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Possible bound states of $D^*\bar{D}^*$ and D^*D^* systems

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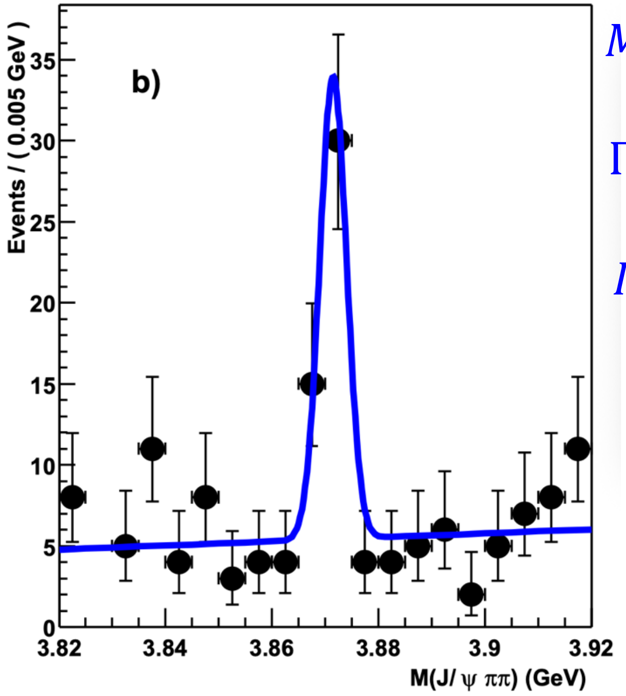
Motivation



[Phys.Rev.Lett. 91 (2003) 262001]

$M = 3871.64 \pm 0.06$ MeV

$\Gamma = 1.19 \pm 0.21$ MeV

$I^G(J^{PC}) = 1^+(1^{++})$

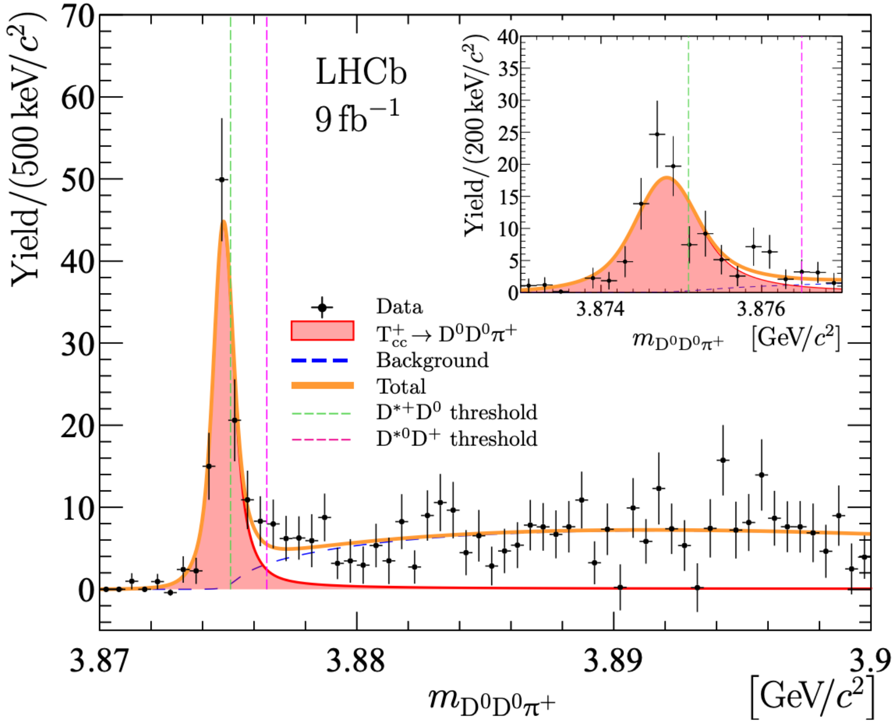
$M = 3874.75 \pm 0.10$ MeV

$\Gamma = 0.048 \pm 0.002$ MeV

$I(J^P) = 0(1^+)$

$M_{\bar{D}^0 D^{*0}} = 3871.71$ MeV

$M_{\bar{D}^0 D^{*+}} = 3875.11$ MeV



[Nature Phys. 18 (2022) 7, 751-754]

$$|\mathrm{X}(3872)\rangle = c_1|D\bar{D}^*\rangle + c_2|[cq][\bar{c}\bar{q}]\rangle + c_3|c\bar{c}\rangle + \cdots$$

Observation of a narrow charmonium-like state in exclusive $B^\pm \to K^\pm\pi^+\pi^- J/\psi$ decays

Belle Collaboration • S.K. Choi (Gyeongsang Natl. U.) et al. (Sep, 2003)

Published in: *Phys.Rev.Lett.* 91 (2003) 262001 • e-Print: hep-ex/0309032 [hep-ex]

Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration • Roel Aaij (Nikhef, Amsterdam) et al. (Sep 2, 2021)

Published in: *Nature Phys.* 18 (2022) 7, 751-754, *Nature Phys.* (2022) • e-Print: 2109.01038 [hep-ex]

Motivation

- **Mass**
  - **Lattice QCD**
  - **Effective field theory**
  - **QCD sum rules**
  - **Quark model、Chiral quark model 、One boson exchange model、Bethe-Salpeter equation、…**
- **Deacy**
  - **Strong decay** $R = \frac{B_{X(3872)\to J/\psi\omega}}{B_{X(3872)\to J/\psi\rho}} \approx 1$
  - **Radiative Decay** $R = \frac{B_{X(3872)\to\psi(2S)\gamma}}{B_{X(3872)\to J/\psi\gamma}} = 1.67 \pm 0.21$
- **Production**
  - $\boldsymbol{B \to KX}$、$\boldsymbol{pp \to X + \cdots}$、$\boldsymbol{e^+e^- \to \gamma X}$、$\boldsymbol{Y(4260) \to \gamma X}$、$\cdots$
- **Heavy quark spin symmetry**
  - X$\boldsymbol{(3872)}$: $\boldsymbol{D\bar{D}^* I(J^{PC}) = 0(1^{++}) \to D^*\bar{D}^* I(J^{PC}) = 0(2^{++})}$
  - $\boldsymbol{T_{cc}(3875): DD^* I(J^P) = 0(1^+) \to D^*D^* I(J^P) = 0(1^+)}$

Bethe-Salpeter equation

- Bethe-Salpeter wave function

$$\chi_P^{\mu\nu}(x_1, x_2) = \langle 0|TD^{*\mu}(x_1)\bar{D}^{*\nu}\,(x_2)|P\rangle$$

- The BS wave function can be derived from a four-point Green function

$$\boxed{P^0 = E_P}$$

$$S^{\mu\nu\beta\alpha}(x_1, x_2; y_2, y_1) = \left\langle 0\middle|TD^{*\mu}(x_1)\bar{D}^{*\nu}\,(x_2)\left(D^{*\alpha}(y_1)\bar{D}^{*\beta}\,(y_2)\right)^{\dagger}\middle|0\right\rangle$$

- Express the above four-point Green function

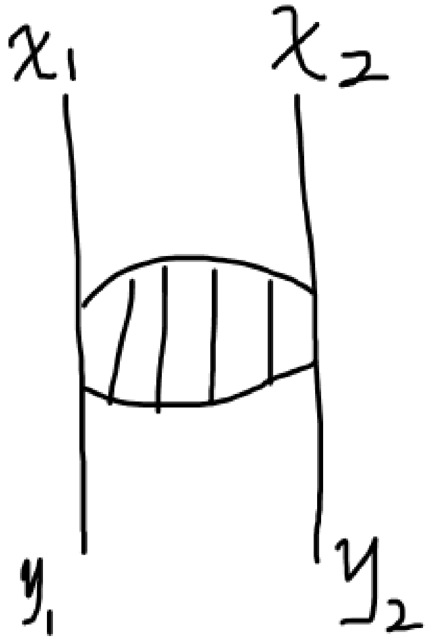
$$S^{\mu\nu\beta\alpha}(x_1, x_2; y_2, y_1) = S_0^{\mu\nu\beta\alpha}(x_1, x_2; y_2, y_1)$$

$$+\int d^4a_1 d^4a_2 d^4b_1 d^4b_2 S_0^{\mu\nu\gamma\delta}(x_1, x_2; a_2, a_1)\bar{K}^{\delta\gamma\iota\kappa}(a_1, a_2; b_2, b_1)S^{\iota\kappa\beta\alpha}(b_1, b_2; y_2, y_1)$$

with $S_0^{\mu\nu\beta\alpha}(x_1, x_2; y_2, y_1) = \Delta_1^{\mu\alpha}(x_1, y_1)\Delta_2^{\nu\beta}(\mathrm{x}_2, \mathrm{y}_2)$

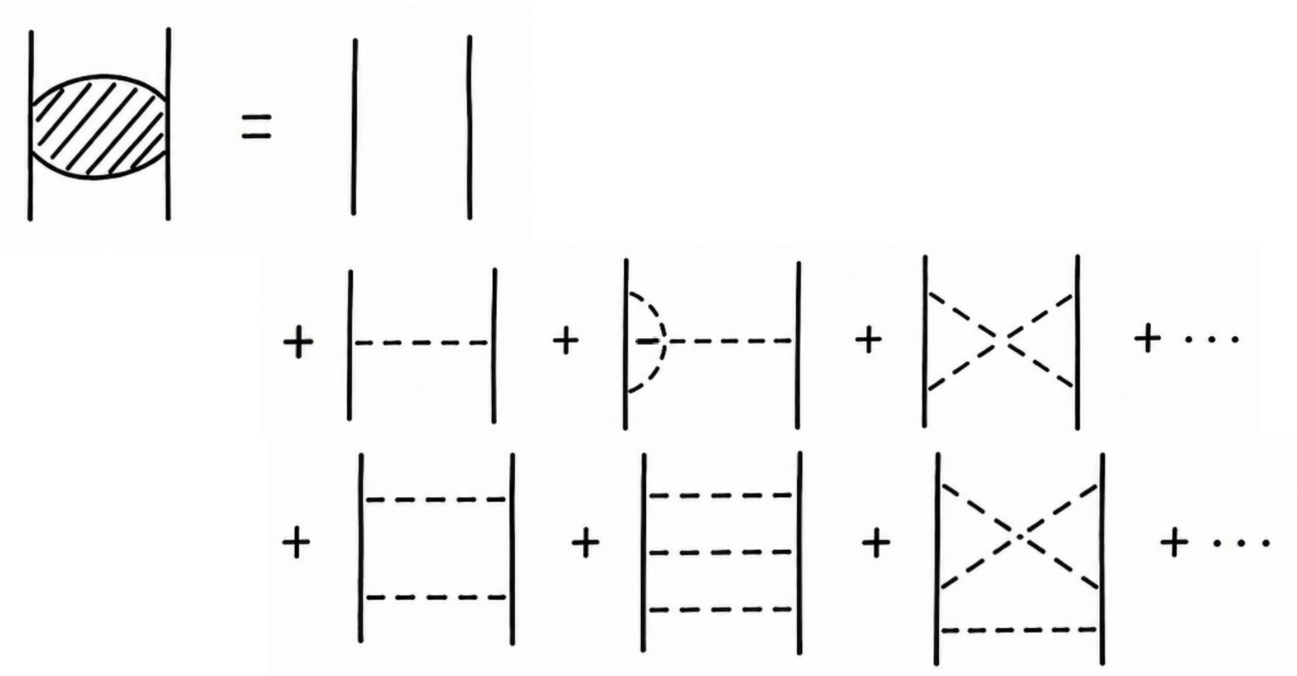
- The Bethe-Salpeter equation of two vector mesons are

$$\chi_P^{\mu\nu}(x_1, x_2) = \int d^4a_1 d^4a_2 d^4b_1 d^4b_2\; S_0^{\mu\nu\gamma\delta}(x_1, x_2; a_2, a_1)\bar{K}^{\delta\gamma\iota\kappa}(a_1, a_2; b_2, b_1)\chi_P^{\kappa\iota}(b_1, b_2)$$



Bethe-Salpeter equation

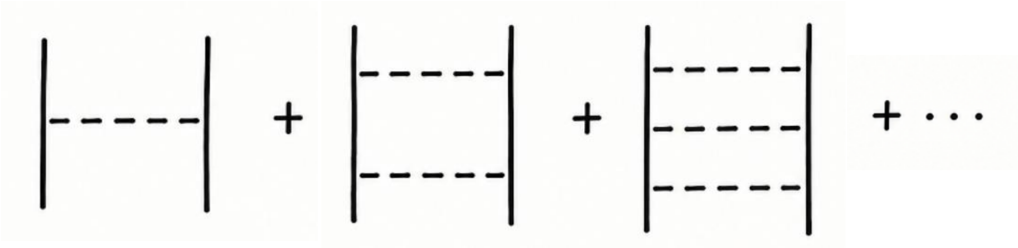
The **Feynman diagram** expansion of the two-body propagator



irreducible diagrams

reducible diagrams

The **interaction kernel** in the ladder approximation



Bethe-Salpeter equation

- In the momentum space

$$\chi_P^{\mu\nu}(x_1, x_2) = e^{-iPX}\chi_P^{\mu\nu}(p)$$

where $P = Mv$, $p_1 = \lambda_1 P + p$ and $p_2 = \lambda_2 P - p$ with $\lambda_{1(2)} = \frac{m_{1(2)}}{m_1+m_2}$.

- The Bethe-Salpeter equation in the momentum space

$$\chi_P^{\mu\nu}(p) = S^{\mu\alpha}(p_1, m_1)\int \frac{d^4q}{(2\pi)^4}\, \bar{K}_{\alpha\beta\lambda\tau}(P, p, q)\chi_P^{\lambda\tau}(q)S^{\nu\beta}(p_2, m_2)$$

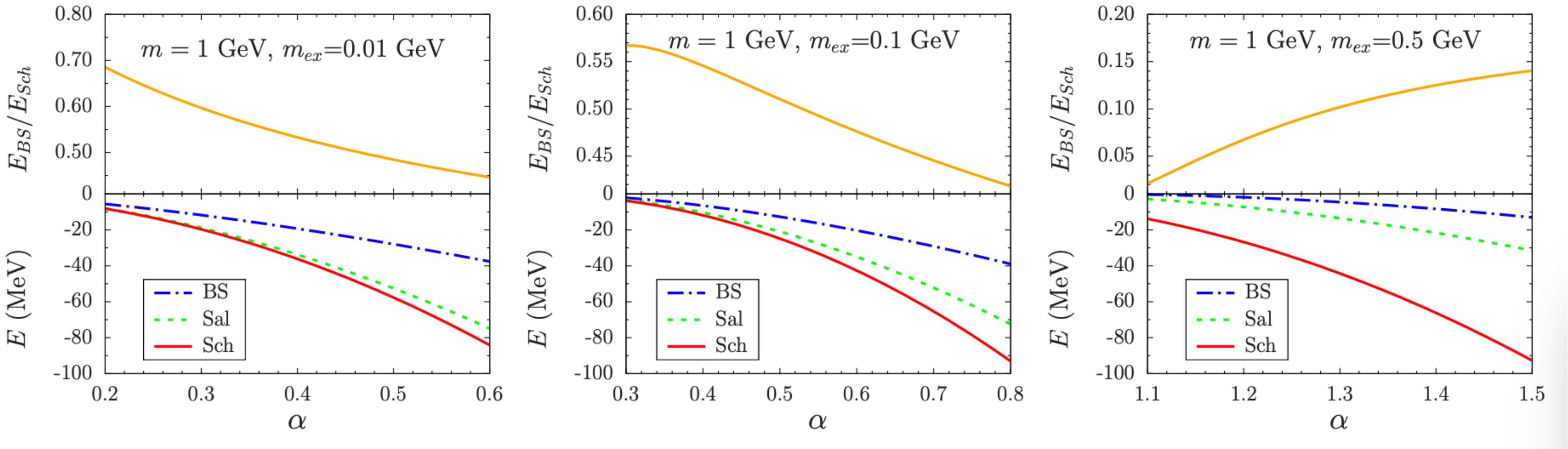
- The normalization condition for the BS wave function

$$i\int \frac{d^4p d^4q}{(2\pi)^8}\bar{\chi}_P^{\mu\nu}(p)\frac{\partial}{\partial P^0}\left[I_{\mu\nu\alpha\beta}(p,q) + \bar{K}_{\mu\nu\alpha\beta}(p,q)\right]\chi_P^{\alpha\beta}(q) = 2E_P$$

with $I_{\mu\nu\alpha\beta}(p,q) = -(2\pi)^4\delta^4(p-q)S_{\mu\nu}^{-1}(p_1, m_1)S_{\alpha\beta}^{-1}(p_2, m_2)$.

Bethe-Salpeter equation

- Provides a relativistic and **Lorentz-covariant** framework for describing two-body bound states.
- Determines the bound state mass and the corresponding covariant **BS wave function**.
- Describes the **internal momentum** and spin structure of the bound state.
- The normalized BS wave function can be used to study **decay properties**, **form factors**, and **production processes**.



Arxiv:2512.02524

Bethe-Salpeter equation

- Light quark isospin conventions

$$|u\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle, \quad |d\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle, \quad |\bar{u}\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle, \quad |\bar{d}\rangle = -\left|\frac{1}{2},\frac{1}{2}\right\rangle.$$

- Flavor wave functions

$$|T_{c\bar{c}}; I=0, I_3=0\rangle = -\frac{1}{\sqrt{2}}\left(|D^{*+}D^{*-}\rangle + |D^{*0}\bar{D}^{*0}\rangle\right),$$
$$|T_{c\bar{c}}; I=1, I_3=1\rangle = -|D^{*+}\bar{D}^{*0}\rangle,$$
$$|T_{c\bar{c}}; I=1, I_3=0\rangle = -\frac{1}{\sqrt{2}}\left(|D^{*+}D^{*-}\rangle - |D^{*0}\bar{D}^{*0}\rangle\right),$$
$$|T_{c\bar{c}}; I=1, I_3=-1\rangle = |D^{*0}D^{*-}\rangle,$$

$$|T_{cc}; I=0, I_3=0\rangle = -\frac{1}{\sqrt{2}}\left(|D^{*+}D^{*0}\rangle - |D^{*0}D^{*+}\rangle\right),$$
$$|T_{cc}; I=1, I_3=1\rangle = |D^{*+}D^{*+}\rangle,$$
$$|T_{cc}; I=1, I_3=0\rangle = -\frac{1}{\sqrt{2}}\left(|D^{*+}D^{*0}\rangle + |D^{*0}D^{*+}\rangle\right),$$
$$|T_{cc}; I=1, I_3=-1\rangle = |D^{*0}D^{*0}\rangle.$$

- Allowed systems

S-wave $D^*\overline{D}^*$

$C = (-1)^{L+S} = (-1)^S$

$I = 0, 1;\ J^{PC} = 0^{++}, 1^{+-}, 2^{++}$

S-wave D^*D^*

Spin $S = 0, 2$ is symmetry, $S = 1$ is antisymmetry

$J(J^P) = 0(1^+), 1(0^+), 1(2^+)$

Bethe-Salpeter equation

For convenience， $p_l = p \cdot v$ and $p_t = p - p_l\, v$.

- The BS wave function can be decomposed into a complete set of linearly independent Lorentz–Dirac covariants

$$\chi_{0^{++}}^{\mu\nu}(P,p) = \phi_1(p_l,p_t^2) g_t^{\mu\nu} + \phi_2(p_l,p_t^2) p_t^\mu p_t^\nu$$

$$\chi_{1^{+-}}^{\mu\nu}(P,p) = \psi_1(p_l,p_t^2)\epsilon^{\mu\nu\alpha\beta} v_\alpha \epsilon_\beta + \psi(p_l,p_t^2)\epsilon^{\mu\nu\alpha\beta} v_\alpha p_{t\beta} p_t \cdot \epsilon$$

$$g_t^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$$

$$\chi_{2^{++}}^{\mu\nu}(P,p) = \eta_1(p_l,p_t^2)\xi^{\mu\nu} + \eta_2(p_l,p_t^2)\left(p_t^\mu p_{t\alpha}\xi^{\alpha\nu} + p_t^\nu p_{t\alpha}\xi^{\alpha\mu}\right) + \eta_3(p_l,p_t^2) p_{t\alpha} p_{t\beta}\xi^{\alpha\beta}\, g_t^{\mu\nu} + \eta_4(p_l,p_t^2) p_{t\alpha} p_{t\beta}\xi^{\alpha\beta} p_t^\mu p_t^\nu$$

- Due to the centrifugal barrier, near-threshold states are generally dominated by S-wave components.

$$\chi_{0^{++}}^{\mu\nu}(P,p) = \phi_1(p_l,p_t^2) g_t^{\mu\nu}$$

$$\chi_{1^{+-}}^{\mu\nu}(P,p) = \psi_1(p_l,p_t^2)\epsilon^{\mu\nu\alpha\beta} v_\alpha \epsilon_\beta$$

$$\chi_{2^{++}}^{\mu\nu}(P,p) = \eta_1(p_l,p_t^2)\xi^{\mu\nu}$$

Bethe-Salpeter equation

- The irreducible kernel $\overline{K}$

For a near-threshold shallow bound state → low characteristic momentum

$$\mathrm{Q} \ll m_1, m_2$$

Consider the exchanged-particle propagator

$$\frac{1}{p^2 - m_{ex}^2} \approx \frac{1}{m_{ex}^2}$$

The BS kernel is dominated by one boson exchange

- Effective Lagrangians

$$\mathcal{L}_{D^*D^*\sigma} = g_{D^*D^*\sigma} D_a^{*\mu} D_{a\mu}^{*\dagger} \sigma, + g_{\bar{D}^*\bar{D}^*\sigma} \bar{D}_a^{*\mu} \bar{D}_{a\mu}^{*\dagger} \sigma,$$

$$\mathcal{L}_{D^*D^*\mathcal{P}} = g_{D^*D^*\mathcal{P}} \epsilon_{\mu\nu\alpha\beta} D_a^{*\nu\dagger} \overleftrightarrow{\partial}^{\beta} D_b^{*\mu} \partial^{\alpha} \mathcal{P}_{ab}, + g_{\bar{D}^*\bar{D}^*\mathcal{P}} \epsilon_{\mu\nu\alpha\beta} \bar{D}_a^{*\mu\dagger} \overleftrightarrow{\partial}^{\beta} \bar{D}_b^{*\nu} \partial^{\alpha} \mathcal{P}_{ab},$$

$$\begin{aligned}\mathcal{L}_{D^*D^*\mathcal{V}} = & \, i g_{D^*D^*\mathcal{V}} \left(D_{b\nu}^{*} \overleftrightarrow{\partial}_{\mu} D_a^{*\nu\dagger} \right) \mathcal{V}_{ba}^{\mu} + i g'_{D^*D^*\mathcal{V}} \left(D_b^{*\mu} D_a^{*\nu\dagger} - D_a^{*\mu\dagger} D_b^{*\nu} \right) (\partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu)_{ba}, \\ & + i g_{\bar{D}^*\bar{D}^*\mathcal{V}} \left(\bar{D}_{b\nu}^{*} \overleftrightarrow{\partial}_{\mu} \bar{D}_a^{*\nu\dagger} \right) \mathcal{V}_{ba}^{\mu} + i g'_{\bar{D}^*\bar{D}^*\mathcal{V}} \left(\bar{D}_b^{*\mu} \bar{D}_a^{*\nu\dagger} - \bar{D}_a^{*\mu\dagger} \bar{D}_b^{*\nu} \right) (\partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu)_{ba},\end{aligned}$$

Bethe-Salpeter equation

- $D^*\overline{D}^*$

$$\bar{K}_{\sigma}^{\alpha\beta\lambda\tau} = - C_{\sigma}^{I} g_{D^*D^*\sigma} g_{\bar{D}^*\bar{D}^*\sigma} g^{\alpha\lambda}\Delta(k, m_\sigma) g^{\beta\tau},$$

$$\bar{K}_{\mathcal{P}}^{\alpha\beta\lambda\tau} = - C_{\mathcal{P}}^{I} g_{D^*D^*\mathcal{P}}^{2} \epsilon_{\alpha\lambda\kappa\delta} \left(p_1 + q_1\right)^{\delta} k^{\kappa} \Delta(k, m_{\mathcal{P}}) \epsilon_{\tau\beta\gamma\zeta} \left(p_2 + q_2\right)^{\zeta} k^{\gamma},$$

$$\begin{aligned}\bar{K}_{\mathcal{V}}^{\alpha\beta\lambda\tau} =& C_{\mathcal{V}}^{I} \left[g_{D^*D^*\mathcal{V}} (p_1 + q_1)_{\sigma} g_{\alpha\lambda} + 2 g'_{D^*D^*\mathcal{V}} \left(k_\alpha g_{\sigma\lambda} - k_\lambda g_{\sigma\alpha}\right)\right] \Delta^{\sigma\delta}(k, m_{\mathcal{V}}) \\ &\times \left[g_{D^*D^*\mathcal{V}} (p_2 + q_2)_{\delta} g_{\tau\beta} + 2 g'_{D^*D^*\mathcal{V}} (k_\tau g_{\delta\beta} - k_\beta g_{\delta\tau})\right].\end{aligned}$$

- D^*D^*

$$\bar{K}_{\sigma}^{\alpha\beta\lambda\tau}(P,p,q) = - C_{\sigma}^{I} g_{D^*D^*\sigma} g^{\alpha\lambda}\Delta(k, m_\sigma) g^{\beta\tau},$$

$$\bar{K}_{\mathcal{P}}^{\alpha\beta\lambda\tau}(P,p,q) = C_{\mathcal{P}}^{I} g_{D^*D^*\mathcal{P}}^{2} \epsilon^{\alpha\lambda\kappa\delta} \left(p_1 + q_1\right)_{\delta} k_{\kappa} \Delta(k, m_{\mathcal{P}}) \epsilon^{\beta\tau\gamma\zeta} \left(p_2 + q_2\right)_{\zeta} k_{\gamma},$$

$$\begin{aligned}\bar{K}_{\mathcal{V}}^{\alpha\beta\lambda\tau}(P,p,q) =& - C_{\mathcal{V}}^{I} \left[g_{D^*D^*\mathcal{V}} \left(p_1 + q_1\right)^{\sigma} g^{\alpha\lambda} + 2 g'_{D^*D^*\mathcal{V}} \left(k^\alpha g^{\lambda\sigma} - k^\lambda g^{\alpha\sigma}\right)\right] \Delta_{\sigma\delta}(k, m_{\mathcal{V}}) \\ &\times \left[g_{D^*D^*\mathcal{V}} \left(p_2 + q_2\right)^{\delta} g^{\beta\tau} + 2 g'_{D^*D^*\mathcal{V}} \left(-k^\beta g^{\delta\tau} + k^\tau g^{\beta\delta}\right)\right].\end{aligned}$$

Bethe-Salpeter equation

TABLE I. Isospin factors.

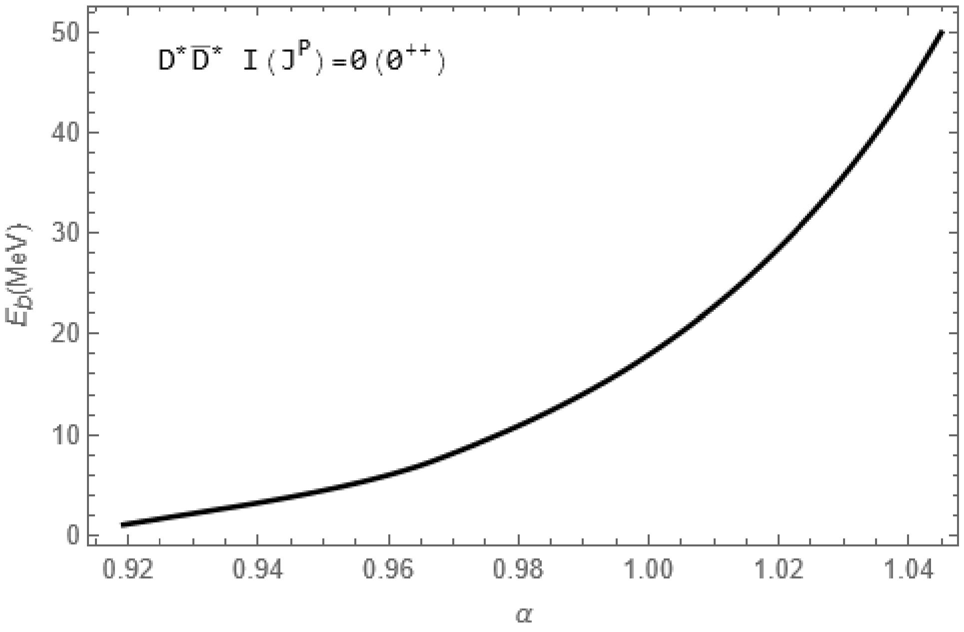
		C_σ	C_π	C_η	C_ρ	C_ω
		σ	π	η	ρ	ω
$D^*\bar{D}^*$	$I=0$	1	3/2	1/6	3/2	1/2
	$I=1$	1	-1/2	1/6	-1/2	1/2
D^*D^*	$I=0$	1	-3/2	1/6	-3/2	1/2
	$I=1$	1	1/2	1/6	1/2	1/2

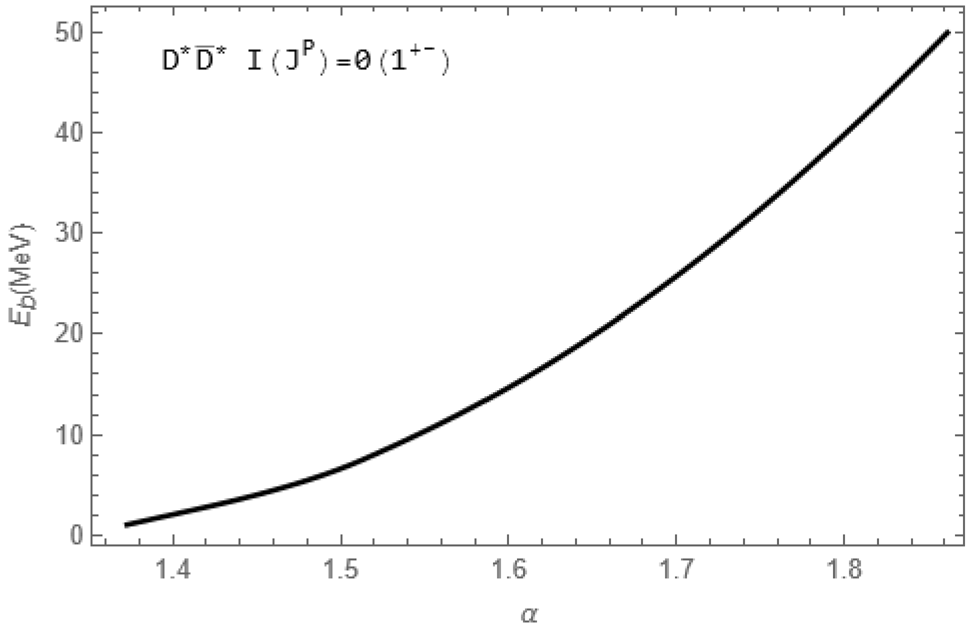
To account phenomenologically for the finite size of the constituent hadrons and to regularize the high-momentum behavior of the interaction kernel

$$F(k^2) = \frac{\Lambda^2 - m^2}{\Lambda^2 - k^2}$$

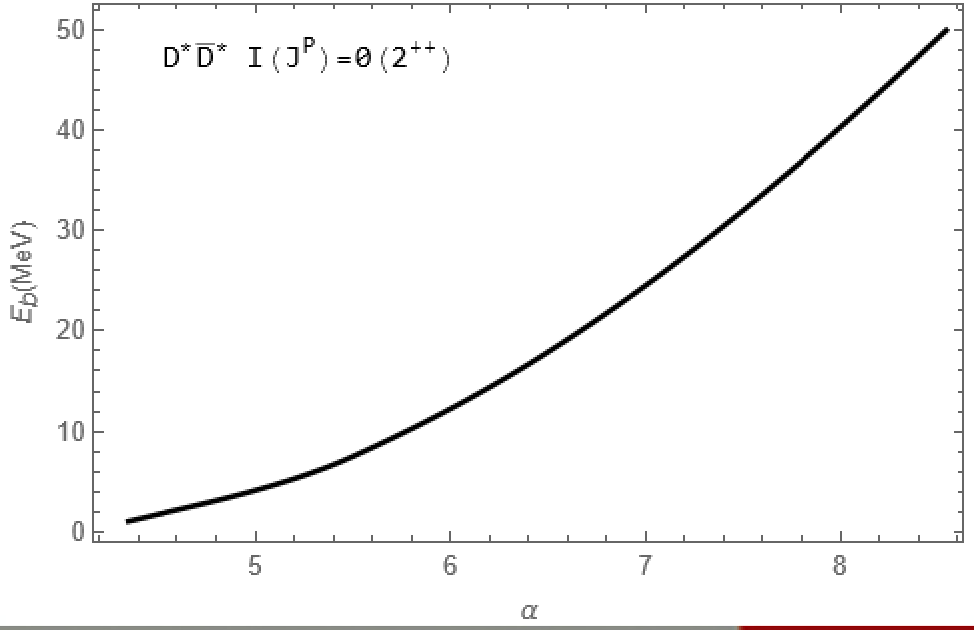
$\Lambda = m + \alpha\, \Lambda_{QCD}$ with Λ_{QCD} =220 MeV

Results





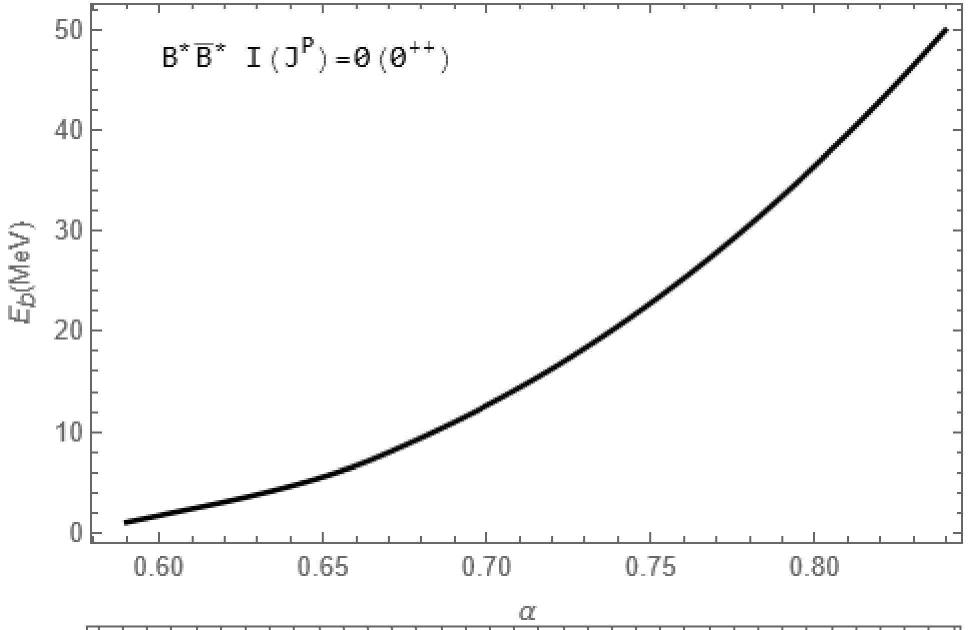
$M_{D^*\bar{D}^*} = 4017.11$ MeV

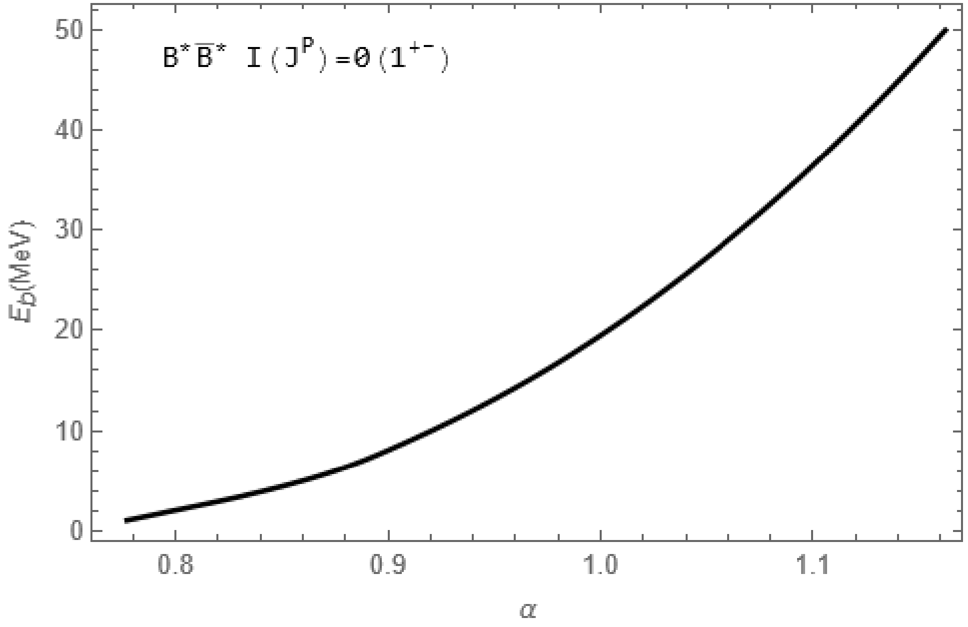


EFT (X(3872), X(3915), Z_b(10610))

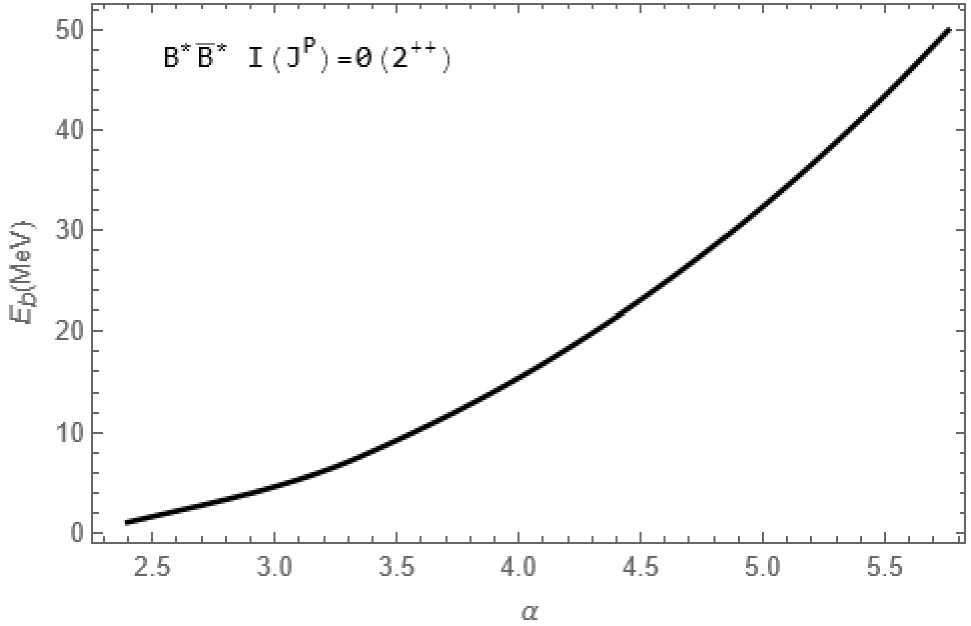
$0(0^{++})$	$0(1^{+-})$	$0(2^{++})$	
	3957 ± 17 MeV	4012 ± 3 MeV	[Phys.Rev. D 86, 056004 (2012)]
		4012^{+4}_{-5} MeV	[Phys. Rev. D 88, 054007 (2013)]
$1(0^{++})$	$1(1^{+-})$	$1(2^{++})$	
	$\cancel{4013^{+4}_{-11}\ \text{MeV}}$		[Phys. Rev. D 88, 054007 (2013)]
3953^{+24}_{-26} MeV	3988^{+15}_{-17} MeV		[Phys. Rev. D 87, 076006 (2013)]

Results





$M_{B^*\bar{B}^*} = 10559.1$ MeV

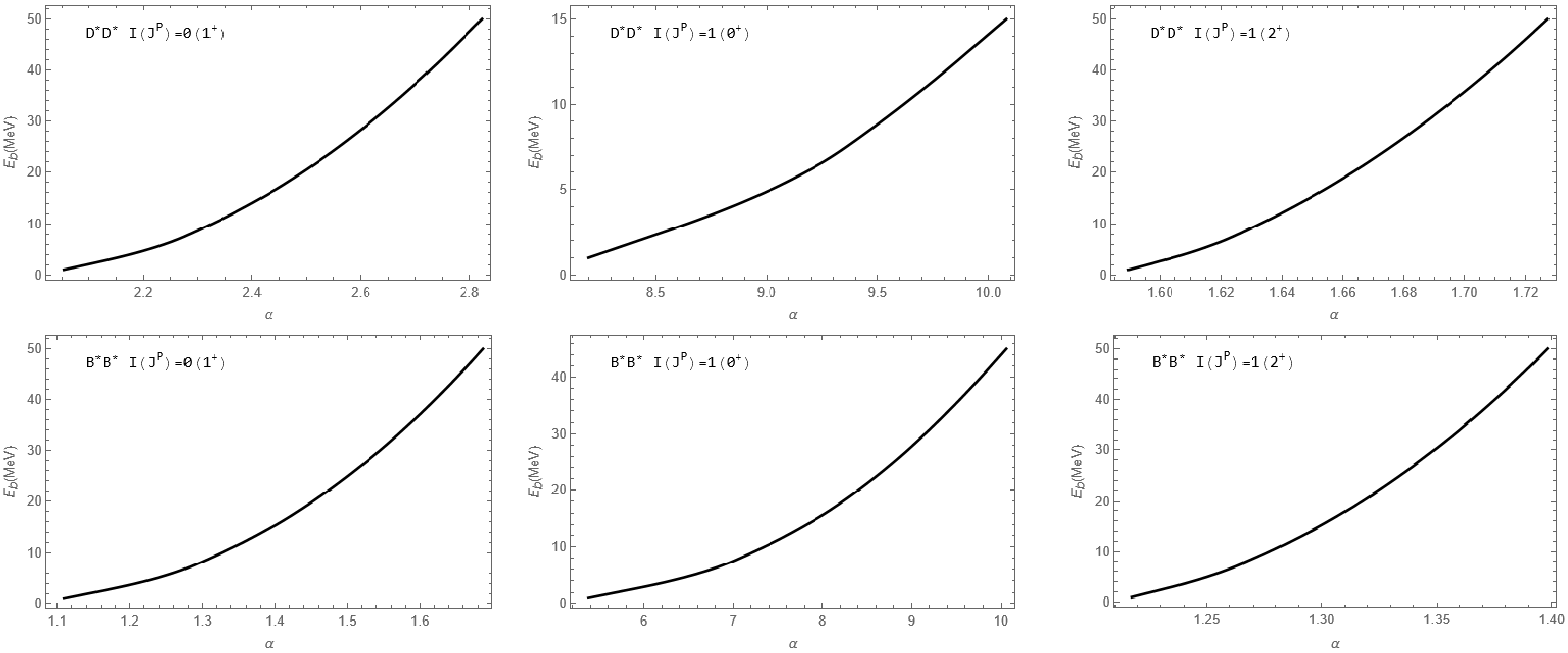


EFT (X(3872), Z_b(10610))

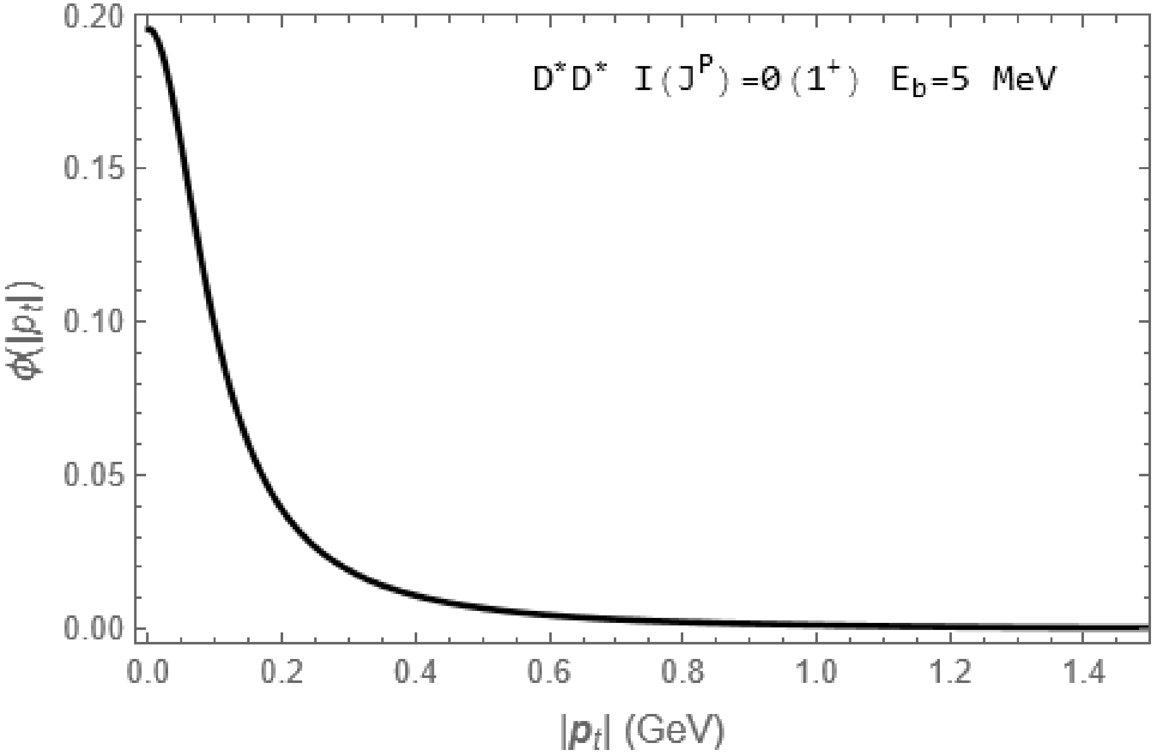
$0(2^{++})$	$1(1^{+-})$
10626^{+6}_{-7} MeV	10648.1 ± 2.1 MeV

[Phys. Rev. D 88, 054007 (2013)]

Results



Numerical result of the BS equation



Summary and Outlook

- ✓ Possible S-wave molecular states in the $D^*\bar{D}^*$, $B^*\bar{B}^*$, $D^*\,D^*$, and $\bar{B}^*\bar{B}^*$ systems were investigated within the Bethe--Salpeter formalism.
- For the hidden-heavy systems, bound state solutions were found in all three isoscalar channels, $J^{PC} = 0^{++}, 1^{+-}, 2^{++}$, while no isovector solutions were obtained within the parameter range considered.
- For the doubly heavy systems, Bose symmetry allows only $I(J^P) = 0(1^+), 1(0^+)$, and $1(2^+)$. Bound state solutions were obtained in all three channels.
- The $0(1^+)$ and $1(2^+)$ doubly heavy states can be generated with moderate cutoff values, whereas the $1(0^+)$ state requires a much larger cutoff and is therefore more model dependent.
- ✓ The resulting BS wave functions can be further used to investigate the decay and production properties of these bound states.

Thank you for your attention !