Examine the Medium-Modified DGLAP Evolution in Small Collision Systems

Workshop on extreme nuclear matter frontiers, YiChang, Apr 24–28, 2026

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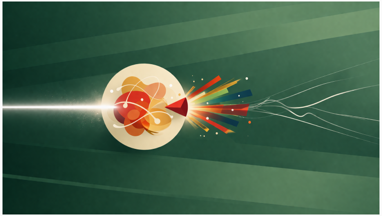
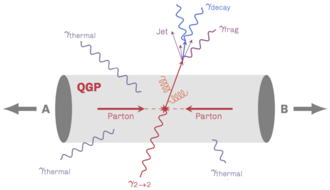
WK, B. Mecaj, I. Vitev, JHEP 04 (2026) 155

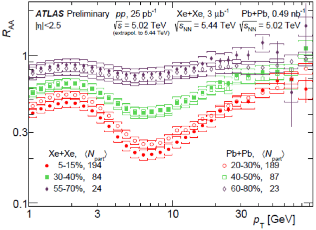
WK, I. Vitev, PLB 854 (2024) 138751

WK, I. Vitev, PRC 107 (2023) 064903



Medium modification of parton propagator





$$R_{AA}(p_T) = \frac{dN_{AA\to h+X}}{\langle T_{AB}\rangle d\sigma_{pp\to h+X}}$$

- In AA collisions, hard partons are surrounded by hot medium. Final-state interactions modify the parton dynamics relative to pp.
- A key signature of QGP, directly related to color d.o.f.
  $\Leftarrow$ dense color charges in QGP is natural to explain R_{AA} suppression.
- **This is the idea, but how to make reliable calculations?**

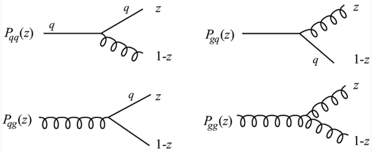
Production in the vacuum

- The factorized calculation of hadron production in hadronic collider

$$\frac{d\sigma_{pp\to hX}}{dp_T^h\, dy\, d\phi} = \sum_{ijX} \int dx_1\, dx_2\, f_{i/p}(x_1,\mu)\, f_{j/p}(x_B,\mu) \int dp_T^k \frac{d\sigma_{ij\to kX}(\mu,\mu')}{dp_T^k\, dy\, d\phi} \int dz\, \delta(p_T^h - z p_T^k)\, D_{h/k}(z,\mu')$$

- Hadronization requires non-perturbative initial data.

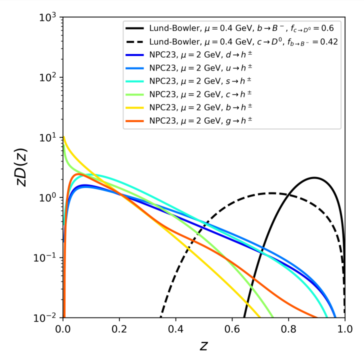
$$D_{h/k}(z,\mu_0,\mu') = \int \frac{dz'}{z'} \underbrace{D_{h/\ell}(z/z',\mu_0)}_{\text{Initial data, NP}} \quad \underbrace{C_{\ell/k}(z',\mu,\mu_0)}_{\text{Parton physics, perturbative}}$$

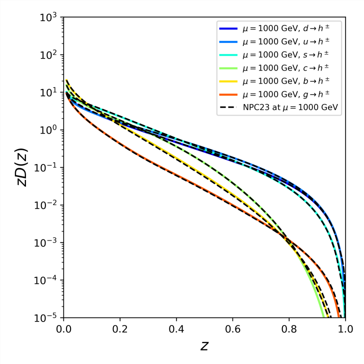


- Parton evolution takes care of leading parton fluctuations between scales μ_0 and μ (DGLAP)

$$\frac{\partial}{\partial \ln \mu^2} C_{ij}(z',\mu,\mu_0) = \frac{\alpha_s(\mu)}{2\pi} \int \frac{dz}{z} \left[P_{ik}(z')\right]_+ C_{kj}(z'/z,\mu,\mu_0)$$

Evolution of fragmentation function in the vacuum





- **Left: initial conditions.** NPC23 [Gao et al., PRD112(2025)5] for light hadrons, Lund-Bowler parametrization for heavy mesons.
- **Right: evolution to $\mu = 1$ TeV** and consistency check with NPC23.

Production and fragmentation in AA collisions

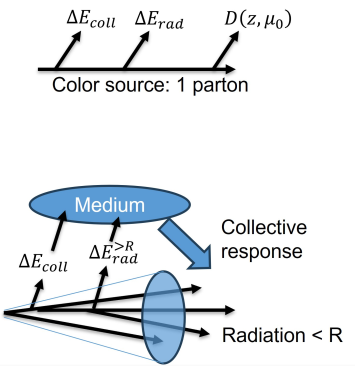
- In $A{+}B$, nuclear PDF is different $f_{i/p}, f_{j/p} \Longrightarrow f_{i/A}, f_{j/B}$.
  There are also final-state effects $D_{h/k}(z,\mu') \Longrightarrow D_{h/k}(z,\mu';\mathrm{medium})$

- Formation time of energetic light hadrons $\tau_h \sim \frac{E_h}{\Lambda^2} \gg L$.
  For $E_h = 10$ GeV, $\tau_h \approx 50$ fm, much larger than size of medium
  $\Rightarrow$ Non-perturbative part of fragmentation is same as vacuum $+\mathcal{O}(L\Lambda^2/E_h)$

- Modification should further factorize into

$$D_{h/k}(z,\mu_0,\mu';\mathrm{medium}) = \int \frac{dz'}{z'} D_{h/\ell}(z/z',\mu_0)\ \underbrace{C_{\ell/k}(z';\mu,\mu_0,\mathrm{medium})}_{\text{parton physics, vac+med effects}}$$

What goes into the "transfer matrix" $C_{i/j}(z; \mu, \mu_0, \text{medium})$?



For hadrons

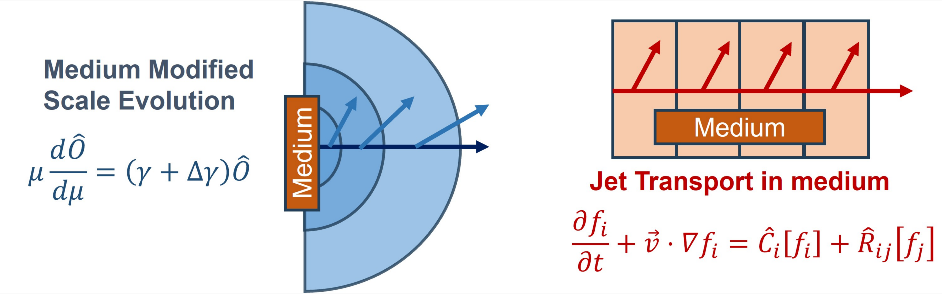
- Energy loss of a single color charge in the medium.

For jets

- Energy loss out of cone R, sourced by a fluctuating number of partons in cone.
- A fraction of the perturbatively lost energy can flow back into the cone via medium collective response.

The space-time picture of medium modification: two limiting cases

- Radiations with formation time $\tau \gg L$ probe the entire medium in a coherent way.
- Radiations with formation time $\tau \ll L$ depends on local properties of the medium.
- Multiple emissions are thus resumed by different strategies.



A quantitative criteria

- Cumulative effect of transport: momentum broadening $\langle k_\perp^2 \rangle = \hat{q}L$, $\hat{q} \equiv \frac{d\langle k_\perp^2 \rangle}{dL}$,
- Typical off-shellness of fluctuations that sees the medium coherently

$$\frac{2k^0}{k^2} \sim L \longrightarrow k^2 \sim \frac{2k^0}{L}.$$

- Together, they define a dimensionless parameter r

$$v = \frac{\hat{q}L}{2E/L} = \begin{cases} \ll 1 & \text{Medium-modified DGLAP (mDGLAP)} \\ \gg 1 & \text{Transport equation} \end{cases}$$

- Another emergence scale $\sqrt{(\hat{q}L) \times (2E/L)} = \sqrt{2E\hat{q}}$, for radiation in transport.

Typical number of v in HIC and EIC

Quark-gluon plasma (LHC)	$\langle \hat{q} L \rangle$	$\frac{2E}{L}$	$v = \frac{\langle \hat{q} L \rangle}{2E/L}$
Central Pb+Pb	8 GeV^2	1 GeV^2, at E=10 GeV	$\mathcal{O}(10)$
Central Pb+Pb	8 GeV^2	10 GeV^2, at E=100 GeV	$\mathcal{O}(1)$
$\star$ O+O, $b = 0$ p+Pb	1.5 GeV^2	10 GeV^2, at E=50 GeV	$\mathcal{O}(0.1)$

Cold nuclear matter	$\langle \hat{q} L \rangle$	$\frac{2E}{L}$ at $y_e = 0.3$	$v = \frac{\langle \hat{q} L \rangle}{2E/L}$
$\star$ ^{131}Kr at HERMES	0.25 GeV^2	1.0 GeV^2	0.25
$\star$ ^{208}Pb at EicC	0.3 GeV^2	3.5 GeV^2	$\mathcal{O}(0.1)$
$\star$ ^{208}Pb at EIC	0.3 GeV^2	33 GeV^2	$\mathcal{O}(0.01)$

Jet modification in small or dilute systems should be better described by mDGLAP equation.

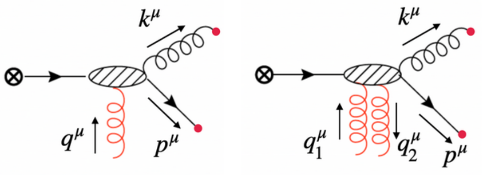
Medium-modified evolution of the parton spectrum

Define the energy spectrum of parton i: $F_i(z; E_0) = z\frac{dN_i}{dz}$

$$\frac{\partial}{\partial \ln \mu^2} F_i(z; E_0) = \sum_j \int \frac{dx}{x} \left[\frac{\alpha_s}{2\pi} x P_{ij}^{\mathrm{vac}}(x) + \int d^2 k_\perp x \frac{dN_{ij}}{dx d^2 k_\perp} \frac{\partial \Theta(x, k_\perp; \mu)}{\partial \ln \mu^2} \right]_+ F_j(z/x; E_0)$$

- $\frac{dN_{ij}}{dxd^2k_\perp}$ are the medium-induced single-emission spectrum.
- $\Theta(x, k_\perp; \mu)$ is a cut-off function, e.g., $\Theta\left(k_\perp^2 < \mu^2\right)$, $\Theta\left(\frac{k_\perp^2}{x(1-x)} < \mu^2\right)$, $\Theta\left(\frac{k_\perp^2}{(1-x)} < \mu^2\right)$, etc
- The medium-induced splitting function to first order in medium opacity are known (both initial and final-state, both collinear and TMD)

The medium-induced real-emission



$$\left(\frac{dN}{dx\,d^2\boldsymbol{k}_\perp}\right)_{q\to qg} = \frac{\alpha_s}{2\pi^2}C_F\frac{1+(1-x)^2}{x}\int\frac{d\Delta z}{\lambda_g(z)}\int d^2\mathbf{q}_\perp\frac{1}{\sigma_{el}}\frac{d\sigma_{el}^{\text{medium}}}{d^2\mathbf{q}_\perp}\left[\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}-\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right)\right.$$
$$\times\left(1-\cos[(\Omega_1-\Omega_2)\Delta z]\right)+\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\cdot\left(2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}-\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\right)\left(1-\cos[(\Omega_1-\Omega_3)\Delta z]\right)$$
$$+\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\left(1-\cos[(\Omega_2-\Omega_3)\Delta z]\right)+\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\cdot\left(\frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2}-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\right)\left(1-\cos[\Omega_4\Delta z]\right)$$
$$\left.-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\cdot\frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2}\left(1-\cos[\Omega_5\Delta z]\right)+\frac{1}{N_c^2}\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}-\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\right)\left(1-\cos[(\Omega_1-\Omega_2)\Delta z]\right)\right],$$

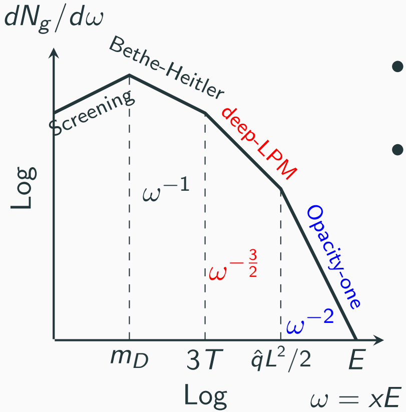
G. Ovanesyan, I. Vitev, PLB 706 (2012) 371-378

- The formula involves a convolution with the parton-medium forward scattering cross-section $\frac{d\sigma_{R,T}}{d^2q_\perp} = \frac{C_R C_T}{d_A}\frac{g^4}{(q_\perp^2+m_D^2)^2}$.
- The formula integrates over the density profile of the scattering center.

$$\text{Opacity: } \chi = \int\frac{d\Delta z}{\lambda_g} \sim \frac{L}{\lambda_{\text{mfp}}} \sim \frac{\hat{q}L}{m_D^2}, \qquad \text{Opacity expansion: } \sum c_n\chi^n$$

- The path-length integral can be numerically performed in hydro background.

The opacity expansion and full radiation spectrum (all opacity)



- They mDGLAP vs Transport has similar form of energy loss, subtile differences in the log factor.
- For example, **in a finite brick medium:**

$$\Delta E_{\mathrm{rad}}^{\mathrm{mDGLAP}} \propto \alpha_s C_R \hat{q} L^2 \ln \frac{2E/L}{m_D^2} \ldots \ln \frac{2E/L}{\hat{q}L}$$

$$\Delta E_{\mathrm{rad}}^{\mathrm{Transport}} \propto \alpha_s C_R \hat{q} L^2 \sqrt{\ln \frac{\sqrt{2E\hat{q}}}{m_D^2}} \to \sqrt{\ln \frac{\hat{q}L}{m_D^2} + \frac{1}{2} \ln \frac{2E/L}{\hat{q}L}}$$

[NLL deep-LPM, P. Arnold, C. Dogan PRD78(2008)065008]

$\star$ For small opacity $\frac{\hat{q}L}{m_D^2} \sim O(1)$, and high energy $\frac{1}{v} = \frac{2E/L}{\hat{q}L} \gg 1$, mDGLAP gives the dominant correction.

Mass effects: more than just the dead cone

Massive splitting function obtained at first-order in opacity from SCET_{M}

[Z.-B. Kang, F. Ringer, I. Vitev, JHEP 03 (2017) 146] .

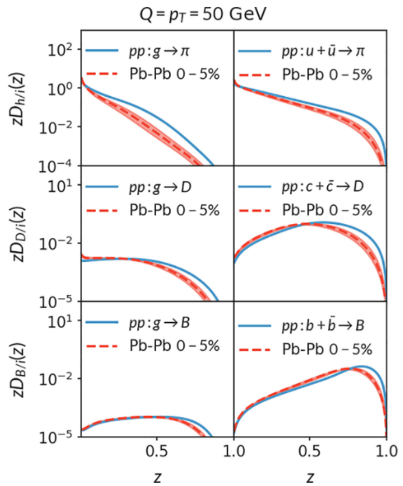
- Mass changes propagator $1/k_\perp^2 \to 1/(k_\perp^2 + \nu M^2)$
- Mass causes new possibility in the splitting amplitude (squared)

$$\text{Light quark to gluon:} \quad \frac{1+(1-x)^2}{x} \frac{\vec{k}_\perp}{k_\perp^2} \cdot \frac{\vec{p}_\perp}{p_\perp^2}$$

$$\text{Heavy quark to gluon:} \quad \frac{1+(1-x)^2}{x} \frac{\vec{k}_\perp}{k_\perp^2 + x^2 M^2} \cdot \frac{\vec{p}_\perp}{p_\perp^2 + x^2 M^2} + x^3 \frac{M}{k_\perp^2 + x^2 M^2} \cdot \frac{M}{p_\perp^2 + x^2 M^2}$$

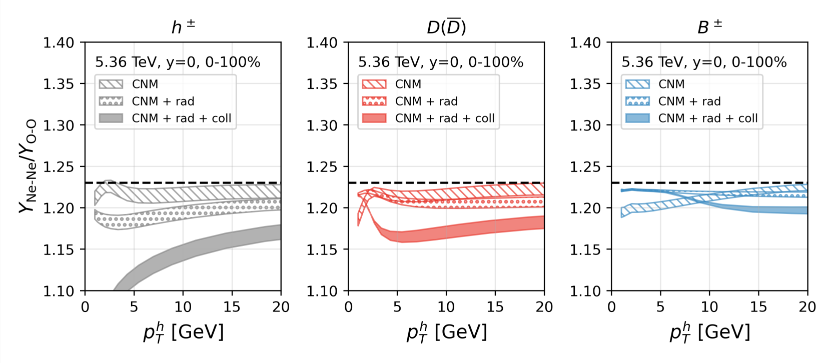
$$\text{Gluon to } Q\bar{Q}\text{:} \quad (x^2 + (1-x)^2) \frac{\vec{k}_\perp}{k_\perp^2 + M^2} \cdot \frac{\vec{p}_\perp}{p_\perp^2 + M^2} + \frac{M}{k_\perp^2 + M^2} \cdot \frac{M}{p_\perp^2 + M^2}$$

The modified fragmentation function



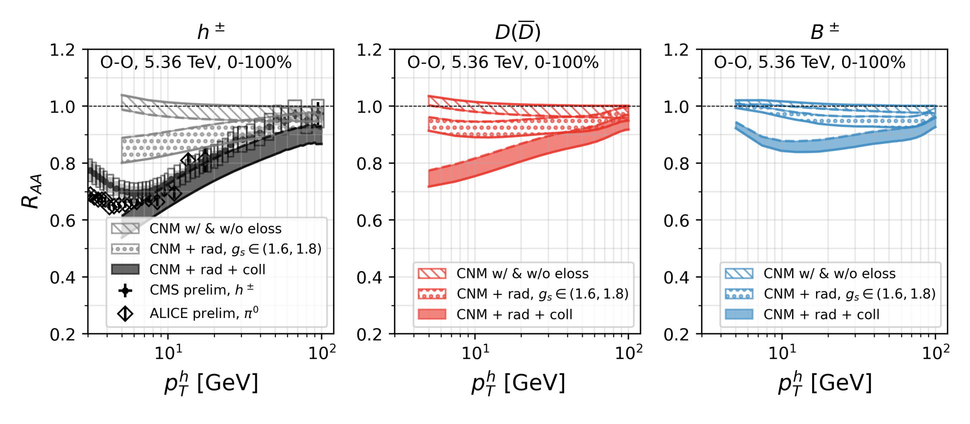
- Calculations are done for Central Pb+Pb.
- The modified evolution: a red-shift of the fragmentation function to smaller z.
- But, note that for $p_T \sim 50$ GeV. $v = \mathcal{O}(1)$. Transport effect can also be important.
- Now look at small systems.

Application to O+O and Ne+Ne



- The ratio of hadron spectrum (w/o $1/N_{\text{coll}}$) between Ne-Ne and O-O at $y = 2.75$.
- However, transport approach also provides a reasonable description (R_{AA} is only sensitive to total energy loss)

Application to O+O and Ne+Ne

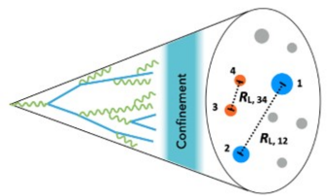


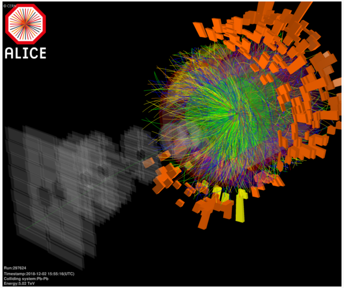
- Cold nuclear matter effects + QGP effects (collisional energy loss + mDGLAP).
- The nuclear modification of hadron, D meson, and B meson in O+O at 5.36 TeV.

Questions we should ask about the mDGLAP approach

- How to test it and differentiate it from the transport approach?
- Is there a more solid foundation of this equation?
  Note that we just write down mDGALP without a proof.

Energy correlators as a scale dependent probe of medium modified shower





https://cds.cern.ch/record/2649643

- Energy correlators [I. Moult, H.-X. Zhu 2506.09119 for a review].
- For example, two-point EEC as E-weighted θ-histogram

$$\Sigma(\theta^2) = \frac{1}{N_{\text{jet}}} \sum_{\text{jets}} \sum_{i \in \text{jet}} \sum_{j \neq i, j \in \text{jet}} E_i E_j \delta(\theta^2 - \theta_{ij}),$$

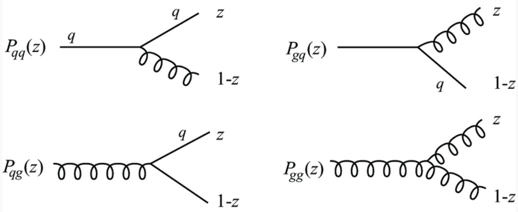
- A direct connection to asymptotic energy flow operator

$$\Sigma(n_1, n_2, \cdots, n_k; E) = \langle \hat{\mathcal{E}}_{n_1} \hat{\mathcal{E}}_{n_2} \cdots \hat{\mathcal{E}}_{n_k} \rangle$$

$$\hat{\mathcal{E}}_n = \lim_{r \to \infty} r^2 n_i \int_0^\infty dt T^{0i}(t, r\vec{n})$$

"The theoretical calorimeter": 1) a detector sitting at direction $\vec{n}$, 2) absorb all energy flow into this direction after the collision, 3) push the detector to infinity.

E2C in a jet at LO

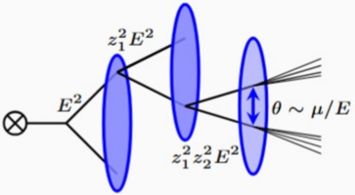


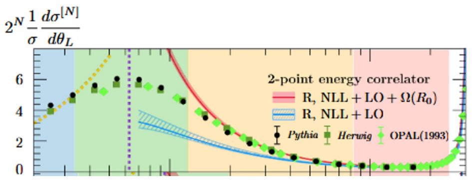
For a perturbative angle $\theta_{\rm EEC} E \gg \Lambda_{\rm QCD}$, the LO correlation is given by the parton splitting with precisely this angle

$$\Sigma_i(\theta^2) = \underbrace{\sum_{(jk)} \frac{\alpha_s}{2\pi^2} \int dx P_{i\to jk}(x) \int \frac{d^2 k_\perp}{k_\perp^2}}_{\text{Sum over all possible splitting}} \underbrace{x(1-x)}_{E \text{ weighting}} \underbrace{\delta\left(\theta^2 - \frac{k_\perp^2}{[x(1-x)E]^2}\right)}_{\text{Angle projection}} \propto \frac{\alpha_s}{\theta^2}$$

This is the normal, expected scaling $\Sigma(\theta^2) \sim 1/\theta^2$

Quantum-corrected scaling, i.e., anomalous dimension





K. Lee, A. Pathak, I. Stewart, Z. Sun PRL133 (2024) 231902

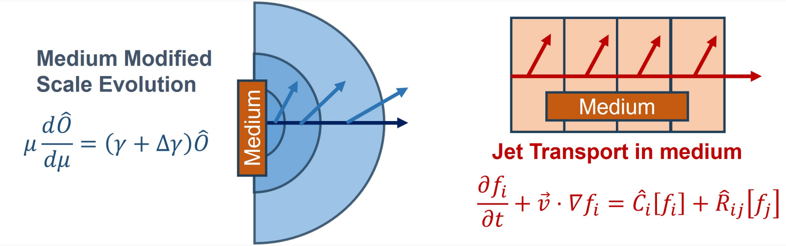
- Emissions below the angle θ_{EEC} cannot change the energy flow at leading power.
- Emissions within $\theta_{\mathrm{EEC}} < \theta_r < R$ reduce the measured energy flows

$$\frac{\partial J_i(E)}{\partial \ln \mu^2} = \frac{\alpha_s}{4\pi}(-2)\int_0^1 dx J_j(zE)[P_{ij}(z)]_+ = J_j(E)\frac{\alpha_s}{4\pi}\underbrace{(-2)\int_0^1 dz z^{N-1}[P_{ij}(z)]_+}_{\gamma_{ij}^{(0)}(N)}$$

- Anomalous dimension in the vacuum are simply moments of the DGLAP kernels!

For example, for pure gluon system: $\Sigma(\theta^2) \propto \frac{\alpha_s}{\theta^2}\left[\frac{\theta^2}{R^2}\right]^{\frac{\alpha_s}{4\pi}\gamma_{gg}^{(0)}(N)}$

EEC is ideal to differentiate transport phenomena versus scale evolution



- If parton dynamics in medium is predominantly described scale evolution, then we should expect a modified anomalous dimension $\gamma^{(0)}(N) + \Delta\gamma_{\text{med}}$.
- If it is better described by a transport equation.

  $\Rightarrow$ The multiple radiations are not necessarily correlated in a scale-dependent way

  $\Rightarrow$ independent mixture of medium-induced splittings at different time.

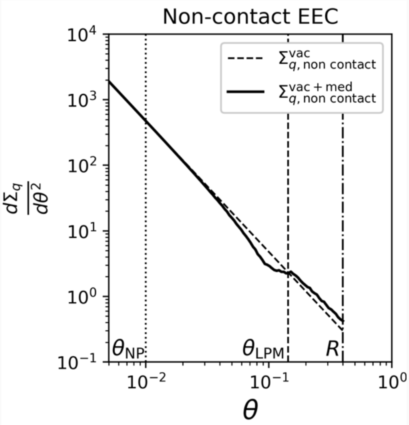
Understand jet EEC in small collision system (p+Pb, O+O)

- Compute medium correction from medium-induced TMD splitting function at first-order in opacity $\Sigma_i(\theta) = \Sigma_{i,\mathrm{vac}}(\theta) + \Sigma_{i,\mathrm{med}}(\theta)$

$$\Sigma_{i,\mathrm{med}}(\theta) = \int dx d^2 k_\perp d^2 q_\perp \sum_{(jk)} \frac{dN^{\mathrm{real\ emission}}_{\mathrm{med,i\to j+k}}}{dx d^2 k_\perp d^2 q_\perp} x(1-x) \delta(\theta^2 - \theta_{ij}^2)$$

- In general, dN depends on medium density profile, and we can do it numerically.
- Analytic insights possible with "exponential-decaying medium" $\rho \propto \rho_0 e^{-t/L}$.

Robust features of the opacity-one correction at LO



[WK, B. Mecaj, I. Vitev, 2512.11952 (JHEP)]

- Magnitude of the correction: $\Sigma_{q,\rm med} \propto \alpha_s^2 \rho L^3 \mathcal{L}_c$
  $\mathcal{L}_c = \ln \frac{E/L}{m_D^2}$ due to large-q tail of the scattering.
- For radiation coherent over the entire medium.

$$|1 - e^{ip^- x^+}|^2 \Rightarrow 2\left(1 - \cos\frac{k_\perp^2 L}{2x(1-x)E}\right)$$
$$= 2\left(1 - \cos\frac{x(1-x)\theta^2 EL}{2}\right)$$

A characteristic angle $\theta_{\rm LPM} = \sqrt{\frac{8\pi}{EL}}$,

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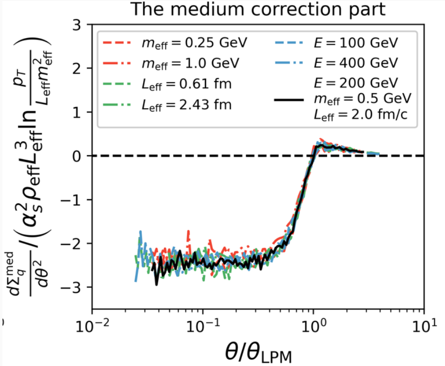

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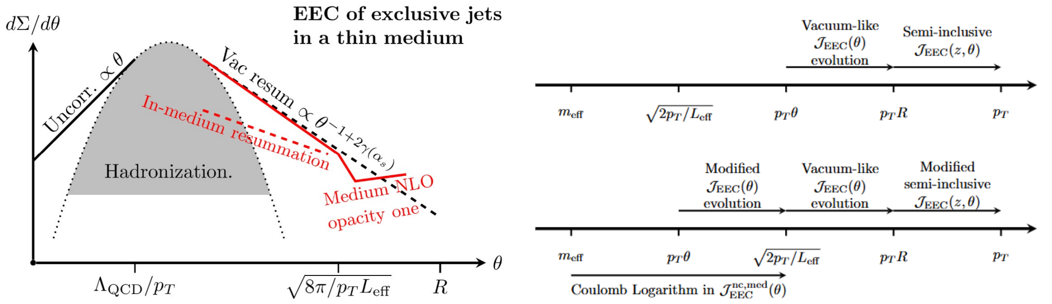
[WK, B. Mecaj, I. Vitev, 2512.11952 (JHEP)]

Medium-modified anomalous dimension



- Opacity-one medium-induced radiation is only logarithmic enhanced in the phase space $p^2 \ll 2E/L \Rightarrow$ a modified anomalous dimension.
- Momentum broadening angle is much smaller $\theta_c \approx \frac{\sqrt{\hat{q}L}}{E} \ll \theta_{\rm LPM} \approx \sqrt{\frac{8\pi}{EL}}$.

Extract the anomalous dimension from the modified splitting function

$$
\begin{aligned}
f_{qq}^{\rm med}(N) =& \int_0^1 dx x^{N-1} \left[F_\epsilon^{q\to qg}(x)\right]_+ \\
=& \frac{2(N-1)C_F C_A + C_F^2}{\epsilon} + C_F(2C_F - C_A)\frac{-N^2+2N+1}{N^2-1} \\
& + C_F C_A \Big\{ 3 + 4N(\gamma_E - 1) + 2(N-2)\left[2\psi_0(N-2) - \psi_0(N-1)\right] \\
& + N\left[3\psi_0(N) - \psi_0(N+1)\right] + (N+2)\psi_0(N+2) \Big\} + \mathcal{O}(\epsilon)\,, \qquad (5.3)
\end{aligned}
$$

$$
\begin{aligned}
f_{gq}^{\rm med}(N) =& \int_0^1 dx (1-x)^2 \left[F_\epsilon^{q\to qg}(x)\right]_+ \\
=& -\frac{C_F^2}{\epsilon} + C_F \Big\{ C_A \frac{N^2+N+6}{(N+1)N(N-1)(N-2)} \\
& + C_A\left[\frac{1}{2} - \gamma_E + \frac{1}{(N-1)(N-2)} - \psi_0(N-2)\right] \\
& + (2C_F - C_A)\left[\frac{1}{2} - \gamma_E + \frac{1}{(N+1)N} - \psi_0(N)\right] \Big\} + \mathcal{O}(\epsilon)\,, \qquad (5.4)
\end{aligned}
$$

$$
\begin{aligned}
f_{gg}^{\rm med}(N) =& \int_0^1 dx x^2 \frac{2}{x}\left[x F_\epsilon^{g\to gg}(x)\right]_+ - 2N_f \int_0^1 dx x F_\epsilon^{g\to q\bar{q}}(x) \\
=& \frac{2C_A^2(N-1) + 2N_f T_F C_F}{\epsilon} + C_A^2 \Big\{ \frac{N^5 + 6N^4 - 5N^3 + 6N^2 + 4N + 96}{3N^5 - 15N^3 + 12N} \\
& + (N-3)\left[-1 + \gamma_E + 2\psi_0(N-3) - \psi_0(N-2)\right] \\
& + (N-1)\left[-1 + \gamma_E + \psi_0(N-1)\right] \\
& + (N+1)\left[-1 + \gamma_E + 2\psi_0(N+1) - \psi_0(N+2)\right] \\
& + (N+3)\left[-1 + \gamma_E + \psi_0(N+3)\right] \Big\} \\
& + 2N_f T_F \frac{2C_A + 6C_F}{3} + \mathcal{O}(\epsilon)\,, \qquad (5.5)
\end{aligned}
$$

$$
\begin{aligned}
f_{qg}^{\rm med}(N) =& 2N_f \int_0^1 dx x^2 F_\epsilon^{g\to q\bar{q}}(x) \\
=& -\frac{2N_f T_F C_F}{\epsilon} + 2N_f T_F \Big\{ -C_F + C_A \frac{N^2 - N + 6}{(N+2)(N+1)N(N-1)} \\
& + (2C_F - C_A)\left[\frac{1}{2} - \gamma_E + \frac{1}{N(N-1)} - \psi_0(N+1)\right] \\
& + C_A\left[\frac{1}{2} - \gamma_E + \frac{1}{(N+2)(N+1)} - \psi_0(N+3)\right] \Big\} + \mathcal{O}(\epsilon)\,. \qquad (5.6)
\end{aligned}
$$

- This can be done analytically using the technique in

[Ke, Mecaj, Vitev, JHEP04(2026)155, Ke, Vitev, PLB854(2024)138751]

$$
\begin{aligned}
\Delta\gamma_{qq}(N) =& w_{\rm med}\left(2(N-1)C_F C_A + C_F^2\right) \\
\Delta\gamma_{gq}(N) =& w_{\rm med}\left(-C_F^2\right) \\
\Delta\gamma_{gg}(N) =& w_{\rm med}\left(2(N-1)C_A^2 + 2N_f T_F C_F\right) \\
\Delta\gamma_{qg}(N) =& w_{\rm med}\left(-2N_f T_F C_F\right)
\end{aligned}
$$

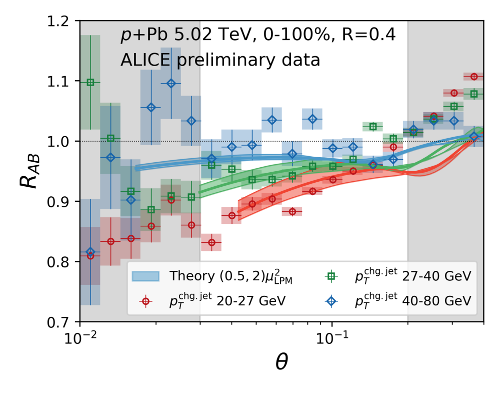
- Correction directly relates to medium properties

$$
w_{\rm med} = \frac{4\pi\alpha_s(\mu^2)\rho_{\rm eff} L_{\rm eff}}{2p_T^{\rm jet}/L_{\rm eff}}
$$

- Is this a possible robust probe of medium property in small system? $E_3C/E_2C \propto \theta^{\frac{\alpha_s}{2\pi} w_{\rm med} 2C_R C_A}$

Comparison with preliminary ALICE measurement in p+Pb

$$\frac{d\Sigma}{d\theta\, dp_T\, dy} = \sum_{a,b,c} \int dx_a\, dx_b\, dz_J\, f_{a/A}(x_a,\mu)\, f_{b/B}(x_b,\mu)\, \mathcal{H}_{ab\to c}\left(\frac{p_T}{z_J}, y, \mu\right)$$
$$\times \left[\mathcal{J}^{\text{vac}}_{\text{EEC},c}(\theta;\, z_J,\, p_T, R, \mu) + \mathcal{J}^{\text{med}}_{\text{EEC},c}(\theta;\, z_J,\, p_T, R, \mu;\, L, m_{\text{eff}})\right],$$



- The middle window correspond to $\frac{\Lambda_{\text{QCD}}}{E} \ll \theta \ll \sqrt{\frac{8\pi}{EL}}$
- The jet p_T dependence seem to be consistent.
- If data are finally confirmed by ALICE, this will be a strong support for the mDGLAP approach.
- Use EEC to extract medium quantities in small system.

The final question: can we derive mDGLAP from fundamental principles?

mDGLAP may require one to start from the real-time formalism

- Consider collinear parton interact with medium $\mathcal{L}_{\rm int} = g\phi_c^2(x)A_{\rm med}(x)$.
- Measure jet observable $\hat{O}[\phi]$, inclusive over the medium. Often treat medium as classical background, and perform ensemble average

$$\overline{O} = \int \mathcal{D}[\phi_+, \phi_-]\hat{O}[\phi] \underbrace{\int \mathcal{D}[A]e^{-W[A]}}_{\text{medium ensemble avg.}} e^{iS[\phi_+]-iS[\phi_-]+ig\int d^4x\left[\phi_+^2(x)-\phi_-^2(x)\right]A(x)}$$

Some promising aspects!

- Medium correction to $\overline{O}$ depends on $-\frac{g^2}{2}(\phi_+^2 - \phi_-^2)_x \Gamma(x, y)(\phi_+^2 - \phi_-^2)_y$, etc, which encodes statistical correlation functions

$$\Gamma(x, y) = \langle A(x)A(y)\rangle = \int \mathcal{D}[A] e^{-W[A]} A(x)A(y)\,, \;\Gamma(q_\perp) \sim \frac{\rho_{\text{eff}}}{(q_\perp^2 + \xi^2)^2}$$

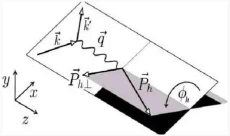
- **Opacity expansion**: an expansion in $\Gamma(x, y)$ can be defined at the action level.
- The effective interaction is color singlet after medium ensemble average, may be useful for showing factorization using SCET technique.

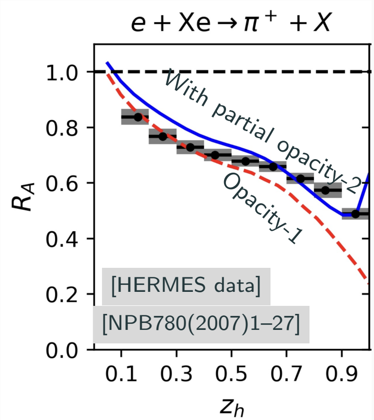
Summary

- High-p_T probes are essential for tomography study of AA and eA.
- The resummation of medium correction require different strategies.
  - Transport equation for $\tau_f \ll L$ and large opacity $\hat{q}L/m_D^2 \gg 1$
  - Modified DGLAP equation for $\tau_f \gg L$ and small opacity $\hat{q}L/m_D^2 \sim 1$.
- mDGLAP provide a good description of R_{AA} in O-O, but we need a more differential test of parton shower in medium.
- EEC sees the scale evolution of parton shower. A shower following mDGLAP will result in a modified anomalous dimension $\Leftrightarrow$ directly related to medium properties.
- An appealing long-term objective: to put mDGLAP-like theory on a more solid foundation, and match with transport theory.

Questions?

Analytic insights into the mDGLAP equation





- The mDGLAP equation can be further expanded under the hierarchy $2E/L \gg m_D^2$ [WK, I. Vitev PLB854(2024)138751] ,

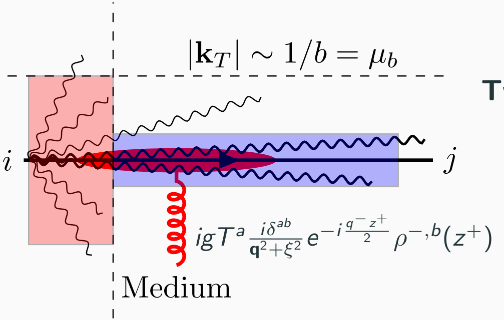
$$\frac{\partial F_q(z,\mu^2)}{\partial \ln \mu^2} = v\left(4C_F C_A \frac{\partial}{\partial z} - \frac{2C_F(2C_A + C_F)}{z}\right) F_q + v\frac{C_F T_R F_g}{z} + \mathcal{O}(v^2),$$

$$\frac{\partial F_g(z,\mu^2)}{\partial \ln \mu^2} = v\left(4C_A^2 \frac{\partial}{\partial z} - \frac{2N_f C_F}{z}\right) F_g + v\frac{2C_F^2 \sum_{q,\bar{q}} F_q}{z} + \mathcal{O}(v^2).$$

- Small opacity + scale separation:
$v = \alpha_s \frac{\alpha_s \rho L}{4E/L} B \ll 1$.
- Resum emissions between $2E/L > p^2 > m_D^2$, and causes energy loss.

$$R_A = \left[\frac{1}{\sigma}\frac{d\sigma}{dx_B\, dy\, dz_h}\right]_{e+A} \Big/ \left[\frac{1}{\sigma}\frac{d\sigma}{dx_B\, dy\, dz_h}\right]_{e+d}$$

Medium modified TMD fragmentation (3D)



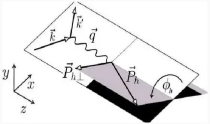
Two leading-log effects at first order in opacity

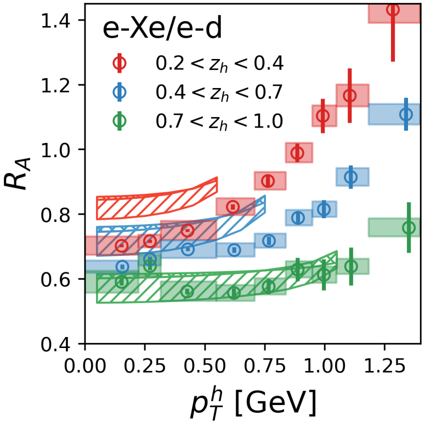
- Medium-induced collinear evolution.
- Momentum broadening due to forward scattering with medium and induced soft radiation.

$$C_{ij}^{\text{vac+med}}(z, b, \mu, \frac{\zeta}{\nu^2}) = e^{\sum_\ell \rho_\ell^- L^+ [\Sigma_{i\ell}(b, \mu_b, \frac{\sqrt{\zeta_{\text{LPM}}\zeta_{\text{med}}}}{\mu_b^2}) - \Sigma_{i\ell}|_{b=0}]} U_{\text{TMD}}(\mu_b, \mu, \frac{\zeta}{\nu^2})$$

$$C_{ik}(z, \alpha_s(\mu_b)) \otimes U_{ks}^{\text{DGLAP}}(z, \mu_b, \mu_0) \otimes M_{si}^{\text{med}}(z, \mu_b, Q^2, 2E/L, \xi^2)$$

Medium (dynamical) Modified TMD fragmentation (3D)





$$R_A = \left[\frac{1}{\sigma}\frac{d\sigma}{dx_B dQ^2 dz_h dp_T^h}\right]_{e+A} \Big/ \left[\frac{1}{\sigma}\frac{d\sigma}{dx_B dQ^2 dz_h dp_T^h}\right]_{e+d}$$

e+Xe to pion at HERMES, three different z_h bins.

- Uncertainty in collinear FF in the vacuum (NNFF1.0nlo).
- Uncertainty in collinear nuclear PDF (nCTEQ15WZnlo).
- Uncertainty in cold nuclear matter parameters (vary by 50%).