Electromagnetic Polarizabilities of the triplet Hadrons in Chiral perturbation theory

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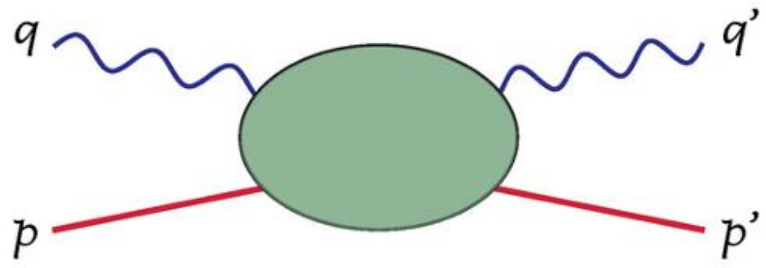
01

Electromagnetic Polarizabilities

Electromagnetic Polarizabilities

Electromagnetic polarizabilities describe the second-order response of hadrons to external electromagnetic fields:

$$H^{(2)} = -\frac{1}{2}4\pi\alpha_E \boldsymbol{E}^2 - \frac{1}{2}4\pi\beta_M \boldsymbol{H}^2 .$$



F. Hagelstein, *Symmetry* **12**, 1407 (2020)

Extracting polarizabilities via low-energy Compton scattering amplitude in forward limit:

$$T^{(2)} = \boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}'\left(\frac{-Z^2e^2}{M} + \boxed{\omega\omega'\alpha_E}\right) + \boxed{(\boldsymbol{\epsilon}\times\boldsymbol{k})\cdot(\boldsymbol{\epsilon}'\times\boldsymbol{k}')\,\omega\omega'\beta_M} + \mathcal{O}\left(\omega^4\right),$$

where

$$\alpha_E = 2\sum_{n\neq 0}\frac{|\langle n|D_z|0\rangle|^2}{E_n - E_0} + \frac{Z^2e^2\langle r_E^2\rangle}{3M}$$

$$\beta_M = 2\sum_{n\neq 0}\frac{|\langle n|M_z|0\rangle|^2}{E_n - E_0} - e^2\sum_i \frac{q_i^2}{6m_i}\langle r_i^2\rangle - \frac{\langle \mathbf{D}^2\rangle}{2M}$$

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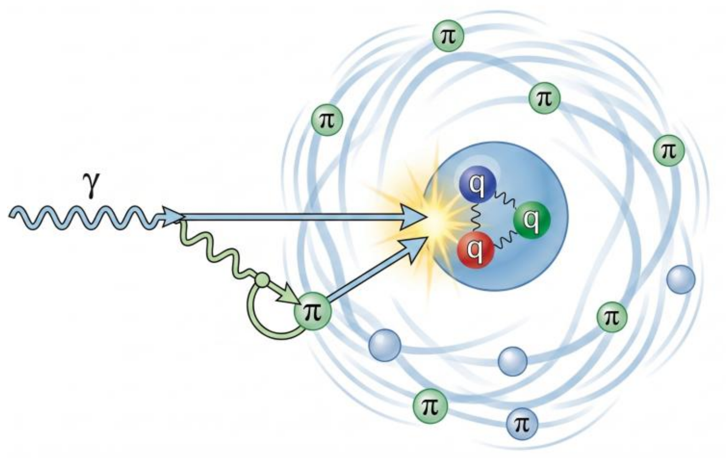
$$\alpha_E = 2\sum_{n\neq 0}\frac{|\langle n|D_z|0\rangle|^2}{E_n - E_0} + \frac{Z^2e^2\langle r_E^2\rangle}{3M}$$

$$\beta_M = 2\sum_{n\neq 0}\frac{|\langle n|M_z|0\rangle|^2}{E_n - E_0} - e^2\sum_i \frac{q_i^2}{6m_i}\langle r_i^2\rangle - \frac{\langle \mathbf{D}^2\rangle}{2M}$$



Experimental data for the nucleon system (10^{-4} fm^3) :

$$\alpha_p \simeq 11.2(4),\ \alpha_n \simeq 11.8(11);\ \ \beta_p \simeq 2.5(4),\ \beta_n \simeq 3.7(12)$$

- A pure constituent-quark picture cannot fully reproduce the observed polarizabilities.
- Pion-cloud fluctuations and low-lying excited states are essential.

ChPT naturally resolves this discrepancy[1]: $\boxed{\alpha_p \simeq \alpha_n\,,\quad \alpha \gg \beta}$

[1] V. Bernard, et al, Nucleons with chiral loops: Electromagnetic polarizabilities, Nucl. Phys. B 373, 346 (1992).

Spin-Dependent Polarizabilities

For a massive particle with spin s, the number of independent polarizabilities is $2(s+1)(2s+1)$. We can organize the structures according to the irreducible spin structures formed by the initial and final polarization vectors.

$$\xi_i'^*\xi_j \quad \Rightarrow \quad \boldsymbol{\xi}'^* \cdot \boldsymbol{\xi} \oplus \boldsymbol{\sigma} \oplus \mathcal{Q}_{ij}$$

$$\boldsymbol{\sigma} = \boldsymbol{\xi}'^* \times \boldsymbol{\xi}$$

$$\mathcal{Q}_{ij} = \xi_i'^*\xi_j + \xi_j'^*\xi_i - \frac{2}{3}\delta_{ij}\,\boldsymbol{\xi}'^* \cdot \boldsymbol{\xi}.$$

In a multipole expansion ordered by the photon energy, the corresponding effective interaction Hamiltonian is written as

$$H_{\text{int}} = H_{\text{scal}} + H_{\text{vec}} + H_{\text{tens}},$$

Spin-Dependent Polarizabilities

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$$H_{\text{int}} = H_{\text{scal}} + H_{\text{vec}} + H_{\text{tens}},$$

$$H_{\text{scal}} = -\frac{1}{2}\,4\pi\left[\alpha_E\,\boldsymbol{E}^2 + \beta_M\,\boldsymbol{H}^2\right],$$

$$\begin{aligned}H_{\text{vec}} = -\frac{1}{2}\,4\pi\Big[&\gamma_{E1E1}\,\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\dot{\boldsymbol{E}}) + \gamma_{M1M1}\,\boldsymbol{\sigma}\cdot(\boldsymbol{H}\times\dot{\boldsymbol{H}})\\ &+ 2\gamma_{E1M2}\,\sigma^i E^j H_{ij} + 2\gamma_{M1E2}\,\sigma^i H^j E_{ij}\Big],\end{aligned}$$

$$\begin{aligned}H_{\text{tens}} = -\frac{1}{2}\,4\pi\Big[&a_{E1E1}\,Q_{ij}E_iE_j + a_{M1M1}\,Q_{ij}H_iH_j\\ &+ a_{E2E2}\,Q_{ij}E_{ik}E_{jk} + a_{M2M2}\,Q_{ij}H_{ik}H_{jk}\\ &+ a_{E1E3}\,Q_{ij}E_kE_{k,ij} + a_{M1M3}\,Q_{ij}H_kH_{k,ij}\Big].\end{aligned}$$

spin 0

spin 1/2

spin 1

Spin-Dependent Polarizabilities

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As in the scalar case, the amplitude can be systematically expanded in terms of a complete basis of 12 independent structures:

$$\mathcal{M} = \sum_{i=1}^{12} A_i(\omega,\theta)\rho_i$$

$$\Pi_i = \frac{1}{4\pi n_i!}\frac{\partial^{n_i}}{\partial\omega^{n_i}} A_i(\omega,\theta)|_{\omega=\theta=0},$$

$$n_i = \begin{cases} 2, & i = 1,2,7,8, \\ 3, & i = 3,4,5,6, \\ 4, & i = 9,10,11,12. \end{cases}$$

$$\begin{aligned}
\rho_1 &= \boldsymbol{\epsilon}'^* \cdot \boldsymbol{\epsilon}\, \boldsymbol{\xi}'^* \cdot \boldsymbol{\xi}, \\
\rho_2 &= \boldsymbol{s}'^* \cdot \boldsymbol{s}\, \boldsymbol{\xi}'^* \cdot \boldsymbol{\xi}, \\
\rho_3 &= (\boldsymbol{\xi}'^* \times \boldsymbol{\xi}) \cdot (\boldsymbol{\epsilon}'^* \times \boldsymbol{\epsilon}), \\
\rho_4 &= (\boldsymbol{\xi}'^* \times \boldsymbol{\xi}) \cdot (\boldsymbol{s}'^* \times \boldsymbol{s}), \\
\rho_5 &= (\boldsymbol{\xi}'^* \times \boldsymbol{\xi}) \cdot \hat{\boldsymbol{k}}\, \boldsymbol{s}'^* \cdot \boldsymbol{\epsilon} - (\boldsymbol{\xi}'^* \times \boldsymbol{\xi}) \cdot \hat{\boldsymbol{k}}'\, \boldsymbol{\epsilon}'^* \cdot \boldsymbol{s}, \\
\rho_6 &= (\boldsymbol{\xi}'^* \times \boldsymbol{\xi}) \cdot \hat{\boldsymbol{k}}'\, \boldsymbol{s}'^* \cdot \boldsymbol{\epsilon} - (\boldsymbol{\xi}'^* \times \boldsymbol{\xi}) \cdot \hat{\boldsymbol{k}}\, \boldsymbol{\epsilon}'^* \cdot \boldsymbol{s},
\end{aligned}$$

Spin-Dependent Polarizabilities

$$\rho_7 = \boldsymbol{\xi}'^* \cdot \boldsymbol{\epsilon}'^* \, \boldsymbol{\xi} \cdot \boldsymbol{\epsilon} + \boldsymbol{\xi}'^* \cdot \boldsymbol{\epsilon} \, \boldsymbol{\xi} \cdot \boldsymbol{\epsilon}'^* - \frac{2}{3} \boldsymbol{\xi}'^* \cdot \boldsymbol{\xi} \, \boldsymbol{\epsilon}'^* \cdot \boldsymbol{\epsilon},$$

$$\rho_8 = \boldsymbol{\xi}'^* \cdot \boldsymbol{s}'^* \, \boldsymbol{\xi} \cdot \boldsymbol{s} + \boldsymbol{\xi}'^* \cdot \boldsymbol{s} \, \boldsymbol{\xi} \cdot \boldsymbol{s}'^* - \frac{2}{3} \boldsymbol{\xi}'^* \cdot \boldsymbol{\xi} \, \boldsymbol{s}'^* \cdot \boldsymbol{s},$$

$$\begin{aligned}\rho_9 = {} & \boldsymbol{\epsilon}'^* \cdot \hat{\boldsymbol{k}} \left(\boldsymbol{\xi}'^* \cdot \hat{\boldsymbol{k}}' \, \boldsymbol{\xi} \cdot \boldsymbol{\epsilon} + \boldsymbol{\xi}'^* \cdot \boldsymbol{\epsilon} \, \boldsymbol{\xi} \cdot \hat{\boldsymbol{k}}' \right) \\ & + \boldsymbol{\epsilon} \cdot \hat{\boldsymbol{k}}' \left(\boldsymbol{\xi}'^* \cdot \hat{\boldsymbol{k}} \, \boldsymbol{\xi} \cdot \boldsymbol{\epsilon}'^* + \boldsymbol{\xi}'^* \cdot \boldsymbol{\epsilon}'^* \, \boldsymbol{\xi} \cdot \hat{\boldsymbol{k}} \right) - \frac{4}{3} \boldsymbol{\xi}'^* \cdot \boldsymbol{\xi} \, \boldsymbol{\epsilon}'^* \cdot \hat{\boldsymbol{k}} \, \boldsymbol{\epsilon} \cdot \hat{\boldsymbol{k}}',\end{aligned}$$

$$\begin{aligned}\rho_{10} = {} & \boldsymbol{s}'^* \cdot \hat{\boldsymbol{k}} \left(\boldsymbol{\xi}'^* \cdot \hat{\boldsymbol{k}}' \, \boldsymbol{\xi} \cdot \boldsymbol{s} + \boldsymbol{\xi}'^* \cdot \boldsymbol{s} \, \boldsymbol{\xi} \cdot \hat{\boldsymbol{k}}' \right) \\ & + \boldsymbol{s} \cdot \hat{\boldsymbol{k}}' \left(\boldsymbol{\xi}'^* \cdot \hat{\boldsymbol{k}} \, \boldsymbol{\xi} \cdot \boldsymbol{s}'^* + \boldsymbol{\xi}'^* \cdot \boldsymbol{s}'^* \, \boldsymbol{\xi} \cdot \hat{\boldsymbol{k}} \right) - \frac{4}{3} \boldsymbol{\xi}'^* \cdot \boldsymbol{\xi} \, \boldsymbol{s}'^* \cdot \hat{\boldsymbol{k}} \, \boldsymbol{s} \cdot \hat{\boldsymbol{k}}',\end{aligned}$$

$$\rho_{11} = \boldsymbol{\epsilon}'^* \cdot \boldsymbol{\epsilon} \left[\boldsymbol{\xi}'^* \cdot \hat{\boldsymbol{k}} \, \boldsymbol{\xi} \cdot \hat{\boldsymbol{k}} + \boldsymbol{\xi}'^* \cdot \hat{\boldsymbol{k}}' \, \boldsymbol{\xi} \cdot \hat{\boldsymbol{k}}' - \frac{2}{3} \boldsymbol{\xi}'^* \cdot \boldsymbol{\xi} \right],$$

$$\rho_{12} = \boldsymbol{s}'^* \cdot \boldsymbol{s} \left[\boldsymbol{\xi}'^* \cdot \hat{\boldsymbol{k}} \, \boldsymbol{\xi} \cdot \hat{\boldsymbol{k}} + \boldsymbol{\xi}'^* \cdot \hat{\boldsymbol{k}}' \, \boldsymbol{\xi} \cdot \hat{\boldsymbol{k}}' - \frac{2}{3} \boldsymbol{\xi}'^* \cdot \boldsymbol{\xi} \right].$$

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02

Heavy Hadron Chiral Perturbation Theory

Chiral Perturbation Theory

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Singly Heavy mesons $\bar{Q}q$:

$J^P = 0^-$ $\quad$ $J^P = 1^-$

$$P = (\bar{D}^0, D^-, D_s^-) \qquad P_\mu^* = (\bar{D}_\mu^{*0}, D_\mu^{*-}, D_{s\mu}^{*-}).$$

$$P = (B^+, B^0, B_s^0) \qquad P_\mu^* = (B_\mu^{*+}, B_\mu^{*0}, B_{s\mu}^{*0})$$

Doubly Heavy baryons QQq:

$J^P = 1/2^+$ $\quad$ $J^P = 3/2^+$

$$\mathcal{B} = (\Xi_{cc}^{++}, \Xi_{cc}^{+}, \Omega_{cc}^{+}), \qquad \mathcal{B}_\mu^* = (\Xi_{cc\mu}^{*++}, \Xi_{cc\mu}^{*+}, \Omega_{cc\mu}^{*+}),$$

Heavy Diquark-Antiquark symmetry (HDAS)

Triplet representation

$$P \to P' = hP$$

$$D_\mu P = \partial_\mu P + \Gamma_\mu P$$

Chiral Perturbation Theory

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Doubly Heavy baryons *bcq*:

$J^P = 1/2^+$ $\qquad$ $J^P = 3/2^+$

bc pair: $\mathcal{T} = (\Xi^{++}_{[bc]u}, \Xi^{+}_{[bc]d}, \Omega^{+}_{[bc]s}),$ $\qquad$ $\mathcal{B}^*_\mu = (\Xi^{*++}_{\{bc\}u\mu}, \Xi^{*+}_{\{bc\}d\mu}, \Omega^{*+}_{\{bc\}s\mu})$

Chiral eigenstate $\mathcal{B} = (\Xi^{++}_{\{bc\}u}, \Xi^{+}_{\{bc\}d}, \Omega^{+}_{\{bc\}s}),$

cq pair: $\mathcal{T} = (\Xi^{++}_{b[cu]}, \Xi^{+}_{b[cd]}, \Omega^{+}_{b[cs]}),$ $\qquad$ $\mathcal{B}^*_\mu = (\Xi^{*++}_{b\{cu\}\mu}, \Xi^{*+}_{b\{cd\}\mu}, \Omega^{*+}_{b\{cs\}\mu})$

Mass eigenstate $\mathcal{B} = (\Xi^{++}_{b\{cu\}}, \Xi^{+}_{b\{cd\}}, \Omega^{+}_{b\{cs\}}),$

TABLE VIII. Masses of the $J^P = \frac{1}{2}^+$ baryons without identical quarks in MeV. The notations are the same as those in Fig. VIII.

$J^P = \frac{1}{2}^+$	channel	$AL1$ DMC	$AL1$ VAR	$AL1$ FAD [15]	FT I DMC	FT II DMC	EXP[65]
$\Xi_c(nsc)$	$\chi^A_s(12;3)$	2466	2465	/	2537	2470	2469
	$\chi^S_s(12;3)$	2470	2569	/	2643	2573	2579
	mixing	2465	2464	2467	2535	2469	2469
$\Xi_b(nsb)$	$\chi^A_s(12;3)$	5803	5802	/	5870	5806	5794
	$\chi^S_s(12;3)$	5942	5941	/	6014	5944	5935
	mixing	5802	5802	5806	5870	5806	5794
$\Xi_{cb}(ncb)$	$\chi^A_s(12;3)$	6915	6914	/	6968	6911	/
	$\chi^S_s(12;3)$	6960	6959	/	7014	6955	/
	mixing	6914	6914	6915	6967	6911	/
$\Omega^0_{cb}(scb)$	$\chi^A_s(12;3)$	7003	7002	/	7052	7004	/
	$\chi^S_s(12;3)$	7041	7040	/	7091	7040	/
	mixing	7002	7002	7003	7052	7003	/

[2] Y. Ma, et al, Ground state baryons in the flux-tube three-body confinement model using diffusion Monte Carlo, Phys. Rev. D 107, 054035 (2023).

Chiral Perturbation Theory

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The Degrees of Freedom

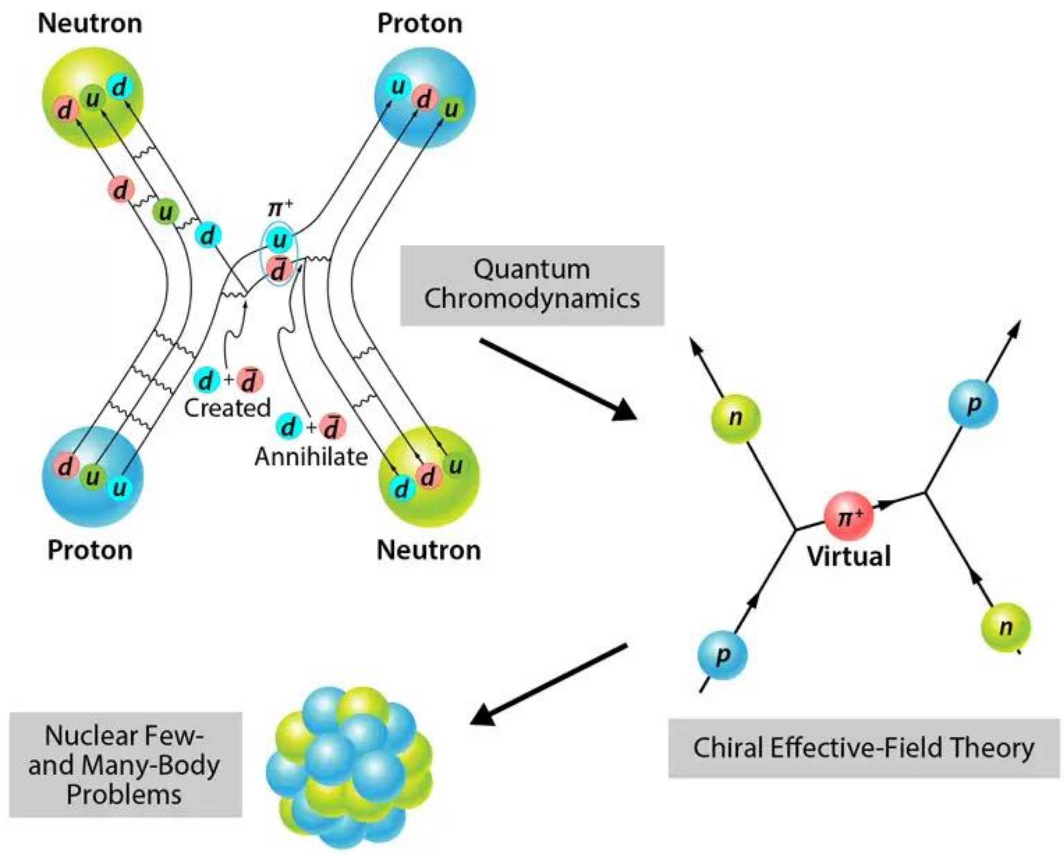

Fig1. The degrees of freedom in ChPT

Perturbation Expansion

$$g \longrightarrow \frac{p}{\Lambda_\chi} \quad \begin{array}{l} \sim m_\pi \quad 0.1\ \text{GeV} \\ \sim 4\pi f_\pi \quad 1\ \text{GeV} \end{array}$$

(QCD coupling constant)

Consistent with the Symmetries of the Underlying Theory (QCD)

- Lorentz Symmetry
- Chiral Symmetry
- Baryon Number Conservation
- C, P-parity invariance
- ……

Chiral Perturbation Theory

Heavy Hadron Chiral Perturbation Theory

$$\mathcal{L} = \bar{\Psi}(i\not{D} - \boxed{m})\Psi + g_A \bar{\Psi}\gamma_\mu\gamma_5 u^\mu \Psi - \frac{e\kappa}{4m}\bar{\Psi}\sigma^{\mu\nu}F_{\mu\nu}\Psi.$$

Heavy-hadron masses $m \gg m_\pi$ break naive chiral power counting.

Non-relativistic expansion

(FW transformation)

$$\begin{cases} B_n(x) = e^{iM_B v\cdot x}\dfrac{1+\not{v}}{2}\psi_n, \\ \boxed{H_n(x) = e^{iM_B v\cdot x}\dfrac{1-\not{v}}{2}\psi_n,} \end{cases}$$

integrate out the heavy component

$$\mathcal{L}_{\mathrm{FW}} = \bar{\Psi}\left(i\not{D}_{||} + 2g_A S\cdot u + \frac{ie(1+\kappa)}{2m}[S^\mu, S^\nu]F_{\mu\nu} - \frac{D_\perp^2}{2m} - \frac{ig_A}{m}\{S\cdot D,\, v\cdot u\} - \frac{g_A^2}{2m}(v\cdot u)^2\right)\Psi.$$

Chiral Perturbation Theory

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Chiral Lagrangian for Singly Heavy mesons

$$\mathcal{L}_{H\phi}^{(1)} = 2iP^\dagger v\cdot DP - 2P^{*\dagger}(iv\cdot D-\Delta)P^* + 2g(iP_\mu^{*\dagger}u^\mu P + \text{H.c.}) - 2\tilde{g}i\epsilon^{\mu\nu\rho\sigma}u_\mu P_\nu^{*\dagger}P_\rho^* v_\sigma,$$

$$\mathcal{L}_{H\gamma}^{(2)} = -P^\dagger\frac{D^2}{m_P}P + P^{*\dagger}\frac{D^2}{m_{P^*}}P^* + 4i\tilde{a}P^{*\dagger\mu}P^{*\nu}\tilde{F}_{\mu\nu}^+ + 4iaP^{*\dagger\mu}P^{*\nu}\text{Tr}(F_{\mu\nu}^+) + 2i\tilde{a}\epsilon^{\mu\nu\rho\sigma}P_\rho^{*\dagger}Pv_\sigma\tilde{F}_{\mu\nu}^+ + \text{H.c.} + 2ia\epsilon^{\mu\nu\rho\sigma}P_\rho^{*\dagger}Pv_\sigma\text{Tr}(F_{\mu\nu}^+) + \text{H.c.},$$

Chiral Lagrangian for Doubly Heavy baryons

$$\mathcal{L}_{\mathcal{B}\phi}^{(1)} = \bar{\mathcal{B}}iv\cdot D\mathcal{B} - \bar{\mathcal{B}}^*(iv\cdot D-\delta)\mathcal{B}^* + 2g_1\bar{\mathcal{B}}S\cdot u\mathcal{B} + g_2(\bar{\mathcal{B}}_\mu^* u^\mu\mathcal{B} + \text{H.c.}) + 2g_3\bar{\mathcal{B}}^* S\cdot u\mathcal{B}^*,$$

$$\mathcal{L}_{\mathcal{B}\phi}^{(2)} = \bar{\mathcal{B}}\frac{(v\cdot D)^2-D^2}{2M}\mathcal{B} - \bar{\mathcal{B}}^{*\mu}\frac{(v\cdot D)^2-D^2}{2M^*}\mathcal{B}_\mu^* - \frac{i\tilde{f}_1}{4M_N}\bar{\mathcal{B}}[S^\mu,S^\nu]\tilde{F}_{\mu\nu}^+\mathcal{B} - \frac{if_1}{4M_N}\bar{\mathcal{B}}[S^\mu,S^\nu]\mathcal{B}\text{Tr}(F_{\mu\nu}^+) + \frac{i\tilde{f}_2}{4M_N}\bar{\mathcal{B}}\tilde{F}_{\mu\nu}^+S^\nu\mathcal{B}^{*\mu} + \text{H.c.} + \frac{if_2}{4M_N}\bar{\mathcal{B}}S^\nu\mathcal{B}^{*\mu}\text{Tr}(F_{\mu\nu}^+) + \text{H.c.} + \frac{i\tilde{f}_3}{4M_N}\bar{\mathcal{B}}^{*\mu}\tilde{F}_{\mu\nu}^+\mathcal{B}^{*\nu} + \frac{if_3}{4M_N}\bar{\mathcal{B}}^{*\mu}\mathcal{B}^{*\nu}\text{Tr}(F_{\mu\nu}^+),$$

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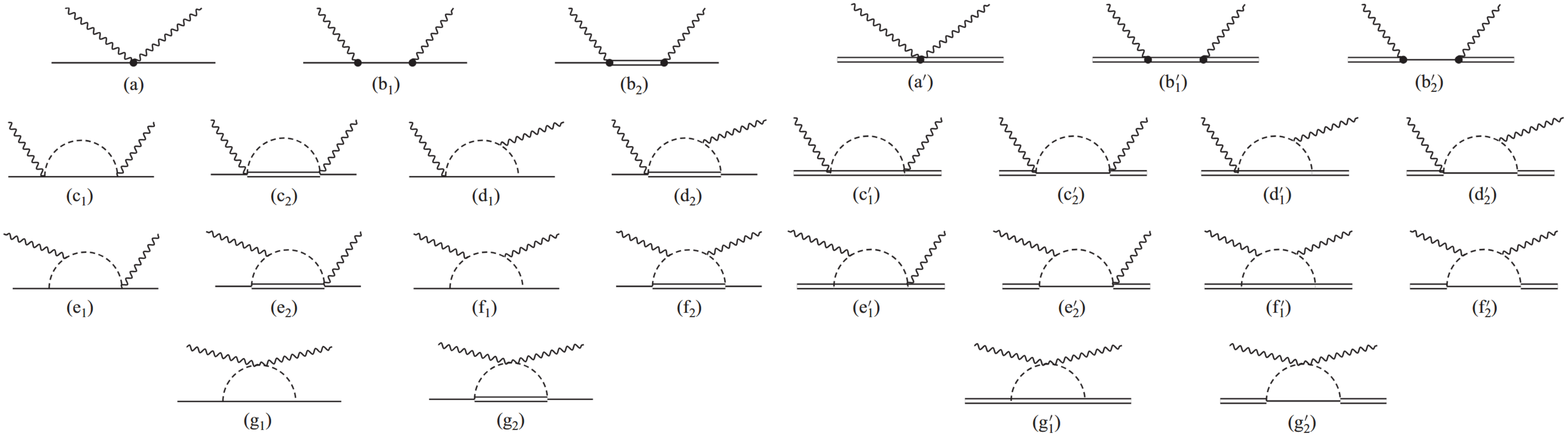

Fig2. The Born and loop diagrams contributing to the electromagnetic polarizabilities up to $O(p^3)$. Wavy, dashed, and solid lines denote photons, Goldstone bosons, and heavy hadrons, respectively. The single and double lines represent the pseudoscalar mesons (spin $\frac{1}{2}$ baryons) and vector mesons (spin $\frac{3}{2}$ baryons).

03

Results and Conclusions

Results

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TABLE III. The numerical results of singly heavy meson electromagnetic polarizabilities (in unit of 10^{-4} fm^3). The values in parentheses represent the uncertainties of the results.

	$\alpha_E^{(c_1-g_1)}$	$\alpha_E^{(c_2-g_2)}$	α_E^{Tot}	$\beta_M^{(b_2)}$	$\beta_M^{(c_1-g_1)}$	$\beta_M^{(c_2-g_2)}$	β_M^{Tot}
$\bar{D}^0$	0	1.50(36)	1.50(36)	11.24(191)	0	0.21(5)	11.45(191)
D^-	0	1.22(29)	1.22(29)	0.69(25)	0	0.17(4)	0.87(26)
D_s^-	0	0.47(11)	0.47(11)	0.09(6)	0	0.05(1)	0.14(7)
B^+	0	1.78(41)	1.78(41)	22.64(470)	0	0.21(5)	22.85(470)
B^0	0	1.49(35)	1.49(35)	7.01(131)	0	0.17(4)	7.19(131)
B_s^0	0	0.44(10)	0.44(10)	2.71(50)	0	0.04(1)	2.75(50)
	$\alpha_E^{(c_1'-g_1')}$	$\alpha_E^{(c_2'-g_2')}$	α_E^{Tot}	$\beta_M^{(b_2')}$	$\beta_M^{(c_1'-g_1')}$	$\beta_M^{(c_2'-g_2')}$	β_M^{Tot}
$\bar{D}^{*0}$	2.15(52)	291(105)	294(106)	−3.75(64)	0.22(5)	0.98(25)	−2.55(70)
D^{*-}	1.81(43)	−0.39(9) −64.5(155)i	1.42(34) −64.5(155)i	−0.23(8)	0.18(4)	−0.06(1) +0.62(15)i	−0.11(4) +0.62(15)i
D_s^{*-}	0.34(8)	0.39(9)	0.74(18)	−0.03(2)	0.03(1)	0.03(1)	0.03(3)
B^{*+}	1.65(38)	1.24(29)	2.89(67)	−7.55(157)	0.16(4)	0.10(2)	−7.28(157)
B^{*0}	1.38(32)	1.12(26)	2.51(58)	−2.34(44)	0.14(3)	0.09(2)	−2.11(44)
B_s^{*0}	0.26(6)	0.19(5)	0.46(11)	−0.90(17)	0.03(1)	0.02	−0.86(17)

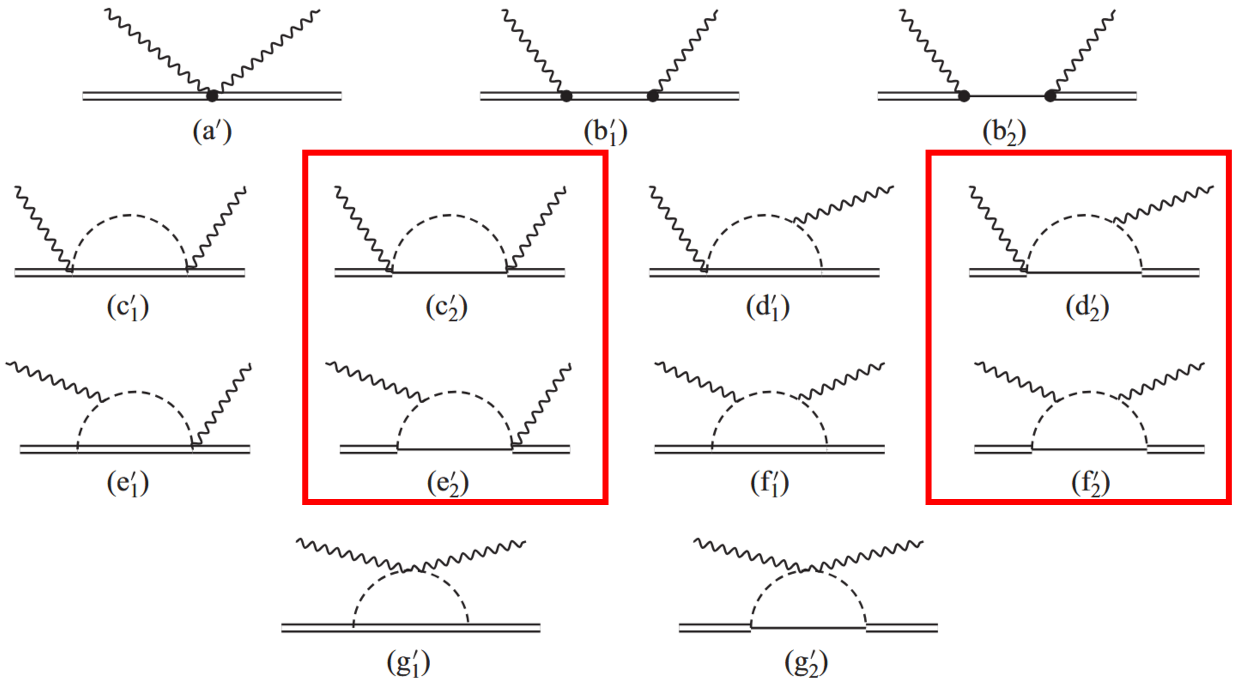
Results

	$\alpha_E^{(c_1'-g_1')}$	$\alpha_E^{(c_2'-g_2')}$	α_E^{Tot}
$\bar{D}^{*0}$	2.15(52)	291(105)	294(106)
D^{*-}	1.81(43)	−0.39(9) −64.5(155)i	1.42(34) −64.5(155)i

Threshold singularity

$$\Delta = m_{D^*} - m_D \approx 142\ MeV$$

$$m_\pi \approx 140\ MeV$$



Results

TABLE V. The numerical results of doubly heavy baryon electromagnetic polarizabilities (in unit of 10^{-4} fm^3). $b[cq]$ and $b\{cq\}$ denote scalar $S_{[cq]}=1$ state $\mathcal{T}$ and axial-vector $S_{\{cq\}}=1$ state $\mathcal{B}$, respectively. Uncertainties are given in parentheses.

	$\alpha_E^{(c_1-g_1)}$	$\alpha_E^{(c_2-g_2)}$	α_E^{Tot}	$\beta_M^{(b_1)}$	$\beta_M^{(b_2)}$	$\beta_M^{(c_1-g_1)}$	$\beta_M^{(c_2-g_2)}$	β_M^{Tot}
Ξ_{cc}^{++}	0.60(12)	2.85(57)	3.44(69)	0	2.40(49)	0.06(1)	0.36(7)	2.82(50)
Ξ_{cc}^{+}	0.50(10)	2.23(45)	2.73(55)	0	1.74(22)	0.05(1)	0.29(6)	2.08(23)
Ω_{cc}^{+}	0.09(2)	0.62(12)	0.71(14)	0	1.38(11)	0.01	0.07(1)	1.45(12)
Ξ_{bb}^{0}	0.60(12)	3.72(74)	4.31(86)	0	9.74(175)	0.06(1)	0.41(8)	10.22(246)
Ξ_{bb}^{-}	0.50(1)	3.03(61)	3.53(71)	0	1.83(50)	0.05(1)	0.34(7)	2.23(71)
Ω_{bb}^{-}	0.09(2)	0.69(14)	0.79(16)	0	1.26(34)	0.01	0.07(1)	1.34(49)
$\Xi_{b\{cu\}}^{+}$	5.00(100)	0.96(19)	5.97(119)	−1.97(40)	4.37(56)	0.46(9)	0.11(2)	2.96(97)
$\Xi_{b\{cd\}}^{0}$	4.31(86)	0.79(16)	5.10(102)	−1.43(18)	0.13(5)	0.39(8)	0.09(2)	−0.83(25)
$\Omega_{b\{cs\}}^{0}$	0.69(14)	0.18(4)	0.87(17)	−1.13(9)	0.05(2)	0.07(1)	0.02	−0.99(11)
$\Xi_{b[cu]}^{+}$	1.36(27)	2.32(46)	3.68(74)	1.97(40)	2.25(46)	0.15(3)	0.28(6)	4.66(87)
$\Xi_{b[cd]}^{0}$	1.11(22)	1.84(37)	2.94(59)	1.43(18)	1.63(20)	0.13(3)	0.23(5)	3.42(39)
$\Omega_{b[cs]}^{0}$	0.26(5)	0.48(10)	0.74(15)	1.13(9)	1.29(11)	0.03(1)	0.05(1)	2.50(20)
	$\alpha_E^{(c_1'-g_1')}$	$\alpha_E^{(c_2'-g_2')}$	α_E^{Tot}	$\beta_M^{(b_1')}$	$\beta_M^{(b_2')}$	$\beta_M^{(c_1'-g_1')}$	$\beta_M^{(c_2'-g_2')}$	β_M^{Tot}
Ξ_{cc}^{*++}	2.98(60)	7.97(159)	10.95(219)	0	−1.20(25)	0.30(6)	0.42(8)	−0.48(28)
Ξ_{cc}^{*+}	2.50(50)	7.49(150)	9.99(200)	0	−0.87(11)	0.25(5)	0.38(8)	−0.24(17)
Ω_{cc}^{*+}	0.47(9)	0.48(10)	0.96(19)	0	−0.69(6)	0.05(1)	0.04(1)	−0.60(6)
Ξ_{bb}^{*0}	2.98(60)	3.34(67)	6.32(126)	0	−4.87(87)	0.30(6)	0.28(6)	−4.29(88)
Ξ_{bb}^{*-}	2.50(50)	2.93(59)	5.43(109)	0	−0.92(25)	0.25(5)	0.24(5)	−0.42(27)
Ω_{bb}^{*-}	0.47(9)	0.42(8)	0.89(18)	0	−0.63(17)	0.05(1)	0.04(1)	−0.54(17)
Ξ_{bc}^{*+}	2.98(60)	4.86(97)	7.84(157)	0	−3.31(5)	0.30(6)	0.34(7)	−2.68(14)
Ξ_{bc}^{*0}	2.50(50)	4.41(88)	6.92(138)	0	−0.88(8)	0.25(5)	0.29(6)	−0.33(13)
Ω_{bc}^{*0}	0.47(9)	0.22(4)	0.69(14)	0	−0.67(4)	0.05(1)	0.04(1)	−0.58(5)

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TABLE II: Scalar, spin, and tensor polarizabilities of the D^* mesons obtained from Eq. (11). The entries for α_E, β_M, a_{E1E1}, and a_{M1M1} are in units of $10^{-4}\,\mathrm{fm}^3$; the four γ polarizabilities are in units of $10^{-4}\,\mathrm{fm}^4$; and a_{E2E2}, a_{M2M2}, a_{E1E3}, and a_{M1M3} are in units of $10^{-4}\,\mathrm{fm}^5$. Numbers in parentheses denote propagated uncertainties.

Polarizability	Meson	Born	$P^*\phi$ loop	$P\phi$ loop	Total
α_E	$\bar{D}^{*0}$	0	2.15(52)	291(70)	294(70)
	D^{*-}	0	1.81(43)	$-0.396(95) - 64.4(154)\,\mathrm{i}$	$1.41(34) - 64.4(154)\,\mathrm{i}$
	D_s^{*-}	0	0.341(82)	0.395(95)	0.74(18)
β_M	$\bar{D}^{*0}$	$-3.75(64)$	0.215(52)	0.98(23)	$-2.55(70)$
	D^{*-}	$-0.229(83)$	0.181(43)	$-0.057(14) + 0.62(15)\,\mathrm{i}$	$-0.105(88) + 0.62(15)\,\mathrm{i}$
	D_s^{*-}	$-0.031(21)$	0.0341(82)	0.0259(62)	0.029(25)
γ_{E1E1}	$\bar{D}^{*0}$	0	$-0.64(15)$	$-1.29(31)\times 10^4$	$-1.29(31)\times 10^4$
	D^{*-}	0	$-0.61(15)$	$+0.180(43) - 1.10(26)\times 10^3\,\mathrm{i}$	$-0.79(19) - 1.10(26)\times 10^3\,\mathrm{i}$
	D_s^{*-}	0	$-0.0325(78)$	$-0.147(35)$	$-0.179(43)$
γ_{M1M1}	$\bar{D}^{*0}$	$-7.81(132)$	$-0.129(31)$	$-67.5(162)$	$-75.4(163)$
	D^{*-}	$-0.48(17)$	$-0.122(29)$	$-0.0390(94) + 16.0(38)\,\mathrm{i}$	$-0.64(18) + 16.0(38)\,\mathrm{i}$
	D_s^{*-}	$-0.064(44)$	$-0.0065(16)$	$-0.0185(44)$	$-0.089(44)$

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γ_{M1E2}	$\bar{D}^{*0}$	0	$-0.129(31)$	$-67.5(162)$	$-67.6(162)$
	D^{*-}	0	$-0.122(29)$	$-0.0390(94)+16.0(38)\,\mathrm{i}$	$-0.161(39)+16.0(38)\,\mathrm{i}$
	D_s^{*-}	0	$-0.0065(16)$	$-0.0185(44)$	$-0.0250(60)$
γ_{E1M2}	$\bar{D}^{*0}$	0	$0.129(31)$	$67.5(162)$	$67.6(162)$
	D^{*-}	0	$0.122(29)$	$0.0390(94)-16.0(38)\,\mathrm{i}$	$0.161(39)-16.0(38)\,\mathrm{i}$
	D_s^{*-}	0	$0.0065(16)$	$0.0185(44)$	$0.0250(60)$
a_{E1E1}	$\bar{D}^{*0}$	0	$-0.65(15)$	$288(69)$	$287(69)$
	D^{*-}	0	$-0.54(13)$	$-0.169(40)-66.8(160)\,\mathrm{i}$	$-0.71(17)-66.8(160)\,\mathrm{i}$
	D_s^{*-}	0	$-0.102(25)$	$0.291(70)$	$0.189(45)$
a_{M1M1}	$\bar{D}^{*0}$	$-5.62(95)$	$0.323(77)$	$-2.94(70)$	$-8.23(114)$
	D^{*-}	$-0.34(12)$	$0.272(65)$	$0.171(41)-1.86(44)\,\mathrm{i}$	$0.10(16)-1.86(44)\,\mathrm{i}$
	D_s^{*-}	$-0.046(31)$	$0.051(12)$	$-0.078(19)$	$-0.072(32)$
a_{E2E2}	$\bar{D}^{*0}$	0	$-0.55(13)$	$9.70(233)\times10^3$	$9.70(233)\times10^3$
	D^{*-}	0	$-0.54(13)$	$-0.116(28)+835(200)\,\mathrm{i}$	$-0.66(16)+835(200)\,\mathrm{i}$
	D_s^{*-}	0	$-0.0081(20)$	$0.0329(79)$	$0.0247(59)$

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a_{M2M2}	$\bar{D}^{*0}$	0	0.220(53)	$1.11(27)\times10^3$	$1.11(27)\times10^3$
	D^{*-}	0	0.217(52)	$0.104(25) + 127(30)\,\mathrm{i}$	$0.321(77) + 127(30)\,\mathrm{i}$
	D_s^{*-}	0	0.00325(78)	$-0.0061(15)$	$-0.00290(69)$
a_{E1E3}	$\bar{D}^{*0}$	0	$-0.220(53)$	$3.64(87)\times10^3$	$3.64(87)\times10^3$
	D^{*-}	0	$-0.217(52)$	$-0.049(12) + 312(75)\,\mathrm{i}$	$-0.266(64) + 312(75)\,\mathrm{i}$
	D_s^{*-}	0	$-0.00325(78)$	0.0128(31)	0.0096(23)
a_{M1M3}	$\bar{D}^{*0}$	0	0.110(26)	38.7(93)	38.8(93)
	D^{*-}	0	0.109(26)	$-0.045(11) - 8.66(208)\,\mathrm{i}$	$0.063(15) - 8.66(208)\,\mathrm{i}$
	D_s^{*-}	0	0.00164(39)	0.00395(95)	0.0056(13)

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In the long-wavelength limit, second-order perturbation theory gives

$$\alpha_E = 2\alpha_{\mathrm{em}} \sum_{n\neq i} \frac{|\langle n|D_z|i\rangle|^2}{E_n - E_i}, \qquad \beta_M = 2\alpha_{\mathrm{em}} \sum_{n\neq i} \frac{|\langle n|M_z|i\rangle|^2}{E_n - E_i}.$$

The dipole operators associated with the pion cloud can then be written classically as

$$\boldsymbol{D}_\pi = \int \mathrm{d}^3 r \, \boldsymbol{r} \, \rho_\pi(\boldsymbol{r}), \qquad \boldsymbol{M}_\pi = \frac{1}{2} \int \mathrm{d}^3 r \, \boldsymbol{r} \times \boldsymbol{j}_\pi(\boldsymbol{r}), \qquad \boldsymbol{j}_\pi(\boldsymbol{r}) \simeq \rho_\pi(\boldsymbol{r}) \boldsymbol{v}_\pi(\boldsymbol{r}).$$

We can estimate the characteristic momentum and velocity scales as

$$q_\pi \sim \sqrt{M_\pi^2 - \Delta^2}, \quad |\boldsymbol{v}_\pi| \sim \frac{q_\pi}{M_\pi} \sim 0.17$$

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$$|\boldsymbol{v}_\pi| \sim \frac{q_\pi}{M_\pi} \sim 0.17 \qquad \frac{|\langle n|M_{\pi,z}|i\rangle|}{|\langle n|D_{\pi,z}|i\rangle|} \sim \mathcal{O}(v_\pi), \qquad \frac{\beta_M^{(P\phi)}}{\alpha_E^{(P\phi)}} \sim \mathcal{O}(v_\pi^2).$$

TABLE II: $P\phi$-loop contributions to the selected scalar, spin, and tensor polarizabilities of the D^* mesons. The entries for α_E, β_M, a_{E1E1}, and a_{M1M1} are in units of $10^{-4}\,\mathrm{fm}^3$; the entries for γ_{E1E1} and γ_{M1M1} are in units of $10^{-4}\,\mathrm{fm}^4$. Numbers in parentheses denote propagated uncertainties.

Polarizability	$\bar{D}^{*0}$	D^{*-}	D_s^{*-}
α_E	291(70)	$-0.396(95) - 64.4(154)\,\mathrm{i}$	0.395(95)
β_M	0.98(23)	$-0.057(14) + 0.62(15)\,\mathrm{i}$	0.0259(62)
γ_{E1E1}	$-1.29(31)\times10^4$	$-0.180(43) - 1.10(26)\times10^3\,\mathrm{i}$	$-0.147(35)$
γ_{M1M1}	$-67.5(162)$	$-0.0390(94) + 16.0(38)\,\mathrm{i}$	$-0.0185(44)$
a_{E1E1}	288(69)	$-0.169(40) - 66.8(160)\,\mathrm{i}$	0.291(70)
a_{M1M1}	$-2.94(70)$	$0.171(41) - 1.86(44)\,\mathrm{i}$	$-0.078(19)$

Conclusions

- **Pion-cloud dynamics:** Chiral loops dominate electric polarizabilities.
- **Giant electric polarizability of D^* :** the exact kinematic coincidence ($\Delta \approx m_\pi$) induces a threshold singularity in the chiral loops, leading to anomalously large polarizabilities.
- **Doubly heavy baryons:** Electromagnetic properties depend strictly on the heavy-flavor composition, with the bcq system dominated by mixing with the scalar state T.
- **Spin-dependent polarizabilities of D^*:** $M1M1$-type polarizabilities are suppressed by momentum of pion.
- **More precise calculation:** Further segregate energy scales in proximity to the threshold and reconduct power counting.
- **Benchmark:** Our rigorous estimations offer explicit quantitative benchmarks for future lattice QCD studies.