Temporal Evolution and Spatial Distribution of a Dynamically Generated Nucleon Resonance

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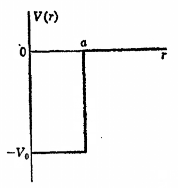
CONTENTS

Brief Review of a Resonance with Quantum Mechanics

Bound states can be well described with quantum mechanics

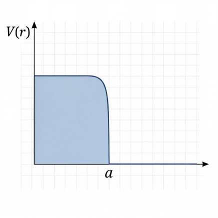


- Schrödinger equation provides
  - mass of bound state
  - wavefunction of bound state
- One can extract everything from the wavefunction.
- The wavefunction has a direct probability interpretation.

A resonance can also be described with Schrödinger equation

- A state χ_k with energy $E = \frac{\hbar^2 k^2}{2m}$ in s wave satisfies:

$$\chi_k''(r) + [k^2 - \frac{2m}{\hbar^2} V(r)]\chi_k(r) = 0$$



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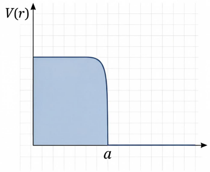

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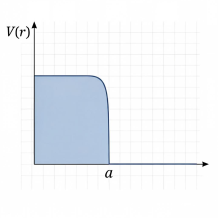

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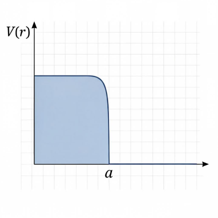

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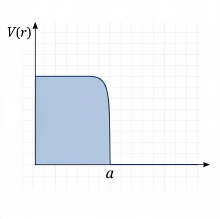

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There are two main problems for the characterization of resonance

Complex pole energy $E_{\text{pole}} = m_{\text{pole}} - i\Gamma_{\text{pole}}/2$ can be obtained by

- searching for the poles of S matrix;
- or directly solving Schrödinger equation with the complex-scaling method $r \to re^{i\theta}$.

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There are two main problems for the characterization of resonance:

- One is related to the experiment.
- The other is from the theory itself.

Complex pole energy $E_{\text{pole}} = m_{\text{pole}} - i\Gamma_{\text{pole}}/2$ cannot be directly measured

- For the peaks in experiments:
  central value $\neq m_{\text{pole}}$, width $\neq \Gamma_{\text{pole}}$

- The peaks of some resonances are not obvious.
- Some peaks are not originated from resonances.

- The line shapes of invariant spectra would be more complicated.

The direct probability interpretation is lost

- When $k \to k_{\text{pole}}$, $\chi_k(r)$ seems divergent because of $S(k_{\text{pole}}) \to \infty$

$$\chi_k(r) = A(k)[e^{-ikr} - S(k)e^{+ikr}]$$

- But one can use different normalization to make the above divergence disappear.

$$\chi_k(r) \cong \frac{1}{S(k)} e^{-ikr} - e^{+ikr} \qquad \to -e^{+ikr}$$

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- However there is an unavoidable divergence as $r \to \infty$ at the pole $k_{\text{pole}} = k_R - ik_I$.

$$\chi_{k_{\text{pole}}}(r) \to -e^{+ik_{\text{pole}}r} = -e^{+ik_R r} e^{k_I r} \to \infty$$

The resonance is always tangled with the emitted scattering states.

How to interpret $\chi_{k_{\text{pole}}}(r) \xrightarrow{r\to\infty} \infty$?

- It is not the resonance that spreads infinitely far away.

- The wavefunction obviously contains the information of the emitted scattering states.

$$\chi_k(r) = A(k)[e^{-ikr} - S(k)e^{+ikr}]$$

- The emitted particles fly infinitely apart.

Representations for the $\Lambda(1405)$ Dynamically Generated by $\pi\Sigma$-$\bar{K}N$

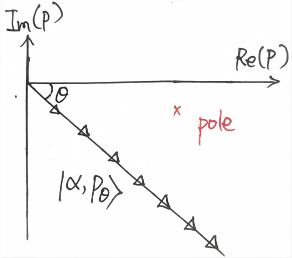
The $\Lambda(1405)$ is closely related to the $\pi\Sigma$-$\bar{K}N$ system

- The $\Lambda(1405)$ is located just below the $\bar{K}N$ threshold.
- It was observed mainly through the $\pi\Sigma$ channel.
- It is widely interpreted as a $\bar{K}N$ quasi-bound state or
  a dynamically generated meson-baryon resonance.

Gamow vector with complex scaling method

- introduce the two-particle basis with complex-scaling momentum $p \to p_\theta \equiv pe^{-i\theta}$

$$|\alpha, p_\theta\rangle = |\pi\Sigma, p_\theta\rangle, |\bar{K}N, p_\theta\rangle$$



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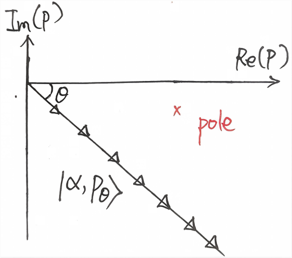
- solve the Hamiltonian equation in the above basis

$$H\,|\psi^{\text{Gamow}}\rangle = (H_0 + V)\,|\psi^{\text{Gamow}}\rangle = E_{\text{pole}}\,|\psi^{\text{Gamow}}\rangle\,.$$



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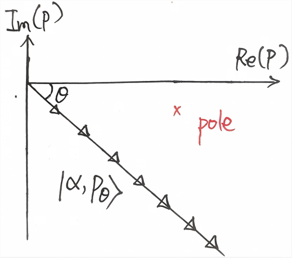
$$H\left|\psi^{\text{Gamow}}\right\rangle = (H_0 + V)\left|\psi^{\text{Gamow}}\right\rangle = E_{\text{pole}}\left|\psi^{\text{Gamow}}\right\rangle .$$

- then obtain resonance pole E_{pole} and corresponding Gamow wavefunction $\left|\psi^{\text{Gamow}}\right\rangle$.



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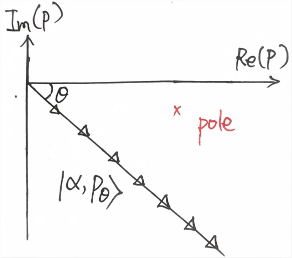
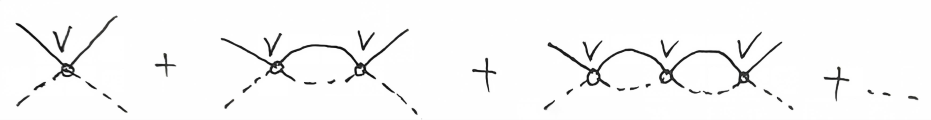


The Gamow vector is spanned in the complex-momentum space which cannot be measured directly.

Alternatively Gamow wavefunction can be obtained by residues of T matrix



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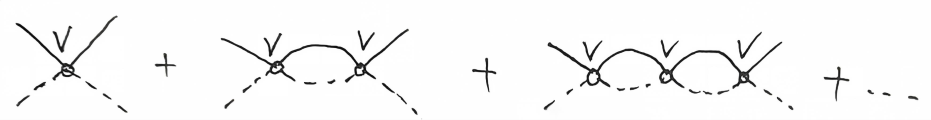

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- extract the residues $\gamma_\alpha(p_\theta)$ of T matrix

$$T_{\alpha,\beta}(p_\theta, p'_\theta; E) \approx \frac{\gamma_\alpha(p_\theta)\,\gamma_\beta(p'_\theta)}{E - E_{\text{pole}}} + \cdots ,$$

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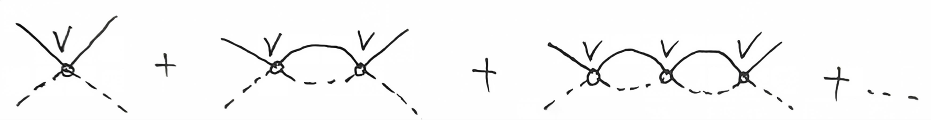

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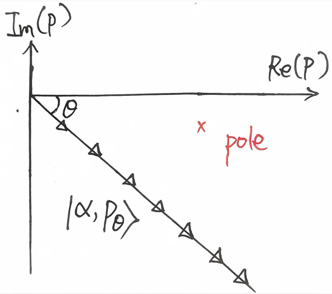
$$T_{\alpha,\beta}(p_\theta, p'_\theta; E) \approx \frac{\gamma_\alpha(p_\theta)\,\gamma_\beta(p'_\theta)}{E - E_{\text{pole}}} + \cdots ,$$

- finally get the relation between the Gamow wavefunction and residues of T matrix

$$\psi_\alpha^{\text{Gamow}}(p_\theta) \equiv \langle \alpha, p_\theta | \psi^{\text{Gamow}} \rangle = \frac{\gamma_\alpha(p_\theta)}{E_{\text{pole}} - \omega_\alpha(p_\theta)} ,$$

A natural transition to the representation in the real world



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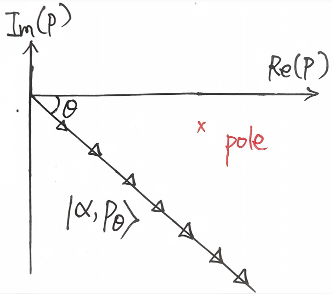

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- Gamow vector in Complex momentum space;
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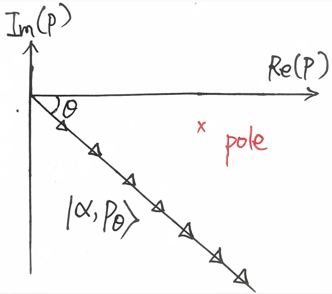

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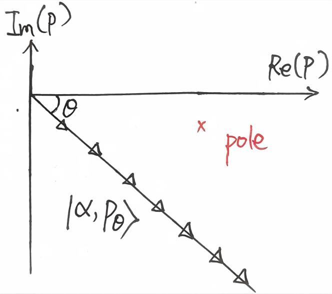

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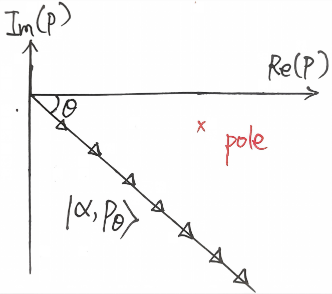

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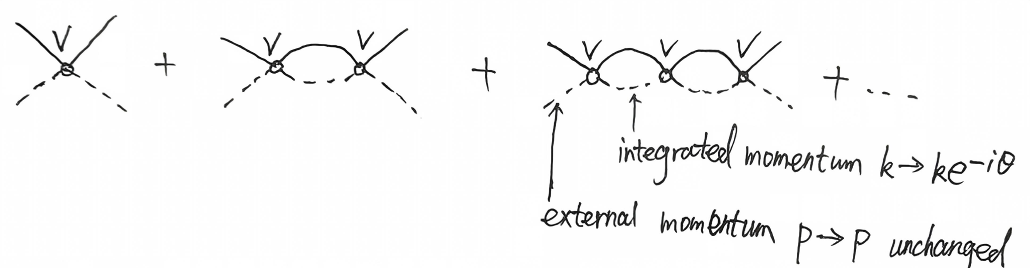
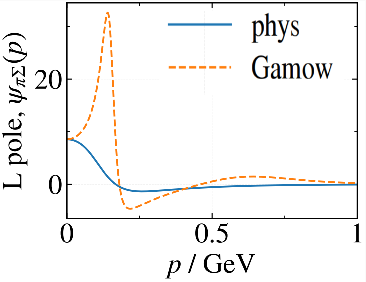


Relations between Gamow Wavefunctions and Physical Representations

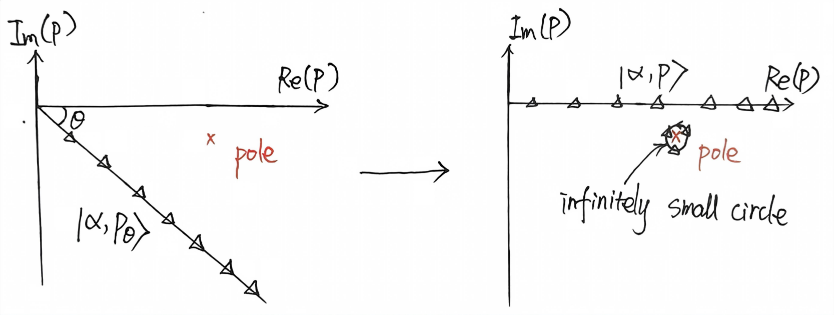
- Two wavefunctions $\psi_\alpha^{\text{Gamow}}(p_\theta)$ and $\psi_\alpha^{\text{phys}}(p)$ look very different



- But each contains the same resonance information
  because they are connected by analytic continuation.
- Thus some nontrivial relations still exist, for example we notice

$$2\,\mathrm{Re}\sum_\alpha \int dp_\theta\, p_\theta^2\, \psi_\alpha^{\text{Gamow}*}(p_\theta^*)\, \psi_\alpha^{\text{Gamow}}(p_\theta) = \sum_\alpha \int dp\, p^2\, \psi_\alpha^{\text{phys}*}(p^*)\, \psi_\alpha^{\text{phys}}(p)\,.$$

A New Relations between Gamow and Physical Representations



- By introducing a virtual vector state spanned around the basis states with the kinetic energy approaching the pole

$$|\psi^{\text{virtual}}\rangle = \sum_{\{\alpha \mid \theta_\alpha \neq 0\}} \oint_{\mathcal{C}_\epsilon^\alpha \to p_\alpha^{\text{on}}} dp\, p^2 \frac{\gamma_\alpha(p)}{E_{\text{pole}} - \omega_\alpha(p)} |\alpha, p\rangle,$$

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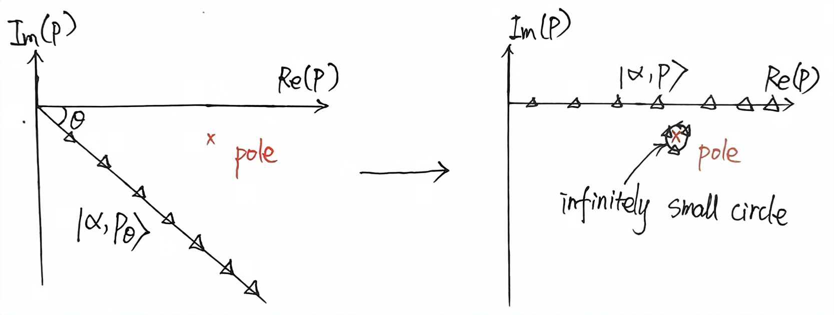

- we can prove

$$H\left(|\psi^{\text{phys}}\rangle + |\psi^{\text{virtual}}\rangle\right) = E_{\text{pole}}\left(|\psi^{\text{phys}}\rangle + |\psi^{\text{virtual}}\rangle\right)$$

  - $|\psi^{\text{phys}}\rangle$ in the ordinary Hilbert space
  - $|\psi^{\text{virtual}}\rangle$ in the special complex space closely around the pole

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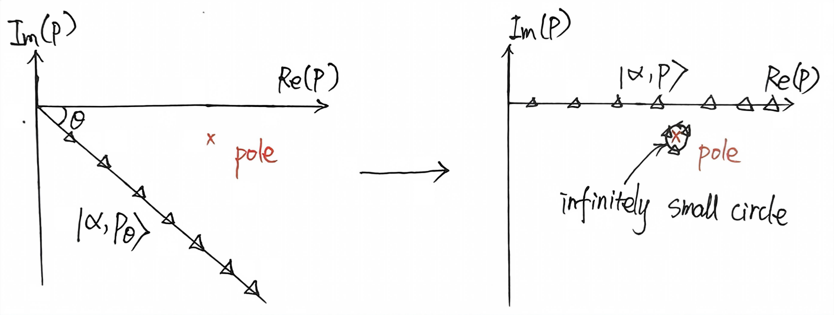


- With respect to the Hamiltonian equation,

$$|\psi^{\text{Gamow}}\rangle \cong |\psi^{\text{phys}}\rangle + |\psi^{\text{virtual}}\rangle \,,$$

which is a new essential relation between $|\psi^{\text{Gamow}}\rangle$ and $|\psi^{\text{phys}}\rangle$ in addition to their analytic connection.

$\Lambda(1405)$ Survival Probabilities and Final $\pi\Sigma$ Invariant Spectrum

Two pole structure associated with the $\Lambda(1405)$

- The $\Lambda(1405)$ region can contain two nearby isoscalar poles with chiral effective field theory *[Kaiser:1995cy,Oset:1997it,Oller:2000fj,Jido:2003cb]*
- Recent amplitude analyses and direct $d(K^-, n)\pi\Sigma$ measurements further constrain the pole positions *[Sadasivan:2022srs,J-PARCE31:2022plu]*.
- The $\pi\Sigma$ line shape has been studied in experiments *[Hemingway:1984pz,Siebenson:2013rpa,Wickramaarachchi:2022mhi,BESIII:2024jgy, Wang:2026daa]*, and these results show that the measured line shape should not be identified with a single Breit-Wigner resonance without considering the underlying pole structures.

Our framework for the dynamics of $\Lambda(1405)$

- We have used Hamiltonian effective field theory to analyze the $\Lambda(1/2^-)$ family by analyzing *[Liu:2023xvy,Liu:2016wxq,Hall:2014uca]*
  - $K^- p$ scattering data,
  - Lattice QCD data of $\Lambda(1/2^-)$ spectrum.

  We noticed
  - $\Lambda(1405)$ is mainly a $\bar{K}N$-$\pi\Sigma$ dynamically generated state.
  - The bare triquark core is important for $\Lambda(1670)$.

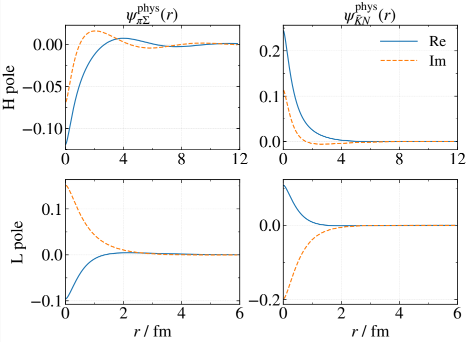
- In this work we use a simple separable interaction in the $\pi\Sigma$ and $\bar{K}N$ channels

$$V_{\alpha\beta}(p,p') = \frac{3v_{\alpha\beta}}{4\pi^2 f_\pi^2}\, u_\alpha(p)\, u_\beta(p')\,, \quad u_\alpha(p) = \left(1 + \frac{p^2}{\Lambda_\alpha^2}\right)^{-2} .$$

  By adjusting the coupling parameters, we obtain
  - the higher-mass (H) pole $E^{\mathrm{H}}_{\mathrm{pole}} = 1430 - 22\, i$ MeV,
  - the lower-mass (L) pole $E^{\mathrm{L}}_{\mathrm{pole}} = 1338 - 89\, i$ MeV.

Coordinate Wavefunctions of $\Lambda(1405)$

The wavefunctions $\psi_\alpha^{\text{phys}}(r) \equiv \langle \alpha, r | \psi^{\text{phys}} \rangle$ in coordinate space can be obtained by Fourier transformation.



They are confined within finite ranges.

Temporal evolution for the two $\Lambda(1405)$ poles

With

$$|\psi^{\text{phys}}\rangle = \sum_{\alpha} \int dp\, p^2 \frac{\gamma_\alpha(p)}{E_{\text{pole}} - \omega_\alpha(p)} |\alpha, p\rangle \,,$$

we can calculate the evolution of the states

$$|\psi^{\text{phys}}, t\rangle = e^{-iHt} |\psi^{\text{phys}}\rangle \,.$$

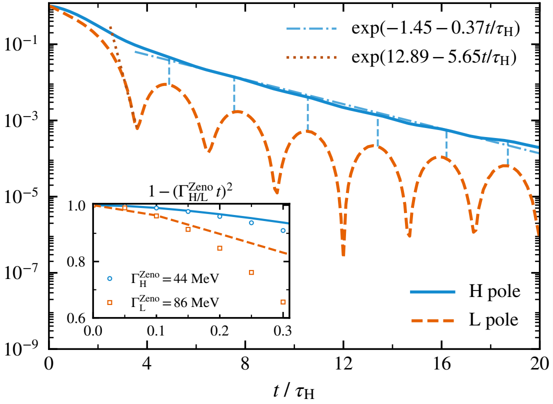
In detail

- insert the completeness relation with $|\alpha, p\rangle$
- discretize the resulting momentum integrals using Gaussian quadrature
- thus reduce time evolution to a finite-dimensional matrix-vector operation for numerical solutions.

Survival probabilities for the two $\Lambda(1405)$ poles

$$|\psi^{\text{phys}}, t\rangle = C(t)\,|\psi^{\text{phys}}\rangle + |\chi_{\text{scatt.}}, t\rangle.$$

Survival probabilities$= |C(t)|^2$.



- At very early times,
  - the survival probability follows the expected quadratic behavior, known as the quantum Zeno effect *[Wigner:1955zz, Chiu:1977ds, LEVITAN1988267]*
  - the fact that $\Gamma_{\text{H}}^{\text{Zeno}} \approx \Gamma_{\text{pole}}^{\text{H}}$ but $\Gamma_{\text{L}}^{\text{Zeno}} < \Gamma_{\text{pole}}^{\text{L}}$ indicates the complicated interference between these two nearby poles and the scattering channels.

Survival probabilities for the two Λ(1405) poles

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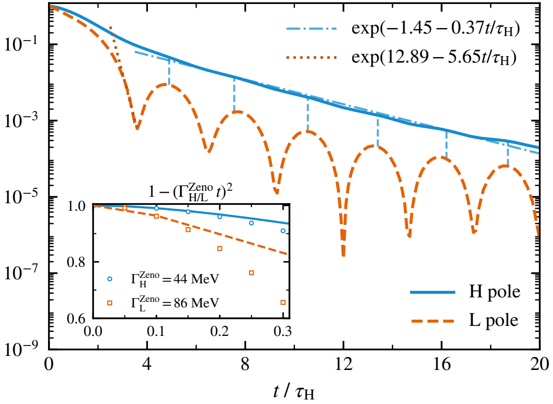

- At late times for the lower-mass pole,

  its survival probability shows an obvious damped oscillatory behavior

  which is a known quantum phenomenon of resonance.

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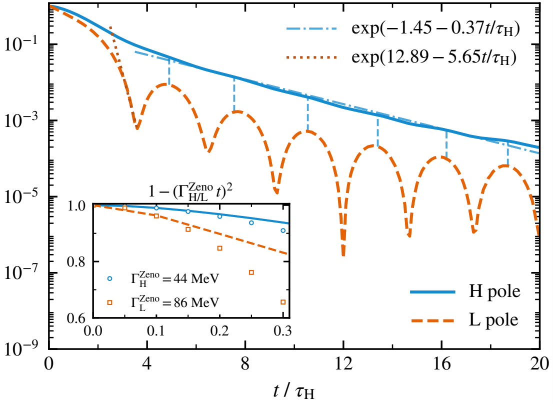


- In the intermediate-time region for the higher-mass pole,
  - we show an exponential fit $\exp(-1.45 - 0.37t/\tau_{\mathrm{H}})$ over $3.5 \leq t/\tau_{\mathrm{H}} \leq 20$
  - the deviation exhibits weak oscillations whose peaks marked by the vertical dashed lines coincide with the corresponding peaks in the survival probability of the lower-mass pole

suggesting these two states are strongly correlated.

Resonances decay with the scattering stated emitted

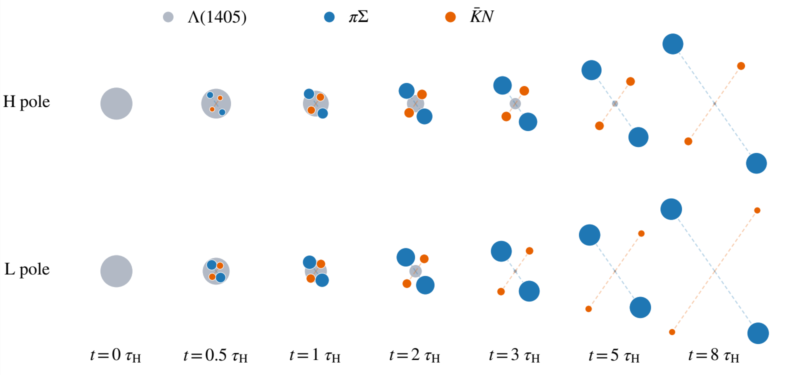


Illustration of the time evolution $|\psi^{\text{phys}}, t\rangle = e^{-iHt}\,|\psi^{\text{phys}}\rangle$ of the resonance components

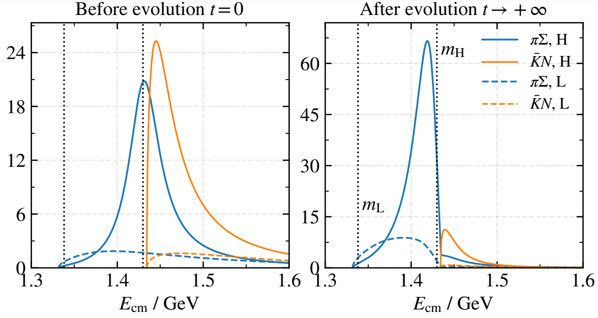
- The gray area is proportional to the survival probability
  and decreases with time in units of $\tau_{\mathrm{H}} = 1\,/\,|\Gamma^{\mathrm{H}}_{\text{pole}}|$
- The emitted $\pi\Sigma$ and $\bar{K}N$ move away from the interaction region.
  Their distances from the center are proportional to their root-mean-square radii.

The asymptotic state at the infinite time

The asymptotic state $|\psi^{\text{phys}}, t \to +\infty\rangle$ in the Schrödinger picture

$$|\psi^{\text{phys}}, t \to +\infty\rangle = \sum_\alpha \int dp\, p^2 \psi_\alpha^{\text{phys}}(p) \times \left(e^{-i\omega_\alpha(p)t} |\alpha, p\rangle + \sum_\beta \int dq\, q^2 \frac{T_{\beta,\alpha}(q,p;E)}{\omega_\beta(q) - \omega_\alpha(p) + i\epsilon} e^{-i\omega_\beta(q)t} |\beta, q\rangle \right) .$$

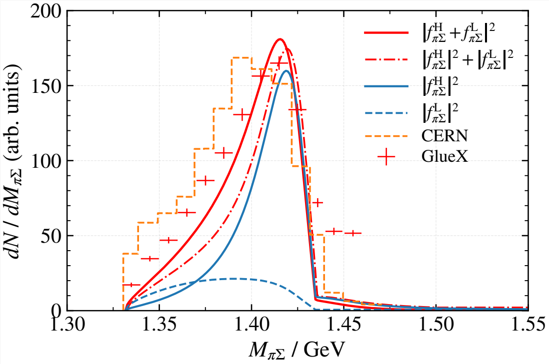
The resulting distribution $|f_\alpha(E)|^2 = |\langle \alpha, E \,|\, \psi^{\text{phys}}(t = +\infty)\rangle|^2$



For the high-mass pole

- the $\bar{K}N$ component is significant when initially formed
- most of it turns into the $\pi\Sigma$ rather than $\bar{K}N$ scattering states because of
  - energy conservation when $t \to \infty$
  - the larger mass of $\bar{K}N$ than m^{H}_{pole}

Comparison with the experimental measurements



We give the full results with both

- the strongest interference scenario $|f^{\mathrm{H}}_{\pi\Sigma}(E)+f^{\mathrm{L}}_{\pi\Sigma}(E)|^2$
- no interference assumption $|f^{\mathrm{H}}_{\pi\Sigma}(E)|^2+|f^{\mathrm{L}}_{\pi\Sigma}(E)|^2$

The currently observed $\pi\Sigma$ spectrum prefers the strongest interference scenario.

Summary

Summary

- We have developed a physical description $|\psi^{\mathrm{phys}}_{\mathrm{H/L}}\rangle$ of the two-pole $\Lambda(1405)$ in the real momentum basis by
  analytically continuing Gamow wavefunction from complex momentum space.

- The $|\psi^{\mathrm{phys}}_{\mathrm{H/L}}\rangle$ is spanned in the ordinary Hilbert space,
  and thus the probability interpretation can be naturally obtained.

- With respect to the Hamiltonian eqaution, $|\psi^{\mathrm{phys}}_{\mathrm{H/L}}\rangle + |\psi^{\mathrm{virtual}}_{\mathrm{H/L}}\rangle$ is equivalent to $|\psi^{\mathrm{Gamow}}_{\mathrm{H/L}}\rangle$.

With this framework, we suggest the final invariant spectrum should include the convolution of resonance wavefunctions in addition to the scattering T matrix.

Thank you for your attention!