X(3872)中的短程和长程贡献

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第十一届“XYZ研讨会”

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研究背景

2003年

VOLUME 91, NUMBER 26 PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2003

Observation of a Narrow Charmoniumlike State in Exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ Decays

S.-K. Choi,[5] S. L. Olsen,[6] K. Abe,[7] T. Abe,[7] I. Adachi,[7] Byoung Sup Ahn,[14] H. Aihara,[43] K. Akai,[7] M. Akatsu,[20] M. Akemoto,[7] Y. Asano,[48] T. Aso,[47] V. Aulchenko,[1] T. Aushev,[11] A. M. Bakich,[38] Y. Ban,[31] S. Banerjee,[39] A. Bondar,[1] A. Bozek,[25] M. Bračko,[18,12] J. Brodzicka,[25] T. E. Browder,[6] P. Chang,[24] Y. Chao,[24] K.-F. Chen,[24] B. G. Cheon,[37] R. Chistov,[11] Y. Choi,[37] Y. K. Choi,[37] M. Danilov,[11] L. Y. Dong,[9] A. Drutskov,[11] S. Eidelman,[1] V. Eiges,[11] J. Flanagan,[7]

$$X(3872):\ J^{PC} = 1^{++}$$

$$M_{X(3872)} = 3871.64\ \text{MeV}$$
$$\Gamma_{X(3872)} = 1.19\ \text{MeV}$$

当时，还没有大量的奇特强子态被发现。人们对强子的认识仍然停留在夸克模型给出的介子和重子。

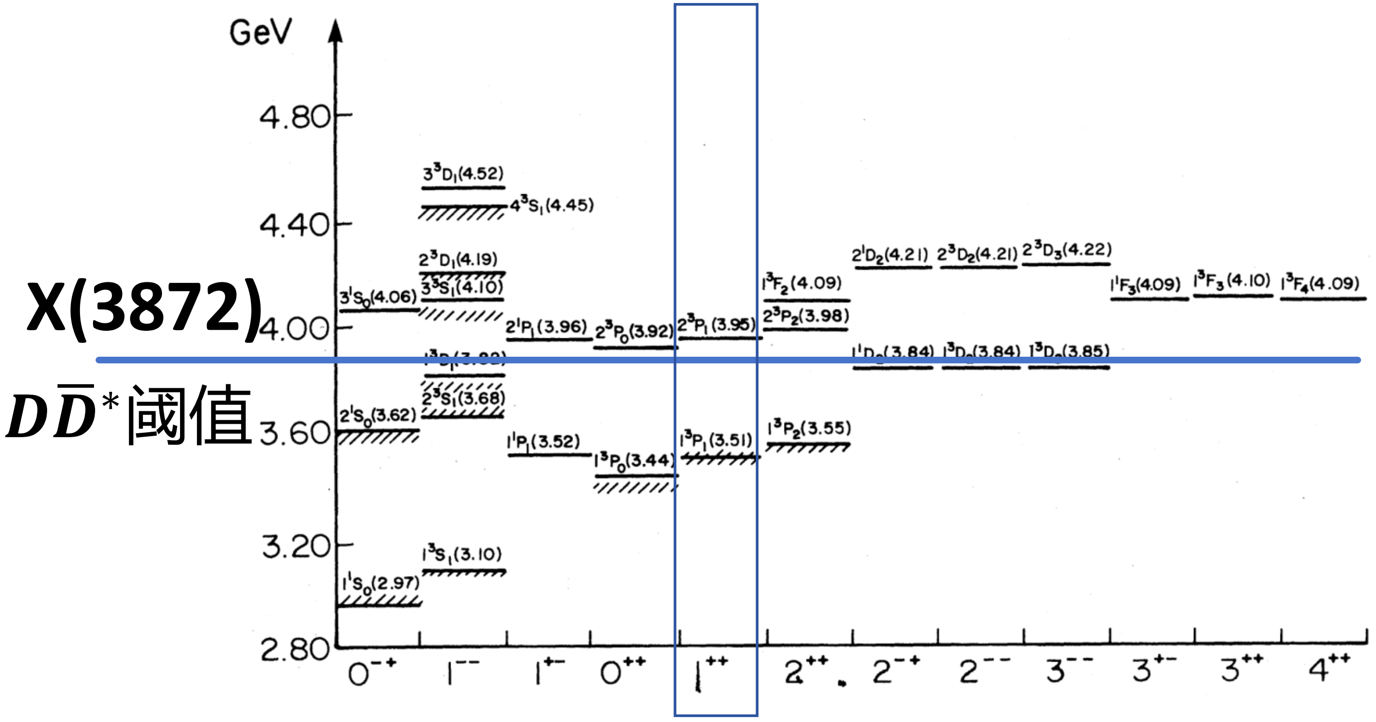
当时注意到X(3872)的特殊之处是：
1.它的质量非常靠近阈值，
2.宽度非常窄，
3.在粲偶素质量谱上缺少对应。

在上述情形下，人们思考X(3872)是奇特强子态的可能性。

研究背景

Meson	State	Mode	Γ_{thy} (MeV)	Amps. ($GeV^{-1/2}$)
$\chi_2(3972)$	2^3P_2	DD	42	$^1D_2 = +0.0992$
		DD*	37	$^3D_2 = -0.1172$
		D_sD_s	0.7	$^1D_2 = +0.0202$
		total	80	
$\chi_1(3925)$	2^3P_1	DD*	165	$^3S_1 = +0.2883$
				$^3D_1 = -0.0525$
$\chi_0(3852)$	2^3P_0	DD	30	$^1S_0 = +0.1025$
$h_c(3934)$	2^1P_1	DD*	87	$^3S_1 = -0.1847$
				$^3D_1 = -0.0851$

势模型中最接近X（3872）质量的1^{++}态



在简单势模型的图像中，X(3872)的质量和宽度都不能被解释。

它的质量和宽度参数引起了众多讨论

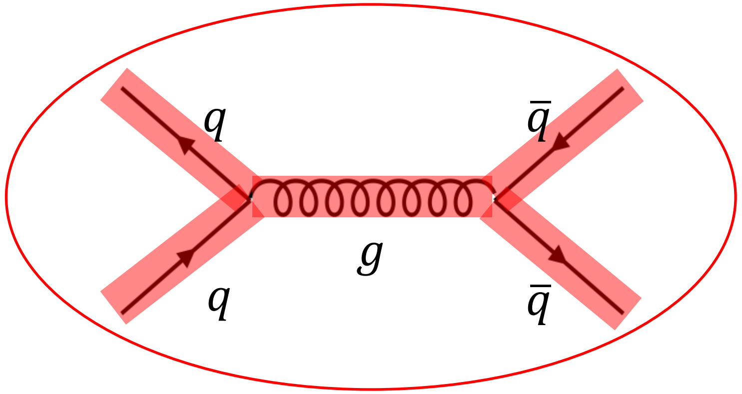
分子态

紧致多夸克态

运动学效应

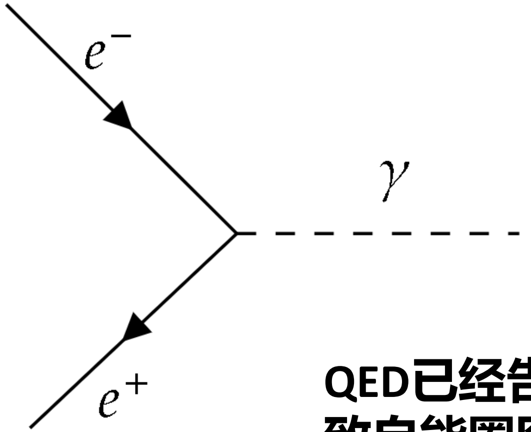
耦合道模型中的X(3872)

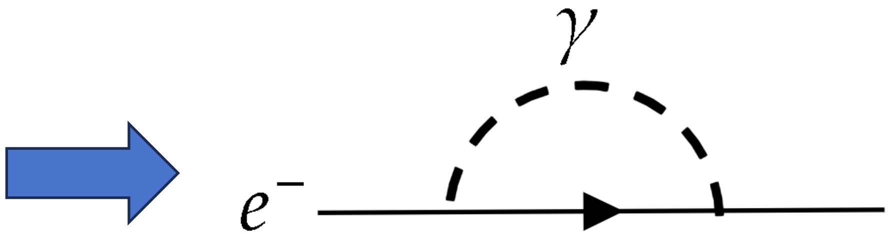
在放弃势模型图像理解X(3872)之前，我们先进行一点仔细地考虑：



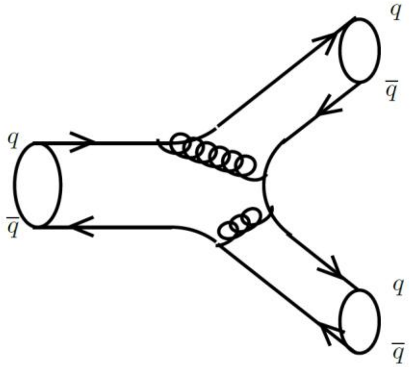
势模型仅考虑强子内夸克反夸克间相互作用

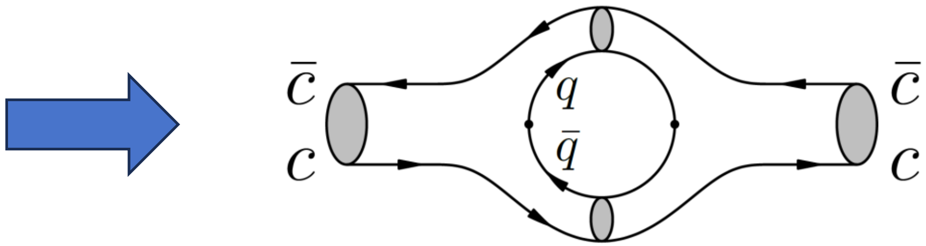
所以简单势模型对强子质量谱的预言还不完备





QED已经告诉我们相互作用顶点的存在会导致自能圈图，进而会存在自能修正





相互作用本身对粒子质量的影响通过自能圈图表达

耦合道模型中的X(3872)

实际上，针对这种问题人们早就做出了许多研究

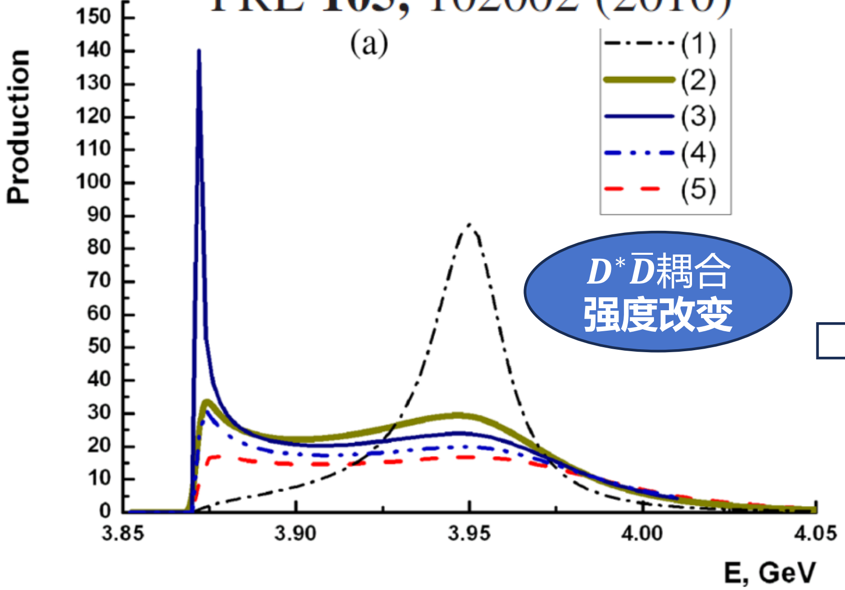
Refs.	bare Mass (MeV)	physical mass (MeV)	mass shift (MeV)
[1]	4180	3990	-190
[2]	3914	3868	-46
[3]	3953	3884	-69
[4]	4030	3875	-155
[5]	4038	3930	-108

此处，裸质量表示考虑自能修正前的势模型质量，物理质量表示考虑自能修正后的结果。

[1] Phys. Rev. D 72, 034010
[2] Phys. Rev. D 76, 077502
[3] Eur. Phys. J. A 50, 165
[4] Phys. Rev. D 80, 014012
[5] Z. Phys. C 23, 59

谱学研究告诉我长程相互作用不能被忽视

PRL **105**, 102002 (2010)



在2010年的PRL文章里，X(3872)被明确地解释为耦合道现象产生的态

TABLE I. Hadronic shift (MeV) of charmonium $2^{3,1}P_J$ bare states E_n for different channels. The bare positions were taken from Badalian *et al.* [7]. δ is the total shift and κ is the closed channel (D^*D^*) suppression coefficient.

State	J^{PC}	E_n	Shifts ($\gamma = 1.1$) DD	DD^*		D^*D^*	δ	E	Exp.
			···	···	$\kappa = 0$	0	0	3.969	
2^3P_2	2^{++}	3969	···	···	$\kappa = 0.25$	−14	−14	3.955	Z(3930)
			···	···	$\kappa = 0.5$	−27	−27	3.942	
2^3P_1	1^{++}	3954			to threshold			3.872	X(3872)
			−2	···	$\kappa = 0$	0	−2	3.916	
2^3P_0	0^{++}	3918	−2	···	$\kappa = 0.25$	−33	−35	3.883	···
			−2	···	$\kappa = 0.5$	−66	−68	3.850	
2^1P_1	1^{+-}	3959	···	−29	$\kappa = 0.25$	−7	−36	3.927	···
			···	−32	$\kappa = 0.5$	−14	−46	3.918	

耦合道模型中的X(3872)

我们的工作：

Phys.Rev.D 101,054029，arXiv:2002.03311

红色实线代表$\chi_{cJ}(2P)$的自能函数$\mathrm{Re}\Pi(m^2)$

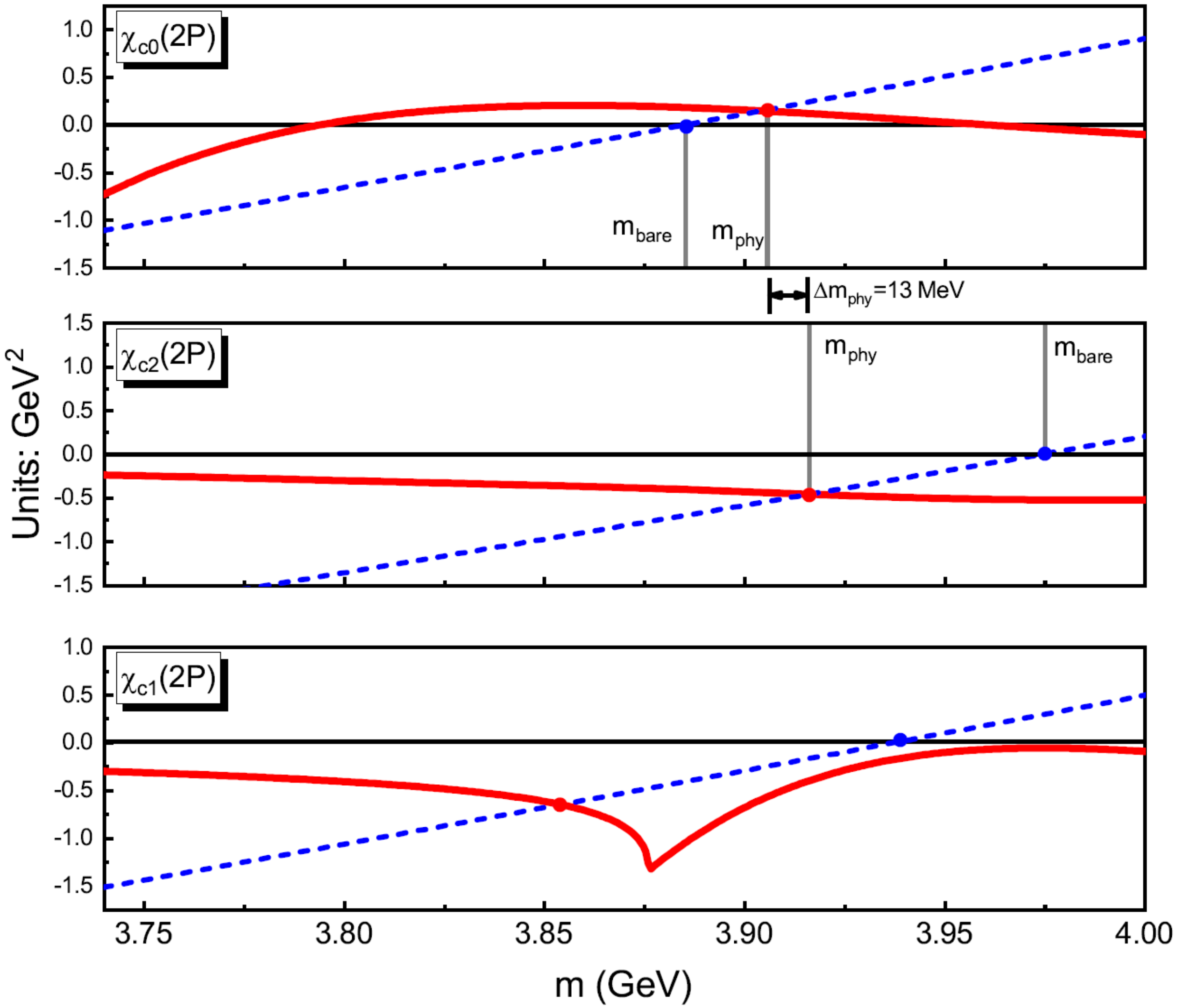
蓝色虚线代表$m^2 - m_{\mathrm{bare}}^2$函数

上述两条线的交点是耦合道方程的解，即物理质量。

State	m_{bare} (MeV)	m_{phy} (MeV)	δm (MeV)	Γ (MeV)
$\chi_{c0}(2P)$	3885	3904	+19	23
$\chi_{c1}(2P)$	3936	3855	−81	0
$\chi_{c2}(2P)$	3974	3917	−57	26

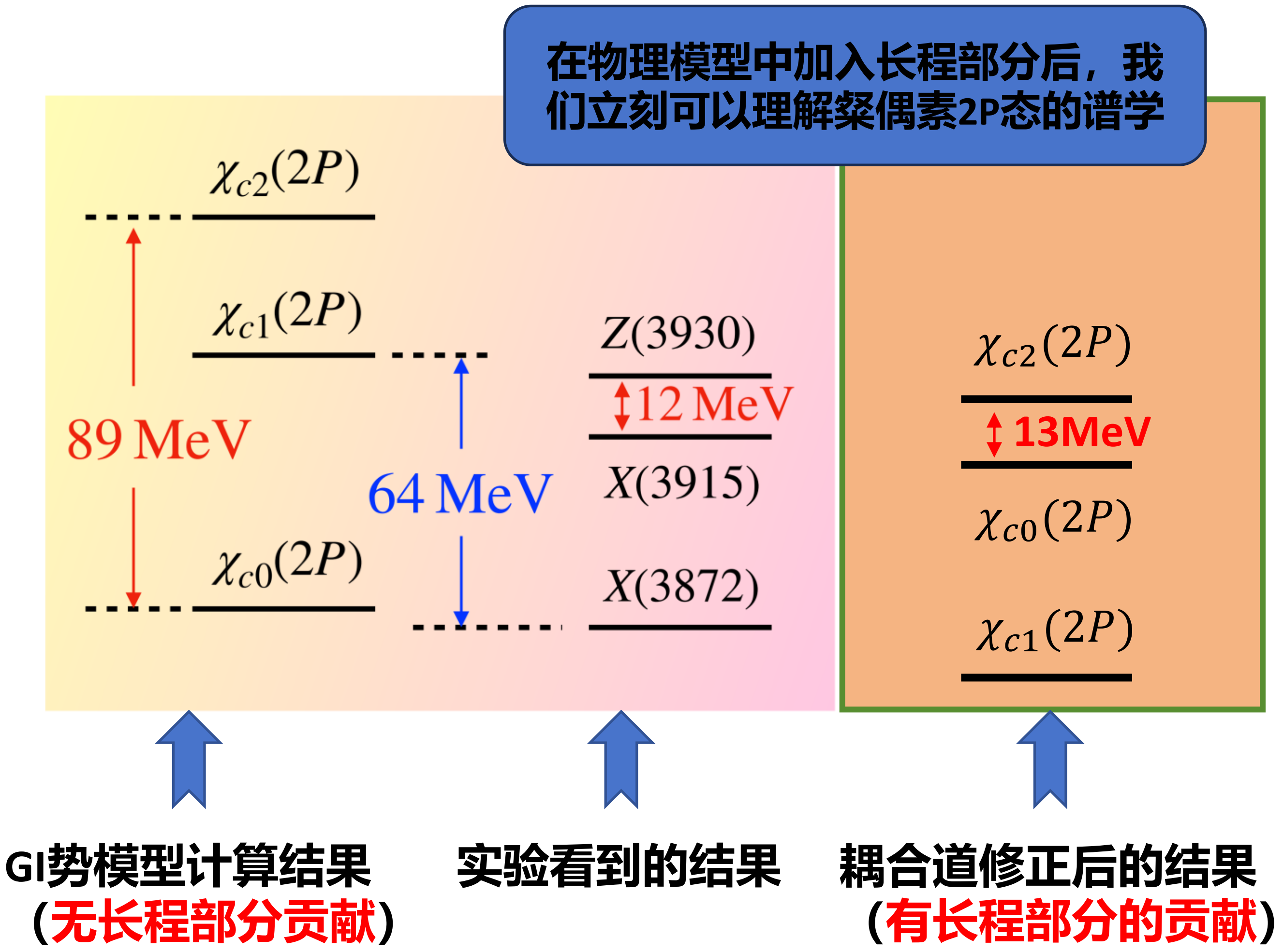
经过计算:

- $\chi_{c1}(2P)$的质量从3.95 GeV附近下降到X(3872)附近。
- $\chi_{c0}(2P)$和$\chi_{c2}(2P)$间的质量劈裂被大幅地削减，以至于仅剩13 MeV的质量差。



耦合道模型中的X(3872)

耦合道图像下——X(3915), X(3872)和Z(3930)的分类问题可以完美解决



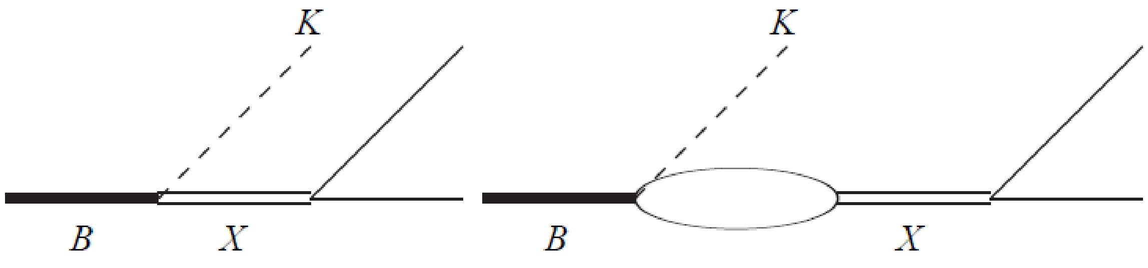
考虑耦合道修正后，2P态粲偶素$\chi_{c0}(2P)$, $\chi_{c1}(2P)$和$\chi_{c2}(2P)$的谱学和衰变与实验探测到的X(3915), X(3872)和Z(3930)粒子的质量和宽度完全吻合。

这说明在耦合道图像下，这三个XYZ粒子完全可以被分类为粲偶素。同时也说明了

- **耦合道修正在粲偶素激发态的研究中不能被忽略。**
- **引入$D^{(*)}\bar{D}^{(*)}$自能圈图意味着阈值效应在X(3872)的研究中亦不能被忽略。也就是说X(3872)物理态既包含短程的$|c\bar{c}\rangle$部分也包含了长程的$|D^{(*)}\bar{D}^{(*)}\rangle$贡献。**

B衰变中X(3872)的产生

考虑X(3872)的产生过程：$B \to K D^{*0}\bar{D}^0$ （C.Phys.C 39,093101 arXiv:1309.2859v4）

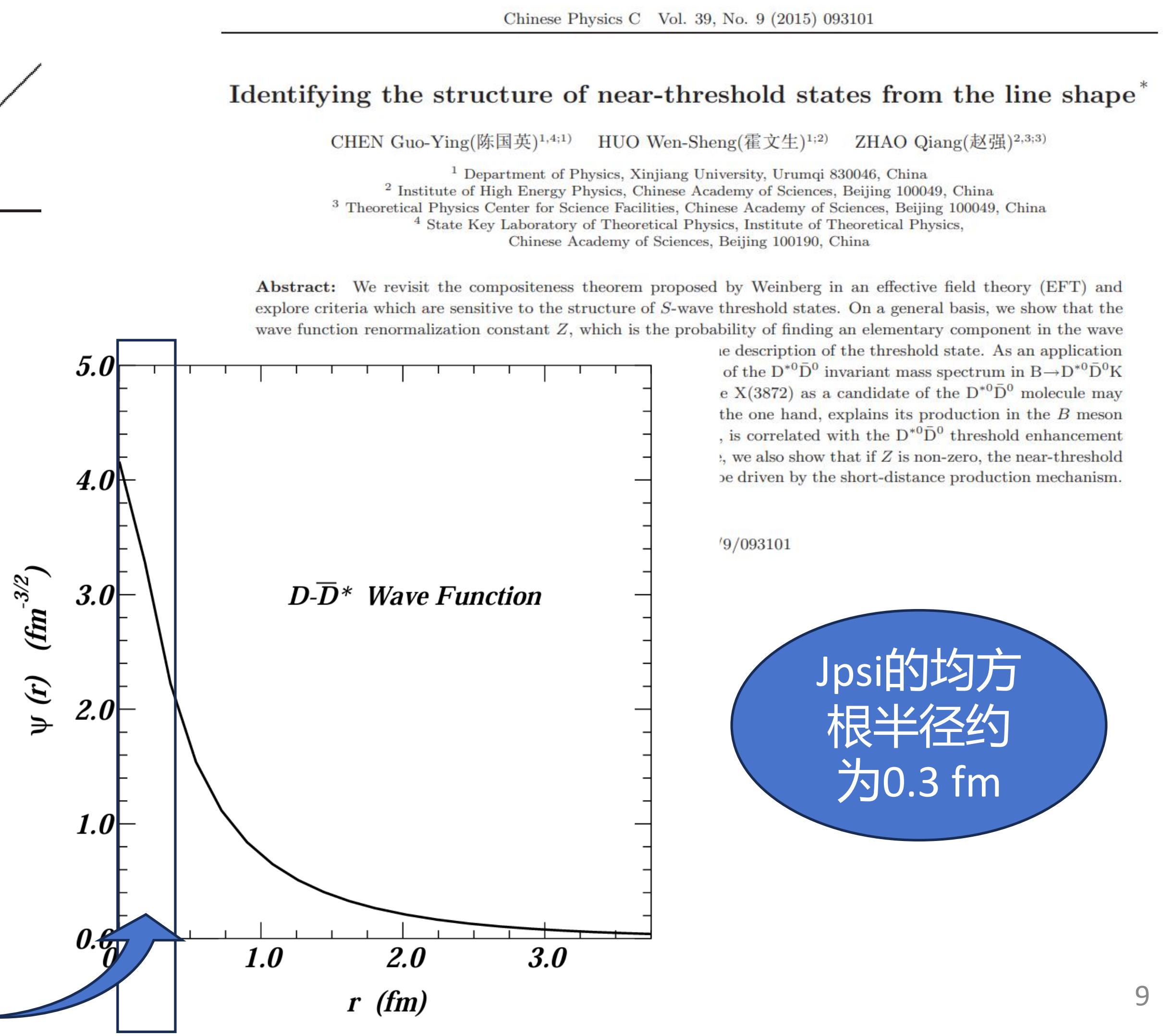


(a) (b)

除了对质量谱和衰变宽度的讨论，在具体产生过程中case by case 的计算和对比更加重要。

右边是一个两体束缚态系统的径向波函数图像，它包含了r从0到无穷大的部分。

在分子态图像中，组成分子态的强子是有大小的，所以当r趋近于0时单纯的分子图像还对么？


Chinese Physics C Vol. 39, No. 9 (2015) 093101

Identifying the structure of near-threshold states from the line shape*

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Abstract: We revisit the compositeness theorem proposed by Weinberg in an effective field theory (EFT) and explore criteria which are sensitive to the structure of S-wave threshold states. On a general basis, we show that the wave function renormalization constant Z, which is the probability of finding an elementary component in the wave ...e description of the threshold state. As an application ... of the $D^{*0}\bar{D}^0$ invariant mass spectrum in $B \to D^{*0}\bar{D}^0K$ ...e X(3872) as a candidate of the $D^{*0}\bar{D}^0$ molecule may ... the one hand, explains its production in the B meson ..., is correlated with the $D^{*0}\bar{D}^0$ threshold enhancement ..., we also show that if Z is non-zero, the near-threshold ...be driven by the short-distance production mechanism.

...9/093101

Jpsi的均方根半径约为0.3 fm

B衰变中X(3872)的产生

X(3872)可以分解为短程和长程的部分：

$$|X(3872)\rangle = \sqrt{Z}|c\bar{c}\rangle + \sqrt{1-Z}|D\bar{D}^*\rangle$$

短程部分以$c\bar{c}$为主　　长程部分以$D\bar{D}^*$为主

★ **大量分子态谱学计算已经说明了仅考虑长程贡献可以很好地解释X(3872)的质量，但是不能很好地理解它的产生和衰变问题**

一个著名的例子

$\Gamma(\gamma\psi(2S))/\Gamma(\gamma J/\psi)$　　Γ_{30}/Γ_{27}

VALUE	CL%	EVTS	DOCUMENT ID	TECN	COMMENT
1.67±0.21±0.13			[1] AAIJ 24AD	LHCB	$B^+ \to \gamma\psi(2S)K^+$ and $\gamma J/\psi K^+$

• • • We do not use the following data for averages, fits, limits, etc. • • •

VALUE	CL%	EVTS	DOCUMENT ID	TECN	COMMENT
<0.59	90		ABLIKIM 20W	BES3	$e^+e^- \to \gamma\chi_{c1}(3872)$
2.46±0.64±0.29		36±9	[2] AAIJ 14AH	LHCB	$B^+ \to \gamma\psi(2S)K^+$
<2.1	90		BHARDWAJ 11	BELL	$B^+ \to \gamma\psi(2S)K^+$
3.4 ±1.4			AUBERT 09B	BABR	$B^+ \to \gamma c\bar{c}K'$

在**考虑粲偶素贡献的情况下**，这个比值范围估计为（*Phys.Rev.D* 79 (2009) 094004, Bai-Qing Li, Kuang-Ta Chao）：

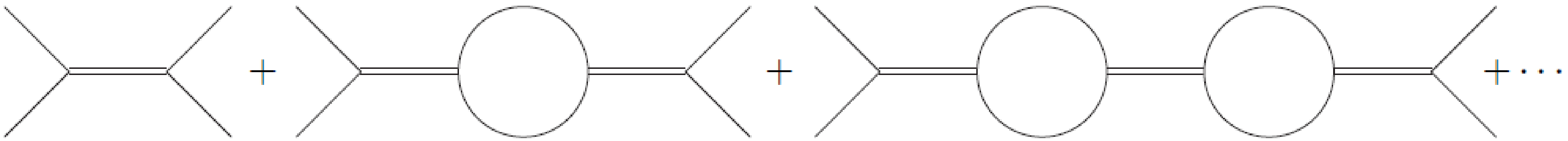
$$R_{\psi'\gamma/\psi\gamma} = 1.3\text{-}6.0.$$

纯粹分子态图像下的结果为（*Phys.Lett.B* 588 (2004) 189, Swanson）：

$$R_{\psi'\gamma/\psi\gamma} = \frac{\mathcal{B}(X\to\psi'\gamma)}{\mathcal{B}(X\to\psi\gamma)} \simeq 4\times10^{-3}$$

还有很多例子，此处不再赘述

B衰变中X(3872)的产生



$$g_0^2 = \frac{2\pi\sqrt{2\mu B}}{\mu^2}\frac{1-Z}{Z} = g^2/Z$$

单圈图振幅：

$$\mathcal{A} = -\frac{g_0^2}{E + B_0 - g_0^2 \frac{\mu}{2\pi}\sqrt{-2\mu E - i\varepsilon}}$$

$$= \frac{\delta'}{E + B + \tilde{\Sigma}'(E)}.$$

$$G(s) = \frac{i}{s - M_0^2 - \Pi(s)},$$

$$\Pi(s) = i g_0^2 \int \frac{dl^4}{(2\pi)^4} \frac{1}{[l^2 - m_1^2 + i\varepsilon][(P-l)^2 - m_2^2 + i\varepsilon]}$$

$$\tilde{\Sigma}'(E) = \delta'[\frac{\mu}{2\pi}\sqrt{-2\mu E} + \frac{\mu\sqrt{2\mu B}}{4\pi B}(E - B)]$$

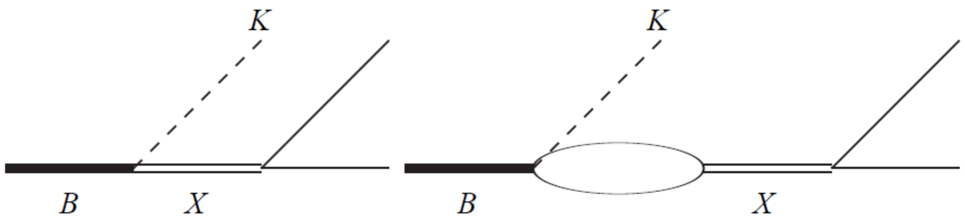
$$G(E) = \frac{1}{2}\frac{i}{E - M_0 - \Pi(E)} = \frac{1}{2}\frac{Z}{E - M - Z\tilde{\Pi}(E)}$$

B衰变中X(3872)的产生

X(3872)的传播子可以写成：

$$G(E)_X = \frac{iZ}{E+B+\tilde{\Sigma}'(E)+i\Gamma/2}$$

$\boldsymbol{B \to K D^{*0} \bar{D}^{0}}$：



(a) (b)

（a）图和（b）图相应的振幅表示为：

$$i\mathcal{M}^{a} = A_{KX} i \frac{g}{\sqrt{2Z}} \frac{iZ}{E+B+\tilde{\Sigma}'(E) - i\frac{\Gamma}{2}} \vec{p}_K \cdot \vec{\varepsilon}$$

$$= -\frac{A_{KX}}{\sqrt{2}} \frac{\sqrt{Z} g}{E+B+\tilde{\Sigma}'(E) - i\frac{\Gamma}{2}} \vec{p}_K \cdot \vec{\varepsilon}.$$

$$i\mathcal{M}_b = B_{KDD} \frac{-g^2}{2Z} \frac{iZ}{E+B+\tilde{\Sigma}'(E) + i\frac{\Gamma}{2}} \cdot i \frac{\mu}{2\pi} \sqrt{-2\mu E - i\varepsilon} \cdot \vec{p}_K \cdot \vec{\varepsilon} \cdot 2$$

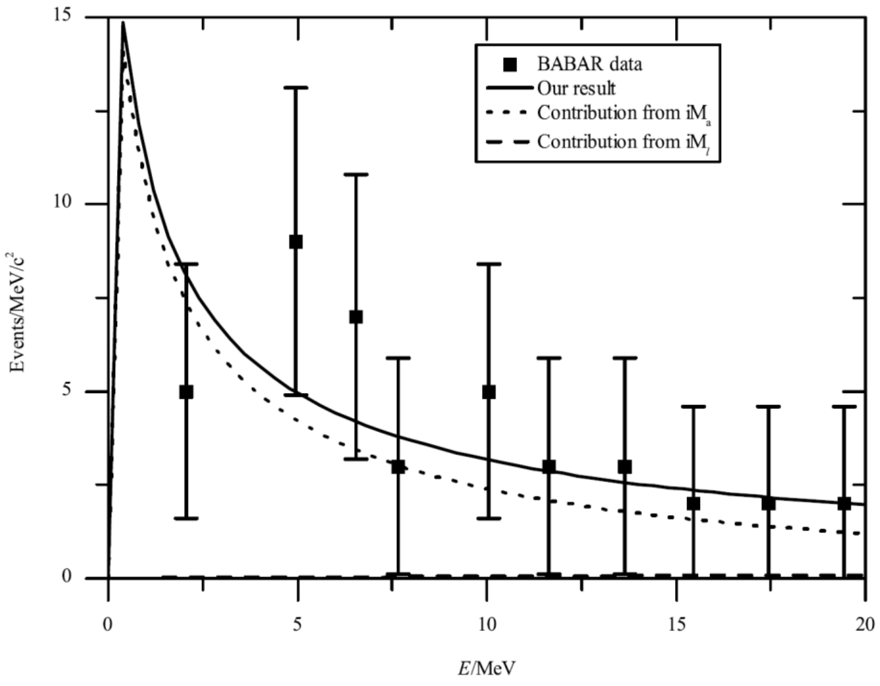
$$= B_{KDD} \frac{g^2 \frac{\mu}{2\pi} \sqrt{-2\mu E - i\varepsilon}}{E+B+\tilde{\Sigma}'(E) + i\frac{\Gamma}{2}} \vec{p}_K \cdot \vec{\varepsilon},$$

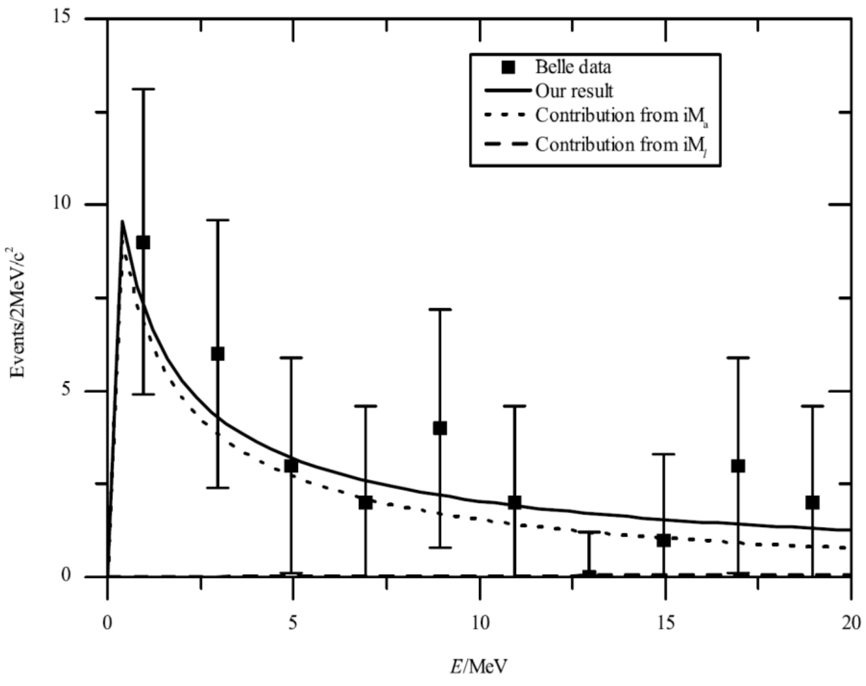
$$i\mathcal{M} = i\mathcal{M}_a + i\mathcal{M}_b + \mathcal{B}_{DDK} \vec{p}_K \cdot \vec{\epsilon}^{\,*}(D^*)$$

背景项

B衰变中X(3872)的产生

上述模型中 Γ, B, Z, $\mathcal{A}_{XK}$和 $\mathcal{B}_{DDK}$ 是模型自由参数，其中Γ和B可以通过实验数据确定下来





拟合结果说明：

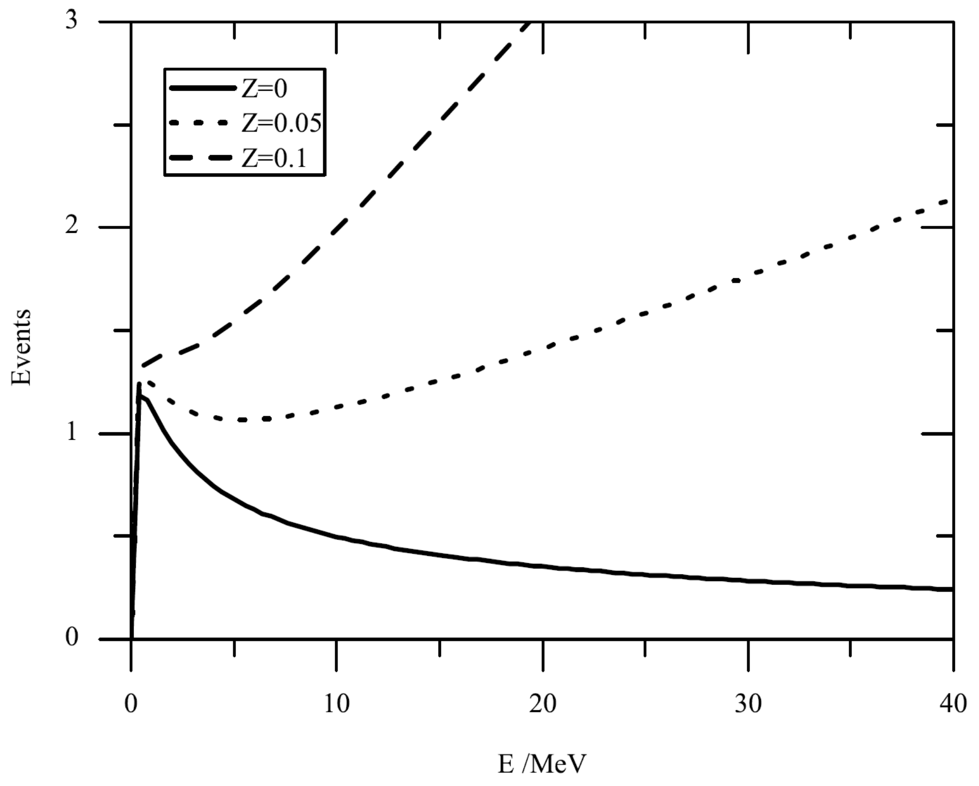
X(3872)中存在紧致的$c\bar{c}$成分，而且在B衰变中起到了重要作用，虽然$c\bar{c}$占比可能不大。

通过上述拟合，作者得到：

$$\frac{\mathcal{A}_{XK}}{\mathcal{B}_{DDK}} = -0.15 \pm 0.65\ \mathrm{GeV}^{3/2}, \qquad Z = 0.19 \pm 0.29.$$

B衰变中X(3872)的产生

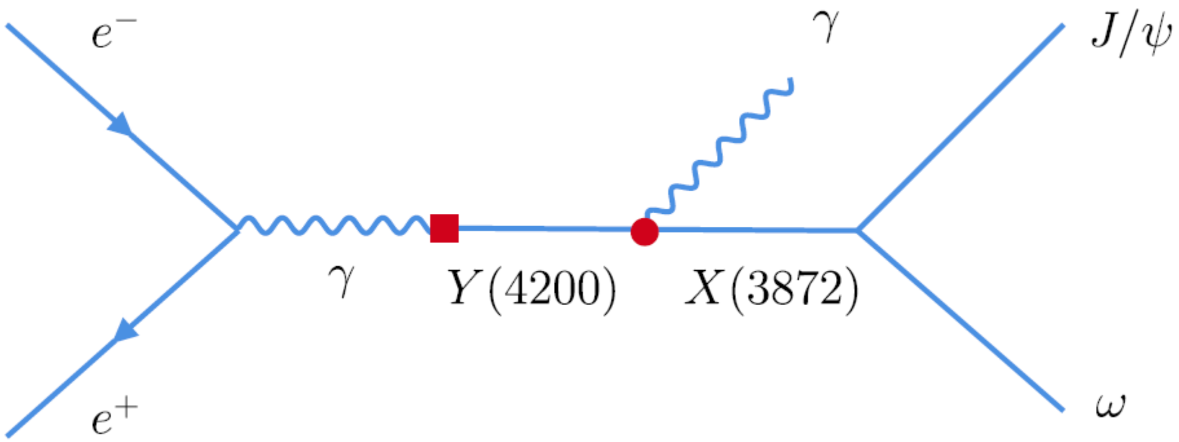
振幅M_l随z改变，线形变化明显：



$$i\mathcal{M}_\ell = i\mathcal{M}_b + \mathcal{B}_{DDK}\vec{p}_K\cdot\vec{\epsilon}^*(D^*)$$
$$= \mathcal{B}_{DDK}\frac{ZE+(2-Z)B}{E+B+\tilde{\Sigma}'(E)}\vec{p}_K\cdot\vec{\epsilon}^*(D^*)$$

仅考虑纯分子态的振幅，线形对z的取值十分敏感，当z不等于0，阈值增强快速消失。
从另一方面讲，在z不可忽略的情形下，阈值处的信号主要来自于短程部分($|c\bar{c}\rangle$)。

辐射跃迁中的X(3872)



Role of the short-distance interaction in $e^+e^- \to \gamma X(3872)$

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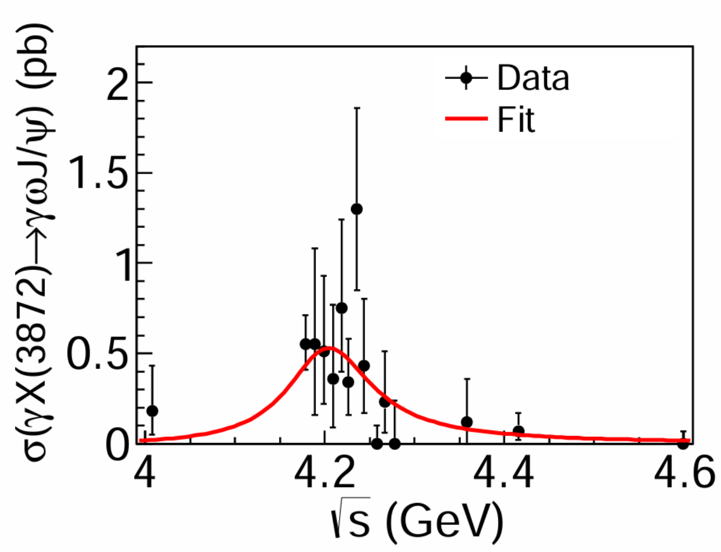
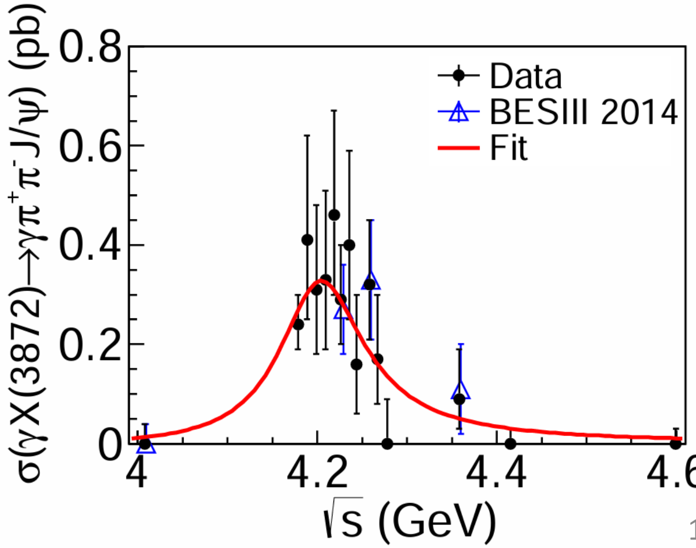
In this study, we analyze the cross-section data from the $e^+e^- \to \gamma J/\psi\omega$ process to explore both short-distance and long-distance interactions for the radiative transition $Y(4200) \to \gamma X(3872)$. We investigate the short-distance effects through the E1 transition among the $c\bar{c}$ components, and the long-distance effects via hadronic loop diagrams. Our numerical analysis reveals that short-distance interactions play a significantly larger role in the radiative transition than that of the long-distance interactions. This finding underscores the importance of the compact $c\bar{c}$ components in both the initial and the final states for accurately understanding the cross-section $\sigma[e^+e^- \to \gamma J/\psi\omega]$. Furthermore, with the help of relative branch ratio $\mathcal{R}$ we estimate the $\Gamma[Y(4200) \to \gamma X(3872)]$ implied in the experimental study. Finally, we also discuss the possible existence of the $\psi(4040)$ signal within the cross-section data.

BESIII实验在正负电子对撞的过程中对$\gamma\omega J/\psi$的测量给我们提供了研究X(3872)的新途径。





辐射跃迁中的X(3872)

$$M[Y(4200)] = 4200.6^{+7.9}_{-13.3} \pm 3.0\ \mathrm{MeV}/c^2$$

$$\Gamma[Y(4200)] = 115^{+38}_{-26} \pm 12\ \mathrm{MeV},$$

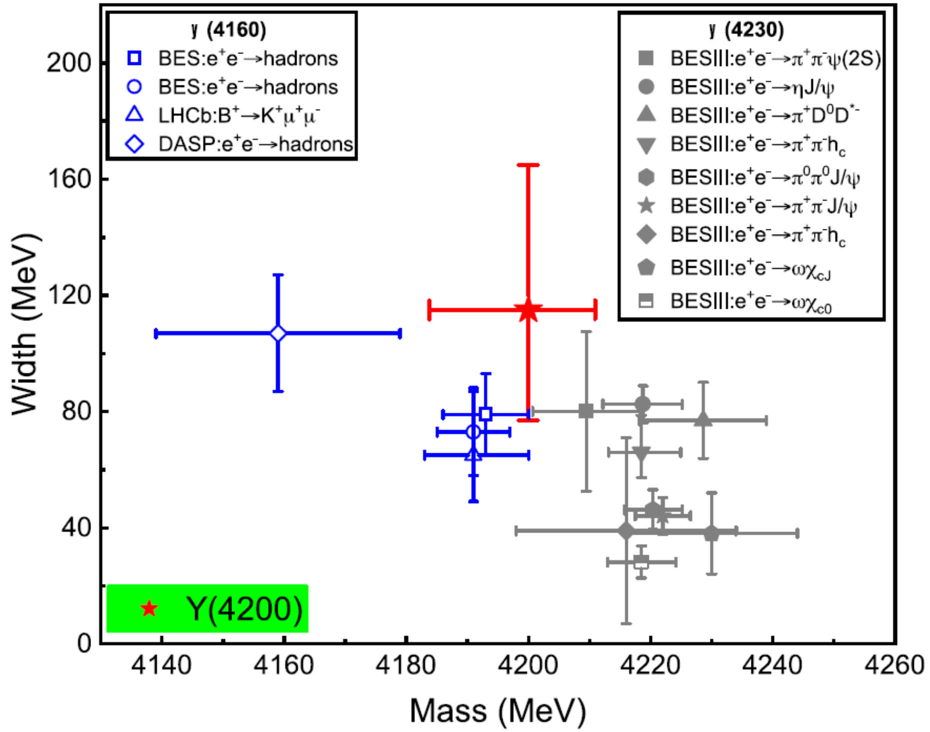
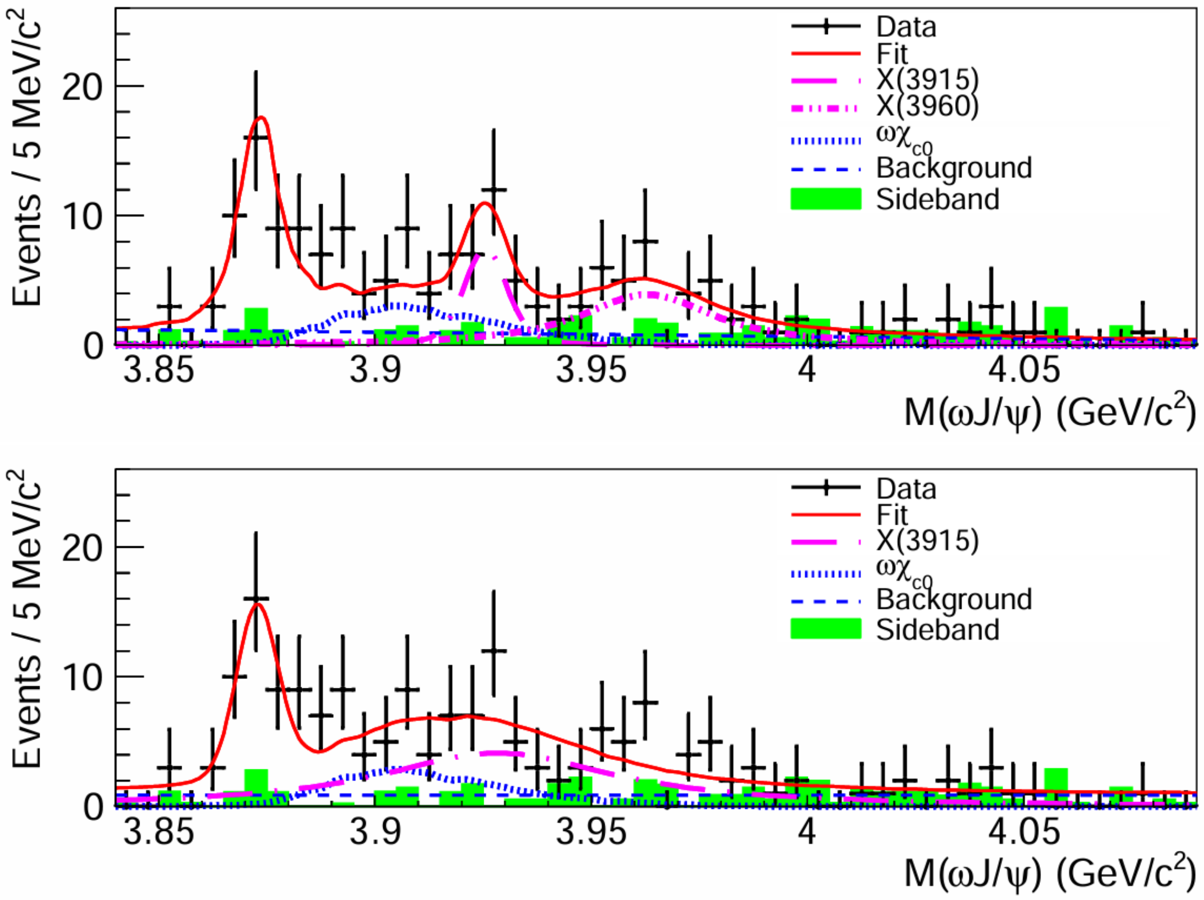


FIG. 1. The comparisons of the measured masses and widths of $\psi(4160)$ and $\psi(4230)$ across different channels with $Y(4200)$.



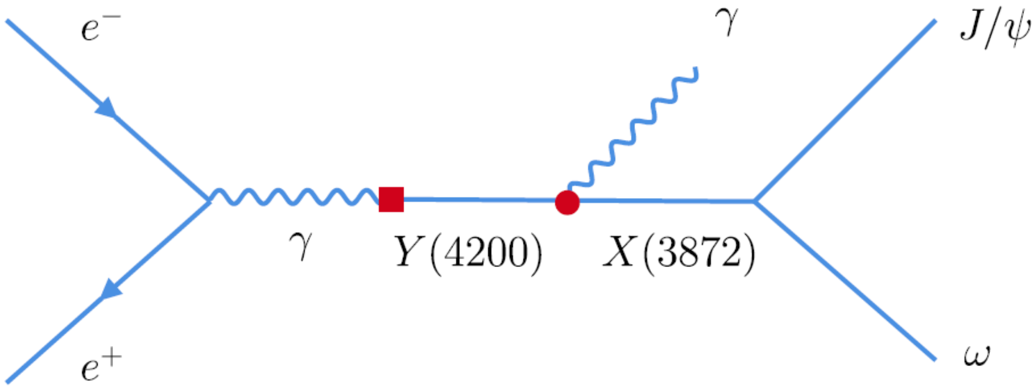
X(3872)信号十分显著

辐射跃迁中的X(3872)

$$\mathcal{L}_{\gamma Y} = -\frac{eM_Y^2}{f_Y} V^\mu A_\mu$$

$$\mathcal{L} = g_{XJ/\psi\omega} \epsilon^{\mu\nu\alpha\beta} \partial_\mu X_\nu \psi_\alpha \omega_\beta$$

$$\mathcal{L} = ie g_{YX\gamma} \varepsilon_{\mu\nu\alpha\beta} F^{\mu\nu} Y^\alpha X^\beta$$



$$|X(3872)\rangle = \sqrt{Z_X}|\chi_{c1}(2P)\rangle + \sqrt{1-Z_X}|D\bar{D}^*\rangle$$

$$\mathcal{P}(E) = \frac{iZ_X}{E + B + \Sigma(E, Z_X) + i\frac{\Gamma_X}{2}} \qquad \Sigma(E, Z_X) = -g_X^2 \left[\frac{\mu}{2\pi}\sqrt{-2\mu E - i\epsilon} + \frac{\mu\sqrt{2\mu B}}{4\pi B}(E - B)\right]$$

过程总振幅如下：

$$i\mathcal{M}^{\rm tot} = \bar{v}(p_2)(-ie\gamma^\mu)u(p_1)\frac{-ig_{\mu\nu}}{q^2}\left(-i\frac{eM_V^2}{f_V}\right)\frac{i\left(-g^{\nu\phi} + \frac{q^\nu q^\phi}{m_Y^2}\right)}{q^2 - m_Y^2 + i\sqrt{s}\Gamma_Y} i\left(\sqrt{Z_X}g_{YX\gamma}^S + \sqrt{1-Z_X}\frac{g_{YX\gamma}^L}{\sqrt{Z_X}}\right)\varepsilon_{\eta\tau\phi\omega}(ip_5^\eta \varepsilon_5^{*\tau} - ip_5^\tau \varepsilon_5^{*\eta})$$

$$\times \frac{i\left(-g^{\omega\lambda} + \frac{k^\omega k^\lambda}{m_X^2}\right)Z_X}{E + B + \Sigma(E, Z_X) + i\frac{\Gamma_X}{2}} ig_{XJ\psi\omega}\varepsilon_{\alpha\lambda\beta\delta}(-ik^\alpha)\varepsilon_3^{*\beta}\varepsilon_4^{*\delta}, \qquad (7$$

辐射跃迁中的X(3872)

A. The E1 transition to the $c\bar{c}$ component in $X(3872)$

$$\Gamma_{\mathrm{E1}}[\psi(mS)/\psi(nD) \to \gamma\chi_{c1}(2P)]$$

$$\Gamma_{\mathrm{E1}}(n^{2S+1}L_J \to n'^{2S'+1}L'_{J'} + \gamma) = \frac{4}{3} C_{fi}\delta_{SS'} e_c^2 \alpha |\langle f|r|i\rangle|^2 E_\gamma^3$$

Processes	Decay width [keV]
$\Gamma[\psi(3S) \to \gamma\chi_{c1}(2P)]$	62
$\Gamma^1[\psi(2D) \to \gamma\chi_{c1}(2P)]$	262
$\Gamma^2[\psi(2D) \to \gamma\chi_{c1}(2P)]$	342
$\Gamma[\psi(4S) \to \gamma\chi_{c1}(2P)]$	0.08
$\Gamma[\psi(3D) \to \gamma\chi_{c1}(2P)]$	0.43

With the above results, we find that the $\Gamma^{1(2)}[\psi(2D) \to \gamma\chi_{c1}(2P)] = 262/342$ keV is significantly larger than the other channels in Table I, which suggests the $\psi(2D) \to \gamma\psi(2P)$ process dominates the short-distance interaction among the different possible E1 transitions. Consequently, the short-distance coupling constant is determined as $g^S_{YX\gamma} = 2.75$ by the $\Gamma^{1(2)}[\psi(2D) \to \gamma\chi_{c1}(2P)]$.[2]

辐射跃迁中的X(3872)

B. Radiative production of $X(3872)$ through the long-distance contributions

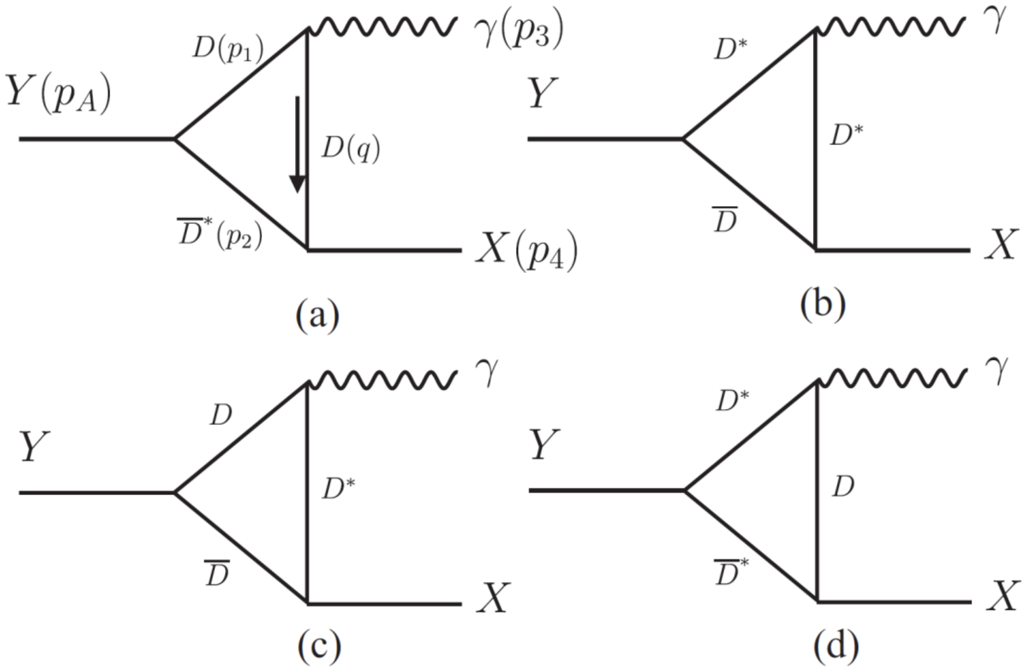
$$\mathcal{L}_{YD^{(*)}\bar{D}^{(*)}} = +ig_{YD\bar{D}} D \overleftrightarrow{\partial}^{\alpha} D^{\dagger} Y_{\alpha}$$
$$+ g_{YD\bar{D}^*}\varepsilon^{\mu\nu\alpha\beta}[D^*_\mu \overleftrightarrow{\partial}_\nu D^\dagger - D\overleftrightarrow{\partial}_\nu D^{*\dagger}_\mu]\partial_\beta Y_\alpha$$
$$+ ig_{YD^*\bar{D}^*} Y_\alpha [D^{*\alpha}\overleftrightarrow{\partial}_\mu D^{*\mu\dagger} + D^*_\mu \overleftrightarrow{\partial}^\mu D^{*\alpha\dagger}$$
$$- D^*_\mu \overleftrightarrow{\partial}^\alpha D^{*\mu\dagger}],$$

$$\mathcal{L}_{D^+D^+\gamma} = ieA_\mu D^- \overleftrightarrow{\partial}^\mu D^+,$$

$$\mathcal{L}_{DD^*\gamma} = \frac{eg_{D^+D^{*-}\gamma}}{4}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}D^{*+}_{\alpha\beta}D^-$$
$$+ \frac{eg_{D^0\bar{D}^{*0}\gamma}}{4}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}D^{*0}_{\alpha\beta}\bar{D}^0,$$

$$\mathcal{L}_{D^{*+}D^{*+}\gamma} = ieA_\mu[g^{\alpha\beta}D^{*-}_\alpha \overleftrightarrow{\partial}^\mu D^{*+}_\beta + g^{\mu\beta}D^{*-}_\alpha \partial^\alpha D^{*+}_\beta$$
$$- g^{\mu\alpha}\partial^\beta D^{*-}_\alpha D^{*+}_\beta],$$

$$\mathcal{L}_{XD\bar{D}^*} = \frac{1}{\sqrt{2}} g_X X_\mu (D^\dagger D^{*\mu} - D^{*\mu\dagger} D) \qquad g_X^2 = \frac{2\pi\sqrt{2\mu B}}{\mu^2}(1 - Z_X).$$



$$\mathcal{M}^a = \int \frac{d^4q}{(2\pi)^4}(-i)g_{YD\bar{D}^*}\varepsilon^{\mu\nu\alpha\beta}p_{A\beta}\varepsilon_{A\alpha}(p_1 - p_2)_\nu$$
$$\times e\varepsilon_3^* \cdot (p_1 + q)\frac{g_X}{\sqrt{2}}\varepsilon_4^{*\rho}\left(-g_{\mu\rho} + \frac{p_{2\mu}p_{2\rho}}{m_2^2}\right)$$
$$\times \frac{\mathcal{F}^2(q^2)}{(p_1^2 - m_1^2)(p_2^2 - m_2^2)(q^2 - m_q^2)},$$

……

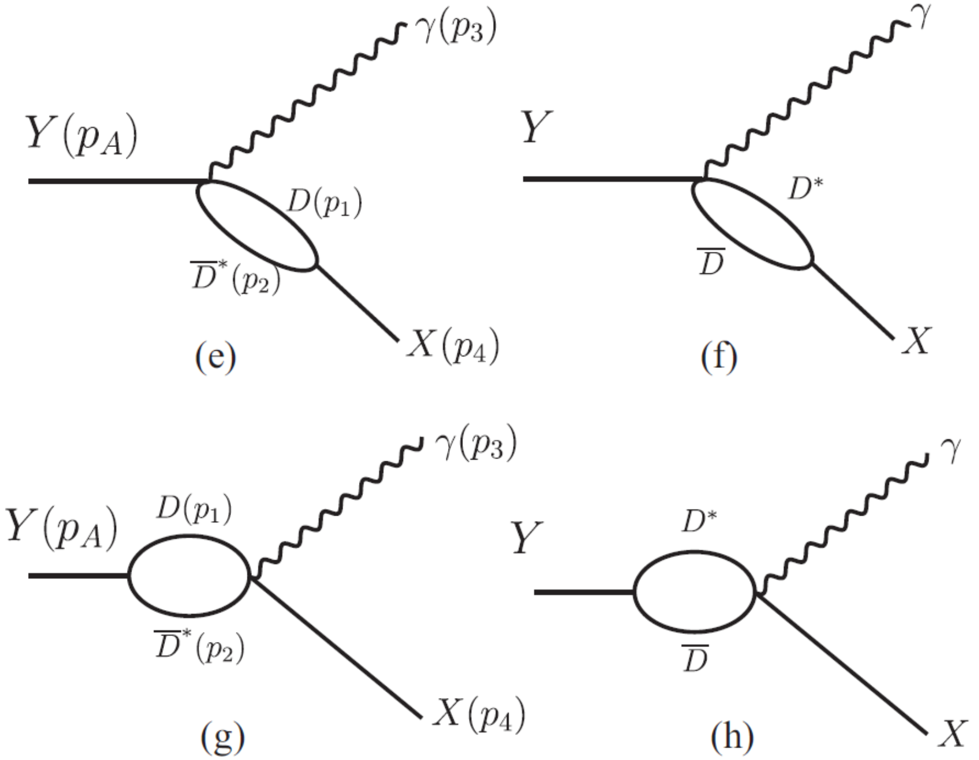
辐射跃迁中的X(3872)

$$\mathcal{L}_{YD\bar{D}^*\gamma} = -ieg_{YD\bar{D}^*\gamma}\varepsilon^{\mu\nu\alpha\beta}A_\nu[D^*_\mu\bar{D} - D\bar{D}^*_\mu]\partial_\beta Y_\alpha,$$

$$\mathcal{L}_{XD\bar{D}^*\gamma} = -\frac{i}{\sqrt{2}}ef_{XD\bar{D}^*\gamma}A^\mu B^\alpha(D^*_\mu\overleftrightarrow{\partial}_\alpha\bar{D} - D\overleftrightarrow{\partial}_\alpha\bar{D}^*_\mu).$$

$$\mathcal{M}^e = \int\frac{d^4p_1}{(2\pi)^4}(-i)eg_{YD\bar{D}^*\gamma}\varepsilon^{\mu\nu\alpha\beta}\varepsilon^*_{3\nu}p_{A\beta}\varepsilon_{A\alpha}$$
$$\times\left(-g_{\mu\rho}+\frac{p_{2\mu}p_{2\rho}}{m_2^2}\right)\left(\frac{g_{YD\bar{D}^*}}{\sqrt{2}}\varepsilon_4^{*\rho}\right)$$
$$\times\frac{\mathcal{F}^2_{\text{cont}}(p_1^2)}{(p_1^2-m_1^2)(p_2^2-m_2^2)},$$

由规范不变性来限制，实际上不需要知道具体的形式



$$p_3^\mu\mathcal{M}_\mu^{\text{tot}} = 0$$

辐射跃迁中的X(3872)

$$\mathcal{M}^a = A_1 \varepsilon_{\mu\nu\alpha\beta} p_3^\mu \varepsilon_3^\nu \varepsilon_4^\alpha \varepsilon_A^\beta + A_2 \varepsilon_{\mu\nu\alpha\beta} p_4^\mu \varepsilon_3^\nu \varepsilon_4^\alpha \varepsilon_A^\beta + A_3 \varepsilon_{\mu\nu\alpha\beta} p_3^\mu p_4^\nu \varepsilon_4^\alpha \varepsilon_A^\beta p_4 \cdot \varepsilon_3,$$

$$\mathcal{M}^b = B_1 \varepsilon_{\mu\nu\alpha\beta} p_3^\mu \varepsilon_3^\nu \varepsilon_4^\alpha \varepsilon_A^\beta + B_2 \varepsilon_{\mu\nu\alpha\beta} p_4^\mu \varepsilon_3^\nu \varepsilon_4^\alpha \varepsilon_A^\beta + B_3 \varepsilon_{\mu\nu\alpha\beta} p_3^\mu p_4^\nu \varepsilon_3^\alpha \varepsilon_A^\beta p_3 \cdot \varepsilon_4 + B_4 \varepsilon_{\mu\nu\alpha\beta} p_3^\mu p_4^\nu \varepsilon_4^\alpha \varepsilon_A^\beta p_4 \cdot \varepsilon_3,$$

$$\mathcal{M}^e = \mathcal{M}^f = E_1 \varepsilon_{\mu\nu\alpha\beta} p_3^\mu \varepsilon_3^\nu \varepsilon_4^\alpha \varepsilon_A^\beta + E_2 \varepsilon_{\mu\nu\alpha\beta} p_4^\mu \varepsilon_3^\nu \varepsilon_4^\alpha \varepsilon_A^\beta,$$

$$\mathcal{M}^g = \mathcal{M}^h = G_1 \varepsilon_{\mu\nu\alpha\beta} p_3^\mu \varepsilon_3^\nu \varepsilon_4^\alpha \varepsilon_A^\beta + G_2 \varepsilon_{\mu\nu\alpha\beta} p_4^\mu \varepsilon_3^\nu \varepsilon_4^\alpha \varepsilon_A^\beta,$$

$A_1 = A_2,\ B_1 = B_2,\ E_1 = E_2,$ and $G_1 = G_2$

$$2E_1 + 2G_1 = -A_2 - B_2 + A_3(p_4 \cdot p_3) + B_4(p_4 \cdot p_3)$$

$$\mathcal{M}^{\mathrm{cont}} = (2E_1 + 2G_1)\varepsilon_{\mu\nu\alpha\beta} p_3^\mu \varepsilon_3^\nu \varepsilon_4^\alpha \varepsilon_A^\beta$$

$$\mathcal{M}^{\mathrm{tri}} = \mathcal{M}^c + \mathcal{M}^d + (A_1 + B_1 + B_3(p_3 \cdot \varepsilon_4)) \times \varepsilon_{\mu\nu\alpha\beta} p_3^\mu \varepsilon_3^\nu \varepsilon_4^\alpha \varepsilon_A^\beta.$$

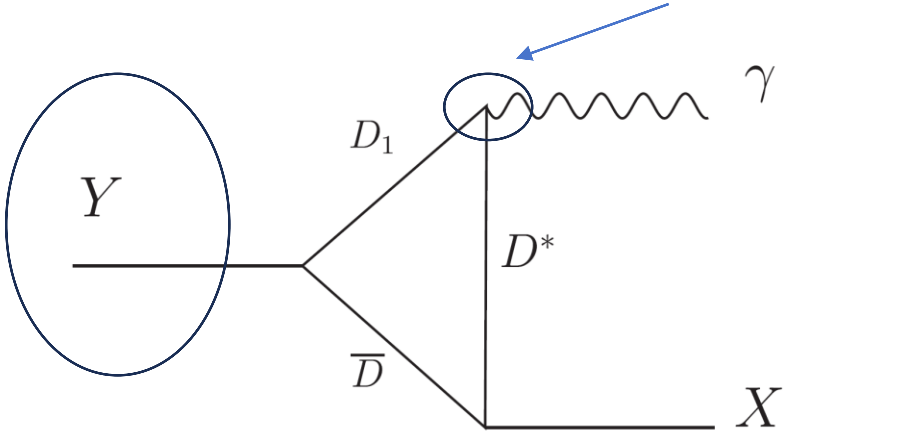
$$\Gamma^{\mathrm{loop}}[Y(4200) \to \gamma X(3872)] = \frac{1}{3}\frac{1}{8\pi}\frac{|\mathbf{p}_3|}{m_A^2}|\mathcal{M}^{\mathrm{loop}}|^2$$

TABLE II. The long-distance contribution of the radiative decay from $\mathcal{M}^a$ to $\mathcal{M}^h$. Here, Y_1 and Y_2 denote $\psi(4160)$ and $\psi(4230)$, respectively.

α [eV]	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\frac{\Gamma^{\mathrm{loop}}[Y_1 \to \gamma X(3872)]}{1-Z_X}$	0.7	1.1	1.6	2.3	3.2	4.5	5.7
$\frac{\Gamma^{\mathrm{loop}}[Y_2 \to \gamma X(3872)]}{1-Z_X}$	0.4	0.7	1.0	1.5	1.9	2.9	3.8

辐射跃迁中的X(3872)

可能的分子态$[D_1\bar{D}]$贡献：



在许多研究中，4.2 GeV处Y态可以作为分子态考虑因为靠近$[D_1\bar{D}]$阈值。

$$\mathcal{M}^{D\bar{D}_1} = \int \frac{d^4q}{(2\pi)^4}(-i)\frac{g_Y}{\sqrt{2}}\varepsilon_{A\rho}\left(-g^{\rho\beta}+\frac{p_1^\rho p_1^\beta}{m_1^2}\right) \times e g_{D_1 D^*\gamma}\varepsilon_{\mu\nu\alpha\beta}(p_3^\mu \varepsilon_3^{*\nu} - p_3^\nu \varepsilon_3^{*\mu})\frac{g_X}{\sqrt{2}}\varepsilon_{4\tau}^* \times \left(-g^{\alpha\tau}+\frac{q^\alpha q^\tau}{m_q^2}\right)\frac{\mathcal{F}^2(q^2)}{(p_1^2-m_1^2)(q^2-m_q^2)(p_2^2-m_2^2)}$$

TABLE III. The radiative decay widths from a molecular $D_1\bar{D}$ initial state and the total radiative decay widths Γ^L in the long-distance interaction. Here, Y_1 and Y_2 represent $\psi(4160)$ and $\psi(4230)$, respectively.

α [eV]	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\frac{\Gamma^{D_1\bar{D}}[Y_1\to\gamma X(3872)]}{1-Z_X}$	4.7	6.6	8.1	9.4	10.6	11.6	12.5
$\frac{\Gamma^{L}[Y_1\to\gamma X(3872)]}{1-Z_X}$	5.0	7.5	10.1	13.4	16.7	20.7	24.1
$\frac{\Gamma^{D_1\bar{D}}[Y_2\to\gamma X(3872)]}{1-Z_X}$	4.9	6.8	8.4	9.7	10.8	11.9	12.8
$\frac{\Gamma^{L}[Y_2\to\gamma X(3872)]}{1-Z_X}$	7.5	10.8	14.1	17.8	21.1	25.8	29.8

辐射跃迁中的X(3872)

拟合散射截面：

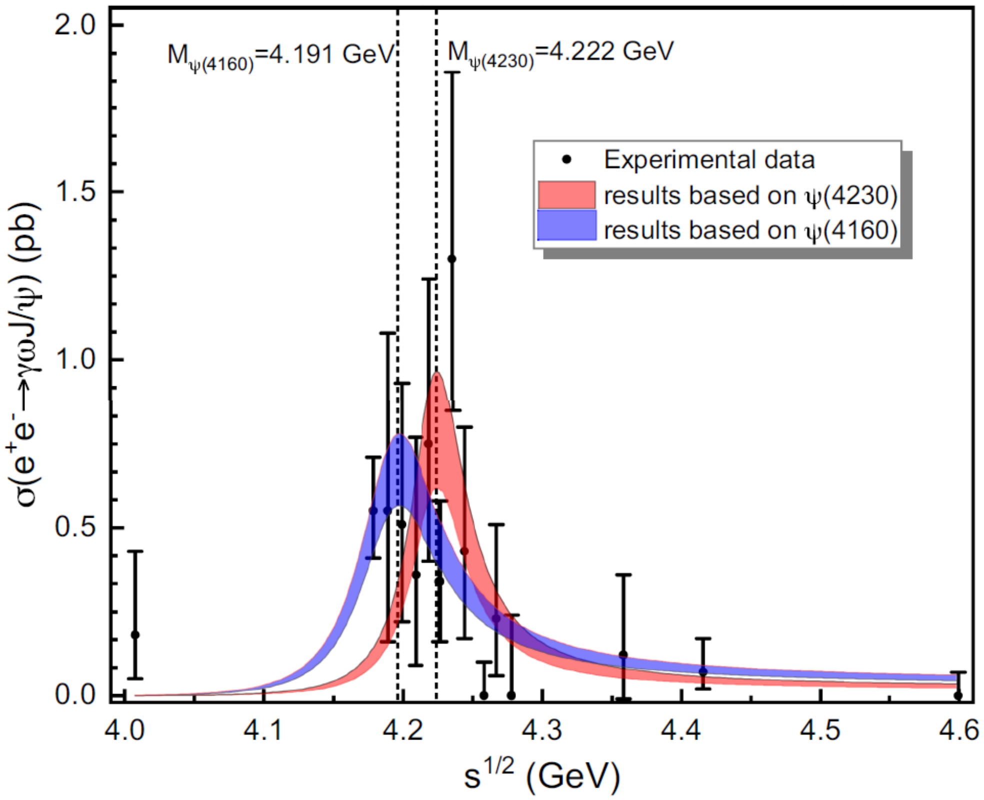


FIG. 6. The cross section of $e^+e^- \to \gamma J/\psi\omega$. The black dots with error bars are the experimental data copied from Ref. [2]. The two peaks with colored bands are the numerical results when $\psi(4160)$ and $\psi(4230)$ are assumed to be the original source of $Y(4200)$.

拟合结果：

对于$\psi(4160)$假设：$Z_X = 0.25\sim0.28$

对于$\psi(4230)$假设：$Z_X = 0.12\sim0.14$

另外，我们进行了进一步的估计：

实验给出了：$\mathcal{R} = \mathcal{B}[X(3872) \to \omega J/\psi]/\mathcal{B}[X(3872) \to \pi^+\pi^- J/\psi]$

$$\Gamma_{Y(4200)}^{e^+e^-\to\gamma J/\psi\pi\pi} = \Gamma^{e^+e^-} \times \mathcal{B}[Y(4200) \to \gamma X(3872)] \times \mathcal{B}[X(3872) \to \pi^+\pi^- J/\psi]$$

$$\Gamma^{\rm est}[Y(4200) \to \gamma X(3872)] = \frac{\mathcal{R} \cdot \Gamma_{Y(4200)}^{e^+e^-\to\gamma J/\psi\pi^+\pi^-} \cdot \Gamma_{Y(4200)}^{\rm tot}}{\Gamma_{Y(4200)}^{e^+e^-} \cdot \mathcal{B}[X(3872) \to \omega J/\psi]}$$

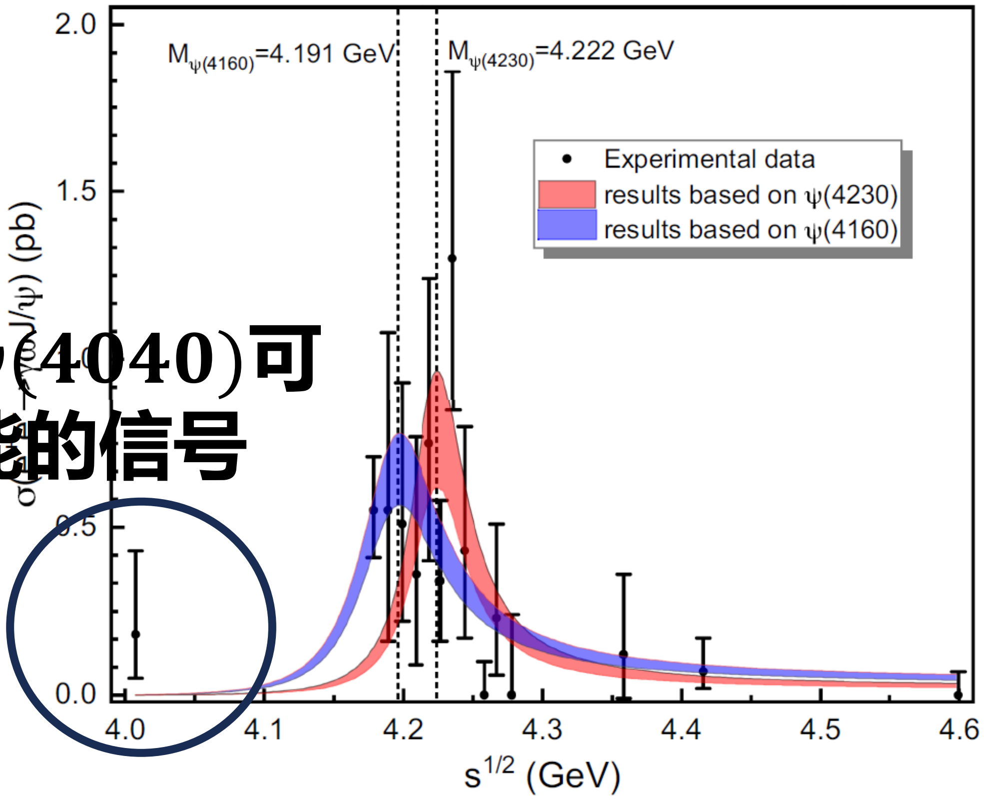
$$\mathcal{R} = 1.6_{-0.3}^{+0.4}, \quad \Gamma_{Y(4200)}^{e^+e^-\to\gamma J/\psi\pi^+\pi^-} = 4.5_{-0.8}^{+1.1} \times 10^{-2}\ \text{eV}$$

$$\mathcal{B}[X(3872) \to \omega J/\psi] = 4.3 \pm 2.1\%$$

$$\Gamma^{\rm est}[Y_1 \to \gamma X(3872)] = 244_{-179}^{+188}\ \text{keV},$$

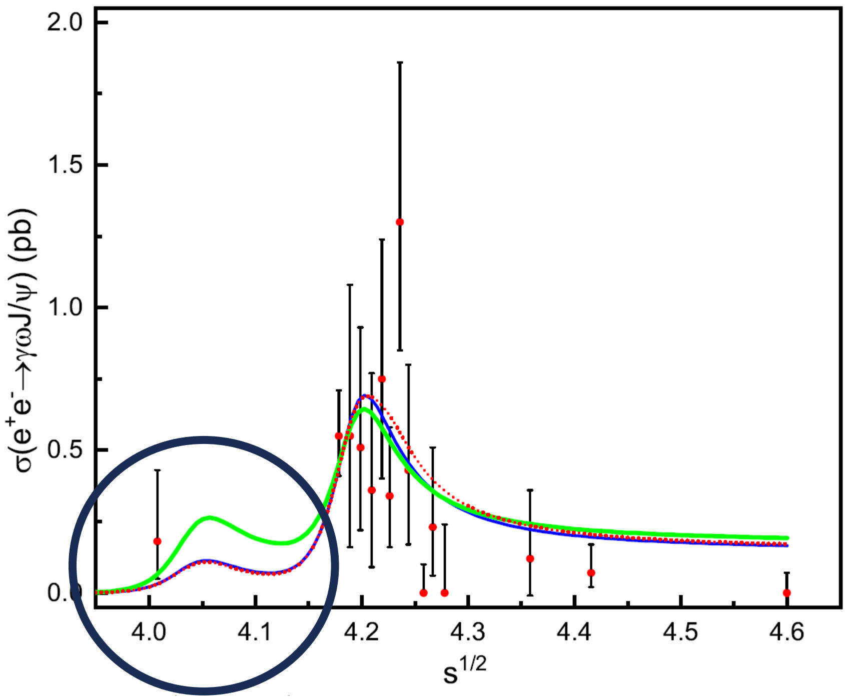
$$\Gamma^{\rm est}[Y_2 \to \gamma X(3872)] = 53_{-56}^{+57}\ \text{keV}.$$

辐射跃迁中的X(3872)



$\psi(4040)$可能的信号

FIG. 6. The cross section of $e^+e^- \to \gamma J/\psi\omega$. The black dots with error bars are the experimental data copied from Ref. [2]. The two peaks with colored bands are the numerical results when $\psi(4160)$ and $\psi(4230)$ are assumed to be the original source of $Y(4200)$.



Processes	Decay width [keV]
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$\Gamma[\psi(3D) \to \gamma\chi_{c1}(2P)]$	0.43

实际上在研究过程中考虑过$\psi(4040)$的贡献，如左图所示，会存在明显峰结构。
结果未公开在论文中。

总结与展望

人们对X(3872)中短程部分贡献的理解是逐步深入的，也渐渐意识到简单的分子态模型并不能正确的理解分子态。通过对X(3872)在B衰变、辐射跃迁过程中产率的研究，我们确定了X(3872)中一定存在$c\bar{c}$短程贡献而且起到了重要作用不能被忽略，虽然计算告诉我们$D\bar{D}^*$仍然占据X(3872)的主要部分。

谢　谢