_A$

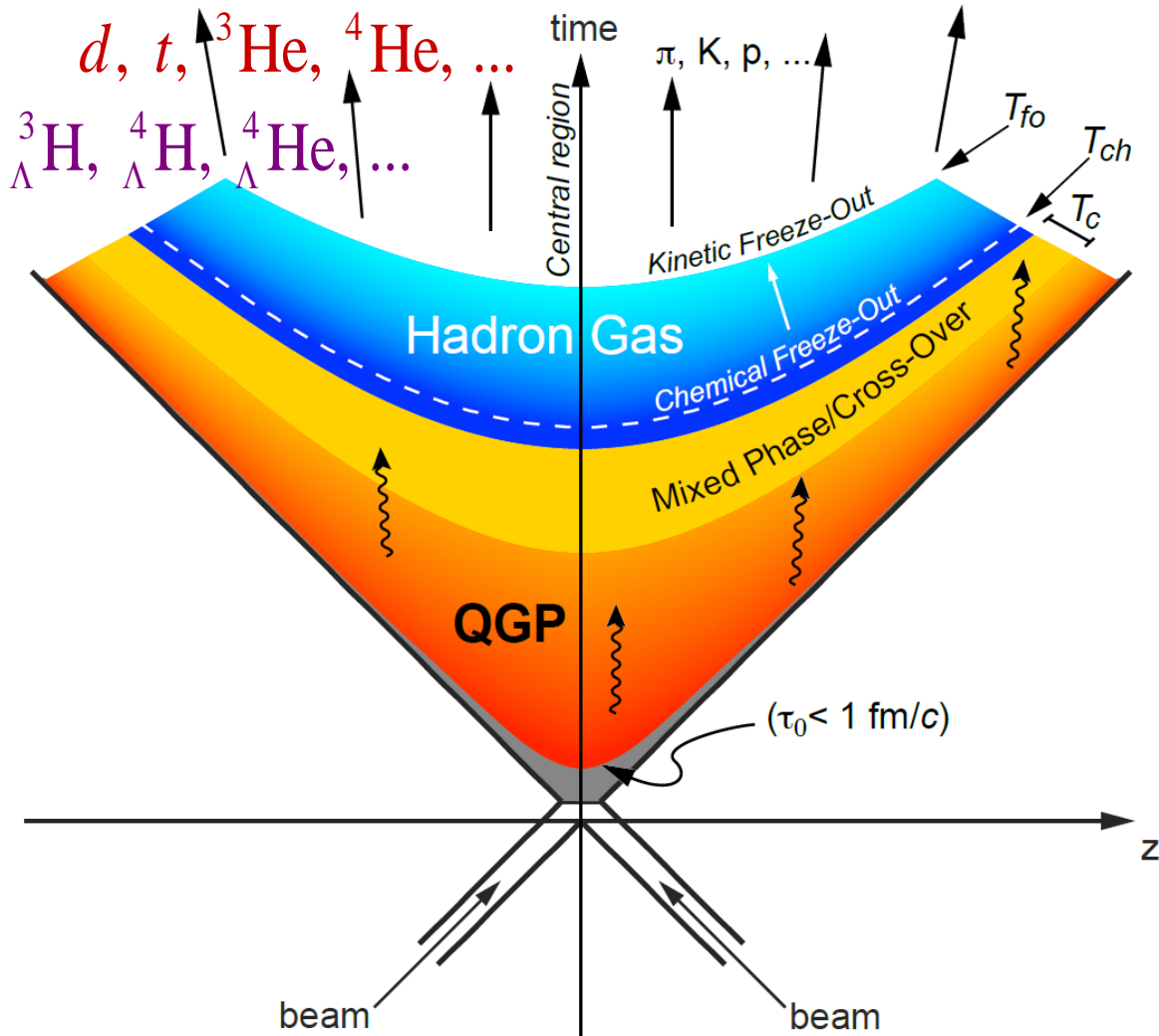
$p_T$  spectra

Production correlations of different light (hyper-)nuclei

## IV. Summary

# I. Introduction

What can **light** and **hyper-nuclei** in high energy heavy-ion collisions tell us?



- Natural tools for elementary nucleon-nucleon, nucleon-hyperon, and three-body interactions
- Probes of system freeze-out properties
- Production mechanisms of composite particles
- Search for more molecular states
- .....

## Thermal models

$$N_i = V \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{g_i}{\exp[(\sqrt{\vec{p}^2 + m_i^2} - \mu_i) / T_{ch}] \pm 1}$$

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P. J. Siemens and J. I. Kapusta, Phys. Rev. Lett. **43**, 1486, 1979;

A. Andronic, P. Braun-Munzinger, J. Stachel, and H. Stoecker, Phys. Lett. B **697**, 203, 2011;

J. Cleymans, S. Kabana, I. Kraus, H. Oeschler, K. Redlich, and N. Sharma, Phys. Rev. C **84**, 054916, 2011;

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... ..

## Coalescence models

$$f_H \sim f_p^Z f_{\Lambda,\Omega}^{N_{\Lambda,\Omega}} f_n^{A-Z-N_{\Lambda,\Omega}} \otimes \mathcal{R}_H$$

S. T. Butler and C.A. Pearson, Phys. Rev. **129**, 836, 1963;

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## Multi-fragmentation

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J.P. Bondorf, A.S. Botvina, A.S. Iljinov, I.N. Mishustin, and K. Sneppen, Phys. Rep. **257**, 133, 1995;

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A.S. Botvina, N. Buyukcizmeci, and M. Bleicher, Phys. Rev. C **103**, 064602, 2021;

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# Experimental measurements

**RHIC: 3~200 GeV**

**Au+Au, Cu+Cu, Ru+Ru, Zr+Zr, ...**

**LHC: 0.9~13.6 TeV**

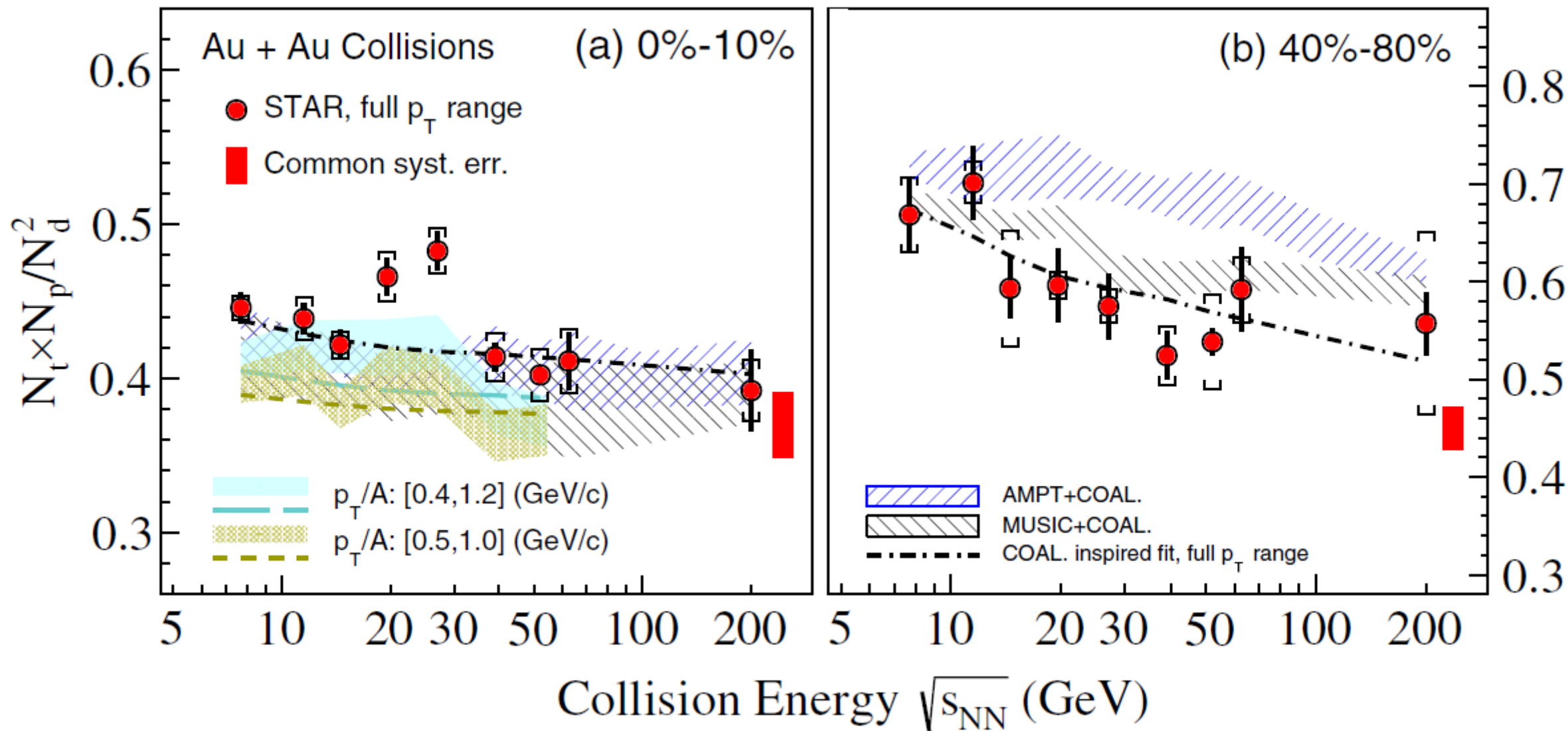
**pp, p+Pb, Xe+Xe, Pb+Pb, ...**

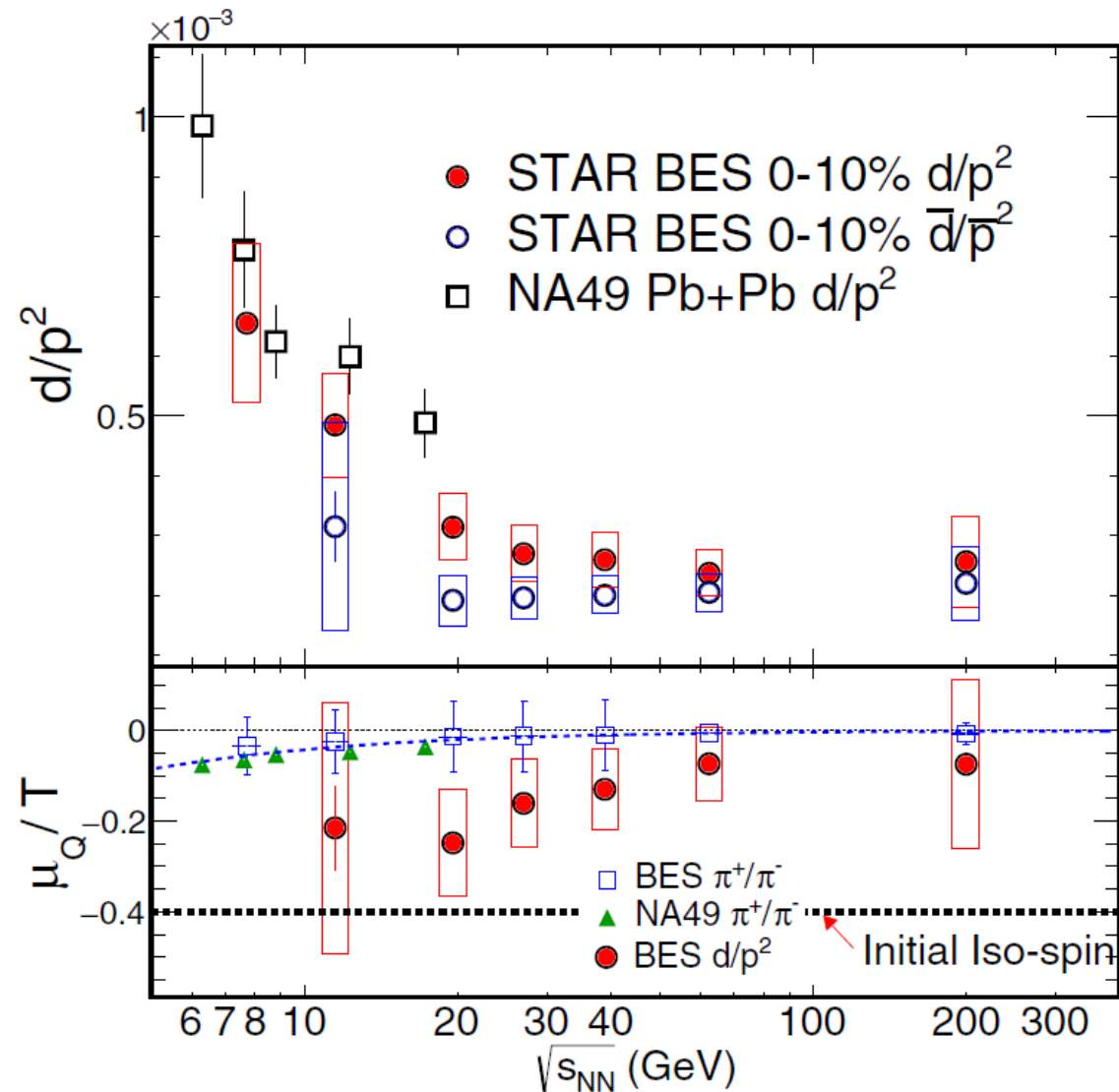
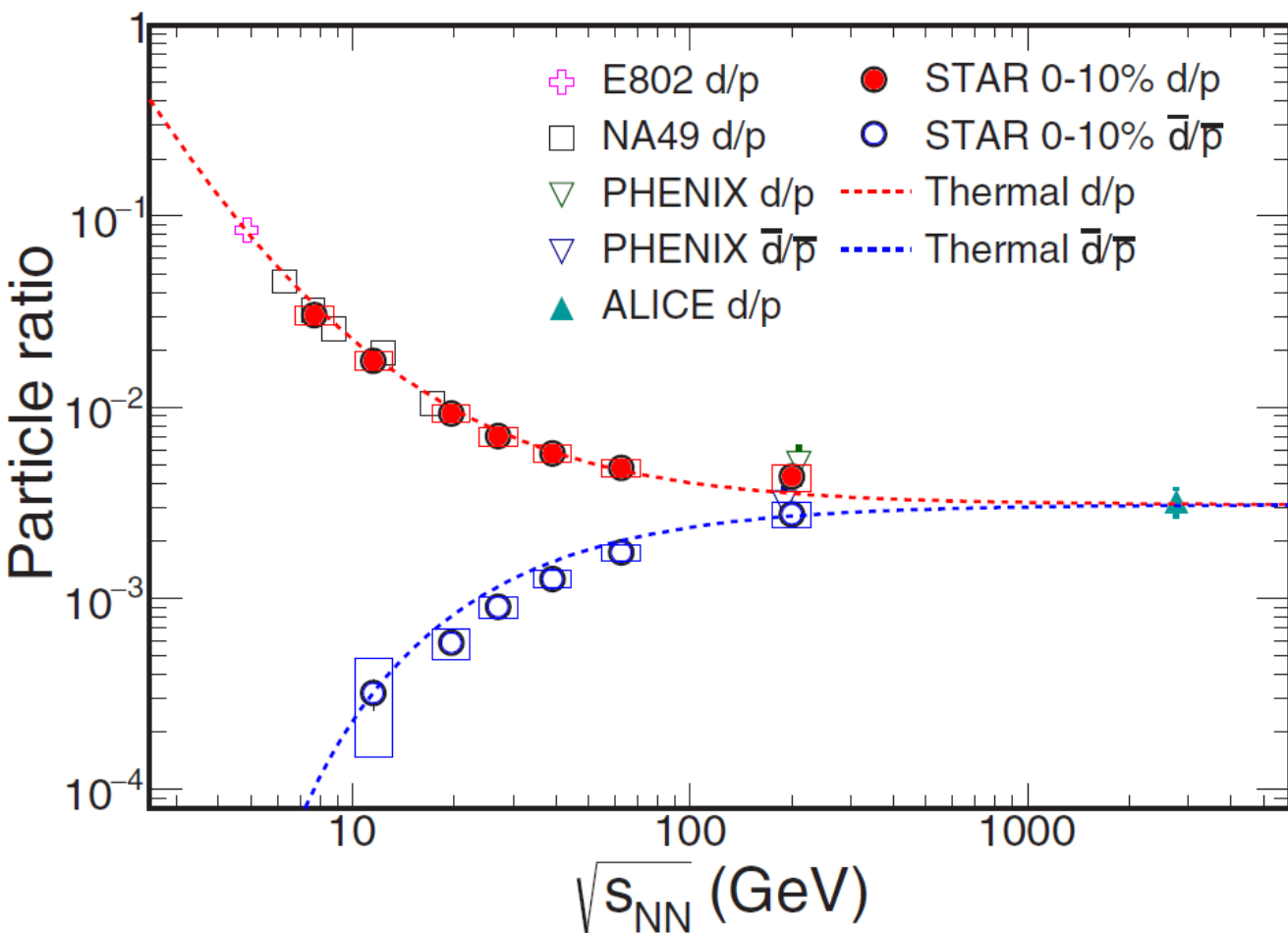
$d, t, {}^3\text{He}, {}^4\text{He}, {}^3_{\Lambda}\text{H}, {}^4_{\Lambda}\text{H}, {}^4_{\Lambda}\text{He}, \dots$

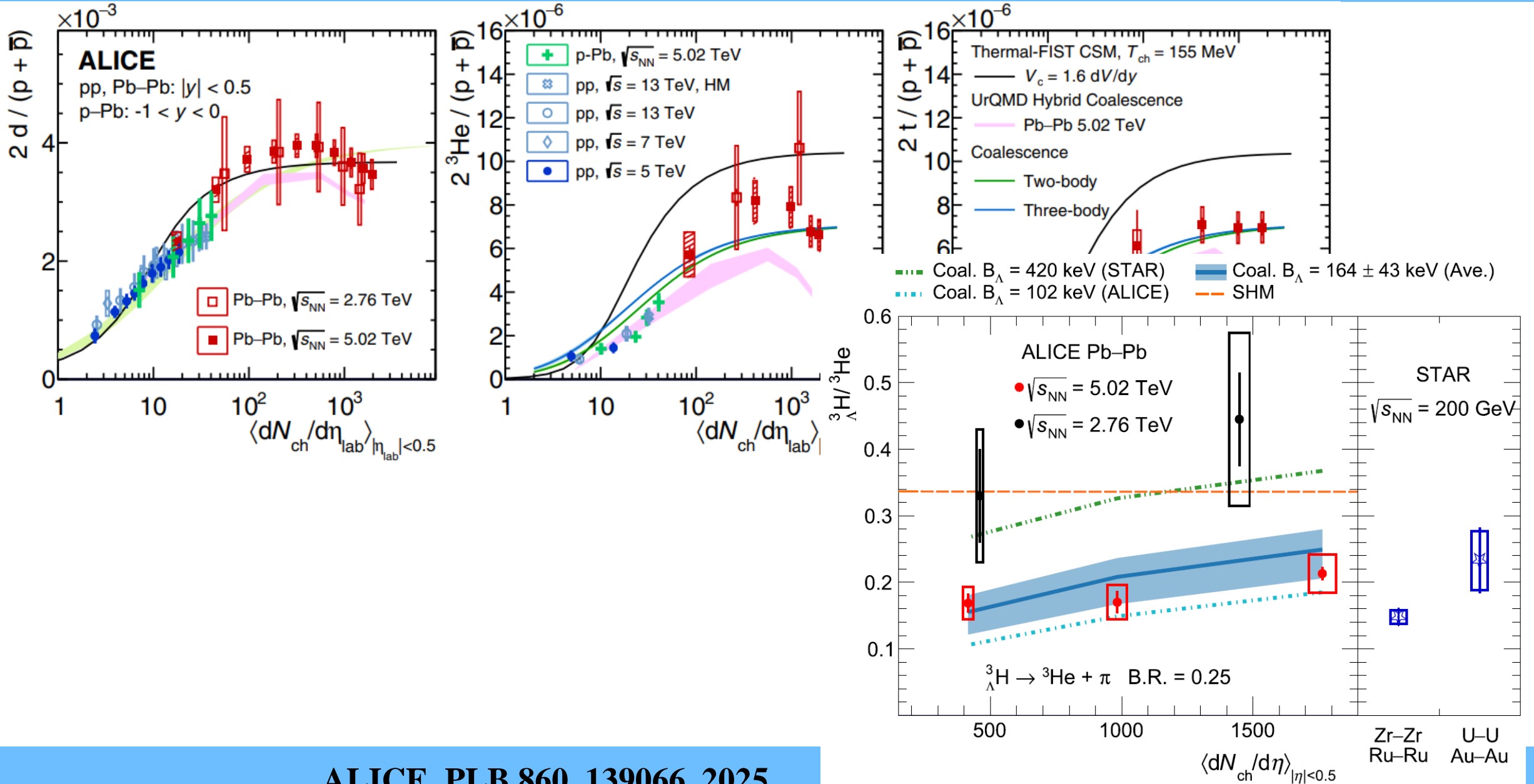
- coalescence factors  $B_2$  and  $B_3$
- $p_T$  spectra
- flows  $v_1, v_2$  and  $v_3$
- yield ratios
- .....

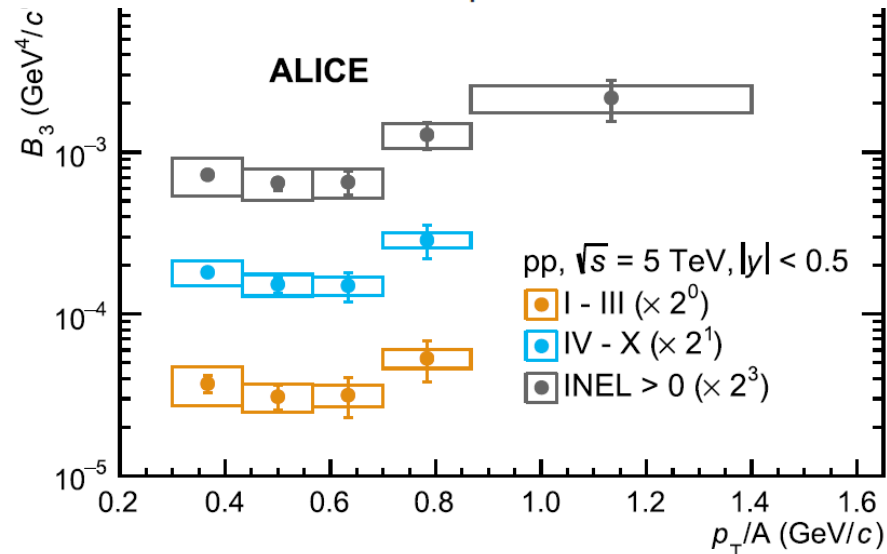
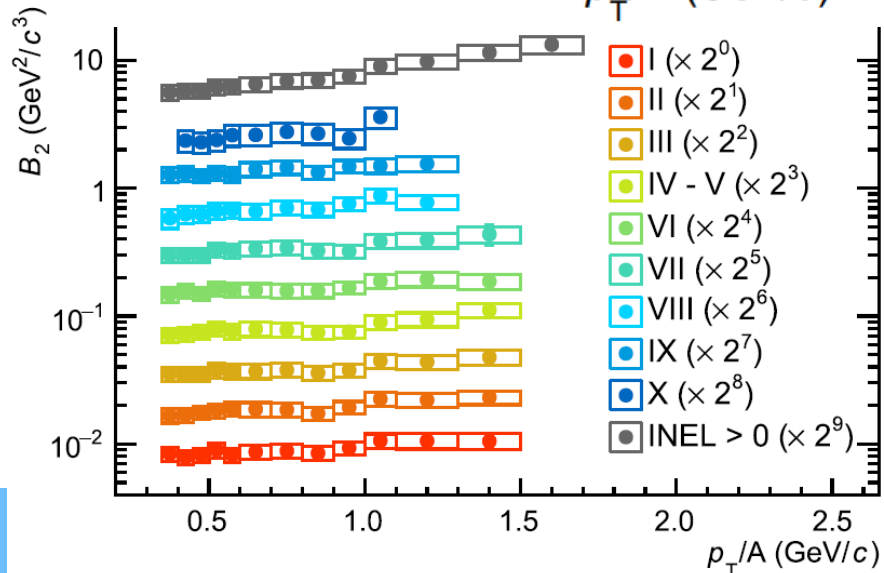
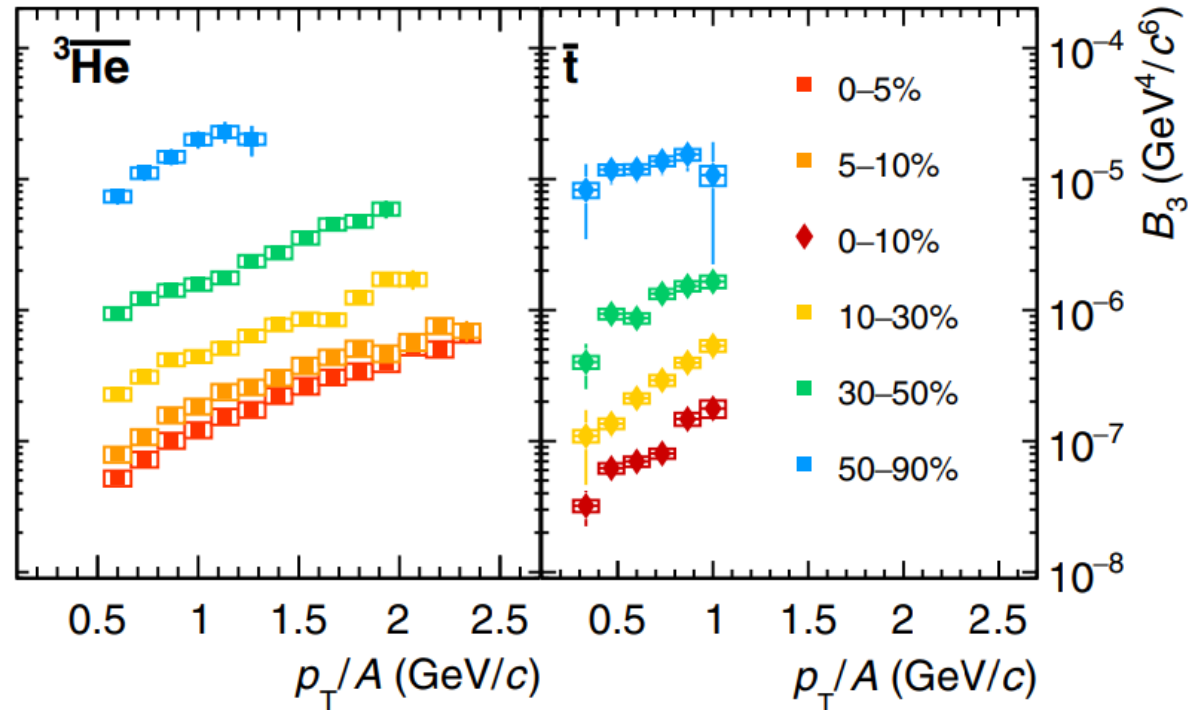
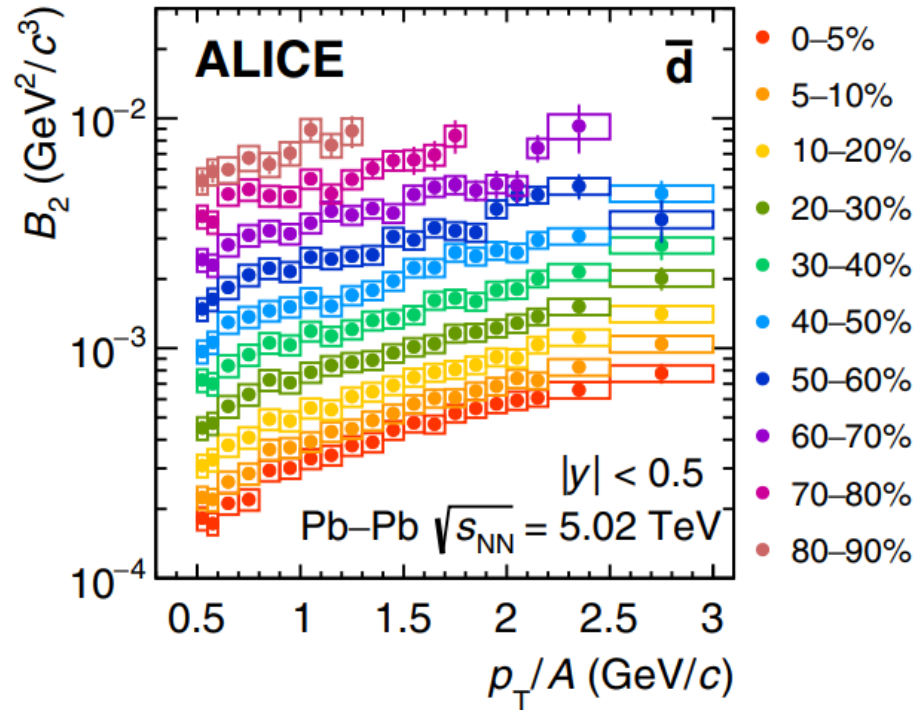
collision energy, system size,  
and  $p_T$  dependence

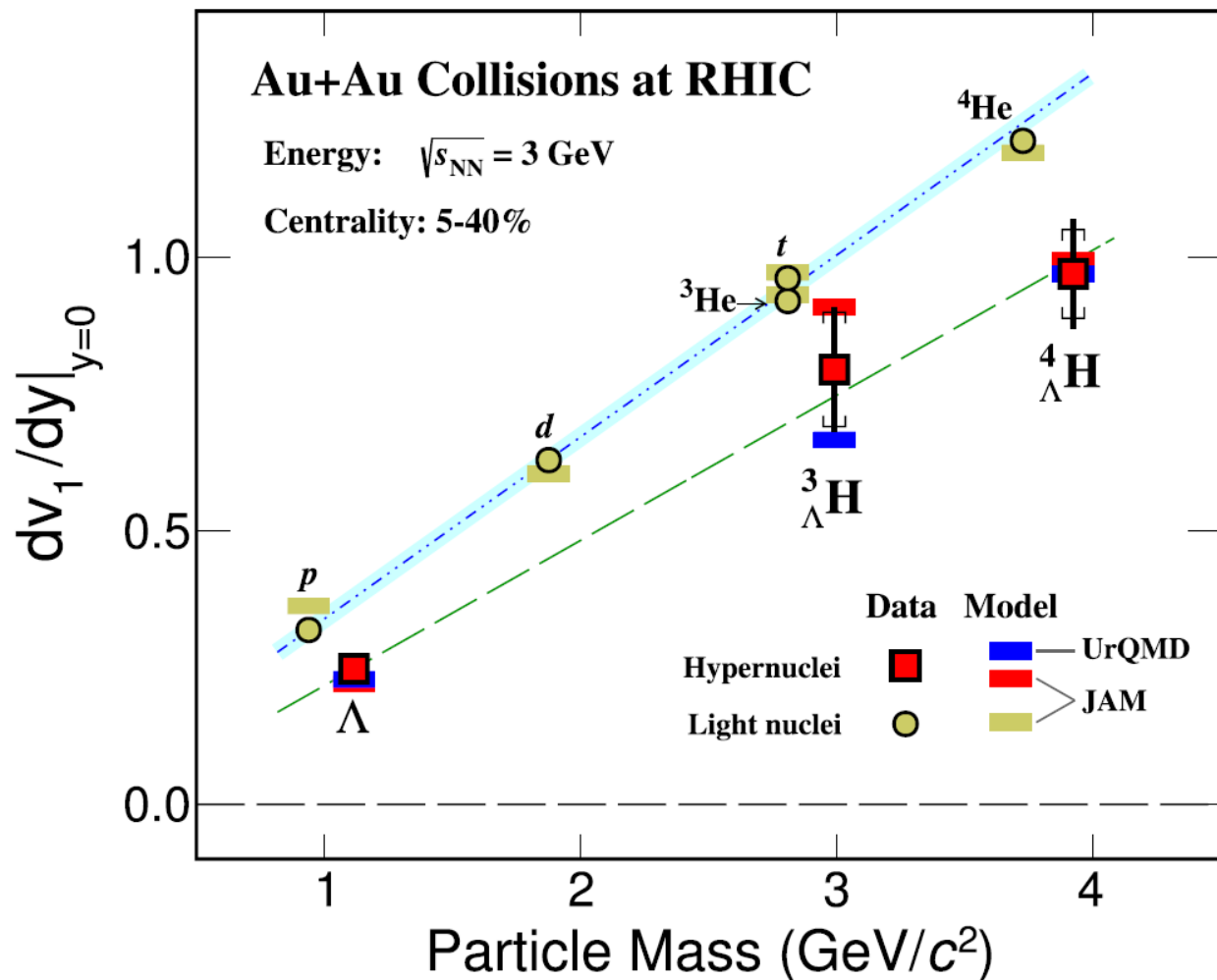
mass number scaling



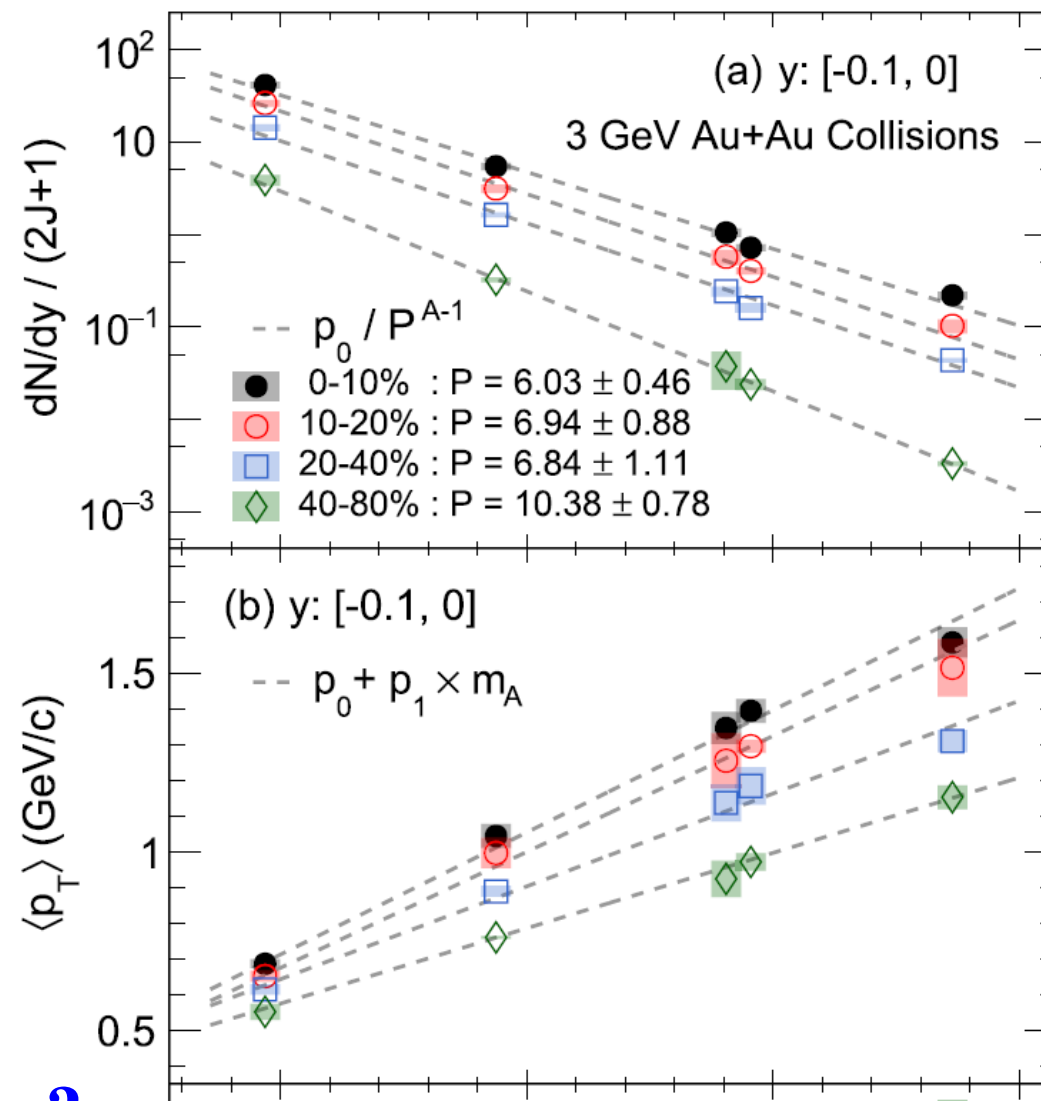








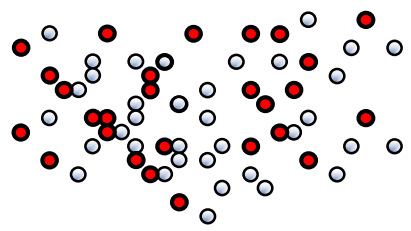
mass number scaling → coalescence ?



# How coalescence works for those RHIC & LHC measurements ?

## Characteristics originated from coalescence itself ?

quark-antiquark system

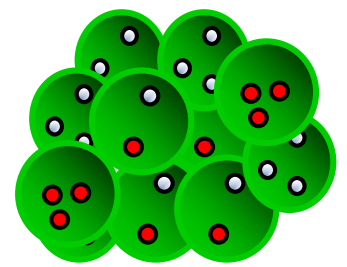


$q = u, d, s, c, b$   
 $\bar{q} = \bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{b}$



Shandong Quark  
Combination Model

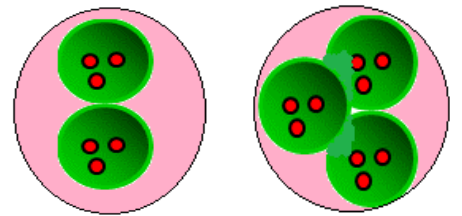
hadronic system



$M(q\bar{q}), B(qqq)$



Coalescence Model



$d, t, {}^3\text{He}, {}^3_{\Lambda}\text{H}, \dots$

an analytical combination/coalescence model

The three-dimensional momentum distribution

$$f_{H_j}(\vec{p}) = \int d\vec{x}_1 d\vec{x}_2 d\vec{p}_1 d\vec{p}_2 f_{h_1 h_2}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2) \mathcal{R}_{H_j}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2, \vec{p})$$

kernel function  $\mathcal{R}_{H_j}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2, \vec{p}) = g_{H_j} \mathcal{R}_{H_j}^{(x,p)}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2) \delta\left(\sum_{i=1}^2 \vec{p}_i - \vec{p}\right)$

$$\mathcal{R}_{H_j}^{(x,p)}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2) = 8e^{-\frac{(\vec{x}'_1 - \vec{x}'_2)^2}{2\sigma_{21}^2}} e^{-\frac{2\sigma_{21}^2(m_2\vec{p}'_1 - m_1\vec{p}'_2)^2}{(m_1 + m_2)^2}}$$

two-hadron joint coordinate-momentum distribution

$$f_{h_1 h_2}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2) = N_{h_1 h_2} f_{h_1 h_2}^{(n)}(\vec{x}_1, \vec{x}_2; \vec{p}_1, \vec{p}_2) = N_{h_1 h_2} f_{h_1 h_2}^{(n)}(\vec{x}_1, \vec{x}_2) f_{h_1 h_2}^{(n)}(\vec{p}_1, \vec{p}_2)$$



**assumption 1: coordinate-momentum factorization**

# two-body coalescence $h_1 + h_2 \rightarrow H_j$

## The three-dimensional momentum distribution

$$f_{H_j}(\vec{p}) = N_{h_1 h_2} g_{H_j} A_{H_j} M_{H_j}(\vec{p})$$

$$\vec{X} = \frac{\sqrt{2}(m_1 \vec{x}_1 + m_2 \vec{x}_2)}{m_1 + m_2}, \quad \vec{r} = \frac{\vec{x}_1 - \vec{x}_2}{\sqrt{2}}$$

$$f_{h_1 h_2}^{(n)}(\vec{r}) = \frac{1}{(\pi C_1 R_f^2)^{3/2}} e^{-\frac{\vec{r}^2}{C_1 R_f^2}}$$

$$A_{H_j} = 8 \int d\vec{x}_1 d\vec{x}_2 f_{h_1 h_2}^{(n)}(\vec{x}_1, \vec{x}_2) e^{-\frac{(\vec{x}_1 - \vec{x}_2)^2}{2\sigma_{21}^2}} = \frac{8\sigma_{21}^3}{(C_1 R_f^2 + \sigma_{21}^2) \sqrt{C_1 (R_f / \gamma)^2 + \sigma_{21}^2}}$$

$$M_{H_j}(\vec{p}) = \int d\vec{p}_1 d\vec{p}_2 f_{h_1 h_2}^{(n)}(\vec{p}_1, \vec{p}_2) e^{-\frac{2\sigma_{21}^2 (m_2 \vec{p}_1 - m_1 \vec{p}_2)^2}{(m_1 + m_2)^2}} \delta\left(\sum_{i=1}^2 \vec{p}_i - \vec{p}\right) = \left(\frac{\sqrt{\pi}}{\sqrt{2}\sigma_{21}}\right)^3 \gamma f_{h_1 h_2}\left(\frac{m_1 \vec{p}}{m_1 + m_2}, \frac{m_2 \vec{p}}{m_1 + m_2}\right)$$

**↑ equal-velocity combination  
assumption 2: delta function approximation**

## two-body coalescence $h_1 + h_2 \rightarrow H_j$

### The three-dimensional momentum distribution

$$f_{H_j}(\vec{p}) = \frac{8\pi^{3/2} g_{H_j} \gamma}{2^{3/2} (C_1 R_f^2 + \sigma_{21}^2) \sqrt{C_1 (R_f / \gamma)^2 + \sigma_{21}^2}} f_{h_1}\left(\frac{m_1 \vec{p}}{m_1 + m_2}\right) f_{h_2}\left(\frac{m_2 \vec{p}}{m_1 + m_2}\right)$$

### The invariant momentum distribution

$$f_{H_j}^{(\text{inv})}(p_T, y) = \frac{8\pi^{3/2} g_{H_j}}{2^{3/2} (C_1 R_f^2 + \sigma_{21}^2) \sqrt{C_1 (R_f / \gamma)^2 + \sigma_{21}^2}} \frac{m_{H_j}}{m_1 m_2} f_{h_1}^{(\text{inv})}\left(\frac{m_1 p_T}{m_1 + m_2}, y\right) f_{h_2}^{(\text{inv})}\left(\frac{m_2 p_T}{m_1 + m_2}, y\right)$$

## $N$ -body coalescence $h_1 + h_2 + \dots + h_N \rightarrow H_j$

$$f_{H_j}^{(\text{inv})}(p_T, y) = \frac{8^{N-1} \pi^{3(N-1)/2} g_{H_j} m_{H_j}}{N^{3/2} \prod_{k=1}^{N-1} (C_k R_f^2 + \sigma_{Nk}^2) \sqrt{C_k (R_f / \gamma)^2 + \sigma_{Nk}^2}} \prod_{i=1}^N \frac{1}{m_i} f_{h_i}^{(\text{inv})}\left(\frac{m_i p_T}{m_1 + m_2 + \dots + m_N}, y\right)$$

### III. Applications in heavy-ion collisions at RHIC and LHC

- Coalescence factor  $B_A$  ( $B_2$  &  $B_3$ )

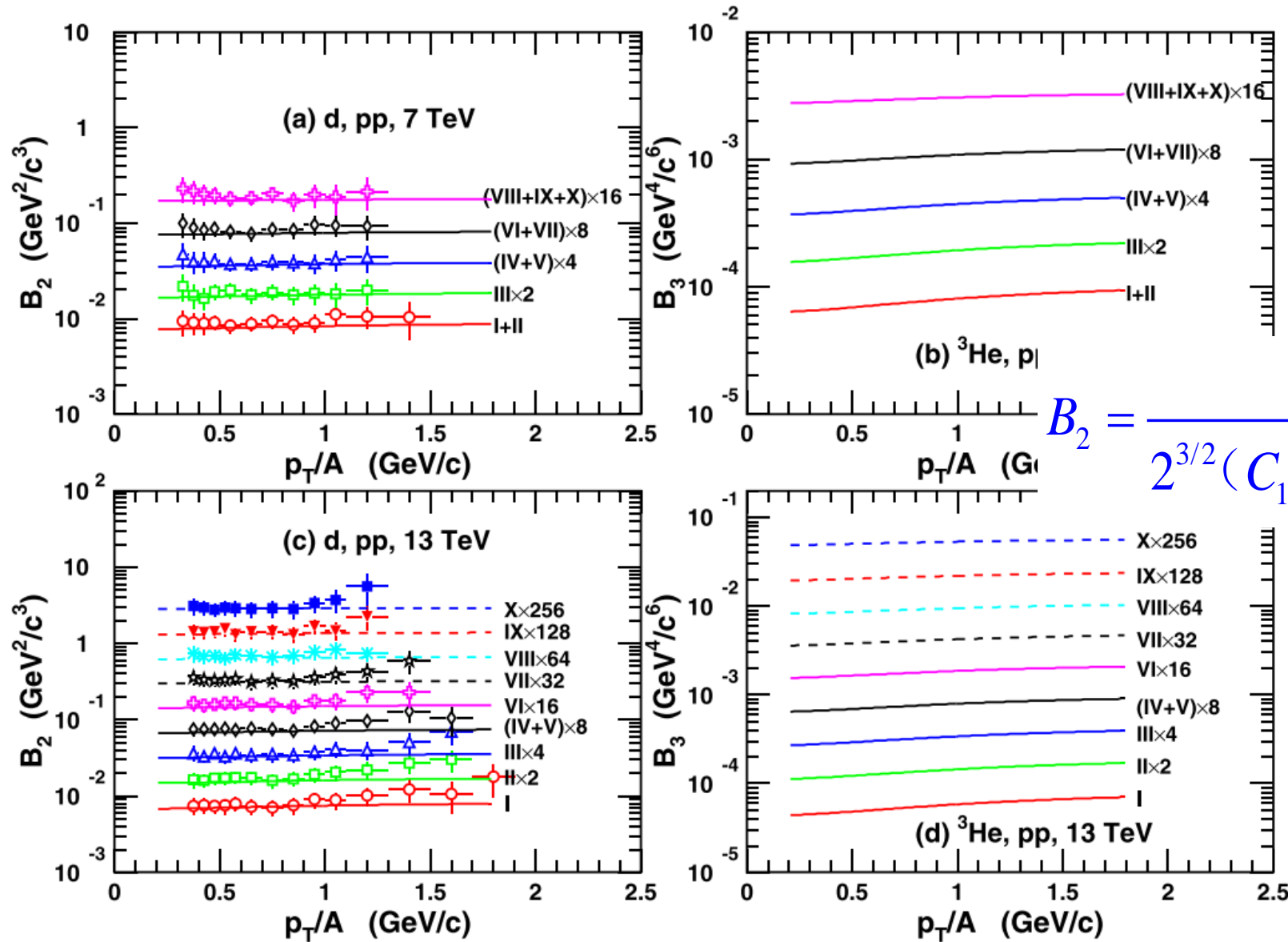
**independent of primordial nucleon/hyperon momentum distributions**

- $p_T$  spectra

**need primordial nucleon/hyperon momentum distributions as inputs**

- Production correlation of different light (hyper-)nuclei

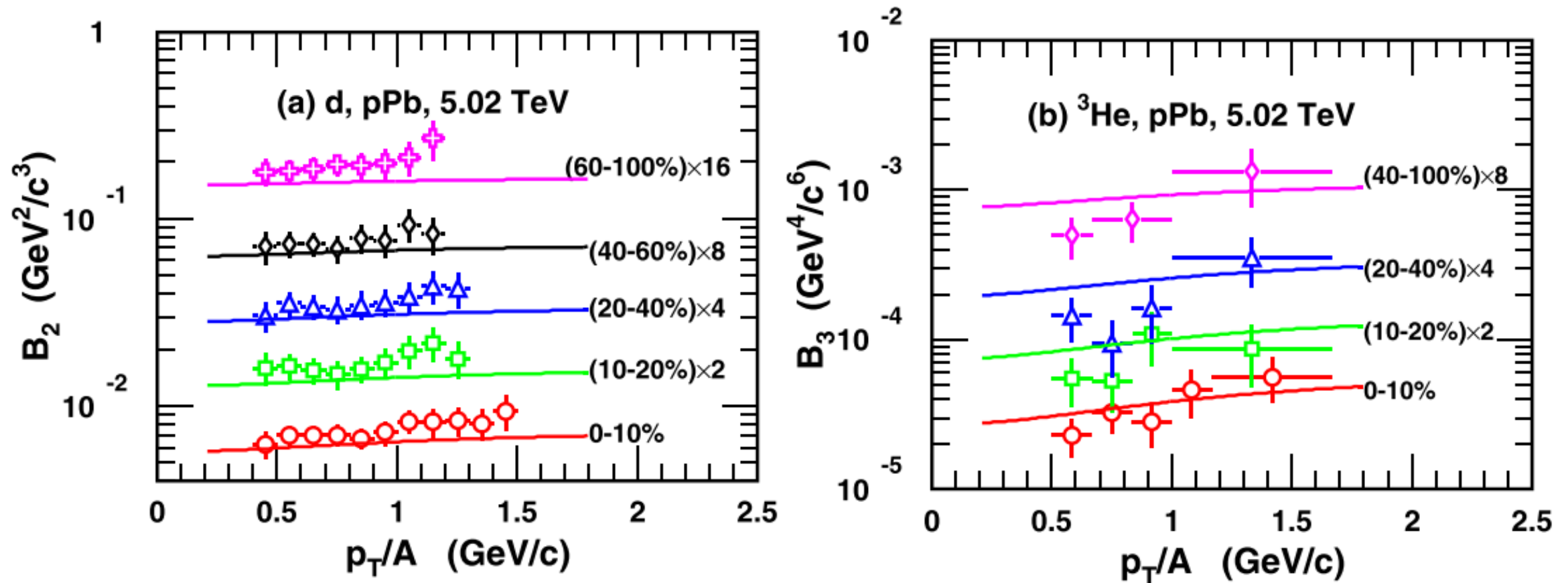
# $B_A$ as function of $p_T/A$ in pp collisions at LHC



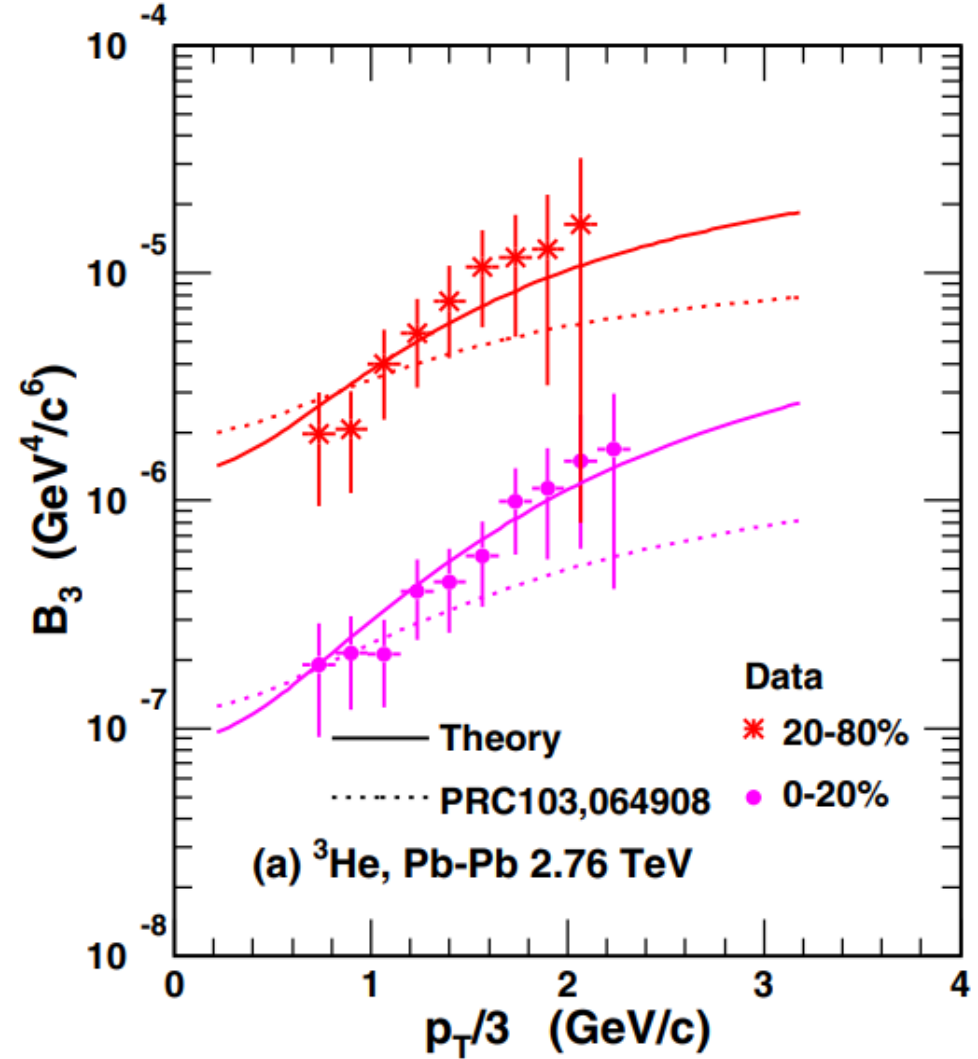
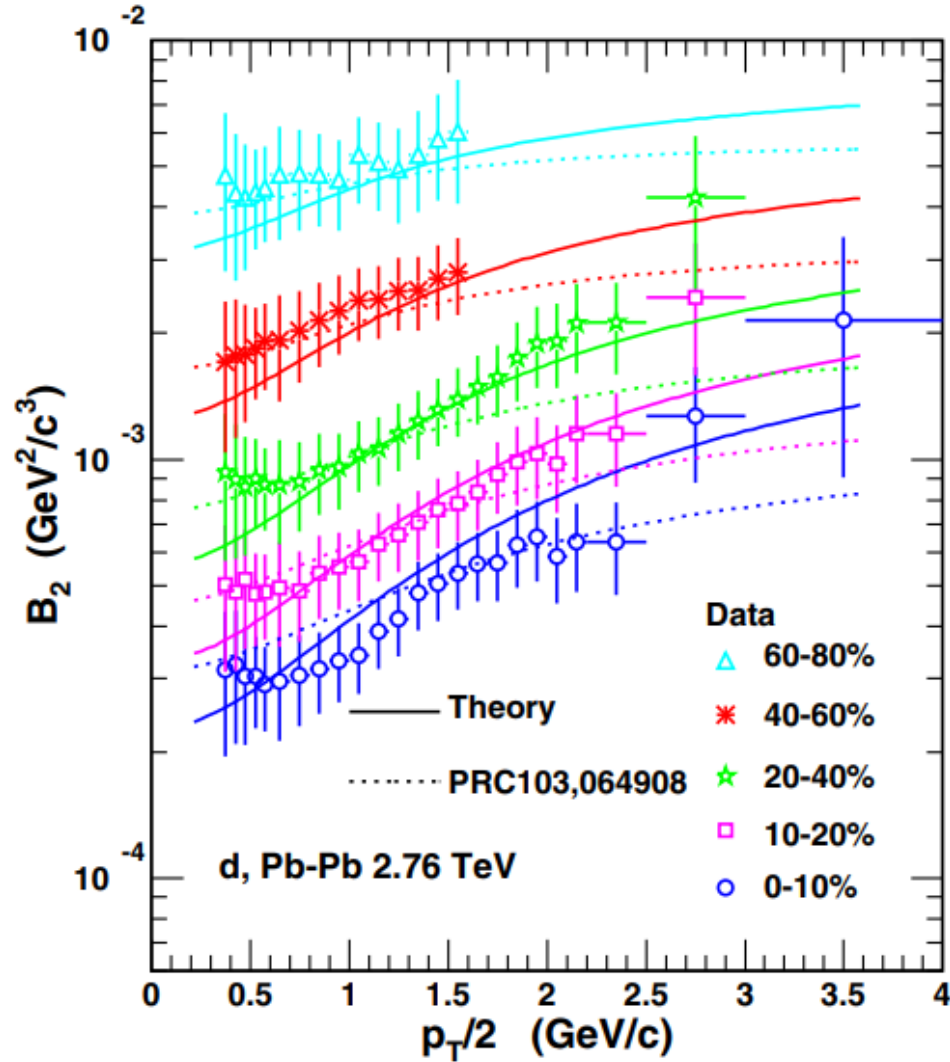
$$B_2 = \frac{8\pi^{3/2} g_d}{2^{3/2} (C_1 R_f^2 + \sigma_d^2) \sqrt{C_1 (R_f / \gamma)^2 + \sigma_d^2}} \frac{m_d}{m_p m_n}$$

weak  $p_T$  dependence of  $B_2$  and  $B_3$  in pp collisions

# $B_A$ as function of $p_T/A$ in p-Pb collisions at LHC



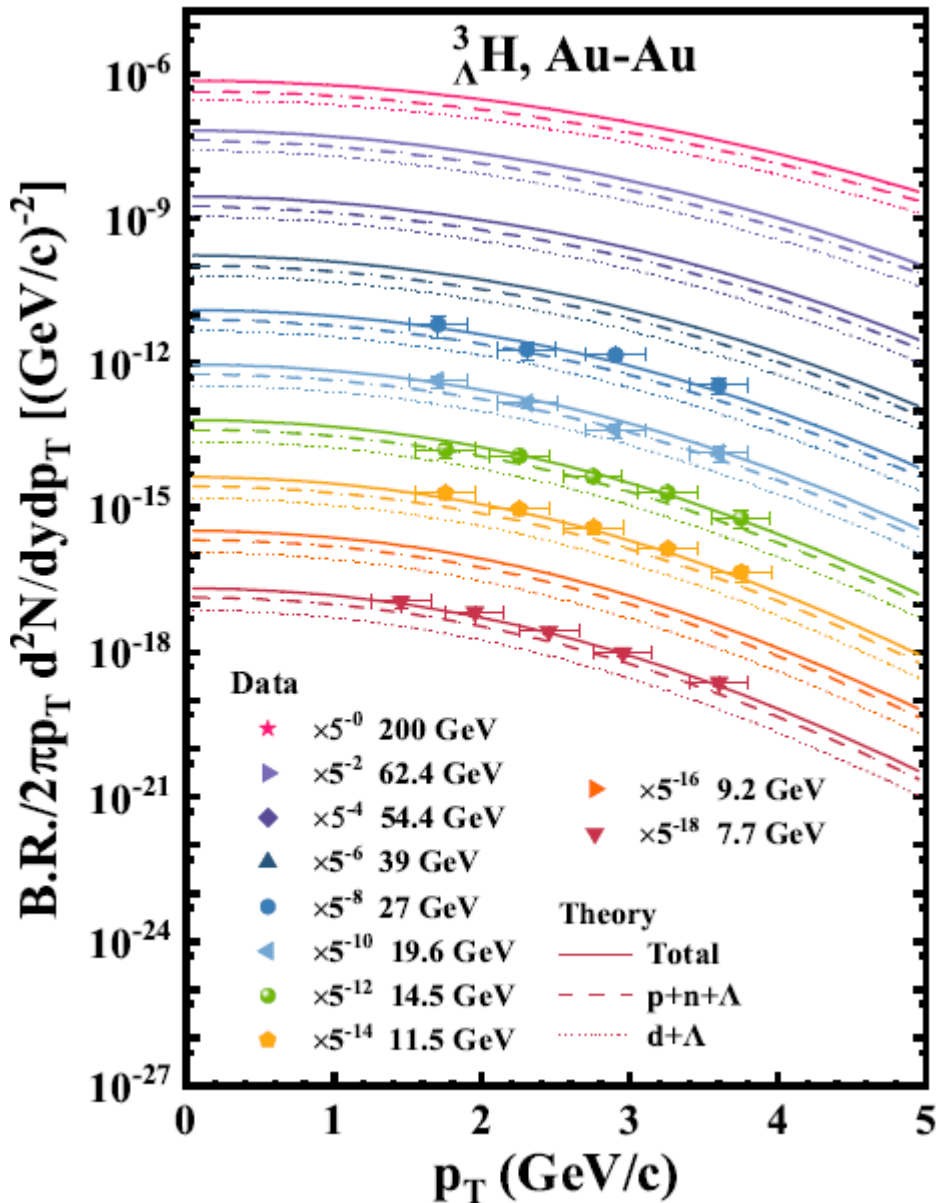
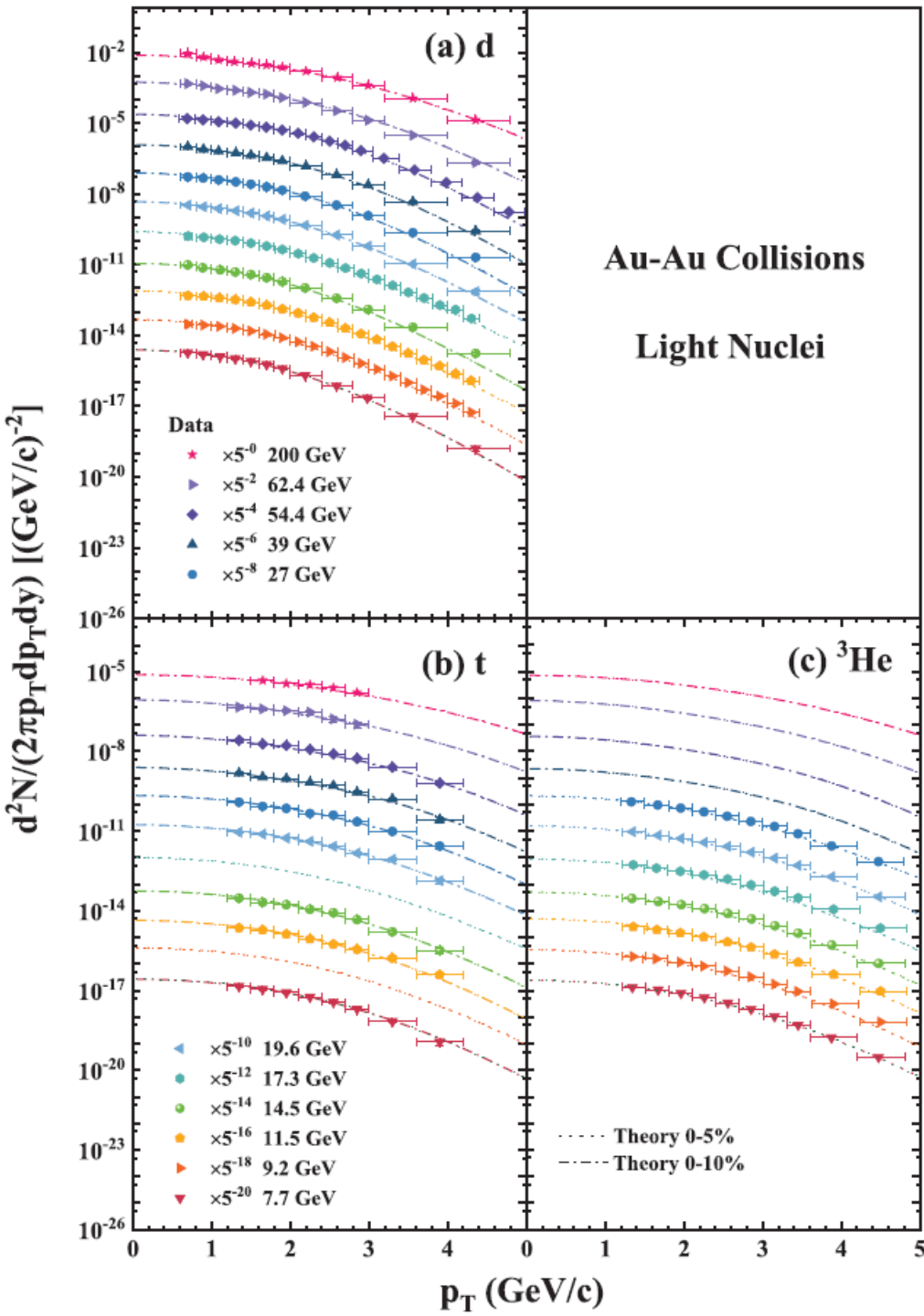
# $B_A$ as function of $p_T/A$ in Pb-Pb collisions at LHC



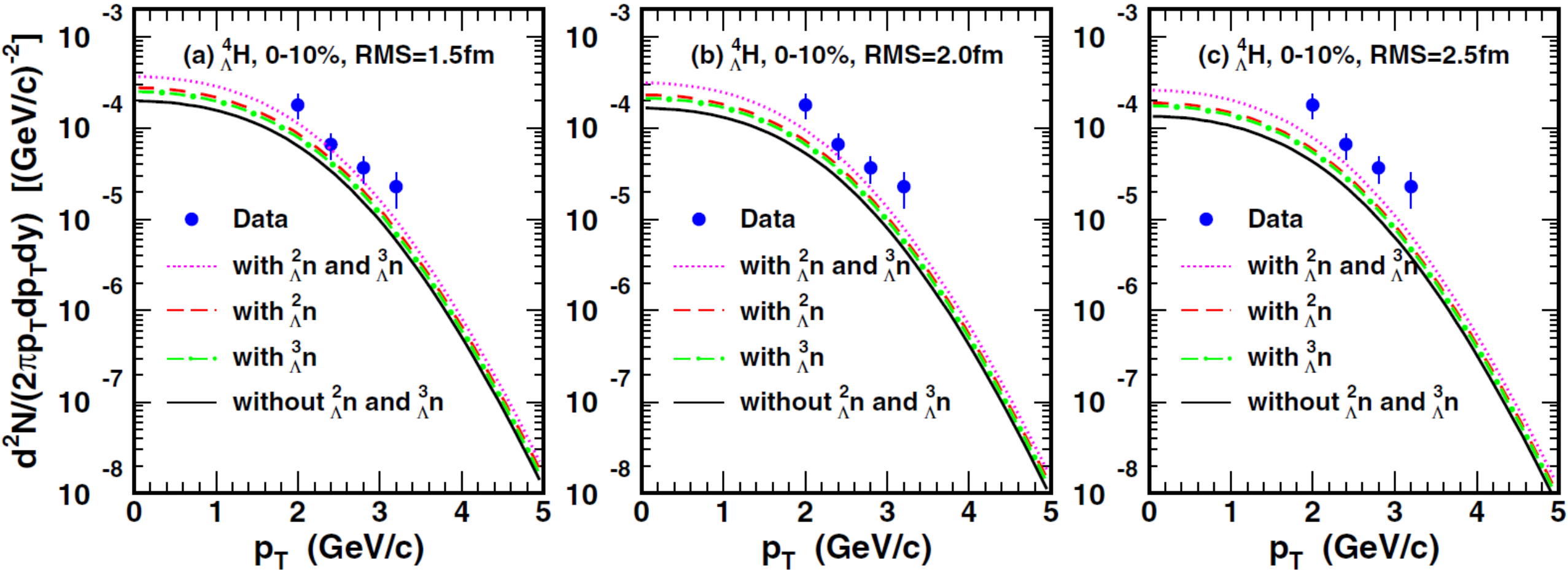
obvious  $p_T$  dependence of  $B_2$  and  $B_3$  in large Pb-Pb collision system

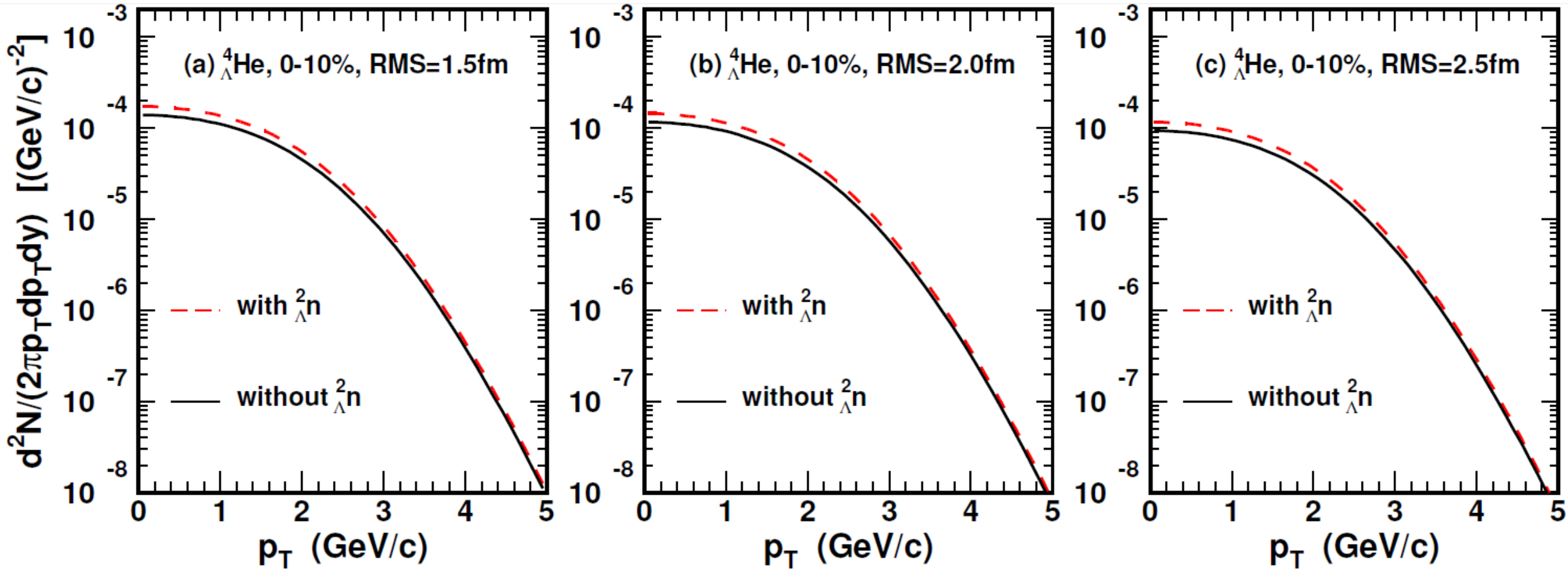
**From results of  $B_A$ , we find:**

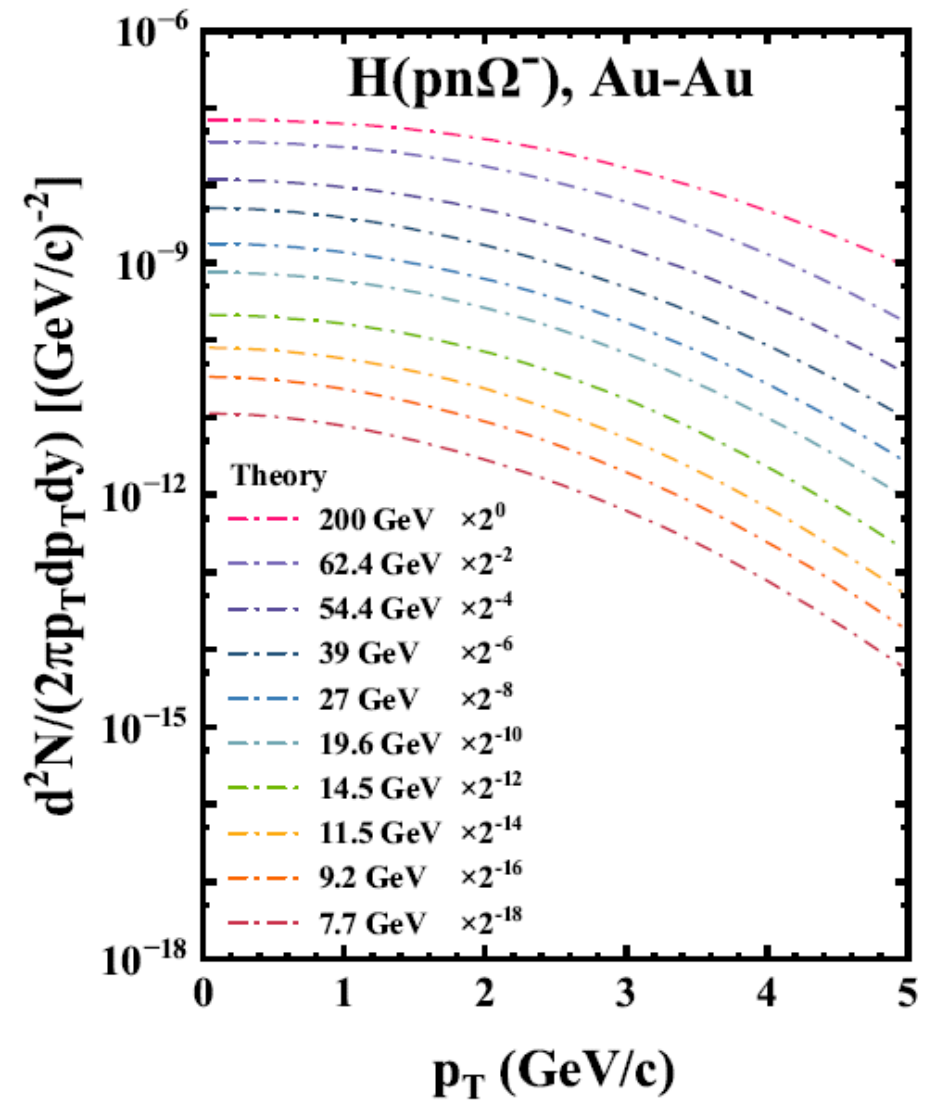
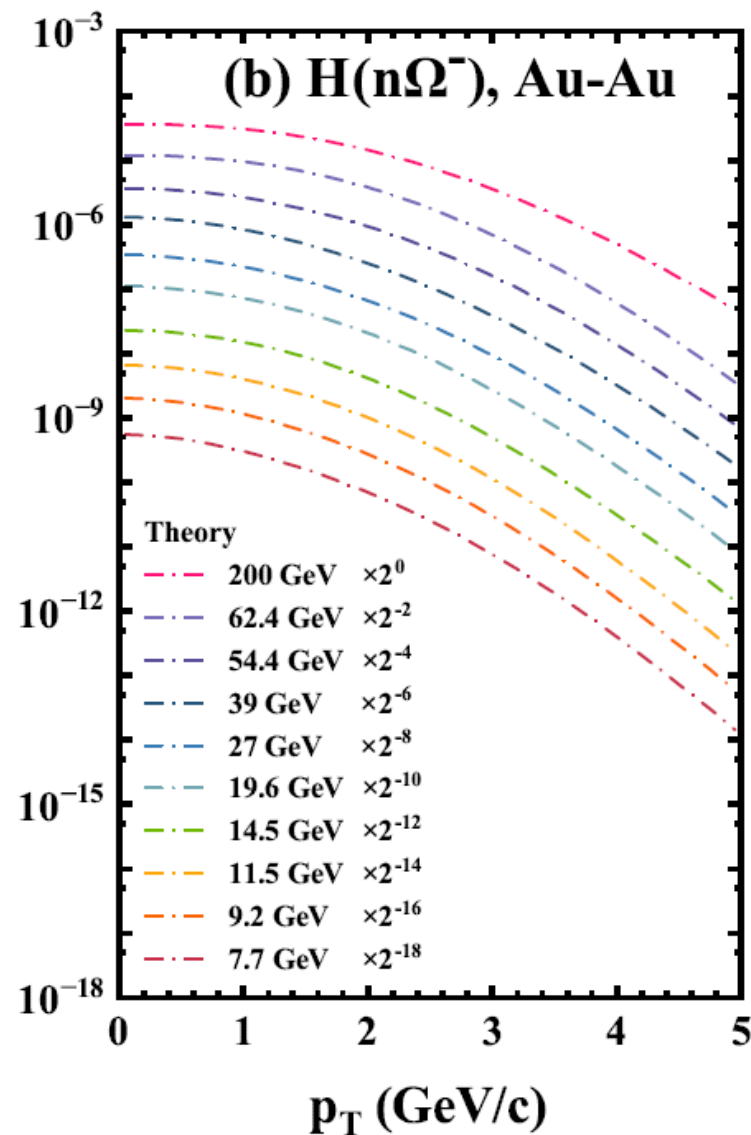
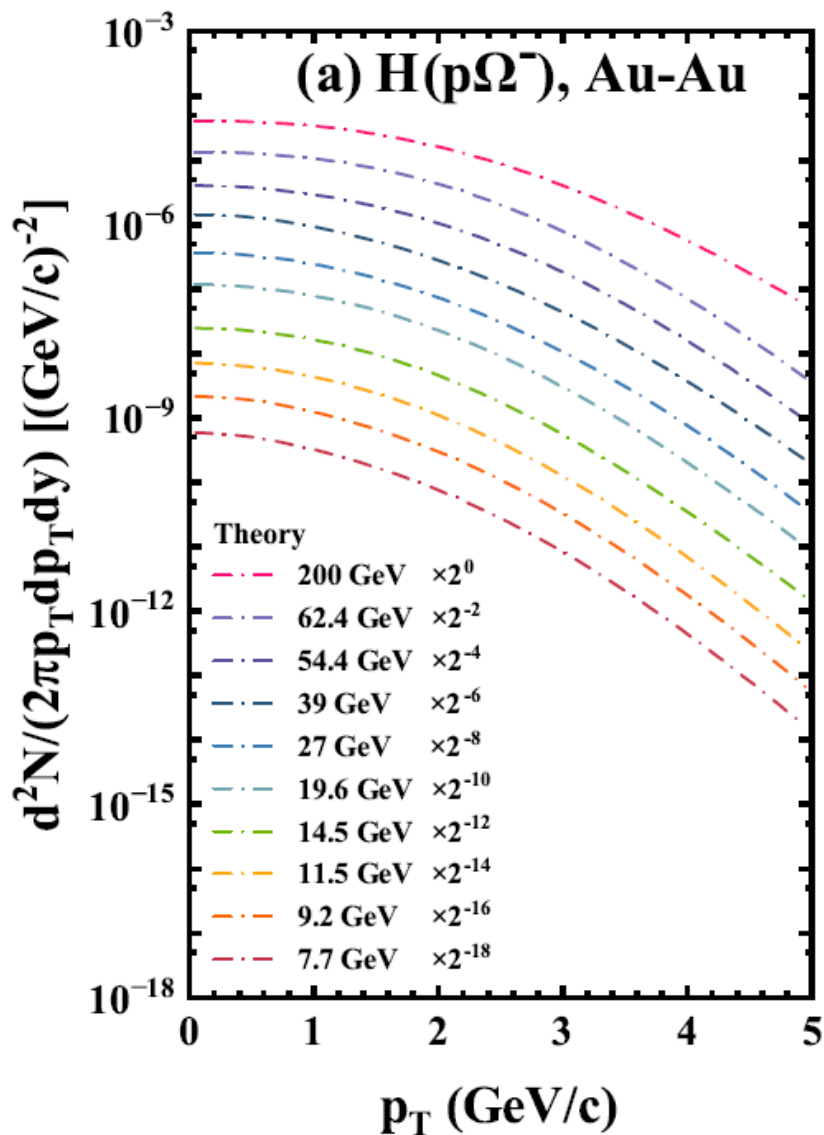
- **Analytical coalescence model can naturally give obvious growth of  $B_A$  against  $p_T$  in Pb-Pb collisions but relatively weak  $p_T$  dependence in pp and p-Pb collisions.**
- **Coordinate-momentum correlation is necessary in large Pb-Pb collisions.**

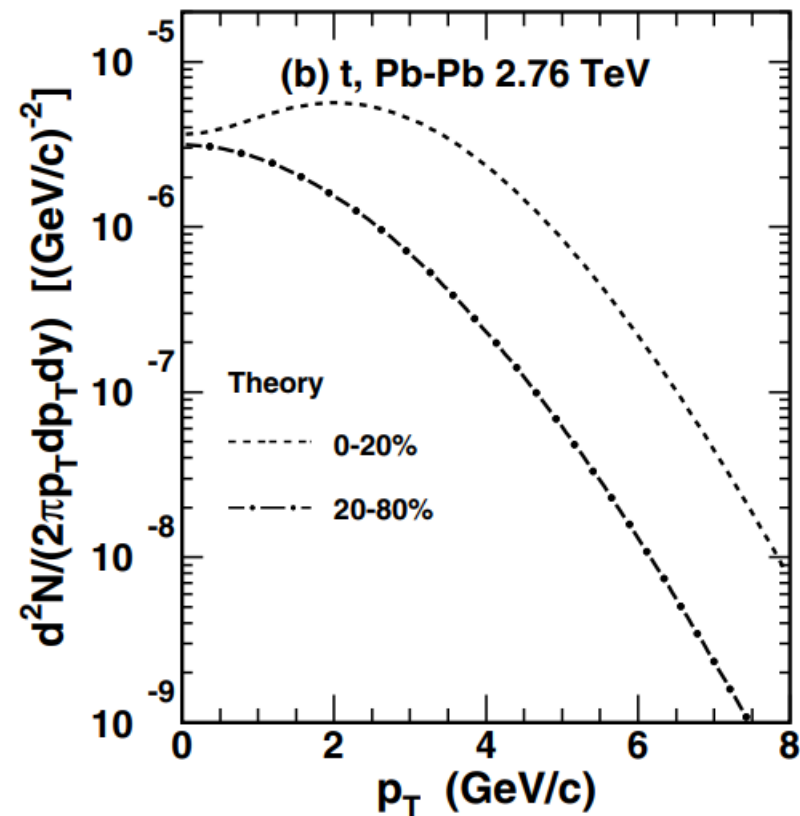
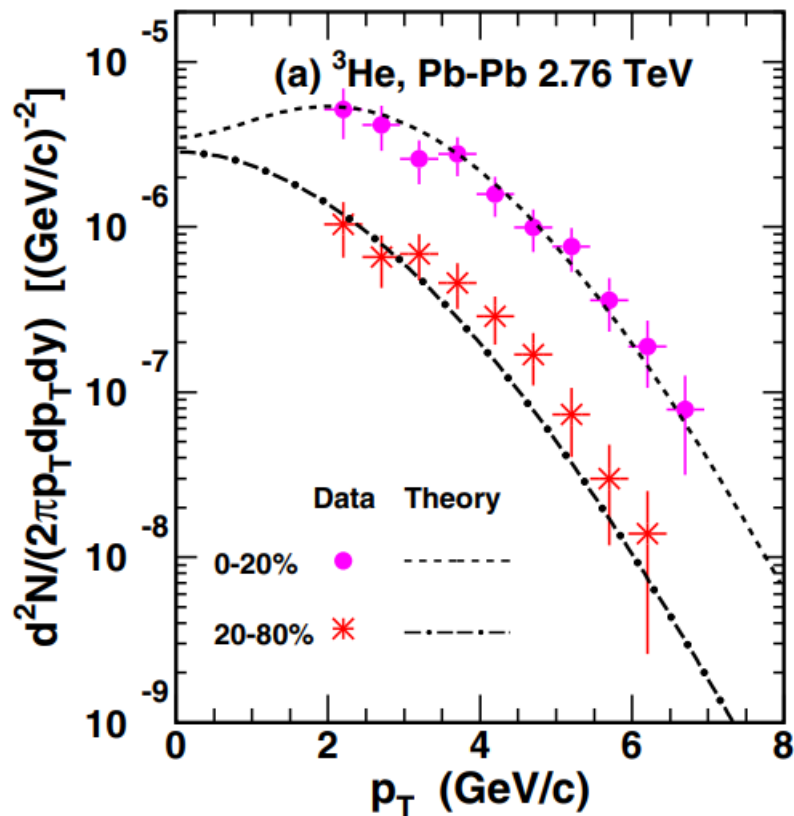
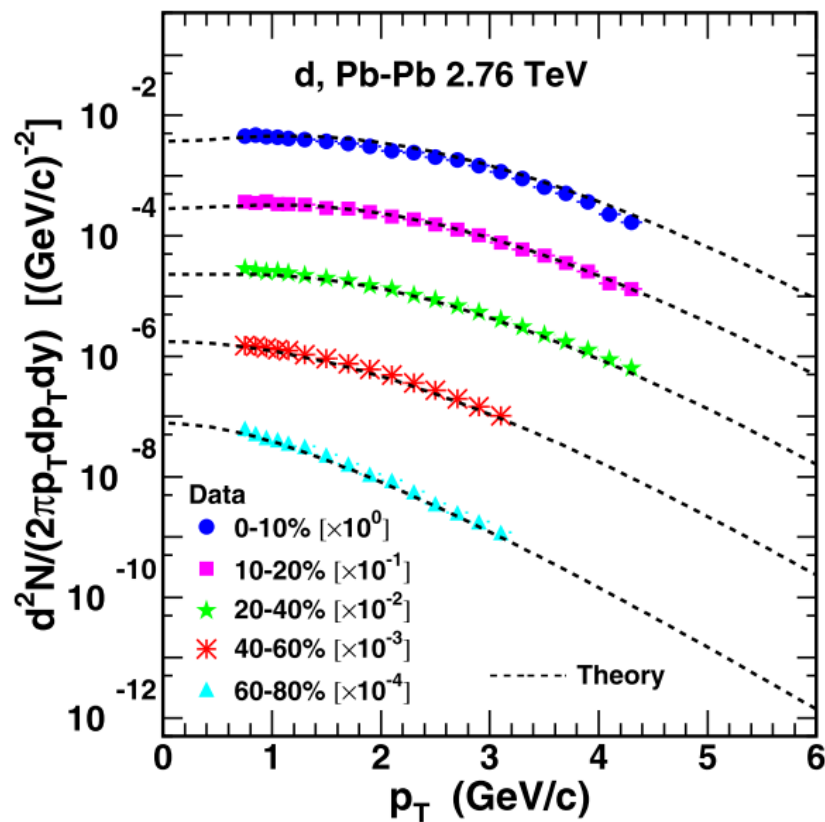


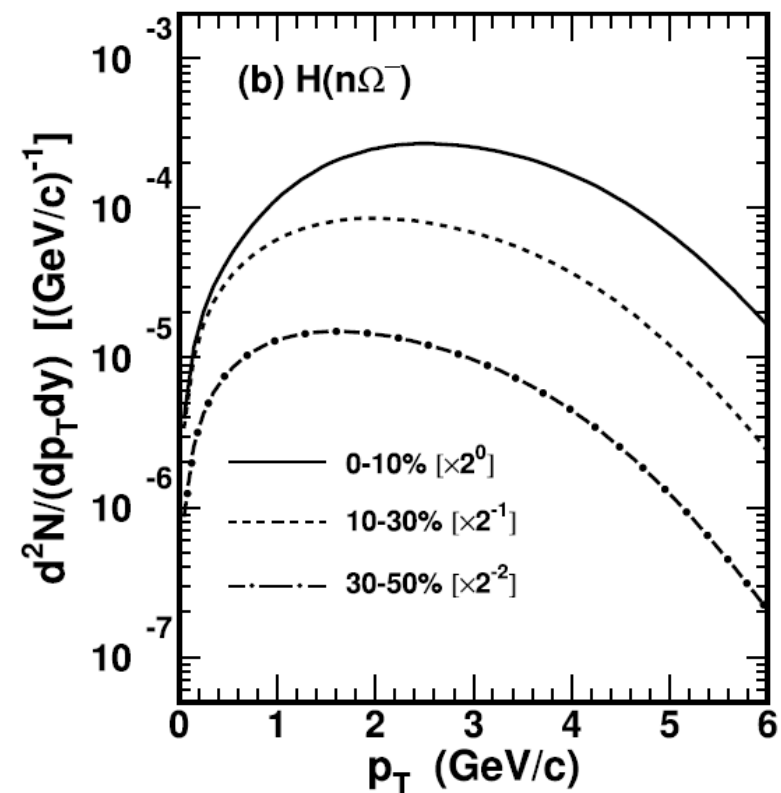
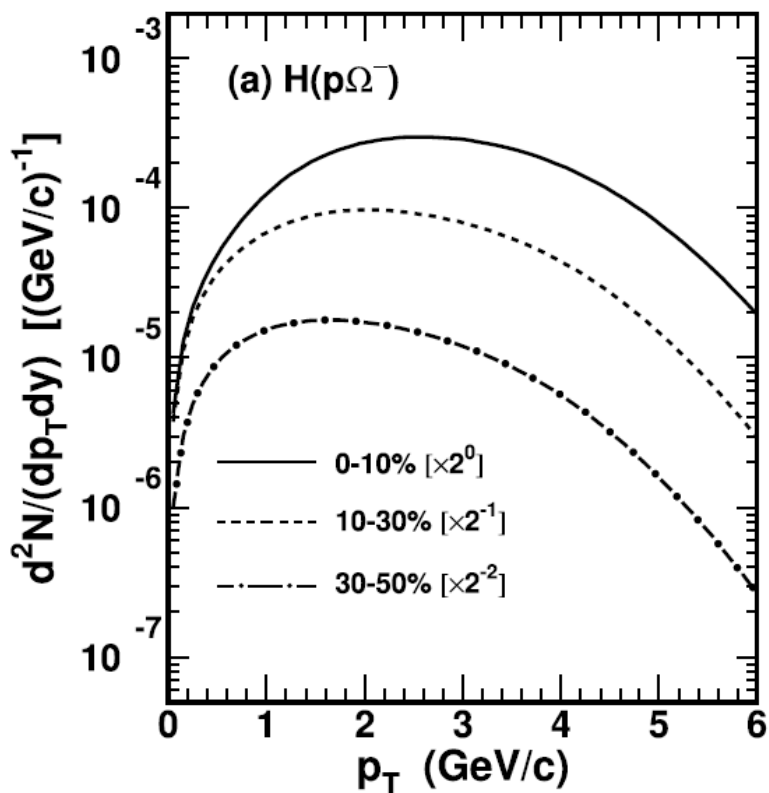
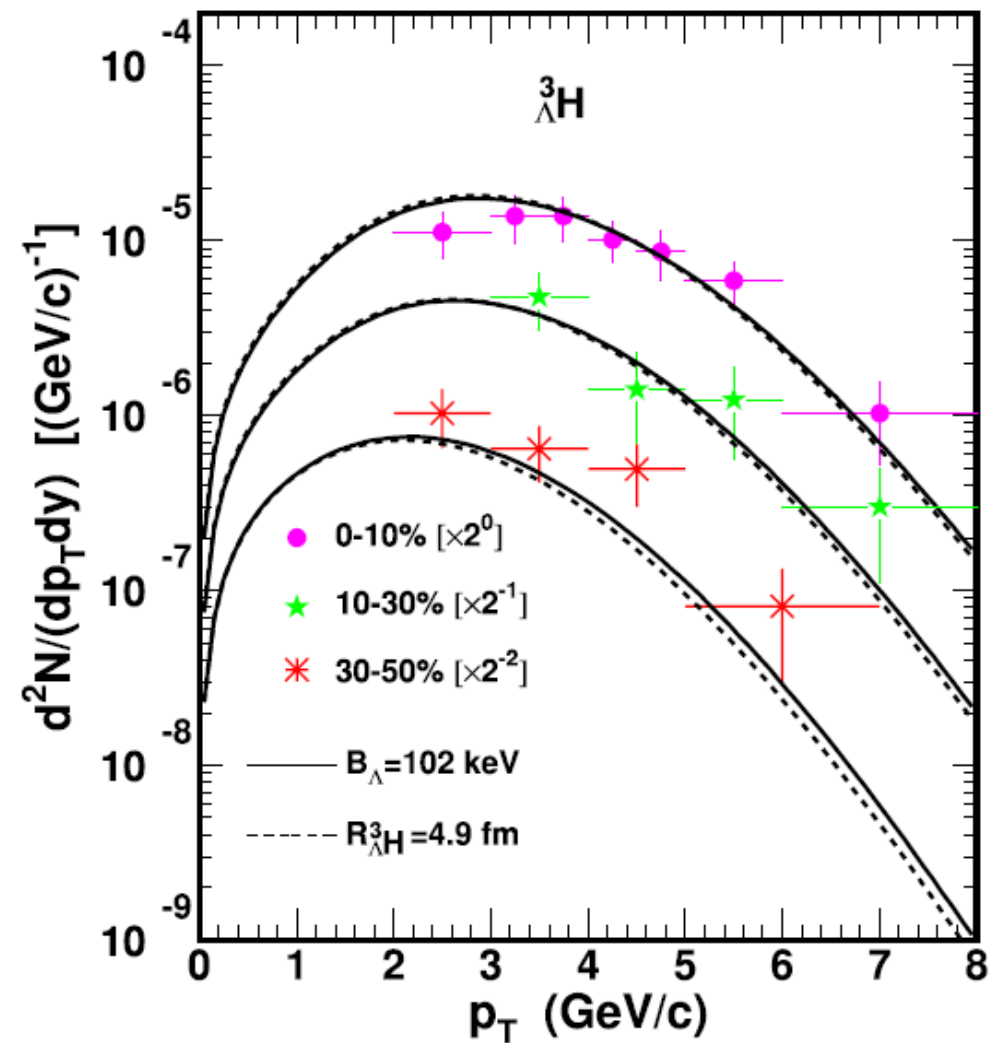
favor halo structure  
with binding energy  
410 keV





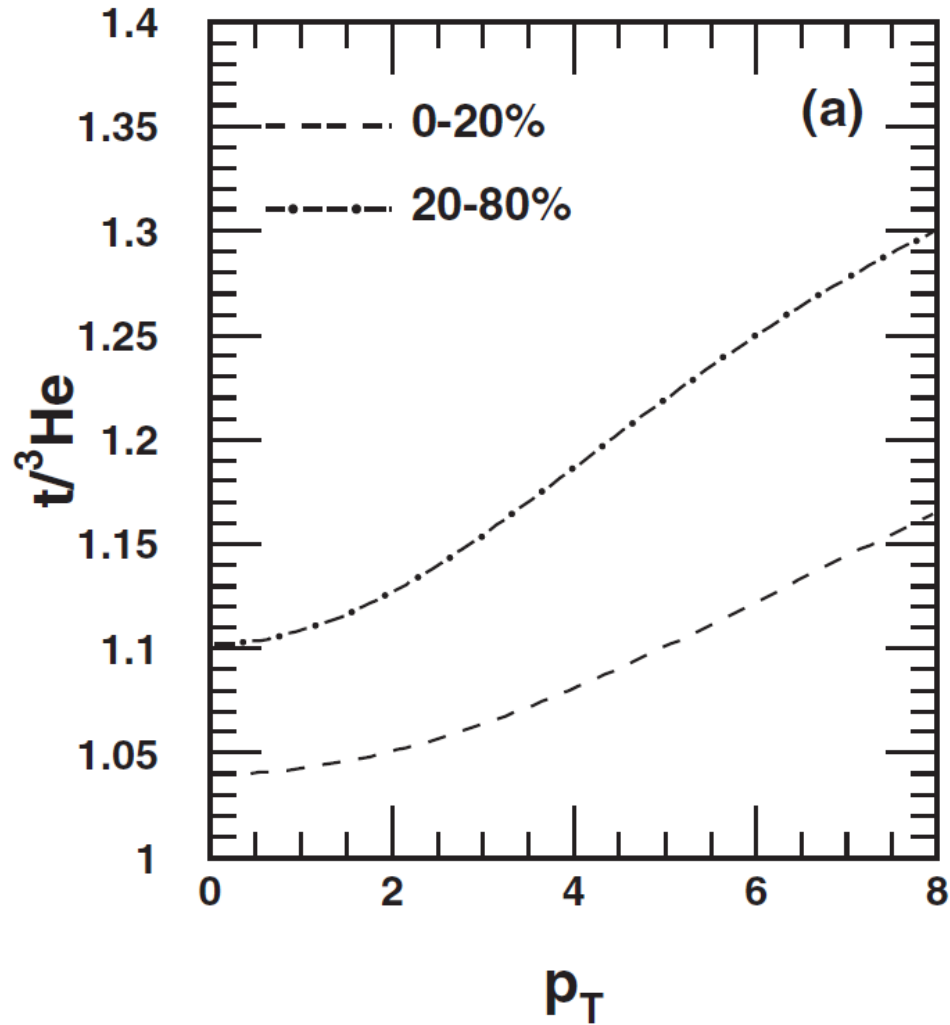




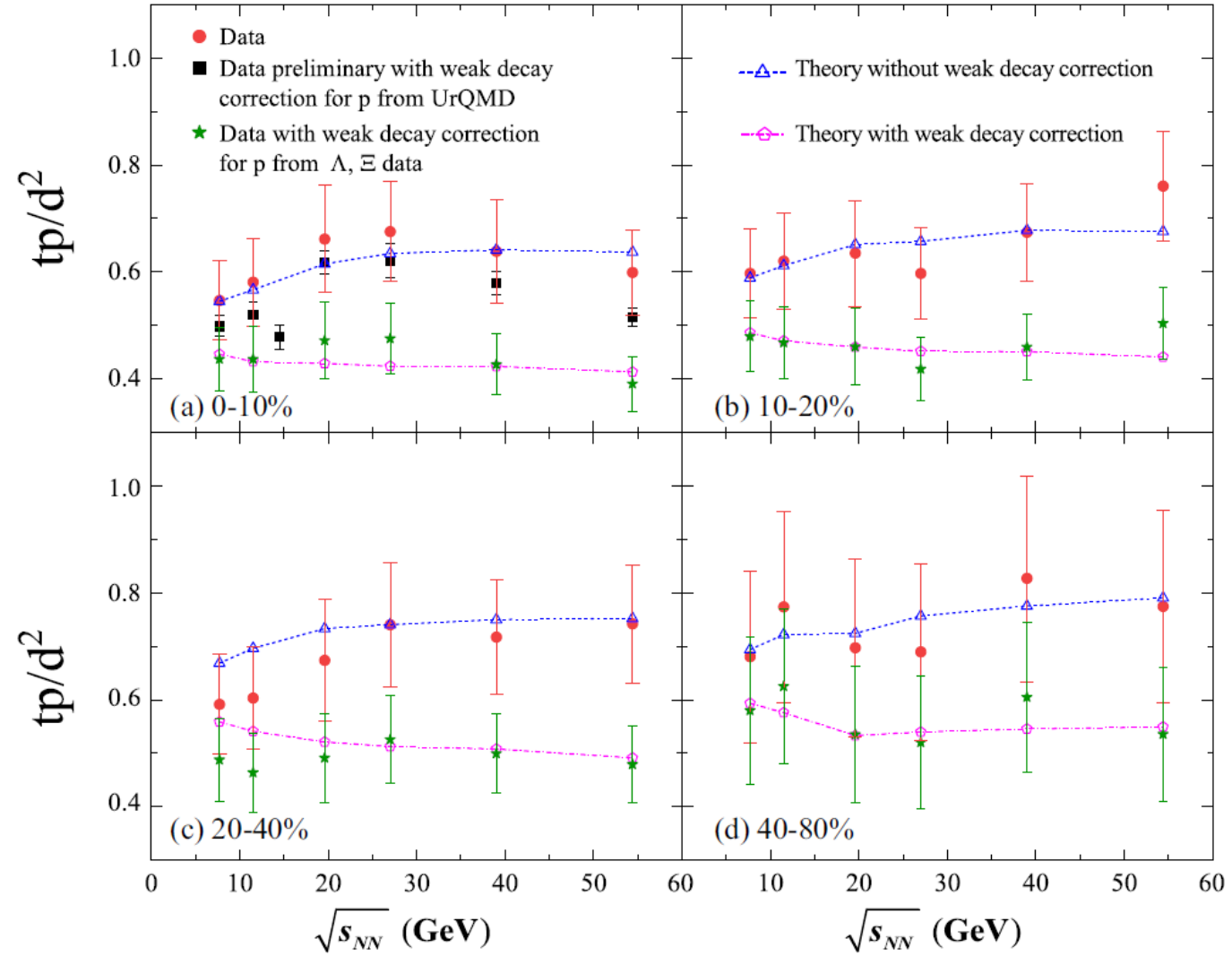


**From results of  $p_T$  spectra,**

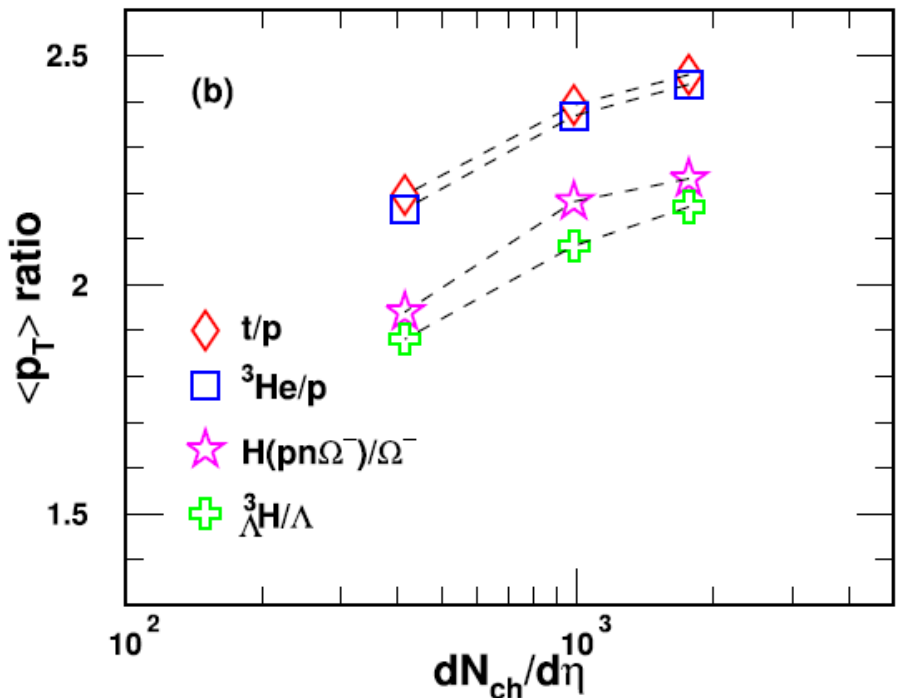
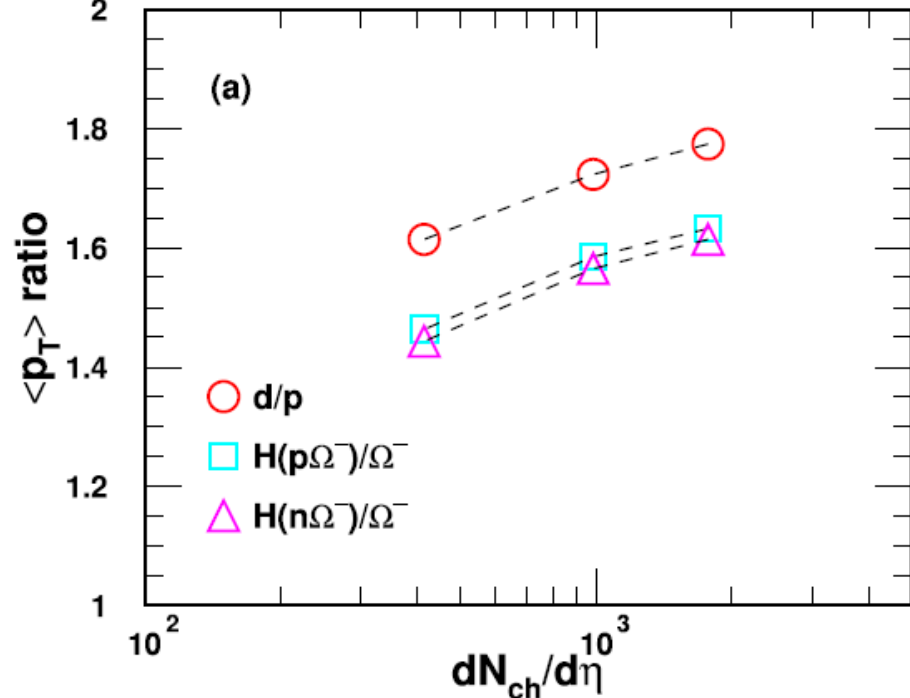
- **Analytical coalescence model can reproduce the available data of  $p_T$  spectra of d, t, He,  ${}^3_{\Lambda}\text{H}$  from 3 GeV to 5.02 TeV.**
- **We give corresponding predictions of  ${}^4_{\Lambda}\text{H}$ ,  ${}^4_{\Lambda}\text{He}$  and  $\Omega$ -hypernuclei.**



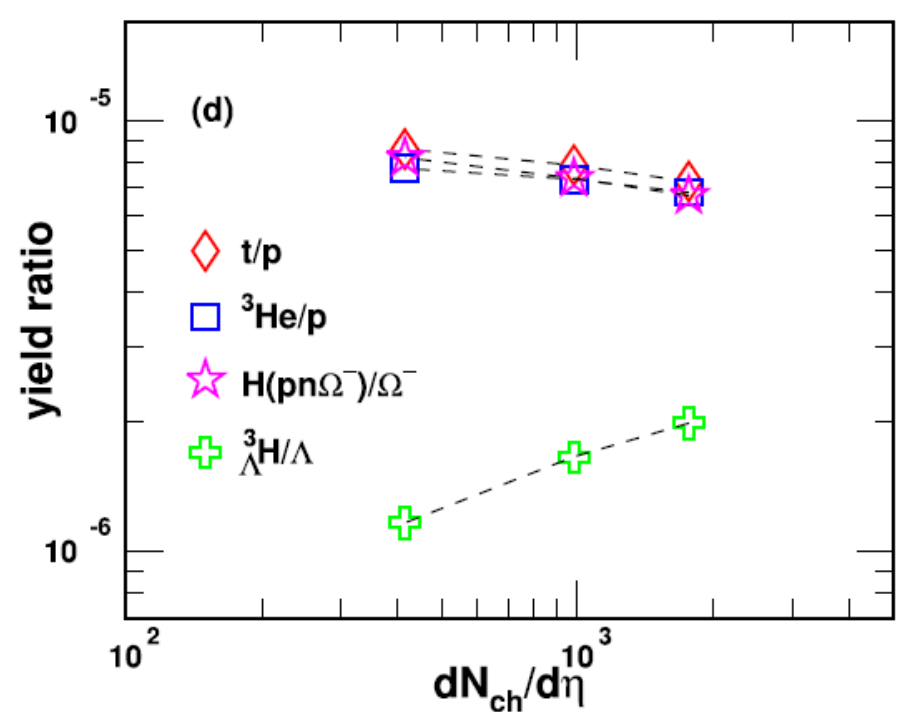
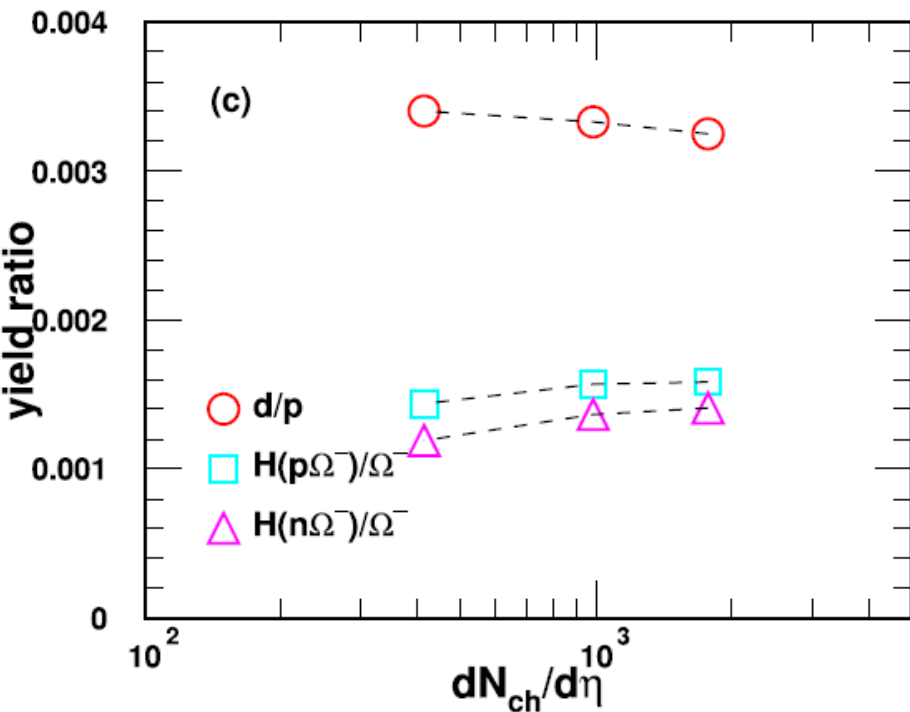
$t^3\text{He}$  can reveal production mechanisms of light nuclei.



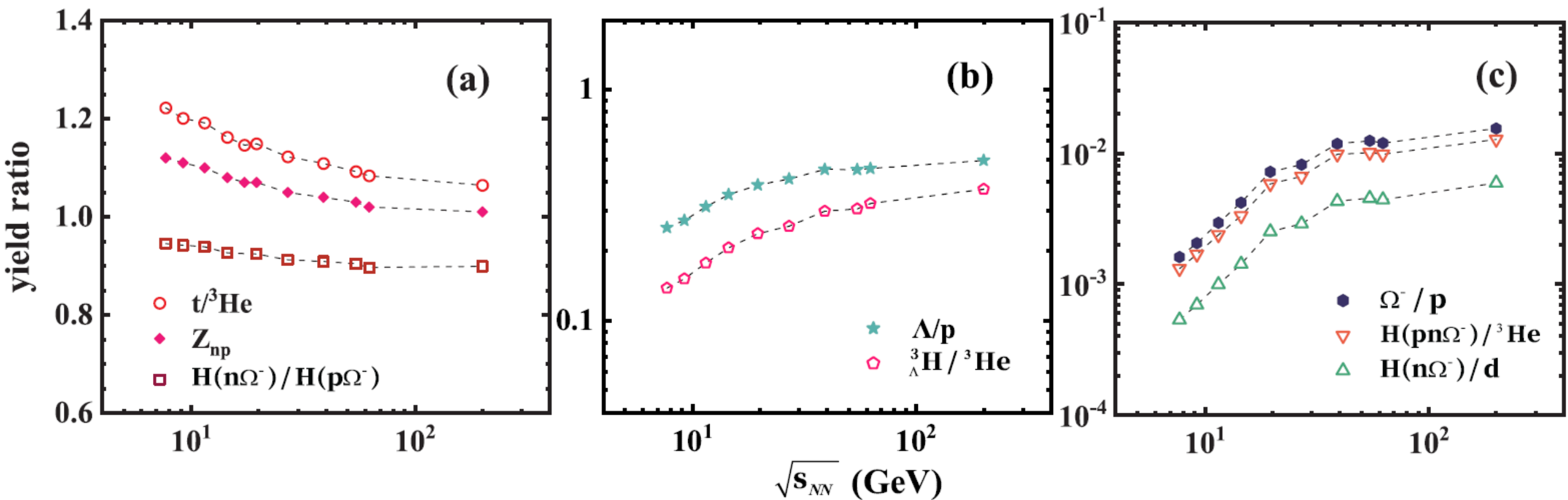
no peak without phase transition



RQW, X.L. Hou, Y.H. Li, J. Song, F.L. Shao, Nucl. Sci. Tech. 36, 185, 2025.



sensitive observables to production mechanism



characteristics from coalescence itself

# Production asymmetry of ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$

\* **four-body coalescence or three-body coalescence**

$$\frac{{}^4_{\Lambda}\text{He}}{{}^4_{\Lambda}\text{H}} = \frac{N_p}{N_n} = \frac{1}{Z_{np}} = 0.746,$$

$$\frac{{}^4_{\Lambda}\text{H} - {}^4_{\Lambda}\text{He}}{{}^4_{\Lambda}\text{H} + {}^4_{\Lambda}\text{He}} = \frac{N_n - N_p}{N_n + N_p} = \frac{Z_{np} - 1}{Z_{np} + 1} = 0.145$$

\*  $t + \Lambda \rightarrow {}^4_{\Lambda}\text{H}$  and  ${}^3\text{He} + \Lambda \rightarrow {}^4_{\Lambda}\text{He}$

$$\frac{{}^4_{\Lambda}\text{He}}{{}^4_{\Lambda}\text{H}} = \frac{N_{{}^3\text{He}}}{N_t} = 0.687,$$

$$\frac{{}^4_{\Lambda}\text{H} - {}^4_{\Lambda}\text{He}}{{}^4_{\Lambda}\text{H} + {}^4_{\Lambda}\text{He}} = \frac{N_t - N_{{}^3\text{He}}}{N_t + N_{{}^3\text{He}}} = 0.186$$

\* **two-body coalescence with  ${}^2_{\Lambda}\text{n}$  and  ${}^3_{\Lambda}\text{n}$**

$$\frac{{}^4_{\Lambda}\text{He}}{{}^4_{\Lambda}\text{H}} = 0,$$

$$\frac{{}^4_{\Lambda}\text{H} - {}^4_{\Lambda}\text{He}}{{}^4_{\Lambda}\text{H} + {}^4_{\Lambda}\text{He}} = 1$$

**sensitive to different coalescence channels**

# Production asymmetry of ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$

\* **Total**  ${}^4_{\Lambda}\text{H}$  and  ${}^4_{\Lambda}\text{He}$

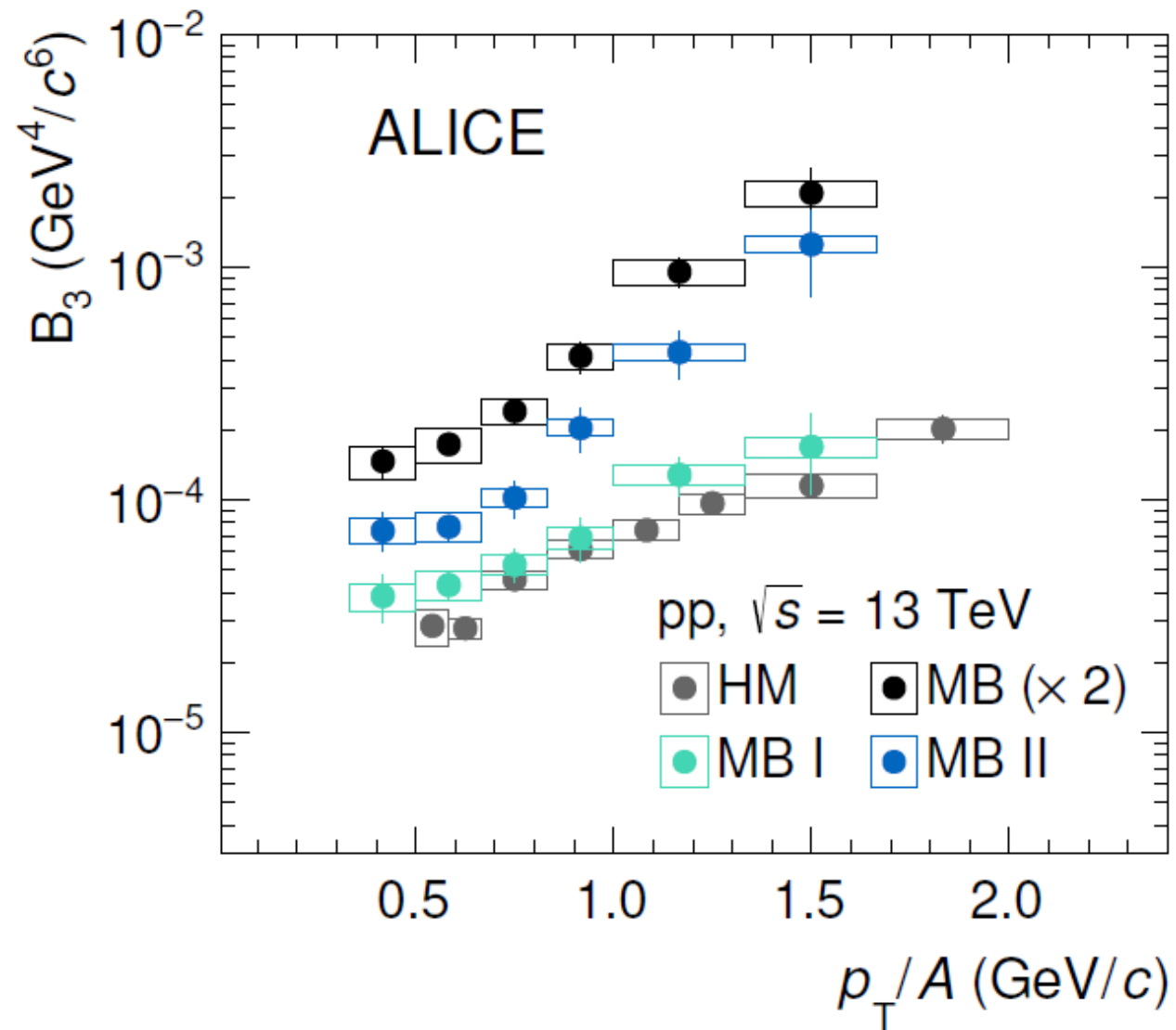
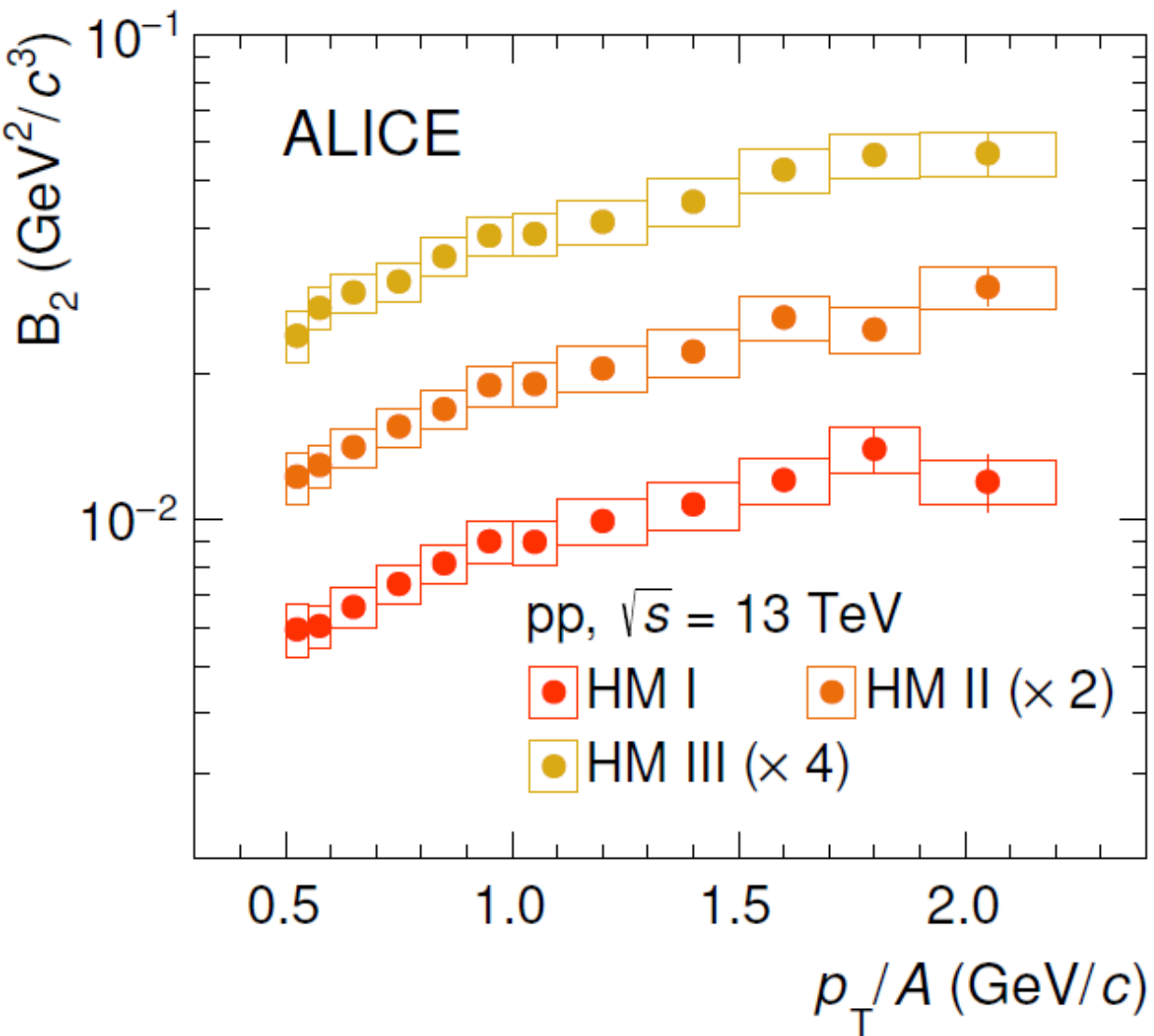
	RMS (fm)	neither ${}^2_{\Lambda}n$ nor ${}^3_{\Lambda}n$	with ${}^2_{\Lambda}n$	with ${}^3_{\Lambda}n$	with ${}^2_{\Lambda}n$ and ${}^3_{\Lambda}n$
$\frac{{}^4_{\Lambda}\text{He}}{{}^4_{\Lambda}\text{H}}$	1.5	0.715	0.639	0.573	0.488
	2.0	0.713	0.634	0.564	0.477
	2.5	0.711	0.629	0.553	0.464
$\frac{{}^4_{\Lambda}\text{H}-{}^4_{\Lambda}\text{He}}{{}^4_{\Lambda}\text{H}+{}^4_{\Lambda}\text{He}}$	1.5	0.166	0.221	0.271	0.344
	2.0	0.167	0.224	0.279	0.354
	2.5	0.169	0.228	0.288	0.366

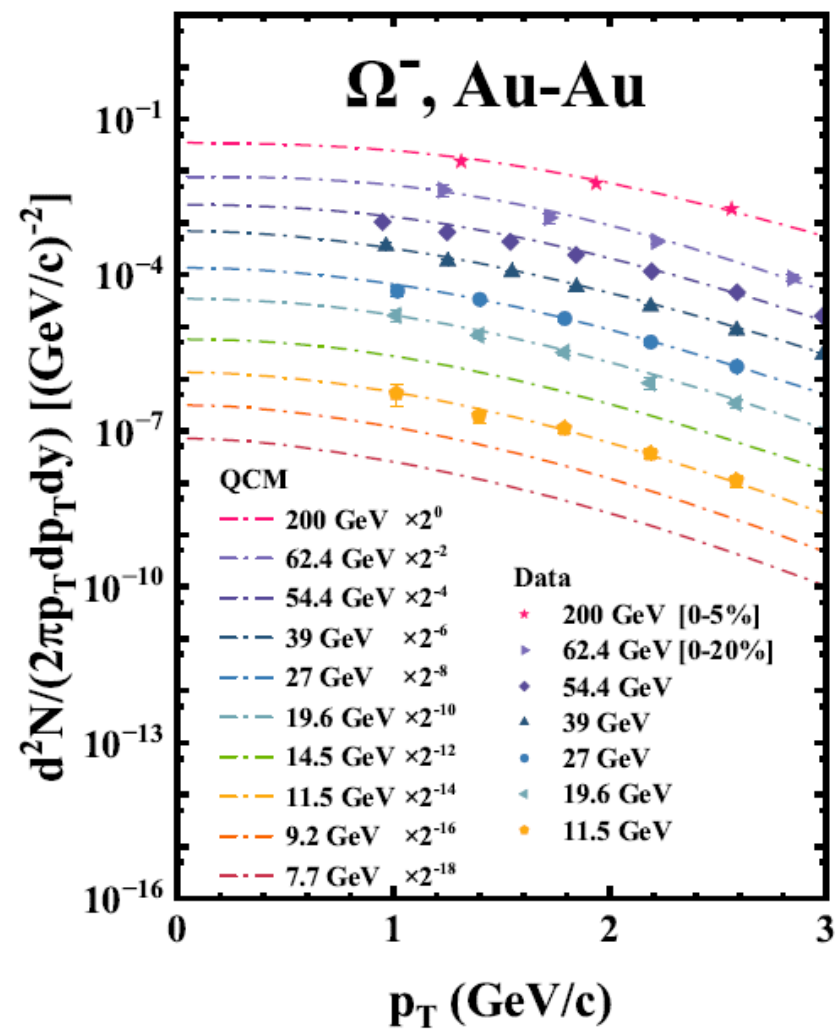
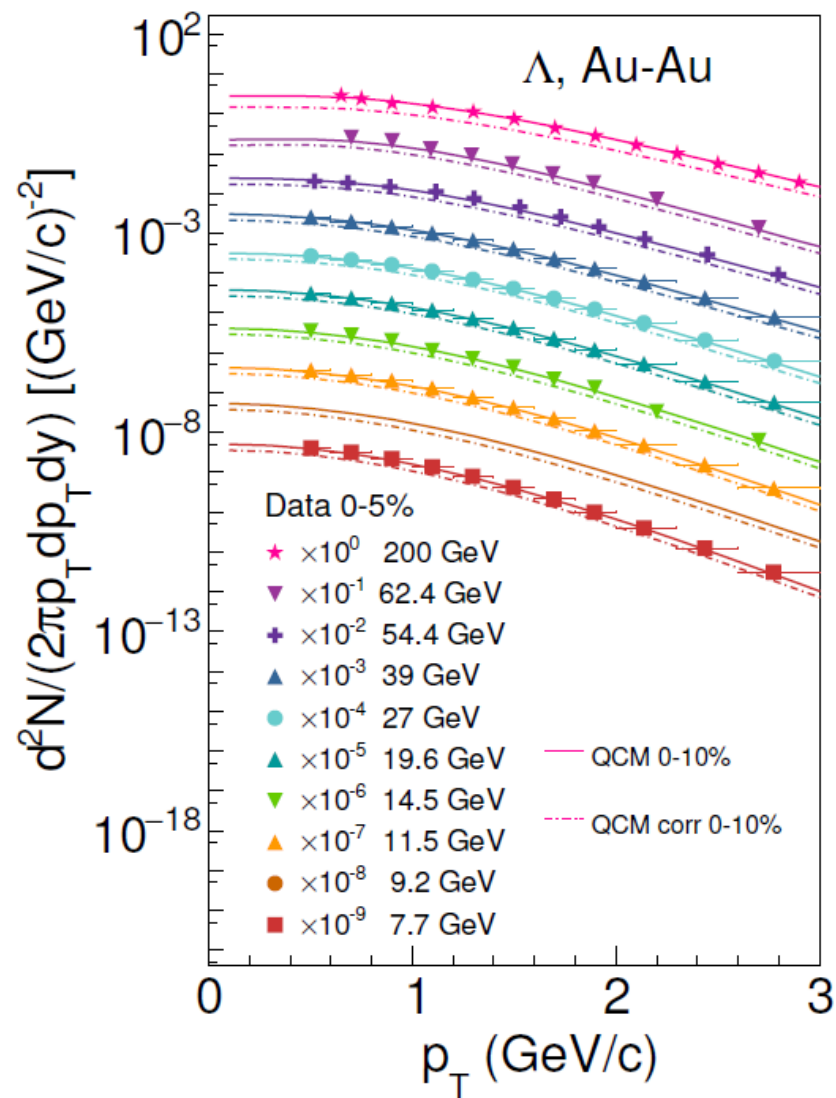
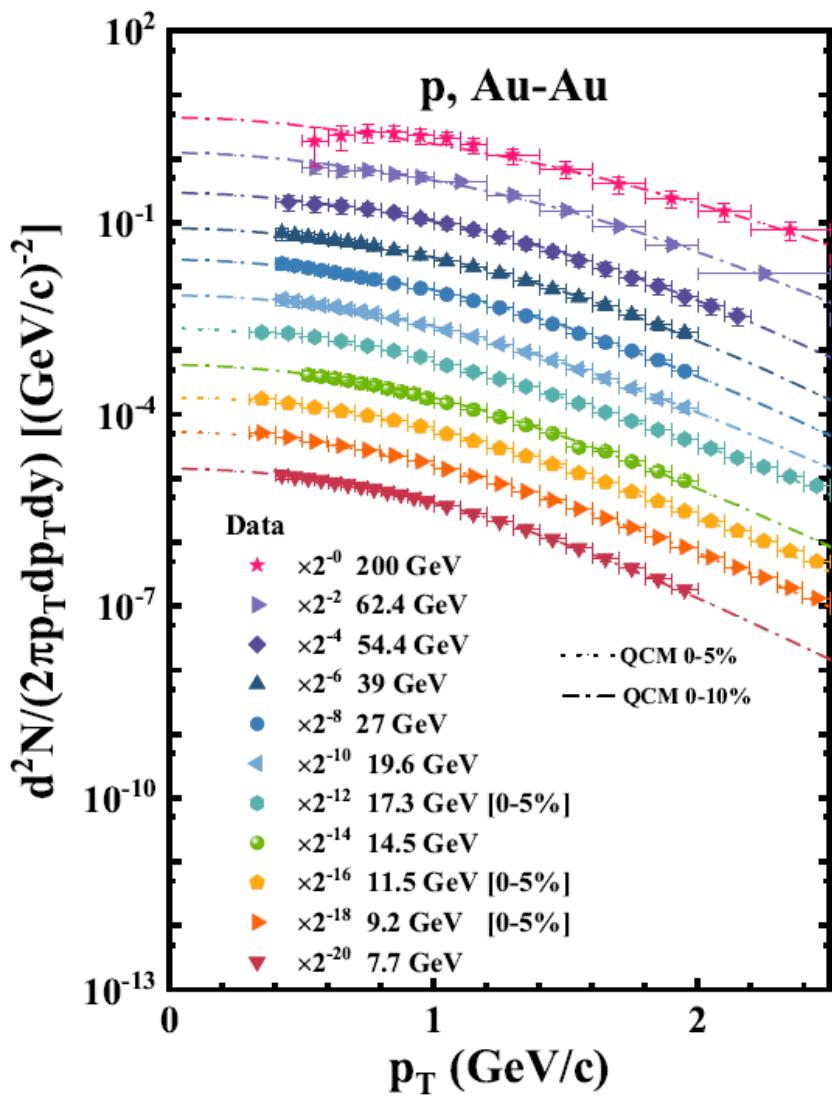
**Comparisons with future measurements can help shed light on the existence constraints of  ${}^2_{\Lambda}n$  and  ${}^3_{\Lambda}n$ .**

## IV. Summary

- ◆ We developed an analytical coalescence model to deal with productions of light and hyper-nuclei in high energy heavy-ion collisions. The relationships of light (hyper-)nuclei with primordial nucleons and hyperons were clearly given.
- ◆ We applied the coalescence model to heavy-ion collisions at RHIC and LHC, and gave explanations for different experimental observables.
- ◆ We presented production correlations of different species of light (hyper-)nuclei from coalescence itself.
- ◆ The analytical coalescence model can be further applied to other molecular states.

**Thank you!**





$\sqrt{s_{NN}}$ (GeV)	dN/dy ( $\times 10^{-4}$ )						
	Theory						Data
	I	II halo			III	Total	
spherical	102 keV	148 keV	410 keV	d+ $\Lambda$			
7.7	7.274	7.282	10.071	18.899	9.895	28.794	37.9 $\pm$ 5.1 $\pm$ 5.8
9.2	4.860	4.839	6.647	12.253	6.534	18.786	— — —
11.5	2.664	2.636	3.592	6.486	3.533	10.018	12.4 $\pm$ 1.8 $\pm$ 2.1
14.5	1.670	1.643	2.222	3.933	2.186	6.120	5.42 $\pm$ 0.73 $\pm$ 0.95
19.6	1.039	1.017	1.363	2.362	1.342	3.704	4.31 $\pm$ 0.59 $\pm$ 0.69
27	0.602	0.587	0.780	1.324	0.768	2.092	2.26 $\pm$ 0.30 $\pm$ 0.31
39	0.340	0.330	0.435	0.722	0.428	1.150	— — —
54.4	0.240	0.232	0.303	0.495	0.299	0.794	— — —
62.4	0.233	0.226	0.294	0.478	0.290	0.768	— — —
200	0.130	0.125	0.159	0.247	0.157	0.404	— — —

$$R_f = k \times \left( \frac{dN_{\text{ch}}}{dy} \right)^{1/3} \times \left( \sqrt{p_T^2 + m_{H_j}^2} \right)^a + b.$$

$\sqrt{s_{NN}}$ (GeV)	$H(p\Omega^-)$		$H(n\Omega^-)$		$H(pn\Omega^-)$	
	$dN/dy (\times 10^{-4})$	$\langle p_T \rangle$ (GeV/c)	$dN/dy (\times 10^{-4})$	$\langle p_T \rangle$ (GeV/c)	$dN/dy (\times 10^{-5})$	$\langle p_T \rangle$
7.7	7.829	1.281	7.402	1.275	2.735	1.569
9.2	7.496	1.295	7.061	1.289	2.091	1.586
11.5	6.641	1.323	6.236	1.318	1.342	1.623
14.5	6.465	1.368	5.992	1.362	0.984	1.671
19.6	7.942	1.392	7.340	1.386	0.909	1.704
27	6.306	1.414	5.754	1.408	0.542	1.729
39	6.046	1.425	5.498	1.419	0.380	1.738
54.4	5.152	1.512	4.660	1.505	0.263	1.843
62.4	4.945	1.574	4.435	1.567	0.235	1.926
200	4.545	1.725	4.087	1.716	0.139	2.102

